Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.2-Cosine/207-4.2.3.1

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**644**]. This is test number [207].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (644)	0.00 (0)
Mathematica	98.60 (635)	1.40 (9)
Maple	97.52 (628)	2.48 (16)
Fricas	72.98 (470)	27.02 (174)
Mupad	35.87 (231)	64.13 (413)
Maxima	32.45 (209)	67.55 (435)
Giac	29.50 (190)	70.50 (454)
Reduce	25.47 (164)	74.53 (480)
Sympy	10.56 (68)	89.44 (576)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

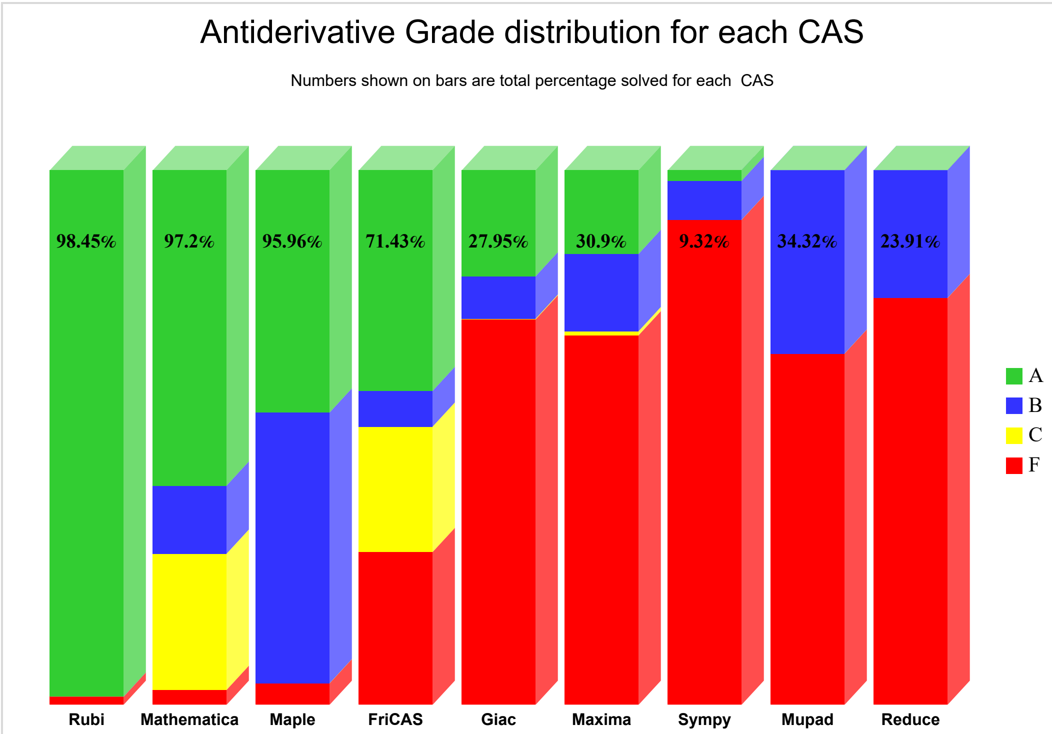
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

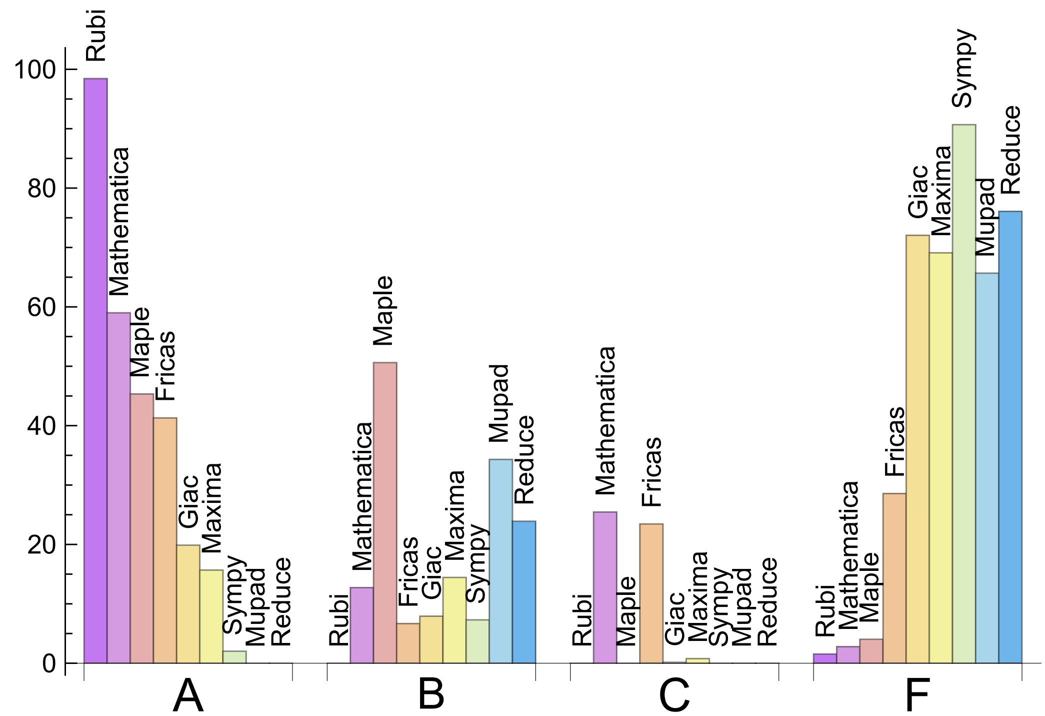
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.447	0.000	0.000	1.553
Mathematica	59.006	12.733	25.466	2.795
Maple	45.342	50.621	0.000	4.037
Fricas	41.304	6.677	23.447	28.571
Giac	19.876	7.919	0.155	72.050
Maxima	15.683	14.441	0.776	69.099
Sympy	2.019	7.298	0.000	90.683
Mupad	0.000	34.317	0.000	65.683
Reduce	0.000	23.913	0.000	76.087

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	9	88.89	11.11	0.00
Maple	16	68.75	31.25	0.00
Fricas	174	56.90	43.10	0.00
Mupad	413	0.00	100.00	0.00
Maxima	435	78.85	9.43	11.72
Giac	454	78.63	15.86	5.51
Reduce	480	100.00	0.00	0.00
Sympy	576	35.76	64.24	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Reduce	0.18
Giac	0.44
Fricas	0.93
Rubi	1.27
Maxima	2.24
Mathematica	6.03
Sympy	13.75
Maple	30.00
Mupad	37.16

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	226.23	1.02	194.00	1.01
Giac	252.34	1.64	182.00	1.43
Reduce	277.74	1.75	157.00	1.29
Fricas	290.91	1.63	201.50	1.26
Mathematica	524.72	2.03	212.00	1.09
Maple	597.95	2.41	319.00	1.95
Sympy	652.56	5.24	258.00	2.51
Mupad	883.30	4.10	202.00	1.43
Maxima	14869.09	85.36	269.00	1.75

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

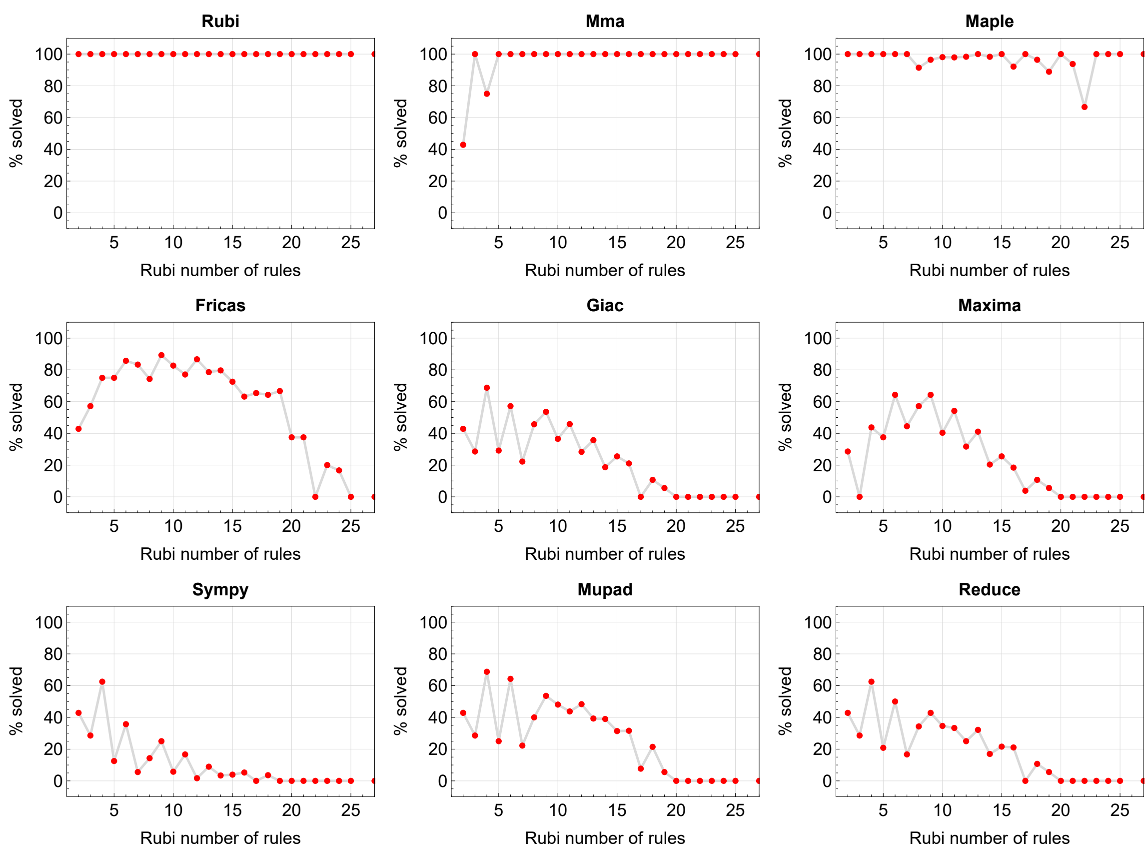


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

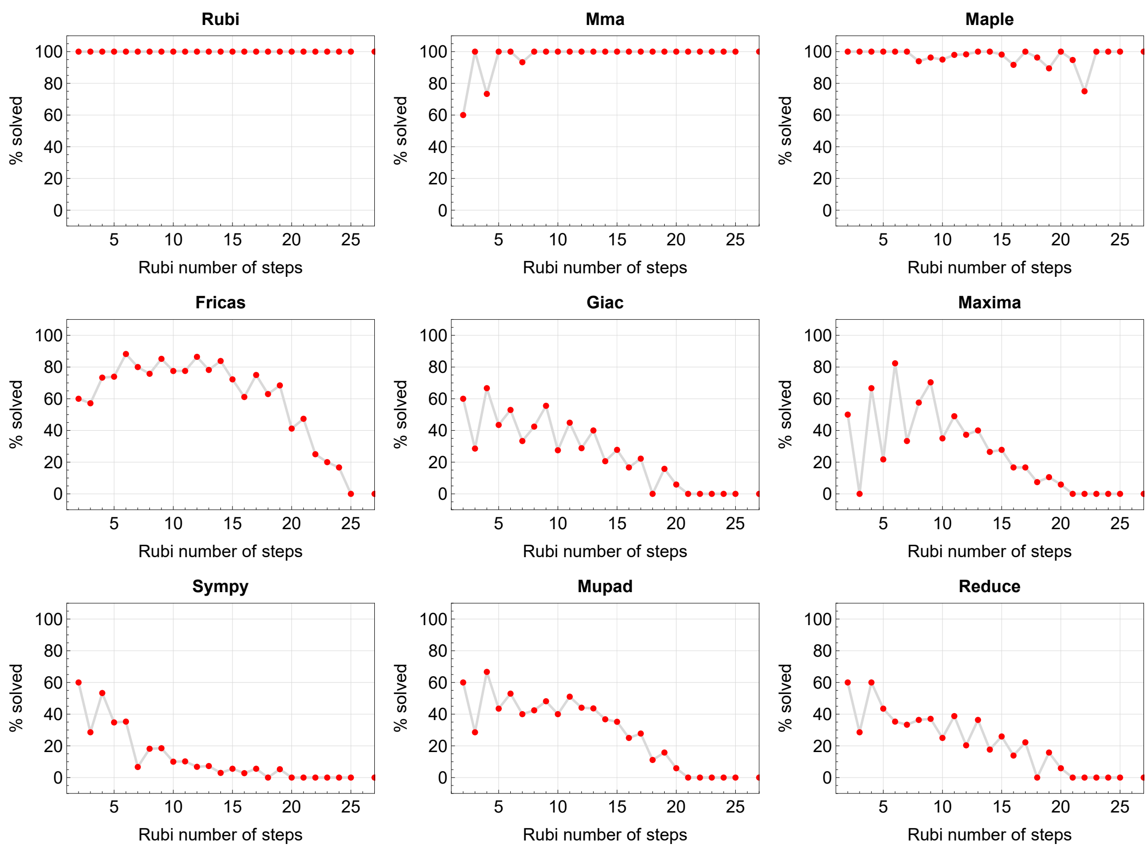


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

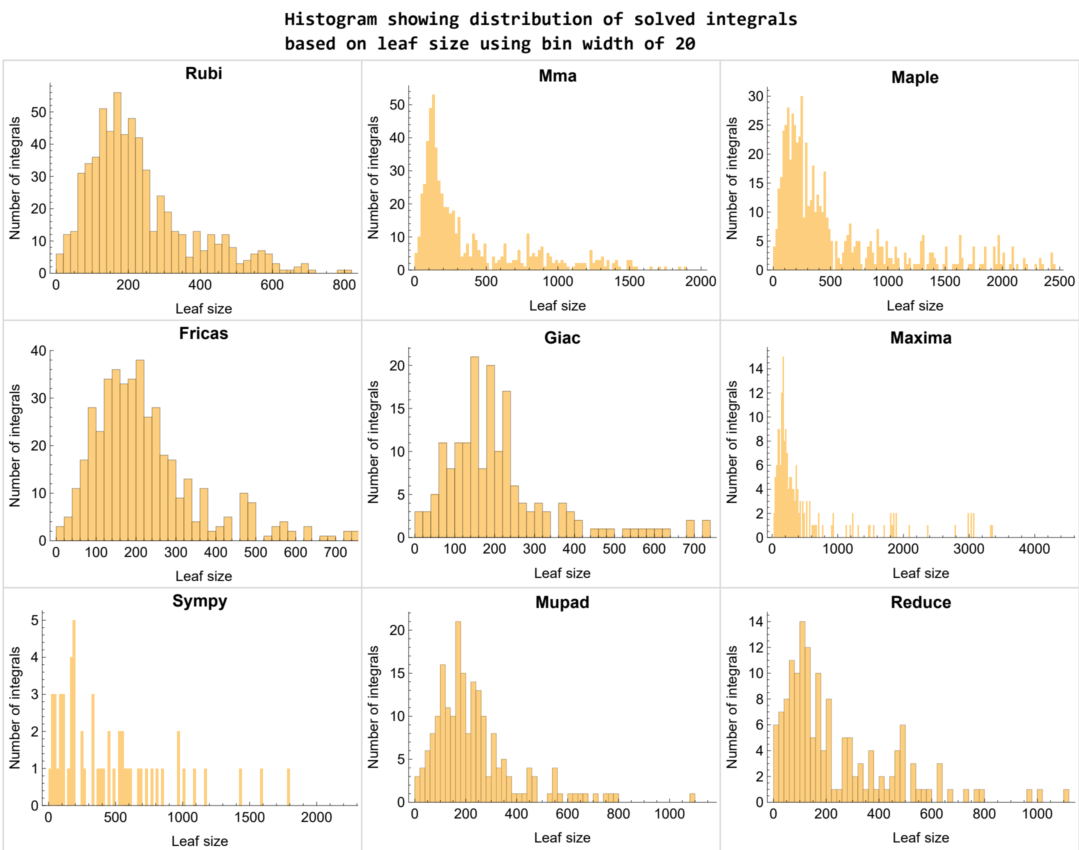


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

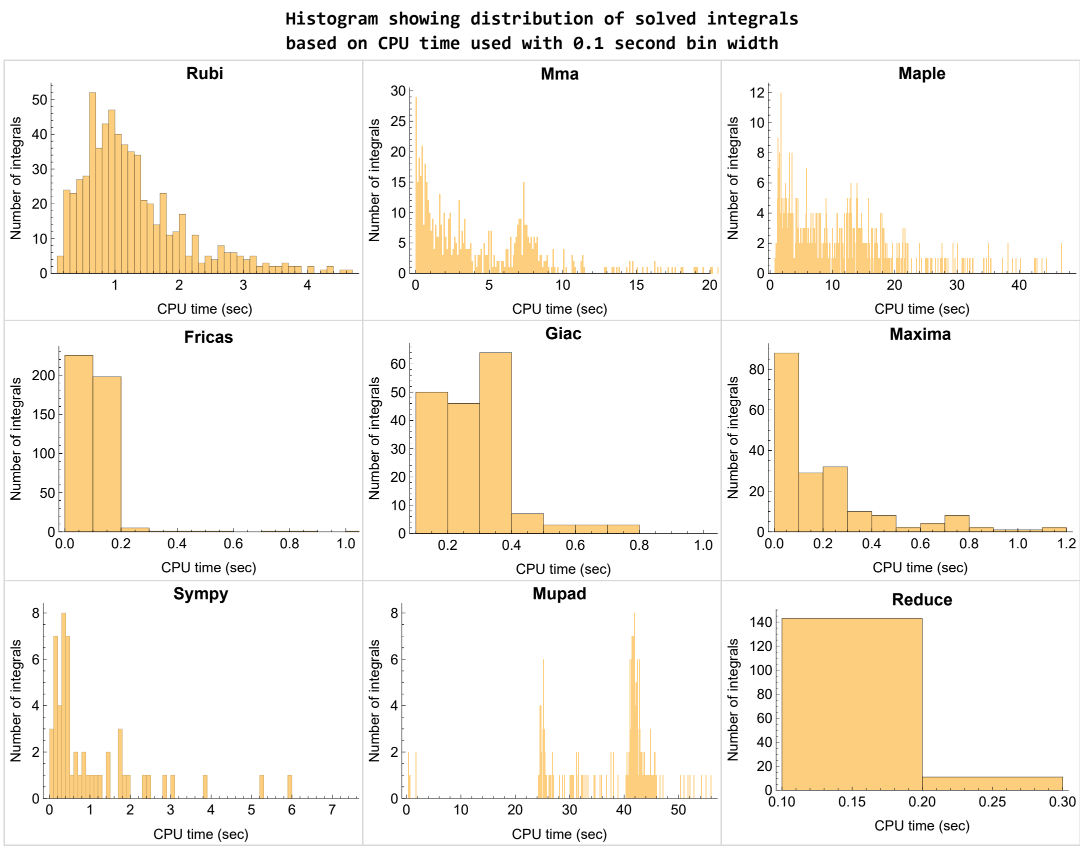


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

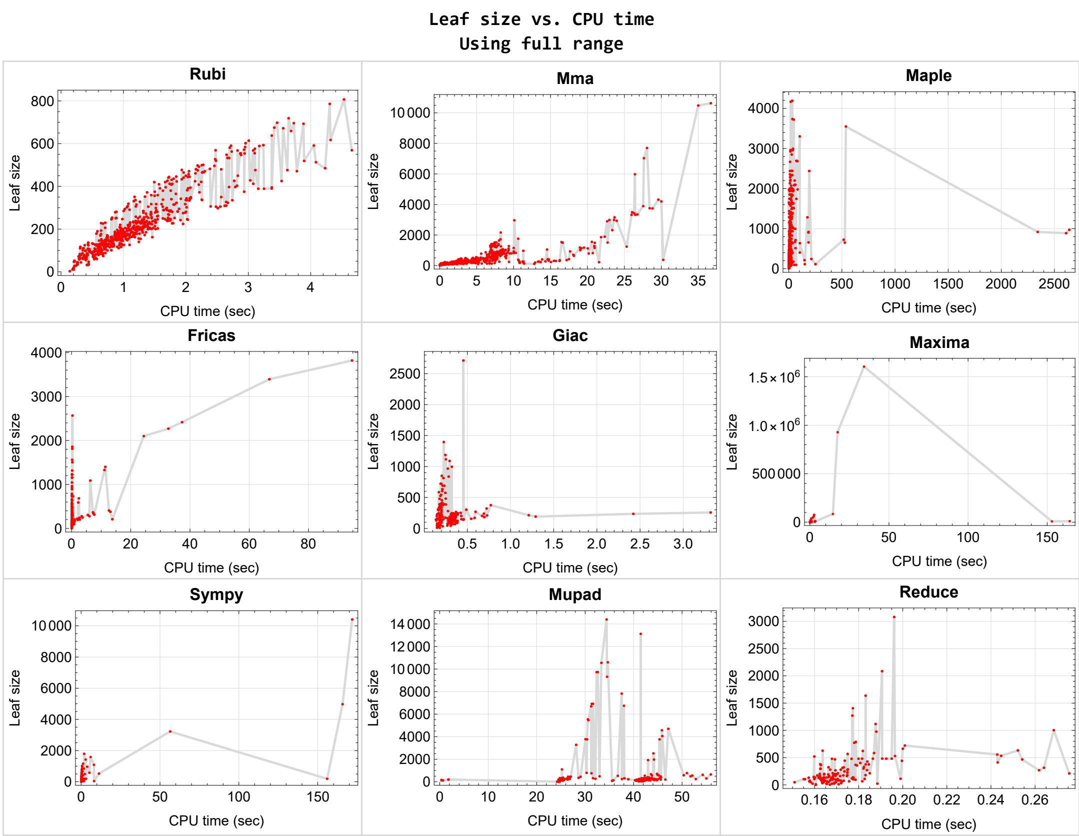


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{449, 455, 456, 457, 458, 635, 641, 642, 643, 644\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {32, 33, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 163, 164, 165, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 303, 310, 316, 317, 318, 325, 332, 338, 339, 340, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 389, 393, 394, 395, 396, 398, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 427, 429, 430, 431, 432, 433, 434, 435, 436, 440, 454, 479, 489, 522, 523, 524, 529, 530, 531, 532, 533, 537, 538, 540, 544, 547, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 590, 591, 592, 593, 595, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 614, 615, 616, 617, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 640}

Maple {122, 164, 165, 198, 283, 284, 302, 303, 309, 310, 316, 317, 318, 333, 334, 335, 336, 337, 338, 339, 340, 382, 419, 487, 534, 583}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

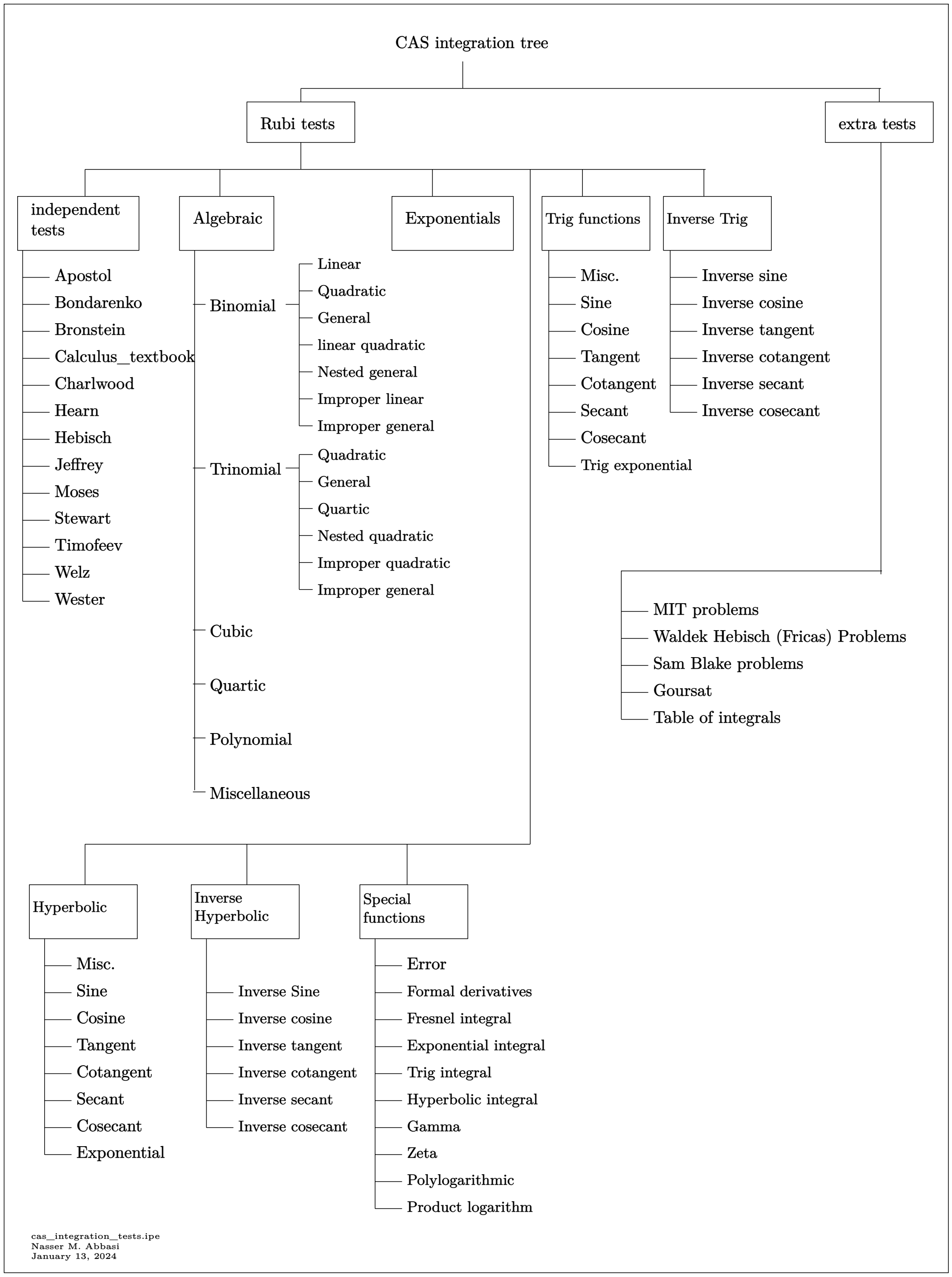
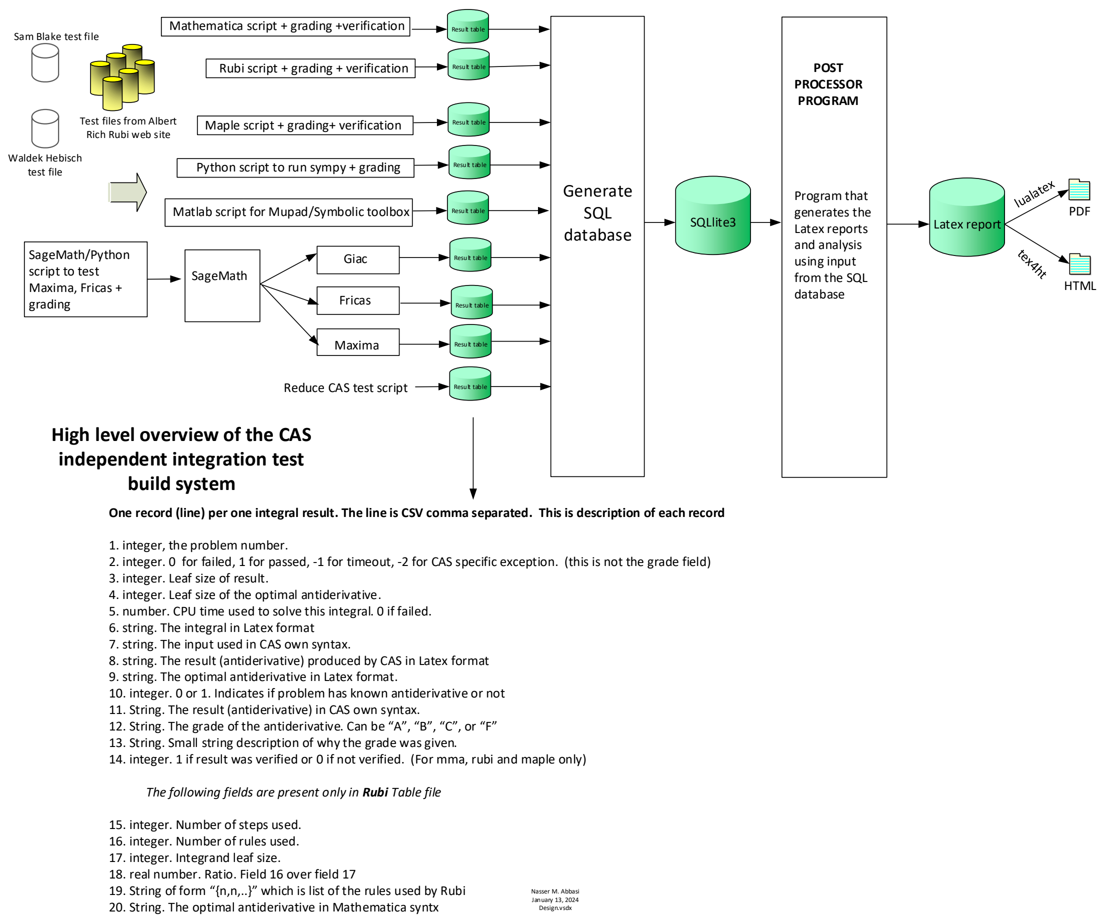


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 35, 36, 37, 41, 49, 51, 60, 61, 62, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 197, 198, 199, 202, 203, 205, 208, 209, 211, 212, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 304, 305, 306, 311, 312, 313, 319, 320, 321, 322, 323, 326, 327, 328, 329, 333, 334, 335, 336, 337, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 397, 398, 399, 421, 422, 423, 424, 437, 438, 439, 440, 450, 451, 452, 453, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 528, 533, 539, 545, 546, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 572, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 592, 593, 594, 601, 613, 615, 616, 617, 618, 622, 631, 632, 633, 634, 636, 637, 638, 639 }

B grade { 23, 32, 33, 34, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 122, 204, 210, 236, 246, 256, 257, 273, 274, 283, 369, 393, 454, 540, 547, 571, 573, 574, 575, 576, 590, 591, 595, 596, 597, 598, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 614, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 640 }

C grade { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 194, 195, 196, 200, 201, 206, 207, 213, 214, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 317, 318, 324, 325, 330, 331, 332, 338, 339, 340, 344, 395, 396, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 520, 521, 522, 523, 524, 526, 527, 529, 530, 531, 532, 534, 535, 536, 537, 538, 541, 542, 543, 544, 548, 549 }

F normal fail { 441, 442, 443, 444, 445, 446, 447, 448 }

F(-1) timedout fail { 641 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 82, 83, 84, 85, 91, 92, 93, 100, 101, 102, 107, 145, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 300, 322, 323, 341, 386, 397, 421, 422, 437, 438, 439, 440, 444, 447, 448, 467, 473, 475, 480, 481, 492, 493, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 593, 594, 617, 618, 631, 632, 633, 634 }

B grade { 78, 79, 80, 81, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 198, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 441, 442, 443, 445, 446, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 474, 476, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 494, 495, 496, 497, 498, 500, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 613, 614, 615, 616, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630 }

C grade { }

F normal fail { 450, 451, 452, 453, 454, 499, 636, 637, 638, 639, 640 }

F(-1) timedout fail { 605, 606, 607, 608, 609 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 109, 110, 114, 115, 116, 117, 122, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 261, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

B grade { 6, 53, 72, 78, 79, 103, 104, 105, 106, 111, 112, 113, 118, 119, 120, 121, 198, 219, 255, 256, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 285, 294 }

C grade { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 296, 297, 298, 299, 304, 305, 306, 311, 312, 313, 319, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 336, 337, 341, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 383, 384, 385, 386, 387, 388, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 584, 585, 586, 587, 588, 589 }

F normal fail { 300, 307, 308, 314, 365, 396, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 411, 412, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 611, 612, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

F(-1) timedout fail { 301, 302, 303, 309, 310, 315, 316, 317, 318, 323, 324, 325, 330, 331, 332, 338, 339, 340, 342, 344, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 389, 390, 391, 392, 393, 394, 395, 397, 402, 410, 413, 414, 431, 432, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 596, 604, 609, 610, 613, 629, 630 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 50, 51, 52, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 91, 92, 93, 94, 104, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249 }

B grade { 6, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 63, 64, 79, 80, 81, 87, 88, 89, 90, 95, 96, 97, 100, 101, 102, 103, 105, 106, 111, 112, 113, 120, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 219, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519 }

C grade { 193, 194, 195, 196, 525 }

F normal fail { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 191, 197, 198, 199, 200, 201, 202, 203, 204, 205, 208, 209, 210, 211, 212, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 377, 378, 379, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 488, 489, 490, 491, 492, 520, 521, 522, 523, 524, 527, 528, 531, 532, 533, 534, 535, 539, 540, 541, 542, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 574, 575, 576, 577, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

F(-1) timedout fail { 98, 99, 107, 108, 109, 110, 114, 115, 116, 117, 118, 119, 122, 156, 163, 165, 206, 207, 213, 214, 331, 332, 340, 375, 376, 380, 382, 482, 487, 516, 529, 530, 536, 537, 538, 543, 544, 572, 573, 578, 579 }

F(-2) exception fail { 121, 164, 192, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 381, 526 }

Giac

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 215, 216, 217, 223, 224, 225, 231, 232, 233, 235, 240, 241, 242, 244, 251, 252, 254, 255, 258, 260, 261, 262, 264, 281, 282, 283, 286, 287, 288, 289, 290, 292, 293, 294 }

B grade { 5, 6, 7, 15, 79, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 234, 236, 237, 238, 239, 243, 245, 246, 247, 248, 249, 250, 253, 256, 257, 259, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 285, 291, 295 }

C grade { 284 }

F normal fail { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 174, 175, 182, 183, 195, 196, 200, 201, 206, 207, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 499, 500, 508, 509, 518, 519, 520, 521, 522, 523, 529, 530, 531, 532, 536, 537, 538, 543, 544, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639 }

F(-1) timedout fail { 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 202, 203, 204, 205, 208, 209, 210, 211, 212, 493, 494, 495, 496, 497, 498, 501, 502, 503, 504, 505, 506, 507, 510, 511, 512, 513, 514, 515, 516, 517, 524, 525, 526, 527, 528, 533, 534, 535, 539, 540, 541, 542, 545, 546, 547, 548, 549 }

F(-2) exception fail { 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 454, 640 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 102, 103, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 171, 172, 173, 179, 180, 181, 188, 189, 190, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 321, 322, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 493, 494, 495, 496, 501, 502, 503, 504, 510, 511, 512, 513 }

C grade { }

F normal fail { }

F(-1) timedout fail { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 497, 498, 499, 500, 505, 506, 507, 508, 509, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

F(-2) exception fail { }

Sympy

A grade { 42, 50, 51, 59, 60, 61, 67, 68, 69, 70, 225, 284, 288 }

B grade { 1, 2, 3, 4, 10, 11, 12, 13, 19, 20, 21, 28, 29, 30, 38, 39, 40, 41, 47, 48, 49, 56, 57, 58, 65, 66, 215, 216, 217, 223, 224, 231, 232, 233, 240, 241, 242, 251, 252, 253, 261, 281, 282, 283, 285, 286, 292 }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 14, 15, 16, 17, 22, 23, 24, 31, 32, 43, 44, 45, 46, 52, 53, 54, 55, 62, 63, 64, 71, 72, 73, 75, 76, 77, 78, 79, 80, 85, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 119, 120, 121, 122, 126, 147, 148, 168, 169, 170, 171, 176, 177, 178, 192, 193, 194, 195, 198, 199, 200, 204, 205, 218, 219, 220, 221, 222, 226, 227, 228, 229, 234, 235, 236, 243, 244, 254, 255, 256, 257, 262, 263, 264, 270, 271, 272, 278, 279, 280, 287, 293, 294, 295, 297, 298, 299, 300, 301, 302, 306, 320, 321, 322, 323, 324, 325, 330, 331, 332, 338, 339, 340, 341, 342, 343, 344, 348, 396, 397, 398, 399, 404, 405, 406, 421, 422, 423, 424, 427, 428, 429, 433, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 453, 462, 463, 464, 468, 469, 474, 475, 478, 479, 480, 483, 484, 488, 498, 499, 500, 525, 526, 527, 528, 533, 534, 553, 554, 555, 559, 560, 565, 566, 568, 569, 570, 571, 574, 575, 579, 585, 586, 587, 588, 594, 595, 596, 617, 618, 619, 622, 623, 632, 633, 637, 638, 639, 640 }

F(-1) timedout fail { 18, 25, 26, 27, 33, 34, 35, 36, 37, 74, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 108, 109, 115, 116, 117, 118, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 174, 175, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 230, 237, 238, 239, 245, 246, 247, 248, 249, 250, 258, 259, 260, 265, 266, 267, 268, 269, 273, 274, 275, 276, 277, 289, 290, 291, 296, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 329, 333, 334, 335, 336, 337, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 430, 431, 432, 434, 435, 436, 450, 451, 452, 454, 459, 460, 461, 465, 466, 467, 470, 471, 472, 473, 476, 477, 481, 482, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 529, 530, 531, 532, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 557, 558, 561, 562, 563, 564, 567, 572, 573, 576, 577, 578, 580, 581, 582, 583, 584, 589, 590, 591, 592, 593, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 634, 635, 636, 641 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295 }

C grade { }

F normal fail { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	120	77	87	124	88	333	112	132	236
N.S.	1	0.96	0.62	0.70	0.99	0.70	2.66	0.90	1.06	1.89
time (sec)	N/A	0.624	0.439	23.928	0.048	0.084	0.280	0.306	0.157	42.027

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	93	75	67	101	74	252	89	104	212
N.S.	1	0.96	0.77	0.69	1.04	0.76	2.60	0.92	1.07	2.19
time (sec)	N/A	0.534	0.312	13.114	0.043	0.084	0.250	0.319	0.156	41.873

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	65	54	79	56	168	68	76	84
N.S.	1	1.03	0.84	0.70	1.03	0.73	2.18	0.88	0.99	1.09
time (sec)	N/A	0.366	0.212	7.842	0.035	0.084	0.177	0.318	0.165	40.514

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	37	55	38	94	45	48	50
N.S.	1	1.00	0.94	0.79	1.17	0.81	2.00	0.96	1.02	1.06
time (sec)	N/A	0.207	0.127	3.553	0.073	0.081	0.091	0.268	0.151	41.440

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	46	48	47	51	0	79	51	100
N.S.	1	1.00	1.44	1.50	1.47	1.59	0.00	2.47	1.59	3.12
time (sec)	N/A	0.414	0.029	3.132	0.043	0.090	0.000	0.285	0.158	40.889

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	50	73	79	0	84	114	100
N.S.	1	1.00	1.34	1.56	2.28	2.47	0.00	2.62	3.56	3.12
time (sec)	N/A	0.434	0.025	5.108	0.042	0.092	0.000	0.304	0.167	41.550

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	59	75	75	95	89	0	124	280	94
N.S.	1	1.05	1.34	1.34	1.70	1.59	0.00	2.21	5.00	1.68
time (sec)	N/A	0.532	0.034	7.444	0.041	0.090	0.000	0.277	0.167	41.354

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	87	56	81	127	105	0	154	297	126
N.S.	1	1.01	0.65	0.94	1.48	1.22	0.00	1.79	3.45	1.47
time (sec)	N/A	0.646	0.368	10.183	0.070	0.087	0.000	0.295	0.163	42.358

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	102	77	119	163	127	0	188	479	166
N.S.	1	0.96	0.73	1.12	1.54	1.20	0.00	1.77	4.52	1.57
time (sec)	N/A	0.659	0.466	10.742	0.042	0.090	0.000	0.340	0.177	43.022

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	177	134	111	216	130	600	166	162	315
N.S.	1	0.93	0.70	0.58	1.13	0.68	3.14	0.87	0.85	1.65
time (sec)	N/A	0.980	0.756	150.435	0.040	0.091	0.424	0.299	0.162	42.125

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	151	108	93	178	110	459	137	134	277
N.S.	1	0.94	0.68	0.58	1.11	0.69	2.87	0.86	0.84	1.73
time (sec)	N/A	0.865	0.401	51.912	0.041	0.083	0.323	0.349	0.160	41.829

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	128	86	74	144	90	338	110	106	134
N.S.	1	0.99	0.67	0.57	1.12	0.70	2.62	0.85	0.82	1.04
time (sec)	N/A	0.541	0.346	19.020	0.056	0.083	0.200	0.311	0.155	41.062

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	83	84	116	110	70	199	85	78	98
N.S.	1	0.88	0.89	1.23	1.17	0.74	2.12	0.90	0.83	1.04
time (sec)	N/A	0.300	0.556	2.227	0.048	0.082	0.138	0.283	0.158	41.547

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	96	74	94	79	0	145	81	141
N.S.	1	1.00	1.17	0.90	1.15	0.96	0.00	1.77	0.99	1.72
time (sec)	N/A	0.670	1.417	3.559	0.046	0.085	0.000	0.328	0.171	41.492

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	143	81	105	108	0	155	142	161
N.S.	1	1.00	1.93	1.09	1.42	1.46	0.00	2.09	1.92	2.18
time (sec)	N/A	0.675	2.201	6.071	0.040	0.099	0.000	0.339	0.166	41.083

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	71	106	142	119	0	154	314	162
N.S.	1	1.00	0.81	1.20	1.61	1.35	0.00	1.75	3.57	1.84
time (sec)	N/A	0.710	0.907	8.951	0.048	0.094	0.000	0.355	0.173	40.797

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	119	72	120	174	125	0	178	299	145
N.S.	1	1.05	0.64	1.06	1.54	1.11	0.00	1.58	2.65	1.28
time (sec)	N/A	0.845	1.165	10.773	0.049	0.087	0.000	0.325	0.182	43.125

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	145	81	150	230	145	0	212	481	183
N.S.	1	1.01	0.56	1.04	1.60	1.01	0.00	1.47	3.34	1.27
time (sec)	N/A	0.988	1.359	12.825	0.046	0.087	0.000	0.351	0.186	44.521

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	197	134	112	262	130	695	166	162	315
N.S.	1	0.98	0.67	0.56	1.30	0.65	3.46	0.83	0.81	1.57
time (sec)	N/A	1.181	0.544	250.811	0.044	0.086	0.417	0.302	0.168	43.265

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	146	108	93	213	110	530	136	134	277
N.S.	1	0.95	0.70	0.60	1.38	0.71	3.44	0.88	0.87	1.80
time (sec)	N/A	0.609	0.386	68.490	0.047	0.084	0.476	0.300	0.160	41.827

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	101	116	176	167	90	371	112	106	134
N.S.	1	0.87	1.00	1.52	1.44	0.78	3.20	0.97	0.91	1.16
time (sec)	N/A	0.349	0.648	3.462	0.047	0.079	0.197	0.383	0.164	40.304

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	117	113	93	141	102	0	180	108	178
N.S.	1	1.05	1.02	0.84	1.27	0.92	0.00	1.62	0.97	1.60
time (sec)	N/A	0.899	1.589	7.256	0.063	0.090	0.000	0.361	0.183	40.561

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	111	272	120	140	127	0	192	178	197
N.S.	1	1.01	2.47	1.09	1.27	1.15	0.00	1.75	1.62	1.79
time (sec)	N/A	0.923	3.801	7.877	0.049	0.093	0.000	0.328	0.175	41.405

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	116	208	129	165	137	0	192	376	207
N.S.	1	1.02	1.82	1.13	1.45	1.20	0.00	1.68	3.30	1.82
time (sec)	N/A	0.962	4.938	9.486	0.048	0.098	0.000	0.367	0.185	41.732

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	131	103	142	212	141	0	189	329	209
N.S.	1	1.05	0.82	1.14	1.70	1.13	0.00	1.51	2.63	1.67
time (sec)	N/A	0.966	1.919	13.321	0.052	0.093	0.000	0.310	0.182	41.577

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	165	91	178	269	145	0	212	481	185
N.S.	1	1.07	0.59	1.16	1.75	0.94	0.00	1.38	3.12	1.20
time (sec)	N/A	1.158	2.314	14.750	0.057	0.084	0.000	0.363	0.182	43.236

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	191	101	196	337	165	0	246	481	224
N.S.	1	1.03	0.55	1.06	1.82	0.89	0.00	1.33	2.60	1.21
time (sec)	N/A	1.302	3.196	17.269	0.046	0.093	0.000	0.335	0.195	44.471

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	242	156	358	356	150	960	193	190	353
N.S.	1	1.00	0.65	1.49	1.48	0.62	3.98	0.80	0.79	1.46
time (sec)	N/A	1.513	0.801	7.549	0.041	0.087	0.614	0.340	0.169	42.861

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	170	134	306	297	130	765	166	162	316
N.S.	1	0.92	0.72	1.65	1.61	0.70	4.14	0.90	0.88	1.71
time (sec)	N/A	0.630	0.481	6.103	0.056	0.083	0.461	0.265	0.172	43.647

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	125	133	248	236	110	544	139	134	278
N.S.	1	0.83	0.89	1.65	1.57	0.73	3.63	0.93	0.89	1.85
time (sec)	N/A	0.380	1.087	4.996	0.040	0.084	0.320	0.360	0.165	42.485

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	162	138	108	198	118	0	214	136	188
N.S.	1	1.07	0.91	0.72	1.31	0.78	0.00	1.42	0.90	1.25
time (sec)	N/A	1.184	2.276	25.786	0.044	0.096	0.000	0.342	0.178	41.857

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	156	312	140	187	150	0	226	213	242
N.S.	1	1.04	2.08	0.93	1.25	1.00	0.00	1.51	1.42	1.61
time (sec)	N/A	1.243	6.464	28.204	0.054	0.092	0.000	0.369	0.185	42.167

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	154	343	168	199	156	0	230	428	243
N.S.	1	0.95	2.12	1.04	1.23	0.96	0.00	1.42	2.64	1.50
time (sec)	N/A	1.274	10.667	27.697	0.042	0.094	0.000	0.384	0.184	42.739

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	172	380	165	235	159	0	227	376	254
N.S.	1	1.04	2.30	1.00	1.42	0.96	0.00	1.38	2.28	1.54
time (sec)	N/A	1.339	10.626	16.872	0.046	0.092	0.000	0.359	0.183	41.803

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	184	132	203	307	157	0	223	530	255
N.S.	1	1.06	0.76	1.17	1.77	0.91	0.00	1.29	3.06	1.47
time (sec)	N/A	1.308	5.693	17.738	0.045	0.093	0.000	0.388	0.196	41.710

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	214	232	217	376	165	0	246	481	224
N.S.	1	1.08	1.17	1.10	1.90	0.83	0.00	1.24	2.43	1.13
time (sec)	N/A	1.504	5.159	17.990	0.059	0.086	0.000	0.320	0.192	38.896

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	240	259	251	464	185	0	280	663	262
N.S.	1	1.05	1.13	1.10	2.03	0.81	0.00	1.22	2.90	1.14
time (sec)	N/A	1.747	5.163	19.289	0.051	0.096	0.000	0.276	0.200	44.264

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	134	311	96	394	120	1794	181	158	170
N.S.	1	0.88	2.03	0.63	2.58	0.78	11.73	1.18	1.03	1.11
time (sec)	N/A	0.650	2.313	1.442	0.154	0.080	1.907	0.307	0.167	42.329

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	107	249	78	310	98	1161	151	130	138
N.S.	1	0.88	2.04	0.64	2.54	0.80	9.52	1.24	1.07	1.13
time (sec)	N/A	0.546	2.089	1.290	0.126	0.080	1.221	0.284	0.162	42.640

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	98	197	61	225	83	665	124	99	107
N.S.	1	1.09	2.19	0.68	2.50	0.92	7.39	1.38	1.10	1.19
time (sec)	N/A	0.390	1.723	1.158	0.118	0.079	0.820	0.289	0.170	41.394

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	47	93	42	143	61	264	78	48	65
N.S.	1	0.87	1.72	0.78	2.65	1.13	4.89	1.44	0.89	1.20
time (sec)	N/A	0.443	0.633	1.080	0.146	0.078	0.556	0.271	0.164	41.162

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	76	28	73	43	49	43	35	30
N.S.	1	1.00	2.24	0.82	2.15	1.26	1.44	1.26	1.03	0.88
time (sec)	N/A	0.261	0.410	0.944	0.147	0.077	0.354	0.306	0.169	41.913

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	109	54	99	74	0	71	60	42
N.S.	1	1.00	2.48	1.23	2.25	1.68	0.00	1.61	1.36	0.95
time (sec)	N/A	0.319	0.784	1.284	0.055	0.082	0.000	0.346	0.172	41.043

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	70	201	93	196	127	0	110	173	78
N.S.	1	1.01	2.91	1.35	2.84	1.84	0.00	1.59	2.51	1.13
time (sec)	N/A	0.511	2.038	1.571	0.045	0.085	0.000	0.322	0.161	35.552

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	100	289	126	282	156	0	157	285	119
N.S.	1	0.93	2.70	1.18	2.64	1.46	0.00	1.47	2.66	1.11
time (sec)	N/A	0.637	4.798	1.620	0.058	0.084	0.000	0.318	0.175	41.306

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	118	490	170	368	168	0	182	363	152
N.S.	1	0.90	3.74	1.30	2.81	1.28	0.00	1.39	2.77	1.16
time (sec)	N/A	0.682	5.848	1.706	0.041	0.085	0.000	0.294	0.163	43.094

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	160	369	108	372	154	1425	192	202	189
N.S.	1	0.94	2.17	0.64	2.19	0.91	8.38	1.13	1.19	1.11
time (sec)	N/A	0.819	2.556	1.441	0.129	0.081	2.831	0.318	0.164	41.313

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	152	315	88	283	138	843	164	174	152
N.S.	1	1.03	2.14	0.60	1.93	0.94	5.73	1.12	1.18	1.03
time (sec)	N/A	0.645	2.347	1.259	0.144	0.090	1.736	0.296	0.169	41.914

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	137	73	191	117	411	119	145	105
N.S.	1	1.01	1.38	0.74	1.93	1.18	4.15	1.20	1.46	1.06
time (sec)	N/A	0.743	1.974	1.181	0.157	0.081	1.185	0.290	0.162	41.980

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	73	153	49	120	91	105	86	65	65
N.S.	1	1.04	2.19	0.70	1.71	1.30	1.50	1.23	0.93	0.93
time (sec)	N/A	0.518	1.159	1.050	0.114	0.080	0.955	0.299	0.166	41.891

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	43	46	93	58	94	60	51	45
N.S.	1	0.98	0.66	0.71	1.43	0.89	1.45	0.92	0.78	0.69
time (sec)	N/A	0.294	0.182	0.970	0.046	0.070	0.704	0.314	0.169	41.587

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	80	170	75	145	131	0	113	90	74
N.S.	1	1.01	2.15	0.95	1.84	1.66	0.00	1.43	1.14	0.94
time (sec)	N/A	0.511	1.018	1.361	0.039	0.082	0.000	0.332	0.163	41.203

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	264	126	244	207	0	155	267	123
N.S.	1	1.07	2.47	1.18	2.28	1.93	0.00	1.45	2.50	1.15
time (sec)	N/A	0.778	2.661	1.600	0.043	0.091	0.000	0.346	0.169	41.041

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	151	496	158	336	228	0	198	412	165
N.S.	1	0.99	3.26	1.04	2.21	1.50	0.00	1.30	2.71	1.09
time (sec)	N/A	0.945	4.494	1.770	0.068	0.094	0.000	0.319	0.243	40.872

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	171	609	196	425	247	0	226	557	203
N.S.	1	0.96	3.40	1.09	2.37	1.38	0.00	1.26	3.11	1.13
time (sec)	N/A	1.010	6.432	1.885	0.053	0.091	0.000	0.335	0.243	41.975

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	216	491	124	412	205	1584	228	268	238
N.S.	1	0.99	2.25	0.57	1.89	0.94	7.27	1.05	1.23	1.09
time (sec)	N/A	1.170	3.475	1.593	0.119	0.087	5.978	0.309	0.262	42.047

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	207	435	106	322	190	966	200	247	203
N.S.	1	1.07	2.25	0.55	1.67	0.98	5.01	1.04	1.28	1.05
time (sec)	N/A	0.954	2.962	1.398	0.124	0.083	3.825	0.314	0.174	41.985

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	154	361	89	231	165	496	155	174	152
N.S.	1	1.05	2.46	0.61	1.57	1.12	3.37	1.05	1.18	1.03
time (sec)	N/A	1.075	2.641	1.261	0.126	0.080	2.379	0.345	0.173	41.914

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	125	241	69	160	137	148	120	94	134
N.S.	1	1.08	2.08	0.59	1.38	1.18	1.28	1.03	0.81	1.16
time (sec)	N/A	0.799	2.002	1.184	0.119	0.078	1.702	0.325	0.179	41.974

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	104	135	56	115	93	117	75	66	66
N.S.	1	1.02	1.32	0.55	1.13	0.91	1.15	0.74	0.65	0.65
time (sec)	N/A	0.547	1.284	1.119	0.048	0.075	1.009	0.312	0.164	41.697

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	99	63	56	115	93	114	75	66	66
N.S.	1	0.97	0.62	0.55	1.13	0.91	1.12	0.74	0.65	0.65
time (sec)	N/A	0.394	0.233	1.155	0.046	0.075	0.894	0.309	0.182	41.695

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	197	95	187	185	0	148	119	130
N.S.	1	1.09	1.68	0.81	1.60	1.58	0.00	1.26	1.02	1.11
time (sec)	N/A	0.753	1.499	1.528	0.044	0.086	0.000	0.314	0.177	35.800

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	161	482	143	286	272	0	190	296	168
N.S.	1	1.11	3.32	0.99	1.97	1.88	0.00	1.31	2.04	1.16
time (sec)	N/A	1.100	4.477	1.731	0.043	0.086	0.000	0.298	0.163	42.554

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	203	610	175	377	295	0	233	441	216
N.S.	1	1.04	3.11	0.89	1.92	1.51	0.00	1.19	2.25	1.10
time (sec)	N/A	1.299	6.512	1.869	0.042	0.091	0.000	0.350	0.200	42.360

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	251	555	123	364	238	1085	233	275	259
N.S.	1	1.10	2.42	0.54	1.59	1.04	4.74	1.02	1.20	1.13
time (sec)	N/A	1.233	7.314	1.597	0.146	0.087	8.121	0.333	0.175	42.324

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	202	481	106	271	213	578	188	202	201
N.S.	1	1.09	2.60	0.57	1.46	1.15	3.12	1.02	1.09	1.09
time (sec)	N/A	1.436	6.937	1.516	0.118	0.083	5.209	0.344	0.162	42.110

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	173	329	89	201	180	192	155	122	162
N.S.	1	1.12	2.14	0.58	1.31	1.17	1.25	1.01	0.79	1.05
time (sec)	N/A	1.101	6.803	1.283	0.136	0.081	3.092	0.322	0.168	41.363

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	147	193	82	175	124	182	117	108	86
N.S.	1	1.08	1.42	0.60	1.29	0.91	1.34	0.86	0.79	0.63
time (sec)	N/A	0.837	5.302	1.295	0.049	0.073	2.426	0.319	0.163	41.856

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	137	163	76	174	124	178	117	108	84
N.S.	1	0.99	1.18	0.55	1.26	0.90	1.29	0.85	0.78	0.61
time (sec)	N/A	0.664	1.787	1.300	0.047	0.072	1.732	0.306	0.160	41.638

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	132	81	84	175	125	177	117	108	87
N.S.	1	0.96	0.59	0.61	1.27	0.91	1.28	0.85	0.78	0.63
time (sec)	N/A	0.508	0.573	1.259	0.048	0.075	1.439	0.335	0.170	41.490

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	167	239	113	228	236	0	182	147	199
N.S.	1	1.14	1.63	0.77	1.55	1.61	0.00	1.24	1.00	1.35
time (sec)	N/A	0.997	2.379	1.602	0.058	0.086	0.000	0.365	0.160	41.563

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	199	595	160	326	337	0	224	324	236
N.S.	1	1.14	3.40	0.91	1.86	1.93	0.00	1.28	1.85	1.35
time (sec)	N/A	1.392	7.509	1.921	0.047	0.090	0.000	0.371	0.166	41.617

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	247	798	192	419	360	0	267	469	273
N.S.	1	1.06	3.44	0.83	1.81	1.55	0.00	1.15	2.02	1.18
time (sec)	N/A	1.663	9.024	2.108	0.053	0.096	0.000	0.368	0.169	41.659

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	103	121	145	99	0	182	48	0
N.S.	1	1.00	0.55	0.65	0.78	0.53	0.00	0.97	0.26	0.00
time (sec)	N/A	0.862	0.698	3.268	0.245	0.080	0.000	0.696	0.169	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	146	80	102	118	82	0	147	48	0
N.S.	1	1.01	0.56	0.71	0.82	0.57	0.00	1.02	0.33	0.00
time (sec)	N/A	0.657	0.286	3.109	0.225	0.081	0.000	0.459	0.176	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	112	64	83	88	64	0	107	46	0
N.S.	1	1.11	0.63	0.82	0.87	0.63	0.00	1.06	0.46	0.00
time (sec)	N/A	0.589	0.174	2.953	0.221	0.076	0.000	0.363	0.172	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	62	57	47	0	70	37	0
N.S.	1	1.00	0.74	1.00	0.92	0.76	0.00	1.13	0.60	0.00
time (sec)	N/A	0.290	0.085	2.715	0.280	0.074	0.000	0.357	0.166	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	219	21	127	0	89	50	0
N.S.	1	1.00	1.00	3.32	0.32	1.92	0.00	1.35	0.76	0.00
time (sec)	N/A	0.372	0.106	4.188	0.201	0.089	0.000	0.382	0.202	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	85	567	710	153	0	121	54	0
N.S.	1	1.00	1.25	8.34	10.44	2.25	0.00	1.78	0.79	0.00
time (sec)	N/A	0.410	0.212	4.479	0.233	0.097	0.000	0.387	0.199	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	109	101	936	3352	178	0	195	54	0
N.S.	1	0.93	0.86	8.00	28.65	1.52	0.00	1.67	0.46	0.00
time (sec)	N/A	0.539	0.879	4.540	2.976	0.105	0.000	0.350	0.204	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	150	129	1282	5021	197	0	244	54	0
N.S.	1	0.94	0.81	8.01	31.38	1.23	0.00	1.52	0.34	0.00
time (sec)	N/A	0.708	1.936	4.471	3.009	0.104	0.000	0.391	0.194	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	238	125	142	185	125	0	235	93	0
N.S.	1	1.02	0.53	0.61	0.79	0.53	0.00	1.00	0.40	0.00
time (sec)	N/A	1.195	0.919	3.295	0.260	0.082	0.000	2.421	0.186	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	197	103	123	154	107	0	191	93	0
N.S.	1	1.04	0.54	0.65	0.81	0.57	0.00	1.01	0.49	0.00
time (sec)	N/A	0.990	0.494	3.086	0.242	0.080	0.000	1.290	0.197	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	146	81	104	123	88	0	155	91	0
N.S.	1	1.06	0.59	0.75	0.89	0.64	0.00	1.12	0.66	0.00
time (sec)	N/A	0.733	0.325	3.074	0.234	0.076	0.000	0.544	0.182	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	99	65	85	93	69	0	115	80	0
N.S.	1	0.98	0.64	0.84	0.92	0.68	0.00	1.14	0.79	0.00
time (sec)	N/A	0.400	0.214	2.678	0.223	0.076	0.000	0.365	0.187	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	85	269	39	149	0	140	105	0
N.S.	1	1.03	0.81	2.56	0.37	1.42	0.00	1.33	1.00	0.00
time (sec)	N/A	0.603	0.236	3.881	0.192	0.090	0.000	0.452	0.230	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	109	98	603	1315	172	0	149	113	0
N.S.	1	1.06	0.95	5.85	12.77	1.67	0.00	1.45	1.10	0.00
time (sec)	N/A	0.640	0.336	4.013	0.238	0.104	0.000	0.430	0.262	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	118	109	938	3339	182	0	201	113	0
N.S.	1	0.99	0.92	7.88	28.06	1.53	0.00	1.69	0.95	0.00
time (sec)	N/A	0.692	0.553	4.160	0.364	0.102	0.000	0.429	0.261	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	160	132	1283	7567	202	0	252	113	0
N.S.	1	0.98	0.80	7.82	46.14	1.23	0.00	1.54	0.69	0.00
time (sec)	N/A	0.863	1.052	4.139	153.022	0.108	0.000	0.423	0.270	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	201	151	1617	10504	220	0	302	113	0
N.S.	1	0.96	0.72	7.74	50.26	1.05	0.00	1.44	0.54	0.00
time (sec)	N/A	1.055	1.705	4.185	164.240	0.127	0.000	0.487	0.355	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	249	127	142	207	137	0	257	141	0
N.S.	1	1.05	0.54	0.60	0.87	0.58	0.00	1.08	0.59	0.00
time (sec)	N/A	1.330	1.107	13.412	0.272	0.081	0.000	3.317	0.285	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	180	105	123	172	116	0	213	139	0
N.S.	1	1.03	0.60	0.70	0.98	0.66	0.00	1.22	0.79	0.00
time (sec)	N/A	0.865	0.605	5.803	0.295	0.081	0.000	1.211	0.255	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	133	83	104	139	95	0	169	128	0
N.S.	1	0.96	0.60	0.75	1.01	0.69	0.00	1.22	0.93	0.00
time (sec)	N/A	0.501	0.358	3.319	0.247	0.075	0.000	0.584	0.194	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	150	104	310	61	177	0	202	165	0
N.S.	1	1.06	0.73	2.18	0.43	1.25	0.00	1.42	1.16	0.00
time (sec)	N/A	0.845	0.436	5.477	0.219	0.090	0.000	0.671	0.303	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	153	120	683	8114	202	0	209	177	0
N.S.	1	1.06	0.83	4.74	56.35	1.40	0.00	1.45	1.23	0.00
time (sec)	N/A	0.921	0.561	15.781	0.479	0.108	0.000	0.727	0.331	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	159	126	995	11782	204	0	239	177	0
N.S.	1	1.02	0.81	6.38	75.53	1.31	0.00	1.53	1.13	0.00
time (sec)	N/A	0.942	0.674	59.413	3.310	0.103	0.000	0.692	0.355	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	169	131	1282	7994	212	0	268	177	0
N.S.	1	1.03	0.80	7.82	48.74	1.29	0.00	1.63	1.08	0.00
time (sec)	N/A	0.980	1.187	175.114	3.327	0.113	0.000	0.594	0.326	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	212	152	1650	0	232	0	322	177	0
N.S.	1	1.01	0.73	7.89	0.00	1.11	0.00	1.54	0.85	0.00
time (sec)	N/A	1.216	1.903	1.305	0.000	0.125	0.000	0.723	0.333	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	253	176	1975	0	252	0	376	177	0
N.S.	1	1.00	0.69	7.78	0.00	0.99	0.00	1.48	0.70	0.00
time (sec)	N/A	1.397	2.407	1.336	0.000	0.126	0.000	0.771	0.322	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	226	111	281	1604723	184	0	0	71	0
N.S.	1	1.12	0.55	1.39	7944.17	0.91	0.00	0.00	0.35	0.00
time (sec)	N/A	1.276	0.774	2.737	34.217	0.090	0.000	0.000	0.172	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	174	94	240	927957	166	0	0	71	0
N.S.	1	1.09	0.59	1.51	5836.21	1.04	0.00	0.00	0.45	0.00
time (sec)	N/A	0.926	0.408	2.642	17.508	0.088	0.000	0.000	0.172	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	125	78	194	38386	149	0	0	69	160
N.S.	1	1.06	0.66	1.64	325.31	1.26	0.00	0.00	0.58	1.36
time (sec)	N/A	0.615	0.190	2.441	0.826	0.083	0.000	0.000	0.176	0.353

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	60	160	19040	135	0	0	61	112
N.S.	1	1.00	0.77	2.05	244.10	1.73	0.00	0.00	0.78	1.44
time (sec)	N/A	0.320	0.083	2.244	0.468	0.081	0.000	0.000	0.167	42.212

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	272	91	171	0	0	73	0
N.S.	1	1.00	0.79	2.99	1.00	1.88	0.00	0.00	0.80	0.00
time (sec)	N/A	0.476	0.087	3.178	0.187	0.095	0.000	0.000	0.172	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	128	95	703	18436	259	0	0	77	0
N.S.	1	1.08	0.80	5.91	154.92	2.18	0.00	0.00	0.65	0.00
time (sec)	N/A	0.736	0.408	3.517	0.387	0.113	0.000	0.000	0.186	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	177	114	1152	76209	284	0	0	77	0
N.S.	1	1.07	0.69	6.98	461.87	1.72	0.00	0.00	0.47	0.00
time (sec)	N/A	1.036	0.873	3.464	2.683	0.122	0.000	0.000	0.176	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	287	167	448	0	241	0	0	91	0
N.S.	1	1.10	0.64	1.72	0.00	0.92	0.00	0.00	0.35	0.00
time (sec)	N/A	1.733	1.288	2.924	0.000	0.091	0.000	0.000	0.186	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	232	142	445	0	224	0	0	91	0
N.S.	1	1.07	0.66	2.06	0.00	1.04	0.00	0.00	0.42	0.00
time (sec)	N/A	1.350	1.055	2.538	0.000	0.088	0.000	0.000	0.169	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	180	97	327	0	205	0	0	91	0
N.S.	1	1.05	0.57	1.91	0.00	1.20	0.00	0.00	0.53	0.00
time (sec)	N/A	0.954	0.788	2.521	0.000	0.100	0.000	0.000	0.172	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	121	104	256	0	189	0	0	89	0
N.S.	1	1.03	0.88	2.17	0.00	1.60	0.00	0.00	0.75	0.00
time (sec)	N/A	0.603	0.433	2.401	0.000	0.086	0.000	0.000	0.184	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	63	220	62254	172	0	0	81	0
N.S.	1	1.00	0.72	2.53	715.56	1.98	0.00	0.00	0.93	0.00
time (sec)	N/A	0.344	0.232	2.399	2.460	0.083	0.000	0.000	0.169	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	133	131	364	15722	281	0	0	93	0
N.S.	1	1.05	1.03	2.87	123.80	2.21	0.00	0.00	0.73	0.00
time (sec)	N/A	0.725	0.690	4.388	0.821	0.099	0.000	0.000	0.181	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	178	141	860	47933	339	0	0	97	0
N.S.	1	1.05	0.83	5.06	281.96	1.99	0.00	0.00	0.57	0.00
time (sec)	N/A	1.069	1.316	3.604	1.906	0.130	0.000	0.000	0.181	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	232	205	1372	0	361	0	0	97	0
N.S.	1	1.05	0.93	6.21	0.00	1.63	0.00	0.00	0.44	0.00
time (sec)	N/A	1.416	1.663	3.595	0.000	0.129	0.000	0.000	0.179	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	287	139	467	0	270	0	0	111	0
N.S.	1	1.10	0.53	1.79	0.00	1.03	0.00	0.00	0.43	0.00
time (sec)	N/A	1.759	1.698	2.868	0.000	0.089	0.000	0.000	0.204	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	232	117	397	0	254	0	0	111	0
N.S.	1	1.07	0.54	1.84	0.00	1.18	0.00	0.00	0.51	0.00
time (sec)	N/A	1.334	1.112	2.503	0.000	0.088	0.000	0.000	0.228	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	178	100	327	0	237	0	0	111	0
N.S.	1	1.05	0.59	1.93	0.00	1.40	0.00	0.00	0.66	0.00
time (sec)	N/A	0.979	0.788	2.436	0.000	0.092	0.000	0.000	0.264	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	87	292	0	223	0	0	109	0
N.S.	1	1.05	0.69	2.32	0.00	1.77	0.00	0.00	0.87	0.00
time (sec)	N/A	0.634	0.601	2.450	0.000	0.085	0.000	0.000	0.289	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	124	80	292	0	223	0	0	101	0
N.S.	1	0.98	0.63	2.32	0.00	1.77	0.00	0.00	0.80	0.00
time (sec)	N/A	0.437	0.524	2.287	0.000	0.088	0.000	0.000	0.182	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	176	126	430	84333	339	0	0	113	0
N.S.	1	1.07	0.77	2.62	514.23	2.07	0.00	0.00	0.69	0.00
time (sec)	N/A	1.019	1.678	4.878	14.558	0.108	0.000	0.000	0.180	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	223	142	925	0	404	0	0	117	0
N.S.	1	1.08	0.69	4.47	0.00	1.95	0.00	0.00	0.57	0.00
time (sec)	N/A	1.457	3.601	3.453	0.000	0.142	0.000	0.000	0.207	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	264	282	870	1438	0	428	0	0	117	0
N.S.	1	1.07	3.30	5.45	0.00	1.62	0.00	0.00	0.44	0.00
time (sec)	N/A	1.866	6.212	3.579	0.000	0.139	0.000	0.000	0.184	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	158	914	411	0	193	0	0	83	177
N.S.	1	0.99	5.75	2.58	0.00	1.21	0.00	0.00	0.52	1.11
time (sec)	N/A	0.800	7.763	19.490	0.000	0.098	0.000	0.000	0.191	42.123

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	130	872	383	0	179	0	0	81	166
N.S.	1	0.98	6.61	2.90	0.00	1.36	0.00	0.00	0.61	1.26
time (sec)	N/A	0.658	7.383	12.428	0.000	0.094	0.000	0.000	0.185	41.438

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	103	830	355	0	161	0	0	70	128
N.S.	1	1.02	8.22	3.51	0.00	1.59	0.00	0.00	0.69	1.27
time (sec)	N/A	0.625	7.330	9.663	0.000	0.096	0.000	0.000	0.217	0.430

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	73	309	321	0	142	0	0	63	79
N.S.	1	1.04	4.41	4.59	0.00	2.03	0.00	0.00	0.90	1.13
time (sec)	N/A	0.534	7.120	5.376	0.000	0.088	0.000	0.000	0.178	41.408

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	783	242	0	173	0	0	74	90
N.S.	1	1.00	11.86	3.67	0.00	2.62	0.00	0.00	1.12	1.36
time (sec)	N/A	0.534	6.935	4.717	0.000	0.088	0.000	0.000	0.171	42.184

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	97	813	399	0	196	0	0	83	150
N.S.	1	1.02	8.56	4.20	0.00	2.06	0.00	0.00	0.87	1.58
time (sec)	N/A	0.634	7.045	5.168	0.000	0.091	0.000	0.000	0.184	42.508

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	126	865	634	0	219	0	0	83	177
N.S.	1	0.95	6.55	4.80	0.00	1.66	0.00	0.00	0.63	1.34
time (sec)	N/A	0.675	7.315	6.869	0.000	0.096	0.000	0.000	0.174	42.810

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	190	944	413	0	223	0	0	125	266
N.S.	1	0.98	4.87	2.13	0.00	1.15	0.00	0.00	0.64	1.37
time (sec)	N/A	0.998	6.871	16.169	0.000	0.099	0.000	0.000	0.195	41.987

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	162	898	385	0	203	0	0	114	231
N.S.	1	1.01	5.58	2.39	0.00	1.26	0.00	0.00	0.71	1.43
time (sec)	N/A	0.960	6.790	13.809	0.000	0.092	0.000	0.000	0.193	43.378

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	852	357	0	179	0	0	105	153
N.S.	1	1.05	6.76	2.83	0.00	1.42	0.00	0.00	0.83	1.21
time (sec)	N/A	0.840	7.807	9.533	0.000	0.090	0.000	0.000	0.182	44.306

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	121	623	244	0	198	0	0	107	134
N.S.	1	1.03	5.28	2.07	0.00	1.68	0.00	0.00	0.91	1.14
time (sec)	N/A	0.831	7.743	5.757	0.000	0.091	0.000	0.000	0.193	44.837

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	124	311	513	0	210	0	0	118	196
N.S.	1	1.03	2.59	4.28	0.00	1.75	0.00	0.00	0.98	1.63
time (sec)	N/A	0.847	7.526	6.096	0.000	0.092	0.000	0.000	0.176	45.601

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	156	883	714	0	239	0	0	127	229
N.S.	1	0.98	5.55	4.49	0.00	1.50	0.00	0.00	0.80	1.44
time (sec)	N/A	0.968	7.894	7.115	0.000	0.097	0.000	0.000	0.189	42.473

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	186	925	824	0	263	0	0	127	235
N.S.	1	0.96	4.77	4.25	0.00	1.36	0.00	0.00	0.65	1.21
time (sec)	N/A	1.020	8.188	9.317	0.000	0.097	0.000	0.000	0.182	37.591

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	238	990	441	0	243	0	0	167	360
N.S.	1	1.00	4.18	1.86	0.00	1.03	0.00	0.00	0.70	1.52
time (sec)	N/A	1.373	7.049	26.997	0.000	0.107	0.000	0.000	0.218	42.816

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	210	944	413	0	223	0	0	156	323
N.S.	1	1.03	4.63	2.02	0.00	1.09	0.00	0.00	0.76	1.58
time (sec)	N/A	1.315	6.966	19.685	0.000	0.098	0.000	0.000	0.201	41.638

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	182	898	385	0	203	0	0	147	255
N.S.	1	1.06	5.25	2.25	0.00	1.19	0.00	0.00	0.86	1.49
time (sec)	N/A	1.165	7.985	13.343	0.000	0.097	0.000	0.000	0.206	41.144

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	175	888	337	0	229	0	0	147	229
N.S.	1	1.04	5.25	1.99	0.00	1.36	0.00	0.00	0.87	1.36
time (sec)	N/A	1.160	8.137	9.987	0.000	0.098	0.000	0.000	0.209	41.061

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	170	879	654	0	223	0	0	149	251
N.S.	1	1.06	5.46	4.06	0.00	1.39	0.00	0.00	0.93	1.56
time (sec)	N/A	1.158	8.472	7.924	0.000	0.096	0.000	0.000	0.240	42.290

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	175	890	916	0	243	0	0	160	287
N.S.	1	1.02	5.20	5.36	0.00	1.42	0.00	0.00	0.94	1.68
time (sec)	N/A	1.181	9.216	9.066	0.000	0.102	0.000	0.000	0.277	43.602

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	206	925	902	0	263	0	0	169	307
N.S.	1	1.01	4.53	4.42	0.00	1.29	0.00	0.00	0.83	1.50
time (sec)	N/A	1.320	10.015	10.368	0.000	0.099	0.000	0.000	0.278	42.805

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	234	967	1151	0	283	0	0	169	552
N.S.	1	0.99	4.08	4.86	0.00	1.19	0.00	0.00	0.71	2.33
time (sec)	N/A	1.368	11.309	12.634	0.000	0.101	0.000	0.000	0.188	43.778

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	148	946	281	0	269	0	0	65	0
N.S.	1	0.95	6.06	1.80	0.00	1.72	0.00	0.00	0.42	0.00
time (sec)	N/A	0.649	8.151	7.513	0.000	0.099	0.000	0.000	0.169	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	120	893	262	0	250	0	0	63	0
N.S.	1	0.98	7.26	2.13	0.00	2.03	0.00	0.00	0.51	0.00
time (sec)	N/A	0.628	7.693	5.495	0.000	0.096	0.000	0.000	0.190	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	862	244	0	237	0	0	55	0
N.S.	1	1.06	10.14	2.87	0.00	2.79	0.00	0.00	0.65	0.00
time (sec)	N/A	0.498	7.450	4.461	0.000	0.091	0.000	0.000	0.164	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	89	858	243	0	241	0	0	56	0
N.S.	1	1.07	10.34	2.93	0.00	2.90	0.00	0.00	0.67	0.00
time (sec)	N/A	0.501	8.038	2.903	0.000	0.089	0.000	0.000	0.172	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	116	894	319	0	290	0	0	65	0
N.S.	1	0.97	7.51	2.68	0.00	2.44	0.00	0.00	0.55	0.00
time (sec)	N/A	0.630	8.195	3.444	0.000	0.095	0.000	0.000	0.173	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	144	931	466	0	320	0	0	67	0
N.S.	1	0.94	6.08	3.05	0.00	2.09	0.00	0.00	0.44	0.00
time (sec)	N/A	0.653	8.714	4.559	0.000	0.099	0.000	0.000	0.207	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	200	1024	465	0	383	0	0	85	0
N.S.	1	0.99	5.04	2.29	0.00	1.89	0.00	0.00	0.42	0.00
time (sec)	N/A	0.989	8.935	10.410	0.000	0.108	0.000	0.000	0.188	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	171	980	435	0	367	0	0	85	0
N.S.	1	1.03	5.90	2.62	0.00	2.21	0.00	0.00	0.51	0.00
time (sec)	N/A	0.930	8.445	8.924	0.000	0.109	0.000	0.000	0.171	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	945	421	0	352	0	0	83	0
N.S.	1	1.04	6.95	3.10	0.00	2.59	0.00	0.00	0.61	0.00
time (sec)	N/A	0.789	8.179	5.543	0.000	0.098	0.000	0.000	0.180	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	130	694	350	0	314	0	0	75	0
N.S.	1	1.07	5.74	2.89	0.00	2.60	0.00	0.00	0.62	0.00
time (sec)	N/A	0.750	7.780	4.803	0.000	0.094	0.000	0.000	0.170	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	130	695	350	0	318	0	0	76	0
N.S.	1	1.07	5.74	2.89	0.00	2.63	0.00	0.00	0.63	0.00
time (sec)	N/A	0.767	8.048	4.224	0.000	0.094	0.000	0.000	0.171	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	979	494	0	407	0	0	85	0
N.S.	1	1.00	5.83	2.94	0.00	2.42	0.00	0.00	0.51	0.00
time (sec)	N/A	0.927	8.226	4.282	0.000	0.098	0.000	0.000	0.187	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	197	1020	723	0	436	0	0	87	0
N.S.	1	0.98	5.07	3.60	0.00	2.17	0.00	0.00	0.43	0.00
time (sec)	N/A	0.982	8.852	5.442	0.000	0.107	0.000	0.000	0.187	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	259	482	493	0	495	0	0	105	0
N.S.	1	1.02	1.90	1.94	0.00	1.95	0.00	0.00	0.41	0.00
time (sec)	N/A	1.370	8.972	18.813	0.000	0.121	0.000	0.000	0.186	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	229	441	465	0	478	0	0	105	0
N.S.	1	1.05	2.01	2.12	0.00	2.18	0.00	0.00	0.48	0.00
time (sec)	N/A	1.271	7.134	17.767	0.000	0.112	0.000	0.000	0.179	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	203	400	451	0	467	0	0	105	0
N.S.	1	1.08	2.13	2.40	0.00	2.48	0.00	0.00	0.56	0.00
time (sec)	N/A	1.137	5.327	16.821	0.000	0.107	0.000	0.000	0.180	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	195	845	451	0	465	0	0	103	0
N.S.	1	1.08	4.69	2.51	0.00	2.58	0.00	0.00	0.57	0.00
time (sec)	N/A	1.105	6.783	6.316	0.000	0.114	0.000	0.000	0.210	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	193	664	451	0	465	0	0	95	0
N.S.	1	1.08	3.73	2.53	0.00	2.61	0.00	0.00	0.53	0.00
time (sec)	N/A	1.073	6.657	6.197	0.000	0.103	0.000	0.000	0.179	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	197	1029	451	0	465	0	0	96	0
N.S.	1	1.08	5.65	2.48	0.00	2.55	0.00	0.00	0.53	0.00
time (sec)	N/A	1.118	8.352	5.689	0.000	0.109	0.000	0.000	0.272	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	221	227	1069	685	0	521	0	0	105	0
N.S.	1	1.03	4.84	3.10	0.00	2.36	0.00	0.00	0.48	0.00
time (sec)	N/A	1.298	8.551	5.588	0.000	0.116	0.000	0.000	0.254	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	254	255	1110	876	0	548	0	0	107	0
N.S.	1	1.00	4.37	3.45	0.00	2.16	0.00	0.00	0.42	0.00
time (sec)	N/A	1.330	9.313	6.777	0.000	0.111	0.000	0.000	0.274	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	209	135	554	8220	170	0	0	62	0
N.S.	1	0.95	0.61	2.51	37.19	0.77	0.00	0.00	0.28	0.00
time (sec)	N/A	0.939	1.144	18.456	0.606	0.162	0.000	0.000	0.210	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	166	118	502	2981	153	0	0	60	0
N.S.	1	0.94	0.67	2.85	16.94	0.87	0.00	0.00	0.34	0.00
time (sec)	N/A	0.749	0.621	14.203	0.463	0.121	0.000	0.000	0.215	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	123	100	450	1851	135	0	0	52	0
N.S.	1	0.94	0.76	3.44	14.13	1.03	0.00	0.00	0.40	0.00
time (sec)	N/A	0.565	0.353	13.158	0.325	0.136	0.000	0.000	0.190	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	350	939	115	0	0	54	0
N.S.	1	1.00	1.06	4.49	12.04	1.47	0.00	0.00	0.69	0.00
time (sec)	N/A	0.426	0.188	12.500	0.293	0.118	0.000	0.000	0.181	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	86	189	245	127	0	0	62	0
N.S.	1	1.00	1.13	2.49	3.22	1.67	0.00	0.00	0.82	0.00
time (sec)	N/A	0.411	0.197	12.201	0.218	0.095	0.000	0.000	0.180	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	128	289	67	0	0	62	112
N.S.	1	1.00	0.67	1.51	3.40	0.79	0.00	0.00	0.73	1.32
time (sec)	N/A	0.424	0.172	12.994	0.177	0.085	0.000	0.000	0.174	43.423

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	128	78	156	428	86	0	0	62	194
N.S.	1	0.98	0.60	1.20	3.29	0.66	0.00	0.00	0.48	1.49
time (sec)	N/A	0.574	0.287	14.191	0.181	0.085	0.000	0.000	0.198	46.543

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	171	102	171	522	104	0	0	62	479
N.S.	1	0.98	0.58	0.98	2.98	0.59	0.00	0.00	0.35	2.74
time (sec)	N/A	0.740	0.455	14.443	0.173	0.084	0.000	0.000	0.177	51.754

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	219	136	227	8904	181	0	0	119	0
N.S.	1	0.96	0.60	1.00	39.22	0.80	0.00	0.00	0.52	0.00
time (sec)	N/A	1.071	1.300	16.382	0.702	0.165	0.000	0.000	0.253	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	176	119	207	3023	163	0	0	109	0
N.S.	1	0.98	0.66	1.15	16.79	0.91	0.00	0.00	0.61	0.00
time (sec)	N/A	0.864	0.752	17.000	0.442	0.132	0.000	0.000	0.230	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	132	101	184	1884	143	0	0	103	0
N.S.	1	0.99	0.76	1.38	14.17	1.08	0.00	0.00	0.77	0.00
time (sec)	N/A	0.722	0.415	21.283	0.337	0.127	0.000	0.000	0.203	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	172	1801	153	0	0	113	0
N.S.	1	1.00	0.85	1.37	14.29	1.21	0.00	0.00	0.90	0.00
time (sec)	N/A	0.708	0.374	21.311	0.331	0.134	0.000	0.000	0.180	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	128	106	137	1124	152	0	0	121	0
N.S.	1	1.02	0.85	1.10	8.99	1.22	0.00	0.00	0.97	0.00
time (sec)	N/A	0.685	0.433	13.581	0.250	0.103	0.000	0.000	0.191	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	139	80	88	344	88	0	0	121	195
N.S.	1	1.04	0.60	0.66	2.57	0.66	0.00	0.00	0.90	1.46
time (sec)	N/A	0.704	0.353	12.985	0.176	0.085	0.000	0.000	0.204	44.710

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	110	481	107	0	0	121	236
N.S.	1	1.00	0.56	0.61	2.66	0.59	0.00	0.00	0.67	1.30
time (sec)	N/A	0.892	0.538	13.253	0.208	0.093	0.000	0.000	0.202	52.836

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	224	124	130	573	126	0	0	121	289
N.S.	1	0.98	0.54	0.57	2.51	0.55	0.00	0.00	0.53	1.27
time (sec)	N/A	1.102	0.711	13.198	0.171	0.095	0.000	0.000	0.252	55.011

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	273	159	249	10042	213	0	0	181	0
N.S.	1	1.00	0.58	0.91	36.65	0.78	0.00	0.00	0.66	0.00
time (sec)	N/A	1.442	2.261	17.788	0.797	0.169	0.000	0.000	0.437	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	230	137	229	9415	193	0	0	171	0
N.S.	1	1.01	0.60	1.01	41.48	0.85	0.00	0.00	0.75	0.00
time (sec)	N/A	1.204	1.336	17.953	0.753	0.161	0.000	0.000	0.327	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	185	121	209	3071	173	0	0	163	0
N.S.	1	1.03	0.67	1.16	17.06	0.96	0.00	0.00	0.91	0.00
time (sec)	N/A	1.007	0.888	21.681	0.477	0.129	0.000	0.000	0.246	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	126	200	2080	182	0	0	165	0
N.S.	1	1.00	0.71	1.12	11.69	1.02	0.00	0.00	0.93	0.00
time (sec)	N/A	1.026	0.752	21.309	0.379	0.132	0.000	0.000	0.220	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	177	130	220	2370	188	0	0	175	0
N.S.	1	1.02	0.75	1.27	13.70	1.09	0.00	0.00	1.01	0.00
time (sec)	N/A	1.012	0.799	21.322	0.353	0.131	0.000	0.000	0.206	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	180	130	161	1548	180	0	0	183	0
N.S.	1	1.05	0.76	0.94	9.00	1.05	0.00	0.00	1.06	0.00
time (sec)	N/A	0.989	0.862	13.830	0.291	0.104	0.000	0.000	0.211	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	191	104	106	396	114	0	0	183	551
N.S.	1	1.06	0.57	0.59	2.19	0.63	0.00	0.00	1.01	3.04
time (sec)	N/A	1.053	0.656	12.952	0.157	0.092	0.000	0.000	0.217	52.103

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	235	126	126	533	135	0	0	183	647
N.S.	1	1.03	0.55	0.55	2.34	0.59	0.00	0.00	0.80	2.84
time (sec)	N/A	1.233	0.845	12.971	0.161	0.091	0.000	0.000	0.225	55.936

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	278	147	146	626	156	0	0	183	773
N.S.	1	1.01	0.53	0.53	2.28	0.57	0.00	0.00	0.67	2.81
time (sec)	N/A	1.510	0.959	12.668	0.180	0.093	0.000	0.000	0.197	51.007

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	203	228	239	0	220	0	0	83	0
N.S.	1	1.07	1.20	1.26	0.00	1.16	0.00	0.00	0.44	0.00
time (sec)	N/A	1.161	0.695	14.787	0.000	1.911	0.000	0.000	0.181	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	167	196	0	203	0	0	75	0
N.S.	1	1.06	1.18	1.39	0.00	1.44	0.00	0.00	0.53	0.00
time (sec)	N/A	0.813	0.469	16.657	0.000	1.022	0.000	0.000	0.178	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	132	1221	130	0	0	76	0
N.S.	1	1.00	0.82	1.32	12.21	1.30	0.00	0.00	0.76	0.00
time (sec)	N/A	0.552	0.181	11.403	0.796	0.796	0.000	0.000	0.179	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	203	165	1188	143	0	0	85	0
N.S.	1	1.00	2.05	1.67	12.00	1.44	0.00	0.00	0.86	0.00
time (sec)	N/A	0.449	2.729	11.567	0.710	0.112	0.000	0.000	0.184	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	150	627	189	1485	163	0	0	87	0
N.S.	1	1.06	4.42	1.33	10.46	1.15	0.00	0.00	0.61	0.00
time (sec)	N/A	0.729	8.218	11.432	0.757	0.108	0.000	0.000	0.187	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	205	1759	217	1826	180	0	0	87	0
N.S.	1	1.10	9.41	1.16	9.76	0.96	0.00	0.00	0.47	0.00
time (sec)	N/A	1.028	10.614	11.665	0.725	0.112	0.000	0.000	0.187	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	205	255	286	0	276	0	0	103	0
N.S.	1	1.04	1.29	1.45	0.00	1.40	0.00	0.00	0.52	0.00
time (sec)	N/A	1.181	1.599	16.858	0.000	3.425	0.000	0.000	0.182	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	145	150	171	259	0	242	0	0	95	0
N.S.	1	1.03	1.18	1.79	0.00	1.67	0.00	0.00	0.66	0.00
time (sec)	N/A	0.832	1.363	8.954	0.000	2.639	0.000	0.000	0.174	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	184	177	0	164	0	0	96	0
N.S.	1	1.00	1.72	1.65	0.00	1.53	0.00	0.00	0.90	0.00
time (sec)	N/A	0.454	1.406	12.846	0.000	0.111	0.000	0.000	0.194	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	158	423	193	0	201	0	0	105	0
N.S.	1	1.01	2.71	1.24	0.00	1.29	0.00	0.00	0.67	0.00
time (sec)	N/A	0.747	4.410	12.615	0.000	0.111	0.000	0.000	0.179	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	213	1054	233	0	221	0	0	107	0
N.S.	1	1.05	5.19	1.15	0.00	1.09	0.00	0.00	0.53	0.00
time (sec)	N/A	1.078	6.936	12.896	0.000	0.119	0.000	0.000	0.255	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	262	311	354	0	341	0	0	125	0
N.S.	1	1.07	1.26	1.44	0.00	1.39	0.00	0.00	0.51	0.00
time (sec)	N/A	1.553	3.724	17.308	0.000	7.347	0.000	0.000	0.257	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	205	252	291	0	306	0	0	123	0
N.S.	1	1.06	1.30	1.50	0.00	1.58	0.00	0.00	0.63	0.00
time (sec)	N/A	1.153	2.057	8.600	0.000	5.514	0.000	0.000	0.273	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	160	491	236	0	215	0	0	115	0
N.S.	1	1.04	3.19	1.53	0.00	1.40	0.00	0.00	0.75	0.00
time (sec)	N/A	0.731	6.425	8.885	0.000	0.113	0.000	0.000	0.179	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	162	150	238	0	217	0	0	116	0
N.S.	1	1.04	0.96	1.53	0.00	1.39	0.00	0.00	0.74	0.00
time (sec)	N/A	0.739	2.928	12.794	0.000	0.113	0.000	0.000	0.207	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	213	728	255	0	248	0	0	125	0
N.S.	1	1.05	3.59	1.26	0.00	1.22	0.00	0.00	0.62	0.00
time (sec)	N/A	1.082	8.412	12.724	0.000	0.119	0.000	0.000	0.190	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	268	239	293	0	270	0	0	127	0
N.S.	1	1.07	0.96	1.17	0.00	1.08	0.00	0.00	0.51	0.00
time (sec)	N/A	1.442	9.375	12.727	0.000	0.118	0.000	0.000	0.191	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	317	378	422	0	407	0	0	145	0
N.S.	1	1.08	1.29	1.44	0.00	1.39	0.00	0.00	0.49	0.00
time (sec)	N/A	1.988	5.133	17.027	0.000	12.627	0.000	0.000	0.189	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	260	319	350	0	366	0	0	145	0
N.S.	1	1.08	1.32	1.45	0.00	1.52	0.00	0.00	0.60	0.00
time (sec)	N/A	1.540	3.567	8.667	0.000	7.209	0.000	0.000	0.206	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	215	613	288	0	266	0	0	143	0
N.S.	1	1.07	3.05	1.43	0.00	1.32	0.00	0.00	0.71	0.00
time (sec)	N/A	1.096	6.613	7.923	0.000	0.117	0.000	0.000	0.204	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	215	167	288	0	264	0	0	135	0
N.S.	1	1.07	0.83	1.43	0.00	1.31	0.00	0.00	0.67	0.00
time (sec)	N/A	1.097	3.160	8.328	0.000	0.120	0.000	0.000	0.190	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	217	168	288	0	266	0	0	136	0
N.S.	1	1.07	0.83	1.42	0.00	1.31	0.00	0.00	0.67	0.00
time (sec)	N/A	1.085	3.027	13.003	0.000	0.119	0.000	0.000	0.192	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	268	798	305	0	298	0	0	145	0
N.S.	1	1.07	3.19	1.22	0.00	1.19	0.00	0.00	0.58	0.00
time (sec)	N/A	1.435	9.686	11.830	0.000	0.119	0.000	0.000	0.189	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	323	262	343	0	319	0	0	147	0
N.S.	1	1.09	0.88	1.15	0.00	1.07	0.00	0.00	0.49	0.00
time (sec)	N/A	1.848	11.210	12.434	0.000	0.125	0.000	0.000	0.198	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	97	91	86	101	81	252	89	95	117
N.S.	1	0.92	0.87	0.82	0.96	0.77	2.40	0.85	0.90	1.11
time (sec)	N/A	0.575	0.316	9.017	0.185	0.080	0.196	0.175	0.164	41.567

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	75	65	79	60	168	68	63	84
N.S.	1	1.02	0.89	0.77	0.94	0.71	2.00	0.81	0.75	1.00
time (sec)	N/A	0.415	0.265	5.365	0.102	0.079	0.135	0.153	0.163	41.273

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	51	51	55	42	94	45	45	50
N.S.	1	1.00	0.98	0.98	1.06	0.81	1.81	0.87	0.87	0.96
time (sec)	N/A	0.214	0.174	2.454	0.215	0.080	0.094	0.173	0.170	41.059

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	48	47	54	0	79	54	100
N.S.	1	1.00	1.31	1.37	1.34	1.54	0.00	2.26	1.54	2.86
time (sec)	N/A	0.445	0.052	2.128	0.179	0.091	0.000	0.150	0.166	41.151

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	50	73	85	0	84	79	114
N.S.	1	1.00	1.23	1.43	2.09	2.43	0.00	2.40	2.26	3.26
time (sec)	N/A	0.437	0.016	3.543	0.081	0.091	0.000	0.191	0.172	41.255

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	64	75	75	95	96	0	151	279	104
N.S.	1	1.05	1.23	1.23	1.56	1.57	0.00	2.48	4.57	1.70
time (sec)	N/A	0.557	0.023	5.037	0.040	0.091	0.000	0.191	0.172	41.658

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	91	67	81	127	115	0	210	194	145
N.S.	1	0.98	0.72	0.87	1.37	1.24	0.00	2.26	2.09	1.56
time (sec)	N/A	0.655	0.334	6.838	0.049	0.090	0.000	0.204	0.170	42.430

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	106	85	119	163	136	0	304	482	194
N.S.	1	0.93	0.75	1.04	1.43	1.19	0.00	2.67	4.23	1.70
time (sec)	N/A	0.714	0.785	7.230	0.044	0.087	0.000	0.160	0.191	43.990

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	164	146	147	176	142	459	156	139	307
N.S.	1	0.87	0.77	0.78	0.93	0.75	2.43	0.83	0.74	1.62
time (sec)	N/A	0.815	1.923	31.687	0.041	0.096	0.312	0.167	0.164	38.081

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	177	118	118	142	114	338	124	112	169
N.S.	1	1.04	0.69	0.69	0.84	0.67	1.99	0.73	0.66	0.99
time (sec)	N/A	0.639	2.840	12.233	0.036	0.086	0.195	0.185	0.163	40.900

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	110	90	114	108	85	199	93	75	115
N.S.	1	1.03	0.84	1.07	1.01	0.79	1.86	0.87	0.70	1.07
time (sec)	N/A	0.357	1.044	1.680	0.043	0.079	0.159	0.200	0.173	40.951

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	120	94	92	87	0	178	82	169
N.S.	1	1.00	1.40	1.09	1.07	1.01	0.00	2.07	0.95	1.97
time (sec)	N/A	0.603	1.700	2.529	0.049	0.093	0.000	0.189	0.167	41.369

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	109	79	103	117	0	152	101	169
N.S.	1	1.00	1.82	1.32	1.72	1.95	0.00	2.53	1.68	2.82
time (sec)	N/A	0.573	2.144	4.465	0.036	0.093	0.000	0.174	0.178	42.430

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	87	104	140	136	0	190	321	176
N.S.	1	1.05	1.09	1.30	1.75	1.70	0.00	2.38	4.01	2.20
time (sec)	N/A	0.640	0.287	6.123	0.050	0.093	0.000	0.185	0.173	42.404

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	122	97	118	172	150	0	294	314	227
N.S.	1	1.05	0.84	1.02	1.48	1.29	0.00	2.53	2.71	1.96
time (sec)	N/A	0.810	0.758	7.829	0.046	0.091	0.000	0.168	0.264	45.012

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	149	120	148	228	180	0	478	533	314
N.S.	1	0.96	0.77	0.95	1.46	1.15	0.00	3.06	3.42	2.01
time (sec)	N/A	0.941	0.852	8.529	0.044	0.109	0.000	0.178	0.245	45.425

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	231	289	213	266	211	721	230	210	352
N.S.	1	0.86	1.07	0.79	0.99	0.78	2.68	0.86	0.78	1.31
time (sec)	N/A	1.226	2.164	147.715	0.044	0.093	0.449	0.197	0.276	42.936

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	252	176	176	217	174	551	188	156	277
N.S.	1	1.04	0.72	0.72	0.89	0.72	2.27	0.77	0.64	1.14
time (sec)	N/A	0.887	3.255	42.582	0.045	0.087	0.305	0.212	0.167	42.565

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	179	140	180	171	136	386	148	124	202
N.S.	1	1.05	0.82	1.05	1.00	0.80	2.26	0.87	0.73	1.18
time (sec)	N/A	0.567	1.600	1.577	0.037	0.084	0.210	0.222	0.163	42.202

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	146	159	135	145	131	0	314	112	1924
N.S.	1	1.07	1.16	0.99	1.06	0.96	0.00	2.29	0.82	14.04
time (sec)	N/A	0.963	2.194	4.769	0.044	0.097	0.000	0.189	0.199	42.979

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	134	217	124	144	152	0	234	137	236
N.S.	1	1.02	1.66	0.95	1.10	1.16	0.00	1.79	1.05	1.80
time (sec)	N/A	0.942	2.632	5.933	0.054	0.104	0.000	0.173	0.168	42.035

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	126	277	133	169	167	0	239	367	249
N.S.	1	1.02	2.23	1.07	1.36	1.35	0.00	1.93	2.96	2.01
time (sec)	N/A	0.982	4.919	7.540	0.048	0.101	0.000	0.166	0.179	42.011

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	154	120	146	216	189	0	336	356	526
N.S.	1	1.06	0.83	1.01	1.49	1.30	0.00	2.32	2.46	3.63
time (sec)	N/A	1.036	0.910	9.013	0.042	0.107	0.000	0.201	0.179	36.734

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	199	144	185	273	211	0	586	722	395
N.S.	1	1.06	0.77	0.98	1.45	1.12	0.00	3.12	3.84	2.10
time (sec)	N/A	1.220	1.311	9.249	0.050	0.098	0.000	0.170	0.201	45.662

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	225	181	200	341	249	0	722	584	470
N.S.	1	0.95	0.77	0.85	1.44	1.06	0.00	3.06	2.47	1.99
time (sec)	N/A	1.392	3.734	12.789	0.053	0.103	0.000	0.190	0.187	45.589

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	315	408	368	366	289	1017	313	270	436
N.S.	1	0.86	1.11	1.01	1.00	0.79	2.78	0.86	0.74	1.19
time (sec)	N/A	1.803	2.528	5.770	0.043	0.100	0.617	0.166	0.184	45.595

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	336	333	242	307	243	811	263	227	403
N.S.	1	1.03	1.02	0.74	0.94	0.75	2.50	0.81	0.70	1.24
time (sec)	N/A	1.192	4.210	212.169	0.042	0.102	0.459	0.198	0.175	42.896

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	254	263	258	246	197	580	212	168	307
N.S.	1	1.05	1.09	1.07	1.02	0.82	2.41	0.88	0.70	1.27
time (sec)	N/A	0.818	1.875	3.617	0.050	0.092	0.323	0.178	0.169	42.403

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	214	210	191	208	183	0	603	161	369
N.S.	1	1.07	1.05	0.96	1.04	0.92	0.00	3.02	0.80	1.84
time (sec)	N/A	1.389	2.764	15.218	0.038	0.095	0.000	0.208	0.175	43.198

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	204	257	164	197	196	0	371	179	2522
N.S.	1	1.05	1.32	0.84	1.01	1.01	0.00	1.90	0.92	12.93
time (sec)	N/A	1.398	3.783	16.238	0.048	0.103	0.000	0.198	0.183	44.086

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	201	310	178	209	202	0	526	445	330
N.S.	1	0.96	1.48	0.85	1.00	0.97	0.00	2.52	2.13	1.58
time (sec)	N/A	1.450	5.296	17.939	0.052	0.105	0.000	0.188	0.174	43.798

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	205	831	175	245	219	0	387	402	636
N.S.	1	1.04	4.20	0.88	1.24	1.11	0.00	1.95	2.03	3.21
time (sec)	N/A	1.474	10.288	19.662	0.043	0.103	0.000	0.252	0.181	44.895

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	230	186	213	317	250	0	635	789	1969
N.S.	1	1.06	0.86	0.99	1.47	1.16	0.00	2.94	3.65	9.12
time (sec)	N/A	1.512	1.582	19.530	0.047	0.104	0.000	0.210	0.179	44.199

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	283	254	237	386	281	0	850	773	555
N.S.	1	1.06	0.95	0.89	1.45	1.05	0.00	3.18	2.90	2.08
time (sec)	N/A	1.753	6.184	21.598	0.052	0.104	0.000	0.204	0.178	45.211

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	302	244	277	474	327	0	1186	1117	706
N.S.	1	0.93	0.75	0.85	1.46	1.01	0.00	3.66	3.45	2.18
time (sec)	N/A	2.011	3.534	23.469	0.046	0.108	0.000	0.245	0.188	44.832

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	195	152	240	0	541	0	360	23	4568
N.S.	1	1.10	0.85	1.35	0.00	3.04	0.00	2.02	0.13	25.66
time (sec)	N/A	1.072	1.689	1.755	0.000	0.120	0.000	0.180	0.159	45.844

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	146	121	169	0	426	10409	227	22	3761
N.S.	1	1.09	0.90	1.26	0.00	3.18	77.68	1.69	0.16	28.07
time (sec)	N/A	0.696	1.195	1.490	0.000	0.111	171.834	0.150	0.168	45.349

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	86	85	110	0	322	3225	142	10	541
N.S.	1	0.97	0.96	1.24	0.00	3.62	36.24	1.60	0.11	6.08
time (sec)	N/A	0.499	0.873	1.383	0.000	0.107	56.425	0.136	0.161	42.302

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	242	524	296	1	344
N.S.	1	1.00	1.01	1.09	0.00	3.61	7.82	4.42	0.01	5.13
time (sec)	N/A	0.299	0.251	1.228	0.000	0.105	11.326	0.202	0.161	42.833

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	304	0	127	31	342
N.S.	1	1.00	1.47	1.21	0.00	4.00	0.00	1.67	0.41	4.50
time (sec)	N/A	0.390	0.634	1.717	0.000	0.418	0.000	0.178	0.172	42.943

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	96	129	144	0	460	0	175	18	675
N.S.	1	0.97	1.30	1.45	0.00	4.65	0.00	1.77	0.18	6.82
time (sec)	N/A	0.521	1.364	1.922	0.000	0.183	0.000	0.243	0.166	43.584

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	154	300	229	0	589	0	269	95	4051
N.S.	1	1.08	2.10	1.60	0.00	4.12	0.00	1.88	0.66	28.33
time (sec)	N/A	0.979	2.999	2.040	0.000	2.267	0.000	0.201	0.164	45.913

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	204	422	335	0	729	0	412	43	4696
N.S.	1	1.09	2.26	1.79	0.00	3.90	0.00	2.20	0.23	25.11
time (sec)	N/A	1.395	3.696	2.188	0.000	0.506	0.000	0.209	0.169	47.153

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	277	184	267	0	964	0	338	214	6744
N.S.	1	1.05	0.70	1.02	0.00	3.67	0.00	1.29	0.81	25.64
time (sec)	N/A	1.318	2.734	1.977	0.000	0.151	0.000	0.172	0.171	38.012

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	185	147	205	0	789	0	1116	108	3276
N.S.	1	1.19	0.95	1.32	0.00	5.09	0.00	7.20	0.70	21.14
time (sec)	N/A	0.868	1.993	1.707	0.000	0.134	0.000	0.252	0.169	28.141

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	149	119	161	0	551	0	199	83	3775
N.S.	1	1.22	0.98	1.32	0.00	4.52	0.00	1.63	0.68	30.94
time (sec)	N/A	0.644	1.360	1.394	0.000	0.117	0.000	0.146	0.164	30.072

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	111	97	128	0	379	4974	159	64	113
N.S.	1	1.11	0.97	1.28	0.00	3.79	49.74	1.59	0.64	1.13
time (sec)	N/A	0.361	0.658	1.297	0.000	0.105	165.708	0.151	0.158	25.342

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	158	191	182	0	684	0	223	136	3763
N.S.	1	1.19	1.44	1.37	0.00	5.14	0.00	1.68	1.02	28.29
time (sec)	N/A	0.735	1.424	2.052	0.000	2.458	0.000	0.202	0.177	30.394

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	208	240	241	0	1088	0	404	200	5464
N.S.	1	1.10	1.27	1.28	0.00	5.76	0.00	2.14	1.06	28.91
time (sec)	N/A	1.212	3.049	2.359	0.000	6.320	0.000	0.198	0.169	30.655

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	284	438	326	0	1329	0	378	479	6692
N.S.	1	1.05	1.62	1.21	0.00	4.92	0.00	1.40	1.77	24.79
time (sec)	N/A	1.732	7.568	2.651	0.000	11.108	0.000	0.219	0.168	31.242

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	424	734	402	0	1812	0	2712	623	10598
N.S.	1	1.07	1.84	1.01	0.00	4.55	0.00	6.81	1.57	26.63
time (sec)	N/A	2.147	6.613	2.716	0.000	0.230	0.000	0.454	0.182	34.682

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	317	232	341	0	1561	0	543	521	5542
N.S.	1	1.13	0.83	1.22	0.00	5.58	0.00	1.94	1.86	19.79
time (sec)	N/A	1.444	4.055	2.224	0.000	0.192	0.000	0.195	0.160	30.560

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	247	204	282	0	1152	0	455	380	6923
N.S.	1	1.17	0.97	1.34	0.00	5.46	0.00	2.16	1.80	32.81
time (sec)	N/A	0.982	2.780	1.715	0.000	0.163	0.000	0.204	0.163	31.617

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	206	172	234	0	740	0	391	192	248
N.S.	1	1.14	0.96	1.30	0.00	4.11	0.00	2.17	1.07	1.38
time (sec)	N/A	0.706	1.767	1.534	0.000	0.127	0.000	0.228	0.164	26.825

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	190	157	232	0	742	0	390	192	248
N.S.	1	1.16	0.96	1.41	0.00	4.52	0.00	2.38	1.17	1.51
time (sec)	N/A	0.583	1.100	1.420	0.000	0.125	0.000	0.177	0.166	26.745

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	258	269	302	0	1400	0	481	568	6913
N.S.	1	1.21	1.26	1.41	0.00	6.54	0.00	2.25	2.65	32.30
time (sec)	N/A	1.197	2.371	2.470	0.000	11.388	0.000	0.249	0.175	31.365

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	335	418	376	0	2100	0	574	977	9312
N.S.	1	1.12	1.40	1.26	0.00	7.02	0.00	1.92	3.27	31.14
time (sec)	N/A	2.018	7.767	3.093	0.000	24.389	0.000	0.244	0.188	34.517

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	426	507	460	0	2416	0	1395	1637	10547
N.S.	1	1.06	1.26	1.14	0.00	6.01	0.00	3.47	4.07	26.24
time (sec)	N/A	2.774	5.136	3.547	0.000	37.318	0.000	0.227	0.183	33.350

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	466	1278	552	0	2567	0	966	1271	7823
N.S.	1	1.14	3.12	1.35	0.00	6.28	0.00	2.36	3.11	19.13
time (sec)	N/A	2.266	9.358	3.049	0.000	0.301	0.000	0.273	0.177	37.540

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	356	717	459	0	1857	0	813	1004	9733
N.S.	1	1.18	2.38	1.52	0.00	6.17	0.00	2.70	3.34	32.34
time (sec)	N/A	1.499	6.405	2.141	0.000	0.229	0.000	0.223	0.268	32.601

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	319	251	371	0	1220	0	689	632	440
N.S.	1	1.16	0.92	1.35	0.00	4.45	0.00	2.51	2.31	1.61
time (sec)	N/A	1.079	2.832	1.928	0.000	0.162	0.000	0.233	0.252	26.476

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	302	252	384	0	1232	0	722	467	451
N.S.	1	1.15	0.96	1.46	0.00	4.68	0.00	2.75	1.78	1.71
time (sec)	N/A	1.055	2.208	1.774	0.000	0.166	0.000	0.197	0.254	27.049

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	279	227	372	0	1228	0	691	627	440
N.S.	1	1.18	0.96	1.57	0.00	5.18	0.00	2.92	2.65	1.86
time (sec)	N/A	0.868	2.935	1.701	0.000	0.161	0.000	0.213	0.164	26.878

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	365	368	479	0	2269	0	837	1406	9727
N.S.	1	1.21	1.22	1.59	0.00	7.54	0.00	2.78	4.67	32.32
time (sec)	N/A	1.839	2.972	3.128	0.000	32.689	0.000	0.283	0.177	32.360

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	476	549	587	0	3393	0	996	2086	13119
N.S.	1	1.13	1.31	1.40	0.00	8.08	0.00	2.37	4.97	31.24
time (sec)	N/A	3.112	4.936	3.994	0.000	66.779	0.000	0.320	0.191	41.461

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	591	781	672	0	3819	0	1090	3080	14398
N.S.	1	1.08	1.43	1.23	0.00	6.98	0.00	1.99	5.63	26.32
time (sec)	N/A	4.050	5.602	5.954	0.000	94.704	0.000	0.294	0.196	34.422

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	27	28	23	0	25	56	25	24	24
N.S.	1	0.96	1.00	0.82	0.00	0.89	2.00	0.89	0.86	0.86
time (sec)	N/A	0.215	0.012	2.358	0.000	0.075	0.416	0.172	0.169	24.379

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	21	0	24	68	33	23	50
N.S.	1	1.00	0.89	0.78	0.00	0.89	2.52	1.22	0.85	1.85
time (sec)	N/A	0.213	0.014	1.848	0.000	0.076	0.356	0.214	0.188	24.449

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	0	11	31	11	11	11
N.S.	1	1.00	2.09	1.09	0.00	1.00	2.82	1.00	1.00	1.00
time (sec)	N/A	0.185	0.004	1.487	0.000	0.076	0.320	0.178	0.171	24.214

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	3	2	10	3	3
N.S.	1	1.00	1.00	1.33	0.00	1.00	0.67	3.33	1.00	1.00
time (sec)	N/A	0.140	0.000	0.737	0.000	0.060	0.062	0.180	0.167	24.537

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	0	31	39	47	32	16
N.S.	1	1.00	1.00	1.67	0.00	2.58	3.25	3.92	2.67	1.33
time (sec)	N/A	0.191	0.005	1.928	0.000	0.085	1.802	0.148	0.164	24.712

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	19	32	11	19	30
N.S.	1	1.00	1.00	1.09	0.00	1.73	2.91	1.00	1.73	2.73
time (sec)	N/A	0.205	0.006	4.000	0.000	0.072	1.452	0.149	0.165	24.805

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	0	64	0	52	96	73
N.S.	1	1.00	1.00	1.03	0.00	1.78	0.00	1.44	2.67	2.03
time (sec)	N/A	0.267	0.011	4.599	0.000	0.083	0.000	0.191	0.168	24.566

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	27	24	25	0	32	42	25	44	39
N.S.	1	0.96	0.86	0.89	0.00	1.14	1.50	0.89	1.57	1.39
time (sec)	N/A	0.221	0.038	4.291	0.000	0.070	8.196	0.152	0.164	24.661

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	122	98	139	0	350	0	185	214	173
N.S.	1	1.07	0.86	1.22	0.00	3.07	0.00	1.62	1.88	1.52
time (sec)	N/A	0.611	0.641	2.837	0.000	0.110	0.000	0.179	0.168	25.535

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	81	73	98	0	281	0	128	108	193
N.S.	1	1.03	0.92	1.24	0.00	3.56	0.00	1.62	1.37	2.44
time (sec)	N/A	0.424	0.330	2.229	0.000	0.104	0.000	0.140	0.163	24.808

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	66	0	231	0	245	80	101
N.S.	1	1.00	0.97	1.08	0.00	3.79	0.00	4.02	1.31	1.66
time (sec)	N/A	0.304	0.152	2.050	0.000	0.095	0.000	0.167	0.170	25.392

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	45	0	177	190	78	65	44
N.S.	1	1.00	0.98	0.90	0.00	3.54	3.80	1.56	1.30	0.88
time (sec)	N/A	0.221	0.058	1.525	0.000	0.100	155.939	0.162	0.176	24.667

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	103	86	0	292	0	122	137	101
N.S.	1	1.00	1.47	1.23	0.00	4.17	0.00	1.74	1.96	1.44
time (sec)	N/A	0.364	0.296	2.634	0.000	0.126	0.000	0.201	0.168	24.736

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	90	116	125	0	398	0	155	201	326
N.S.	1	1.02	1.32	1.42	0.00	4.52	0.00	1.76	2.28	3.70
time (sec)	N/A	0.496	0.716	2.860	0.000	0.131	0.000	0.191	0.170	25.180

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	132	239	194	0	487	0	221	480	1099
N.S.	1	1.07	1.94	1.58	0.00	3.96	0.00	1.80	3.90	8.93
time (sec)	N/A	0.839	1.684	3.144	0.000	0.203	0.000	0.197	0.180	25.218

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	408	292	1635	0	639	0	0	49	0
N.S.	1	1.06	0.76	4.24	0.00	1.66	0.00	0.00	0.13	0.00
time (sec)	N/A	2.206	2.331	29.649	0.000	0.131	0.000	0.000	0.182	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	317	232	1305	0	561	0	0	49	0
N.S.	1	1.05	0.77	4.31	0.00	1.85	0.00	0.00	0.16	0.00
time (sec)	N/A	1.610	1.652	16.412	0.000	0.114	0.000	0.000	0.186	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	237	179	993	0	492	0	0	47	0
N.S.	1	1.03	0.77	4.30	0.00	2.13	0.00	0.00	0.20	0.00
time (sec)	N/A	1.235	3.555	15.572	0.000	0.106	0.000	0.000	0.174	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	172	146	600	0	437	0	0	38	0
N.S.	1	1.01	0.85	3.51	0.00	2.56	0.00	0.00	0.22	0.00
time (sec)	N/A	0.860	0.915	10.843	0.000	0.101	0.000	0.000	0.173	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	179	107	247	0	0	0	0	51	0
N.S.	1	1.01	0.60	1.39	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.257	12.802	7.994	0.000	0.000	0.000	0.000	0.200	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	227	372	746	0	0	0	0	55	0
N.S.	1	1.07	1.75	3.50	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.795	30.251	10.093	0.000	0.000	0.000	0.000	0.206	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	292	298	420	1290	0	0	0	0	55	0
N.S.	1	1.02	1.44	4.42	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.511	5.576	13.635	0.000	0.000	0.000	0.000	0.211	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	378	389	635	2213	0	0	0	0	55	0
N.S.	1	1.03	1.68	5.85	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	3.251	7.192	16.370	0.000	0.000	0.000	0.000	0.202	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	394	291	1635	0	639	0	0	79	0
N.S.	1	1.04	0.77	4.33	0.00	1.69	0.00	0.00	0.21	0.00
time (sec)	N/A	2.072	3.125	25.269	0.000	0.138	0.000	0.000	0.192	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	305	233	1305	0	562	0	0	77	0
N.S.	1	1.03	0.78	4.39	0.00	1.89	0.00	0.00	0.26	0.00
time (sec)	N/A	1.630	4.932	20.628	0.000	0.125	0.000	0.000	0.180	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	231	203	993	0	493	0	0	68	0
N.S.	1	1.03	0.90	4.41	0.00	2.19	0.00	0.00	0.30	0.00
time (sec)	N/A	1.159	1.557	16.269	0.000	0.113	0.000	0.000	0.184	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	244	406	701	0	0	0	0	87	0
N.S.	1	1.03	1.72	2.97	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	2.032	3.738	11.951	0.000	0.000	0.000	0.000	0.225	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	246	398	991	0	0	0	0	93	0
N.S.	1	1.06	1.72	4.27	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	2.020	3.389	12.234	0.000	0.000	0.000	0.000	0.238	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	295	309	422	1403	0	0	0	0	93	0
N.S.	1	1.05	1.43	4.76	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	2.620	6.107	14.722	0.000	0.000	0.000	0.000	0.241	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	375	390	634	2327	0	0	0	0	93	0
N.S.	1	1.04	1.69	6.21	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	3.374	7.194	18.171	0.000	0.000	0.000	0.000	0.233	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	480	357	1983	0	726	0	0	109	0
N.S.	1	1.04	0.77	4.29	0.00	1.57	0.00	0.00	0.24	0.00
time (sec)	N/A	2.566	3.807	39.416	0.000	0.150	0.000	0.000	0.199	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	382	291	1635	0	639	0	0	107	0
N.S.	1	1.03	0.78	4.40	0.00	1.72	0.00	0.00	0.29	0.00
time (sec)	N/A	2.021	5.974	27.936	0.000	0.136	0.000	0.000	0.196	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	299	254	1305	0	562	0	0	98	0
N.S.	1	1.04	0.88	4.53	0.00	1.95	0.00	0.00	0.34	0.00
time (sec)	N/A	1.518	2.353	20.299	0.000	0.117	0.000	0.000	0.254	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	305	453	1067	0	0	0	0	123	0
N.S.	1	1.04	1.55	3.65	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	2.551	4.375	18.586	0.000	0.000	0.000	0.000	0.367	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	310	442	1491	0	0	0	0	131	0
N.S.	1	1.05	1.49	5.04	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	2.645	4.910	25.993	0.000	0.000	0.000	0.000	0.362	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	315	323	589	1742	0	0	0	0	131	0
N.S.	1	1.03	1.87	5.53	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	2.672	7.180	70.807	0.000	0.000	0.000	0.000	0.280	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	376	395	639	2438	0	0	0	0	131	0
N.S.	1	1.05	1.70	6.48	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	3.377	7.351	192.190	0.000	0.000	0.000	0.000	0.280	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	465	485	729	3548	0	0	0	0	131	0
N.S.	1	1.04	1.57	7.63	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	4.230	7.514	537.626	0.000	0.000	0.000	0.000	0.289	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	340	230	1305	0	562	0	0	22	0
N.S.	1	1.06	0.72	4.08	0.00	1.76	0.00	0.00	0.07	0.00
time (sec)	N/A	1.767	2.100	18.134	0.000	0.122	0.000	0.000	0.193	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	258	180	993	0	493	0	0	22	0
N.S.	1	1.05	0.73	4.04	0.00	2.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.263	1.847	14.192	0.000	0.115	0.000	0.000	0.167	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	187	154	671	0	437	0	0	20	199
N.S.	1	1.02	0.84	3.67	0.00	2.39	0.00	0.00	0.11	1.09
time (sec)	N/A	0.973	2.369	9.661	0.000	0.099	0.000	0.000	0.170	24.507

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	93	249	0	371	0	0	13	135
N.S.	1	1.00	0.72	1.92	0.00	2.85	0.00	0.00	0.10	1.04
time (sec)	N/A	0.615	5.180	5.828	0.000	0.092	0.000	0.000	0.162	25.011

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0	20	0
N.S.	1	1.00	0.69	1.64	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.818	0.695	5.239	0.000	0.000	0.000	0.000	0.167	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	227	320	639	0	0	0	0	22	0
N.S.	1	1.05	1.48	2.96	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.780	14.744	7.003	0.000	0.000	0.000	0.000	0.171	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	305	556	1182	0	0	0	0	22	0
N.S.	1	1.02	1.86	3.95	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	2.482	7.234	7.886	0.000	0.000	0.000	0.000	0.165	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	391	304	1312	0	890	0	0	34	0
N.S.	1	1.01	0.79	3.39	0.00	2.30	0.00	0.00	0.09	0.00
time (sec)	N/A	2.035	3.164	16.437	0.000	0.160	0.000	0.000	0.164	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	280	189	1275	0	767	0	0	34	0
N.S.	1	1.07	0.72	4.87	0.00	2.93	0.00	0.00	0.13	0.00
time (sec)	N/A	1.440	2.558	15.993	0.000	0.135	0.000	0.000	0.169	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	219	170	519	0	663	0	0	32	0
N.S.	1	1.07	0.83	2.54	0.00	3.25	0.00	0.00	0.16	0.00
time (sec)	N/A	1.102	1.622	12.371	0.000	0.118	0.000	0.000	0.163	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	196	151	432	0	588	0	0	26	0
N.S.	1	1.06	0.82	2.34	0.00	3.18	0.00	0.00	0.14	0.00
time (sec)	N/A	0.900	0.842	6.699	0.000	0.110	0.000	0.000	0.164	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	198	460	433	0	0	0	0	32	0
N.S.	1	1.04	2.42	2.28	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.282	4.830	8.325	0.000	0.000	0.000	0.000	0.167	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	335	482	912	0	0	0	0	34	0
N.S.	1	1.11	1.59	3.01	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.744	6.888	13.002	0.000	0.000	0.000	0.000	0.163	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	425	678	1568	0	0	0	0	34	0
N.S.	1	1.07	1.70	3.94	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	3.533	7.416	16.988	0.000	0.000	0.000	0.000	0.173	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	550	555	372	1750	0	1504	0	0	50	0
N.S.	1	1.01	0.68	3.18	0.00	2.73	0.00	0.00	0.09	0.00
time (sec)	N/A	3.102	5.488	20.352	0.000	0.270	0.000	0.000	0.162	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	413	421	334	1412	0	1316	0	0	50	0
N.S.	1	1.02	0.81	3.42	0.00	3.19	0.00	0.00	0.12	0.00
time (sec)	N/A	2.245	4.296	20.773	0.000	0.206	0.000	0.000	0.165	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	331	338	274	954	0	1159	0	0	50	0
N.S.	1	1.02	0.83	2.88	0.00	3.50	0.00	0.00	0.15	0.00
time (sec)	N/A	1.633	3.491	17.210	0.000	0.167	0.000	0.000	0.172	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	307	317	224	864	0	1044	0	0	48	0
N.S.	1	1.03	0.73	2.81	0.00	3.40	0.00	0.00	0.16	0.00
time (sec)	N/A	1.524	3.004	13.330	0.000	0.133	0.000	0.000	0.166	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	275	285	193	754	0	924	0	0	42	0
N.S.	1	1.04	0.70	2.74	0.00	3.36	0.00	0.00	0.15	0.00
time (sec)	N/A	1.317	1.996	10.922	0.000	0.127	0.000	0.000	0.165	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	349	375	743	858	0	0	0	0	48	0
N.S.	1	1.07	2.13	2.46	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.933	7.145	12.940	0.000	0.000	0.000	0.000	0.169	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	437	471	750	1346	0	0	0	0	50	0
N.S.	1	1.08	1.72	3.08	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	3.778	7.833	17.547	0.000	0.000	0.000	0.000	0.164	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	532	569	820	2005	0	0	0	0	50	0
N.S.	1	1.07	1.54	3.77	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	4.659	7.967	21.602	0.000	0.000	0.000	0.000	0.175	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	58	0	149	0	0	28	0
N.S.	1	1.00	1.00	1.00	0.00	2.57	0.00	0.00	0.48	0.00
time (sec)	N/A	0.292	0.133	5.336	0.000	0.088	0.000	0.000	0.161	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	167	0	0	0	0	34	0
N.S.	1	1.00	1.00	2.83	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.403	0.269	6.496	0.000	0.000	0.000	0.000	0.165	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	84	218	0	481	0	0	44	0
N.S.	1	1.00	0.78	2.02	0.00	4.45	0.00	0.00	0.41	0.00
time (sec)	N/A	0.429	0.325	8.922	0.000	0.103	0.000	0.000	0.170	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	186	403	377	0	0	0	0	50	0
N.S.	1	1.04	2.25	2.11	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.148	6.249	11.981	0.000	0.000	0.000	0.000	0.169	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	162	125	451	0	211	0	0	67	177
N.S.	1	0.95	0.74	2.65	0.00	1.24	0.00	0.00	0.39	1.04
time (sec)	N/A	0.797	2.933	25.835	0.000	0.111	0.000	0.000	0.182	25.073

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	134	103	413	0	192	0	0	65	166
N.S.	1	0.96	0.74	2.95	0.00	1.37	0.00	0.00	0.46	1.19
time (sec)	N/A	0.654	2.335	15.144	0.000	0.100	0.000	0.000	0.182	25.117

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	107	86	371	0	175	0	0	56	128
N.S.	1	0.99	0.80	3.44	0.00	1.62	0.00	0.00	0.52	1.19
time (sec)	N/A	0.618	1.798	10.509	0.000	0.098	0.000	0.000	0.185	0.374

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	78	67	326	0	156	0	0	56	85
N.S.	1	1.04	0.89	4.35	0.00	2.08	0.00	0.00	0.75	1.13
time (sec)	N/A	0.529	1.148	6.997	0.000	0.096	0.000	0.000	0.179	24.632

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	246	0	185	0	0	58	96
N.S.	1	1.00	0.90	3.46	0.00	2.61	0.00	0.00	0.82	1.35
time (sec)	N/A	0.524	0.783	6.601	0.000	0.096	0.000	0.000	0.269	24.533

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	102	107	401	0	213	0	0	67	150
N.S.	1	0.99	1.04	3.89	0.00	2.07	0.00	0.00	0.65	1.46
time (sec)	N/A	0.637	0.945	6.783	0.000	0.095	0.000	0.000	0.263	25.624

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	130	134	636	0	235	0	0	67	177
N.S.	1	0.93	0.96	4.54	0.00	1.68	0.00	0.00	0.48	1.26
time (sec)	N/A	0.687	1.307	8.742	0.000	0.099	0.000	0.000	0.248	25.335

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	231	196	666	0	299	0	0	93	275
N.S.	1	0.88	0.74	2.52	0.00	1.13	0.00	0.00	0.35	1.04
time (sec)	N/A	1.107	3.347	57.683	0.000	0.119	0.000	0.000	0.183	24.769

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	203	167	610	0	271	0	0	91	264
N.S.	1	0.91	0.75	2.74	0.00	1.22	0.00	0.00	0.41	1.18
time (sec)	N/A	0.954	2.917	20.772	0.000	0.110	0.000	0.000	0.199	25.122

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	175	139	548	0	243	0	0	82	229
N.S.	1	0.96	0.76	3.01	0.00	1.34	0.00	0.00	0.45	1.26
time (sec)	N/A	0.905	2.490	11.407	0.000	0.098	0.000	0.000	0.194	25.278

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	146	106	487	0	216	0	0	82	177
N.S.	1	1.04	0.76	3.48	0.00	1.54	0.00	0.00	0.59	1.26
time (sec)	N/A	0.789	1.974	10.995	0.000	0.093	0.000	0.000	0.176	25.162

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	124	102	405	0	240	0	0	82	158
N.S.	1	1.02	0.84	3.35	0.00	1.98	0.00	0.00	0.68	1.31
time (sec)	N/A	0.723	1.660	7.984	0.000	0.100	0.000	0.000	0.185	25.140

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	129	105	574	0	255	0	0	84	194
N.S.	1	1.02	0.83	4.56	0.00	2.02	0.00	0.00	0.67	1.54
time (sec)	N/A	0.762	2.186	8.033	0.000	0.101	0.000	0.000	0.202	26.358

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	161	175	723	0	286	0	0	93	227
N.S.	1	0.94	1.02	4.20	0.00	1.66	0.00	0.00	0.54	1.32
time (sec)	N/A	0.876	2.269	9.191	0.000	0.102	0.000	0.000	0.206	25.778

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	272	235	825	0	358	0	0	117	364
N.S.	1	0.89	0.77	2.70	0.00	1.17	0.00	0.00	0.38	1.19
time (sec)	N/A	1.361	3.699	29.322	0.000	0.120	0.000	0.000	0.198	25.439

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	244	197	745	0	321	0	0	108	328
N.S.	1	0.96	0.77	2.92	0.00	1.26	0.00	0.00	0.42	1.29
time (sec)	N/A	1.326	2.700	21.042	0.000	0.115	0.000	0.000	0.183	24.773

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	216	158	664	0	284	0	0	108	275
N.S.	1	1.05	0.77	3.24	0.00	1.39	0.00	0.00	0.53	1.34
time (sec)	N/A	1.171	3.349	16.232	0.000	0.108	0.000	0.000	0.188	25.421

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	208	150	641	0	302	0	0	108	248
N.S.	1	1.03	0.74	3.17	0.00	1.50	0.00	0.00	0.53	1.23
time (sec)	N/A	1.158	2.953	12.421	0.000	0.115	0.000	0.000	0.188	25.071

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	191	165	771	0	306	0	0	108	255
N.S.	1	0.99	0.86	4.02	0.00	1.59	0.00	0.00	0.56	1.33
time (sec)	N/A	1.121	2.927	10.099	0.000	0.109	0.000	0.000	0.180	25.125

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	208	176	950	0	326	0	0	110	291
N.S.	1	1.02	0.86	4.66	0.00	1.60	0.00	0.00	0.54	1.43
time (sec)	N/A	1.152	4.677	11.111	0.000	0.121	0.000	0.000	0.185	26.200

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	200	260	1074	0	0	0	0	18	0
N.S.	1	1.10	1.43	5.90	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.514	13.315	9.271	0.000	0.000	0.000	0.000	0.160	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	148	207	822	0	0	0	0	16	0
N.S.	1	1.08	1.51	6.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.059	1.873	8.009	0.000	0.000	0.000	0.000	0.168	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	86	128	295	0	0	0	0	9	0
N.S.	1	0.97	1.44	3.31	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.589	11.421	5.829	0.000	0.000	0.000	0.000	0.158	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0	18	0
N.S.	1	1.00	0.95	3.56	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.406	0.604	3.621	0.000	0.000	0.000	0.000	0.159	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	206	300	0	0	0	0	18	0
N.S.	1	1.00	2.40	3.49	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.668	2.910	4.318	0.000	0.000	0.000	0.000	0.166	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	162	260	441	0	0	0	0	18	0
N.S.	1	1.08	1.73	2.94	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.462	2.688	5.759	0.000	0.000	0.000	0.000	0.158	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	295	318	1066	0	0	0	0	30	0
N.S.	1	0.97	1.05	3.52	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.891	4.008	20.852	0.000	0.000	0.000	0.000	0.161	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	220	280	849	0	0	0	0	28	0
N.S.	1	0.98	1.25	3.79	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.339	3.268	8.066	0.000	0.000	0.000	0.000	0.159	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	192	260	808	0	0	0	0	22	0
N.S.	1	0.97	1.31	4.08	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.220	2.915	6.278	0.000	0.000	0.000	0.000	0.158	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	185	274	721	0	0	0	0	31	0
N.S.	1	0.92	1.37	3.60	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.361	3.139	5.890	0.000	0.000	0.000	0.000	0.177	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	242	316	856	0	0	0	0	33	0
N.S.	1	0.95	1.23	3.34	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.913	5.191	6.503	0.000	0.000	0.000	0.000	0.170	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	327	427	1005	0	0	0	0	33	0
N.S.	1	0.95	1.24	2.91	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.646	7.347	8.004	0.000	0.000	0.000	0.000	0.200	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	372	390	1977	0	0	0	0	46	0
N.S.	1	1.01	1.06	5.39	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.216	6.109	83.546	0.000	0.000	0.000	0.000	0.234	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	349	360	1937	0	0	0	0	44	0
N.S.	1	1.01	1.05	5.63	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.078	4.336	8.406	0.000	0.000	0.000	0.000	0.229	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	334	365	1850	0	0	0	0	38	0
N.S.	1	0.99	1.08	5.49	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.057	4.962	8.184	0.000	0.000	0.000	0.000	0.219	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	336	383	1744	0	0	0	0	47	0
N.S.	1	0.97	1.11	5.06	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.259	5.388	7.032	0.000	0.000	0.000	0.000	0.159	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	412	458	1975	0	0	0	0	49	0
N.S.	1	0.98	1.09	4.70	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	3.079	5.950	8.944	0.000	0.000	0.000	0.000	0.180	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	523	513	570	2132	0	0	0	0	49	0
N.S.	1	0.98	1.09	4.08	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	4.084	7.681	10.338	0.000	0.000	0.000	0.000	0.163	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	203	0	77	0	0	20	0
N.S.	1	1.00	0.93	4.61	0.00	1.75	0.00	0.00	0.45	0.00
time (sec)	N/A	0.263	0.082	5.234	0.000	0.087	0.000	0.000	0.167	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	180	0	71	0	0	18	0
N.S.	1	1.00	0.84	4.09	0.00	1.61	0.00	0.00	0.41	0.00
time (sec)	N/A	0.270	0.070	3.582	0.000	0.086	0.000	0.000	0.173	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	134	0	59	0	0	11	0
N.S.	1	1.00	1.00	7.88	0.00	3.47	0.00	0.00	0.65	0.00
time (sec)	N/A	0.196	0.036	2.388	0.000	0.083	0.000	0.000	0.178	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	0	53	0	0	20	0
N.S.	1	1.00	1.00	1.12	0.00	3.12	0.00	0.00	1.18	0.00
time (sec)	N/A	0.198	0.043	0.887	0.000	0.082	0.000	0.000	0.217	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	183	0	96	0	0	20	0
N.S.	1	1.00	1.00	4.58	0.00	2.40	0.00	0.00	0.50	0.00
time (sec)	N/A	0.261	0.081	1.494	0.000	0.084	0.000	0.000	0.161	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	214	0	95	0	0	20	0
N.S.	1	1.00	0.84	4.86	0.00	2.16	0.00	0.00	0.45	0.00
time (sec)	N/A	0.261	0.090	1.737	0.000	0.085	0.000	0.000	0.160	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	125	159	553	0	0	0	0	32	0
N.S.	1	1.08	1.37	4.77	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.935	2.231	7.317	0.000	0.000	0.000	0.000	0.169	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	81	82	228	0	0	0	0	30	0
N.S.	1	1.04	1.05	2.92	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.561	0.414	4.917	0.000	0.000	0.000	0.000	0.163	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	189	0	0	0	0	24	0
N.S.	1	1.00	0.89	3.44	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.395	0.252	3.260	0.000	0.000	0.000	0.000	0.168	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	151	0	0	0	0	33	0
N.S.	1	1.00	1.00	5.03	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.245	0.256	1.780	0.000	0.000	0.000	0.000	0.165	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	196	355	0	0	0	0	35	0
N.S.	1	0.99	2.45	4.44	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.601	3.324	4.123	0.000	0.000	0.000	0.000	0.158	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	140	211	425	0	0	0	0	35	0
N.S.	1	1.05	1.59	3.20	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.247	4.677	5.536	0.000	0.000	0.000	0.000	0.169	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	563	1224	1700	0	0	0	0	61	0
N.S.	1	1.01	2.19	3.04	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.734	6.885	25.005	0.000	0.000	0.000	0.000	0.201	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	471	1175	1187	0	0	0	0	53	0
N.S.	1	1.00	2.48	2.51	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.049	18.975	12.804	0.000	0.000	0.000	0.000	0.184	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	391	408	666	0	0	0	0	55	0
N.S.	1	1.02	1.06	1.73	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.585	14.759	15.947	0.000	0.000	0.000	0.000	0.174	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	273	654	0	0	0	0	63	0
N.S.	1	1.00	0.78	1.86	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.177	14.300	21.257	0.000	0.000	0.000	0.000	0.178	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	285	407	1022	0	0	0	0	63	0
N.S.	1	1.00	1.43	3.60	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.003	14.597	30.074	0.000	0.000	0.000	0.000	0.186	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	359	1315	1505	0	0	0	0	63	0
N.S.	1	1.03	3.76	4.30	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.445	6.852	38.191	0.000	0.000	0.000	0.000	0.173	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	450	1408	2083	0	0	0	0	63	0
N.S.	1	1.04	3.25	4.81	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.000	6.992	51.954	0.000	0.000	0.000	0.000	0.186	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	670	675	1284	2336	0	0	0	0	98	0
N.S.	1	1.01	1.92	3.49	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	3.416	7.685	32.501	0.000	0.000	0.000	0.000	0.286	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	568	1227	1803	0	0	0	0	90	0
N.S.	1	1.00	2.17	3.19	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.827	7.451	26.540	0.000	0.000	0.000	0.000	0.306	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	475	1198	1389	0	0	0	0	90	0
N.S.	1	1.01	2.54	2.94	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.189	7.644	21.749	0.000	0.000	0.000	0.000	0.303	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	461	1196	1239	0	0	0	0	92	0
N.S.	1	1.03	2.66	2.76	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.153	6.937	27.581	0.000	0.000	0.000	0.000	0.183	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	420	1236	1363	0	0	0	0	100	0
N.S.	1	1.00	2.95	3.25	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.669	6.867	31.032	0.000	0.000	0.000	0.000	0.183	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	359	1314	1621	0	0	0	0	100	0
N.S.	1	1.02	3.72	4.59	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.548	7.022	38.100	0.000	0.000	0.000	0.000	0.212	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	444	1407	2085	0	0	0	0	100	0
N.S.	1	1.03	3.25	4.82	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	2.032	7.101	54.745	0.000	0.000	0.000	0.000	0.179	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	533	1515	2683	0	0	0	0	100	0
N.S.	1	1.02	2.90	5.14	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.692	7.442	77.431	0.000	0.000	0.000	0.000	0.189	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	779	786	1353	2992	0	0	0	0	135	0
N.S.	1	1.01	1.74	3.84	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	4.304	7.973	38.848	0.000	0.000	0.000	0.000	0.270	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	672	1287	2439	0	0	0	0	127	0
N.S.	1	1.01	1.94	3.67	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	3.558	7.773	31.958	0.000	0.000	0.000	0.000	0.267	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	572	1251	2004	0	0	0	0	127	0
N.S.	1	1.01	2.22	3.55	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	2.932	8.296	30.727	0.000	0.000	0.000	0.000	0.217	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	555	1241	1848	0	0	0	0	127	0
N.S.	1	1.01	2.27	3.38	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	2.877	7.066	31.280	0.000	0.000	0.000	0.000	0.203	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	547	1269	1831	0	0	0	0	129	0
N.S.	1	1.02	2.37	3.42	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	2.884	7.013	35.032	0.000	0.000	0.000	0.000	0.187	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	499	1319	1958	0	0	0	0	137	0
N.S.	1	1.01	2.68	3.97	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	2.254	7.219	42.410	0.000	0.000	0.000	0.000	0.188	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	445	1409	2199	0	0	0	0	137	0
N.S.	1	1.03	3.25	5.07	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	2.058	7.341	55.119	0.000	0.000	0.000	0.000	0.202	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	533	1517	2683	0	0	0	0	137	0
N.S.	1	1.02	2.91	5.14	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	2.666	7.650	80.106	0.000	0.000	0.000	0.000	0.189	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	622	638	1640	3301	0	0	0	0	137	0
N.S.	1	1.03	2.64	5.31	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	3.375	8.023	102.977	0.000	0.000	0.000	0.000	0.198	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	418	423	1236	1387	0	0	0	0	207	0
N.S.	1	1.01	2.96	3.32	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.853	25.305	35.356	0.000	0.000	0.000	0.000	0.216	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	480	1175	1087	0	0	0	0	27	0
N.S.	1	1.00	2.45	2.27	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.161	19.535	25.193	0.000	0.000	0.000	0.000	0.169	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	436	402	476	0	0	0	0	21	0
N.S.	1	1.02	0.94	1.11	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.803	13.854	16.753	0.000	0.000	0.000	0.000	0.213	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	231	173	0	0	0	0	29	0
N.S.	1	1.00	1.01	0.76	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.642	15.051	17.483	0.000	0.000	0.000	0.000	0.230	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	299	480	0	0	0	0	29	0
N.S.	1	1.00	1.30	2.09	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.709	15.718	23.937	0.000	0.000	0.000	0.000	0.238	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	294	416	934	0	0	0	0	29	0
N.S.	1	1.01	1.43	3.22	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.021	17.491	34.377	0.000	0.000	0.000	0.000	0.163	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	375	1319	1507	0	0	0	0	29	0
N.S.	1	1.03	3.63	4.15	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.502	6.974	46.755	0.000	0.000	0.000	0.000	0.168	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	527	1234	1628	0	0	0	0	39	0
N.S.	1	1.05	2.47	3.26	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.470	7.429	25.306	0.000	0.000	0.000	0.000	0.173	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	420	1012	1173	0	0	0	0	33	0
N.S.	1	1.01	2.43	2.82	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.494	19.016	17.023	0.000	0.000	0.000	0.000	0.177	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	307	1223	957	0	0	0	0	42	0
N.S.	1	1.08	4.31	3.37	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.067	6.944	18.053	0.000	0.000	0.000	0.000	0.156	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	331	1281	1293	0	0	0	0	44	0
N.S.	1	1.09	4.20	4.24	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.165	7.064	30.063	0.000	0.000	0.000	0.000	0.173	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	412	1357	1932	0	0	0	0	44	0
N.S.	1	1.05	3.45	4.92	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.739	7.362	40.187	0.000	0.000	0.000	0.000	0.179	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	698	1396	4190	0	0	0	0	57	0
N.S.	1	1.04	2.07	6.22	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	3.461	7.948	32.499	0.000	0.000	0.000	0.000	0.203	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	569	1342	2848	0	0	0	0	55	0
N.S.	1	1.04	2.46	5.23	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.476	7.477	27.513	0.000	0.000	0.000	0.000	0.170	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	410	1335	2072	0	0	0	0	49	0
N.S.	1	1.05	3.41	5.30	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.569	7.395	22.655	0.000	0.000	0.000	0.000	0.159	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	444	1384	2442	0	0	0	0	58	0
N.S.	1	1.03	3.23	5.69	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.730	7.196	29.327	0.000	0.000	0.000	0.000	0.162	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	477	1431	2956	0	0	0	0	60	0
N.S.	1	1.05	3.14	6.48	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.934	7.335	37.220	0.000	0.000	0.000	0.000	0.169	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	567	581	1499	3720	0	0	0	0	60	0
N.S.	1	1.02	2.64	6.56	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.702	7.615	46.709	0.000	0.000	0.000	0.000	0.168	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	426	225	388	0	0	0	0	41	0
N.S.	1	1.02	0.54	0.93	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.703	4.894	22.106	0.000	0.000	0.000	0.000	0.164	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	131	138	0	0	0	0	35	0
N.S.	1	1.00	1.12	1.18	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.309	1.593	15.704	0.000	0.000	0.000	0.000	0.174	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	171	112	0	0	0	0	44	0
N.S.	1	1.00	1.55	1.02	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.305	2.044	12.839	0.000	0.000	0.000	0.000	0.165	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	212	380	0	0	0	0	46	0
N.S.	1	1.00	0.94	1.68	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.672	6.744	28.039	0.000	0.000	0.000	0.000	0.167	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	234	0	0	0	0	87	0
N.S.	1	1.00	0.00	3.25	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.282	0.000	13.029	0.000	0.000	0.000	0.000	0.174	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	216	0	0	0	0	87	0
N.S.	1	1.00	0.00	3.09	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.296	0.000	11.938	0.000	0.000	0.000	0.000	0.178	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	225	0	0	0	0	91	0
N.S.	1	1.00	0.00	2.42	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.473	0.000	13.572	0.000	0.000	0.000	0.000	0.183	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	0	160	0	0	0	0	91	0
N.S.	1	1.00	0.00	1.68	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.458	0.000	13.938	0.000	0.000	0.000	0.000	0.175	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	236	0	0	0	0	87	0
N.S.	1	1.00	0.00	3.28	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.283	0.000	13.901	0.000	0.000	0.000	0.000	0.184	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	242	0	0	0	0	91	0
N.S.	1	1.00	0.00	3.27	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.295	0.000	12.611	0.000	0.000	0.000	0.000	0.178	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	0	154	0	0	0	0	87	0
N.S.	1	1.00	0.00	1.57	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.463	0.000	14.085	0.000	0.000	0.000	0.000	0.190	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	166	0	0	0	0	91	0
N.S.	1	1.00	0.00	1.73	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.452	0.000	14.025	0.000	0.000	0.000	0.000	0.190	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	32	35	61	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.97	1.06	1.85	1.06
time (sec)	N/A	0.262	26.449	0.970	3.746	0.117	121.208	0.823	0.197	25.524

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	595	614	479	0	0	0	0	0	144	0
N.S.	1	1.03	0.81	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	3.007	6.236	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	406	408	263	0	0	0	0	0	117	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.768	3.247	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	291	211	0	0	0	0	0	90	0
N.S.	1	1.01	0.74	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.097	1.792	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	193	162	0	0	0	0	0	63	0
N.S.	1	0.98	0.83	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.641	0.911	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	281	10482	0	0	0	0	0	14	0
N.S.	1	0.98	36.65	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.887	34.999	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	54	34	35	97	35
N.S.	1	1.00	1.06	0.94	1.00	1.54	0.97	1.00	2.77	1.00
time (sec)	N/A	0.842	107.894	0.943	3.205	0.116	139.351	16.684	0.226	25.731

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	35	34	35	59	35
N.S.	1	1.00	1.06	0.94	1.00	1.00	0.97	1.00	1.69	1.00
time (sec)	N/A	0.304	46.623	0.967	2.170	0.103	7.321	0.869	0.228	25.448

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	35	34	35	26	35
N.S.	1	1.00	1.06	0.94	1.00	1.00	0.97	1.00	0.74	1.00
time (sec)	N/A	0.312	43.028	1.085	2.027	0.100	3.687	0.607	0.168	25.742

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	63	34	35	38	35
N.S.	1	1.00	1.06	0.94	1.00	1.80	0.97	1.00	1.09	1.00
time (sec)	N/A	0.873	40.026	1.110	2.241	0.115	9.048	1.137	0.184	26.497

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	166	292	634	0	219	0	0	103	0
N.S.	1	0.97	1.70	3.69	0.00	1.27	0.00	0.00	0.60	0.00
time (sec)	N/A	0.874	3.359	35.451	0.000	0.094	0.000	0.000	0.299	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	137	225	399	0	188	0	0	103	0
N.S.	1	1.01	1.67	2.96	0.00	1.39	0.00	0.00	0.76	0.00
time (sec)	N/A	0.804	1.986	35.011	0.000	0.095	0.000	0.000	0.344	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	157	242	0	141	0	0	95	0
N.S.	1	1.00	1.48	2.28	0.00	1.33	0.00	0.00	0.90	0.00
time (sec)	N/A	0.698	2.235	5.701	0.000	0.102	0.000	0.000	0.275	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	113	148	321	0	142	0	0	70	0
N.S.	1	1.03	1.35	2.92	0.00	1.29	0.00	0.00	0.64	0.00
time (sec)	N/A	0.703	2.547	6.522	0.000	0.093	0.000	0.000	0.198	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	143	148	355	0	169	0	0	103	0
N.S.	1	1.01	1.05	2.52	0.00	1.20	0.00	0.00	0.73	0.00
time (sec)	N/A	0.804	2.798	10.063	0.000	0.096	0.000	0.000	0.192	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	170	182	383	0	187	0	0	103	0
N.S.	1	0.99	1.06	2.23	0.00	1.09	0.00	0.00	0.60	0.00
time (sec)	N/A	0.887	3.494	15.069	0.000	0.101	0.000	0.000	0.211	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	196	299	714	0	239	0	0	163	0
N.S.	1	0.98	1.50	3.59	0.00	1.20	0.00	0.00	0.82	0.00
time (sec)	N/A	1.187	4.284	62.869	0.000	0.093	0.000	0.000	0.402	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	164	188	513	0	202	0	0	163	0
N.S.	1	1.02	1.18	3.21	0.00	1.26	0.00	0.00	1.02	0.00
time (sec)	N/A	1.038	3.255	62.822	0.000	0.120	0.000	0.000	0.396	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	164	152	244	0	166	0	0	151	0
N.S.	1	1.02	0.95	1.52	0.00	1.04	0.00	0.00	0.94	0.00
time (sec)	N/A	1.003	3.388	7.583	0.000	0.120	0.000	0.000	0.343	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	172	153	357	0	187	0	0	114	0
N.S.	1	1.04	0.92	2.15	0.00	1.13	0.00	0.00	0.69	0.00
time (sec)	N/A	1.022	3.704	10.599	0.000	0.108	0.000	0.000	0.225	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	202	193	385	0	211	0	0	163	0
N.S.	1	1.00	0.96	1.92	0.00	1.05	0.00	0.00	0.81	0.00
time (sec)	N/A	1.197	4.189	15.770	0.000	0.115	0.000	0.000	0.211	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	246	435	902	0	263	0	0	221	0
N.S.	1	1.01	1.78	3.70	0.00	1.08	0.00	0.00	0.91	0.00
time (sec)	N/A	1.547	5.896	184.507	0.000	0.103	0.000	0.000	0.498	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	215	268	916	0	243	0	0	221	0
N.S.	1	1.02	1.27	4.34	0.00	1.15	0.00	0.00	1.05	0.00
time (sec)	N/A	1.320	4.983	186.372	0.000	0.118	0.000	0.000	0.501	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	201	202	654	0	215	0	0	221	0
N.S.	1	1.01	1.02	3.29	0.00	1.08	0.00	0.00	1.11	0.00
time (sec)	N/A	1.301	3.579	185.510	0.000	0.117	0.000	0.000	0.518	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	215	207	337	0	197	0	0	205	0
N.S.	1	1.02	0.98	1.60	0.00	0.93	0.00	0.00	0.97	0.00
time (sec)	N/A	1.364	3.116	11.595	0.000	0.110	0.000	0.000	0.494	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	222	194	385	0	211	0	0	156	0
N.S.	1	1.05	0.92	1.82	0.00	1.00	0.00	0.00	0.74	0.00
time (sec)	N/A	1.390	4.613	15.724	0.000	0.105	0.000	0.000	0.358	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	250	196	413	0	231	0	0	221	0
N.S.	1	1.02	0.80	1.69	0.00	0.95	0.00	0.00	0.91	0.00
time (sec)	N/A	1.629	4.798	18.198	0.000	0.139	0.000	0.000	0.335	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	185	650	466	0	308	0	0	71	0
N.S.	1	0.96	3.37	2.41	0.00	1.60	0.00	0.00	0.37	0.00
time (sec)	N/A	0.992	8.999	7.348	0.000	0.116	0.000	0.000	0.172	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	157	400	319	0	250	0	0	67	0
N.S.	1	0.99	2.52	2.01	0.00	1.57	0.00	0.00	0.42	0.00
time (sec)	N/A	0.924	6.449	3.756	0.000	0.106	0.000	0.000	0.190	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	200	243	0	241	0	0	55	0
N.S.	1	1.05	1.63	1.98	0.00	1.96	0.00	0.00	0.45	0.00
time (sec)	N/A	0.759	3.020	3.044	0.000	0.106	0.000	0.000	0.172	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	130	422	244	0	237	0	0	79	0
N.S.	1	1.04	3.38	1.95	0.00	1.90	0.00	0.00	0.63	0.00
time (sec)	N/A	0.753	4.638	4.632	0.000	0.114	0.000	0.000	0.179	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	159	444	262	0	261	0	0	87	0
N.S.	1	0.98	2.72	1.61	0.00	1.60	0.00	0.00	0.53	0.00
time (sec)	N/A	0.933	7.581	5.697	0.000	0.164	0.000	0.000	0.191	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	187	518	281	0	278	0	0	87	0
N.S.	1	0.95	2.64	1.43	0.00	1.42	0.00	0.00	0.44	0.00
time (sec)	N/A	0.952	5.522	7.024	0.000	0.114	0.000	0.000	0.178	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	303	494	0	367	0	0	87	0
N.S.	1	1.00	1.46	2.38	0.00	1.76	0.00	0.00	0.42	0.00
time (sec)	N/A	1.282	4.774	4.484	0.000	0.108	0.000	0.000	0.218	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	171	256	350	0	326	0	0	75	0
N.S.	1	1.06	1.59	2.17	0.00	2.02	0.00	0.00	0.47	0.00
time (sec)	N/A	1.075	3.438	4.398	0.000	0.102	0.000	0.000	0.176	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	175	256	350	0	324	0	0	111	0
N.S.	1	1.04	1.52	2.08	0.00	1.93	0.00	0.00	0.66	0.00
time (sec)	N/A	1.080	4.082	4.868	0.000	0.107	0.000	0.000	0.182	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	182	732	421	0	362	0	0	123	0
N.S.	1	1.03	4.16	2.39	0.00	2.06	0.00	0.00	0.70	0.00
time (sec)	N/A	1.117	8.394	5.820	0.000	0.125	0.000	0.000	0.189	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	211	777	435	0	376	0	0	123	0
N.S.	1	1.02	3.77	2.11	0.00	1.83	0.00	0.00	0.60	0.00
time (sec)	N/A	1.351	8.787	6.504	0.000	0.144	0.000	0.000	0.183	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	261	267	358	685	0	481	0	0	107	0
N.S.	1	1.02	1.37	2.62	0.00	1.84	0.00	0.00	0.41	0.00
time (sec)	N/A	1.727	7.328	5.948	0.000	0.102	0.000	0.000	0.187	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	237	793	451	0	476	0	0	95	0
N.S.	1	1.07	3.57	2.03	0.00	2.14	0.00	0.00	0.43	0.00
time (sec)	N/A	1.505	8.608	5.879	0.000	0.113	0.000	0.000	0.177	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	231	792	451	0	472	0	0	143	0
N.S.	1	1.07	3.67	2.09	0.00	2.19	0.00	0.00	0.66	0.00
time (sec)	N/A	1.519	8.566	6.388	0.000	0.112	0.000	0.000	0.189	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	237	793	451	0	474	0	0	159	0
N.S.	1	1.07	3.57	2.03	0.00	2.14	0.00	0.00	0.72	0.00
time (sec)	N/A	1.465	9.452	6.499	0.000	0.130	0.000	0.000	0.200	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	243	817	451	0	478	0	0	159	0
N.S.	1	1.07	3.58	1.98	0.00	2.10	0.00	0.00	0.70	0.00
time (sec)	N/A	1.507	10.021	7.148	0.000	0.108	0.000	0.000	0.194	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	269	589	465	0	489	0	0	159	0
N.S.	1	1.04	2.27	1.80	0.00	1.89	0.00	0.00	0.61	0.00
time (sec)	N/A	1.718	8.860	7.786	0.000	0.122	0.000	0.000	0.195	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	235	124	366	659	121	0	0	68	479
N.S.	1	1.07	0.56	1.66	3.00	0.55	0.00	0.00	0.31	2.18
time (sec)	N/A	1.155	0.630	21.129	0.240	0.091	0.000	0.000	0.230	32.966

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	192	102	327	568	104	0	0	68	441
N.S.	1	1.10	0.58	1.87	3.25	0.59	0.00	0.00	0.39	2.52
time (sec)	N/A	0.948	0.499	18.292	0.255	0.093	0.000	0.000	0.244	29.042

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	149	78	288	475	86	0	0	68	196
N.S.	1	1.15	0.60	2.22	3.65	0.66	0.00	0.00	0.52	1.51
time (sec)	N/A	0.757	0.307	16.435	0.218	0.096	0.000	0.000	0.238	1.773

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	106	57	249	380	65	0	0	68	114
N.S.	1	1.25	0.67	2.93	4.47	0.76	0.00	0.00	0.80	1.34
time (sec)	N/A	0.607	0.202	15.547	0.224	0.089	0.000	0.000	0.240	0.664

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	86	319	906	91	0	0	64	0
N.S.	1	1.01	0.90	3.32	9.44	0.95	0.00	0.00	0.67	0.00
time (sec)	N/A	0.609	0.206	15.219	0.477	0.099	0.000	0.000	0.214	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	99	103	457	939	97	0	0	52	0
N.S.	1	1.01	1.05	4.66	9.58	0.99	0.00	0.00	0.53	0.00
time (sec)	N/A	0.600	0.249	14.910	0.352	0.132	0.000	0.000	0.203	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	144	120	0	1851	127	0	0	68	0
N.S.	1	0.95	0.79	0.00	12.26	0.84	0.00	0.00	0.45	0.00
time (sec)	N/A	0.764	0.457	0.000	0.447	0.128	0.000	0.000	0.175	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	187	138	740	2981	146	0	0	68	0
N.S.	1	0.95	0.70	3.78	15.21	0.74	0.00	0.00	0.35	0.00
time (sec)	N/A	0.918	0.804	16.971	0.751	0.133	0.000	0.000	0.184	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	288	146	156	712	144	0	0	141	348
N.S.	1	1.05	0.53	0.57	2.59	0.52	0.00	0.00	0.51	1.27
time (sec)	N/A	1.525	0.820	14.030	0.237	0.090	0.000	0.000	0.326	28.637

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	245	124	136	619	126	0	0	141	316
N.S.	1	1.07	0.54	0.60	2.71	0.55	0.00	0.00	0.62	1.39
time (sec)	N/A	1.302	0.791	13.928	0.266	0.096	0.000	0.000	0.342	31.607

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	202	102	116	527	107	0	0	141	259
N.S.	1	1.12	0.56	0.64	2.91	0.59	0.00	0.00	0.78	1.43
time (sec)	N/A	1.087	0.635	14.001	0.228	0.087	0.000	0.000	0.336	32.108

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	160	80	94	436	88	0	0	141	197
N.S.	1	1.19	0.60	0.70	3.25	0.66	0.00	0.00	1.05	1.47
time (sec)	N/A	0.914	0.402	14.018	0.341	0.090	0.000	0.000	0.338	1.792

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	149	106	148	1462	130	0	0	141	0
N.S.	1	1.03	0.73	1.02	10.08	0.90	0.00	0.00	0.97	0.00
time (sec)	N/A	0.891	0.469	14.372	0.369	0.104	0.000	0.000	0.339	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	147	107	197	1801	119	0	0	133	0
N.S.	1	1.01	0.73	1.35	12.34	0.82	0.00	0.00	0.91	0.00
time (sec)	N/A	0.900	0.380	21.621	0.508	0.136	0.000	0.000	0.290	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	190	1884	133	0	0	109	0
N.S.	1	1.00	0.79	1.24	12.31	0.87	0.00	0.00	0.71	0.00
time (sec)	N/A	0.898	0.492	21.783	0.614	0.141	0.000	0.000	0.236	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	197	139	207	3023	153	0	0	141	0
N.S.	1	0.98	0.70	1.04	15.12	0.76	0.00	0.00	0.70	0.00
time (sec)	N/A	1.093	0.978	18.295	0.667	0.137	0.000	0.000	0.197	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	240	156	227	8901	171	0	0	141	0
N.S.	1	0.97	0.63	0.92	36.04	0.69	0.00	0.00	0.57	0.00
time (sec)	N/A	1.297	1.588	18.608	1.125	0.169	0.000	0.000	0.197	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	342	171	172	763	176	0	0	219	789
N.S.	1	1.06	0.53	0.53	2.37	0.55	0.00	0.00	0.68	2.45
time (sec)	N/A	1.919	1.451	3.695	0.203	0.098	0.000	0.000	0.443	30.295

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	299	147	152	672	156	0	0	219	751
N.S.	1	1.09	0.53	0.55	2.44	0.57	0.00	0.00	0.80	2.73
time (sec)	N/A	1.694	1.252	3.142	0.246	0.094	0.000	0.000	0.448	31.295

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	256	126	132	579	135	0	0	219	617
N.S.	1	1.12	0.55	0.58	2.54	0.59	0.00	0.00	0.96	2.71
time (sec)	N/A	1.450	0.969	3.124	0.226	0.095	0.000	0.000	0.446	54.254

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	212	104	112	488	114	0	0	219	579
N.S.	1	1.17	0.57	0.62	2.70	0.63	0.00	0.00	1.21	3.20
time (sec)	N/A	1.241	0.795	3.138	0.227	0.093	0.000	0.000	0.451	50.334

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	201	130	169	1713	162	0	0	219	0
N.S.	1	1.05	0.68	0.88	8.92	0.84	0.00	0.00	1.14	0.00
time (sec)	N/A	1.202	0.946	3.285	0.413	0.107	0.000	0.000	0.440	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	198	130	234	2780	166	0	0	219	0
N.S.	1	1.03	0.67	1.21	14.40	0.86	0.00	0.00	1.13	0.00
time (sec)	N/A	1.248	0.819	3.352	0.531	0.132	0.000	0.000	0.437	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	199	126	223	0	147	0	0	207	0
N.S.	1	1.01	0.64	1.13	0.00	0.74	0.00	0.00	1.05	0.00
time (sec)	N/A	1.272	0.679	21.979	0.000	0.132	0.000	0.000	0.379	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	206	141	215	3071	163	0	0	171	0
N.S.	1	1.03	0.70	1.08	15.36	0.82	0.00	0.00	0.86	0.00
time (sec)	N/A	1.238	1.106	22.112	0.674	0.135	0.000	0.000	0.296	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	251	157	229	9390	183	0	0	219	0
N.S.	1	1.02	0.64	0.93	38.02	0.74	0.00	0.00	0.89	0.00
time (sec)	N/A	1.452	1.908	18.713	1.048	0.173	0.000	0.000	0.210	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	179	249	10042	203	0	0	219	0
N.S.	1	1.00	0.61	0.85	34.16	0.69	0.00	0.00	0.74	0.00
time (sec)	N/A	1.681	3.192	18.750	0.905	0.177	0.000	0.000	0.215	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	336	272	265	0	198	0	0	91	0
N.S.	1	1.14	0.92	0.90	0.00	0.67	0.00	0.00	0.31	0.00
time (sec)	N/A	1.998	10.768	14.137	0.000	0.121	0.000	0.000	0.187	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	281	250	245	0	181	0	0	91	0
N.S.	1	1.12	1.00	0.98	0.00	0.72	0.00	0.00	0.36	0.00
time (sec)	N/A	1.540	8.060	13.984	0.000	0.126	0.000	0.000	0.204	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	226	1719	225	0	164	0	0	91	0
N.S.	1	1.09	8.30	1.09	0.00	0.79	0.00	0.00	0.44	0.00
time (sec)	N/A	1.207	8.216	13.907	0.000	0.139	0.000	0.000	0.177	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	171	617	204	0	143	0	0	91	0
N.S.	1	1.06	3.81	1.26	0.00	0.88	0.00	0.00	0.56	0.00
time (sec)	N/A	0.894	7.030	13.919	0.000	0.119	0.000	0.000	0.304	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	120	203	177	0	110	0	0	87	0
N.S.	1	1.01	1.71	1.49	0.00	0.92	0.00	0.00	0.73	0.00
time (sec)	N/A	0.618	1.788	13.872	0.000	0.110	0.000	0.000	0.257	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	121	102	138	1221	96	0	0	75	0
N.S.	1	0.86	0.73	0.99	8.72	0.69	0.00	0.00	0.54	0.00
time (sec)	N/A	0.746	0.253	13.897	1.108	0.851	0.000	0.000	0.267	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	171	467	196	0	168	0	0	99	0
N.S.	1	0.94	2.58	1.08	0.00	0.93	0.00	0.00	0.55	0.00
time (sec)	N/A	1.015	2.881	18.007	0.000	1.100	0.000	0.000	0.189	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	224	412	239	0	194	0	0	107	0
N.S.	1	0.97	1.79	1.04	0.00	0.84	0.00	0.00	0.47	0.00
time (sec)	N/A	1.340	2.629	17.965	0.000	2.081	0.000	0.000	0.187	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	183	143	281	0	208	0	0	120	0
N.S.	1	0.95	0.74	1.46	0.00	1.08	0.00	0.00	0.62	0.00
time (sec)	N/A	1.198	0.528	24.218	0.000	13.760	0.000	0.000	0.189	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	344	2966	285	0	237	0	0	111	0
N.S.	1	1.09	9.36	0.90	0.00	0.75	0.00	0.00	0.35	0.00
time (sec)	N/A	2.060	10.089	14.485	0.000	0.127	0.000	0.000	0.189	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	289	2166	265	0	220	0	0	111	0
N.S.	1	1.07	8.02	0.98	0.00	0.81	0.00	0.00	0.41	0.00
time (sec)	N/A	1.640	8.256	14.642	0.000	0.116	0.000	0.000	0.198	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	234	981	245	0	197	0	0	111	0
N.S.	1	1.05	4.40	1.10	0.00	0.88	0.00	0.00	0.50	0.00
time (sec)	N/A	1.260	7.088	14.528	0.000	0.114	0.000	0.000	0.220	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	179	443	199	0	163	0	0	107	0
N.S.	1	1.02	2.52	1.13	0.00	0.93	0.00	0.00	0.61	0.00
time (sec)	N/A	0.940	5.159	14.268	0.000	0.115	0.000	0.000	0.187	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	128	191	192	0	144	0	0	95	0
N.S.	1	1.01	1.50	1.51	0.00	1.13	0.00	0.00	0.75	0.00
time (sec)	N/A	0.656	2.273	14.322	0.000	0.116	0.000	0.000	0.186	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	185	171	243	256	0	203	0	0	131	0
N.S.	1	0.92	1.31	1.38	0.00	1.10	0.00	0.00	0.71	0.00
time (sec)	N/A	1.051	3.396	10.754	0.000	2.778	0.000	0.000	0.180	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	226	836	284	0	246	0	0	143	0
N.S.	1	0.95	3.53	1.20	0.00	1.04	0.00	0.00	0.60	0.00
time (sec)	N/A	1.438	8.213	18.250	0.000	3.690	0.000	0.000	0.191	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	344	261	325	0	266	0	0	131	0
N.S.	1	1.09	0.82	1.03	0.00	0.84	0.00	0.00	0.41	0.00
time (sec)	N/A	2.010	13.436	14.802	0.000	0.129	0.000	0.000	0.195	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	289	1152	305	0	246	0	0	131	0
N.S.	1	1.07	4.27	1.13	0.00	0.91	0.00	0.00	0.49	0.00
time (sec)	N/A	1.646	10.009	14.653	0.000	0.122	0.000	0.000	0.189	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	234	732	261	0	210	0	0	127	0
N.S.	1	1.05	3.28	1.17	0.00	0.94	0.00	0.00	0.57	0.00
time (sec)	N/A	1.289	7.396	14.311	0.000	0.119	0.000	0.000	0.196	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	183	162	244	0	207	0	0	115	0
N.S.	1	1.04	0.92	1.39	0.00	1.18	0.00	0.00	0.65	0.00
time (sec)	N/A	0.931	3.237	14.450	0.000	0.121	0.000	0.000	0.197	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	181	474	238	0	205	0	0	163	0
N.S.	1	1.04	2.72	1.37	0.00	1.18	0.00	0.00	0.94	0.00
time (sec)	N/A	0.944	6.475	10.821	0.000	0.139	0.000	0.000	0.203	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	226	264	288	0	277	0	0	179	0
N.S.	1	0.97	1.13	1.23	0.00	1.18	0.00	0.00	0.76	0.00
time (sec)	N/A	1.419	4.444	10.823	0.000	6.012	0.000	0.000	0.206	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	283	929	354	0	313	0	0	179	0
N.S.	1	0.99	3.25	1.24	0.00	1.09	0.00	0.00	0.63	0.00
time (sec)	N/A	1.869	8.580	18.576	0.000	7.653	0.000	0.000	0.194	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	344	267	355	0	295	0	0	151	0
N.S.	1	1.09	0.84	1.12	0.00	0.93	0.00	0.00	0.48	0.00
time (sec)	N/A	2.079	11.326	14.835	0.000	0.136	0.000	0.000	0.193	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	289	802	311	0	260	0	0	147	0
N.S.	1	1.07	2.97	1.15	0.00	0.96	0.00	0.00	0.54	0.00
time (sec)	N/A	1.697	8.023	14.576	0.000	0.124	0.000	0.000	0.187	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	238	180	294	0	257	0	0	135	0
N.S.	1	1.07	0.81	1.32	0.00	1.15	0.00	0.00	0.61	0.00
time (sec)	N/A	1.297	3.344	14.662	0.000	0.120	0.000	0.000	0.201	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	236	179	288	0	255	0	0	195	0
N.S.	1	1.07	0.81	1.30	0.00	1.15	0.00	0.00	0.88	0.00
time (sec)	N/A	1.289	3.320	10.747	0.000	0.123	0.000	0.000	0.204	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	236	561	284	0	257	0	0	215	0
N.S.	1	1.07	2.54	1.29	0.00	1.16	0.00	0.00	0.97	0.00
time (sec)	N/A	1.288	6.769	10.689	0.000	0.126	0.000	0.000	0.222	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	281	352	0	338	0	0	215	0
N.S.	1	1.00	1.00	1.25	0.00	1.20	0.00	0.00	0.77	0.00
time (sec)	N/A	1.781	6.226	10.820	0.000	7.398	0.000	0.000	0.222	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	338	1017	422	0	379	0	0	215	0
N.S.	1	1.02	3.05	1.27	0.00	1.14	0.00	0.00	0.65	0.00
time (sec)	N/A	2.254	9.332	18.941	0.000	13.106	0.000	0.000	0.215	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	170	132	636	0	235	0	0	81	0
N.S.	1	0.94	0.73	3.53	0.00	1.31	0.00	0.00	0.45	0.00
time (sec)	N/A	0.859	2.955	104.357	0.000	0.092	0.000	0.000	0.268	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	142	104	401	0	205	0	0	81	0
N.S.	1	0.99	0.73	2.80	0.00	1.43	0.00	0.00	0.57	0.00
time (sec)	N/A	0.822	1.681	103.412	0.000	0.086	0.000	0.000	0.267	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	85	246	0	153	0	0	75	0
N.S.	1	1.00	0.77	2.22	0.00	1.38	0.00	0.00	0.68	0.00
time (sec)	N/A	0.692	2.374	7.162	0.000	0.084	0.000	0.000	0.248	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	118	90	326	0	156	0	0	56	0
N.S.	1	1.03	0.78	2.83	0.00	1.36	0.00	0.00	0.49	0.00
time (sec)	N/A	0.721	1.437	8.527	0.000	0.087	0.000	0.000	0.197	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	147	108	371	0	184	0	0	81	0
N.S.	1	0.99	0.73	2.51	0.00	1.24	0.00	0.00	0.55	0.00
time (sec)	N/A	0.817	1.751	11.535	0.000	0.097	0.000	0.000	0.186	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	174	125	413	0	203	0	0	81	0
N.S.	1	0.97	0.69	2.29	0.00	1.13	0.00	0.00	0.45	0.00
time (sec)	N/A	0.921	2.420	14.925	0.000	0.103	0.000	0.000	0.194	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	206	171	723	0	286	0	0	115	0
N.S.	1	0.93	0.77	3.27	0.00	1.29	0.00	0.00	0.52	0.00
time (sec)	N/A	1.421	4.377	516.694	0.000	0.093	0.000	0.000	0.309	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	178	125	650	0	247	0	0	115	0
N.S.	1	1.01	0.71	3.67	0.00	1.40	0.00	0.00	0.65	0.00
time (sec)	N/A	1.215	7.041	527.681	0.000	0.093	0.000	0.000	0.318	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	164	124	405	0	208	0	0	107	0
N.S.	1	1.02	0.77	2.52	0.00	1.29	0.00	0.00	0.66	0.00
time (sec)	N/A	1.130	6.398	9.130	0.000	0.092	0.000	0.000	0.276	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	174	128	487	0	226	0	0	82	0
N.S.	1	1.02	0.75	2.85	0.00	1.32	0.00	0.00	0.48	0.00
time (sec)	N/A	1.166	6.719	12.226	0.000	0.091	0.000	0.000	0.224	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	202	161	548	0	254	0	0	115	0
N.S.	1	0.95	0.76	2.57	0.00	1.19	0.00	0.00	0.54	0.00
time (sec)	N/A	1.413	7.430	16.744	0.000	0.102	0.000	0.000	0.200	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	277	225	917	0	364	0	0	149	0
N.S.	1	0.94	0.76	3.11	0.00	1.23	0.00	0.00	0.51	0.00
time (sec)	N/A	1.990	6.150	2342.573	0.000	0.108	0.000	0.000	0.389	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	248	192	970	0	326	0	0	149	0
N.S.	1	1.02	0.79	3.98	0.00	1.34	0.00	0.00	0.61	0.00
time (sec)	N/A	1.717	12.903	2639.665	0.000	0.102	0.000	0.000	0.363	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	238	166	886	0	298	0	0	149	0
N.S.	1	1.00	0.69	3.71	0.00	1.25	0.00	0.00	0.62	0.00
time (sec)	N/A	1.709	11.342	2609.825	0.000	0.100	0.000	0.000	0.365	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	241	172	641	0	270	0	0	139	0
N.S.	1	1.02	0.73	2.70	0.00	1.14	0.00	0.00	0.59	0.00
time (sec)	N/A	1.693	10.849	13.826	0.000	0.106	0.000	0.000	0.329	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	253	180	664	0	295	0	0	108	0
N.S.	1	1.03	0.73	2.71	0.00	1.20	0.00	0.00	0.44	0.00
time (sec)	N/A	1.715	10.787	20.325	0.000	0.101	0.000	0.000	0.221	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	281	219	745	0	332	0	0	149	0
N.S.	1	0.95	0.74	2.53	0.00	1.13	0.00	0.00	0.51	0.00
time (sec)	N/A	2.001	11.297	24.900	0.000	0.124	0.000	0.000	0.201	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	224	225	441	0	0	0	0	18	0
N.S.	1	1.07	1.07	2.10	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.954	13.654	15.178	0.000	0.000	0.000	0.000	0.188	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	125	300	0	0	0	0	16	0
N.S.	1	1.00	0.99	2.38	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.097	3.630	5.154	0.000	0.000	0.000	0.000	0.171	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	217	0	0	0	0	9	0
N.S.	1	1.00	0.75	2.15	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.806	1.686	4.263	0.000	0.000	0.000	0.000	0.166	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	126	220	295	0	0	0	0	18	0
N.S.	1	0.85	1.48	1.98	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.009	21.565	6.437	0.000	0.000	0.000	0.000	0.158	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	203	542	822	0	0	0	0	18	0
N.S.	1	1.03	2.75	4.17	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.550	7.804	8.743	0.000	0.000	0.000	0.000	0.173	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	389	735	1005	0	0	0	0	30	0
N.S.	1	0.96	1.81	2.48	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	3.160	7.565	42.842	0.000	0.000	0.000	0.000	0.173	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	306	681	856	0	0	0	0	28	0
N.S.	1	0.97	2.16	2.71	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.397	7.324	7.309	0.000	0.000	0.000	0.000	0.164	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	252	639	721	0	0	0	0	22	0
N.S.	1	0.97	2.46	2.77	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.819	7.229	6.513	0.000	0.000	0.000	0.000	0.162	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	247	626	808	0	0	0	0	35	0
N.S.	1	0.96	2.43	3.13	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.755	7.194	6.849	0.000	0.000	0.000	0.000	0.175	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	275	655	849	0	0	0	0	39	0
N.S.	1	0.97	2.31	2.99	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.955	7.399	9.047	0.000	0.000	0.000	0.000	0.169	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	350	701	1066	0	0	0	0	39	0
N.S.	1	0.96	1.93	2.94	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.565	7.514	11.371	0.000	0.000	0.000	0.000	0.210	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	476	844	1975	0	0	0	0	44	0
N.S.	1	0.99	1.76	4.11	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	3.625	7.829	9.565	0.000	0.000	0.000	0.000	0.242	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	403	797	1744	0	0	0	0	38	0
N.S.	1	1.00	1.97	4.31	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.819	7.387	7.690	0.000	0.000	0.000	0.000	0.249	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	397	784	1850	0	0	0	0	57	0
N.S.	1	0.99	1.95	4.60	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.691	7.450	8.786	0.000	0.000	0.000	0.000	0.262	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	397	786	1937	0	0	0	0	63	0
N.S.	1	0.99	1.96	4.84	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.791	7.571	6.892	0.000	0.000	0.000	0.000	0.168	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	428	820	1977	0	0	0	0	63	0
N.S.	1	1.00	1.92	4.63	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	3.030	7.845	11.176	0.000	0.000	0.000	0.000	0.153	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	521	519	865	2195	0	0	0	0	63	0
N.S.	1	1.00	1.66	4.21	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	3.894	8.026	13.546	0.000	0.000	0.000	0.000	0.161	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	214	0	91	0	0	20	0
N.S.	1	1.00	0.73	3.34	0.00	1.42	0.00	0.00	0.31	0.00
time (sec)	N/A	0.343	0.090	2.661	0.000	0.087	0.000	0.000	0.184	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	46	183	0	76	0	0	18	0
N.S.	1	1.00	0.77	3.05	0.00	1.27	0.00	0.00	0.30	0.00
time (sec)	N/A	0.356	0.067	1.881	0.000	0.080	0.000	0.000	0.180	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	53	0	0	11	0
N.S.	1	1.00	1.00	3.62	0.00	1.43	0.00	0.00	0.30	0.00
time (sec)	N/A	0.265	0.054	1.737	0.000	0.082	0.000	0.000	0.190	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	59	0	0	20	0
N.S.	1	1.00	1.00	3.62	0.00	1.59	0.00	0.00	0.54	0.00
time (sec)	N/A	0.266	0.061	2.832	0.000	0.083	0.000	0.000	0.198	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	180	0	71	0	0	20	0
N.S.	1	1.00	0.78	2.81	0.00	1.11	0.00	0.00	0.31	0.00
time (sec)	N/A	0.348	0.065	4.204	0.000	0.085	0.000	0.000	0.162	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	203	0	77	0	0	20	0
N.S.	1	1.00	0.88	3.17	0.00	1.20	0.00	0.00	0.31	0.00
time (sec)	N/A	0.347	0.103	5.862	0.000	0.092	0.000	0.000	0.162	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	471	3321	2095	0	0	0	0	69	0
N.S.	1	1.00	7.02	4.43	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.303	25.964	56.475	0.000	0.000	0.000	0.000	0.233	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	380	2899	1515	0	0	0	0	69	0
N.S.	1	0.97	7.43	3.88	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.632	22.664	44.335	0.000	0.000	0.000	0.000	0.261	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	306	346	1036	0	0	0	0	69	0
N.S.	1	0.94	1.07	3.20	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.198	16.252	34.799	0.000	0.000	0.000	0.000	0.256	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	372	635	667	0	0	0	0	65	0
N.S.	1	0.91	1.55	1.62	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.378	18.162	27.329	0.000	0.000	0.000	0.000	0.245	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	412	787	666	0	0	0	0	53	0
N.S.	1	0.93	1.77	1.50	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.782	20.167	21.931	0.000	0.000	0.000	0.000	0.197	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	492	1121	1145	0	0	0	0	69	0
N.S.	1	0.92	2.10	2.15	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.209	20.043	20.662	0.000	0.000	0.000	0.000	0.189	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	584	1533	1656	0	0	0	0	69	0
N.S.	1	0.94	2.47	2.67	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.948	16.489	27.438	0.000	0.000	0.000	0.000	0.180	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	554	3739	2693	0	0	0	0	114	0
N.S.	1	0.99	6.65	4.79	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.870	28.805	70.526	0.000	0.000	0.000	0.000	0.302	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	465	3318	2095	0	0	0	0	114	0
N.S.	1	0.98	7.01	4.43	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	2.260	26.477	60.625	0.000	0.000	0.000	0.000	0.320	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	380	2931	1631	0	0	0	0	114	0
N.S.	1	0.97	7.46	4.15	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.757	23.951	43.641	0.000	0.000	0.000	0.000	0.299	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	441	5981	1371	0	0	0	0	114	0
N.S.	1	0.92	12.49	2.86	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.963	26.428	15.346	0.000	0.000	0.000	0.000	0.304	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	482	927	1245	0	0	0	0	108	0
N.S.	1	0.95	1.82	2.45	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	2.385	17.647	14.680	0.000	0.000	0.000	0.000	0.287	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	496	1134	1389	0	0	0	0	90	0
N.S.	1	0.93	2.13	2.61	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.456	19.174	12.402	0.000	0.000	0.000	0.000	0.236	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	626	589	1489	1739	0	0	0	0	114	0
N.S.	1	0.94	2.38	2.78	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	3.175	20.981	11.607	0.000	0.000	0.000	0.000	0.191	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	696	1888	2280	0	0	0	0	114	0
N.S.	1	0.95	2.59	3.12	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	3.728	22.340	13.075	0.000	0.000	0.000	0.000	0.212	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F(-1)	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	662	659	4198	0	0	0	0	0	159	0
N.S.	1	1.00	6.34	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	3.685	30.005	180.000	0.000	0.000	0.000	0.000	0.353	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F(-1)	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	562	554	3755	0	0	0	0	0	159	0
N.S.	1	0.99	6.68	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	2.941	28.412	180.000	0.000	0.000	0.000	0.000	0.350	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F(-1)	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	474	466	3348	0	0	0	0	0	159	0
N.S.	1	0.98	7.06	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	2.287	26.752	180.000	0.000	0.000	0.000	0.000	0.368	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F(-1)	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	553	520	7032	0	0	0	0	0	159	0
N.S.	1	0.94	12.72	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	2.482	27.647	180.000	0.000	0.000	0.000	0.000	0.354	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F(-1)	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	596	568	7700	0	0	0	0	0	159	0
N.S.	1	0.95	12.92	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	3.077	28.062	180.000	0.000	0.000	0.000	0.000	0.351	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	576	1278	1854	0	0	0	0	151	0
N.S.	1	0.95	2.11	3.05	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	3.105	20.927	20.537	0.000	0.000	0.000	0.000	0.322	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	593	1504	2004	0	0	0	0	127	0
N.S.	1	0.95	2.41	3.21	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	3.242	22.725	12.481	0.000	0.000	0.000	0.000	0.294	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	724	693	1857	2353	0	0	0	0	159	0
N.S.	1	0.96	2.56	3.25	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	3.884	21.841	11.881	0.000	0.000	0.000	0.000	0.188	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	839	807	703	2924	0	0	0	0	159	0
N.S.	1	0.96	0.84	3.49	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	4.533	18.084	15.783	0.000	0.000	0.000	0.000	0.201	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	396	2987	1517	0	0	0	0	29	0
N.S.	1	0.98	7.41	3.76	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.676	23.587	32.020	0.000	0.000	0.000	0.000	0.177	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	315	355	940	0	0	0	0	29	0
N.S.	1	0.95	1.08	2.85	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.209	17.172	15.308	0.000	0.000	0.000	0.000	0.171	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	251	302	486	0	0	0	0	27	0
N.S.	1	0.93	1.12	1.80	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.877	16.223	10.220	0.000	0.000	0.000	0.000	0.175	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	249	157	179	0	0	0	0	21	0
N.S.	1	0.93	0.59	0.67	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.813	4.857	7.375	0.000	0.000	0.000	0.000	0.185	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	457	814	507	0	0	0	0	29	0
N.S.	1	0.94	1.67	1.04	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.043	20.712	9.119	0.000	0.000	0.000	0.000	0.174	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	501	1157	1054	0	0	0	0	29	0
N.S.	1	0.93	2.15	1.96	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.276	20.963	11.359	0.000	0.000	0.000	0.000	0.164	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	3433	1938	0	0	0	0	41	0
N.S.	1	1.00	7.93	4.48	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.933	26.313	30.927	0.000	0.000	0.000	0.000	0.177	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	352	2988	1300	0	0	0	0	39	0
N.S.	1	1.02	8.66	3.77	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.344	22.964	12.939	0.000	0.000	0.000	0.000	0.166	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	328	305	916	0	0	0	0	33	0
N.S.	1	1.01	0.94	2.83	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.263	15.447	5.618	0.000	0.000	0.000	0.000	0.155	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	441	1050	1098	0	0	0	0	46	0
N.S.	1	0.93	2.21	2.31	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.720	14.527	8.880	0.000	0.000	0.000	0.000	0.167	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	548	1551	1597	0	0	0	0	50	0
N.S.	1	0.98	2.77	2.85	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.651	20.591	11.391	0.000	0.000	0.000	0.000	0.165	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	602	4316	3734	0	0	0	0	57	0
N.S.	1	0.99	7.11	6.15	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.955	29.596	34.253	0.000	0.000	0.000	0.000	0.162	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	498	3891	2960	0	0	0	0	55	0
N.S.	1	1.00	7.84	5.97	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.188	27.543	15.615	0.000	0.000	0.000	0.000	0.189	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	465	3493	2439	0	0	0	0	49	0
N.S.	1	0.99	7.45	5.20	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.963	26.049	11.176	0.000	0.000	0.000	0.000	0.183	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	3155	1970	0	0	0	0	68	0
N.S.	1	1.00	7.32	4.57	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.771	23.661	7.430	0.000	0.000	0.000	0.000	0.174	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	590	1496	2667	0	0	0	0	74	0
N.S.	1	0.98	2.49	4.43	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	2.722	16.653	11.912	0.000	0.000	0.000	0.000	0.182	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	733	719	2318	4167	0	0	0	0	74	0
N.S.	1	0.98	3.16	5.68	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	3.645	23.352	17.741	0.000	0.000	0.000	0.000	0.206	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	246	298	382	0	0	0	0	41	0
N.S.	1	0.92	1.12	1.44	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.807	6.073	28.578	0.000	0.000	0.000	0.000	0.282	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	104	110	0	0	0	0	35	0
N.S.	1	1.00	0.80	0.85	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.432	0.117	13.697	0.000	0.000	0.000	0.000	0.257	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	147	140	0	0	0	0	48	0
N.S.	1	1.00	1.07	1.02	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.436	0.156	11.740	0.000	0.000	0.000	0.000	0.222	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	446	236	373	0	0	0	0	52	0
N.S.	1	0.93	0.49	0.78	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.845	1.653	14.588	0.000	0.000	0.000	0.000	0.181	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	0	35	61	37
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.00	1.06	1.85	1.12
time (sec)	N/A	0.435	31.282	0.217	4.080	0.131	0.000	0.644	0.218	42.359

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	644	617	317	0	0	0	0	0	144	0
N.S.	1	0.96	0.49	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	4.317	4.631	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	455	450	259	0	0	0	0	0	117	0
N.S.	1	0.99	0.57	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	2.832	2.292	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	315	205	0	0	0	0	0	90	0
N.S.	1	0.96	0.63	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.818	1.000	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	220	163	0	0	0	0	0	63	0
N.S.	1	1.01	0.75	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.947	0.417	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	305	10630	0	0	0	0	0	14	0
N.S.	1	1.02	35.55	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.397	36.705	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	0	33	35	54	0	35	97	37
N.S.	1	1.00	0.00	0.94	1.00	1.54	0.00	1.00	2.77	1.06
time (sec)	N/A	1.100	0.000	0.232	2.093	0.141	0.000	1.119	0.267	45.437

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	35	34	35	59	37
N.S.	1	1.00	1.06	0.94	1.00	1.00	0.97	1.00	1.69	1.06
time (sec)	N/A	0.479	72.609	0.230	2.678	0.097	8.419	0.705	0.215	42.785

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	35	34	35	26	37
N.S.	1	1.00	1.06	0.94	1.00	1.00	0.97	1.00	0.74	1.06
time (sec)	N/A	0.513	53.186	0.238	2.887	0.109	3.112	0.673	0.184	44.113

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	63	34	35	38	37
N.S.	1	1.00	1.06	0.94	1.00	1.80	0.97	1.00	1.09	1.06
time (sec)	N/A	1.111	39.688	0.244	1.783	0.107	11.129	1.205	0.179	48.341

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [318] had the largest ratio of [.8181819999999999965]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	14	13	0.96	29	0.448
2	A	12	11	0.96	29	0.379
3	A	6	6	1.03	27	0.222
4	A	2	2	1.00	21	0.095
5	A	8	8	1.00	27	0.296
6	A	8	8	1.00	29	0.276
7	A	11	10	1.05	29	0.345
8	A	13	12	1.01	29	0.414
9	A	13	12	0.96	29	0.414
10	A	16	15	0.93	31	0.484
11	A	14	13	0.94	31	0.419
12	A	8	8	0.99	29	0.276
13	A	4	4	0.88	23	0.174
14	A	10	10	1.00	29	0.345
15	A	10	10	1.00	31	0.323
16	A	10	10	1.00	31	0.323
17	A	13	12	1.05	31	0.387
18	A	15	14	1.01	31	0.452
19	A	17	16	0.98	31	0.516
20	A	9	9	0.95	29	0.310
21	A	5	5	0.87	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	13	13	1.05	29	0.448
23	A	12	12	1.01	31	0.387
24	A	12	12	1.02	31	0.387
25	A	13	13	1.05	31	0.419
26	A	15	14	1.07	31	0.452
27	A	17	16	1.03	31	0.516
28	A	19	18	1.00	31	0.581
29	A	9	9	0.92	29	0.310
30	A	5	5	0.83	23	0.217
31	A	15	15	1.07	29	0.517
32	A	15	15	1.04	31	0.484
33	A	15	15	0.95	31	0.484
34	A	15	15	1.04	31	0.484
35	A	15	15	1.06	31	0.484
36	A	17	16	1.08	31	0.516
37	A	20	19	1.05	31	0.613
38	A	12	11	0.88	31	0.355
39	A	10	9	0.88	31	0.290
40	A	4	4	1.09	31	0.129
41	A	9	9	0.87	29	0.310
42	A	4	4	1.00	23	0.174
43	A	5	5	1.00	29	0.172
44	A	9	8	1.01	31	0.258
45	A	11	10	0.93	31	0.323
46	A	11	10	0.90	31	0.323
47	A	13	12	0.94	31	0.387
48	A	6	6	1.03	31	0.194
49	A	11	11	1.01	31	0.355
50	A	9	9	1.04	29	0.310
51	A	4	4	0.98	23	0.174
52	A	7	7	1.01	29	0.241
53	A	11	10	1.07	31	0.323

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	13	12	0.99	31	0.387
55	A	14	13	0.96	31	0.419
56	A	15	14	0.99	31	0.452
57	A	8	8	1.07	31	0.258
58	A	13	13	1.05	31	0.419
59	A	11	11	1.08	31	0.355
60	A	9	9	1.02	29	0.310
61	A	6	6	0.97	23	0.261
62	A	9	9	1.09	29	0.310
63	A	13	12	1.11	31	0.387
64	A	15	14	1.04	31	0.452
65	A	10	10	1.10	31	0.323
66	A	15	15	1.09	31	0.484
67	A	13	13	1.12	31	0.419
68	A	11	11	1.08	31	0.355
69	A	11	11	0.99	29	0.379
70	A	8	8	0.96	23	0.348
71	A	11	11	1.14	29	0.379
72	A	15	14	1.14	31	0.452
73	A	17	16	1.06	31	0.516
74	A	11	11	1.00	33	0.333
75	A	9	9	1.01	33	0.273
76	A	9	9	1.11	31	0.290
77	A	4	4	1.00	25	0.160
78	A	6	5	1.00	31	0.161
79	A	6	5	1.00	33	0.152
80	A	8	7	0.93	33	0.212
81	A	10	9	0.94	33	0.273
82	A	14	14	1.02	33	0.424
83	A	12	12	1.04	33	0.364
84	A	11	11	1.06	31	0.355
85	A	6	6	0.98	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	8	1.03	31	0.258
87	A	9	8	1.06	33	0.242
88	A	9	8	0.99	33	0.242
89	A	11	10	0.98	33	0.303
90	A	13	12	0.96	33	0.364
91	A	15	15	1.05	33	0.455
92	A	13	13	1.03	31	0.419
93	A	8	8	0.96	25	0.320
94	A	12	11	1.06	31	0.355
95	A	12	11	1.06	33	0.333
96	A	12	11	1.02	33	0.333
97	A	12	11	1.03	33	0.333
98	A	14	13	1.01	33	0.394
99	A	16	15	1.00	33	0.455
100	A	17	16	1.12	33	0.485
101	A	14	13	1.09	33	0.394
102	A	11	10	1.06	31	0.323
103	A	6	5	1.00	25	0.200
104	A	8	7	1.00	31	0.226
105	A	11	10	1.08	33	0.303
106	A	14	13	1.07	33	0.394
107	A	20	19	1.10	33	0.576
108	A	17	16	1.07	33	0.485
109	A	14	13	1.05	33	0.394
110	A	11	10	1.03	31	0.323
111	A	6	5	1.00	25	0.200
112	A	11	10	1.05	31	0.323
113	A	14	13	1.05	33	0.394
114	A	17	16	1.05	33	0.485
115	A	20	19	1.10	33	0.576
116	A	17	16	1.07	33	0.485
117	A	14	13	1.05	33	0.394

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
118	A	11	10	1.05	31	0.323
119	A	8	7	0.98	25	0.280
120	A	14	13	1.07	31	0.419
121	A	17	16	1.08	33	0.485
122	A	20	19	1.07	33	0.576
123	A	14	14	0.99	31	0.452
124	A	12	12	0.98	31	0.387
125	A	12	12	1.02	31	0.387
126	A	10	10	1.04	31	0.323
127	A	10	10	1.00	31	0.323
128	A	12	12	1.02	31	0.387
129	A	12	12	0.95	31	0.387
130	A	14	14	0.98	33	0.424
131	A	14	14	1.01	33	0.424
132	A	12	12	1.05	33	0.364
133	A	12	12	1.03	33	0.364
134	A	12	12	1.03	33	0.364
135	A	14	14	0.98	33	0.424
136	A	14	14	0.96	33	0.424
137	A	17	17	1.00	33	0.515
138	A	18	18	1.03	33	0.545
139	A	15	15	1.06	33	0.455
140	A	15	15	1.04	33	0.455
141	A	16	16	1.06	33	0.485
142	A	15	15	1.02	33	0.455
143	A	17	17	1.01	33	0.515
144	A	18	18	0.99	33	0.545
145	A	10	10	0.95	33	0.303
146	A	10	10	0.98	33	0.303
147	A	8	8	1.06	33	0.242
148	A	8	8	1.07	33	0.242
149	A	10	10	0.97	33	0.303

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	10	10	0.94	33	0.303
151	A	12	12	0.99	33	0.364
152	A	13	13	1.03	33	0.394
153	A	10	10	1.04	33	0.303
154	A	10	10	1.07	33	0.303
155	A	10	10	1.07	33	0.303
156	A	12	12	1.00	33	0.364
157	A	13	13	0.98	33	0.394
158	A	15	15	1.02	33	0.455
159	A	15	15	1.05	33	0.455
160	A	13	13	1.08	33	0.394
161	A	13	13	1.08	33	0.394
162	A	13	13	1.08	33	0.394
163	A	13	13	1.08	33	0.394
164	A	15	15	1.03	33	0.455
165	A	15	15	1.00	33	0.455
166	A	12	11	0.95	35	0.314
167	A	10	9	0.94	35	0.257
168	A	8	7	0.94	35	0.200
169	A	6	5	1.00	35	0.143
170	A	6	5	1.00	35	0.143
171	A	4	4	1.00	35	0.114
172	A	6	6	0.98	35	0.171
173	A	8	8	0.98	35	0.229
174	A	13	12	0.96	35	0.343
175	A	11	10	0.98	35	0.286
176	A	9	8	0.99	35	0.229
177	A	9	8	1.00	35	0.229
178	A	9	8	1.02	35	0.229
179	A	7	7	1.04	35	0.200
180	A	9	9	1.00	35	0.257
181	A	11	11	0.98	35	0.314

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
182	A	16	15	1.00	35	0.429
183	A	14	13	1.01	35	0.371
184	A	12	11	1.03	35	0.314
185	A	12	11	1.00	35	0.314
186	A	12	11	1.02	35	0.314
187	A	12	11	1.05	35	0.314
188	A	10	10	1.06	35	0.286
189	A	12	12	1.03	35	0.343
190	A	14	14	1.01	35	0.400
191	A	14	13	1.07	35	0.371
192	A	11	10	1.06	35	0.286
193	A	8	7	1.00	35	0.200
194	A	7	6	1.00	35	0.171
195	A	10	9	1.06	35	0.257
196	A	13	12	1.10	35	0.343
197	A	14	13	1.04	35	0.371
198	A	11	10	1.03	35	0.286
199	A	7	6	1.00	35	0.171
200	A	10	9	1.01	35	0.257
201	A	13	12	1.05	35	0.343
202	A	17	16	1.07	35	0.457
203	A	14	13	1.06	35	0.371
204	A	10	9	1.04	35	0.257
205	A	10	9	1.04	35	0.257
206	A	13	12	1.05	35	0.343
207	A	16	15	1.07	35	0.429
208	A	20	19	1.08	35	0.543
209	A	17	16	1.08	35	0.457
210	A	13	12	1.07	35	0.343
211	A	13	12	1.07	35	0.343
212	A	13	12	1.07	35	0.343
213	A	16	15	1.07	35	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	19	18	1.09	35	0.514
215	A	12	11	0.92	29	0.379
216	A	6	6	1.02	27	0.222
217	A	2	2	1.00	21	0.095
218	A	8	8	1.00	27	0.296
219	A	8	8	1.00	29	0.276
220	A	11	10	1.05	29	0.345
221	A	13	12	0.98	29	0.414
222	A	13	12	0.93	29	0.414
223	A	12	11	0.87	31	0.355
224	A	8	8	1.04	29	0.276
225	A	4	4	1.03	23	0.174
226	A	8	8	1.00	29	0.276
227	A	9	9	1.00	31	0.290
228	A	9	9	1.05	31	0.290
229	A	12	11	1.05	31	0.355
230	A	14	13	0.96	31	0.419
231	A	15	14	0.86	31	0.452
232	A	10	10	1.04	29	0.345
233	A	6	6	1.05	23	0.261
234	A	11	11	1.07	29	0.379
235	A	10	10	1.02	31	0.323
236	A	11	11	1.02	31	0.355
237	A	12	12	1.06	31	0.387
238	A	14	13	1.06	31	0.419
239	A	16	15	0.95	31	0.484
240	A	17	16	0.86	31	0.516
241	A	13	13	1.03	29	0.448
242	A	8	8	1.05	23	0.348
243	A	13	13	1.07	29	0.448
244	A	13	13	1.05	31	0.419
245	A	13	13	0.96	31	0.419

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	14	14	1.04	31	0.452
247	A	14	14	1.06	31	0.452
248	A	16	15	1.06	31	0.484
249	A	19	18	0.93	31	0.581
250	A	13	12	1.10	31	0.387
251	A	10	9	1.09	31	0.290
252	A	11	10	0.97	29	0.345
253	A	6	5	1.00	23	0.217
254	A	7	6	1.00	29	0.207
255	A	11	10	0.97	31	0.323
256	A	13	12	1.08	31	0.387
257	A	16	15	1.09	31	0.484
258	A	14	13	1.05	31	0.419
259	A	10	9	1.19	31	0.290
260	A	10	9	1.22	29	0.310
261	A	8	7	1.11	23	0.304
262	A	9	8	1.19	29	0.276
263	A	12	11	1.10	31	0.355
264	A	15	14	1.05	31	0.452
265	A	17	16	1.07	31	0.516
266	A	13	12	1.13	31	0.387
267	A	11	10	1.17	31	0.323
268	A	12	11	1.14	29	0.379
269	A	11	10	1.16	23	0.435
270	A	11	10	1.21	29	0.345
271	A	14	13	1.12	31	0.419
272	A	17	16	1.06	31	0.516
273	A	17	16	1.14	31	0.516
274	A	15	14	1.18	31	0.452
275	A	12	11	1.16	31	0.355
276	A	14	13	1.15	29	0.448
277	A	13	12	1.18	23	0.522

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	14	13	1.21	29	0.448
279	A	16	15	1.13	31	0.484
280	A	19	18	1.08	31	0.581
281	A	5	4	0.96	34	0.118
282	A	4	4	1.00	34	0.118
283	A	3	3	1.00	32	0.094
284	A	2	2	1.00	26	0.077
285	A	3	3	1.00	32	0.094
286	A	5	4	1.00	34	0.118
287	A	5	5	1.00	34	0.147
288	A	5	4	0.96	34	0.118
289	A	11	10	1.07	34	0.294
290	A	11	10	1.03	34	0.294
291	A	7	6	1.00	32	0.188
292	A	5	4	1.00	26	0.154
293	A	8	7	1.00	32	0.219
294	A	12	11	1.02	34	0.324
295	A	14	13	1.07	34	0.382
296	A	21	21	1.06	33	0.636
297	A	18	18	1.05	33	0.545
298	A	17	17	1.03	31	0.548
299	A	12	12	1.01	25	0.480
300	A	14	14	1.01	31	0.452
301	A	18	18	1.07	33	0.545
302	A	21	21	1.02	33	0.636
303	A	24	24	1.03	33	0.727
304	A	21	21	1.04	33	0.636
305	A	20	20	1.03	31	0.645
306	A	15	15	1.03	25	0.600
307	A	18	18	1.03	31	0.581
308	A	18	18	1.06	33	0.545
309	A	21	21	1.05	33	0.636

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	24	24	1.04	33	0.727
311	A	24	24	1.04	33	0.727
312	A	23	23	1.03	31	0.742
313	A	18	18	1.04	25	0.720
314	A	21	21	1.04	31	0.677
315	A	21	21	1.05	33	0.636
316	A	21	21	1.03	33	0.636
317	A	24	24	1.05	33	0.727
318	A	27	27	1.04	33	0.818
319	A	18	18	1.06	33	0.545
320	A	15	15	1.05	33	0.455
321	A	14	14	1.02	31	0.452
322	A	9	9	1.00	25	0.360
323	A	9	9	1.00	31	0.290
324	A	18	18	1.05	33	0.545
325	A	21	21	1.02	33	0.636
326	A	18	18	1.01	33	0.545
327	A	15	15	1.07	33	0.455
328	A	14	14	1.07	31	0.452
329	A	12	12	1.06	25	0.480
330	A	14	14	1.04	31	0.452
331	A	21	21	1.11	33	0.636
332	A	24	24	1.07	33	0.727
333	A	21	21	1.01	33	0.636
334	A	18	18	1.02	33	0.545
335	A	15	15	1.02	33	0.455
336	A	17	17	1.03	31	0.548
337	A	15	15	1.04	25	0.600
338	A	21	21	1.07	31	0.677
339	A	24	24	1.08	33	0.727
340	A	27	27	1.07	33	0.818
341	A	5	5	1.00	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	5	5	1.00	34	0.147
343	A	8	8	1.00	28	0.286
344	A	15	15	1.04	34	0.441
345	A	14	14	0.95	31	0.452
346	A	12	12	0.96	31	0.387
347	A	12	12	0.99	31	0.387
348	A	10	10	1.04	31	0.323
349	A	10	10	1.00	31	0.323
350	A	12	12	0.99	31	0.387
351	A	12	12	0.93	31	0.387
352	A	15	15	0.88	33	0.455
353	A	13	13	0.91	33	0.394
354	A	13	13	0.96	33	0.394
355	A	11	11	1.04	33	0.333
356	A	11	11	1.02	33	0.333
357	A	11	11	1.02	33	0.333
358	A	13	13	0.94	33	0.394
359	A	16	16	0.89	33	0.485
360	A	16	16	0.96	33	0.485
361	A	14	14	1.05	33	0.424
362	A	14	14	1.03	33	0.424
363	A	14	14	0.99	33	0.424
364	A	14	14	1.02	33	0.424
365	A	15	15	1.10	33	0.455
366	A	12	12	1.08	33	0.364
367	A	8	8	0.97	33	0.242
368	A	5	5	1.00	33	0.152
369	A	10	10	1.00	33	0.303
370	A	15	15	1.08	33	0.455
371	A	15	15	0.97	33	0.455
372	A	12	12	0.98	33	0.364
373	A	12	12	0.97	33	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	12	12	0.92	33	0.364
375	A	15	15	0.95	33	0.455
376	A	18	18	0.95	33	0.545
377	A	15	15	1.01	33	0.455
378	A	15	15	1.01	33	0.455
379	A	15	15	0.99	33	0.455
380	A	15	15	0.97	33	0.455
381	A	18	18	0.98	33	0.545
382	A	21	21	0.98	33	0.636
383	A	5	5	1.00	36	0.139
384	A	5	5	1.00	36	0.139
385	A	3	3	1.00	36	0.083
386	A	3	3	1.00	36	0.083
387	A	5	5	1.00	36	0.139
388	A	5	5	1.00	36	0.139
389	A	13	13	1.08	36	0.361
390	A	9	9	1.04	36	0.250
391	A	6	6	1.00	36	0.167
392	A	3	3	1.00	36	0.083
393	A	11	11	0.99	36	0.306
394	A	16	16	1.05	36	0.444
395	A	17	17	1.01	35	0.486
396	A	13	13	1.00	35	0.371
397	A	11	11	1.02	35	0.314
398	A	8	8	1.00	35	0.229
399	A	8	8	1.00	35	0.229
400	A	11	11	1.03	35	0.314
401	A	14	14	1.04	35	0.400
402	A	20	20	1.01	35	0.571
403	A	17	17	1.00	35	0.486
404	A	14	14	1.01	35	0.400
405	A	13	13	1.03	35	0.371

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	11	11	1.00	35	0.314
407	A	11	11	1.02	35	0.314
408	A	14	14	1.03	35	0.400
409	A	17	17	1.02	35	0.486
410	A	23	23	1.01	35	0.657
411	A	20	20	1.01	35	0.571
412	A	17	17	1.01	35	0.486
413	A	16	16	1.01	35	0.457
414	A	16	16	1.02	35	0.457
415	A	14	14	1.01	35	0.400
416	A	14	14	1.03	35	0.400
417	A	17	17	1.02	35	0.486
418	A	20	20	1.03	35	0.571
419	A	11	11	1.01	43	0.256
420	A	14	14	1.00	35	0.400
421	A	14	14	1.02	35	0.400
422	A	5	5	1.00	35	0.143
423	A	5	5	1.00	35	0.143
424	A	8	8	1.01	35	0.229
425	A	11	11	1.03	35	0.314
426	A	13	13	1.05	35	0.371
427	A	10	10	1.01	35	0.286
428	A	7	7	1.08	35	0.200
429	A	8	8	1.09	35	0.229
430	A	11	11	1.05	35	0.314
431	A	16	16	1.04	35	0.457
432	A	13	13	1.04	35	0.371
433	A	10	10	1.05	35	0.286
434	A	10	10	1.03	35	0.286
435	A	11	11	1.05	35	0.314
436	A	14	14	1.02	35	0.400
437	A	16	16	1.02	38	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	3	3	1.00	38	0.079
439	A	3	3	1.00	38	0.079
440	A	6	6	1.00	38	0.158
441	A	2	2	1.00	33	0.061
442	A	2	2	1.00	33	0.061
443	A	4	4	1.00	33	0.121
444	A	4	4	1.00	33	0.121
445	A	2	2	1.00	33	0.061
446	A	2	2	1.00	33	0.061
447	A	4	4	1.00	33	0.121
448	A	4	4	1.00	33	0.121
449	N/A	2	0	1.00	33	0.000
450	A	12	12	1.03	33	0.364
451	A	10	10	1.00	33	0.303
452	A	8	8	1.01	33	0.242
453	A	8	8	0.98	31	0.258
454	A	9	8	0.98	33	0.242
455	N/A	5	0	1.00	35	0.000
456	N/A	2	0	1.00	35	0.000
457	N/A	2	0	1.00	35	0.000
458	N/A	5	0	1.00	35	0.000
459	A	14	14	0.97	31	0.452
460	A	14	14	1.01	31	0.452
461	A	12	12	1.00	31	0.387
462	A	12	12	1.03	31	0.387
463	A	14	14	1.01	31	0.452
464	A	14	14	0.99	31	0.452
465	A	16	16	0.98	33	0.485
466	A	14	14	1.02	33	0.424
467	A	14	14	1.02	33	0.424
468	A	15	15	1.04	33	0.455
469	A	17	17	1.00	33	0.515

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	19	19	1.01	33	0.576
471	A	17	17	1.02	33	0.515
472	A	18	18	1.01	33	0.545
473	A	17	17	1.02	33	0.515
474	A	17	17	1.05	33	0.515
475	A	20	20	1.02	33	0.606
476	A	14	14	0.96	33	0.424
477	A	14	14	0.99	33	0.424
478	A	12	12	1.05	33	0.364
479	A	12	12	1.04	33	0.364
480	A	14	14	0.98	33	0.424
481	A	14	14	0.95	33	0.424
482	A	16	16	1.00	33	0.485
483	A	15	15	1.06	33	0.455
484	A	15	15	1.04	33	0.455
485	A	14	14	1.03	33	0.424
486	A	17	17	1.02	33	0.515
487	A	19	19	1.02	33	0.576
488	A	17	17	1.07	33	0.515
489	A	18	18	1.07	33	0.545
490	A	17	17	1.07	33	0.515
491	A	17	17	1.07	33	0.515
492	A	19	19	1.04	33	0.576
493	A	12	12	1.07	35	0.343
494	A	10	10	1.10	35	0.286
495	A	8	8	1.15	35	0.229
496	A	6	6	1.25	35	0.171
497	A	8	7	1.01	35	0.200
498	A	8	7	1.01	35	0.200
499	A	10	9	0.95	35	0.257
500	A	12	11	0.95	35	0.314
501	A	15	15	1.05	35	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	13	13	1.07	35	0.371
503	A	11	11	1.12	35	0.314
504	A	9	9	1.19	35	0.257
505	A	11	10	1.03	35	0.286
506	A	11	10	1.01	35	0.286
507	A	11	10	1.00	35	0.286
508	A	13	12	0.98	35	0.343
509	A	15	14	0.97	35	0.400
510	A	18	18	1.06	35	0.514
511	A	16	16	1.09	35	0.457
512	A	14	14	1.12	35	0.400
513	A	12	12	1.17	35	0.343
514	A	14	13	1.05	35	0.371
515	A	14	13	1.03	35	0.371
516	A	14	13	1.01	35	0.371
517	A	14	13	1.03	35	0.371
518	A	16	15	1.02	35	0.429
519	A	18	17	1.00	35	0.486
520	A	21	20	1.14	35	0.571
521	A	18	17	1.12	35	0.486
522	A	15	14	1.09	35	0.400
523	A	12	11	1.06	35	0.314
524	A	9	8	1.01	35	0.229
525	A	10	9	0.86	35	0.257
526	A	13	12	0.94	35	0.343
527	A	16	15	0.97	35	0.429
528	A	13	12	0.95	54	0.222
529	A	21	20	1.09	35	0.571
530	A	18	17	1.07	35	0.486
531	A	15	14	1.05	35	0.400
532	A	12	11	1.02	35	0.314
533	A	9	8	1.01	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	13	12	0.92	35	0.343
535	A	16	15	0.95	35	0.429
536	A	21	20	1.09	35	0.571
537	A	18	17	1.07	35	0.486
538	A	15	14	1.05	35	0.400
539	A	12	11	1.04	35	0.314
540	A	12	11	1.04	35	0.314
541	A	16	15	0.97	35	0.429
542	A	19	18	0.99	35	0.514
543	A	21	20	1.09	35	0.571
544	A	18	17	1.07	35	0.486
545	A	15	14	1.07	35	0.400
546	A	15	14	1.07	35	0.400
547	A	15	14	1.07	35	0.400
548	A	19	18	1.00	35	0.514
549	A	22	21	1.02	35	0.600
550	A	14	14	0.94	31	0.452
551	A	14	14	0.99	31	0.452
552	A	12	12	1.00	31	0.387
553	A	12	12	1.03	31	0.387
554	A	14	14	0.99	31	0.452
555	A	14	14	0.97	31	0.452
556	A	18	18	0.93	33	0.545
557	A	16	16	1.01	33	0.485
558	A	16	16	1.02	33	0.485
559	A	16	16	1.02	33	0.485
560	A	18	18	0.95	33	0.545
561	A	21	21	0.94	33	0.636
562	A	19	19	1.02	33	0.576
563	A	19	19	1.00	33	0.576
564	A	19	19	1.02	33	0.576
565	A	19	19	1.03	33	0.576

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
566	A	21	21	0.95	33	0.636
567	A	20	20	1.07	33	0.606
568	A	15	15	1.00	33	0.455
569	A	11	11	1.00	33	0.333
570	A	14	14	0.85	33	0.424
571	A	17	17	1.03	33	0.515
572	A	23	23	0.96	33	0.697
573	A	20	20	0.97	33	0.606
574	A	17	17	0.97	33	0.515
575	A	17	17	0.96	33	0.515
576	A	17	17	0.97	33	0.515
577	A	20	20	0.96	33	0.606
578	A	23	23	0.99	33	0.697
579	A	20	20	1.00	33	0.606
580	A	20	20	0.99	33	0.606
581	A	20	20	0.99	33	0.606
582	A	20	20	1.00	33	0.606
583	A	23	23	1.00	33	0.697
584	A	7	7	1.00	36	0.194
585	A	7	7	1.00	36	0.194
586	A	5	5	1.00	36	0.139
587	A	5	5	1.00	36	0.139
588	A	7	7	1.00	36	0.194
589	A	7	7	1.00	36	0.194
590	A	16	16	1.00	35	0.457
591	A	13	13	0.97	35	0.371
592	A	10	10	0.94	35	0.286
593	A	10	10	0.91	35	0.286
594	A	13	13	0.93	35	0.371
595	A	15	15	0.92	35	0.429
596	A	19	19	0.94	35	0.543
597	A	19	19	0.99	35	0.543

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	16	16	0.98	35	0.457
599	A	13	13	0.97	35	0.371
600	A	13	13	0.92	35	0.371
601	A	15	15	0.95	35	0.429
602	A	16	16	0.93	35	0.457
603	A	19	19	0.94	35	0.543
604	A	22	22	0.95	35	0.629
605	A	22	22	1.00	35	0.629
606	A	19	19	0.99	35	0.543
607	A	16	16	0.98	35	0.457
608	A	16	16	0.94	35	0.457
609	A	18	18	0.95	35	0.514
610	A	18	18	0.95	35	0.514
611	A	19	19	0.95	35	0.543
612	A	22	22	0.96	35	0.629
613	A	25	25	0.96	35	0.714
614	A	13	13	0.98	35	0.371
615	A	10	10	0.95	35	0.286
616	A	7	7	0.93	35	0.200
617	A	7	7	0.93	35	0.200
618	A	16	16	0.94	35	0.457
619	A	16	16	0.93	35	0.457
620	A	13	13	1.00	35	0.371
621	A	10	10	1.02	35	0.286
622	A	9	9	1.01	35	0.257
623	A	12	12	0.93	35	0.343
624	A	15	15	0.98	35	0.429
625	A	16	16	0.99	35	0.457
626	A	13	13	1.00	35	0.371
627	A	12	12	0.99	35	0.343
628	A	12	12	1.00	35	0.343
629	A	15	15	0.98	35	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	18	18	0.98	35	0.514
631	A	8	8	0.92	38	0.211
632	A	5	5	1.00	38	0.132
633	A	5	5	1.00	38	0.132
634	A	18	18	0.93	38	0.474
635	N/A	4	0	1.00	33	0.000
636	A	21	21	0.96	33	0.636
637	A	19	19	0.99	33	0.576
638	A	16	16	0.96	33	0.485
639	A	11	11	1.01	31	0.355
640	A	15	14	1.02	33	0.424
641	N/A	7	0	1.00	35	0.000
642	N/A	4	0	1.00	35	0.000
643	N/A	4	0	1.00	35	0.000
644	N/A	7	0	1.00	35	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 125

$$\begin{aligned}&\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx\\&=\frac{3}{8}a(A+B)x+\frac{a(5A+4B)\sin(c+dx)}{5d}\\&\quad+\frac{3a(A+B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d}\\&\quad+\frac{aB\cos^4(c+dx)\sin(c+dx)}{5d}-\frac{a(5A+4B)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
3/8*a*(A+B)*x+1/5*a*(5*A+4*B)*sin(d*x+c)/d+3/8*a*(A+B)*cos(d*x+c)*sin(d*x+
c)/d+1/4*a*(A+B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a*B*cos(d*x+c)^4*sin(d*x+c)
/d-1/15*a*(5*A+4*B)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{a(480(A+B)\sin(c+dx)-160(A+2B)\sin^3(c+dx)+96B\sin^5(c+dx)+15(A+B)(12(c+dx)+8}{480d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*(480*(A + B)*Sin[c + d*x] - 160*(A + 2*B)*Sin[c + d*x]^3 + 96*B*Sin[c +
 d*x]^5 + 15*(A + B)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
)))/(480*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {3042, 3447, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{3447}$$
$$\int \cos^3(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left((aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{5}\int \cos^3(c+dx)(a(5A+4B)+5a(A+B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a(5A+4B)+5a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(5a(A+B)\int \cos^4(c+dx)dx+a(5A+4B)\int \cos^3(c+dx)dx\right)+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(a(5A+4B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+5a(A+B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{5}\left(5a(A+B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a(5A+4B)\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(5a(A+B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a(5A+4B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a(5A+4B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a(5A+4B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+\right.}{d}\right.$$
$$\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$\Big\downarrow$ 3115

$$\frac{1}{5}\left(5a(A+B)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a(5A+4B)\left(\frac{1}{3}\sin^3(c+d\right.}{d}\right.$$
$$\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

$\Big\downarrow$ 24

$$\frac{1}{5}\left(5a(A+B)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a(5A+4B)\left(\frac{1}{3}\sin^3(c+dx)-\right.}{d}\right.$$
$$\frac{aB\sin(c+dx)\cos^4(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + (-((a*(5*A + 4*B)*(-Sin[c + d*x]
 + Sin[c + d*x]^3/3))/d) + 5*a*(A + B)*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d
) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 23.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a\left(8(A+B)\sin(2dx+2c)+\frac{2(4A+5B)\sin(3dx+3c)}{3}+(A+B)\sin(4dx+4c)+\frac{2B\sin(5dx+5c)}{5}+4(6A+5B)\sin(dx+c)+12(A+\right.}{32d}$
parts	$\frac{(Aa+Ba)\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{Aa\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}+\frac{Ba\left(\frac{8}{3}+\cos(dx+\right.}{}$
derivativedivides	$\frac{\frac{Ba\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+Aa\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+Ba\left(\frac{\left(\cos(dx+c)^3\right.}{}\right.}{d}$
default	$\frac{\frac{Ba\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+Aa\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+Ba\left(\frac{\left(\cos(dx+c)^3\right.}{}\right.}{d}$
risch	$\frac{3axA}{8}+\frac{3aBx}{8}+\frac{3\sin(dx+c)Aa}{4d}+\frac{5aB\sin(dx+c)}{8d}+\frac{Ba\sin(5dx+5c)}{80d}+\frac{\sin(4dx+4c)Aa}{32d}+\frac{\sin(4dx+4c)Ba}{32d}$
norman	$\frac{3a(A+B)x}{8}+\frac{15a(A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8}+\frac{15a(A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{15a(A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4}+\frac{15a(A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+$
orering	Expression too large to display

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/32*a*(8*(A+B)*sin(2*d*x+2*c)+2/3*(4*A+5*B)*sin(3*d*x+3*c)+(A+B)*sin(4*d*
x+4*c)+2/5*B*sin(5*d*x+5*c)+4*(6*A+5*B)*sin(d*x+c)+12*(A+B)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{45\,(A+B)adx+\left(24\,Ba\cos\left(dx+c\right)^4+30\,(A+B)a\cos\left(dx+c\right)^3+8\,(5\,A+4\,B)a\cos\left(dx+c\right)^2+45\,(\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/120*(45*(A + B)*a*d*x + (24*B*a*cos(d*x + c)^4 + 30*(A + B)*a*cos(d*x +
c)^3 + 8*(5*A + 4*B)*a*cos(d*x + c)^2 + 45*(A + B)*a*cos(d*x + c) + 16*(5*
A + 4*B)*a)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(117) = 234$.

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.66

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{3Aax\sin^4(c+dx)}{8} + \frac{3Aax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Aax\cos^4(c+dx)}{8} + \frac{3Aa\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Aa\sin^3(c+dx)}{3d} + \frac{5Aa\sin \\ x(A+B\cos(c))\,(a\cos(c)+a)\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((3*A*a*x*sin(c + d*x)**4/8 + 3*A*a*x*sin(c + d*x)**2*cos(c + d*x
)**2/4 + 3*A*a*x*cos(c + d*x)**4/8 + 3*A*a*sin(c + d*x)**3*cos(c + d*x)/(8
*d) + 2*A*a*sin(c + d*x)**3/(3*d) + 5*A*a*sin(c + d*x)*cos(c + d*x)**3/(8*
d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*
a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 8*B*a*
sin(c + d*x)**5/(15*d) + 4*B*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B
*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**4/d
 + 5*B*a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*
(a*cos(c) + a)*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx =$$

$$-\frac{160\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa-}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 15*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 32*(3*sin(d*x + c)^5 - 10*sin(d
*x + c)^3 + 15*sin(d*x + c))*B*a - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) +
8*sin(2*d*x + 2*c))*B*a)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{3}{8}\,(Aa+Ba)x+\frac{Ba\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{(Aa+Ba)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(4\,Aa+5\,Ba)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(Aa+Ba)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(6\,Aa+5\,Ba)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
3/8*(A*a + B*a)*x + 1/80*B*a*sin(5*d*x + 5*c)/d + 1/32*(A*a + B*a)*sin(4*d
*x + 4*c)/d + 1/48*(4*A*a + 5*B*a)*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*si
n(2*d*x + 2*c)/d + 1/8*(6*A*a + 5*B*a)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.89

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(\frac{3Aa}{4}+\frac{3Ba}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{29Aa}{6}+\frac{13Ba}{6}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{20Aa}{3}+\frac{116Ba}{15}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{35Aa}{6}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right.}$$
$$-\frac{3a\left(\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)(A+B)}{4d}+\frac{3a\operatorname{atan}\left(\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(A+B)}{4\left(\frac{3Aa}{4}+\frac{3Ba}{4}\right)}\right)(A+B)}{4d}$$

input
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)*((13*A*a)/4 + (13*B*a)/4) + tan(c/2 + (d*x)/2)^9*((3*A
*a)/4 + (3*B*a)/4) + tan(c/2 + (d*x)/2)^7*((29*A*a)/6 + (13*B*a)/6) + tan(
c/2 + (d*x)/2)^3*((35*A*a)/6 + (19*B*a)/6) + tan(c/2 + (d*x)/2)^5*((20*A*a
)/3 + (116*B*a)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4
+ 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
 + 1)) - (3*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(A + B))/(4*d) + (3*a*a
tan((3*a*tan(c/2 + (d*x)/2)*(A + B))/(4*((3*A*a)/4 + (3*B*a)/4)))*(A + B))
/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{a\left(-30\cos(dx+c)\sin(dx+c)^3a-30\cos(dx+c)\sin(dx+c)^3b+75\cos(dx+c)\sin(dx+c)a+75\,\mathrm{c}\right.}{}$$

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
(a*( - 30*cos(c + d*x)*sin(c + d*x)**3*a - 30*cos(c + d*x)*sin(c + d*x)**3
*b + 75*cos(c + d*x)*sin(c + d*x)*a + 75*cos(c + d*x)*sin(c + d*x)*b + 24*
sin(c + d*x)**5*b - 40*sin(c + d*x)**3*a - 80*sin(c + d*x)**3*b + 120*sin(
c + d*x)*a + 120*sin(c + d*x)*b + 45*a*d*x + 45*b*d*x))/(120*d)
```

3.2 $\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 97

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{1}{8}a(4A+3B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a(A+B)\sin^3(c+dx)}{3d}$$

output
```
1/8*a*(4*A+3*B)*x+a*(A+B)*sin(d*x+c)/d+1/8*a*(4*A+3*B)*cos(d*x+c)*sin(d*x+
c)/d+1/4*a*B*cos(d*x+c)^3*sin(d*x+c)/d-1/3*a*(A+B)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{a(48Ac+36Bc+48Adx+36Bdx+96(A+B)\sin(c+dx)-32(A+B)\sin^3(c+dx)+24(A+B)\sin}{96d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*(48*A*c + 36*B*c + 48*A*d*x + 36*B*d*x + 96*(A + B)*Sin[c + d*x] - 32*(
A + B)*Sin[c + d*x]^3 + 24*(A + B)*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]
))/(96*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 3447, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos^2(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left((aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{4}\int \cos^2(c+dx)(a(4A+3B)+4a(A+B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a(4A+3B)+4a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(4a(A+B)\int \cos^3(c+dx)dx+a(4A+3B)\int \cos^2(c+dx)dx\right)+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(a(4A+3B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4a(A+B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{4}\left(a(4A+3B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4a(A+B)\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(a(4A+3B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4a(A+B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(a(4A+3B)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4a(A+B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(a(4A+3B)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4a(A+B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a*(4*A + 3*B)*(x/2 + (Cos[c + d
*x]*Sin[c + d*x])/(2*d)) - (4*a*(A + B)*(-Sin[c + d*x] + Sin[c + d*x]^3/3)
)/d)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 13.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{a\left(\frac{(A+B)\sin(2dx+2c)}{2}+\frac{(A+B)\sin(3dx+3c)}{6}+\frac{\sin(4dx+4c)B}{16}+\frac{3(A+B)\sin(dx+c)}{2}+dx\left(A+\frac{3B}{4}\right)\right)}{2d}$
parts	$\frac{(Aa+Ba)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}+\frac{Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{Ba\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx}{4}\right.}{d}$
derivativedivides	$\frac{Ba\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+\frac{Ba\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Aa}{d}$
default	$\frac{Ba\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+\frac{Ba\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Aa}{d}$
risch	$\frac{axA}{2}+\frac{3aBx}{8}+\frac{3\sin(dx+c)Aa}{4d}+\frac{3aB\sin(dx+c)}{4d}+\frac{\sin(4dx+4c)Ba}{32d}+\frac{\sin(3dx+3c)Aa}{12d}+\frac{\sin(3dx+3c)Ba}{12d}+$
norman	$\frac{\frac{a(4A+3B)x}{8}+\frac{a(4A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{a(4A+3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{3a(4A+3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{a(4A+3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*a*(1/2*(A+B)*sin(2*d*x+2*c)+1/6*(A+B)*sin(3*d*x+3*c)+1/16*sin(4*d*x+4*
c)*B+3/2*(A+B)*sin(d*x+c)+d*x*(A+3/4*B))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{3\,(4\,A+3\,B)adx+\left(6\,Ba\cos\left(dx+c\right)^3+8\,(A+B)a\cos\left(dx+c\right)^2+3\,(4\,A+3\,B)a\cos\left(dx+c\right)+16\,(A\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/24*(3*(4*A + 3*B)*a*d*x + (6*B*a*cos(d*x + c)^3 + 8*(A + B)*a*cos(d*x +
c)^2 + 3*(4*A + 3*B)*a*cos(d*x + c) + 16*(A + B)*a)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(88) = 176.

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.60

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\begin{cases}\frac{Aax\sin^2\left(c+dx\right)}{2}+\frac{Aax\cos^2\left(c+dx\right)}{2}+\frac{2Aa\sin^3\left(c+dx\right)}{3d}+\frac{Aa\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{Aa\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{3Bax\sin^4\left(c+dx\right.}{8} \\ x(A+B\cos\left(c\right))\left(a\cos\left(c\right)+a\right)\cos^2\left(c\right)\end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + 2*A*a*sin(c
 + d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d + A*a*sin(c + d*x)*c
os(c + d*x)/(2*d) + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*sin(c + d*x)**2*co
s(c + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 3*B*a*sin(c + d*x)**3*cos(c
+ d*x)/(8*d) + 2*B*a*sin(c + d*x)**3/(3*d) + 5*B*a*sin(c + d*x)*cos(c + d*
x)**3/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos
(c))*(a*cos(c) + a)*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = -\frac{32\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa+32\left(\sin\left(dx+c\right)^3-3\sin\right.}{96\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 24*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 3*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a)/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{1}{8}\left(4\,Aa+3\,Ba\right)x+\frac{Ba\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{\left(Aa+Ba\right)\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{\left(Aa+Ba\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{3\left(Aa+Ba\right)\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/8*(4*A*a + 3*B*a)*x + 1/32*B*a*sin(4*d*x + 4*c)/d + 1/12*(A*a + B*a)*sin
(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*x + 2*c)/d + 3/4*(A*a + B*a)*sin
(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 41.87 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.19

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(A\,a+\frac{3\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{7\,A\,a}{3}+\frac{49\,B\,a}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{13\,A\,a}{3}+\frac{31\,B\,a}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(3\,A\,a+\frac{1}{}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$
$$+\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(4\,A+3\,B)}{4\left(A\,a+\frac{3\,B\,a}{4}\right)}\right)\,(4\,A+3\,B)}{4\,d}$$
$$-\frac{a\,(4\,A+3\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)*(3*A*a + (13*B*a)/4) + tan(c/2 + (d*x)/2)^7*(A*a + (3*
B*a)/4) + tan(c/2 + (d*x)/2)^3*((13*A*a)/3 + (31*B*a)/12) + tan(c/2 + (d*x
)/2)^5*((7*A*a)/3 + (49*B*a)/12))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 +
 (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*ata
n((a*tan(c/2 + (d*x)/2)*(4*A + 3*B))/(4*(A*a + (3*B*a)/4)))*(4*A + 3*B))/(
4*d) - (a*(4*A + 3*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{a\left(-6\cos(dx+c)\sin(dx+c)^3b+12\cos(dx+c)\sin(dx+c)\,a+15\cos(dx+c)\sin(dx+c)\,b-8\sin(\right.}{24d}$$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
(a*( - 6*cos(c + d*x)*sin(c + d*x)**3*b + 12*cos(c + d*x)*sin(c + d*x)*a +
 15*cos(c + d*x)*sin(c + d*x)*b - 8*sin(c + d*x)**3*a - 8*sin(c + d*x)**3*
b + 24*sin(c + d*x)*a + 24*sin(c + d*x)*b + 12*a*d*x + 9*b*d*x))/(24*d)
```

3.3 $\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{1}{2}a(A+B)x + \frac{a(3A+2B)\sin(c+dx)}{3d} + \frac{a(A+B)\cos(c+dx)\sin(c+dx)}{2d} + \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d}$$

output
```
1/2*a*(A+B)*x+1/3*a*(3*A+2*B)*sin(d*x+c)/d+1/2*a*(A+B)*cos(d*x+c)*sin(d*x+
c)/d+1/3*a*B*cos(d*x+c)^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{a(6Ac+6Bc+6Adx+6Bdx+3(4A+3B)\sin(c+dx)+3(A+B)\sin(2(c+dx))+B\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*(6*A*c + 6*B*c + 6*A*d*x + 6*B*d*x + 3*(4*A + 3*B)*Sin[c + d*x] + 3*(A
+ B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3447, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left((aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{3}\int \cos(c+dx)(a(3A+2B)+3a(A+B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a(3A+2B)+3a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\frac{a(3A+2B)\sin(c+dx)}{d}+\frac{3a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ax(A+B)\right)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*a*(A + B)*x)/2 + (a*(3*A + 2
*B)*Sin[c + d*x])/d + (3*a*(A + B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.84 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{\left(\frac{(A+B)\sin(2dx+2c)}{2}+\frac{B\sin(3dx+3c)}{6}+\left(2A+\frac{3B}{2}\right)\sin(dx+c)+(A+B)xd\right)a}{2d}$
parts	$\frac{(Aa+Ba)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{\sin(dx+c)Aa}{d}+\frac{Ba\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}$
derivativedivides	$\frac{\frac{Ba\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)}{d}$
default	$\frac{\frac{Ba\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)}{d}$
risch	$\frac{axA}{2}+\frac{aBx}{2}+\frac{\sin(dx+c)Aa}{d}+\frac{3aB\sin(dx+c)}{4d}+\frac{\sin(3dx+3c)Ba}{12d}+\frac{\sin(2dx+2c)Aa}{4d}+\frac{\sin(2dx+2c)Ba}{4d}$
norman	$\frac{\frac{(A+B)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{a(A+B)x}{2}+\frac{3a(A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{3a(A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{a(A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+\frac{4a(3A+B}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$
orering	Expression too large to display

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1/2*(A+B)*sin(2*d*x+2*c)+1/6*B*sin(3*d*x+3*c)+(2*A+3/2*B)*sin(d*x+c)+
(A+B)*x*d)*a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{3\,(A+B)adx+\left(2\,Ba\cos\left(dx+c\right)^2+3\,(A+B)a\cos\left(dx+c\right)+2\,(3\,A+2\,B)a\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
1/6*(3*(A + B)*a*d*x + (2*B*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) +
2*(3*A + 2*B)*a)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{Aa\sin(c+dx)}{d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{2Ba\sin \\ x(A+B\cos(c))(a\cos(c)+a)\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
 d*x)*cos(c + d*x)/(2*d) + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 +
B*a*x*cos(c + d*x)**2/2 + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*c
os(c + d*x)**2/d + B*a*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A +
 B*cos(c))*(a*cos(c) + a)*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{3(2dx+2c+\sin(2dx+2c))Aa - 4\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Ba + 3(2dx+2c+\sin(2dx+2}{12d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 4*(sin(d*x + c)^3 - 3*sin(d
*x + c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 12*A*a*sin(d*x + c
))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{1}{2}\,(Aa+Ba)x+\frac{Ba\sin\left(3\,dx+3\,c\right)}{12\,d}$$
$$+\frac{(Aa+Ba)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(4\,Aa+3\,Ba)\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
1/2*(A*a + B*a)*x + 1/12*B*a*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*
x + 2*c)/d + 1/4*(4*A*a + 3*B*a)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 40.51 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{A\,a\,x}{2}+\frac{B\,a\,x}{2}+\frac{A\,a\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,a\,\sin\left(c+d\,x\right)}{4\,d}$$
$$+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
(A*a*x)/2 + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (3*B*a*sin(c + d*x))/(4*d)
+ (A*a*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(3
*c + 3*d*x))/(12*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{a\big(3\cos(dx+c)\sin(dx+c)\,a+3\cos(dx+c)\sin(dx+c)\,b-2\sin(dx+c)^3\,b+6\sin(dx+c)\,a+6\sin}{6d}$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
(a*(3*cos(c + d*x)*sin(c + d*x)*a + 3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(
c + d*x)**3*b + 6*sin(c + d*x)*a + 6*sin(c + d*x)*b + 3*a*d*x + 3*b*d*x))/
(6*d)
```

3.4 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{1}{2}a(2A+B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{aB\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*(2*A+B)*x+a*(A+B)*sin(d*x+c)/d+1/2*a*B*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{a(2Bc+4Adx+2Bdx+4(A+B)\sin(c+dx)+B\sin(2(c+dx)))}{4d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*(2*B*c + 4*A*d*x + 2*B*d*x + 4*(A + B)*Sin[c + d*x] + B*Sin[2*(c + d*x)
]))/(4*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3213}$$

$$\frac{a(A+B)\sin(c+dx)}{d}+\frac{1}{2}ax(2A+B)+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(a*(2*A + B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{a\left(\frac{B\sin(2dx+2c)}{4}+(A+B)\sin(dx+c)+dx\left(A+\frac{B}{2}\right)\right)}{d}$
risch	$axA+\frac{aBx}{2}+\frac{\sin(dx+c)Aa}{d}+\frac{aB\sin(dx+c)}{d}+\frac{\sin(2dx+2c)Ba}{4d}$
parts	$axA+\frac{(Aa+Ba)\sin(dx+c)}{d}+\frac{Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
derivativedivides	$\frac{Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)+Ba\sin(dx+c)+Aa(dx+c)}{d}$
default	$\frac{Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)+Ba\sin(dx+c)+Aa(dx+c)}{d}$
norman	$\frac{\frac{a(2A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}+a(2A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\frac{a(2A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(2A+B)x}{2}+\frac{a(2A+B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$
orering	$x(a+a\cos(dx+c))(A+B\cos(dx+c))-\frac{5(-d\sin(dx+c)a(A+B\cos(dx+c))-(a+a\cos(dx+c))B}{4d^{2}}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*B*sin(2*d*x+2*c)+(A+B)*sin(d*x+c)+d*x*(A+1/2*B))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{(2\,A+B)adx+(Ba\cos(dx+c)+2\,(A+B)a)\sin(dx+c)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((2*A + B)*a*d*x + (B*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx = \begin{cases} Aax + \frac{Aa\sin(c+dx)}{d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{Ba\sin(c+dx)\cos(c+dx)}{2d} + \frac{Ba\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B\cos(c))(a\cos(c) + a) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a*x + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*a*x*co
s(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/d
, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx = \frac{4(dx + c)Aa + (2dx + 2c + \sin(2dx + 2c))Ba + 4Aa\sin(dx + c) + 4Ba\sin(dx + c)}{4d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 4*A*a*sin(d*
x + c) + 4*B*a*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx$$
$$= \frac{1}{2}(2\,Aa + Ba)x + \frac{Ba\sin(2\,dx + 2\,c)}{4\,d} + \frac{(Aa + Ba)\sin(dx + c)}{d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*A*a + B*a)*x + 1/4*B*a*sin(2*d*x + 2*c)/d + (A*a + B*a)*sin(d*x + c
)/d
```

Mupad [B] (verification not implemented)

Time = 41.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx$$
$$= A\,a\,x + \frac{B\,a\,x}{2} + \frac{A\,a\,\sin(c + d\,x)}{d} + \frac{B\,a\,\sin(c + d\,x)}{d} + \frac{B\,a\,\sin(2\,c + 2\,d\,x)}{4\,d}$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
A*a*x + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (B*a*sin
(2*c + 2*d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx$$
$$= \frac{a(\cos(dx + c)\sin(dx + c)\,b + 2\sin(dx + c)\,a + 2\sin(dx + c)\,b + 2adx + bdx)}{2d}$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
(a*(cos(c + d*x)*sin(c + d*x)*b + 2*sin(c + d*x)*a + 2*sin(c + d*x)*b + 2*
a*d*x + b*d*x))/(2*d)
```

3.5 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 32

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = a(A+B)x+\frac{aA\text{arctanh}(\sin(c+dx))}{d}+\frac{aB\sin(c+dx)}{d}$$

output
```
a*(A+B)*x+a*A*arctanh(sin(d*x+c))/d+a*B*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = aAx+aBx+\frac{aA\coth^{-1}(\sin(c+dx))}{d}+\frac{aB\cos(dx)\sin(c)}{d}+\frac{aB\cos(c)\sin(dx)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
a*A*x + a*B*x + (a*A*ArcCoth[Sin[c + d*x]])/d + (a*B*Cos[d*x]*Sin[c])/d +
(a*B*Cos[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3447}$$

$$\int \sec(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3502}$$

$$\int (aA+a(A+B)\cos(c+dx))\sec(c+dx)dx+\frac{aB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{aA+a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 3214}$$

$$aA\int \sec(c+dx)dx+ax(A+B)+\frac{aB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$aA\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+ax(A+B)+\frac{aB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{aA\text{arctanh}(\sin(c+dx))}{d}+ax(A+B)+\frac{aB\sin(c+dx)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
a*(A + B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{Aa(dx+c)+Ba\sin(dx+c)+Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba(dx+c)}{d}$
default	$\frac{Aa(dx+c)+Ba\sin(dx+c)+Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba(dx+c)}{d}$
parallelrisch	$\frac{\left(-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+B\sin(dx+c)+(A+B)xd\right)a}{d}$
parts	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(Aa+Ba)(dx+c)}{d}+\frac{aB\sin(dx+c)}{d}$
risch	$axA+aBx-\frac{iBa\,e^{i(dx+c)}}{2d}+\frac{iBa\,e^{-i(dx+c)}}{2d}+\frac{Aa\ln(e^{i(dx+c)}+i)}{d}-\frac{Aa\ln(e^{i(dx+c)}-i)}{d}$
norman	$\frac{(Aa+Ba)x+(Aa+Ba)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(2Aa+2Ba)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{2Ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2Ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{Aa\ln\left(t\right.}{}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(A*a*(d*x+c)+B*a*sin(d*x+c)+A*a*ln(sec(d*x+c)+tan(d*x+c))+B*a*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int(a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{2\,(A+B)adx+Aa\log\left(\sin\left(dx+c\right)+1\right)-Aa\log\left(-\sin\left(dx+c\right)+1\right)+2\,Ba\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="frica
s")
```

output
```
1/2*(2*(A + B)*a*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c) +
 1) + 2*B*a*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = a\left(\int A\sec(c+dx)\,dx + \int A\cos(c+dx)\sec(c+dx)\,dx + \int B\cos(c+dx)\sec(c+dx)\,dx + \int B\cos^2(c+dx)\sec(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
a*(Integral(A*sec(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x)
+ Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*se
c(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{(dx+c)Aa + (dx+c)Ba + Aa\log(\sec(dx+c)+\tan(dx+c)) + Ba\sin(dx+c)}{d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxim
a")
```

output
```
((d*x + c)*A*a + (d*x + c)*B*a + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*
a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec(c + dx)\,dx = \frac{Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + (Aa + Ba)(dx + c) + \frac{2\,Ba\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1}}{d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac"
)
```

output
```
(A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) + (A*a + B*a)*(d*x + c) + 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x +
 1/2*c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 40.89 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.12

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec(c + dx)\,dx = \frac{B\,a\,\sin\left(c + d\,x\right)}{d} + \frac{2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
(B*a*sin(c + d*x))/d + (2*A*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec(c + dx)\,dx$$
$$= \frac{a\left(-\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)a + \log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)a + \sin(dx + c)\,b + adx + bdx\right)}{d}$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
(a*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + sin(c +
 d*x)*b + a*d*x + b*d*x))/d
```

3.6 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 32

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = aBx+\frac{a(A+B)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}$$

output
```
a*B*x+a*(A+B)*arctanh(sin(d*x+c))/d+a*A*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = aBx+\frac{aA\coth^{-1}(\sin(c+dx))}{d}+\frac{aB\coth^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
a*B*x + (a*A*ArcCoth[Sin[c + d*x]])/d + (a*B*ArcCoth[Sin[c + d*x]])/d + (a
*A*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 3447, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sec^2(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3500}$$

$$\int (a(A+B)+aB\cos(c+dx))\sec(c+dx)dx+\frac{aA\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a(A+B)+aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aA\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3214}$$

$$a(A+B)\int \sec(c+dx)dx+\frac{aA\tan(c+dx)}{d}+aBx$$

$$\Big\downarrow \text{3042}$$

$$a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{aA\tan(c+dx)}{d}+aBx$$

$$\Big\downarrow \text{4257}$$

$$\frac{a(A+B)\text{arctanh}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+aBx$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
a*B*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

method	result
parts	$\frac{aA\tan(dx+c)}{d}+\frac{(Aa+Ba)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba(dx+c)}{d}$
derivativedivides	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba(dx+c)+Aa\tan(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba(dx+c)+Aa\tan(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$-\frac{\left((A+B)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-(A+B)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-dxB\cos(dx+c)-A\sin(dx+c)\right)a}{\cos(dx+c)d}$
risch	$aBx+\frac{2iAa}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{Aa\ln(\mathrm{e}^{i(dx+c)}+i)}{d}+\frac{a\ln(\mathrm{e}^{i(dx+c)}+i)B}{d}-\frac{Aa\ln(\mathrm{e}^{i(dx+c)}-i)}{d}-\frac{a\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$
norman	$\frac{aBx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+aBx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-aBx-\frac{2Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{2Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-aBx\tan\left(\frac{dx}{2}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOSE
)
```

output
```
a*A*tan(d*x+c)/d+(A*a+B*a)/d*ln(sec(d*x+c)+tan(d*x+c))+B*a/d*(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int(a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{2\,Badx\cos(dx+c)+(A+B)a\cos(dx+c)\log(\sin(dx+c)+1)-(A+B)a\cos(dx+c)\log(-\sin(d}{2\,d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fri
cas")
```

output
```
1/2*(2*B*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
 - (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/(d*
cos(d*x + c))
```

Sympy [F]

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= a\left(\int A\sec^2(c + dx)\,dx + \int A\cos(c + dx)\sec^2(c + dx)\,dx + \int B\cos(c + dx)\sec^2(c + dx)\,dx + \int B\cos^2(c + dx)\sec^2(c + dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**
2, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d
*x)**2*sec(c + d*x)**2, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.28

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \frac{2(dx + c)Ba + Aa(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + Ba(\log(\sin(dx + c) + 1) - \log(\sin}{2d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="max
ima")
```

output
```
1/2*(2*(d*x + c)*B*a + A*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c
))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(32) = 64$.

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.62

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \frac{(dx + c)Ba + (Aa + Ba)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (Aa + Ba)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2Aa\tan}{\tan(\frac{1}{2}}}{d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gia
c")
```

output
```
((d*x + c)*B*a + (A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + B
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1
/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 41.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.12

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \frac{A\,a\tan(c + d\,x)}{d} + \frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
(A*a*tan(c + d*x))/d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/d + (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atanh
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.56

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{a\left(-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\cos(dx+c)\log\left(\tan\left(\right.\right.\right.}{\cos(dx+c)\,d}$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
(a*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a - cos(c + d*x)*log(tan((c
+ d*x)/2) - 1)*b + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + cos(c + d*x)
*log(tan((c + d*x)/2) + 1)*b + cos(c + d*x)*b*d*x + sin(c + d*x)*a))/(cos(
c + d*x)*d)
```

3.7 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 56

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a(A+2B)\text{arctanh}(\sin(c+dx))}{2d} + \frac{a(A+B)\tan(c+dx)}{d} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*(A+2*B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/2*a*A*sec(d*x+c
)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aB\coth^{-1}(\sin(c+dx))}{d} + \frac{aA\text{arctanh}(\sin(c+dx))}{2d} + \frac{aA\tan(c+dx)}{d} + \frac{aB\tan(c+dx)}{d} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*B*ArcCoth[Sin[c + d*x]])/d + (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (a*A*T
an[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2
*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sec^3(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{2}\int (2a(A+B)+a(A+2B)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2a(A+B)+a(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{2}\left(2a(A+B)\int \sec^2(c+dx)dx + a(A+2B)\int \sec(c+dx)dx\right) + \frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(a(A+2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx + 2a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2 dx\right) + \frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{2}\left(a(A+2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx - \frac{2a(A+B)\int 1d(-\tan(c+dx))}{d}\right) + \frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(a(A+2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx + \frac{2a(A+B)\tan(c+dx)}{d}\right) + \frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{a(A+2B)\text{arctanh}(\sin(c+dx))}{d} + \frac{2a(A+B)\tan(c+dx)}{d}\right) + \frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((a*(A + 2*B)*ArcTanh[Sin[c + d*x]
])/d + (2*a*(A + B)*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{Aa\tan(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))+Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba\tan(dx+c)}{d}$
default	$\frac{Aa\tan(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))+Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba\tan(dx+c)}{d}$
parts	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{(Aa+Ba)\tan(dx+c)}{d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$-\frac{\left((\cos(2dx+2c)+1)(A+2B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-(\cos(2dx+2c)+1)(A+2B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(-2A-2B)\sin(\right.}{2d(\cos(2dx+2c)+1)}$
risch	$-\frac{ia\left(A\,e^{3i(dx+c)}-2A\,e^{2i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2A-2B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{Aa\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)B}{d}$
norman	$\frac{\frac{a(A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{a(3A+2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(5A+2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{a(A+2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2}-\frac{a(A+2B)\ln\left(\tan\right.}{2d}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(A*a*tan(d*x+c)+B*a*ln(sec(d*x+c)+tan(d*x+c))+A*a*(1/2*sec(d*x+c)*tan(
d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{(A+2\,B)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(A+2\,B)a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(2}{4\,d\cos(dx+c)^2}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fri
cas")
```

output
```
1/4*((A + 2*B)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*B)*a*cos(d*
x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(A + B)*a*cos(d*x + c) + A*a)*sin(d
*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = a\left(\int A\sec^3(c+dx)\,dx+\int A\cos(c+dx)\sec^3(c+dx)\,dx + \int B\cos(c+dx)\sec^3(c+dx)\,dx+\int B\cos^2(c+dx)\sec^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**
3, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(B*cos(c + d
*x)**2*sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = -\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2\,Ba(\log(\sin(dx+c)+1)-\log(\ldots}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="max
ima")
```

output
```
-1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 2*B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) - 4*A*a*tan(d*x + c) - 4*B*a*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.21

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{(Aa+2\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+2\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{}}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gia
c")
```

output
```
1/2*((A*a + 2*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*a)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan
(1/2*d*x + 1/2*c)^3 - 3*A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 41.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.68

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(3\,A\,a+2\,B\,a)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(A\,a+2\,B\,a)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(A+2\,B)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(3*A*a + 2*B*a) - tan(c/2 + (d*x)/2)^3*(A*a + 2*B*a))/
(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2
 + (d*x)/2))*(A + 2*B))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 280, normalized size of antiderivative = 5.00

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^3(c + dx)\,dx$$
$$= \frac{a\left(-\cos(dx + c)\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\sin(dx + c)^2 a - 2\cos(dx + c)\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\sin(dx + c\right.}{}$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
(a*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*cos(c
+ d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + cos(c + d*x)*log(tan(
(c + d*x)/2) - 1)*a + 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*sin(c + d*x)**2*b - cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*a - 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b - cos(c + d*x)*sin(c +
d*x)*a + 2*sin(c + d*x)**3*a + 2*sin(c + d*x)**3*b - 2*sin(c + d*x)*a - 2*
sin(c + d*x)*b))/(2*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.8 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 86

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a(A+B)\text{arctanh}(\sin(c+dx))}{2d}+\frac{a(2A+3B)\tan(c+dx)}{3d}+\frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
1/2*a*(A+B)*arctanh(sin(d*x+c))/d+1/3*a*(2*A+3*B)*tan(d*x+c)/d+1/2*a*(A+B)
*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.65

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a(3(A+B)\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(6(A+B)+3(A+B)\sec(c+dx)+2A\tan^2(c+dx)))}{6d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*(3*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B) + 3*(A + B)*
Sec[c + d*x] + 2*A*Tan[c + d*x]^2)))/(6*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sec^4(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\int (3a(A+B)+a(2A+3B)\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a(A+B)+a(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(3a(A+B)\int \sec^3(c+dx)dx+a(2A+3B)\int \sec^2(c+dx)dx\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(a(2A+3B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\Big\downarrow$ 4254

$$\frac{1}{3}\left(3a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a(2A+3B)\int 1d(-\tan(c+dx))}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\Big\downarrow$ 24

$$\frac{1}{3}\left(3a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a(2A+3B)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\Big\downarrow$ 4255

$$\frac{1}{3}\left(3a(A+B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a(2A+3B)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(3a(A+B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a(2A+3B)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(3a(A+B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a(2A+3B)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((a*(2*A + 3*B)*Tan[c + d*x])/d
+ 3*a*(A + B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(
2*d)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 10.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result
parts	$-\frac{Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{(Aa+Ba)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{Ba\tan(dx+c)}{d}$
derivativedivides	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba\tan(dx+c)-Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+Ba\left(\frac{\sec(dx+c}{\right.}}{d}$
default	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba\tan(dx+c)-Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+Ba\left(\frac{\sec(dx+c}{\right.}}{d}$
parallelrisch	$\frac{a\left(-\frac{3\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)(A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{3\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)(A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+(A+\right.}{d(3\cos(dx+c)+\cos(3dx+3c))}$
norman	$\frac{-\frac{4a(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}-\frac{2a(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{2a(7A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{3(A+B)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(A+B)a\tan\left(\frac{dx}{2}+\right.}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}$
risch	$-\frac{ia\left(3A\,e^{5i(dx+c)}+3B\,e^{5i(dx+c)}-6B\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12B\,e^{2i(dx+c)}-3A\,e^{i(dx+c)}-3B\,e^{i(dx+c)}-4A-6B\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOSE
)
```

output
```
-A*a/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*a+B*a)/d*(1/2*sec(d*x+c)*tan(
d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*a/d*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.22

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^4(c + dx)\,dx$$
$$= \frac{3\,(A+B)a\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)} - 3\,(A+B)a\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1)} + 2\,(2}{12\,d\cos{(dx+c)}^3}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fri
cas")
```

output
```
1/12*(3*(A + B)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + B)*a*cos(d
*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A + 3*B)*a*cos(d*x + c)^2 + 3*(
A + B)*a*cos(d*x + c) + 2*A*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^4(c + dx)\,dx$$
$$= a\left(\int A\sec^4{(c+dx)}\,dx + \int A\cos{(c+dx)}\sec^4{(c+dx)}\,dx + \int B\cos{(c+dx)}\sec^4{(c+dx)}\,dx + \int B\cos^2{(c+dx)}\sec^4{(c+dx)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**
4, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(B*cos(c + d
*x)**2*sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.48

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^4(c + dx)\,dx$$
$$= \frac{4\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)Aa - 3\,Aa\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right.\right.}{12\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="max
ima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*a*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) + 12*B*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.79

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^4(c + dx)\,dx$$
$$= \frac{3\left(Aa + Ba\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)+1\right|\right) - 3\left(Aa + Ba\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gia
c")
```

output
```
1/6*(3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*a)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*t
an(1/2*d*x + 1/2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*tan(1/2*d*x
+ 1/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 42.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(A+B)}{d} - \frac{(A\,a+B\,a)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{4\,A\,a}{3}-4\,B\,a\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(3\,A\,a+3\,B\,a)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^4,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2))*(A + B))/d - (tan(c/2 + (d*x)/2)*(3*A*a + 3*B
*a) + tan(c/2 + (d*x)/2)^5*(A*a + B*a) - tan(c/2 + (d*x)/2)^3*((4*A*a)/3 +
 4*B*a))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (
d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.45

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a\left(-3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2a-3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+\right.\right.}{}$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
(a*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 3*cos(
c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + 3*cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*a + 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 3*c
os(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 3*cos(c + d*x)*l
og(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*a - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b - 3*cos(c + d*x
)*sin(c + d*x)*a - 3*cos(c + d*x)*sin(c + d*x)*b + 4*sin(c + d*x)**3*a + 6
*sin(c + d*x)**3*b - 6*sin(c + d*x)*a - 6*sin(c + d*x)*b))/(6*cos(c + d*x)
*d*(sin(c + d*x)**2 - 1))
```

3.9 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 106

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{a(3A+4B)\text{arctanh}(\sin(c+dx))}{8d} + \frac{a(A+B)\tan(c+dx)}{d} + \frac{a(3A+4B)\sec(c+dx)\tan(c+dx)}{8d} + \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a(A+B)\tan^3(c+dx)}{3d}$$

output
```
1/8*a*(3*A+4*B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/8*a*(3*A+4*B)
*sec(d*x+c)*tan(d*x+c)/d+1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*(A+B)*tan
(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^5(c + dx)\,dx$$
$$= \frac{a(3(3A + 4B)\text{arctanh}(\sin(c + dx)) + \sec(c + dx)\,(9A + 12B + 8(A + B)(2 + \cos(2(c + dx)))\sec(c + d}{24d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*(3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*A + 12*B + 8*(A
+ B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*A*Sec[c + d*x]^2)*Tan[c + d*x
]))/(24*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c + dx)(a\cos(c + dx) + a)(A + B\cos(c + dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c + dx + \frac{\pi}{2}\right) + a\right)\left(A + B\sin\left(c + dx + \frac{\pi}{2}\right)\right)}{\sin\left(c + dx + \frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{ 3447}$$

$$\int \sec^5(c + dx)\left((aA + aB)\cos(c + dx) + aA + aB\cos^2(c + dx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(aA + aB)\sin\left(c + dx + \frac{\pi}{2}\right) + aA + aB\sin\left(c + dx + \frac{\pi}{2}\right)^2}{\sin\left(c + dx + \frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{4}\int (4a(A+B)+a(3A+4B)\cos(c+dx))\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{4a(A+B)+a(3A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(4a(A+B)\int \sec^4(c+dx)dx+a(3A+4B)\int \sec^3(c+dx)dx\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(a(3A+4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}\left(a(3A+4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a(A+B)\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(a(3A+4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a(A+B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(a(3A+4B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a(A+B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(a(3A+4B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a(A+B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\Big\downarrow$ 4257

$$\frac{1}{4}\left(a(3A+4B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a(A+B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*(3*A + 4*B)*(ArcTanh[Sin[c +
d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*a*(A + B)*(-Tan[c +
d*x] - Tan[c + d*x]^3/3))/d)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 10.74 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12

method	result
parts	$\frac{Aa\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\frac{(Aa+Ba)\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx\cdots}{d}$
derivativedivides	$\frac{-Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Aa\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec\cdots}{}\right.\right.}{d}$
default	$\frac{-Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Aa\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec\cdots}{}\right.\right.}{d}$
parallelrisch	$\frac{8\left(-\frac{9\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{4B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{16}+\frac{9\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{4B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}{16}\right.}{3d(\cos(4dx+4c)+4\cos(2dx\cdots}$
norman	$\frac{-\frac{a(3A+4B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{4d}+\frac{a(13A-20B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{6d}+\frac{a(13A+12B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a(29A-4B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}+\frac{a(31A+4B)\cdots}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4}$
risch	$-\frac{ia\left(9A\,e^{7i(dx+c)}+12B\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12B\,e^{5i(dx+c)}-48A\,e^{4i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12\cdots\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOSE
)
```

output
```
A*a/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))-(A*a+B*a)/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*a/d*(1/2*sec(d
*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.20

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{3\,(3\,A+4\,B)a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,A+4\,B)a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)\cdots}{48\,d\,\mathrm{co}\cdots}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fri
cas")
```

output
```
1/48*(3*(3*A + 4*B)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A + 4*B)
*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(A + B)*a*cos(d*x + c)^3
+ 3*(3*A + 4*B)*a*cos(d*x + c)^2 + 8*(A + B)*a*cos(d*x + c) + 6*A*a)*sin(d
*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= a\left(\int A\sec^5(c+dx)\,dx + \int A\cos(c+dx)\sec^5(c+dx)\,dx + \int B\cos(c+dx)\sec^5(c+dx)\,dx + \int B\cos^2(c+dx)\sec^5(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
a*(Integral(A*sec(c + d*x)**5, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**
5, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**5, x) + Integral(B*cos(c + d
*x)**2*sec(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.54

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{16\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Aa + 16\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ba - 3\,Aa\left(\frac{2\left(3\sin(dx+c)^3 - 5\,\mathrm{s}\cdots\right.}{\sin(dx+c)^4 - 2\sin(\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="max
ima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(tan(d*x + c)^3 + 3*ta
n(d*x + c))*B*a - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.77

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{3\,(3\,Aa + 4\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(3\,Aa + 4\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(9\,Aa\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gia
c")
```

output
```
1/24*(3*(3*A*a + 4*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*
B*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7
+ 12*B*a*tan(1/2*d*x + 1/2*c)^7 - 49*A*a*tan(1/2*d*x + 1/2*c)^5 - 28*B*a*t
an(1/2*d*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*x
 + 1/2*c)^3 - 39*A*a*tan(1/2*d*x + 1/2*c) - 36*B*a*tan(1/2*d*x + 1/2*c))/(
tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 43.02 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.57

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{\left(-\frac{3\,A\,a}{4} - B\,a\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^7 + \left(\frac{49\,A\,a}{12} + \frac{7\,B\,a}{3}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 + \left(-\frac{31\,A\,a}{12} - \frac{13\,B\,a}{3}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + \left(\frac{13\,A\,a}{4}\right.}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^8 - 4\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 - 4\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}$$
$$+ \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)\,(3\,A + 4\,B)}{4\,d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*((13*A*a)/4 + 3*B*a) - tan(c/2 + (d*x)/2)^7*((3*A*a)/4
 + B*a) - tan(c/2 + (d*x)/2)^3*((31*A*a)/12 + (13*B*a)/3) + tan(c/2 + (d*x
)/2)^5*((49*A*a)/12 + (7*B*a)/3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 +
 (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*ata
nh(tan(c/2 + (d*x)/2))*(3*A + 4*B))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.52

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Too large to display}$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
(a*( - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 12*cos
(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b + 18*cos(c + d*x)*lo
g(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 24*cos(c + d*x)*log(tan((c + d
*x)/2) - 1)*sin(c + d*x)**2*b - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a
 - 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 9*cos(c + d*x)*log(tan((c
 + d*x)/2) + 1)*sin(c + d*x)**4*a + 12*cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*sin(c + d*x)**4*b - 18*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c +
d*x)**2*a - 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b +
9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 12*cos(c + d*x)*log(tan((c +
d*x)/2) + 1)*b - 9*cos(c + d*x)*sin(c + d*x)**3*a - 12*cos(c + d*x)*sin(c
+ d*x)**3*b + 15*cos(c + d*x)*sin(c + d*x)*a + 12*cos(c + d*x)*sin(c + d*x
)*b + 16*sin(c + d*x)**5*a + 16*sin(c + d*x)**5*b - 40*sin(c + d*x)**3*a -
 40*sin(c + d*x)**3*b + 24*sin(c + d*x)*a + 24*sin(c + d*x)*b))/(24*cos(c
+ d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.10 $\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 191

$$\begin{aligned}\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx &= \frac{1}{16}a^2(12A+11B)x+\frac{a^2(9A+8B)\sin(c+dx)}{5d}\\ &\quad+\frac{a^2(12A+11B)\cos(c+dx)\sin(c+dx)}{16d}\\ &\quad+\frac{a^2(12A+11B)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^2(6A+7B)\cos^4(c+dx)\sin(c+dx)}{30d}\\ &\quad+\frac{B\cos^4(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{6d}-\frac{a^2(9A+8B)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
1/16*a^2*(12*A+11*B)*x+1/5*a^2*(9*A+8*B)*sin(d*x+c)/d+1/16*a^2*(12*A+11*B)
*cos(d*x+c)*sin(d*x+c)/d+1/24*a^2*(12*A+11*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/
30*a^2*(6*A+7*B)*cos(d*x+c)^4*sin(d*x+c)/d+1/6*B*cos(d*x+c)^4*(a^2+a^2*cos
(d*x+c))*sin(d*x+c)/d-1/15*a^2*(9*A+8*B)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{a^2(660Bc+720Adx+660Bdx+120(11A+10B)\sin(c+dx)+15(32A+31B)\sin(2(c+dx))+180A}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(a^2*(660*B*c + 720*A*d*x + 660*B*d*x + 120*(11*A + 10*B)*Sin[c + d*x] + 1
5*(32*A + 31*B)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 200*B*Sin[3*(c
 + d*x)] + 60*A*Sin[4*(c + d*x)] + 75*B*Sin[4*(c + d*x)] + 12*A*Sin[5*(c +
 d*x)] + 24*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{6}\int \cos^3(c+dx)(\cos(c+dx)a+a)(2a(3A+2B)+a(6A+7B)\cos(c+dx))dx+\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(2a(3A+2B)+a(6A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3447

$$\frac{1}{6}\int \cos^3(c+dx)\left((6A+7B)\cos^2(c+dx)a^2+2(3A+2B)a^2+\left(2(3A+2B)a^2+(6A+7B)a^2\right)\cos(c+dx)\right)dx+\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left((6A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+2(3A+2B)a^2+\left(2(3A+2B)a^2+(6A+7B)a^2\right)\sin\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3502

$$\frac{1}{6}\left(\frac{1}{5}\int \cos^3(c+dx)\left(6(9A+8B)a^2+5(12A+11B)\cos(c+dx)a^2\right)dx+\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c+dx)}{5d}\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(6(9A+8B)a^2+5(12A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx+\frac{a^2(6A+7B)\sin(c+dx)}{5d}\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3227

$$\frac{1}{6}\left(\frac{1}{5}\left(5a^2(12A+11B)\int \cos^4(c+dx)dx+6a^2(9A+8B)\int \cos^3(c+dx)dx\right)+\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(6a^2(9A+8B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3 dx+5a^2(12A+11B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4 dx\right)+\frac{a^2(6A+7B)\sin(c}{5}\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3113

$$\frac{1}{6}\left(\frac{1}{5}\left(5a^2(12A+11B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4 dx-\frac{6a^2(9A+8B)\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{a^2(6}{}\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 2009

$$\frac{1}{6}\left(\frac{1}{5}\left(5a^2(12A+11B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4 dx-\frac{6a^2(9A+8B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{a^2(6A+7}{}\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3115

$$\frac{1}{6}\left(\frac{1}{5}\left(5a^2(12A+11B)\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{6a^2(9A+8B)\left(\frac{1}{3}\sin^3(c+dx)-\text{si}\right.}{d}\right.\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(5a^2(12A+11B)\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{6a^2(9A+8B)\left(\frac{1}{3}\sin^3(c+d\right.}{d}\right.\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 3115

$$\frac{1}{6}\left(\frac{1}{5}\left(5a^2(12A+11B)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{6a^2(9A+8B)\left(\right.}{}\right.\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

$\downarrow$ 24

$$\frac{1}{6}\left(\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{1}{5}\left(5a^2(12A+11B)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos}{2d}\right.\right.\right.\right.$$
$$\frac{B\sin(c+dx)\cos^4(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{6d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(B*Cos[c + d*x]^4*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(6*d) + ((a^2*(6*
A + 7*B)*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + ((-6*a^2*(9*A + 8*B)*(-Sin[c
 + d*x] + Sin[c + d*x]^3/3))/d + 5*a^2*(12*A + 11*B)*((Cos[c + d*x]^3*Sin[
c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 150.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{a^{2}\left(\left(8A+\frac{31B}{4}\right)\sin(2dx+2c)+\left(3A+\frac{10B}{3}\right)\sin(3dx+3c)+\left(A+\frac{5B}{4}\right)\sin(4dx+4c)+\frac{(A+2B)\sin(5dx+5c)}{5}+\frac{B\sin(6dx+6c)}{12}+\right.}{16d}$
parts	$\frac{(a^{2}A+2a^{2}B)\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5d}+\frac{(2a^{2}A+a^{2}B)\left(\frac{\left(\cos(dx+c)^{3}+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3d}{8}\right.}{d}$
risch	$\frac{3a^{2}xA}{4}+\frac{11a^{2}Bx}{16}+\frac{11\sin(dx+c)a^{2}A}{8d}+\frac{5\sin(dx+c)a^{2}B}{4d}+\frac{a^{2}B\sin(6dx+6c)}{192d}+\frac{\sin(5dx+5c)a^{2}A}{80d}+\frac{\sin(5dx}{4}$
derivativedivides	$\frac{\frac{a^{2}A\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5}+a^{2}B\left(\frac{\left(\cos(dx+c)^{5}+\frac{5\cos(dx+c)^{3}}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+}{}$
default	$\frac{\frac{a^{2}A\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5}+a^{2}B\left(\frac{\left(\cos(dx+c)^{5}+\frac{5\cos(dx+c)^{3}}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+}{}$
norman	$\frac{\frac{a^{2}(12A+11B)x}{16}+\frac{17a^{2}(12A+11B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{24d}+\frac{a^{2}(12A+11B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}+\frac{3a^{2}(12A+11B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8}+\frac{15a^{2}(12A+1}{}}{}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/16*a^2*((8*A+31/4*B)*sin(2*d*x+2*c)+(3*A+10/3*B)*sin(3*d*x+3*c)+(A+5/4*B
)*sin(4*d*x+4*c)+1/5*(A+2*B)*sin(5*d*x+5*c)+1/12*B*sin(6*d*x+6*c)+2*(11*A+
10*B)*sin(d*x+c)+12*x*d*(A+11/12*B))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$\int\cos^{3}(c+dx)(a+a\cos(c+dx))^{2}(A+B\cos(c+dx))\,dx$$
$$=\frac{15\,(12\,A+11\,B)a^{2}dx+\left(40\,Ba^{2}\cos\left(dx+c\right)^{5}+48\,(A+2\,B)a^{2}\cos\left(dx+c\right)^{4}+10\,(12\,A+11\,B)a^{2}\cos\left(\right.\right.}{2}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/240*(15*(12*A + 11*B)*a^2*d*x + (40*B*a^2*cos(d*x + c)^5 + 48*(A + 2*B)*
a^2*cos(d*x + c)^4 + 10*(12*A + 11*B)*a^2*cos(d*x + c)^3 + 16*(9*A + 8*B)*
a^2*cos(d*x + c)^2 + 15*(12*A + 11*B)*a^2*cos(d*x + c) + 32*(9*A + 8*B)*a^
2)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(173) = 346$.

Time = 0.42 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.14

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((3*A*a**2*x*sin(c + d*x)**4/4 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
 + d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/4 + 8*A*a**2*sin(c + d*x)**5/(15
*d) + 4*A*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**2*sin(c + d*
x)**3*cos(c + d*x)/(4*d) + 2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c +
 d*x)*cos(c + d*x)**4/d + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + A*
a**2*sin(c + d*x)*cos(c + d*x)**2/d + 5*B*a**2*x*sin(c + d*x)**6/16 + 15*B
*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B*a**2*x*sin(c + d*x)**4/8
+ 15*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**2*x*sin(c + d*x)
**2*cos(c + d*x)**2/4 + 5*B*a**2*x*cos(c + d*x)**6/16 + 3*B*a**2*x*cos(c +
 d*x)**4/8 + 5*B*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 16*B*a**2*sin(
c + d*x)**5/(15*d) + 5*B*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*B*
a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)**3*cos(
c + d*x)/(8*d) + 11*B*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**2*
sin(c + d*x)*cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*
d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**2*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.13

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{64\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Aa^2-320\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^2-}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 60*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 + 128*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*B*a^2 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c
 - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^2 + 30*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{Ba^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{1}{16}\left(12\,Aa^2+11\,Ba^2\right)x+\frac{\left(Aa^2+2\,Ba^2\right)\sin\left(5\,dx+5\,c\right)}{80\,d}$$
$$+\frac{\left(4\,Aa^2+5\,Ba^2\right)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{\left(9\,Aa^2+10\,Ba^2\right)\sin\left(3\,dx+3\,c\right)}{48\,d}$$
$$+\frac{\left(32\,Aa^2+31\,Ba^2\right)\sin\left(2\,dx+2\,c\right)}{64\,d}+\frac{\left(11\,Aa^2+10\,Ba^2\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/192*B*a^2*sin(6*d*x + 6*c)/d + 1/16*(12*A*a^2 + 11*B*a^2)*x + 1/80*(A*a^
2 + 2*B*a^2)*sin(5*d*x + 5*c)/d + 1/64*(4*A*a^2 + 5*B*a^2)*sin(4*d*x + 4*c
)/d + 1/48*(9*A*a^2 + 10*B*a^2)*sin(3*d*x + 3*c)/d + 1/64*(32*A*a^2 + 31*B
*a^2)*sin(2*d*x + 2*c)/d + 1/8*(11*A*a^2 + 10*B*a^2)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.13 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.65

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(\frac{3\,A\,a^2}{2}+\frac{11\,B\,a^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{17\,A\,a^2}{2}+\frac{187\,B\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{107\,A\,a^2}{5}+\frac{331\,B\,a^2}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\right.}$$
$$-\frac{a^2\,(12\,A+11\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}$$
$$+\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(12\,A+11\,B)}{8\left(\frac{3\,A\,a^2}{2}+\frac{11\,B\,a^2}{8}\right)}\right)\,(12\,A+11\,B)}{8\,d}$$

input
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)*((13*A*a^2)/2 + (53*B*a^2)/8) + tan(c/2 + (d*x)/2)^11*
((3*A*a^2)/2 + (11*B*a^2)/8) + tan(c/2 + (d*x)/2)^3*((31*A*a^2)/2 + (87*B*
a^2)/8) + tan(c/2 + (d*x)/2)^9*((17*A*a^2)/2 + (187*B*a^2)/24) + tan(c/2 +
 (d*x)/2)^7*((107*A*a^2)/5 + (331*B*a^2)/20) + tan(c/2 + (d*x)/2)^5*((117*
A*a^2)/5 + (501*B*a^2)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x
)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (
d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (a^2*(12*A + 11*B)*(atan(tan(c/
2 + (d*x)/2)) - (d*x)/2))/(8*d) + (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(12*A
+ 11*B))/(8*((3*A*a^2)/2 + (11*B*a^2)/8)))*(12*A + 11*B))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{a^2\left(40\cos\left(dx+c\right)\sin\left(dx+c\right)^5b-120\cos\left(dx+c\right)\sin\left(dx+c\right)^3a-190\cos\left(dx+c\right)\sin\left(dx+c\right)^3b+30\right.}{}$$

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
(a**2*(40*cos(c + d*x)*sin(c + d*x)**5*b - 120*cos(c + d*x)*sin(c + d*x)**
3*a - 190*cos(c + d*x)*sin(c + d*x)**3*b + 300*cos(c + d*x)*sin(c + d*x)*a
 + 315*cos(c + d*x)*sin(c + d*x)*b + 48*sin(c + d*x)**5*a + 96*sin(c + d*x
)**5*b - 240*sin(c + d*x)**3*a - 320*sin(c + d*x)**3*b + 480*sin(c + d*x)*
a + 480*sin(c + d*x)*b + 180*a*d*x + 165*b*d*x))/(240*d)
```

3.11 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 160

$$\begin{aligned}
&\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx \\
&= \frac{1}{8}a^2(7A+6B)x + \frac{a^2(10A+9B)\sin(c+dx)}{5d} \\
&\quad + \frac{a^2(7A+6B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2(5A+6B)\cos^3(c+dx)\sin(c+dx)}{20d} \\
&\quad + \frac{B\cos^3(c+dx)\,(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d} - \frac{a^2(10A+9B)\sin^3(c+dx)}{15d}
\end{aligned}$$

output
```
1/8*a^2*(7*A+6*B)*x+1/5*a^2*(10*A+9*B)*sin(d*x+c)/d+1/8*a^2*(7*A+6*B)*cos(
d*x+c)*sin(d*x+c)/d+1/20*a^2*(5*A+6*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*B*cos
(d*x+c)^3*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d-1/15*a^2*(10*A+9*B)*sin(d*x+c)
^3/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\frac{a^2(360Bc+420Adx+360Bdx+60(12A+11B)\sin(c+dx)+240(A+B)\sin(2(c+dx))+80A\sin(3}{480d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(a^2*(360*B*c + 420*A*d*x + 360*B*d*x + 60*(12*A + 11*B)*Sin[c + d*x] + 24
0*(A + B)*Sin[2*(c + d*x)] + 80*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)]
 + 15*A*Sin[4*(c + d*x)] + 30*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/
(480*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{5}\int \cos^2(c+dx)(\cos(c+dx)a+a)(a(5A+3B)+a(5A+6B)\cos(c+dx))dx+\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(5A+3B)+a(5A+6B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{5}\int \cos^2(c+dx)\left((5A+6B)\cos^2(c+dx)a^2+(5A+3B)a^2+\left((5A+3B)a^2+(5A+6B)a^2\right)\cos(c+dx)\right)dx+\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left((5A+6B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+(5A+3B)a^2+\left((5A+3B)a^2+(5A+6B)a^2\right)\sin\left(\right.\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{5}\left(\frac{1}{4}\int \cos^2(c+dx)\left(5(7A+6B)a^2+4(10A+9B)\cos(c+dx)a^2\right)dx+\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(5(7A+6B)a^2+4(10A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx+\frac{a^2(5A+6B)\sin(c+dx)\,c}{4d}\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(4a^2(10A+9B)\int \cos^3(c+dx)dx+5a^2(7A+6B)\int \cos^2(c+dx)dx\right)+\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c}{4d}\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5a^2(7A+6B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4a^2(10A+9B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{a^2(5A+6B)\sin(c\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$\Big\downarrow$ 3113

$$\frac{1}{5}\left(\frac{1}{4}\left(5a^2(7A+6B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4a^2(10A+9B)\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{a^2(5\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$\Big\downarrow$ 2009

$$\frac{1}{5}\left(\frac{1}{4}\left(5a^2(7A+6B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4a^2(10A+9B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{a^2(5A+6B\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$\Big\downarrow$ 3115

$$\frac{1}{5}\left(\frac{1}{4}\left(5a^2(7A+6B)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4a^2(10A+9B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$\Big\downarrow$ 24

$$\frac{1}{5}\left(\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{4}\left(5a^2(7A+6B)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4a^2(10A+9B\right.\right.$$
$$\frac{B\sin(c+dx)\cos^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(B*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d) + ((a^2*(5*
A + 6*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (5*a^2*(7*A + 6*B)*(x/2 + (C
os[c + d*x]*Sin[c + d*x])/(2*d)) - (4*a^2*(10*A + 9*B)*(-Sin[c + d*x] + Si
n[c + d*x]^3/3))/d)/4)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 51.91 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{a^2\left(16(A+B)\sin(2dx+2c)+2\left(\frac{8A}{3}+3B\right)\sin(3dx+3c)+(A+2B)\sin(4dx+4c)+\frac{2B\sin(5dx+5c)}{5}+4(12A+11B)\sin(dx+c\cdots\right.}{32d}$
parts	$\frac{(a^2A+2a^2B)\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{(2a^2A+a^2B)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}+\frac{a^2\cdots}{\cdots}$
risch	$\frac{7a^2xA}{8}+\frac{3a^2Bx}{4}+\frac{3\sin(dx+c)a^2A}{2d}+\frac{11\sin(dx+c)a^2B}{8d}+\frac{\sin(5dx+5c)a^2B}{80d}+\frac{\sin(4dx+4c)a^2A}{32d}+\frac{\sin(4dx+\cdots}{16\cdots}$
derivativedivides	$\frac{a^2A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+\frac{2a^2A\left(\cos(dx+c)\cdots\right.}{3\cdots}}{\cdots}$
default	$\frac{a^2A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+\frac{2a^2A\left(\cos(dx+c)\cdots\right.}{3\cdots}}{\cdots}$
norman	$\frac{\frac{a^2(7A+6B)x}{8}+\frac{7a^2(7A+6B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}+\frac{a^2(7A+6B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}+\frac{5a^2(7A+6B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8}+\frac{5a^2(7A+6B)x\tan\left(\frac{dx}{2}+\cdots\right.}{4}\cdots}{\cdots}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/32*a^2*(16*(A+B)*sin(2*d*x+2*c)+2*(8/3*A+3*B)*sin(3*d*x+3*c)+(A+2*B)*sin
(4*d*x+4*c)+2/5*B*sin(5*d*x+5*c)+4*(12*A+11*B)*sin(d*x+c)+28*x*(A+6/7*B)*d
)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{15\,(7\,A+6\,B)a^2dx+\left(24\,Ba^2\cos\left(dx+c\right)^4+30\,(A+2\,B)a^2\cos\left(dx+c\right)^3+8\,(10\,A+9\,B)a^2\cos\left(dx+\right.\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/120*(15*(7*A + 6*B)*a^2*d*x + (24*B*a^2*cos(d*x + c)^4 + 30*(A + 2*B)*a^
2*cos(d*x + c)^3 + 8*(10*A + 9*B)*a^2*cos(d*x + c)^2 + 15*(7*A + 6*B)*a^2*
cos(d*x + c) + 16*(10*A + 9*B)*a^2)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(144) = 288.

Time = 0.32 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.87

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{3Aa^2x\sin^4(c+dx)}{8}+\frac{3Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{Aa^2x\sin^2(c+dx)}{2}+\frac{3Aa^2x\cos^4(c+dx)}{8}+\frac{Aa^2x\cos^2(c+dx)}{2}+\frac{3Aa^2\sin^3(c\cdots}{} \\ x(A+B\cos(c))\,(a\cos(c)+a)^2\cos^2(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/8 +
 A*a**2*x*cos(c + d*x)**2/2 + 3*A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d)
+ 4*A*a**2*sin(c + d*x)**3/(3*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(
8*d) + 2*A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c + d*x)*cos(c
 + d*x)/(2*d) + 3*B*a**2*x*sin(c + d*x)**4/4 + 3*B*a**2*x*sin(c + d*x)**2*
cos(c + d*x)**2/2 + 3*B*a**2*x*cos(c + d*x)**4/4 + 8*B*a**2*sin(c + d*x)**
5/(15*d) + 4*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c
 + d*x)**3*cos(c + d*x)/(4*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*si
n(c + d*x)*cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d)
 + B*a**2*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*
cos(c) + a)**2*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = -\frac{320\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^2-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^2}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/480*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 15*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 120*(2*d*x + 2*c + sin(2*d*
x + 2*c))*A*a^2 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
c))*B*a^2 + 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 30*(12*d*x + 12*
c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{Ba^2 \sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\,(7\,Aa^2+6\,Ba^2)x + \frac{(Aa^2+2\,Ba^2)\sin(4\,dx+4\,c)}{32\,d} + \frac{(8\,Aa^2+9\,Ba^2)\sin(3\,dx+3\,c)}{48\,d} + \frac{(Aa^2+Ba^2)\sin(2\,dx+2\,c)}{2\,d} + \frac{(12\,Aa^2+11\,Ba^2)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/80*B*a^2*sin(5*d*x + 5*c)/d + 1/8*(7*A*a^2 + 6*B*a^2)*x + 1/32*(A*a^2 +
2*B*a^2)*sin(4*d*x + 4*c)/d + 1/48*(8*A*a^2 + 9*B*a^2)*sin(3*d*x + 3*c)/d
+ 1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/8*(12*A*a^2 + 11*B*a^2)*sin(d
*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 41.83 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.73

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(\frac{7\,A\,a^2}{4}+\frac{3\,B\,a^2}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{49\,A\,a^2}{6}+7\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{40\,A\,a^2}{3}+\frac{72\,B\,a^2}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\tan\left(\frac{c}{2}-\right.\right.}$$
$$-\frac{a^2\,(7\,A+6\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d} + \frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(7\,A+6\,B)}{4\,\left(\frac{7\,A\,a^2}{4}+\frac{3\,B\,a^2}{2}\right)}\right)\,(7\,A+6\,B)}{4\,d}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)*((25*A*a^2)/4 + (13*B*a^2)/2) + tan(c/2 + (d*x)/2)^9*(
(7*A*a^2)/4 + (3*B*a^2)/2) + tan(c/2 + (d*x)/2)^7*((49*A*a^2)/6 + 7*B*a^2)
 + tan(c/2 + (d*x)/2)^3*((79*A*a^2)/6 + 9*B*a^2) + tan(c/2 + (d*x)/2)^5*((
40*A*a^2)/3 + (72*B*a^2)/5))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*
x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*
x)/2)^10 + 1)) - (a^2*(7*A + 6*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4
*d) + (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(7*A + 6*B))/(4*((7*A*a^2)/4 + (3*
B*a^2)/2)))*(7*A + 6*B))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{a^2\left(-30\cos\left(dx+c\right)\sin\left(dx+c\right)^3 a-60\cos\left(dx+c\right)\sin\left(dx+c\right)^3 b+135\cos\left(dx+c\right)\sin\left(dx+c\right) a+15\right.}{}$$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
(a**2*( - 30*cos(c + d*x)*sin(c + d*x)**3*a - 60*cos(c + d*x)*sin(c + d*x)
**3*b + 135*cos(c + d*x)*sin(c + d*x)*a + 150*cos(c + d*x)*sin(c + d*x)*b
+ 24*sin(c + d*x)**5*b - 80*sin(c + d*x)**3*a - 120*sin(c + d*x)**3*b + 24
0*sin(c + d*x)*a + 240*sin(c + d*x)*b + 105*a*d*x + 90*b*d*x))/(120*d)
```

3.12 $\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 129

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx \\ = \frac{1}{8}a^2(8A+7B)x+\frac{a^2(8A+7B)\sin(c+dx)}{6d}+\frac{a^2(8A+7B)\cos(c+dx)\sin(c+dx)}{24d} \\ +\frac{(4A-B)(a+a\cos(c+dx))^2\sin(c+dx)}{12d}+\frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4ad}\end{aligned}$$

output
```
1/8*a^2*(8*A+7*B)*x+1/6*a^2*(8*A+7*B)*sin(d*x+c)/d+1/24*a^2*(8*A+7*B)*cos(
d*x+c)*sin(d*x+c)/d+1/12*(4*A-B)*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/4*B*(a+
a*cos(d*x+c))^3*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\frac{a^2(84Bc+96Adx+84Bdx+24(7A+6B)\sin(c+dx)+48(A+B)\sin(2(c+dx))+8A\sin(3(c+dx)}{96d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(a^2*(84*B*c + 96*A*d*x + 84*B*d*x + 24*(7*A + 6*B)*Sin[c + d*x] + 48*(A +
 B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 16*B*Sin[3*(c + d*x)] + 3*B*
Sin[4*(c + d*x)]))/(96*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a\cos(c+dx)+a)^2\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (\cos(c+dx)a+a)^2(3aB+a(4A-B)\cos(c+dx))dx}{4a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(3aB+a(4A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{4a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{3}a(8A+7B)\int (\cos(c+dx)a+a)^2dx+\frac{a(4A-B)\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}}{4a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}a(8A+7B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx+\frac{a(4A-B)\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}}{4a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4ad}$$

$$\Big\downarrow \text{3123}$$

$$\frac{\frac{1}{3}a(8A+7B)\left(\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}\right)+\frac{a(4A-B)\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}}{4a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4ad}$$

input

```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output

```
(B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d) + ((a*(4*A - B)*(a + a*Cos
[c + d*x])^2*Sin[c + d*x])/(3*d) + (a*(8*A + 7*B)*((3*a^2*x)/2 + (2*a^2*Si
n[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)))/3)/(4*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 19.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{a^{2}\left(\frac{(A+B)\sin(2dx+2c)}{2}+\frac{\left(\frac{A}{2}+B\right)\sin(3dx+3c)}{6}+\frac{\sin(4dx+4c)B}{32}+\frac{\left(\frac{7A}{2}+3B\right)\sin(dx+c)}{2}+dx\left(A+\frac{7B}{8}\right)\right)}{d}$
parts	$\frac{(a^{2}A+2a^{2}B)\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3d}+\frac{(2a^{2}A+a^{2}B)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{\sin(dx+c)a^{2}A}{d}+$
risch	$a^{2}xA+\frac{7a^{2}Bx}{8}+\frac{7\sin(dx+c)a^{2}A}{4d}+\frac{3\sin(dx+c)a^{2}B}{2d}+\frac{\sin(4dx+4c)a^{2}B}{32d}+\frac{\sin(3dx+3c)a^{2}A}{12d}+$
derivativedivides	$\frac{\frac{a^{2}A\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3}+a^{2}B\left(\frac{\left(\cos(dx+c)^{3}+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^{2}A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.}{d}$
default	$\frac{\frac{a^{2}A\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3}+a^{2}B\left(\frac{\left(\cos(dx+c)^{3}+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^{2}A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.}{d}$
norman	$\frac{\frac{a^{2}(8A+7B)x}{8}+\frac{11a^{2}(8A+7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{12d}+\frac{a^{2}(8A+7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4d}+\frac{a^{2}(8A+7B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2}+\frac{3a^{2}(8A+7B)x\tan\left(\frac{dx}{2}+\right.}{4}}{\left(1+\tan\left(\frac{dx}{2}\right.\right.}$
orering	Expression too large to display

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
a^2*(1/2*(A+B)*sin(2*d*x+2*c)+1/6*(1/2*A+B)*sin(3*d*x+3*c)+1/32*sin(4*d*x+
4*c)*B+1/2*(7/2*A+3*B)*sin(d*x+c)+d*x*(A+7/8*B))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int \cos(c+dx)(a+a\cos(c+dx))^{2}(A+B\cos(c+dx))\,dx$$
$$=\frac{3\,(8\,A+7\,B)a^{2}dx+\left(6\,Ba^{2}\cos\left(dx+c\right)^{3}+8\,(A+2\,B)a^{2}\cos\left(dx+c\right)^{2}+3\,(8\,A+7\,B)a^{2}\cos\left(dx+c\right)+\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/24*(3*(8*A + 7*B)*a^2*d*x + (6*B*a^2*cos(d*x + c)^3 + 8*(A + 2*B)*a^2*co
s(d*x + c)^2 + 3*(8*A + 7*B)*a^2*cos(d*x + c) + 8*(5*A + 4*B)*a^2)*sin(d*x
 + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(112) = 224$.

Time = 0.20 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.62

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} Aa^2x\sin^2(c+dx) + Aa^2x\cos^2(c+dx) + \frac{2Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{d} \\ x(A+B\cos(c))\,(a\cos(c)+a)^2\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**2*x*sin(c + d*x)**2 + A*a**2*x*cos(c + d*x)**2 + 2*A*a**2*
sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin
(c + d*x)*cos(c + d*x)/d + A*a**2*sin(c + d*x)/d + 3*B*a**2*x*sin(c + d*x)
**4/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**2*x*sin(c + d*
x)**2/2 + 3*B*a**2*x*cos(c + d*x)**4/8 + B*a**2*x*cos(c + d*x)**2/2 + 3*B*
a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*B*a**2*sin(c + d*x)**3/(3*d) +
 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*B*a**2*sin(c + d*x)*cos(c
 + d*x)**2/d + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A +
B*cos(c))*(a*cos(c) + a)**2*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx =$$

$$-\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^2-48\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^2+64\left(\sin\left(dx+c\right)^3-3\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 48*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^2 + 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 3*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 24*(2*d*x + 2*c
 + sin(2*d*x + 2*c))*B*a^2 - 96*A*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\frac{Ba^2\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{1}{8}\left(8\,Aa^2+7\,Ba^2\right)x+\frac{\left(Aa^2+2\,Ba^2\right)\sin\left(3\,dx+3\,c\right)}{12\,d}$$
$$+\frac{\left(Aa^2+Ba^2\right)\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(7\,Aa^2+6\,Ba^2\right)\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/32*B*a^2*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^2 + 7*B*a^2)*x + 1/12*(A*a^2 +
2*B*a^2)*sin(3*d*x + 3*c)/d + 1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/4
*(7*A*a^2 + 6*B*a^2)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 41.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = A\,a^2\,x+\frac{7\,B\,a^2\,x}{8}+\frac{7\,A\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,B\,a^2\,\sin\left(c+d\,x\right)}{2\,d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{A\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{B\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}+\frac{B\,a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

output
```
A*a^2*x + (7*B*a^2*x)/8 + (7*A*a^2*sin(c + d*x))/(4*d) + (3*B*a^2*sin(c +
d*x))/(2*d) + (A*a^2*sin(2*c + 2*d*x))/(2*d) + (A*a^2*sin(3*c + 3*d*x))/(1
2*d) + (B*a^2*sin(2*c + 2*d*x))/(2*d) + (B*a^2*sin(3*c + 3*d*x))/(6*d) + (
B*a^2*sin(4*c + 4*d*x))/(32*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{a^2\left(-6\cos\left(dx+c\right)\sin\left(dx+c\right)^3b+24\cos\left(dx+c\right)\sin\left(dx+c\right)a+27\cos\left(dx+c\right)\sin\left(dx+c\right)b-8\sin\left(\right.\right.}{24d}$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
(a**2*( - 6*cos(c + d*x)*sin(c + d*x)**3*b + 24*cos(c + d*x)*sin(c + d*x)*
a + 27*cos(c + d*x)*sin(c + d*x)*b - 8*sin(c + d*x)**3*a - 16*sin(c + d*x)
**3*b + 48*sin(c + d*x)*a + 48*sin(c + d*x)*b + 24*a*d*x + 21*b*d*x))/(24*
d)
```

3.13 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{1}{2}a^2(3A+2B)x+\frac{2a^2(3A+2B)\sin(c+dx)}{3d} + \frac{a^2(3A+2B)\cos(c+dx)\sin(c+dx)}{6d}+\frac{B(a+a\cos(c+dx))^2\sin(c+dx)}{3d}$$

output
```
1/2*a^2*(3*A+2*B)*x+2/3*a^2*(3*A+2*B)*sin(d*x+c)/d+1/6*a^2*(3*A+2*B)*cos(d
*x+c)*sin(d*x+c)/d+1/3*B*(a+a*cos(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{a^2\sin(c+dx)\left(12A+11B+3(A+2B)\cos(c+dx)+B\cos(2(c+dx))+\frac{6(3A+2B)\arcsin\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)}{\sqrt{\sin^2(c+dx)}}\right)}{6d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(a^2*Sin[c + d*x]*(12*A + 11*B + 3*(A + 2*B)*Cos[c + d*x] + B*Cos[2*(c + d
*x)] + (6*(3*A + 2*B)*ArcSin[Sqrt[Sin[(c + d*x)/2]^2]])/Sqrt[Sin[c + d*x]^
2]))/(6*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3230, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{3}(3A+2B)\int (\cos(c+dx)a+a)^2dx+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3A+2B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{3}(3A+2B)\left(\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}\right)+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*A + 2*B)*((3*a^2*x)/2
+ (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\frac{a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^2B\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+2a^2A\sin\left(dx+c\right)+2a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\cdots\right.}{d}$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^2*B*(cos(d*x+c)
^2+2)*sin(d*x+c)+2*a^2*A*sin(d*x+c)+2*a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a^2*A*(d*x+c)+a^2*B*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\,dx = \frac{3\,(3\,A + 2\,B)a^2dx + \left(2\,Ba^2\cos\left(dx + c\right)^2 + 3\,(A + 2\,B)a^2\cos\left(dx + c\right) + 2\,(6\,A + 5\,B)a^2\right)\sin\left(dx + c\right)}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*(3*A + 2*B)*a^2*d*x + (2*B*a^2*cos(d*x + c)^2 + 3*(A + 2*B)*a^2*cos
(d*x + c) + 2*(6*A + 5*B)*a^2)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\,dx = \begin{cases} \frac{Aa^2x\sin^2(c+dx)}{2} + \frac{Aa^2x\cos^2(c+dx)}{2} + Aa^2x + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Aa^2\sin(c+dx)}{d} + Ba^2x\sin^2(c + dx) + B \\ x(A + B\cos(c))\,(a\cos(c) + a)^2 \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**
2*x + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*sin(c + d*x)/d + B
*a**2*x*sin(c + d*x)**2 + B*a**2*x*cos(c + d*x)**2 + 2*B*a**2*sin(c + d*x)
**3/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*sin(c + d*x)*co
s(c + d*x)/d + B*a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(
c) + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\,dx$$
$$= \frac{3\,(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right))Aa^2 + 12\,(dx + c)Aa^2 - 4\left(\sin\left(dx + c\right)^3 - 3\,\sin\left(dx + c\right)\right)Ba^2 + 6\,(2\,dx -}{12\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 12*(d*x + c)*A*a^2 - 4*(s
in(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))
*B*a^2 + 24*A*a^2*sin(d*x + c) + 12*B*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\,dx$$
$$= \frac{Ba^2\sin\left(3\,dx + 3\,c\right)}{12\,d} + \frac{1}{2}\left(3\,Aa^2 + 2\,Ba^2\right)x + \frac{\left(Aa^2 + 2\,Ba^2\right)\sin\left(2\,dx + 2\,c\right)}{4\,d} + \frac{\left(8\,Aa^2 + 7\,Ba^2\right)\sin\left(dx + c\right)}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/12*B*a^2*sin(3*d*x + 3*c)/d + 1/2*(3*A*a^2 + 2*B*a^2)*x + 1/4*(A*a^2 + 2
*B*a^2)*sin(2*d*x + 2*c)/d + 1/4*(8*A*a^2 + 7*B*a^2)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 41.55 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\, dx$$
$$= \frac{3\,A\,a^2\,x}{2} + B\,a^2\,x + \frac{2\,A\,a^2\,\sin\left(c + d\,x\right)}{d} + \frac{7\,B\,a^2\,\sin\left(c + d\,x\right)}{4\,d} + \frac{A\,a^2\,\sin\left(2\,c + 2\,d\,x\right)}{4\,d} + \frac{B\,a^2\,\sin\left(2\,c + 2\,d\,x\right)}{2\,d} + \frac{B\,a^2\,\sin\left(3\,c + 3\,d\,x\right)}{12\,d}$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

output
```
(3*A*a^2*x)/2 + B*a^2*x + (2*A*a^2*sin(c + d*x))/d + (7*B*a^2*sin(c + d*x)
)/(4*d) + (A*a^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*sin(2*c + 2*d*x))/(2*d)
+ (B*a^2*sin(3*c + 3*d*x))/(12*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\, dx$$
$$= \frac{a^2 \left(3\cos\left(dx + c\right)\sin\left(dx + c\right) a + 6\cos\left(dx + c\right)\sin\left(dx + c\right) b - 2\sin\left(dx + c\right)^3 b + 12\sin\left(dx + c\right) a + 12\right.}{6d}$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
(a**2*(3*cos(c + d*x)*sin(c + d*x)*a + 6*cos(c + d*x)*sin(c + d*x)*b - 2*s
in(c + d*x)**3*b + 12*sin(c + d*x)*a + 12*sin(c + d*x)*b + 9*a*d*x + 6*b*d
*x))/(6*d)
```

3.14 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 82

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{1}{2}a^2(4A+3B)x+\frac{a^2A\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B(a^2+a^2\cos(c+dx))\sin(c+dx)}{2d}$$

output
```
1/2*a^2*(4*A+3*B)*x+a^2*A*arctanh(sin(d*x+c))/d+1/2*a^2*(2*A+3*B)*sin(d*x+
c)/d+1/2*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{a^2\left(8Adx+6Bdx-4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+d\right.\right.}{4d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(a^2*(8*A*d*x + 6*B*d*x - 4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(A + 2*B)*Sin[c + d*x] + B
*Sin[2*(c + d*x)]))/(4*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{2}\int (\cos(c+dx)a+a)(2aA+a(2A+3B)\cos(c+dx))\sec(c+dx)dx+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(2aA+a(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{2}\int \left((2A+3B)\cos^2(c+dx)a^2+2Aa^2+\left(2Aa^2+(2A+3B)a^2\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+2Aa^2+\left(2Aa^2+(2A+3B)a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(2Aa^2+(4A+3B)\cos(c+dx)a^2\right)\sec(c+dx)dx+\frac{a^2(2A+3B)\sin(c+dx)}{d}\right)+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{2Aa^2+(4A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2(2A+3B)\sin(c+dx)}{d}\right)+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(2a^2A\int \sec(c+dx)dx+\frac{a^2(2A+3B)\sin(c+dx)}{d}+a^2x(4A+3B)\right)+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2a^2A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2(2A+3B)\sin(c+dx)}{d}+a^2x(4A+3B)\right)+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{2a^2A\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2(2A+3B)\sin(c+dx)}{d}+a^2x(4A+3B)\right)+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(2*d) + (a^2*(4*A + 3*B)*x + (2*
a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A + 3*B)*Sin[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

method	result
parallelrisch	$-\frac{\left(A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{B\sin(2dx+2c)}{4}+(-A-2B)\sin(dx+c)-2dx\left(A+\frac{3B}{4}\right)\right)a^2}{d}$
derivativedivides	$\frac{a^2A\sin(dx+c)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2A(dx+c)+2a^2B\sin(dx+c)+a^2A\ln(\sec(dx+c)+\tan(dx+c))+}{d}$
default	$\frac{a^2A\sin(dx+c)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2A(dx+c)+2a^2B\sin(dx+c)+a^2A\ln(\sec(dx+c)+\tan(dx+c))+}{d}$
parts	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(a^2A+2a^2B)\sin(dx+c)}{d}+\frac{(2a^2A+a^2B)(dx+c)}{d}+\frac{a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\right.}{d}$
risch	$2a^2xA+\frac{3a^2Bx}{2}-\frac{ie^{i(dx+c)}a^2A}{2d}-\frac{ie^{i(dx+c)}a^2B}{d}+\frac{ie^{-i(dx+c)}a^2A}{2d}+\frac{ie^{-i(dx+c)}a^2B}{d}+\frac{a^2A\ln(e^{i(dx+c)}+}{d}$
norman	$\frac{(2a^2A+\frac{3}{2}a^2B)x+(2a^2A+\frac{3}{2}a^2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(6a^2A+\frac{9}{2}a^2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(6a^2A+\frac{9}{2}a^2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
-(A*ln(tan(1/2*d*x+1/2*c)-1)-A*ln(tan(1/2*d*x+1/2*c)+1)-1/4*B*sin(2*d*x+2*
c)+(-A-2*B)*sin(d*x+c)-2*d*x*(A+3/4*B))*a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{(4A+3B)a^2dx+Aa^2\log(\sin(dx+c)+1)-Aa^2\log(-\sin(dx+c)+1)+(Ba^2\cos(dx+c)+2(A+}{2d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fri
cas")
```

output
```
1/2*((4*A + 3*B)*a^2*d*x + A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*
x + c) + 1) + (B*a^2*cos(d*x + c) + 2*(A + 2*B)*a^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= a^2\left(\int A\sec(c+dx)\,dx + \int 2A\cos(c+dx)\sec(c+dx)\,dx + \int A\cos^2(c+dx)\sec(c+dx)\,dx + \int B\cos(c+dx)\sec(c+dx)\,dx + \int 2B\cos^2(c+dx)\sec(c+dx)\,dx + \int B\cos^3(c+dx)\sec(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*cos(c + d*x)*sec(c + d*x)
, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*
x)*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x), x) + Inte
gral(B*cos(c + d*x)**3*sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{8(dx+c)Aa^2 + (2dx+2c+\sin(2dx+2c))Ba^2 + 4(dx+c)Ba^2 + 4Aa^2\log(\sec(dx+c)+\tan(dx-}{4d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="max
ima")
```

output
```
1/4*(8*(d*x + c)*A*a^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 4*(d*x +
 c)*B*a^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 4*A*a^2*sin(d*x + c
) + 8*B*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.77

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$
$$= \frac{2\, Aa^2 \log\left(\left|\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) + 1\right|\right) - 2\, Aa^2 \log\left(\left|\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) - 1\right|\right) + (4\, Aa^2 + 3\, Ba^2)(dx + c) + \frac{2\,(2\ldots}{\ldots}}{2\, d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gia
c")
```

output
```
1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) + (4*A*a^2 + 3*B*a^2)*(d*x + c) + 2*(2*A*a^2*tan(1/2*d*
x + 1/2*c)^3 + 3*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^2*tan(1/2*d*x + 1/2*
c) + 5*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 41.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$
$$= \frac{A\, a^2\, \sin\left(c + d\, x\right)}{d} + \frac{2\, B\, a^2\, \sin\left(c + d\, x\right)}{d} + \frac{4\, A\, a^2\, \mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\, x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\, x}{2}\right)}\right)}{d} + \frac{2\, A\, a^2\, \mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\, x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\, x}{2}\right)}\right)}{d} + \frac{3\, B\, a^2\, \mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\, x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\, x}{2}\right)}\right)}{d} + \frac{B\, a^2\, \sin\left(2\, c + 2\, d\, x\right)}{4\, d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x),x)
```

output
```
(A*a^2*sin(c + d*x))/d + (2*B*a^2*sin(c + d*x))/d + (4*A*a^2*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)))/d + (3*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (B*a^2*sin(2*c + 2*d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$
$$= \frac{a^2 \left(\cos(dx + c)\sin(dx + c)\, b - 2\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) a + 2\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) a + 2\sin(dx + c)\, a\right.}{2d}$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
(a**2*(cos(c + d*x)*sin(c + d*x)*b - 2*log(tan((c + d*x)/2) - 1)*a + 2*log
(tan((c + d*x)/2) + 1)*a + 2*sin(c + d*x)*a + 4*sin(c + d*x)*b + 4*a*d*x +
 3*b*d*x))/(2*d)
```

3.15 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 74

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = a^2(A+2B)x+\frac{a^2(2A+B)\text{arctanh}(\sin(c+dx))}{d} - \frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A(a^2+a^2\cos(c+dx))\tan(c+dx)}{d}$$

output
```
a^2*(A+2*B)*x+a^2*(2*A+B)*arctanh(sin(d*x+c))/d-a^2*(A-B)*sin(d*x+c)/d+A*(
a^2+a^2*cos(d*x+c))*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.93

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a^2\left(Ac+2Bc+Adx+2Bdx-2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\text{s}\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(a^2*(A*c + 2*B*c + A*d*x + 2*B*d*x - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
B*Sin[c + d*x] + A*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3454, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3454}$$

$$\int (\cos(c+dx)a+a)(a(2A+B)-a(A-B)\cos(c+dx))\sec(c+dx)dx+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(2A+B)-a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$$\Big\downarrow \text{3447}$$

$$\int \left(-\left((A-B)\cos^2(c+dx)a^2\right)+(2A+B)a^2+\left(a^2(2A+B)-a^2(A-B)\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$\downarrow$ 3042

$$\int \frac{-\left((A-B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2\right)+(2A+B)a^2+\left(a^2(2A+B)-a^2(A-B)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$\downarrow$ 3502

$$\int \left((2A+B)a^2+(A+2B)\cos(c+dx)a^2\right)\sec(c+dx)dx-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$\downarrow$ 3042

$$\int \frac{(2A+B)a^2+(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$\downarrow$ 3214

$$a^2(2A+B)\int \sec(c+dx)dx-\frac{a^2(A-B)\sin(c+dx)}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$\downarrow$ 3042

$$a^2(2A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^2(A-B)\sin(c+dx)}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

$\downarrow$ 4257

$$\frac{a^2(2A+B)\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2(A-B)\sin(c+dx)}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
a^2*(A + 2*B)*x + (a^2*(2*A + B)*ArcTanh[Sin[c + d*x]])/d - (a^2*(A - B)*S
in[c + d*x])/d + (A*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 6.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

method	result
parts	$\frac{a^2A\tan(dx+c)}{d}+\frac{(a^2A+2a^2B)(dx+c)}{d}+\frac{(2a^2A+a^2B)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{\sin(dx+c)a^2B}{d}$
derivativedivides	$\frac{a^2A(dx+c)+a^2B\sin(dx+c)+2a^2A\ln(\sec(dx+c)+\tan(dx+c))+2a^2B(dx+c)+a^2A\tan(dx+c)+a^2B\ln(\sec(dx+c)+\text{ta}}{d}$
default	$\frac{a^2A(dx+c)+a^2B\sin(dx+c)+2a^2A\ln(\sec(dx+c)+\tan(dx+c))+2a^2B(dx+c)+a^2A\tan(dx+c)+a^2B\ln(\sec(dx+c)+\text{ta}}{d}$
parallelrisch	$-\frac{2a^2\left(\cos(dx+c)\left(A+\frac{B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\cos(dx+c)\left(A+\frac{B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{B\sin(2dx+2c)}{4}-\frac{dx(A+2B)}{2}\right.}{\cos(dx+c)d}$
risch	$a^2xA+2a^2Bx-\frac{ie^{i(dx+c)}a^2B}{2d}+\frac{ie^{-i(dx+c)}a^2B}{2d}+\frac{2ia^2A}{d(e^{2i(dx+c)}+1)}+\frac{2a^2A\ln(e^{i(dx+c)}+i)}{d}+\frac{a^2\ln(e^{i(}}{}$
norman	$\frac{(-a^2A-2a^2B)x+(-2a^2A-4a^2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(a^2A+2a^2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(2a^2A+4a^2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}\right.\right.}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBO
SE)
```

output
```
a^2*A*tan(d*x+c)/d+(A*a^2+2*B*a^2)/d*(d*x+c)+(2*A*a^2+B*a^2)/d*ln(sec(d*x+
c)+tan(d*x+c))+1/d*sin(d*x+c)*a^2*B
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{2\,(A+2\,B)a^2dx\cos\left(dx+c\right)+(2\,A+B)a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-(2\,A+B)a^2\cos\left(dx+c\right. \cdots}{2\,d\cos\left(dx+c\right)}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="f
ricas")
```

output
```
1/2*(2*(A + 2*B)*a^2*d*x*cos(d*x + c) + (2*A + B)*a^2*cos(d*x + c)*log(sin
(d*x + c) + 1) - (2*A + B)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*
a^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= a^2\left(\int A\sec^2\left(c+dx\right)dx+\int 2A\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int A\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int 2B\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int B\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*cos(c + d*x)*sec(c + d
*x)**2, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(B*c
os(c + d*x)*sec(c + d*x)**2, x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x
)**2, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{2\,(dx + c)Aa^2 + 4\,(dx + c)Ba^2 + 2\,Aa^2(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + Ba^2(\log(\sin(d}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="m
axima")
```

output
```
1/2*(2*(d*x + c)*A*a^2 + 4*(d*x + c)*B*a^2 + 2*A*a^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 2*B*a^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(74) = 148.

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.09

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{(Aa^2 + 2\,Ba^2)(dx + c) + (2\,Aa^2 + Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (2\,Aa^2 + Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \right.\right.\right.}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="g
iac")
```

output
```
((A*a^2 + 2*B*a^2)*(d*x + c) + (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^
2*tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*
x + 1/2*c) + B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 41.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.18

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{B\,a^2\,\sin\left(c+d\,x\right)}{d} + \frac{2\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{A\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

output
```
(B*a^2*sin(c + d*x))/d + (2*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (4*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a
^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^2*sin(c + d*x))/(d*cos(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.92

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{a^2\left(-2\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+2\cos\left(dx+c\right)\log\left(\mathrm{ta}\right.\right.}{}$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
(a**2*( - 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a - cos(c + d*x)*log(ta
n((c + d*x)/2) - 1)*b + 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*b + cos(c + d*x)*sin(c + d*x)*b + cos(c
+ d*x)*a*d*x + 2*cos(c + d*x)*b*d*x + sin(c + d*x)*a))/(cos(c + d*x)*d)
```

3.16 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 88

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = a^2Bx+\frac{a^2(3A+4B)\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^2(3A+2B)\tan(c+dx)}{2d} + \frac{A(a^2+a^2\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
a^2*B*x+1/2*a^2*(3*A+4*B)*arctanh(sin(d*x+c))/d+1/2*a^2*(3*A+2*B)*tan(d*x+
c)/d+1/2*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a^2\left(2Bdx+2(A+2B)\coth^{-1}(\sin(c+dx))+A\text{arctanh}(\sin(c+dx))+4A\tan(c+dx)+2B\tan(c+dx\right.}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a^2*(2*B*d*x + 2*(A + 2*B)*ArcCoth[Sin[c + d*x]] + A*ArcTanh[Sin[c + d*x]
] + 4*A*Tan[c + d*x] + 2*B*Tan[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(2
*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3454, 3042, 3447, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{2}\int(\cos(c+dx)a+a)(a(3A+2B)+2aB\cos(c+dx))\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(3A+2B)+2aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{2}\int \left(2B\cos^2(c+dx)a^2+(3A+2B)a^2+\left(2Ba^2+(3A+2B)a^2\right)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{2B\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+(3A+2B)a^2+\left(2Ba^2+(3A+2B)a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \ 3500$$

$$\frac{1}{2}\left(\int \left((3A+4B)a^2+2B\cos(c+dx)a^2\right)\sec(c+dx)dx+\frac{a^2(3A+2B)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\left(\int \frac{(3A+4B)a^2+2B\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2(3A+2B)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \ 3214$$

$$\frac{1}{2}\left(a^2(3A+4B)\int \sec(c+dx)dx+\frac{a^2(3A+2B)\tan(c+dx)}{d}+2a^2Bx\right)+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\left(a^2(3A+4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2(3A+2B)\tan(c+dx)}{d}+2a^2Bx\right)+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

$$\Big\downarrow \ 4257$$

$$\frac{1}{2}\left(\frac{a^2(3A+4B)\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2(3A+2B)\tan(c+dx)}{d}+2a^2Bx\right)+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

input

```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (2*a^2*B*x
+ (a^2*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*A + 2*B)*Tan[c + d*x
])/d)/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 8.95 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

method	result
parts	$\frac{a^2 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{(a^2A+2a^2B)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(2a^2A+a^2B)\,\mathrm{ta}}{d}$
derivativedivides	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))+a^2B(dx+c)+2a^2A\tan(dx+c)+2a^2B\ln(\sec(dx+c)+\tan(dx+c))+a^2A\left(\frac{\sec(dx+c)\tan}{2}\right.}{d}$
default	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))+a^2B(dx+c)+2a^2A\tan(dx+c)+2a^2B\ln(\sec(dx+c)+\tan(dx+c))+a^2A\left(\frac{\sec(dx+c)\tan}{2}\right.}{d}$
parallelrisch	$-\frac{3a^2\left(\left(A+\frac{4B}{3}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\left(A+\frac{4B}{3}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2dxB\cos(2}{3}\right.}{2d(\cos(2dx+2c)+1)}$
risch	$a^2Bx-\frac{ia^2\left(A\,e^{3i(dx+c)}-4A\,e^{2i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-4A-2B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3a^2A\ln\left(e^{i(dx+c)}-i\right)}{2d}-\frac{2a^2\ln(e}{}$
norman	$\frac{a^2Bx+a^2Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a^2Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+a^2Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\frac{a^2(5A+2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{6a^2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBO
SE)
```

output
```
a^2*A/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+(A*a^2+2
*B*a^2)/d*ln(sec(d*x+c)+tan(d*x+c))+(2*A*a^2+B*a^2)/d*tan(d*x+c)+a^2*B/d*(
d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{4\,Ba^2dx\cos\left(dx+c\right)^2+(3\,A+4\,B)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(3\,A+4\,B)a^2\cos\left(dx+c\right)^2\,\mathrm{l}}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="f
ricas")
```

output
```
1/4*(4*B*a^2*d*x*cos(d*x + c)^2 + (3*A + 4*B)*a^2*cos(d*x + c)^2*log(sin(d
*x + c) + 1) - (3*A + 4*B)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(
2*(2*A + B)*a^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = a^2\left(\int A\sec^3(c+dx)\,dx+\int 2A\cos(c+dx)\sec^3(c+dx)\,dx + \int A\cos^2(c+dx)\sec^3(c+dx)\,dx+\int B\cos(c+dx)\sec^3(c+dx)\,dx + \int 2B\cos^2(c+dx)\sec^3(c+dx)\,dx+\int B\cos^3(c+dx)\sec^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*cos(c + d*x)*sec(c + d
*x)**3, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(B*c
os(c + d*x)*sec(c + d*x)**3, x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x
)**3, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^3(c + dx)\, dx$$

$$= \frac{4\,(dx + c)Ba^2 - Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx + c\right) + 1\right) + \log\left(\sin\left(dx + c\right) - 1\right)\right) + 2\,Aa^2(\log\left(\sin\left(dx\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="m
axima")
```

output
```
1/4*(4*(d*x + c)*B*a^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(
sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 4*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 8*A*a^2*tan(d*x + c) + 4*B*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.75

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^3(c + dx)\, dx$$

$$= \frac{2\,(dx + c)Ba^2 + (3\,Aa^2 + 4\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (3\,Aa^2 + 4\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \right.\right.}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="g
iac")
```

output
```
1/2*(2*(d*x + c)*B*a^2 + (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c)
+ 1)) - (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^
2*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*A*a^2*tan(1/
2*d*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^2)/d
```

Mupad [B] (verification not implemented)

Time = 40.80 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^3(c + dx)\, dx$$
$$= \frac{3\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,A\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)} + \frac{A\,a^2\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2} + \frac{B\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

output
```
(3*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2*atanh(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^2*sin(c + d*x))/(d*cos(c + d*x)) + (A*a
^2*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (B*a^2*sin(c + d*x))/(d*cos(c + d*
x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.57

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^3(c + dx)\, dx$$
$$= \frac{a^2\left(-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a - 4\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+\right.}{}$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
(a**2*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 4*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + 3*cos(c + d*x)*l
og(tan((c + d*x)/2) - 1)*a + 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b +
3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 4*cos(c + d*x
)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c
+ d*x)/2) + 1)*a - 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b + 2*cos(c +
d*x)*sin(c + d*x)**2*b*d*x - cos(c + d*x)*sin(c + d*x)*a - 2*cos(c + d*x)*
b*d*x + 4*sin(c + d*x)**3*a + 2*sin(c + d*x)**3*b - 4*sin(c + d*x)*a - 2*s
in(c + d*x)*b))/(2*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.17 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 113

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a^2(2A+3B)\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^2(5A+6B)\tan(c+dx)}{3d} + \frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d} + \frac{A(a^2+a^2\cos(c+dx))\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
1/2*a^2*(2*A+3*B)*arctanh(sin(d*x+c))/d+1/3*a^2*(5*A+6*B)*tan(d*x+c)/d+1/6
*a^2*(4*A+3*B)*sec(d*x+c)*tan(d*x+c)/d+1/3*A*(a^2+a^2*cos(d*x+c))*sec(d*x+
c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{a^2\left(6B\coth^{-1}(\sin(c+dx))+3(2A+B)\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(12(A+B)+3(2A+B)\text{se}\right.\right.}{6d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a^2*(6*B*ArcCoth[Sin[c + d*x]] + 3*(2*A + B)*ArcTanh[Sin[c + d*x]] + Tan[
c + d*x]*(12*(A + B) + 3*(2*A + B)*Sec[c + d*x] + 2*A*Tan[c + d*x]^2)))/(6
*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {3042, 3454, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{3}\int (\cos(c+dx)a+a)(a(4A+3B)+a(A+3B)\cos(c+dx))\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin \left(c+dx+\frac{\pi}{2}\right) a+a\right) \left(a(4A+3B)+a(A+3B)\sin \left(c+dx+\frac{\pi}{2}\right)\right)}{\sin \left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \ 3447$$

$$\frac{1}{3}\int \left((A+3B)\cos^2(c+dx)a^2+(4A+3B)a^2+\left((A+3B)a^2+(4A+3B)a^2\right)\cos(c+dx)\right)\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\int \frac{(A+3B)\sin \left(c+dx+\frac{\pi}{2}\right)^2 a^2+(4A+3B)a^2+\left((A+3B)a^2+(4A+3B)a^2\right)\sin \left(c+dx+\frac{\pi}{2}\right)}{\sin \left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \ 3500$$

$$\frac{1}{3}\left(\frac{1}{2}\int \left(2(5A+6B)a^2+3(2A+3B)\cos(c+dx)a^2\right)\sec^2(c+dx)dx+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{2(5A+6B)a^2+3(2A+3B)\sin \left(c+dx+\frac{\pi}{2}\right) a^2}{\sin \left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \ 3227$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2a^2(5A+6B)\int \sec^2(c+dx)dx+3a^2(2A+3B)\int \sec(c+dx)dx\right)+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+ }{2d}\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3a^2(2A+3B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2a^2(5A+6B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{a^2(4A+3B)\tan(c+\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$\big\downarrow$ 4254

$$\frac{1}{3}\left(\frac{1}{2}\left(3a^2(2A+3B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2a^2(5A+6B)\int 1d(-\tan(c+dx))}{d}\right)+\frac{a^2(4A+3B)\tan(c+dx)}{2d}\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$\big\downarrow$ 24

$$\frac{1}{3}\left(\frac{1}{2}\left(3a^2(2A+3B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2a^2(5A+6B)\tan(c+dx)}{d}\right)+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+}{2d}\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

$\big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3a^2(2A+3B)\text{arctanh}(\sin(c+dx))}{d}+\frac{2a^2(5A+6B)\tan(c+dx)}{d}\right)+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+dx}{2d}\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((a^2*(4*
A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((3*a^2*(2*A + 3*B)*ArcTanh[Si
n[c + d*x]])/d + (2*a^2*(5*A + 6*B)*Tan[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 10.77 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

method	result
parts	$-\frac{a^2A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{(a^2A+2a^2B)\tan(dx+c)}{d}+\frac{(2a^2A+a^2B)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+\ldots}{}\right.}{d}$
derivativedivides	$\frac{a^2A\tan(dx+c)+a^2B\ln(\sec(dx+c)+\tan(dx+c))+2a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2B\tan(d\ldots}{d}$
default	$\frac{a^2A\tan(dx+c)+a^2B\ln(\sec(dx+c)+\tan(dx+c))+2a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2B\tan(d\ldots}{d}$
parallelrisch	$\frac{2a^2\left(-\frac{3\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\left(A+\frac{3B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{3\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\left(A+\frac{3B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}\ldots\right.}{d(3\cos(dx+c)+\cos(3dx+3c))}$
risch	$-\frac{ia^2\left(6A\,e^{5i(dx+c)}+3B\,e^{5i(dx+c)}-6A\,e^{4i(dx+c)}-12B\,e^{4i(dx+c)}-24A\,e^{2i(dx+c)}-24B\,e^{2i(dx+c)}-6A\,e^{i(dx+c)}-3B\,e^{i(\ldots}\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{-\frac{2a^2(2A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{a^2(2A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{3d}-\frac{a^2(2A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{d}+\frac{2a^2(2A+5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{a^2(6A+5\ldots}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBO
SE)
```

output
```
-a^2*A/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*a^2+2*B*a^2)/d*tan(d*x+c)+(
2*A*a^2+B*a^2)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))
+1/d*B*ln(sec(d*x+c)+tan(d*x+c))*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$
$$= \frac{3\,(2\,A + 3\,B)a^2 \cos{(dx + c)}^3 \log{(\sin{(dx + c)} + 1)} - 3\,(2\,A + 3\,B)a^2 \cos{(dx + c)}^3 \log{(-\sin{(dx + c)} + 1)} \cdots}{12\,d \cos{(dx + c)}^3}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="f
ricas")
```

output
```
1/12*(3*(2*A + 3*B)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A + 3*
B)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(5*A + 6*B)*a^2*cos(d*
x + c)^2 + 3*(2*A + B)*a^2*cos(d*x + c) + 2*A*a^2)*sin(d*x + c))/(d*cos(d*
x + c)^3)
```

Sympy [F]

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$
$$= a^2 \left(\int A \sec^4{(c + dx)}\, dx + \int 2A \cos{(c + dx)} \sec^4{(c + dx)}\, dx + \int A \cos^2{(c + dx)} \sec^4{(c + dx)}\, dx + \int B \cos{(c + dx)} \sec^4{(c + dx)}\, dx + \int 2B \cos^2{(c + dx)} \sec^4{(c + dx)}\, dx + \int B \cos^3{(c + dx)} \sec^4{(c + dx)}\, dx \right)$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
a**2*(Integral(A*sec(c + d*x)**4, x) + Integral(2*A*cos(c + d*x)*sec(c + d
*x)**4, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**4, x) + Integral(B*c
os(c + d*x)*sec(c + d*x)**4, x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x
)**4, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.54

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^4(c + dx)\,dx$$
$$= \frac{4\left(\tan\left(dx + c\right)^3 + 3\tan\left(dx + c\right)\right)Aa^2 - 6\,Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx + c\right) + 1\right) + \log\left(\sin\left(dx + c\right) - \right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="m
axima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 6*A*a^2*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*
B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(s
in(d*x + c) - 1)) + 6*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 12*A*a^2*tan(d*x + c) + 24*B*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.58

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^4(c + dx)\,dx$$
$$= \frac{3\left(2\,Aa^2 + 3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(2\,Aa^2 + 3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(6\,Aa^2\right.}{}}{6}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="g
iac")
```

output
```
1/6*(3*(2*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a^2
 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*tan(1/2*d*x +
1/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 16*A*a^2*tan(1/2*d*x + 1/2*c)^
3 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) + 15*B
*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 43.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+\frac{3\,B}{2}\right)}{d} - \frac{\left(2\,A\,a^2+3\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{16\,A\,a^2}{3}-8\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(6\,A\,a^2+5\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^4,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2))*(A + (3*B)/2))/d - (tan(c/2 + (d*x)/2)*(6
*A*a^2 + 5*B*a^2) + tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 3*B*a^2) - tan(c/2 + (
d*x)/2)^3*((16*A*a^2)/3 + 8*B*a^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2
 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.65

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{a^2\left(-6\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2a-9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+\right.}{}$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
(a**2*( - 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 9*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + 6*cos(c + d*x)*l
og(tan((c + d*x)/2) - 1)*a + 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b +
6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 9*cos(c + d*x
)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 6*cos(c + d*x)*log(tan((c
+ d*x)/2) + 1)*a - 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b - 6*cos(c +
d*x)*sin(c + d*x)*a - 3*cos(c + d*x)*sin(c + d*x)*b + 10*sin(c + d*x)**3*a
 + 12*sin(c + d*x)**3*b - 12*sin(c + d*x)*a - 12*sin(c + d*x)*b))/(6*cos(c
 + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.18 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 144

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \frac{a^2(7A+8B)\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^2(4A+5B)\tan(c+dx)}{3d}\\ &\quad+\frac{a^2(7A+8B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(5A+4B)\sec^2(c+dx)\tan(c+dx)}{12d}\\ &\quad+\frac{A(a^2+a^2\cos(c+dx))\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

output
```
1/8*a^2*(7*A+8*B)*arctanh(sin(d*x+c))/d+1/3*a^2*(4*A+5*B)*tan(d*x+c)/d+1/8
*a^2*(7*A+8*B)*sec(d*x+c)*tan(d*x+c)/d+1/12*a^2*(5*A+4*B)*sec(d*x+c)^2*tan
(d*x+c)/d+1/4*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.56

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{a^2(3(7A+8B)\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(48(A+B)+3(7A+8B)\sec(c+dx)+6A\sec^3(c+\right.}{24d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a^2*(3*(7*A + 8*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(48*(A + B) + 3*(
7*A + 8*B)*Sec[c + d*x] + 6*A*Sec[c + d*x]^3 + 8*(2*A + B)*Tan[c + d*x]^2)
))/(24*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {3042, 3454, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\int (\cos(c+dx)a+a)(a(5A+4B)+2a(A+2B)\cos(c+dx))\sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(\sin \left(c+dx+\frac{\pi}{2}\right) a+a\right) \left(a(5A+4B)+2a(A+2B) \sin \left(c+dx+\frac{\pi}{2}\right)\right)}{\sin \left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A \tan(c+dx) \sec^3(c+dx) \left(a^2 \cos(c+dx)+a^2\right)}{4d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{4}\int \left(2(A+2B) \cos^2(c+dx)a^2+(5A+4B)a^2+\left(2(A+2B)a^2+(5A+4B)a^2\right) \cos(c+dx)\right) \sec^4(c+dx)dx+\frac{A \tan(c+dx) \sec^3(c+dx) \left(a^2 \cos(c+dx)+a^2\right)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{2(A+2B) \sin \left(c+dx+\frac{\pi}{2}\right)^2 a^2+(5A+4B)a^2+\left(2(A+2B)a^2+(5A+4B)a^2\right) \sin \left(c+dx+\frac{\pi}{2}\right)}{\sin \left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A \tan(c+dx) \sec^3(c+dx) \left(a^2 \cos(c+dx)+a^2\right)}{4d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \left(3(7A+8B)a^2+4(4A+5B) \cos(c+dx)a^2\right) \sec^3(c+dx)dx+\frac{a^2(5A+4B) \tan(c+dx) \sec^2(c+dx)}{3d}\right)+\frac{A \tan(c+dx) \sec^3(c+dx) \left(a^2 \cos(c+dx)+a^2\right)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{3(7A+8B)a^2+4(4A+5B) \sin \left(c+dx+\frac{\pi}{2}\right) a^2}{\sin \left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2(5A+4B) \tan(c+dx) \sec^2(c+dx)}{3d}\right)+\frac{A \tan(c+dx) \sec^3(c+dx) \left(a^2 \cos(c+dx)+a^2\right)}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(3a^2(7A+8B) \int \sec^3(c+dx)dx+4a^2(4A+5B) \int \sec^2(c+dx)dx\right)+\frac{a^2(5A+4B) \tan(c+dx) \sec^2(c+dx)}{3d}\right)+\frac{A \tan(c+dx) \sec^3(c+dx) \left(a^2 \cos(c+dx)+a^2\right)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(4a^2(4A+5B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3a^2(7A+8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{a^2(5A+4B)\tan(c-}{3d}\right.$$
$$\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

$\Big\downarrow$ 4254

$$\frac{1}{4}\left(\frac{1}{3}\left(3a^2(7A+8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a^2(4A+5B)\int 1d(-\tan(c+dx))}{d}\right)+\frac{a^2(5A+4B)\tan(c+dx}{3d}\right.$$
$$\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

$\Big\downarrow$ 24

$$\frac{1}{4}\left(\frac{1}{3}\left(3a^2(7A+8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{4a^2(4A+5B)\tan(c+dx)}{d}\right)+\frac{a^2(5A+4B)\tan(c+dx)\sec^2(c}{3d}\right.$$
$$\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

$\Big\downarrow$ 4255

$$\frac{1}{4}\left(\frac{1}{3}\left(3a^2(7A+8B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a^2(4A+5B)\tan(c+dx)}{d}\right)+\frac{a^2(5A-}{}\right.$$
$$\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(3a^2(7A+8B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a^2(4A+5B)\tan(c+dx)}{d}\right)+\right.$$
$$\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

$\Big\downarrow$ 4257

$$\frac{1}{4}\left(\frac{1}{3}\left(3a^2(7A+8B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a^2(4A+5B)\tan(c+dx)}{d}\right)+\frac{a^2(5}{}\right.$$
$$\frac{A\tan(c+dx)\sec^3(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((a^2*(5*
A + 4*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((4*a^2*(4*A + 5*B)*Tan[c +
d*x])/d + 3*a^2*(7*A + 8*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*T
an[c + d*x])/(2*d)))/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 12.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04

method	result
parts	$\frac{a^2 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{(a^2A+2a^2B)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\right.}{d}$
parallelrisch	$\frac{16\left(-\frac{21\left(A+\frac{8B}{7}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32}+\frac{21\left(A+\frac{8B}{7}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\right.}{32}\right.}{3d(\cos(4dx+4c)+4\cos}$
derivativedivides	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2B\tan(dx+c)-2a^2A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2a^2B\left(\right.}{}$
default	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2B\tan(dx+c)-2a^2A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2a^2B\left(\right.}{}$
norman	$\frac{\frac{a^2(3A-8B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{a^2(7A+8B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{6d}-\frac{a^2(7A+8B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{4d}+\frac{a^2(25A+24B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a^2(53A-1}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4}$
risch	$-\frac{ia^2\left(21A\,e^{7i(dx+c)}+24B\,e^{7i(dx+c)}-24B\,e^{6i(dx+c)}+45A\,e^{5i(dx+c)}+24B\,e^{5i(dx+c)}-96A\,e^{4i(dx+c)}-120B\,e^{4i(dx+c)}-\right.}{12d\left(e^{2i(dx+c)}\right.}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBO
SE)
```

output
```
a^2*A/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+(A*a^2+2*B*a^2)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c
)+tan(d*x+c)))-(2*A*a^2+B*a^2)/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*B/
d*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{3\,(7\,A+8\,B)a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(7\,A+8\,B)a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right.}{4}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="f
ricas")
```

output
```
1/48*(3*(7*A + 8*B)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(7*A + 8*
B)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(4*A + 5*B)*a^2*cos(d*
x + c)^3 + 3*(7*A + 8*B)*a^2*cos(d*x + c)^2 + 8*(2*A + B)*a^2*cos(d*x + c)
 + 6*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.60

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx$$
$$= \frac{32\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Aa^2 + 16\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ba^2 - 3Aa^2\left(\frac{2\left(3\sin(dx+c)^3\right.}{\sin(dx+c)^4 - 2\right.}}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="m
axima")
```

output
```
1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 16*(tan(d*x + c)^3 + 3*
tan(d*x + c))*B*a^2 - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^2*tan
(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$

$$= \frac{3\left(7\,Aa^2+8\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Aa^2+8\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,Aa^2\right.}{}}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="g
iac")
```

output
```
1/24*(3*(7*A*a^2 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(7*A*a^
2 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*a^2*tan(1/2*d*x
+ 1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*A*a^2*tan(1/2*d*x + 1/2*
c)^5 - 88*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 +
 136*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*A*a^2*tan(1/2*d*x + 1/2*c) - 72*B*a
^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 44.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$

$$= \frac{\left(-\frac{7\,A\,a^2}{4}-2\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{77\,A\,a^2}{12}+\frac{22\,B\,a^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{83\,A\,a^2}{12}-\frac{34\,B\,a^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$
$$+\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{7\,A}{8}+B\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*((25*A*a^2)/4 + 6*B*a^2) - tan(c/2 + (d*x)/2)^7*((7*A*
a^2)/4 + 2*B*a^2) + tan(c/2 + (d*x)/2)^5*((77*A*a^2)/12 + (22*B*a^2)/3) -
tan(c/2 + (d*x)/2)^3*((83*A*a^2)/12 + (34*B*a^2)/3))/(d*(6*tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/
2)^8 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2))*((7*A)/8 + B))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.34

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Too large to display}$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
(a**2*( - 21*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 24
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b + 42*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 48*cos(c + d*x)*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**2*b - 21*cos(c + d*x)*log(tan((c + d*x)/2) -
 1)*a - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 21*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a + 24*cos(c + d*x)*log(tan((c + d*x
)/2) + 1)*sin(c + d*x)**4*b - 42*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**2*a - 48*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**
2*b + 21*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 24*cos(c + d*x)*log(ta
n((c + d*x)/2) + 1)*b - 21*cos(c + d*x)*sin(c + d*x)**3*a - 24*cos(c + d*x
)*sin(c + d*x)**3*b + 27*cos(c + d*x)*sin(c + d*x)*a + 24*cos(c + d*x)*sin
(c + d*x)*b + 32*sin(c + d*x)**5*a + 40*sin(c + d*x)**5*b - 80*sin(c + d*x
)**3*a - 88*sin(c + d*x)**3*b + 48*sin(c + d*x)*a + 48*sin(c + d*x)*b))/(2
4*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.19 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 201

$$\begin{aligned}&\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx\\
&=\frac{1}{16}a^3(26A+23B)x+\frac{a^3(19A+17B)\sin(c+dx)}{5d}\\
&\quad+\frac{a^3(26A+23B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^3(22A+21B)\cos^3(c+dx)\sin(c+dx)}{40d}\\
&\quad+\frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d}\\
&\quad+\frac{(3A+4B)\cos^3(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{15d}\\
&\quad-\frac{a^3(19A+17B)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
1/16*a^3*(26*A+23*B)*x+1/5*a^3*(19*A+17*B)*sin(d*x+c)/d+1/16*a^3*(26*A+23*
B)*cos(d*x+c)*sin(d*x+c)/d+1/40*a^3*(22*A+21*B)*cos(d*x+c)^3*sin(d*x+c)/d+
1/6*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/15*(3*A+4*B)*cos(d*
x+c)^3*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d-1/15*a^3*(19*A+17*B)*sin(d*x+c)^3
/d
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.67

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{a^3(1380Bc+1560Adx+1380Bdx+120(23A+21B)\sin(c+dx)+15(64A+63B)\sin(2(c+dx))+34\ldots}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(a^3*(1380*B*c + 1560*A*d*x + 1380*B*d*x + 120*(23*A + 21*B)*Sin[c + d*x]
 + 15*(64*A + 63*B)*Sin[2*(c + d*x)] + 340*A*Sin[3*(c + d*x)] + 380*B*Sin[3
*(c + d*x)] + 90*A*Sin[4*(c + d*x)] + 135*B*Sin[4*(c + d*x)] + 12*A*Sin[5*
(c + d*x)] + 36*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3455$$

$$\frac{1}{6}\int \cos^2(c+dx)(\cos(c+dx)a+a)^2(3a(2A+B)+2a(3A+4B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(3a(2A+B)+2a(3A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+ \frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{6}\left(\frac{1}{5}\int 3\cos^2(c+dx)(\cos(c+dx)a+a)\left((16A+13B)a^2+(22A+21B)\cos(c+dx)a^2\right)dx+\frac{2(3A+4B)\sin(c\right. \frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{3}{5}\int \cos^2(c+dx)(\cos(c+dx)a+a)\left((16A+13B)a^2+(22A+21B)\cos(c+dx)a^2\right)dx+\frac{2(3A+4B)\sin(c+\right. \frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((16A+13B)a^2+(22A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx\right. \frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{6}\left(\frac{3}{5}\int \cos^2(c+dx)\left((22A+21B)\cos^2(c+dx)a^3+(16A+13B)a^3+\left((16A+13B)a^3+(22A+21B)a^3\right)\cos(\right.\right. \frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left((22A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(16A+13B)a^3+\left((16A+13B)a^3+(22A+\right.\right.\right. \frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\int \cos^2(c+dx)\left(5(26A+23B)a^3+8(19A+17B)\cos(c+dx)a^3\right)dx+\frac{a^3(22A+21B)\sin(c+dx)\cos^3}{4d}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(5(26A+23B)a^3+8(19A+17B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx+\frac{a^3(22A+21B)\sin}{}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$\downarrow$ 3227

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(8a^3(19A+17B)\int \cos^3(c+dx)dx+5a^3(26A+23B)\int \cos^2(c+dx)dx\right)+\frac{a^3(22A+21B)\sin(c+d}{4d}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5a^3(26A+23B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+8a^3(19A+17B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{a^3(22A+21}{}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$\downarrow$ 3113

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5a^3(26A+23B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8a^3(19A+17B)\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$\downarrow$ 2009

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5a^3(26A+23B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8a^3(19A+17B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{a^3(22}{}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$\downarrow$ 3115

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5a^3(26A+23B)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{8a^3(19A+17B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx\right.}{d}\right.\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{6}\left(\frac{3}{5}\left(\frac{a^3(22A+21B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{4}\left(5a^3(26A+23B)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{8a^3(19\right.\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^2}{6d}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + ((2*(3*A
+ 4*B)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5*d) + (3*((
a^3*(22*A + 21*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (5*a^3*(26*A + 23*B
)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (8*a^3*(19*A + 17*B)*(-Sin[c
 + d*x] + Sin[c + d*x]^3/3))/d)/4))/5)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 250.81 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.56

method	result
parallelrisch	$\frac{3\left(\left(\frac{32A}{3}+\frac{21B}{2}\right)\sin(2dx+2c)+\frac{2(17A+19B)\sin(3dx+3c)}{9}+\left(A+\frac{3B}{2}\right)\sin(4dx+4c)+\frac{2\left(\frac{A}{3}+B\right)\sin(5dx+5c)}{5}+\frac{B\sin(6dx+6c)}{18}\right.}{32d}$
risch	$\frac{13a^3Ax}{8}+\frac{23a^3Bx}{16}+\frac{23a^3A\sin(dx+c)}{8d}+\frac{21a^3B\sin(dx+c)}{8d}+\frac{a^3B\sin(6dx+6c)}{192d}+\frac{\sin(5dx+5c)a^3A}{80d}+\frac{3\sin(}{}$
parts	$\frac{(a^3A+3a^3B)\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}+\frac{(3a^3A+a^3B)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}+\frac{(3a^3A+3}{}$
derivativedivides	$\frac{\frac{a^3A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+a^3B\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+}{}$
default	$\frac{\frac{a^3A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+a^3B\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+}{}$
norman	$\frac{\frac{a^3(26A+23B)x}{16}+\frac{33a^3(26A+23B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20d}+\frac{17a^3(26A+23B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{24d}+\frac{a^3(26A+23B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}+\frac{3a^3(26A+23}{}}{}$
orering	Expression too large to display

input

```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBO
SE)
```

output

```
3/32*((32/3*A+21/2*B)*sin(2*d*x+2*c)+2/9*(17*A+19*B)*sin(3*d*x+3*c)+(A+3/2
*B)*sin(4*d*x+4*c)+2/5*(1/3*A+B)*sin(5*d*x+5*c)+1/18*B*sin(6*d*x+6*c)+4*(2
3/3*A+7*B)*sin(d*x+c)+52/3*x*(A+23/26*B)*d)*a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{15\,(26\,A+23\,B)a^3dx+\left(40\,Ba^3\cos\left(dx+c\right)^5+48\,(A+3\,B)a^3\cos\left(dx+c\right)^4+10\,(18\,A+23\,B)a^3\cos\left(\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/240*(15*(26*A + 23*B)*a^3*d*x + (40*B*a^3*cos(d*x + c)^5 + 48*(A + 3*B)*
a^3*cos(d*x + c)^4 + 10*(18*A + 23*B)*a^3*cos(d*x + c)^3 + 16*(19*A + 17*B
)*a^3*cos(d*x + c)^2 + 15*(26*A + 23*B)*a^3*cos(d*x + c) + 32*(19*A + 17*B
)*a^3)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(184) = 368.

Time = 0.42 (sec) , antiderivative size = 695, normalized size of antiderivative = 3.46

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((9*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 9*A*a**3*x*cos(c + d*x)**4/8 +
 A*a**3*x*cos(c + d*x)**2/2 + 8*A*a**3*sin(c + d*x)**5/(15*d) + 4*A*a**3*s
in(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*a**3*sin(c + d*x)**3*cos(c + d*
x)/(8*d) + 2*A*a**3*sin(c + d*x)**3/d + A*a**3*sin(c + d*x)*cos(c + d*x)**
4/d + 15*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*A*a**3*sin(c + d*x)
*cos(c + d*x)**2/d + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a**3*x*s
in(c + d*x)**6/16 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*B*a
**3*x*sin(c + d*x)**4/8 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
 9*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*B*a**3*x*cos(c + d*x)**6
/16 + 9*B*a**3*x*cos(c + d*x)**4/8 + 5*B*a**3*sin(c + d*x)**5*cos(c + d*x)
/(16*d) + 8*B*a**3*sin(c + d*x)**5/(5*d) + 5*B*a**3*sin(c + d*x)**3*cos(c
+ d*x)**3/(6*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*B*a**3*si
n(c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/(3*d) + 11*B*a
**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 3*B*a**3*sin(c + d*x)*cos(c + d*
x)**4/d + 15*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*sin(c + d*
x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c
)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.30

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{64\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Aa^3-960\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^3 -}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c
))*A*a^3 + 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*
a^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*B*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 90*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{Ba^3\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\,(26\,Aa^3+23\,Ba^3)x+\frac{(Aa^3+3\,Ba^3)\sin(5\,dx+5\,c)}{80\,d} + \frac{3\,(2\,Aa^3+3\,Ba^3)\sin(4\,dx+4\,c)}{64\,d}+\frac{(17\,Aa^3+19\,Ba^3)\sin(3\,dx+3\,c)}{48\,d} + \frac{(64\,Aa^3+63\,Ba^3)\sin(2\,dx+2\,c)}{64\,d}+\frac{(23\,Aa^3+21\,Ba^3)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/192*B*a^3*sin(6*d*x + 6*c)/d + 1/16*(26*A*a^3 + 23*B*a^3)*x + 1/80*(A*a^
3 + 3*B*a^3)*sin(5*d*x + 5*c)/d + 3/64*(2*A*a^3 + 3*B*a^3)*sin(4*d*x + 4*c
)/d + 1/48*(17*A*a^3 + 19*B*a^3)*sin(3*d*x + 3*c)/d + 1/64*(64*A*a^3 + 63*
B*a^3)*sin(2*d*x + 2*c)/d + 1/8*(23*A*a^3 + 21*B*a^3)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 43.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.57

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{\left(\frac{13\,A\,a^3}{4}+\frac{23\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{221\,A\,a^3}{12}+\frac{391\,B\,a^3}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{429\,A\,a^3}{10}+\frac{759\,B\,a^3}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\right.} - \frac{a^3\,\left(26\,A+23\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d} + \frac{a^3\,\mathrm{atan}\left(\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(26\,A+23\,B\right)}{8\,\left(\frac{13\,A\,a^3}{4}+\frac{23\,B\,a^3}{8}\right)}\right)\,\left(26\,A+23\,B\right)}{8\,d}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*((51*A*a^3)/4 + (105*B*a^3)/8) + tan(c/2 + (d*x)/2)^11
*((13*A*a^3)/4 + (23*B*a^3)/8) + tan(c/2 + (d*x)/2)^3*((419*A*a^3)/12 + (2
11*B*a^3)/8) + tan(c/2 + (d*x)/2)^9*((221*A*a^3)/12 + (391*B*a^3)/24) + ta
n(c/2 + (d*x)/2)^7*((429*A*a^3)/10 + (759*B*a^3)/20) + tan(c/2 + (d*x)/2)^
5*((499*A*a^3)/10 + (969*B*a^3)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c
/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*ta
n(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (a^3*(26*A + 23*B)*(at
an(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d) + (a^3*atan((a^3*tan(c/2 + (d*x)/
2)*(26*A + 23*B))/(8*((13*A*a^3)/4 + (23*B*a^3)/8)))*(26*A + 23*B))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{a^3\left(40\cos\left(dx+c\right)\sin\left(dx+c\right)^5 b - 180\cos\left(dx+c\right)\sin\left(dx+c\right)^3 a - 310\cos\left(dx+c\right)\sin\left(dx+c\right)^3 b + 57\right.}{}$$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
(a**3*(40*cos(c + d*x)*sin(c + d*x)**5*b - 180*cos(c + d*x)*sin(c + d*x)**
3*a - 310*cos(c + d*x)*sin(c + d*x)**3*b + 570*cos(c + d*x)*sin(c + d*x)*a
 + 615*cos(c + d*x)*sin(c + d*x)*b + 48*sin(c + d*x)**5*a + 144*sin(c + d*
x)**5*b - 400*sin(c + d*x)**3*a - 560*sin(c + d*x)**3*b + 960*sin(c + d*x)
*a + 960*sin(c + d*x)*b + 390*a*d*x + 345*b*d*x))/(240*d)
```

3.20 $\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 154

$$\begin{aligned}
&\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx \\
&= \frac{1}{8}a^3(15A+13B)x + \frac{a^3(15A+13B)\sin(c+dx)}{5d} \\
&\quad + \frac{3a^3(15A+13B)\cos(c+dx)\sin(c+dx)}{40d} \\
&\quad + \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} \\
&\quad + \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5ad} - \frac{a^3(15A+13B)\sin^3(c+dx)}{60d}
\end{aligned}$$

output
```
1/8*a^3*(15*A+13*B)*x+1/5*a^3*(15*A+13*B)*sin(d*x+c)/d+3/40*a^3*(15*A+13*B
)*cos(d*x+c)*sin(d*x+c)/d+1/20*(5*A-B)*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/5
*B*(a+a*cos(d*x+c))^4*sin(d*x+c)/a/d-1/60*a^3*(15*A+13*B)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.70

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{a^3(780Bc+900Adx+780Bdx+60(26A+23B)\sin(c+dx)+480(A+B)\sin(2(c+dx))+120A\sin(3}{480d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(a^3*(780*B*c + 900*A*d*x + 780*B*d*x + 60*(26*A + 23*B)*Sin[c + d*x] + 48
0*(A + B)*Sin[2*(c + d*x)] + 120*A*Sin[3*(c + d*x)] + 170*B*Sin[3*(c + d*x
)] + 15*A*Sin[4*(c + d*x)] + 45*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)])
)/(480*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a\cos(c+dx)+a)^3\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (\cos(c+dx)a+a)^3(4aB+a(5A-B)\cos(c+dx))dx}{5a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3\left(4aB+a(5A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{4}a(15A+13B)\int (\cos(c+dx)a+a)^3dx+\frac{a(5A-B)\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}}{5a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}a(15A+13B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3dx+\frac{a(5A-B)\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}}{5a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5ad}$$

$$\Big\downarrow \text{3124}$$

$$\frac{\frac{1}{4}a(15A+13B)\int \left(\cos^3(c+dx)a^3+3\cos^2(c+dx)a^3+3\cos(c+dx)a^3+a^3\right)dx+\frac{a(5A-B)\sin(c+dx)(a\cos(c+dx)+a}{4d}}{5a}$$
$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5ad}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}a(15A+13B)\left(-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}\right)+\frac{a(5A-B)\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}}{5a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5ad}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a*d) + ((a*(5*A - B)*(a + a*Cos
[c + d*x])^3*Sin[c + d*x])/(4*d) + (a*(15*A + 13*B)*((5*a^3*x)/2 + (4*a^3*
Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a^3*Sin[c + d
*x]^3)/(3*d)))/4)/(5*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 68.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{\left(32(A+B)\sin(2dx+2c)+2\left(4A+\frac{17B}{3}\right)\sin(3dx+3c)+(A+3B)\sin(4dx+4c)+\frac{2B\sin(5dx+5c)}{5}+4(26A+23B)\sin(dx+c)\right.}{32d}$
risch	$\frac{15a^3Ax}{8}+\frac{13a^3Bx}{8}+\frac{13a^3A\sin(dx+c)}{4d}+\frac{23a^3B\sin(dx+c)}{8d}+\frac{\sin(5dx+5c)a^3B}{80d}+\frac{\sin(4dx+4c)a^3A}{32d}+\frac{3\sin(}{}$
parts	$\frac{(a^3A+3a^3B)\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{(3a^3A+a^3B)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+$
derivativedivides	$\frac{a^3A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^3B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+a^3A\left(\cos(dx+c\right.}{}$
default	$\frac{a^3A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^3B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+a^3A\left(\cos(dx+c\right.}{}$
norman	$\frac{\frac{a^3(15A+13B)x}{8}+\frac{32a^3(15A+13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{7a^3(15A+13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}+\frac{a^3(15A+13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}+\frac{5a^3(15A+13B)}{}}{}$
orering	Expression too large to display

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/32*(32*(A+B)*sin(2*d*x+2*c)+2*(4*A+17/3*B)*sin(3*d*x+3*c)+(A+3*B)*sin(4*
d*x+4*c)+2/5*B*sin(5*d*x+5*c)+4*(26*A+23*B)*sin(d*x+c)+60*x*(A+13/15*B)*d)
*a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$=\frac{15\,(15\,A+13\,B)a^3dx+\left(24\,Ba^3\cos\left(dx+c\right)^4+30\,(A+3\,B)a^3\cos\left(dx+c\right)^3+8\,(15\,A+19\,B)a^3\cos\left(d\right.\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/120*(15*(15*A + 13*B)*a^3*d*x + (24*B*a^3*cos(d*x + c)^4 + 30*(A + 3*B)*
a^3*cos(d*x + c)^3 + 8*(15*A + 19*B)*a^3*cos(d*x + c)^2 + 15*(15*A + 13*B)
*a^3*cos(d*x + c) + 8*(45*A + 38*B)*a^3)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(136) = 272.

Time = 0.48 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.44

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{3Aa^3x\sin^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Aa^3x\sin^2(c+dx)}{2} + \frac{3Aa^3x\cos^4(c+dx)}{8} + \frac{3Aa^3x\cos^2(c+dx)}{2} + \frac{3Aa^3\sin^3}{} \\ x(A+B\cos(c))(a\cos(c)+a)^3\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8
 + 3*A*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8
*d) + 2*A*a**3*sin(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(
8*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**3*sin(c + d*x)*cos
(c + d*x)/(2*d) + A*a**3*sin(c + d*x)/d + 9*B*a**3*x*sin(c + d*x)**4/8 + 9
*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**3*x*sin(c + d*x)**2/2 +
 9*B*a**3*x*cos(c + d*x)**4/8 + B*a**3*x*cos(c + d*x)**2/2 + 8*B*a**3*sin(
c + d*x)**5/(15*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*
a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/d + B*a
**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*B*a**3*sin(c + d*x)*cos(c + d*x)**
3/(8*d) + 3*B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + B*a**3*sin(c + d*x)*co
s(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.38

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx =$$
$$-\frac{480\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^3-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^3 \cdots}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/480*(480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 15*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 360*(2*d*x + 2*c + sin(2*d*
x + 2*c))*A*a^3 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
c))*B*a^3 + 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 45*(12*d*x + 12*
c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 120*(2*d*x + 2*c + sin(
2*d*x + 2*c))*B*a^3 - 480*A*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx &= \frac{Ba^3\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{1}{8}\left(15\,Aa^3+13\,Ba^3\right)x \\ &+\frac{\left(Aa^3+3\,Ba^3\right)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{\left(12\,Aa^3+17\,Ba^3\right)\sin\left(3\,dx+3\,c\right)}{48\,d} \\ &+\frac{\left(Aa^3+Ba^3\right)\sin\left(2\,dx+2\,c\right)}{d}+\frac{\left(26\,Aa^3+23\,Ba^3\right)\sin\left(dx+c\right)}{8\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/80*B*a^3*sin(5*d*x + 5*c)/d + 1/8*(15*A*a^3 + 13*B*a^3)*x + 1/32*(A*a^3
+ 3*B*a^3)*sin(4*d*x + 4*c)/d + 1/48*(12*A*a^3 + 17*B*a^3)*sin(3*d*x + 3*c
)/d + (A*a^3 + B*a^3)*sin(2*d*x + 2*c)/d + 1/8*(26*A*a^3 + 23*B*a^3)*sin(d
*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 41.83 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.80

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(\frac{15\,A\,a^3}{4}+\frac{13\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{35\,A\,a^3}{2}+\frac{91\,B\,a^3}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(32\,A\,a^3+\frac{416\,B\,a^3}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\tan\left(\frac{c}{2}\right.\right.}$$
$$-\frac{a^3\,(15\,A+13\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}$$
$$+\frac{a^3\,\mathrm{atan}\left(\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(15\,A+13\,B)}{4\left(\frac{15\,A\,a^3}{4}+\frac{13\,B\,a^3}{4}\right)}\right)\,(15\,A+13\,B)}{4\,d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*((49*A*a^3)/4 + (51*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*(
(15*A*a^3)/4 + (13*B*a^3)/4) + tan(c/2 + (d*x)/2)^7*((35*A*a^3)/2 + (91*B*
a^3)/6) + tan(c/2 + (d*x)/2)^3*((61*A*a^3)/2 + (133*B*a^3)/6) + tan(c/2 +
(d*x)/2)^5*(32*A*a^3 + (416*B*a^3)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*ta
n(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + ta
n(c/2 + (d*x)/2)^10 + 1)) - (a^3*(15*A + 13*B)*(atan(tan(c/2 + (d*x)/2)) -
 (d*x)/2))/(4*d) + (a^3*atan((a^3*tan(c/2 + (d*x)/2)*(15*A + 13*B))/(4*((1
5*A*a^3)/4 + (13*B*a^3)/4)))*(15*A + 13*B))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{a^3\left(-30\cos\left(dx+c\right)\sin\left(dx+c\right)^3a-90\cos\left(dx+c\right)\sin\left(dx+c\right)^3b+255\cos\left(dx+c\right)\sin\left(dx+c\right)a+28\right.}{}$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
(a**3*( - 30*cos(c + d*x)*sin(c + d*x)**3*a - 90*cos(c + d*x)*sin(c + d*x)
**3*b + 255*cos(c + d*x)*sin(c + d*x)*a + 285*cos(c + d*x)*sin(c + d*x)*b
+ 24*sin(c + d*x)**5*b - 120*sin(c + d*x)**3*a - 200*sin(c + d*x)**3*b + 4
80*sin(c + d*x)*a + 480*sin(c + d*x)*b + 225*a*d*x + 195*b*d*x))/(120*d)
```

3.21 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx &= \frac{5}{8}a^3(4A+3B)x+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\cos(c+dx)\sin(c+dx)}{8d} \\ &\quad +\frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}-\frac{a^3(4A+3B)\sin^3(c+dx)}{12d}\end{aligned}$$

output
```
5/8*a^3*(4*A+3*B)*x+a^3*(4*A+3*B)*sin(d*x+c)/d+3/8*a^3*(4*A+3*B)*cos(d*x+c
)*sin(d*x+c)/d+1/4*B*(a+a*cos(d*x+c))^3*sin(d*x+c)/d-1/12*a^3*(4*A+3*B)*si
n(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{a^3\sin(c+dx)\left(30(4A+3B)\arcsin\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)+(88A+72B+9(4A+5B)\cos(c+dx)+8\right.}{24d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(a^3*Sin[c + d*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[Sin[(c + d*x)/2]^2]] + (88*A
 + 72*B + 9*(4*A + 5*B)*Cos[c + d*x] + 8*(A + 3*B)*Cos[c + d*x]^2 + 6*B*Co
s[c + d*x]^3)*Sqrt[Sin[c + d*x]^2]))/(24*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{1}{4}(4A+3B)\int (\cos(c+dx)a+a)^3dx+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}(4A+3B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3dx+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{ 3124}$$

$$\frac{1}{4}(4A+3B)\int \left(\cos^3(c+dx)a^3+3\cos^2(c+dx)a^3+3\cos(c+dx)a^3+a^3\right)dx+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}(4A+3B)\left(-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}\right)+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + ((4*A + 3*B)*((5*a^3*x)/2
+ (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a^3*
Sin[c + d*x]^3)/(3*d)))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.52

$$\frac{\frac{a^3A\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+a^3B\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^3A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(1/3*a^3*A*(cos(d*x+c)^2+2)*sin(d*x+c)+a^3*B*(1/4*(cos(d*x+c)^3+3/2*co
s(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*a^3*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a^3*B*(cos(d*x+c)^2+2)*sin(d*x+c)+3*a^3*A*sin(d*x+c)+3*a^3*B*(
1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*A*(d*x+c)+a^3*B*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$=\frac{15\,(4\,A+3\,B)a^3dx+\left(6\,Ba^3\cos\left(dx+c\right)^3+8\,(A+3\,B)a^3\cos\left(dx+c\right)^2+9\,(4\,A+5\,B)a^3\cos\left(dx+c\right)\right.}{24\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(15*(4*A + 3*B)*a^3*d*x + (6*B*a^3*cos(d*x + c)^3 + 8*(A + 3*B)*a^3*c
os(d*x + c)^2 + 9*(4*A + 5*B)*a^3*cos(d*x + c) + 8*(11*A + 9*B)*a^3)*sin(d
*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(107) = 214$.

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.20

$$\int (a + a\cos(c + dx))^3(A + B\cos(c + dx))\,dx$$
$$= \begin{cases} \frac{3Aa^3x\sin^2(c+dx)}{2} + \frac{3Aa^3x\cos^2(c+dx)}{2} + Aa^3x + \frac{2Aa^3\sin^3(c+dx)}{3d} + \frac{Aa^3\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Aa^3\sin(c+dx)\cos(c+dx)}{2d} \\ x(A + B\cos(c))\,(a\cos(c) + a)^3 \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**2/2 + A
*a**3*x + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c + d*x
)**2/d + 3*A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*A*a**3*sin(c + d*x)/
d + 3*B*a**3*x*sin(c + d*x)**4/8 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*B*a**3*x*sin(c + d*x)**2/2 + 3*B*a**3*x*cos(c + d*x)**4/8 + 3*B*
a**3*x*cos(c + d*x)**2/2 + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2
*B*a**3*sin(c + d*x)**3/d + 5*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) +
3*B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a**3*sin(c + d*x)*cos(c + d*
x)/(2*d) + B*a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) +
 a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.44

$$\int (a + a\cos(c + dx))^3(A + B\cos(c + dx))\,dx =$$
$$-\frac{32\left(\sin(dx + c)^3 - 3\sin(dx + c)\right)Aa^3 - 72\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^3 - 96\,(dx + c)Aa^3 + 96}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 72*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 96*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*B*a^3 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*
a^3 - 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 288*A*a^3*sin(d*x + c) -
 96*B*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{Ba^3\sin(4\,dx+4\,c)}{32\,d}+\frac{5}{8}\left(4\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\sin(3\,dx+3\,c)}{12\,d}+\frac{\left(3\,Aa^3+4\,Ba^3\right)\sin(2\,dx+2\,c)}{4\,d}+\frac{\left(15\,Aa^3+13\,Ba^3\right)\sin(dx+c)}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/32*B*a^3*sin(4*d*x + 4*c)/d + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 +
3*B*a^3)*sin(3*d*x + 3*c)/d + 1/4*(3*A*a^3 + 4*B*a^3)*sin(2*d*x + 2*c)/d +
 1/4*(15*A*a^3 + 13*B*a^3)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 40.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{5\,A\,a^3\,x}{2}+\frac{15\,B\,a^3\,x}{8}+\frac{15\,A\,a^3\,\sin(c+d\,x)}{4\,d}+\frac{13\,B\,a^3\,\sin(c+d\,x)}{4\,d}+\frac{3\,A\,a^3\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{A\,a^3\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{B\,a^3\,\sin(2\,c+2\,d\,x)}{d}+\frac{B\,a^3\,\sin(3\,c+3\,d\,x)}{4\,d}+\frac{B\,a^3\,\sin(4\,c+4\,d\,x)}{32\,d}$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

output
```
(5*A*a^3*x)/2 + (15*B*a^3*x)/8 + (15*A*a^3*sin(c + d*x))/(4*d) + (13*B*a^3
*sin(c + d*x))/(4*d) + (3*A*a^3*sin(2*c + 2*d*x))/(4*d) + (A*a^3*sin(3*c +
 3*d*x))/(12*d) + (B*a^3*sin(2*c + 2*d*x))/d + (B*a^3*sin(3*c + 3*d*x))/(4
*d) + (B*a^3*sin(4*c + 4*d*x))/(32*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int (a + a\cos(c + dx))^3(A + B\cos(c + dx))\,dx$$
$$= \frac{a^3(-6\cos(dx+c)\sin(dx+c)^3 b + 36\cos(dx+c)\sin(dx+c)\,a + 51\cos(dx+c)\sin(dx+c)\,b - 8\sin(}{24d}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
(a**3*( - 6*cos(c + d*x)*sin(c + d*x)**3*b + 36*cos(c + d*x)*sin(c + d*x)*
a + 51*cos(c + d*x)*sin(c + d*x)*b - 8*sin(c + d*x)**3*a - 24*sin(c + d*x)
**3*b + 96*sin(c + d*x)*a + 96*sin(c + d*x)*b + 60*a*d*x + 45*b*d*x))/(24*
d)
```

3.22 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{1}{2}a^3(7A+5B)x+\frac{a^3A\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{5a^3(A+B)\sin(c+dx)}{2d}\\ &\quad+\frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3A+5B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{6d}\end{aligned}$$

output
```
1/2*a^3*(7*A+5*B)*x+a^3*A*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*sin(d*x+c)/d
+1/3*a*B*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/6*(3*A+5*B)*(a^3+a^3*cos(d*x+c)
)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int (a + a\cos(c + dx))^3(A + B\cos(c + dx))\sec(c + dx)\,dx$$
$$= \frac{a^3\left(42Adx + 30Bdx - 12A\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + 12A\log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(\right.\right.\right.}{12d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(a^3*(42*A*d*x + 30*B*d*x - 12*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 12*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(4*A + 5*B)*Sin[c + d*
x] + 3*(A + 3*B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c + dx)(a\cos(c + dx) + a)^3(A + B\cos(c + dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c + dx + \frac{\pi}{2}\right) + a\right)^3\left(A + B\sin\left(c + dx + \frac{\pi}{2}\right)\right)}{\sin\left(c + dx + \frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\int (\cos(c + dx)a + a)^2(3aA + a(3A + 5B)\cos(c + dx))\sec(c + dx)dx + \frac{aB\sin(c + dx)(a\cos(c + dx) + a)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(3aA+a(3A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\left(\frac{1}{2}\int 3(\cos(c+dx)a+a)\left(2Aa^2+5(A+B)\cos(c+dx)a^2\right)\sec(c+dx)dx+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c\right.}{2d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3}{2}\int(\cos(c+dx)a+a)\left(2Aa^2+5(A+B)\cos(c+dx)a^2\right)\sec(c+dx)dx+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+\right.}{2d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(2Aa^2+5(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+\right.}{2d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{3}\left(\frac{3}{2}\int\left(5(A+B)\cos^2(c+dx)a^3+2Aa^3+\left(2Aa^3+5(A+B)a^3\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{(3A+5B)\sin(}{}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\int\frac{5(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+2Aa^3+\left(2Aa^3+5(A+B)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{(3A+5B)\sin(c+}{}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\int \left(2Aa^3+(7A+5B)\cos(c+dx)a^3\right)\sec(c+dx)dx+\frac{5a^3(A+B)\sin(c+dx)}{d}\right)+\frac{(3A+5B)\sin(c+dx}{2}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{2Aa^3+(7A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5a^3(A+B)\sin(c+dx)}{d}\right)+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos\right.}{2d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3214

$$\frac{1}{3}\left(\frac{3}{2}\left(2a^3A\int \sec(c+dx)dx+\frac{5a^3(A+B)\sin(c+dx)}{d}+a^3x(7A+5B)\right)+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+\right.}{2d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\left(2a^3A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^3(A+B)\sin(c+dx)}{d}+a^3x(7A+5B)\right)+\frac{(3A+5B)\sin(c+dx)\left(a^3c\right.}{2d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{2a^3A\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^3(A+B)\sin(c+dx)}{d}+a^3x(7A+5B)\right)+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos\right.}{2d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(a*B*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (((3*A + 5*B)*(a^3 + a^3
*Cos[c + d*x])*Sin[c + d*x])/(2*d) + (3*(a^3*(7*A + 5*B)*x + (2*a^3*A*ArcT
anh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Sin[c + d*x])/d))/2)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

method	result
parallelrisch	$-\frac{a^{3}\left(A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{(-A-3B)\sin(2dx+2c)}{4}-\frac{B\sin(3dx+3c)}{12}+3\left(-A-\frac{5B}{4}\right)\sin(dx+c)\right.}{d}$
parts	$\frac{a^{3}A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(a^{3}A+3a^{3}B)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{(3a^{3}A+a^{3}B)(dx+c)}{d}+\frac{(3a^{3}A+$
derivativedivides	$\frac{a^{3}A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^{3}B\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3}+3a^{3}A\sin(dx+c)+3a^{3}B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
default	$\frac{a^{3}A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^{3}B\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3}+3a^{3}A\sin(dx+c)+3a^{3}B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
risch	$\frac{7a^{3}Ax}{2}+\frac{5a^{3}Bx}{2}-\frac{3i\mathrm{e}^{i(dx+c)}a^{3}A}{2d}-\frac{15i\mathrm{e}^{i(dx+c)}a^{3}B}{8d}+\frac{3i\mathrm{e}^{-i(dx+c)}a^{3}A}{2d}+\frac{15i\mathrm{e}^{-i(dx+c)}a^{3}B}{8d}+\frac{a^{3}A\ln(\mathrm{e}^{i(dx}}{d}$
norman	$\left(\frac{7}{2}a^{3}A+\frac{5}{2}a^{3}B\right)x+\left(\frac{7}{2}a^{3}A+\frac{5}{2}a^{3}B\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(14a^{3}A+10a^{3}B\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(14a^{3}A+10a^{3}B\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
-a^3*(A*ln(tan(1/2*d*x+1/2*c)-1)-A*ln(tan(1/2*d*x+1/2*c)+1)+1/4*(-A-3*B)*s
in(2*d*x+2*c)-1/12*B*sin(3*d*x+3*c)+3*(-A-5/4*B)*sin(d*x+c)-7/2*x*d*(A+5/7
*B))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{3\,(7\,A+5\,B)a^3dx+3\,Aa^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Aa^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Ba^3\cos\left(dx+c\right)^2\right.}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fri
cas")
```

output
```
1/6*(3*(7*A + 5*B)*a^3*d*x + 3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-
sin(d*x + c) + 1) + (2*B*a^3*cos(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c)
 + 2*(9*A + 11*B)*a^3)*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=a^3\bigg(\int A\sec\left(c+dx\right)dx+\int 3A\cos\left(c+dx\right)\sec\left(c+dx\right)dx$$
$$+\int 3A\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx$$
$$+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3B\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx$$
$$+\int 3B\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx\bigg)$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*cos(c + d*x)*sec(c + d*x)
, x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(A*cos(c +
d*x)**3*sec(c + d*x), x) + Integral(B*cos(c + d*x)*sec(c + d*x), x) + Inte
gral(3*B*cos(c + d*x)**2*sec(c + d*x), x) + Integral(3*B*cos(c + d*x)**3*s
ec(c + d*x), x) + Integral(B*cos(c + d*x)**4*sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^3+36\,(dx+c)Aa^3-4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^3+9\,(2\,dx-}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="max
ima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 36*(d*x + c)*A*a^3 - 4*(s
in(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 9*(2*d*x + 2*c + sin(2*d*x + 2*c))
*B*a^3 + 12*(d*x + c)*B*a^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)) +
36*A*a^3*sin(d*x + c) + 36*B*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.62

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(7\,Aa^3+5\,Ba^3)(dx+c)+\frac{2(}{}}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gia
c")
```

output
```
1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) + 3*(7*A*a^3 + 5*B*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2
*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^3*tan(1/2*d*x +
 1/2*c)^3 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^3*tan(1/2*d*x + 1/2*c
) + 33*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 40.56 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.60

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{3\,A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{15\,B\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{7\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{5\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,B\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x),x)
```

output
```
(3*A*a^3*sin(c + d*x))/d + (15*B*a^3*sin(c + d*x))/(4*d) + (7*A*a^3*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^3*atanh(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/d + (5*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/d + (A*a^3*sin(2*c + 2*d*x))/(4*d) + (3*B*a^3*sin(2*c + 2*d*x))/(4
*d) + (B*a^3*sin(3*c + 3*d*x))/(12*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{a^3\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)a+9\cos\left(dx+c\right)\sin\left(dx+c\right)b-6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+6\log\left(\tan\left(\frac{dx}{2}\right.\right.\right.}{6d}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
(a**3*(3*cos(c + d*x)*sin(c + d*x)*a + 9*cos(c + d*x)*sin(c + d*x)*b - 6*l
og(tan((c + d*x)/2) - 1)*a + 6*log(tan((c + d*x)/2) + 1)*a - 2*sin(c + d*x
)**3*b + 18*sin(c + d*x)*a + 24*sin(c + d*x)*b + 21*a*d*x + 15*b*d*x))/(6*
d)
```

3.23 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{1}{2}a^3(6A+7B)x+\frac{a^3(3A+B)\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^3B\sin(c+dx)}{2d} - \frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d}+\frac{aA(a+a\cos(c+dx))^2\tan(c+dx)}{d}$$

output
```
1/2*a^3*(6*A+7*B)*x+a^3*(3*A+B)*arctanh(sin(d*x+c))/d+5/2*a^3*B*sin(d*x+c)
/d-1/2*(2*A-B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d+a*A*(a+a*cos(d*x+c))^2*ta
n(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. $2(110) = 220$.

Time = 3.80 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.47

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{1}{32} a^3 (1 + \cos(c + dx))^3 \sec^6\left(\frac{1}{2}(c + dx)\right) \left(2(6A + 7B)x - \frac{4(3A + B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{4(3A + B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{4(A + 3B)\cos(dx)\sin(c)}{d} + \frac{B\cos(2dx)\sin(2c)}{d} + \frac{4(A + 3B)\cos(c)\sin(dx)}{d} + \frac{B\cos(2c)\sin(2dx)}{d} + \frac{4A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} + \frac{4A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}\right)$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*(6*A + 7*B)*x - (4*(3*A +
B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (4*(3*A + B)*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 3*B)*Cos[d*x]*Sin[c])/d + (B*Cos
[2*d*x]*Sin[2*c])/d + (4*(A + 3*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin[2*
d*x])/d + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])) + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]))))/32
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3454, 3042, 3455, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3454}$$

$$\int (\cos(c+dx)a+a)^2(a(3A+B)-a(2A-B)\cos(c+dx))\sec(c+dx)dx+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(3A+B)-a(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{2}\int (\cos(c+dx)a+a)\left(2(3A+B)a^2+5B\cos(c+dx)a^2\right)\sec(c+dx)dx-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(2(3A+B)a^2+5B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{2}\int \left(5B\cos^2(c+dx)a^3+2(3A+B)a^3+\left(5Ba^3+2(3A+B)a^3\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{5B\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+2(3A+B)a^3+\left(5Ba^3+2(3A+B)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(2(3A+B)a^3+(6A+7B)\cos(c+dx)a^3\right)\sec(c+dx)dx+\frac{5a^3B\sin(c+dx)}{d}\right)-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{2(3A+B)a^3+(6A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5a^3B\sin(c+dx)}{d}\right)-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(2a^3(3A+B)\int \sec(c+dx)dx+a^3x(6A+7B)+\frac{5a^3B\sin(c+dx)}{d}\right)-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2a^3(3A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^3x(6A+7B)+\frac{5a^3B\sin(c+dx)}{d}\right)-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{2a^3(3A+B)\text{arctanh}(\sin(c+dx))}{d}+a^3x(6A+7B)+\frac{5a^3B\sin(c+dx)}{d}\right)-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^2}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
-1/2*((2*A - B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/d + (a^3*(6*A + 7*B
)*x + (2*a^3*(3*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*B*Sin[c + d*x])/d
)/2 + (a*A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.88 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

method	result
parts	$\frac{a^3 A \tan(dx+c)}{d}+\frac{(a^3A+3a^3B)\sin(dx+c)}{d}+\frac{(3a^3A+a^3B)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(3a^3A+3a^3B)(dx+c)}{d}$
parallelrisch	$-\frac{3\left(\cos(dx+c)\left(A+\frac{B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\cos(dx+c)\left(A+\frac{B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(-\frac{A}{6}-\frac{B}{2}\right)\sin(2dx+2c)-\frac{B}{}\right.}{d\cos(dx+c)}$
derivativedivides	$\frac{a^3A\sin(dx+c)+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3A(dx+c)+3a^3B\sin(dx+c)+3a^3A\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^3A\sin(dx+c)+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3A(dx+c)+3a^3B\sin(dx+c)+3a^3A\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$3a^3Ax+\frac{7a^3Bx}{2}-\frac{ia^3B\,e^{2i(dx+c)}}{8d}-\frac{ie^{i(dx+c)}a^3A}{2d}-\frac{3ie^{i(dx+c)}a^3B}{2d}+\frac{ie^{-i(dx+c)}a^3A}{2d}+\frac{3ie^{-i(dx+c)}a^3B}{2d}$
norman	$\frac{(-\frac{7}{2}a^3B-3a^3A)x+(-\frac{21}{2}a^3B-9a^3A)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{7}{2}a^3B+3a^3A)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(\frac{21}{2}a^3B+9a^3A)x\tan\left(\frac{dx}{2}+\right.}{}$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBO
SE)
```

output
```
a^3*A/d*tan(d*x+c)+(A*a^3+3*B*a^3)/d*sin(d*x+c)+(3*A*a^3+B*a^3)/d*ln(sec(d
*x+c)+tan(d*x+c))+(3*A*a^3+3*B*a^3)/d*(d*x+c)+a^3*B/d*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{(6\,A+7\,B)a^3dx\cos(dx+c)+(3\,A+B)a^3\cos(dx+c)\log(\sin(dx+c)+1)-(3\,A+B)a^3\cos(dx+c}{2\,d\cos(dx}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="f
ricas")
```

output
```
1/2*((6*A + 7*B)*a^3*d*x*cos(d*x + c) + (3*A + B)*a^3*cos(d*x + c)*log(sin
(d*x + c) + 1) - (3*A + B)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (B*a^
3*cos(d*x + c)^2 + 2*(A + 3*B)*a^3*cos(d*x + c) + 2*A*a^3)*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [F]

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^2(c + dx) \, dx$$
$$= a^3 \left(\int A \sec^2(c + dx) \, dx + \int 3A \cos(c + dx) \sec^2(c + dx) \, dx + \int 3A \cos^2(c + dx) \sec^2(c + dx) \, dx + \int A \cos^3(c + dx) \sec^2(c + dx) \, dx + \int B \cos(c + dx) \sec^2(c + dx) \, dx + \int 3B \cos^2(c + dx) \sec^2(c + dx) \, dx + \int 3B \cos^3(c + dx) \sec^2(c + dx) \, dx + \int B \cos^4(c + dx) \sec^2(c + dx) \, dx \right)$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*cos(c + d*x)*sec(c + d
*x)**2, x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(A
*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)*sec(c + d*x
)**2, x) + Integral(3*B*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*B
*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)**4*sec(c +
d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.27

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^2(c + dx) \, dx$$
$$= \frac{12\,(dx + c)Aa^3 + (2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ba^3 + 12\,(dx + c)Ba^3 + 6\,Aa^3(\log(\sin(dx + c) + 1) - \log \ldots}{\ldots}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="m
axima")
```

output
```
1/4*(12*(d*x + c)*A*a^3 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 12*(d*x
 + c)*B*a^3 + 6*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*
B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x +
c) + 12*B*a^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.75

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx =$$

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - (6\,Aa^3+7\,Ba^3)(dx+c) - 2\,(3\,Aa^3+Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 2\,(3\,Aa^3}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="g
iac")
```

output
```
-1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*A*a^3
 + 7*B*a^3)*(d*x + c) - 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) + 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^3
*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan(1/2
*d*x + 1/2*c) + 7*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

Mupad [B] (verification not implemented)

Time = 41.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.79

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

output
```
(A*a^3*sin(c + d*x))/d + (3*B*a^3*sin(c + d*x))/d + (6*A*a^3*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)))/d + (7*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (2*B*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^3*sin
(c + d*x))/(d*cos(c + d*x)) + (B*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.62

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{a^3\left(\cos\left(dx+c\right)^2\sin\left(dx+c\right)b-6\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a-2\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
(a**3*(cos(c + d*x)**2*sin(c + d*x)*b - 6*cos(c + d*x)*log(tan((c + d*x)/2
) - 1)*a - 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 6*cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*a + 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b + 2*
cos(c + d*x)*sin(c + d*x)*a + 6*cos(c + d*x)*sin(c + d*x)*b + 6*cos(c + d*
x)*a*d*x + 7*cos(c + d*x)*b*d*x + 2*sin(c + d*x)*a))/(2*cos(c + d*x)*d)
```

3.24 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 114

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= a^3(A+3B)x+\frac{a^3(7A+6B)\text{arctanh}(\sin(c+dx))}{2d} \\ &\quad-\frac{5a^3A\sin(c+dx)}{2d}+\frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d} \\ &\quad+\frac{aA(a+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
a^3*(A+3*B)*x+1/2*a^3*(7*A+6*B)*arctanh(sin(d*x+c))/d-5/2*a^3*A*sin(d*x+c)
/d+(2*A+B)*(a^3+a^3*cos(d*x+c))*tan(d*x+c)/d+1/2*a*A*(a+a*cos(d*x+c))^2*se
c(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 4.94 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.82

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{a^3\left(4Ac+12Bc+4Adx+12Bdx-14A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-12B\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a^3*(4*A*c + 12*B*c + 4*A*d*x + 12*B*d*x - 14*A*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 14*A*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*B*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + A/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2 + 4*B*Sin[c + d*x] + 4*(3*A + B)*Tan[c + d*x]))/
(4*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3454, 3042, 3454, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 3454$$

$$\frac{1}{2}\int (\cos(c+dx)a+a)^2(2a(2A+B)-a(A-2B)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(2a(2A+B)-a(A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3454

$$\frac{1}{2}\left(\int (\cos(c+dx)a+a)\left(a^2(7A+6B)-5a^2A\cos(c+dx)\right)\sec(c+dx)dx+\frac{2(2A+B)\tan(c+dx)\left(a^3\cos(c+\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a^2(7A+6B)-5a^2A\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2(2A+B)\tan(c+dx)\left(a^3\cos(c+\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3447

$$\frac{1}{2}\left(\int \left(-5A\cos^2(c+dx)a^3+(7A+6B)a^3+\left(a^3(7A+6B)-5a^3A\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{2(2A+B)\text{ta}}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{-5A\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(7A+6B)a^3+\left(a^3(7A+6B)-5a^3A\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2(2A+B)\tan(}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3502

$$\frac{1}{2}\left(\int \left((7A+6B)a^3+2(A+3B)\cos(c+dx)a^3\right)\sec(c+dx)dx+\frac{2(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{(7A+6B)a^3+2(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}-\frac{5a^3A}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3214

$$\frac{1}{2}\left(a^3(7A+6B)\int \sec(c+dx)dx+\frac{2(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+2a^3x(A+3B)-\frac{5a^3A\sin(c}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(a^3(7A+6B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+2a^3x(A+3B)-\frac{5a^3}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

$\big\downarrow$ 4257

$$\frac{1}{2}\left(\frac{a^3(7A+6B)\text{arctanh}(\sin(c+dx))}{d}+\frac{2(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+2a^3x(A+3B)-\frac{5a^3A}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^2}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (2*a^3*(A +
 3*B)*x + (a^3*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/d - (5*a^3*A*Sin[c + d*x
])/d + (2*(2*A + B)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 9.49 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

method	result
parts	$\frac{a^3A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{(a^3A+3a^3B)(dx+c)}{d}+\frac{(3a^3A+a^3B)\tan(dx+c)}{d}+\frac{(3a^3A}{}$
derivativedivides	$\frac{a^3A(dx+c)+a^3B\sin(dx+c)+3a^3A\ln(\sec(dx+c)+\tan(dx+c))+3a^3B(dx+c)+3a^3A\tan(dx+c)+3a^3B\ln(\sec(dx+c)+}{d}$
default	$\frac{a^3A(dx+c)+a^3B\sin(dx+c)+3a^3A\ln(\sec(dx+c)+\tan(dx+c))+3a^3B(dx+c)+3a^3A\tan(dx+c)+3a^3B\ln(\sec(dx+c)+}{d}$
parallelrisch	$-\frac{7\left(\left(A+\frac{6B}{7}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\left(A+\frac{6B}{7}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2dx(A+3B)c}{7}\right.}{2d(\cos(2dx+2c)+1)}$
risch	$a^3Ax+3a^3Bx-\frac{ie^{i(dx+c)}a^3B}{2d}+\frac{ie^{-i(dx+c)}a^3B}{2d}-\frac{ia^3\left(A\,e^{3i(dx+c)}-6A\,e^{2i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+}\right.}{d\left(e^{2i(dx+c)}+1\right)^2}$
norman	$\frac{(a^3A+3a^3B)x+(-4a^3A-12a^3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-a^3A-3a^3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-a^3A-3a^3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+}{}$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBO
SE)
```

output
```
a^3*A/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+(A*a^3+3
*B*a^3)/d*(d*x+c)+(3*A*a^3+B*a^3)/d*tan(d*x+c)+(3*A*a^3+3*B*a^3)/d*ln(sec(
d*x+c)+tan(d*x+c))+a^3*B*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{4\,(A+3\,B)a^3dx\cos\left(dx+c\right)^2+(7\,A+6\,B)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(7\,A+6\,B)a^3\cos\left(d\right.}{4\,d\cos\left(\right.}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="f
ricas")
```

output
```
1/4*(4*(A + 3*B)*a^3*d*x*cos(d*x + c)^2 + (7*A + 6*B)*a^3*cos(d*x + c)^2*l
og(sin(d*x + c) + 1) - (7*A + 6*B)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) +
1) + 2*(2*B*a^3*cos(d*x + c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= a^3\left(\int A\sec^3(c+dx)\,dx+\int 3A\cos(c+dx)\sec^3(c+dx)\,dx+\int 3A\cos^2(c+dx)\sec^3(c+dx)\,dx+\int A\cos^3(c+dx)\sec^3(c+dx)\,dx+\int B\cos(c+dx)\sec^3(c+dx)\,dx+\int 3B\cos^2(c+dx)\sec^3(c+dx)\,dx+\int 3B\cos^3(c+dx)\sec^3(c+dx)\,dx+\int B\cos^4(c+dx)\sec^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
a**3*(Integral(A*sec(c + d*x)**3, x) + Integral(3*A*cos(c + d*x)*sec(c + d
*x)**3, x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(A
*cos(c + d*x)**3*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)*sec(c + d*x
)**3, x) + Integral(3*B*cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(3*B
*cos(c + d*x)**3*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)**4*sec(c +
d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.45

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{4(dx+c)Aa^3+12(dx+c)Ba^3-Aa^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="m
axima")
```

output
```
1/4*(4*(d*x + c)*A*a^3 + 12*(d*x + c)*B*a^3 - A*a^3*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c
) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c) + 12*A*a^3*tan(d*x
+ c) + 4*B*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.68

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^3(c + dx)\, dx$$
$$= \frac{\frac{4\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 2\,(Aa^3 + 3\,Ba^3)(dx + c) + (7\,Aa^3 + 6\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (7\,Aa^3 + 6}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="g
iac")
```

output
```
1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(A*a^3
+ 3*B*a^3)*(d*x + c) + (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^3*tan(1/2*
d*x + 1/2*c) - 2*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
2)/d
```

Mupad [B] (verification not implemented)

Time = 41.73 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.82

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{B\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$+\frac{6\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$+\frac{3\,A\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{B\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^3,x)
```

output
```
(B*a^3*sin(c + d*x))/d + (2*A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (7*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*B*a
^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*B*a^3*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*A*a^3*sin(c + d*x))/(d*cos(c + d*x
)) + (A*a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (B*a^3*sin(c + d*x))/(d*c
os(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.30

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{a^3\left(-7\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a-6\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+\right.}{}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
(a**3*( - 7*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 6*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + 7*cos(c + d*x)*l
og(tan((c + d*x)/2) - 1)*a + 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b +
7*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 6*cos(c + d*x
)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 7*cos(c + d*x)*log(tan((c
+ d*x)/2) + 1)*a - 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b + 2*cos(c +
d*x)*sin(c + d*x)**3*b + 2*cos(c + d*x)*sin(c + d*x)**2*a*d*x + 6*cos(c +
d*x)*sin(c + d*x)**2*b*d*x - cos(c + d*x)*sin(c + d*x)*a - 2*cos(c + d*x)*
sin(c + d*x)*b - 2*cos(c + d*x)*a*d*x - 6*cos(c + d*x)*b*d*x + 6*sin(c + d
*x)**3*a + 2*sin(c + d*x)**3*b - 6*sin(c + d*x)*a - 2*sin(c + d*x)*b))/(2*
cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.25 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 125

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= a^3Bx+\frac{a^3(5A+7B)\text{arctanh}(\sin(c+dx))}{2d}+\frac{5a^3(A+B)\tan(c+dx)}{2d}\\&\quad+\frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{6d}\\&\quad+\frac{aA(a+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}\end{aligned}$$

output
```
a^3*B*x+1/2*a^3*(5*A+7*B)*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*tan(d*x+c)/d
+1/6*(5*A+3*B)*(a^3+a^3*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*(a+a*c
os(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{a^3(6Bdx+6(A+3B)\coth^{-1}(\sin(c+dx))+3(3A+B)\text{arctanh}(\sin(c+dx))+24A\tan(c+dx)+18B}{6d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a^3*(6*B*d*x + 6*(A + 3*B)*ArcCoth[Sin[c + d*x]] + 3*(3*A + B)*ArcTanh[Si
n[c + d*x]] + 24*A*Tan[c + d*x] + 18*B*Tan[c + d*x] + 9*A*Sec[c + d*x]*Tan
[c + d*x] + 3*B*Sec[c + d*x]*Tan[c + d*x] + 2*A*Tan[c + d*x]^3))/(6*d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3454, 3042, 3454, 27, 3042, 3447, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{3}\int (\cos(c+dx)a+a)^2(a(5A+3B)+3aB\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(5A+3B)+3aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3454

$$\frac{1}{3}\left(\frac{1}{2}\int 3(\cos(c+dx)a+a)\left(5(A+B)a^2+2B\cos(c+dx)a^2\right)\sec^2(c+dx)dx+\frac{(5A+3B)\tan(c+dx)\sec(c+}{2d}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{3}{2}\int(\cos(c+dx)a+a)\left(5(A+B)a^2+2B\cos(c+dx)a^2\right)\sec^2(c+dx)dx+\frac{(5A+3B)\tan(c+dx)\sec(c+d}{2d}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(5(A+B)a^2+2B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{(5A+3B)\tan(c+dx)\sec(c+dx}{2d}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3447

$$\frac{1}{3}\left(\frac{3}{2}\int\left(2B\cos^2(c+dx)a^3+5(A+B)a^3+\left(2Ba^3+5(A+B)a^3\right)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{(5A+3B)\tan}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\int\frac{2B\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+5(A+B)a^3+\left(2Ba^3+5(A+B)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{(5A+3B)\tan(c}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\Big\downarrow$ 3500

$$\frac{1}{3}\left(\frac{3}{2}\left(\int \left((5A+7B)a^3+2B\cos(c+dx)a^3\right)\sec(c+dx)dx+\frac{5a^3(A+B)\tan(c+dx)}{d}\right)+\frac{(5A+3B)\tan(c+d\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{(5A+7B)a^3+2B\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5a^3(A+B)\tan(c+dx)}{d}\right)+\frac{(5A+3B)\tan(c+dx)\sec(c}{2\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\big\downarrow$ 3214

$$\frac{1}{3}\left(\frac{3}{2}\left(a^3(5A+7B)\int\sec(c+dx)dx+\frac{5a^3(A+B)\tan(c+dx)}{d}+2a^3Bx\right)+\frac{(5A+3B)\tan(c+dx)\sec(c+dx}{2d}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\left(a^3(5A+7B)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^3(A+B)\tan(c+dx)}{d}+2a^3Bx\right)+\frac{(5A+3B)\tan(c+dx)\sec(}{}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$\big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{a^3(5A+7B)\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^3(A+B)\tan(c+dx)}{d}+2a^3Bx\right)+\frac{(5A+3B)\tan(c+dx)\sec(c}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((5*A +
3*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (3*(2*a^3
*B*x + (a^3*(5*A + 7*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Tan[c +
d*x])/d))/2)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 13.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{a^3 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{(a^3A+3a^3B)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(3a^3A+a^3B)\left(\frac{\sec(dx+c)\tan(d}{2}\right.}{}$
parallelrisch	$\frac{3\left(-\frac{5\left(A+\frac{7B}{5}\right)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{5\left(A+\frac{7B}{5}\right)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\right.}{d(3\cos(dx+c)+\cos(}$
derivativedivides	$\frac{a^3A\ln(\sec(dx+c)+\tan(dx+c))+a^3B(dx+c)+3a^3A\tan(dx+c)+3a^3B\ln(\sec(dx+c)+\tan(dx+c))+3a^3A\left(\frac{\sec(dx+c)\tan}{2}\right.}{}$
default	$\frac{a^3A\ln(\sec(dx+c)+\tan(dx+c))+a^3B(dx+c)+3a^3A\tan(dx+c)+3a^3B\ln(\sec(dx+c)+\tan(dx+c))+3a^3A\left(\frac{\sec(dx+c)\tan}{2}\right.}{}$
risch	$a^3Bx-\frac{ia^3(9A\,e^{5i(dx+c)}+3B\,e^{5i(dx+c)}-18A\,e^{4i(dx+c)}-18B\,e^{4i(dx+c)}-48A\,e^{2i(dx+c)}-36B\,e^{2i(dx+c)}-9A\,e^{i(dx+}}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{a^3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+a^3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-a^3Bx-a^3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+3a^3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+3a^3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{}$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBO
SE)
```

output
```
-a^3*A/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*a^3+3*B*a^3)/d*ln(sec(d*x+c
)+tan(d*x+c))+(3*A*a^3+B*a^3)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+
c)+tan(d*x+c)))+(3*A*a^3+3*B*a^3)/d*tan(d*x+c)+a^3*B/d*(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.13

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{12\,Ba^3dx\cos{(dx+c)}^3 + 3\,(5\,A+7\,B)a^3\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)} - 3\,(5\,A+7\,B)a^3\cos{(dx+}}{12\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="f
ricas")
```

output
```
1/12*(12*B*a^3*d*x*cos(d*x + c)^3 + 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(s
in(d*x + c) + 1) - 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
 + 2*(2*(11*A + 9*B)*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + 2
*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.70

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$
$$= \frac{4\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)Aa^3 + 12\,(dx+c)Ba^3 - 9\,Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="m
axima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*a^3 - 9*A
*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(si
n(d*x + c) - 1)) - 3*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 18*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 36*A*a^3*tan(d*x + c) + 36*B*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.51

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$
$$= \frac{6\,(dx+c)Ba^3 + 3\left(5\,Aa^3 + 7\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(5\,Aa^3 + 7\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right.\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="g
iac")
```

output
```
1/6*(6*(d*x + c)*B*a^3 + 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) - 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15
*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^3
*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1
/2*d*x + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 41.58 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.67

$$\int (a + a\cos(c+dx))^3 (A + B\cos(c+dx)) \sec^4(c+dx)\,dx$$
$$= \frac{5\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{7\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{11\,A\,a^3\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{3\,A\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2} + \frac{A\,a^3\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3} + \frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)} + \frac{B\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^4,x)
```

output
```
(5*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^3*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (7*B*a^3*atanh(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/d + (11*A*a^3*sin(c + d*x))/(3*d*cos(c + d*x)) + (
3*A*a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (A*a^3*sin(c + d*x))/(3*d*cos
(c + d*x)^3) + (3*B*a^3*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^3*sin(c + d*
x))/(2*d*cos(c + d*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.63

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{a^3\left(-15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a-21\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx\right.}{}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
(a**3*( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 21
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + 15*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*a + 21*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*
b + 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 21*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 15*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*a - 21*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b + 6*
cos(c + d*x)*sin(c + d*x)**2*b*d*x - 9*cos(c + d*x)*sin(c + d*x)*a - 3*cos
(c + d*x)*sin(c + d*x)*b - 6*cos(c + d*x)*b*d*x + 22*sin(c + d*x)**3*a + 1
8*sin(c + d*x)**3*b - 24*sin(c + d*x)*a - 18*sin(c + d*x)*b))/(6*cos(c + d
*x)*d*(sin(c + d*x)**2 - 1))
```

3.26 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 154

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \frac{5a^3(3A+4B)\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3(9A+11B)\tan(c+dx)}{3d}\\&\quad+\frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d}\\&\quad+\frac{(3A+2B)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{6d}\\&\quad+\frac{aA(a+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

output
```
5/8*a^3*(3*A+4*B)*arctanh(sin(d*x+c))/d+1/3*a^3*(9*A+11*B)*tan(d*x+c)/d+1/
24*a^3*(27*A+28*B)*sec(d*x+c)*tan(d*x+c)/d+1/6*(3*A+2*B)*(a^3+a^3*cos(d*x+
c))*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+a*cos(d*x+c))^2*sec(d*x+c)^3*tan(
d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{a^3\left(24B\coth^{-1}(\sin(c+dx))+9(5A+4B)\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(96(A+B)+9(5A+4B\right.\right.}{24d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a^3*(24*B*ArcCoth[Sin[c + d*x]] + 9*(5*A + 4*B)*ArcTanh[Sin[c + d*x]] + T
an[c + d*x]*(96*(A + B) + 9*(5*A + 4*B)*Sec[c + d*x] + 6*A*Sec[c + d*x]^3
+ 8*(3*A + B)*Tan[c + d*x]^2)))/(24*d)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {3042, 3454, 3042, 3454, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\int (\cos(c+dx)a+a)^2(2a(3A+2B)+a(A+4B)\cos(c+dx))\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(2a(3A+2B)+a(A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$\downarrow$ 3454

$$\frac{1}{4}\left(\frac{1}{3}\int(\cos(c+dx)a+a)\left((27A+28B)a^2+(9A+16B)\cos(c+dx)a^2\right)\sec^3(c+dx)dx+\frac{2(3A+2B)\tan(c+\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((27A+28B)a^2+(9A+16B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{2(3A+2B)\tan(c+\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$\downarrow$ 3447

$$\frac{1}{4}\left(\frac{1}{3}\int\left((9A+16B)\cos^2(c+dx)a^3+(27A+28B)a^3+\left((9A+16B)a^3+(27A+28B)a^3\right)\cos(c+dx)\right)\sec^3(c\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int\frac{(9A+16B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(27A+28B)a^3+\left((9A+16B)a^3+(27A+28B)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$\downarrow$ 3500

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(8(9A+11B)a^3+15(3A+4B)\cos(c+dx)a^3\right)\sec^2(c+dx)dx+\frac{a^3(27A+28B)\tan(c+dx)\sec(c+}{2d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{8(9A+11B)a^3+15(3A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^3(27A+28B)\tan(c+dx)\sec(c+dx)}{2d}\right)+\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(8a^3(9A+11B)\int \sec^2(c+dx)dx+15a^3(3A+4B)\int \sec(c+dx)dx\right)+\frac{a^3(27A+28B)\tan(c+dx)\text{s}}{2d}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15a^3(3A+4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+8a^3(9A+11B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{a^3(27A+28B)}{}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15a^3(3A+4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{8a^3(9A+11B)\int 1d(-\tan(c+dx))}{d}\right)+\frac{a^3(27A+28B)\tan}{}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15a^3(3A+4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{8a^3(9A+11B)\tan(c+dx)}{d}\right)+\frac{a^3(27A+28B)\tan(c+dx)}{2d}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{15a^3(3A+4B)\text{arctanh}(\sin(c+dx))}{d}+\frac{8a^3(9A+11B)\tan(c+dx)}{d}\right)+\frac{a^3(27A+28B)\tan(c+dx)\text{se}}{2d}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^2}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((2*(3*A
+ 2*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((a^3
*(27*A + 28*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((15*a^3*(3*A + 4*B)*Arc
Tanh[Sin[c + d*x]])/d + (8*a^3*(9*A + 11*B)*Tan[c + d*x])/d)/2)/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 14.75 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{10\left(-\frac{3\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{4B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4}+\frac{3\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{4B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}\ldots\right.\right.}{4}\ldots\right.}{d(\cos(4dx+4c)+4\ldots}$
parts	$\frac{a^3A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{(a^3A+3a^3B)\tan(dx+c)}{d}-\frac{(3a^3A\ldots}{\ldots}$
derivativedivides	$\frac{a^3A\tan(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3a^3A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3B\tan(dx\ldots}{\ldots}$
default	$\frac{a^3A\tan(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3a^3A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3B\tan(dx\ldots}{\ldots}$
risch	$-\frac{ia^3\left(45A\,e^{7i(dx+c)}+36B\,e^{7i(dx+c)}-24A\,e^{6i(dx+c)}-72B\,e^{6i(dx+c)}+69A\,e^{5i(dx+c)}+36B\,e^{5i(dx+c)}-216A\,e^{4i(dx+c)}-\ldots\right.}{12d(e\ldots}$
norman	$\frac{\frac{19a^3(3A+4B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{4d}-\frac{5a^3(3A+4B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{12d}-\frac{5a^3(3A+4B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}}{4d}+\frac{a^3(49A+44B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a^3(81\ldots}{\ldots}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4\left(\mathrm{ta}\ldots\right.}$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBO
SE)
```

output
```
10*(-3/4*(3/4+1/4*cos(4*d*x+4*c)+cos(2*d*x+2*c))*(A+4/3*B)*ln(tan(1/2*d*x+
1/2*c)-1)+3/4*(3/4+1/4*cos(4*d*x+4*c)+cos(2*d*x+2*c))*(A+4/3*B)*ln(tan(1/2
*d*x+1/2*c)+1)+(A+13/15*B)*sin(2*d*x+2*c)+(3/8*A+3/10*B)*sin(3*d*x+3*c)+(3
/10*A+11/30*B)*sin(4*d*x+4*c)+23/40*sin(d*x+c)*(A+12/23*B))*a^3/d/(cos(4*d
*x+4*c)+4*cos(2*d*x+2*c)+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx$$
$$= \frac{15\,(3\,A + 4\,B)a^3\cos\left(dx + c\right)^4\log\left(\sin\left(dx + c\right) + 1\right) - 15\,(3\,A + 4\,B)a^3\cos\left(dx + c\right)^4\log\left(-\sin\left(dx + c\right) \cdot \right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="f
ricas")
```

output
```
1/48*(15*(3*A + 4*B)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(3*A +
4*B)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(9*A + 11*B)*a^3*cos
(d*x + c)^3 + 9*(5*A + 4*B)*a^3*cos(d*x + c)^2 + 8*(3*A + B)*a^3*cos(d*x +
 c) + 6*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.75

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx$$

$$= \frac{48\left(\tan(dx + c)^3 + 3\tan(dx + c)\right)Aa^3 + 16\left(\tan(dx + c)^3 + 3\tan(dx + c)\right)Ba^3 - 3Aa^3\left(\frac{2\left(3\sin(dx+c)^3 \cdots}{\sin(dx+c)^4 - 2 \cdots}\right.}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="m
axima")
```

output
```
1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(tan(d*x + c)^3 + 3*
tan(d*x + c))*B*a^3 - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*a^3*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A*a^3*tan(d*x + c) + 144
*B*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.38

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx$$

$$= \frac{15\left(3Aa^3 + 4Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15\left(3Aa^3 + 4Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(45 \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="g
iac")
```

output
```
1/24*(15*(3*A*a^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*
a^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a^3*tan(1/2*d*
x + 1/2*c)^7 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 165*A*a^3*tan(1/2*d*x + 1
/2*c)^5 - 220*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*A*a^3*tan(1/2*d*x + 1/2*c
)^3 + 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 147*A*a^3*tan(1/2*d*x + 1/2*c) -
132*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 43.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{\left(-\frac{15\,A\,a^3}{4}-5\,B\,a^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{55\,A\,a^3}{4}+\frac{55\,B\,a^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{73\,A\,a^3}{4}-\frac{73\,B\,a^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$
$$+\frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,A+4\,B\right)}{4\,d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*((49*A*a^3)/4 + 11*B*a^3) - tan(c/2 + (d*x)/2)^7*((15*
A*a^3)/4 + 5*B*a^3) + tan(c/2 + (d*x)/2)^5*((55*A*a^3)/4 + (55*B*a^3)/3) -
 tan(c/2 + (d*x)/2)^3*((73*A*a^3)/4 + (73*B*a^3)/3))/(d*(6*tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/
2)^8 + 1)) + (5*a^3*atanh(tan(c/2 + (d*x)/2))*(3*A + 4*B))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.12

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
(a**3*( - 45*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 60
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b + 90*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 120*cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**2*b - 45*cos(c + d*x)*log(tan((c + d*x)/2)
- 1)*a - 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 45*cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a + 60*cos(c + d*x)*log(tan((c + d*
x)/2) + 1)*sin(c + d*x)**4*b - 90*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*a - 120*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**2*b + 45*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 60*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*b - 45*cos(c + d*x)*sin(c + d*x)**3*a - 36*cos(c + d
*x)*sin(c + d*x)**3*b + 51*cos(c + d*x)*sin(c + d*x)*a + 36*cos(c + d*x)*s
in(c + d*x)*b + 72*sin(c + d*x)**5*a + 88*sin(c + d*x)**5*b - 168*sin(c +
d*x)**3*a - 184*sin(c + d*x)**3*b + 96*sin(c + d*x)*a + 96*sin(c + d*x)*b)
)/(24*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.27 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 185

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx &= \frac{a^3(13A+15B)\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3(38A+45B)\tan(c+dx)}{15d}\\ &+\frac{a^3(13A+15B)\sec(c+dx)\tan(c+dx)}{8d}\\ &+\frac{a^3(43A+45B)\sec^2(c+dx)\tan(c+dx)}{60d}\\ &+\frac{(7A+5B)\left(a^3+a^3\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{20d}\\ &+\frac{aA(a+a\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}\end{aligned}$$

output
```
1/8*a^3*(13*A+15*B)*arctanh(sin(d*x+c))/d+1/15*a^3*(38*A+45*B)*tan(d*x+c)/
d+1/8*a^3*(13*A+15*B)*sec(d*x+c)*tan(d*x+c)/d+1/60*a^3*(43*A+45*B)*sec(d*x
+c)^2*tan(d*x+c)/d+1/20*(7*A+5*B)*(a^3+a^3*cos(d*x+c))*sec(d*x+c)^3*tan(d*
x+c)/d+1/5*a*A*(a+a*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$=\frac{a^3(15(13A+15B)\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15(13A+15B)\sec(c+dx)+30(3A+B)\sec^3(c\right.}{120d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(a^3*(15*(13*A + 15*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(13*A + 15
*B)*Sec[c + d*x] + 30*(3*A + B)*Sec[c + d*x]^3 + 8*(60*(A + B) + 5*(5*A +
3*B)*Tan[c + d*x]^2 + 3*A*Tan[c + d*x]^4))))/(120*d)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 3454, 3042, 3454, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{5}\int (\cos(c+dx)a+a)^2(a(7A+5B)+a(2A+5B)\cos(c+dx))\sec^5(c+dx)dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(7A+5B)+a(2A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3454

$$\frac{1}{5}\left(\frac{1}{4}\int(\cos(c+dx)a+a)\left((43A+45B)a^2+2(11A+15B)\cos(c+dx)a^2\right)\sec^4(c+dx)dx+\frac{(7A+5B)\tan(c+\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((43A+45B)a^2+2(11A+15B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{(7A+5B)\tan(c+\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3447

$$\frac{1}{5}\left(\frac{1}{4}\int\left(2(11A+15B)\cos^2(c+dx)a^3+(43A+45B)a^3+\left(2(11A+15B)a^3+(43A+45B)a^3\right)\cos(c+dx)\right)\mathrm{se}\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\int\frac{2(11A+15B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(43A+45B)a^3+\left(2(11A+15B)a^3+(43A+45B)a^3\right)\sin\left(c+dx\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3500

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int\left(15(13A+15B)a^3+4(38A+45B)\cos(c+dx)a^3\right)\sec^3(c+dx)dx+\frac{a^3(43A+45B)\tan(c+dx)\sec}{3d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int\frac{15(13A+15B)a^3+4(38A+45B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^3(43A+45B)\tan(c+dx)\sec^2(c+dx)}{3d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3227

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15a^3(13A+15B)\int\sec^3(c+dx)dx+4a^3(38A+45B)\int\sec^2(c+dx)dx\right)+\frac{a^3(43A+45B)\tan(c+}{3d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(4a^3(38A+45B)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+15a^3(13A+15B)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{a^3(43A+}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 4254

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15a^3(13A+15B)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a^3(38A+45B)\int 1d(-\tan(c+dx))}{d}\right)+\frac{a^3(43A+45B}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 24

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15a^3(13A+15B)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{4a^3(38A+45B)\tan(c+dx)}{d}\right)+\frac{a^3(43A+45B)\tan(c+}{3d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 4255

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15a^3(13A+15B)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a^3(38A+45B)\tan(c+dx)}{d}\right)\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15a^3(13A+15B)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a^3(38A+45B)\tan(c+}{d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15a^3(13A+15B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a^3(38A+45B)\tan(c+dx)}{d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^2}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (((7*A +
5*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((a^3*(
43*A + 45*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((4*a^3*(38*A + 45*B)*Ta
n[c + d*x])/d + 15*a^3*(13*A + 15*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c
 + d*x]*Tan[c + d*x])/(2*d)))/3)/4)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 17.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.06

method	result
parts	$-\frac{a^{3}A\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}+\frac{\left(a^{3}A+3a^{3}B\right)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
parallelrisch	$\frac{40a^{3}\left(-\frac{39\left(A+\frac{15B}{13}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{32}+\frac{39\left(A+\frac{15B}{13}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c}{2}\right.}{32}\right.}{}$
derivativedivides	$\frac{a^{3}A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^{3}B\tan(dx+c)-3a^{3}A\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+3a^{3}B\left(\right.}{}$
default	$\frac{a^{3}A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^{3}B\tan(dx+c)-3a^{3}A\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+3a^{3}B\left(\right.}{}$
risch	$-\frac{ia^{3}\left(195A\,e^{9i(dx+c)}+225B\,e^{9i(dx+c)}-120B\,e^{8i(dx+c)}+750A\,e^{7i(dx+c)}+570B\,e^{7i(dx+c)}-720A\,e^{6i(dx+c)}-1200B\,e^{6\right.}{}$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBO
SE)
```

output
```
-a^3*A/d*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+(A*a^3+3*B*
a^3)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+(3*A*a^3+
B*a^3)/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))-(3*A*a^3+3*B*a^3)/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*B
/d*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.89

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$=\frac{15\,(13\,A+15\,B)a^3\cos{(dx+c)}^5\log{(\sin{(dx+c)}+1)}-15\,(13\,A+15\,B)a^3\cos{(dx+c)}^5\log{(-\sin{(dx+}}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="f
ricas")
```

output
```
1/240*(15*(13*A + 15*B)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(13*
A + 15*B)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(38*A + 45*B)*a
^3*cos(d*x + c)^4 + 15*(13*A + 15*B)*a^3*cos(d*x + c)^3 + 8*(19*A + 15*B)*
a^3*cos(d*x + c)^2 + 30*(3*A + B)*a^3*cos(d*x + c) + 24*A*a^3)*sin(d*x + c
))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.82

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$

$$=\frac{16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Aa^3+240\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="m
axima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
 240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 240*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a^3 - 45*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 60*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
+ 1) + log(sin(d*x + c) - 1)) - 180*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*a^3*tan(d*x
+ c))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.33

$$\int (a + a\cos(c+dx))^3 (A + B\cos(c+dx)) \sec^6(c+dx)\, dx$$

$$= \frac{15\,(13\,Aa^3 + 15\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\,(13\,Aa^3 + 15\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) -}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="g
iac")
```

output
```
1/120*(15*(13*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(1
3*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(195*A*a^3*tan(
1/2*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*A*a^3*tan(1/2*
d*x + 1/2*c)^7 - 1050*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*
x + 1/2*c)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*A*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 1830*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1
/2*c) + 735*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 44.47 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.21

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(13\,A+15\,B\right)}{4\,d} - \frac{\left(\frac{13\,A\,a^3}{4}+\frac{15\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{91\,A\,a^3}{6}-\frac{35\,B\,a^3}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{416\,A\,a^3}{15}+32\,B\,a^3\right)\,\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\mathrm{ta}\right.}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^6,x)
```

output
```
(a^3*atanh(tan(c/2 + (d*x)/2))*(13*A + 15*B))/(4*d) - (tan(c/2 + (d*x)/2)*
((51*A*a^3)/4 + (49*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((13*A*a^3)/4 + (15*B
*a^3)/4) - tan(c/2 + (d*x)/2)^7*((91*A*a^3)/6 + (35*B*a^3)/2) - tan(c/2 +
(d*x)/2)^3*((133*A*a^3)/6 + (61*B*a^3)/2) + tan(c/2 + (d*x)/2)^5*((416*A*a
^3)/15 + 32*B*a^3))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 +
 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
- 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.60

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

output

```
(a**3*( - 195*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 2
25*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b + 390*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 450*cos(c + d*x)*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)**2*b - 195*cos(c + d*x)*log(tan((c + d*x)
/2) - 1)*a - 225*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 195*cos(c + d*
x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a + 225*cos(c + d*x)*log(tan(
(c + d*x)/2) + 1)*sin(c + d*x)**4*b - 390*cos(c + d*x)*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**2*a - 450*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(
c + d*x)**2*b + 195*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 225*cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*b - 195*cos(c + d*x)*sin(c + d*x)**3*a - 2
25*cos(c + d*x)*sin(c + d*x)**3*b + 285*cos(c + d*x)*sin(c + d*x)*a + 255*
cos(c + d*x)*sin(c + d*x)*b + 304*sin(c + d*x)**5*a + 360*sin(c + d*x)**5*
b - 760*sin(c + d*x)**3*a - 840*sin(c + d*x)**3*b + 480*sin(c + d*x)*a + 4
80*sin(c + d*x)*b))/(120*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)*
*2 + 1))
```

3.28 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 241

$$\begin{aligned}
&\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx \\
&= \frac{1}{16}a^4(49A+44B)x+\frac{a^4(252A+227B)\sin(c+dx)}{35d} \\
&\quad+\frac{a^4(49A+44B)\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad+\frac{a^4(301A+276B)\cos^3(c+dx)\sin(c+dx)}{280d} \\
&\quad+\frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} \\
&\quad+\frac{(7A+10B)\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{42d} \\
&\quad+\frac{7(A+B)\cos^3(c+dx)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{15d} \\
&\quad-\frac{a^4(252A+227B)\sin^3(c+dx)}{105d}
\end{aligned}$$

output
```
1/16*a^4*(49*A+44*B)*x+1/35*a^4*(252*A+227*B)*sin(d*x+c)/d+1/16*a^4*(49*A+
44*B)*cos(d*x+c)*sin(d*x+c)/d+1/280*a^4*(301*A+276*B)*cos(d*x+c)^3*sin(d*x
+c)/d+1/7*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/42*(7*A+10*B)
*cos(d*x+c)^3*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d+7/15*(A+B)*cos(d*x+c)^3*
(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d-1/105*a^4*(252*A+227*B)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.65

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{a^4(18480Bc+20580Adx+18480Bdx+105(352A+323B)\sin(c+dx)+105(127A+124B)\sin(2(c+$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(a^4*(18480*B*c + 20580*A*d*x + 18480*B*d*x + 105*(352*A + 323*B)*Sin[c +
d*x] + 105*(127*A + 124*B)*Sin[2*(c + d*x)] + 5040*A*Sin[3*(c + d*x)] + 54
95*B*Sin[3*(c + d*x)] + 1575*A*Sin[4*(c + d*x)] + 2100*B*Sin[4*(c + d*x)]
+ 336*A*Sin[5*(c + d*x)] + 651*B*Sin[5*(c + d*x)] + 35*A*Sin[6*(c + d*x)]
+ 140*B*Sin[6*(c + d*x)] + 15*B*Sin[7*(c + d*x)]))/(6720*d)
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 3455, 3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4 \left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3455

$$\frac{1}{7}\int \cos^2(c+dx)(\cos(c+dx)a+a)^3(a(7A+3B)+a(7A+10B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3 \left(a(7A+3B)+a(7A+10B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 3455

$$\frac{1}{7}\left(\frac{1}{6}\int \cos^2(c+dx)(\cos(c+dx)a+a)^2\left(3(21A+16B)a^2+98(A+B)\cos(c+dx)a^2\right)dx+\frac{(7A+10B)\sin(c+\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2 \left(3(21A+16B)a^2+98(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 3455

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\int 3\cos^2(c+dx)(\cos(c+dx)a+a)\left((203A+178B)a^3+(301A+276B)\cos(c+dx)a^3\right)dx+\frac{98(A+\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\int \cos^2(c+dx)(\cos(c+dx)a+a)\left((203A+178B)a^3+(301A+276B)\cos(c+dx)a^3\right)dx+\frac{98(A+B)}{}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((203A+178B)a^3+(301A+276B)\sin\left(c+dx+\frac{\pi}{2}\right)\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$$\Big\downarrow \text{ 3447}$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\int \cos^2(c+dx)\left((301A+276B)\cos^2(c+dx)a^4+(203A+178B)a^4+\left((203A+178B)a^4+(301A+2\right.\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left((301A+276B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+(203A+178B)a^4+\left((203A+178B)a^4\right.\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\int \cos^2(c+dx)\left(35(49A+44B)a^4+8(252A+227B)\cos(c+dx)a^4\right)dx+\frac{a^4(301A+276B)\sin(c+}{4d}\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(35(49A+44B)a^4+8(252A+227B)\sin\left(c+dx+\frac{\pi}{2}\right)a^4\right)dx+\frac{a^4(301A+}{}\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(8a^4(252A+227B)\int \cos^3(c+dx)dx+35a^4(49A+44B)\int \cos^2(c+dx)dx\right)+\frac{a^4(301A+276B)}{}\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(35a^4(49A+44B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+8a^4(252A+227B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{a^4(30\right.\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 3113

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(35a^4(49A+44B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8a^4(252A+227B)\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+\right.}{d}\right.\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 2009

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(35a^4(49A+44B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8a^4(252A+227B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 3115

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(35a^4(49A+44B)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{8a^4(252A+227B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(\right.}{d}\right.\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

$\downarrow$ 24

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{3}{5}\left(\frac{a^4(301A+276B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{4}\left(35a^4(49A+44B)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\right.\right.\right.\right.$$

$$\frac{aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^3}{7d}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) + (((7*A +
10*B)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + ((98
*(A + B)*Cos[c + d*x]^3*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(5*d) + (3*
((a^4*(301*A + 276*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (35*a^4*(49*A +
 44*B)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (8*a^4*(252*A + 227*B)*
(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d)/4))/5)/6)/7
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.55 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.49

$$a^4 A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{B\,a^4\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}$$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(a^4*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)
+5/16*d*x+5/16*c)+1/7*B*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*
x+c)^2)*sin(d*x+c)+4/5*a^4*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c
)+4*B*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+6*a^4*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8
*d*x+3/8*c)+6/5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/3*a
^4*A*(cos(d*x+c)^2+2)*sin(d*x+c)+4*B*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c)
)*sin(d*x+c)+3/8*d*x+3/8*c)+a^4*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+1/3*B*a^4*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.62

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{105\,(49\,A+44\,B)a^4dx+\left(240\,Ba^4\cos{(dx+c)}^6+280\,(A+4\,B)a^4\cos{(dx+c)}^5+192\,(7\,A+12\,B)a^4\cos\cdots\right.}{\cdots}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/1680*(105*(49*A + 44*B)*a^4*d*x + (240*B*a^4*cos(d*x + c)^6 + 280*(A + 4
*B)*a^4*cos(d*x + c)^5 + 192*(7*A + 12*B)*a^4*cos(d*x + c)^4 + 70*(41*A +
44*B)*a^4*cos(d*x + c)^3 + 16*(252*A + 227*B)*a^4*cos(d*x + c)^2 + 105*(49
*A + 44*B)*a^4*cos(d*x + c) + 32*(252*A + 227*B)*a^4)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. 2(226) = 452.

Time = 0.61 (sec) , antiderivative size = 960, normalized size of antiderivative = 3.98

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((5*A*a**4*x*sin(c + d*x)**6/16 + 15*A*a**4*x*sin(c + d*x)**4*cos
(c + d*x)**2/16 + 9*A*a**4*x*sin(c + d*x)**4/4 + 15*A*a**4*x*sin(c + d*x)*
*2*cos(c + d*x)**4/16 + 9*A*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + A*a
**4*x*sin(c + d*x)**2/2 + 5*A*a**4*x*cos(c + d*x)**6/16 + 9*A*a**4*x*cos(c
 + d*x)**4/4 + A*a**4*x*cos(c + d*x)**2/2 + 5*A*a**4*sin(c + d*x)**5*cos(c
 + d*x)/(16*d) + 32*A*a**4*sin(c + d*x)**5/(15*d) + 5*A*a**4*sin(c + d*x)*
*3*cos(c + d*x)**3/(6*d) + 16*A*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d)
 + 9*A*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**4*sin(c + d*x)**3/
(3*d) + 11*A*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*A*a**4*sin(c + d
*x)*cos(c + d*x)**4/d + 15*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*A
*a**4*sin(c + d*x)*cos(c + d*x)**2/d + A*a**4*sin(c + d*x)*cos(c + d*x)/(2
*d) + 5*B*a**4*x*sin(c + d*x)**6/4 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d
*x)**2/4 + 3*B*a**4*x*sin(c + d*x)**4/2 + 15*B*a**4*x*sin(c + d*x)**2*cos(
c + d*x)**4/4 + 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 5*B*a**4*x*co
s(c + d*x)**6/4 + 3*B*a**4*x*cos(c + d*x)**4/2 + 16*B*a**4*sin(c + d*x)**7
/(35*d) + 8*B*a**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 5*B*a**4*sin(c
+ d*x)**5*cos(c + d*x)/(4*d) + 16*B*a**4*sin(c + d*x)**5/(5*d) + 2*B*a**4*
sin(c + d*x)**3*cos(c + d*x)**4/d + 10*B*a**4*sin(c + d*x)**3*cos(c + d*x)
**3/(3*d) + 8*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a**4*sin(c +
d*x)**3*cos(c + d*x)/(2*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*si...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.48

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{1792\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)Aa^4-35\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
1/6720*(1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^
4 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*A*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 1260
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 1680*(2*d
*x + 2*c + sin(2*d*x + 2*c))*A*a^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x +
c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*B*a^4 + 2688*(3*sin(d*x + c)^5
 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 140*(4*sin(2*d*x + 2*c)^3
- 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^4 - 2240*(
sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 840*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*B*a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.80

$$\begin{aligned}
&\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx \\
&= \frac{Ba^4\sin(7\,dx+7\,c)}{448\,d} + \frac{1}{16}\,(49\,Aa^4+44\,Ba^4)x \\
&\quad + \frac{(Aa^4+4\,Ba^4)\sin(6\,dx+6\,c)}{192\,d} + \frac{(16\,Aa^4+31\,Ba^4)\sin(5\,dx+5\,c)}{320\,d} \\
&\quad + \frac{5\,(3\,Aa^4+4\,Ba^4)\sin(4\,dx+4\,c)}{64\,d} + \frac{(144\,Aa^4+157\,Ba^4)\sin(3\,dx+3\,c)}{192\,d} \\
&\quad + \frac{(127\,Aa^4+124\,Ba^4)\sin(2\,dx+2\,c)}{64\,d} + \frac{(352\,Aa^4+323\,Ba^4)\sin(dx+c)}{64\,d}
\end{aligned}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/448*B*a^4*sin(7*d*x + 7*c)/d + 1/16*(49*A*a^4 + 44*B*a^4)*x + 1/192*(A*a
^4 + 4*B*a^4)*sin(6*d*x + 6*c)/d + 1/320*(16*A*a^4 + 31*B*a^4)*sin(5*d*x +
 5*c)/d + 5/64*(3*A*a^4 + 4*B*a^4)*sin(4*d*x + 4*c)/d + 1/192*(144*A*a^4 +
 157*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(127*A*a^4 + 124*B*a^4)*sin(2*d*x +
2*c)/d + 1/64*(352*A*a^4 + 323*B*a^4)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.86 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.46

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(\frac{49\,A\,a^4}{8}+\frac{11\,B\,a^4}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{245\,A\,a^4}{6}+\frac{110\,B\,a^4}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{13867\,A\,a^4}{120}+\frac{3113\,B\,a^4}{30}\right)\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\right.}$$
$$-\frac{a^4\,(49\,A+44\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}$$
$$+\frac{a^4\,\mathrm{atan}\left(\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(49\,A+44\,B)}{8\left(\frac{49\,A\,a^4}{8}+\frac{11\,B\,a^4}{2}\right)}\right)\,(49\,A+44\,B)}{8\,d}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + (53*B*a^4)/2) + tan(c/2 + (d*x)/2)^13
*((49*A*a^4)/8 + (11*B*a^4)/2) + tan(c/2 + (d*x)/2)^11*((245*A*a^4)/6 + (1
10*B*a^4)/3) + tan(c/2 + (d*x)/2)^3*((523*A*a^4)/6 + 70*B*a^4) + tan(c/2 +
 (d*x)/2)^7*((896*A*a^4)/5 + (5632*B*a^4)/35) + tan(c/2 + (d*x)/2)^9*((138
67*A*a^4)/120 + (3113*B*a^4)/30) + tan(c/2 + (d*x)/2)^5*((19157*A*a^4)/120
 + (1501*B*a^4)/10))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4
+ 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2
)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) - (a^4*(49*A
+ 44*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d) + (a^4*atan((a^4*tan(c
/2 + (d*x)/2)*(49*A + 44*B))/(8*((49*A*a^4)/8 + (11*B*a^4)/2)))*(49*A + 44
*B))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{a^4\left(280\cos(dx+c)\sin(dx+c)^5a+1120\cos(dx+c)\sin(dx+c)^5b-3430\cos(dx+c)\sin(dx+c)^3a\right.}{}$$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
(a**4*(280*cos(c + d*x)*sin(c + d*x)**5*a + 1120*cos(c + d*x)*sin(c + d*x)
**5*b - 3430*cos(c + d*x)*sin(c + d*x)**3*a - 5320*cos(c + d*x)*sin(c + d*
x)**3*b + 8295*cos(c + d*x)*sin(c + d*x)*a + 8820*cos(c + d*x)*sin(c + d*x
)*b - 240*sin(c + d*x)**7*b + 1344*sin(c + d*x)**5*a + 3024*sin(c + d*x)**
5*b - 6720*sin(c + d*x)**3*a - 8960*sin(c + d*x)**3*b + 13440*sin(c + d*x)
*a + 13440*sin(c + d*x)*b + 5145*a*d*x + 4620*b*d*x))/(1680*d)
```

3.29 $\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 185

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx &= \frac{7}{16}a^4(8A+7B)x+\frac{4a^4(8A+7B)\sin(c+dx)}{5d}\\ &+\frac{27a^4(8A+7B)\cos(c+dx)\sin(c+dx)}{80d}+\frac{a^4(8A+7B)\cos^3(c+dx)\sin(c+dx)}{40d}\\ &+\frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx)}{30d}\\ &+\frac{B(a+a\cos(c+dx))^5\sin(c+dx)}{6ad}-\frac{2a^4(8A+7B)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
7/16*a^4*(8*A+7*B)*x+4/5*a^4*(8*A+7*B)*sin(d*x+c)/d+27/80*a^4*(8*A+7*B)*co
s(d*x+c)*sin(d*x+c)/d+1/40*a^4*(8*A+7*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/30*(6
*A-B)*(a+a*cos(d*x+c))^4*sin(d*x+c)/d+1/6*B*(a+a*cos(d*x+c))^5*sin(d*x+c)/
a/d-2/15*a^4*(8*A+7*B)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{a^4(2940Bc+3360Adx+2940Bdx+120(49A+44B)\sin(c+dx)+15(128A+127B)\sin(2(c+dx))+$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(a^4*(2940*B*c + 3360*A*d*x + 2940*B*d*x + 120*(49*A + 44*B)*Sin[c + d*x]
+ 15*(128*A + 127*B)*Sin[2*(c + d*x)] + 580*A*Sin[3*(c + d*x)] + 720*B*Sin
[3*(c + d*x)] + 120*A*Sin[4*(c + d*x)] + 225*B*Sin[4*(c + d*x)] + 12*A*Sin
[5*(c + d*x)] + 48*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{3447}$$
$$\int (a\cos(c+dx)+a)^4\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (\cos(c+dx)a+a)^4(5aB+a(6A-B)\cos(c+dx))dx}{6a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^5}{6ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4\left(5aB+a(6A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{6a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^5}{6ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{3}{5}a(8A+7B)\int (\cos(c+dx)a+a)^4dx+\frac{a(6A-B)\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}}{6a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^5}{6ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}a(8A+7B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4dx+\frac{a(6A-B)\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}}{6a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^5}{6ad}$$

$$\Big\downarrow \text{3124}$$

$$\frac{\frac{3}{5}a(8A+7B)\int \left(\cos^4(c+dx)a^4+4\cos^3(c+dx)a^4+6\cos^2(c+dx)a^4+4\cos(c+dx)a^4+a^4\right)dx+\frac{a(6A-B)\sin(c}{6a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^5}{6ad}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{5}a(8A+7B)\left(-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}\right)+\frac{a(6A-B)\mathrm{si}}{6a}+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^5}{6ad}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(6*a*d) + ((a*(6*A - B)*(a + a*Cos
[c + d*x])^4*Sin[c + d*x])/(5*d) + (3*a*(8*A + 7*B)*((35*a^4*x)/8 + (8*a^4
*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^4*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3*d)))/5)/(6*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 6.10 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.65

$$\frac{\frac{a^4A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+B\,a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+4a^4A\Bigg(}{}$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(1/5*a^4*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^4*(1/6*(
cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)
+4*a^4*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*
B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a^4*A*(cos(d*x+c)^2
+2)*sin(d*x+c)+6*B*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+4*a^4*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4/3*B*a^4*(cos
(d*x+c)^2+2)*sin(d*x+c)+a^4*A*sin(d*x+c)+B*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+
1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{105\,(8\,A+7\,B)a^4dx+\left(40\,Ba^4\cos\left(dx+c\right)^5+48\,(A+4\,B)a^4\cos\left(dx+c\right)^4+10\,(24\,A+41\,B)a^4\cos\left(d\right.\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/240*(105*(8*A + 7*B)*a^4*d*x + (40*B*a^4*cos(d*x + c)^5 + 48*(A + 4*B)*a
^4*cos(d*x + c)^4 + 10*(24*A + 41*B)*a^4*cos(d*x + c)^3 + 32*(17*A + 18*B)
*a^4*cos(d*x + c)^2 + 105*(8*A + 7*B)*a^4*cos(d*x + c) + 16*(83*A + 72*B)*
a^4)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(170) = 340$.

Time = 0.46 (sec) , antiderivative size = 765, normalized size of antiderivative = 4.14

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((3*A*a**4*x*sin(c + d*x)**4/2 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
 + d*x)**2 + 2*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/2 + 2
*A*a**4*x*cos(c + d*x)**2 + 8*A*a**4*sin(c + d*x)**5/(15*d) + 4*A*a**4*sin
(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x)
/(2*d) + 4*A*a**4*sin(c + d*x)**3/d + A*a**4*sin(c + d*x)*cos(c + d*x)**4/
d + 5*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 6*A*a**4*sin(c + d*x)*co
s(c + d*x)**2/d + 2*A*a**4*sin(c + d*x)*cos(c + d*x)/d + A*a**4*sin(c + d*
x)/d + 5*B*a**4*x*sin(c + d*x)**6/16 + 15*B*a**4*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 9*B*a**4*x*sin(c + d*x)**4/4 + 15*B*a**4*x*sin(c + d*x)**2*c
os(c + d*x)**4/16 + 9*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + B*a**4*
x*sin(c + d*x)**2/2 + 5*B*a**4*x*cos(c + d*x)**6/16 + 9*B*a**4*x*cos(c + d
*x)**4/4 + B*a**4*x*cos(c + d*x)**2/2 + 5*B*a**4*sin(c + d*x)**5*cos(c + d
*x)/(16*d) + 32*B*a**4*sin(c + d*x)**5/(15*d) + 5*B*a**4*sin(c + d*x)**3*c
os(c + d*x)**3/(6*d) + 16*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9
*B*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*B*a**4*sin(c + d*x)**3/(3*d
) + 11*B*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*B*a**4*sin(c + d*x)*
cos(c + d*x)**4/d + 15*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*B*a**
4*sin(c + d*x)*cos(c + d*x)**2/d + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d),
 Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**4*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.61

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$= \frac{64\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Aa^4-1920\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^4}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 120*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 960*(2*d*x + 2*c + sin(2*d*x + 2
*c))*A*a^4 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*
B*a^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*
sin(2*d*x + 2*c))*B*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 1
80*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 240*(2*
d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 960*A*a^4*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.90

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{Ba^4\sin(6\,dx+6\,c)}{192\,d} + \frac{7}{16}\,(8\,Aa^4+7\,Ba^4)x + \frac{(Aa^4+4\,Ba^4)\sin(5\,dx+5\,c)}{80\,d} + \frac{(8\,Aa^4+15\,Ba^4)\sin(4\,dx+4\,c)}{64\,d} + \frac{(29\,Aa^4+36\,Ba^4)\sin(3\,dx+3\,c)}{48\,d} + \frac{(128\,Aa^4+127\,Ba^4)\sin(2\,dx+2\,c)}{64\,d} + \frac{(49\,Aa^4+44\,Ba^4)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/192*B*a^4*sin(6*d*x + 6*c)/d + 7/16*(8*A*a^4 + 7*B*a^4)*x + 1/80*(A*a^4
+ 4*B*a^4)*sin(5*d*x + 5*c)/d + 1/64*(8*A*a^4 + 15*B*a^4)*sin(4*d*x + 4*c)
/d + 1/48*(29*A*a^4 + 36*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(128*A*a^4 + 127
*B*a^4)*sin(2*d*x + 2*c)/d + 1/8*(49*A*a^4 + 44*B*a^4)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 43.65 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.71

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(7\,A\,a^4+\frac{49\,B\,a^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{119\,A\,a^4}{3}+\frac{833\,B\,a^4}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{462\,A\,a^4}{5}+\frac{1617\,B\,a^4}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$
$$-\frac{7\,a^4\,(8\,A+7\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}$$
$$+\frac{7\,a^4\,\mathrm{atan}\left(\frac{7\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(8\,A+7\,B)}{8\left(7\,A\,a^4+\frac{49\,B\,a^4}{8}\right)}\right)\,(8\,A+7\,B)}{8\,d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*(25*A*a^4 + (207*B*a^4)/8) + tan(c/2 + (d*x)/2)^11*(7*
A*a^4 + (49*B*a^4)/8) + tan(c/2 + (d*x)/2)^9*((119*A*a^4)/3 + (833*B*a^4)/
24) + tan(c/2 + (d*x)/2)^3*((233*A*a^4)/3 + (1471*B*a^4)/24) + tan(c/2 + (
d*x)/2)^7*((462*A*a^4)/5 + (1617*B*a^4)/20) + tan(c/2 + (d*x)/2)^5*((562*A
*a^4)/5 + (1967*B*a^4)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x
)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (
d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (7*a^4*(8*A + 7*B)*(atan(tan(c/
2 + (d*x)/2)) - (d*x)/2))/(8*d) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(8
*A + 7*B))/(8*(7*A*a^4 + (49*B*a^4)/8)))*(8*A + 7*B))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.88

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{a^4\left(40\cos(dx+c)\sin(dx+c)^5 b-240\cos(dx+c)\sin(dx+c)^3 a-490\cos(dx+c)\sin(dx+c)^3 b+10\right.}{}$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
(a**4*(40*cos(c + d*x)*sin(c + d*x)**5*b - 240*cos(c + d*x)*sin(c + d*x)**
3*a - 490*cos(c + d*x)*sin(c + d*x)**3*b + 1080*cos(c + d*x)*sin(c + d*x)*
a + 1185*cos(c + d*x)*sin(c + d*x)*b + 48*sin(c + d*x)**5*a + 192*sin(c +
d*x)**5*b - 640*sin(c + d*x)**3*a - 960*sin(c + d*x)**3*b + 1920*sin(c + d
*x)*a + 1920*sin(c + d*x)*b + 840*a*d*x + 735*b*d*x))/(240*d)
```

3.30 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\begin{aligned}\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx &= \frac{7}{8}a^4(5A+4B)x+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}\\ &+\frac{27a^4(5A+4B)\cos(c+dx)\sin(c+dx)}{40d}+\frac{a^4(5A+4B)\cos^3(c+dx)\sin(c+dx)}{20d}\\ &+\frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}-\frac{4a^4(5A+4B)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
7/8*a^4*(5*A+4*B)*x+8/5*a^4*(5*A+4*B)*sin(d*x+c)/d+27/40*a^4*(5*A+4*B)*cos
(d*x+c)*sin(d*x+c)/d+1/20*a^4*(5*A+4*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*B*(a
+a*cos(d*x+c))^4*sin(d*x+c)/d-4/15*a^4*(5*A+4*B)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\, dx$$
$$= \frac{a^4 \sin(c + dx) \left(210(5A + 4B) \arcsin\left(\sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}\right) + (800A + 664B + 15(27A + 28B) \cos(c + d\right.}{120d\sqrt{\sin^2(}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(a^4*Sin[c + d*x]*(210*(5*A + 4*B)*ArcSin[Sqrt[Sin[(c + d*x)/2]^2]] + (800
*A + 664*B + 15*(27*A + 28*B)*Cos[c + d*x] + 16*(10*A + 17*B)*Cos[c + d*x]
^2 + 30*(A + 4*B)*Cos[c + d*x]^3 + 24*B*Cos[c + d*x]^4)*Sqrt[Sin[c + d*x]^
2]))/(120*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c + dx) + a)^4 (A + B\cos(c + dx))\, dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\sin\left(c + dx + \frac{\pi}{2}\right) + a\right)^4 \left(A + B\sin\left(c + dx + \frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \; 3230$$

$$\frac{1}{5}(5A + 4B)\int (\cos(c + dx)a + a)^4 dx + \frac{B\sin(c + dx)(a\cos(c + dx) + a)^4}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}(5A+4B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4dx+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

$\big\downarrow$ 3124

$$\frac{1}{5}(5A+4B)\int \left(\cos^4(c+dx)a^4+4\cos^3(c+dx)a^4+6\cos^2(c+dx)a^4+4\cos(c+dx)a^4+a^4\right)dx+\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

$\big\downarrow$ 2009

$$\frac{1}{5}(5A+4B)\left(-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4}{8}\right.$$
$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + ((5*A + 4*B)*((35*a^4*x)/8
 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^
4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3*d)))/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.65

$$\frac{a^4 A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\,a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+\frac{4a^4A\left(\cos(dx+c)^2+2\right)\sin}{3}}{}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(a^4*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/
5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/3*a^4*A*(cos(d*x+
c)^2+2)*sin(d*x+c)+4*B*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/8*d*x+3/8*c)+6*a^4*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*B*a^4*(c
os(d*x+c)^2+2)*sin(d*x+c)+4*a^4*A*sin(d*x+c)+4*B*a^4*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+a^4*A*(d*x+c)+B*a^4*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$=\frac{105\,(5\,A+4\,B)a^4dx+\left(24\,Ba^4\cos\left(dx+c\right)^4+30\,(A+4\,B)a^4\cos\left(dx+c\right)^3+16\,(10\,A+17\,B)a^4\cos\left(d\right.\right.}{120\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/120*(105*(5*A + 4*B)*a^4*d*x + (24*B*a^4*cos(d*x + c)^4 + 30*(A + 4*B)*a
^4*cos(d*x + c)^3 + 16*(10*A + 17*B)*a^4*cos(d*x + c)^2 + 15*(27*A + 28*B)
*a^4*cos(d*x + c) + 8*(100*A + 83*B)*a^4)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(141) = 282$.

Time = 0.32 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.63

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\,dx$$
$$= \begin{cases} \frac{3Aa^4x\sin^4(c+dx)}{8} + \frac{3Aa^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + 3Aa^4x\sin^2(c+dx) + \frac{3Aa^4x\cos^4(c+dx)}{8} + 3Aa^4x\cos^2(c+dx) \\ x(A + B\cos(c))(a\cos(c) + a)^4 \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((3*A*a**4*x*sin(c + d*x)**4/8 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/8 +
 3*A*a**4*x*cos(c + d*x)**2 + A*a**4*x + 3*A*a**4*sin(c + d*x)**3*cos(c +
d*x)/(8*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) + 5*A*a**4*sin(c + d*x)*cos(c
+ d*x)**3/(8*d) + 4*A*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**4*sin(c
 + d*x)*cos(c + d*x)/d + 4*A*a**4*sin(c + d*x)/d + 3*B*a**4*x*sin(c + d*x)
**4/2 + 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 2*B*a**4*x*sin(c + d*
x)**2 + 3*B*a**4*x*cos(c + d*x)**4/2 + 2*B*a**4*x*cos(c + d*x)**2 + 8*B*a*
*4*sin(c + d*x)**5/(15*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d)
 + 3*B*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*B*a**4*sin(c + d*x)**3/
d + B*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c +
d*x)**3/(2*d) + 6*B*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*B*a**4*sin(c +
 d*x)*cos(c + d*x)/d + B*a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))
*(a*cos(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.57

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx =$$
$$-\frac{640\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^4-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^4 \cdots}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/480*(640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 15*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 720*(2*d*x + 2*c + sin(2*d*
x + 2*c))*A*a^4 - 480*(d*x + c)*A*a^4 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x
+ c)^3 + 15*sin(d*x + c))*B*a^4 + 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*
a^4 - 60*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 4
80*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 1920*A*a^4*sin(d*x + c) - 480*
B*a^4*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\begin{aligned}\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx &= \frac{Ba^4\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{7}{8}\left(5\,Aa^4+4\,Ba^4\right)x \\ &+\frac{\left(Aa^4+4\,Ba^4\right)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{\left(16\,Aa^4+29\,Ba^4\right)\sin\left(3\,dx+3\,c\right)}{48\,d} \\ &+\frac{\left(7\,Aa^4+8\,Ba^4\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{7\left(8\,Aa^4+7\,Ba^4\right)\sin\left(dx+c\right)}{8\,d}\end{aligned}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/80*B*a^4*sin(5*d*x + 5*c)/d + 7/8*(5*A*a^4 + 4*B*a^4)*x + 1/32*(A*a^4 +
4*B*a^4)*sin(4*d*x + 4*c)/d + 1/48*(16*A*a^4 + 29*B*a^4)*sin(3*d*x + 3*c)/
d + 1/4*(7*A*a^4 + 8*B*a^4)*sin(2*d*x + 2*c)/d + 7/8*(8*A*a^4 + 7*B*a^4)*s
in(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.85

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\, dx$$
$$= \frac{\left(\frac{35\,A\,a^4}{4} + 7\,B\,a^4\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^9 + \left(\frac{245\,A\,a^4}{6} + \frac{98\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^7 + \left(\frac{224\,A\,a^4}{3} + \frac{896\,B\,a^4}{15}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} + 5\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^8 + 10\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 + 10\tan\right.}$$
$$- \frac{7\,a^4\,(5\,A + 4\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right) - \frac{d\,x}{2}\right)}{4\,d}$$
$$+ \frac{7\,a^4\,\mathrm{atan}\left(\frac{7\,a^4\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,(5\,A + 4\,B)}{4\,\left(\frac{35\,A\,a^4}{4} + 7\,B\,a^4\right)}\right)\,(5\,A + 4\,B)}{4\,d}$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*((93*A*a^4)/4 + 25*B*a^4) + tan(c/2 + (d*x)/2)^9*((35*
A*a^4)/4 + 7*B*a^4) + tan(c/2 + (d*x)/2)^7*((245*A*a^4)/6 + (98*B*a^4)/3)
+ tan(c/2 + (d*x)/2)^3*((395*A*a^4)/6 + (158*B*a^4)/3) + tan(c/2 + (d*x)/2
)^5*((224*A*a^4)/3 + (896*B*a^4)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(
c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(
c/2 + (d*x)/2)^10 + 1)) - (7*a^4*(5*A + 4*B)*(atan(tan(c/2 + (d*x)/2)) - (
d*x)/2))/(4*d) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(5*A + 4*B))/(4*((3
5*A*a^4)/4 + 7*B*a^4)))*(5*A + 4*B))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\, dx$$
$$= \frac{a^4\left(-30\cos(dx + c)\sin(dx + c)^3 a - 120\cos(dx + c)\sin(dx + c)^3 b + 435\cos(dx + c)\sin(dx + c)\,a + 5\right.}{}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output

```
(a**4*( - 30*cos(c + d*x)*sin(c + d*x)**3*a - 120*cos(c + d*x)*sin(c + d*x
)**3*b + 435*cos(c + d*x)*sin(c + d*x)*a + 540*cos(c + d*x)*sin(c + d*x)*b
 + 24*sin(c + d*x)**5*b - 160*sin(c + d*x)**3*a - 320*sin(c + d*x)**3*b +
960*sin(c + d*x)*a + 960*sin(c + d*x)*b + 525*a*d*x + 420*b*d*x))/(120*d)
```

3.31 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 151

$$\begin{aligned}\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{1}{8}a^4(48A+35B)x+\frac{a^4A\operatorname{arctanh}(\sin(c+dx))}{d}\\ &\quad+\frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}\\ &\quad+\frac{(4A+7B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{12d}\\ &\quad+\frac{(32A+35B)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{24d}\end{aligned}$$

output
```
1/8*a^4*(48*A+35*B)*x+a^4*A*arctanh(sin(d*x+c))/d+5/8*a^4*(8*A+7*B)*sin(d*
x+c)/d+1/4*a*B*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/12*(4*A+7*B)*(a^2+a^2*cos
(d*x+c))^2*sin(d*x+c)/d+1/24*(32*A+35*B)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{a^4\left(576Adx+420Bdx-96A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+96A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(a^4*(576*A*d*x + 420*B*d*x - 96*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] + 96*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(27*A + 28*B)*Sin[c
 + d*x] + 24*(4*A + 7*B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 32*B*Si
n[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {3042, 3455, 3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{4}\int (\cos(c+dx)a+a)^3(4aA+a(4A+7B)\cos(c+dx))\sec(c+dx)dx+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3\left(4aA+a(4A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\Big\downarrow$ 3455

$$\frac{1}{4}\left(\frac{1}{3}\int(\cos(c+dx)a+a)^2\left(12Aa^2+(32A+35B)\cos(c+dx)a^2\right)\sec(c+dx)dx+\frac{(4A+7B)\sin(c+dx)\left(a^2\text{co}\right.}{3d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(12Aa^2+(32A+35B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{(4A+7B)\sin(c+dx)\left(a^2\text{c}\right.}{3d}\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\Big\downarrow$ 3455

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int 3(\cos(c+dx)a+a)\left(8Aa^3+5(8A+7B)\cos(c+dx)a^3\right)\sec(c+dx)dx+\frac{(32A+35B)\sin(c+dx)\left(\right.}{2d}\right.\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int(\cos(c+dx)a+a)\left(8Aa^3+5(8A+7B)\cos(c+dx)a^3\right)\sec(c+dx)dx+\frac{(32A+35B)\sin(c+dx)\left(a^4\right.}{2d}\right.\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(8Aa^3+5(8A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{(32A+35B)\sin(c+dx)\left(a^4\right.}{2d}\right.\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\Big\downarrow$ 3447

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \left(5(8A+7B)\cos^2(c+dx)a^4+8Aa^4+\left(8Aa^4+5(8A+7B)a^4\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{(32A-\right.\right.$$

$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{5(8A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+8Aa^4+\left(8Aa^4+5(8A+7B)a^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{(32A+35\right.\right.$$

$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\int \left(8Aa^4+(48A+35B)\cos(c+dx)a^4\right)\sec(c+dx)dx+\frac{5a^4(8A+7B)\sin(c+dx)}{d}\right)+\frac{(32A+35B)\right.\right.$$

$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{8Aa^4+(48A+35B)\sin\left(c+dx+\frac{\pi}{2}\right)a^4}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5a^4(8A+7B)\sin(c+dx)}{d}\right)+\frac{(32A+35B)\sin(c+\right.\right.$$

$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(8a^4A\int \sec(c+dx)dx+\frac{5a^4(8A+7B)\sin(c+dx)}{d}+a^4x(48A+35B)\right)+\frac{(32A+35B)\sin(c+dx)(}{2d}\right.\right.$$

$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(8a^4A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^4(8A+7B)\sin(c+dx)}{d}+a^4x(48A+35B)\right)+\frac{(32A+35B)\sin(c\right.\right.$$

$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\frac{8a^4A\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^4(8A+7B)\sin(c+dx)}{d}+a^4x(48A+35B)\right)+\frac{(32A+35B)\sin(c+\right.\right.$$
$$\frac{aB\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(a*B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (((4*A + 7*B)*(a^2 + a^2
*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (((32*A + 35*B)*(a^4 + a^4*Cos[c +
d*x])*Sin[c + d*x])/(2*d) + (3*(a^4*(48*A + 35*B)*x + (8*a^4*A*ArcTanh[Sin
[c + d*x]])/d + (5*a^4*(8*A + 7*B)*Sin[c + d*x])/d))/2)/3)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 25.79 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

method	result
parallelrisch	$-\frac{\left(A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(-A-\frac{7B}{4}\right)\sin(2dx+2c)+\left(-\frac{A}{12}-\frac{B}{3}\right)\sin(3dx+3c)-\frac{\sin(4dx+4c)B}{32}\right.}{d}$
parts	$\frac{a^4A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(a^4A+4Ba^4)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}+\frac{(4a^4A+Ba^4)(dx+c)}{d}+\frac{(4a^4A+6}{}$
derivativedivides	$\frac{\frac{a^4A\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Ba^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.}{}$
default	$\frac{\frac{a^4A\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Ba^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.}{}$
risch	$6a^4xA+\frac{35a^4Bx}{8}-\frac{27ie^{i(dx+c)}a^4A}{8d}-\frac{7ie^{i(dx+c)}Ba^4}{2d}+\frac{27ie^{-i(dx+c)}a^4A}{8d}+\frac{7ie^{-i(dx+c)}Ba^4}{2d}+\frac{a^4A\ln(e}{}$
norman	$\frac{(6a^4A+\frac{35}{8}Ba^4)x+(6a^4A+\frac{35}{8}Ba^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(30a^4A+\frac{175}{8}Ba^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(30a^4A+\frac{175}{8}Ba^4)x\tan\left(\right.}{}$

input

```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE
)
```

output

```
-(A*ln(tan(1/2*d*x+1/2*c)-1)-A*ln(tan(1/2*d*x+1/2*c)+1)+(-A-7/4*B)*sin(2*d
*x+2*c)+(-1/12*A-1/3*B)*sin(3*d*x+3*c)-1/32*sin(4*d*x+4*c)*B+(-27/4*A-7*B)
*sin(d*x+c)-6*(A+35/48*B)*x*d)*a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.78

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{3\,(48\,A+35\,B)a^4dx+12\,Aa^4\log\left(\sin\left(dx+c\right)+1\right)-12\,Aa^4\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Ba^4\cos\left(dx+\right.\right.}{24\,d}$$

input

```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fri
cas")
```

output
```
1/24*(3*(48*A + 35*B)*a^4*d*x + 12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*
log(-sin(d*x + c) + 1) + (6*B*a^4*cos(d*x + c)^3 + 8*(A + 4*B)*a^4*cos(d*x
 + c)^2 + 3*(16*A + 27*B)*a^4*cos(d*x + c) + 160*(A + B)*a^4)*sin(d*x + c)
)/d
```

Sympy [F]

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$
$$= a^4 \left(\int A\sec(c + dx)\, dx + \int 4A\cos(c + dx)\sec(c + dx)\, dx + \int 6A\cos^2(c + dx)\sec(c + dx)\, dx + \int 4A\cos^3(c + dx)\sec(c + dx)\, dx + \int A\cos^4(c + dx)\sec(c + dx)\, dx + \int B\cos(c + dx)\sec(c + dx)\, dx + \int 4B\cos^2(c + dx)\sec(c + dx)\, dx + \int 6B\cos^3(c + dx)\sec(c + dx)\, dx + \int 4B\cos^4(c + dx)\sec(c + dx)\, dx + \int B\cos^5(c + dx)\sec(c + dx)\, dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*cos(c + d*x)*sec(c + d*x)
, x) + Integral(6*A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(4*A*cos(c
+ d*x)**3*sec(c + d*x), x) + Integral(A*cos(c + d*x)**4*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(4*B*cos(c + d*x)**2*s
ec(c + d*x), x) + Integral(6*B*cos(c + d*x)**3*sec(c + d*x), x) + Integral
(4*B*cos(c + d*x)**4*sec(c + d*x), x) + Integral(B*cos(c + d*x)**5*sec(c +
 d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.31

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec(c + dx)\, dx =$$

$$-\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^4-96\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^4-384\left(dx+c\right)Aa^4+12$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="max
ima")
```

output
```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 96*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^4 - 384*(d*x + c)*A*a^4 + 128*(sin(d*x + c)^3 - 3*sin(d*
x + c))*B*a^4 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*
B*a^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 96*(d*x + c)*B*a^4 -
96*A*a^4*log(sec(d*x + c) + tan(d*x + c)) - 576*A*a^4*sin(d*x + c) - 384*B
*a^4*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.42

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$

$$= \frac{24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(48\,Aa^4+35\,Ba^4\right)\left(dx+c\right)$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gia
c")
```

output
```
1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) + 3*(48*A*a^4 + 35*B*a^4)*(d*x + c) + 2*(120*A*a^4*t
an(1/2*d*x + 1/2*c)^7 + 105*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 424*A*a^4*tan(1
/2*d*x + 1/2*c)^5 + 385*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 520*A*a^4*tan(1/2*d
*x + 1/2*c)^3 + 511*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 216*A*a^4*tan(1/2*d*x +
 1/2*c) + 279*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/
d
```

Mupad [B] (verification not implemented)

Time = 41.86 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$
$$= \frac{144\, A\, a^4\, \mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) + 24\, A\, a^4\, \mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) + 105\, B\, a^4\, \mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) + 12\, A\, a^4\, \sin\left(2\, c\right.}{}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x),x)
```

output
```
(144*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 24*A*a^4*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 105*B*a^4*atan(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2)) + 12*A*a^4*sin(2*c + 2*d*x) + A*a^4*sin(3*c + 3*d*x) +
21*B*a^4*sin(2*c + 2*d*x) + 4*B*a^4*sin(3*c + 3*d*x) + (3*B*a^4*sin(4*c +
4*d*x))/8 + 81*A*a^4*sin(c + d*x) + 84*B*a^4*sin(c + d*x))/(12*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$
$$= \frac{a^4\left(-6\cos(dx+c)\sin(dx+c)^3 b + 48\cos(dx+c)\sin(dx+c)\, a + 87\cos(dx+c)\sin(dx+c)\, b - 24\log\right.}{}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
(a**4*( - 6*cos(c + d*x)*sin(c + d*x)**3*b + 48*cos(c + d*x)*sin(c + d*x)*
a + 87*cos(c + d*x)*sin(c + d*x)*b - 24*log(tan((c + d*x)/2) - 1)*a + 24*l
og(tan((c + d*x)/2) + 1)*a - 8*sin(c + d*x)**3*a - 32*sin(c + d*x)**3*b +
168*sin(c + d*x)*a + 192*sin(c + d*x)*b + 144*a*d*x + 105*b*d*x))/(24*d)
```

3.32 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 150

$$\begin{aligned}&\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx\\&=\frac{1}{2}a^4(13A+12B)x+\frac{a^4(4A+B)\text{arctanh}(\sin(c+dx))}{d}\\&\quad+\frac{5a^4(A+2B)\sin(c+dx)}{2d}-\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d}\\&\quad-\frac{(3A-8B)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{6d}+\frac{aA(a+a\cos(c+dx))^3\tan(c+dx)}{d}\end{aligned}$$

output
```
1/2*a^4*(13*A+12*B)*x+a^4*(4*A+B)*arctanh(sin(d*x+c))/d+5/2*a^4*(A+2*B)*si
n(d*x+c)/d-1/3*(3*A-B)*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d-1/6*(3*A-8*B)*(
a^4+a^4*cos(d*x+c))*sin(d*x+c)/d+a*A*(a+a*cos(d*x+c))^3*tan(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 312 vs. $2(150) = 300$.

Time = 6.46 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.08

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\,dx$$
$$= \frac{1}{192} a^4 (1 + \cos(c + dx))^4 \sec^8\left(\frac{1}{2}(c + dx)\right) \left(78Ax + 72Bx - \frac{12(4A + B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{12(4A + B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{3(16A + 27B)\cos(dx)\sin(c)}{d} + \frac{3(A + 4B)\cos(2dx)\sin(2c)}{d} + \frac{B\cos(3dx)\sin(3c)}{d} + \frac{3(16A + 27B)\cos(c)\sin(dx)}{d} + \frac{3(A + 4B)\cos(2c)\sin(2dx)}{d} + \frac{B\cos(3c)\sin(3dx)}{d} + \frac{12A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} + \frac{12A\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}\right)$$

input

```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output

```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(78*A*x + 72*B*x - (12*(4*A +
 B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (12*(4*A + B)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]])/d + (3*(16*A + 27*B)*Cos[d*x]*Sin[c])/d +
(3*(A + 4*B)*Cos[2*d*x]*Sin[2*c])/d + (B*Cos[3*d*x]*Sin[3*c])/d + (3*(16*A
 + 27*B)*Cos[c]*Sin[d*x])/d + (3*(A + 4*B)*Cos[2*c]*Sin[2*d*x])/d + (B*Cos
[3*c]*Sin[3*d*x])/d + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])) + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/192
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {3042, 3454, 3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3454}$$

$$\int (\cos(c+dx)a+a)^3(a(4A+B)-a(3A-B)\cos(c+dx))\sec(c+dx)dx+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3\left(a(4A+B)-a(3A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\int (\cos(c+dx)a+a)^2\left(3a^2(4A+B)-a^2(3A-8B)\cos(c+dx)\right)\sec(c+dx)dx-\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(3a^2(4A+B)-a^2(3A-8B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\left(\frac{1}{2}\int 3(\cos(c+dx)a+a)\left(2(4A+B)a^3+5(A+2B)\cos(c+dx)a^3\right)\sec(c+dx)dx-\frac{(3A-8B)\sin(c+dx)(}{2d}\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3}{2}\int(\cos(c+dx)a+a)\left(2(4A+B)a^3+5(A+2B)\cos(c+dx)a^3\right)\sec(c+dx)dx-\frac{(3A-8B)\sin(c+dx)(a}{2d}\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(2(4A+B)a^3+5(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{(3A-8B)\sin(c+dx)(a}{2d}\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{3}\left(\frac{3}{2}\int\left(5(A+2B)\cos^2(c+dx)a^4+2(4A+B)a^4+\left(2(4A+B)a^4+5(A+2B)a^4\right)\cos(c+dx)\right)\sec(c+dx)dx\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\int\frac{5(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+2(4A+B)a^4+\left(2(4A+B)a^4+5(A+2B)a^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\int\left(2(4A+B)a^4+(13A+12B)\cos(c+dx)a^4\right)\sec(c+dx)dx+\frac{5a^4(A+2B)\sin(c+dx)}{d}\right)-\frac{(3A-8B}{}\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{2(4A+B)a^4+(13A+12B)\sin\left(c+dx+\frac{\pi}{2}\right)a^4}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5a^4(A+2B)\sin(c+dx)}{d}\right)-\frac{(3A-8B)\sin(c+\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$\Big\downarrow$ 3214

$$\frac{1}{3}\left(\frac{3}{2}\left(2a^4(4A+B)\int \sec(c+dx)dx+\frac{5a^4(A+2B)\sin(c+dx)}{d}+a^4x(13A+12B)\right)-\frac{(3A-8B)\sin(c+dx)}{2a}\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\left(2a^4(4A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^4(A+2B)\sin(c+dx)}{d}+a^4x(13A+12B)\right)-\frac{(3A-8B)\sin(c}{}\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{2a^4(4A+B)\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^4(A+2B)\sin(c+dx)}{d}+a^4x(13A+12B)\right)-\frac{(3A-8B)\sin(c+}{}\right.$$
$$\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^3}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
-1/3*((3*A - B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/d + (-1/2*((3*A -
 8*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/d + (3*(a^4*(13*A + 12*B)*x +
 (2*a^4*(4*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(A + 2*B)*Sin[c + d*x]
)/d))/2)/3 + (a*A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 28.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

method	result
parallelrisch	$\frac{a^4\left(-32\cos(dx+c)\left(A+\frac{B}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+32\cos(dx+c)\left(A+\frac{B}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(16A+\frac{82B}{3}\right)\sin(2dx+\right.}{8d\cos(dx+c)}$
parts	$\frac{a^4A\tan(dx+c)}{d}+\frac{(a^4A+4B\,a^4)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{(4a^4A+B\,a^4)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(4a}{}$
derivativedivides	$\frac{a^4A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{B\,a^4\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+4a^4A\sin(dx+c)+4B\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{}$
default	$\frac{a^4A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{B\,a^4\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+4a^4A\sin(dx+c)+4B\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{}$
risch	$\frac{13a^4xA}{2}+6a^4Bx+\frac{27ie^{-i(dx+c)}B\,a^4}{8d}+\frac{2ie^{-i(dx+c)}a^4A}{d}+\frac{2ia^4A}{d(e^{2i(dx+c)}+1)}-\frac{27ie^{i(dx+c)}B\,a^4}{8d}-\frac{2ie^{i(dx}}{}$
norman	$\frac{(-\frac{13}{2}a^4A-6B\,a^4)x+(-\frac{65}{2}a^4A-30B\,a^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(\frac{13}{2}a^4A+6B\,a^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(\frac{65}{2}a^4A+30B\,a^4)x\tan}{}$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBO
SE)
```

output
```
1/8*a^4*(-32*cos(d*x+c)*(A+1/4*B)*ln(tan(1/2*d*x+1/2*c)-1)+32*cos(d*x+c)*(
A+1/4*B)*ln(tan(1/2*d*x+1/2*c)+1)+(16*A+82/3*B)*sin(2*d*x+2*c)+(A+4*B)*sin
(3*d*x+3*c)+1/3*sin(4*d*x+4*c)*B+52*x*(A+12/13*B)*d*cos(d*x+c)+9*sin(d*x+c
)*(A+4/9*B))/d/cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{3\,(13\,A+12\,B)a^4dx\cos\left(dx+c\right)+3\,(4\,A+B)a^4\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,A+B)a^4\cos}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="f
ricas")
```

output
```
1/6*(3*(13*A + 12*B)*a^4*d*x*cos(d*x + c) + 3*(4*A + B)*a^4*cos(d*x + c)*l
og(sin(d*x + c) + 1) - 3*(4*A + B)*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1)
 + (2*B*a^4*cos(d*x + c)^3 + 3*(A + 4*B)*a^4*cos(d*x + c)^2 + 8*(3*A + 5*B
)*a^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= a^4 \bigg(\int A\sec^2(c + dx)\, dx + \int 4A\cos(c + dx)\sec^2(c + dx)\, dx + \int 6A\cos^2(c + dx)\sec^2(c + dx)\, dx + \int 4A\cos^3(c + dx)\sec^2(c + dx)\, dx + \int A\cos^4(c + dx)\sec^2(c + dx)\, dx + \int B\cos(c + dx)\sec^2(c + dx)\, dx + \int 4B\cos^2(c + dx)\sec^2(c + dx)\, dx + \int 6B\cos^3(c + dx)\sec^2(c + dx)\, dx + \int 4B\cos^4(c + dx)\sec^2(c + dx)\, dx + \int B\cos^5(c + dx)\sec^2(c + dx)\, dx \bigg)$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
a**4*(Integral(A*sec(c + d*x)**2, x) + Integral(4*A*cos(c + d*x)*sec(c + d
*x)**2, x) + Integral(6*A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(4
*A*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(A*cos(c + d*x)**4*sec(c
+ d*x)**2, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(4*B
*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(6*B*cos(c + d*x)**3*sec(c
+ d*x)**2, x) + Integral(4*B*cos(c + d*x)**4*sec(c + d*x)**2, x) + Integra
l(B*cos(c + d*x)**5*sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.25

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^4 + 72\,(dx + c)Aa^4 - 4\,\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Ba^4 + 12\,(2\,dx}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="m
axima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 72*(d*x + c)*A*a^4 - 4*(s
in(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 12*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*B*a^4 + 48*(d*x + c)*B*a^4 + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 6*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
48*A*a^4*sin(d*x + c) + 72*B*a^4*sin(d*x + c) + 12*A*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.51

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx =$$
$$-\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\,(13\,Aa^4 + 12\,Ba^4)(dx + c) - 6\,(4\,Aa^4 + Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 6\,(4}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="g
iac")
```

output
```
-1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(13*A
*a^4 + 12*B*a^4)*(d*x + c) - 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) + 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21
*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4
*tan(1/2*d*x + 1/2*c)^3 + 76*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1
/2*d*x + 1/2*c) + 54*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 42.17 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.61

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{4\,A\,a^4\,\sin\left(c + d\,x\right)}{d} + \frac{20\,B\,a^4\,\sin\left(c + d\,x\right)}{3\,d} + \frac{13\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{8\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{12\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{A\,a^4\,\sin\left(c + d\,x\right)}{d\,\cos\left(c + d\,x\right)} + \frac{B\,a^4\,{\cos\left(c + d\,x\right)}^2\,\sin\left(c + d\,x\right)}{3\,d} + \frac{A\,a^4\,\cos\left(c + d\,x\right)\,\sin\left(c + d\,x\right)}{2\,d} + \frac{2\,B\,a^4\,\cos\left(c + d\,x\right)\,\sin\left(c + d\,x\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

output
```
(4*A*a^4*sin(c + d*x))/d + (20*B*a^4*sin(c + d*x))/(3*d) + (13*A*a^4*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*A*a^4*atanh(sin(c/2 + (d*x)
/2)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/d + (2*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (
A*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*cos(c + d*x)^2*sin(c + d*x))
/(3*d) + (A*a^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (2*B*a^4*cos(c + d*x)*s
in(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.42

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx$$
$$= \frac{a^4 \left(3\cos\left(dx + c\right)^2 \sin\left(dx + c\right) a + 12\cos\left(dx + c\right)^2 \sin\left(dx + c\right) b - 24\cos\left(dx + c\right) \log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\right.}{}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
(a**4*(3*cos(c + d*x)**2*sin(c + d*x)*a + 12*cos(c + d*x)**2*sin(c + d*x)*
b - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a - 6*cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*b + 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 6*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*b - 2*cos(c + d*x)*sin(c + d*x)**3*b + 2
4*cos(c + d*x)*sin(c + d*x)*a + 42*cos(c + d*x)*sin(c + d*x)*b + 39*cos(c
+ d*x)*a*d*x + 36*cos(c + d*x)*b*d*x + 6*sin(c + d*x)*a))/(6*cos(c + d*x)*
d)
```

3.33 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 162

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{1}{2}a^4(8A+13B)x+\frac{a^4(13A+8B)\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{5a^4(A-B)\sin(c+dx)}{2d} - \frac{(6A+B)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{(5A+2B)\left(a^2+a^2\cos(c+dx)\right)^2\tan(c+dx)}{2d} + \frac{aA(a+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a^4*(8*A+13*B)*x+1/2*a^4*(13*A+8*B)*arctanh(sin(d*x+c))/d-5/2*a^4*(A-B
)*sin(d*x+c)/d-1/2*(6*A+B)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d+1/2*(5*A+2*B)
*(a^2+a^2*cos(d*x+c))^2*tan(d*x+c)/d+1/2*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)
*tan(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(162) = 324$.

Time = 10.67 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.12

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\sec^3(c + dx)\, dx$$
$$= \frac{1}{64} a^4 (1 + \cos(c + dx))^4 \sec^8\left(\frac{1}{2}(c + dx)\right)\left(2(8A + 13B)x - \frac{2(13A + 8B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{2(13A + 8B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{4(A + 4B)\cos(dx)\sin(c)}{d} + \frac{B\cos(2dx)\sin(2c)}{d} + \frac{4(A + 4B)\cos(c)\sin(dx)}{d} + \frac{B\cos(2c)\sin(2dx)}{d} + \frac{A}{d\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{4(4A + B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} - \frac{A}{d\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{4(4A + B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}\right)$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(2*(8*A + 13*B)*x - (2*(13*A
+ 8*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(13*A + 8*B)*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 4*B)*Cos[d*x]*Sin[c])/d +
 (B*Cos[2*d*x]*Sin[2*c])/d + (4*(A + 4*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]
*Sin[2*d*x])/d + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(4*A +
 B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])) - A/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(4*A + B)*Sin
[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
))/64
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 3454, 3042, 3454, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{2}\int (\cos(c+dx)a+a)^3(a(5A+2B)-2a(A-B)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3\left(a(5A+2B)-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{2}\left(\int (\cos(c+dx)a+a)^2\left(a^2(13A+8B)-2a^2(6A+B)\cos(c+dx)\right)\sec(c+dx)dx+\frac{(5A+2B)\tan(c+dx)\,(\ldots}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a^2(13A+8B)-2a^2(6A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{(5A+2B)\tan(c+dx)\,(\ldots}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{2}\left(\frac{1}{2}\int 2(\cos(c+dx)a+a)\left(a^3(13A+8B)-5a^3(A-B)\cos(c+dx)\right)\sec(c+dx)dx-\frac{(6A+B)\sin(c+dx)\left(c\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int (\cos(c+dx)a+a)\left(a^3(13A+8B)-5a^3(A-B)\cos(c+dx)\right)\sec(c+dx)dx-\frac{(6A+B)\sin(c+dx)\left(a^4\,\mathrm{co}\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a^3(13A+8B)-5a^3(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{(6A+B)\sin(c+dx)\left(a^4\,\mathrm{co}\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{2}\left(\int \left(-5(A-B)\cos^2(c+dx)a^4+(13A+8B)a^4+\left(a^4(13A+8B)-5a^4(A-B)\right)\cos(c+dx)\right)\sec(c+dx)dx\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{-5(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+(13A+8B)a^4+\left(a^4(13A+8B)-5a^4(A-B)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left((13A+8B)a^4+(8A+13B)\cos(c+dx)a^4\right)\sec(c+dx)dx-\frac{5a^4(A-B)\sin(c+dx)}{d}-\frac{(6A+B)\sin(c+\right.}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{(13A+8B)a^4+(8A+13B)\sin\left(c+dx+\frac{\pi}{2}\right)a^4}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{5a^4(A-B)\sin(c+dx)}{d}-\frac{(6A+B)\sin(c+dx)\left(a^4\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$\big\downarrow$ 3214

$$\frac{1}{2}\left(a^4(13A+8B)\int\sec(c+dx)dx-\frac{5a^4(A-B)\sin(c+dx)}{d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{d}+a\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(a^4(13A+8B)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{5a^4(A-B)\sin(c+dx)}{d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

$\big\downarrow$ 4257

$$\frac{1}{2}\left(\frac{a^4(13A+8B)\text{arctanh}(\sin(c+dx))}{d}-\frac{5a^4(A-B)\sin(c+dx)}{d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a^4*(8*A +
 13*B)*x + (a^4*(13*A + 8*B)*ArcTanh[Sin[c + d*x]])/d - (5*a^4*(A - B)*Sin
[c + d*x])/d - ((6*A + B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/d + ((5*A
 + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 27.70 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{\left(-13(\cos(2dx+2c)+1)\left(A+\frac{8B}{13}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+13(\cos(2dx+2c)+1)\left(A+\frac{8B}{13}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+8x\left(A+\frac{13}{}\right.}{2d(\cos}$
parts	$\frac{a^4A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{\left(a^4A+4B\,a^4\right)\sin(dx+c)}{d}+\frac{\left(4a^4A+B\,a^4\right)\tan(dx+c)}{d}+\frac{(4}{}$
derivativedivides	$\frac{a^4A\sin(dx+c)+B\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4A(dx+c)+4B\,a^4\sin(dx+c)+6a^4A\ln(\sec(dx+c)+\tan(dx+c)}{}$
default	$\frac{a^4A\sin(dx+c)+B\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4A(dx+c)+4B\,a^4\sin(dx+c)+6a^4A\ln(\sec(dx+c)+\tan(dx+c)}{}$
risch	$4a^4xA+\frac{13a^4Bx}{2}-\frac{i\mathrm{e}^{2i(dx+c)}B\,a^4}{8d}-\frac{i\mathrm{e}^{i(dx+c)}a^4A}{2d}-\frac{2i\mathrm{e}^{i(dx+c)}B\,a^4}{d}+\frac{i\mathrm{e}^{-i(dx+c)}a^4A}{2d}+\frac{2i\mathrm{e}^{-i(dx+c)}B\,a}{d}$
norman	$\frac{\left(4a^4A+\frac{13}{2}B\,a^4\right)x+\left(-20a^4A-\frac{65}{2}B\,a^4\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(-20a^4A-\frac{65}{2}B\,a^4\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(4a^4A+\frac{13}{2}B\,a^4\right)x\tan\left(\right.}{}$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBO
SE)
```

output
```
1/2*(-13*(cos(2*d*x+2*c)+1)*(A+8/13*B)*ln(tan(1/2*d*x+1/2*c)-1)+13*(cos(2*
d*x+2*c)+1)*(A+8/13*B)*ln(tan(1/2*d*x+1/2*c)+1)+8*x*(A+13/8*B)*d*cos(2*d*x
+2*c)+(8*A+5/2*B)*sin(2*d*x+2*c)+(A+4*B)*sin(3*d*x+3*c)+1/4*sin(4*d*x+4*c)
*B+(3*A+4*B)*sin(d*x+c)+8*x*(A+13/8*B)*d)*a^4/d/(cos(2*d*x+2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{2\,(8\,A+13\,B)a^4dx\cos{(dx+c)}^2+(13\,A+8\,B)a^4\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-(13\,A+8\,B)a^4 c$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="f
ricas")
```

output
```
1/4*(2*(8*A + 13*B)*a^4*d*x*cos(d*x + c)^2 + (13*A + 8*B)*a^4*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - (13*A + 8*B)*a^4*cos(d*x + c)^2*log(-sin(d*x +
c) + 1) + 2*(B*a^4*cos(d*x + c)^3 + 2*(A + 4*B)*a^4*cos(d*x + c)^2 + 2*(4*
A + B)*a^4*cos(d*x + c) + A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.23

$$\int (a + a\cos(c + dx))^4(A + B\cos(c + dx))\sec^3(c + dx)\,dx$$

$$= \frac{16\,(dx + c)Aa^4 + (2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right))Ba^4 + 24\,(dx + c)Ba^4 - Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx + \right.\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="m
axima")
```

output
```
1/4*(16*(d*x + c)*A*a^4 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 24*(d*x
 + c)*B*a^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) + 12*A*a^4*(log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1)) + 8*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + 4*A*a^4*sin(d*x + c) + 16*B*a^4*sin(d*x + c) + 16*A*a^4*tan(d*x + c) +
4*B*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.42

$$\int (a + a\cos(c + dx))^4(A + B\cos(c + dx))\sec^3(c + dx)\,dx$$

$$= \frac{(8\,Aa^4 + 13\,Ba^4)(dx + c) + (13\,Aa^4 + 8\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (13\,Aa^4 + 8\,Ba^4)\log\left(\left|\tan\left(\right.\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="g
iac")
```

output
```
1/2*((8*A*a^4 + 13*B*a^4)*(d*x + c) + (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(5*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 5*B*a^4*tan(1/2*d*x + 1/2*c)^7 +
 7*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 7*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^4
*tan(1/2*d*x + 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 11*A*a^4*tan(1/
2*d*x + 1/2*c) - 11*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 -
1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 42.74 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.50

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{4\,B\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{8\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^3,x)
```

output
```
(A*a^4*sin(c + d*x))/d + (4*B*a^4*sin(c + d*x))/d + (8*A*a^4*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d + (13*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
))/d + (8*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*A*a^4
*sin(c + d*x))/(d*cos(c + d*x)) + (A*a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2
) + (B*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*cos(c + d*x)*sin(c + d*
x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.64

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{a^4\left(\cos\left(dx+c\right)^2\sin\left(dx+c\right)^3b-\cos\left(dx+c\right)^2\sin\left(dx+c\right)b-13\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\right.}{}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
(a**4*(cos(c + d*x)**2*sin(c + d*x)**3*b - cos(c + d*x)**2*sin(c + d*x)*b
- 13*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 8*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + 13*cos(c + d*x)*log(tan
((c + d*x)/2) - 1)*a + 8*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 13*cos
(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 8*cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 13*cos(c + d*x)*log(tan((c + d*
x)/2) + 1)*a - 8*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b + 2*cos(c + d*x)
*sin(c + d*x)**3*a + 8*cos(c + d*x)*sin(c + d*x)**3*b + 8*cos(c + d*x)*sin
(c + d*x)**2*a*d*x + 13*cos(c + d*x)*sin(c + d*x)**2*b*d*x - 3*cos(c + d*x
)*sin(c + d*x)*a - 8*cos(c + d*x)*sin(c + d*x)*b - 8*cos(c + d*x)*a*d*x -
13*cos(c + d*x)*b*d*x + 8*sin(c + d*x)**3*a + 2*sin(c + d*x)**3*b - 8*sin(
c + d*x)*a - 2*sin(c + d*x)*b))/(2*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.34 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 165

$$\begin{aligned}\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= a^4(A+4B)x+\frac{a^4(12A+13B)\text{arctanh}(\sin(c+dx))}{2d}\\ &\quad-\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{(11A+9B)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{3d}\\ &\quad+\frac{(2A+B)\left(a^2+a^2\cos(c+dx)\right)^2\sec(c+dx)\tan(c+dx)}{2d}\\ &\quad+\frac{aA(a+a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d}\end{aligned}$$

output
```
a^4*(A+4*B)*x+1/2*a^4*(12*A+13*B)*arctanh(sin(d*x+c))/d-5/2*a^4*(2*A+B)*si
n(d*x+c)/d+1/3*(11*A+9*B)*(a^4+a^4*cos(d*x+c))*tan(d*x+c)/d+1/2*(2*A+B)*(a
^2+a^2*cos(d*x+c))^2*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*(a+a*cos(d*x+c))^3*se
c(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 380 vs. $2(165) = 330$.

Time = 10.63 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.30

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$
$$= a^4 \left(\frac{(A + 4B)(c + dx)}{d} + \frac{(-12A - 13B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + \frac{(12A + 13B)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + \frac{13A + 3B}{12d\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{A\sin\left(\frac{1}{2}(c + dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)^3} + \frac{A\sin\left(\frac{1}{2}(c + dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)^3} + \frac{-13A - 3B}{12d\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} + \frac{4\left(5A\sin\left(\frac{1}{2}(c + dx)\right) + 3B\sin\left(\frac{1}{2}(c + dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} + \frac{4\left(5A\sin\left(\frac{1}{2}(c + dx)\right) + 3B\sin\left(\frac{1}{2}(c + dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)} + \frac{B\sin(c + dx)}{d} \right)$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
a^4*(((A + 4*B)*(c + d*x))/d + ((-12*A - 13*B)*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]])/(2*d) + ((12*A + 13*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]])/(2*d) + (13*A + 3*B)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
+ (A*Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (A*
Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (-13*A -
 3*B)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*A*Sin[(c + d*
x)/2] + 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + (4*(5*A*Sin[(c + d*x)/2] + 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])) + (B*Sin[c + d*x])/d)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 3454, 3042, 3454, 3042, 3454, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{3}\int (\cos(c+dx)a+a)^3(3a(2A+B)-a(A-3B)\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3\left(3a(2A+B)-a(A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{3}\left(\frac{1}{2}\int (\cos(c+dx)a+a)^2\left(2a^2(11A+9B)-a^2(8A-3B)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{3(2A+B)\tan(c+d}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(2a^2(11A+9B)-a^2(8A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{3(2A+B)\tan(c+dx}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$\Big\downarrow$ 3454

$$\frac{1}{3}\left(\frac{1}{2}\left(\int 3(\cos(c+dx)a+a)\left(a^3(12A+13B)-5a^3(2A+B)\cos(c+dx)\right)\sec(c+dx)dx+\frac{2(11A+9B)\tan(c\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int(\cos(c+dx)a+a)\left(a^3(12A+13B)-5a^3(2A+B)\cos(c+dx)\right)\sec(c+dx)dx+\frac{2(11A+9B)\tan(c\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a^3(12A+13B)-5a^3(2A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2(11A+9B)\tan(c}{}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$\Big\downarrow$ 3447

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int\left(-5(2A+B)\cos^2(c+dx)a^4+(12A+13B)a^4+\left(a^4(12A+13B)-5a^4(2A+B)\right)\cos(c+dx)\right)\sec(\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int\frac{-5(2A+B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+(12A+13B)a^4+\left(a^4(12A+13B)-5a^4(2A+B)\right)\sin\left(c+dx+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$\Big\downarrow$ 3502

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(\int\left((12A+13B)a^4+2(A+4B)\cos(c+dx)a^4\right)\sec(c+dx)dx-\frac{5a^4(2A+B)\sin(c+dx)}{d}\right)+\frac{2(11A}{}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(\int \frac{(12A+13B)a^4+2(A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)a^4}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{5a^4(2A+B)\sin(c+dx)}{d}\right)+\frac{2(11A+9B)\text{ta}}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(a^4(12A+13B)\int \sec(c+dx)dx-\frac{5a^4(2A+B)\sin(c+dx)}{d}+2a^4x(A+4B)\right)+\frac{2(11A+9B)\tan(c}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(a^4(12A+13B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{5a^4(2A+B)\sin(c+dx)}{d}+2a^4x(A+4B)\right)+\frac{2(11A+9B)}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(\frac{a^4(12A+13B)\text{arctanh}(\sin(c+dx))}{d}-\frac{5a^4(2A+B)\sin(c+dx)}{d}+2a^4x(A+4B)\right)+\frac{2(11A+9B)\text{ta}}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((3*(2*A
+ B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (3*(2*a
^4*(A + 4*B)*x + (a^4*(12*A + 13*B)*ArcTanh[Sin[c + d*x]])/d - (5*a^4*(2*A
 + B)*Sin[c + d*x])/d) + (2*(11*A + 9*B)*(a^4 + a^4*Cos[c + d*x])*Tan[c +
d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 16.87 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00

method	result
parts	$-\frac{a^4 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{(a^4A+4B\,a^4)(dx+c)}{d}+\frac{(4a^4A+B\,a^4)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)\cdots}{2}\cdots\right)}{d}\cdots$
parallelrisch	$\frac{4\left(-\frac{9\left(A+\frac{13B}{12}\right)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{9\left(A+\frac{13B}{12}\right)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}\cdots\right)}{d(3\cos\cdots}$
derivativedivides	$\frac{a^4A(dx+c)+B\,a^4\sin(dx+c)+4a^4A\ln(\sec(dx+c)+\tan(dx+c))+4B\,a^4(dx+c)+6a^4A\tan(dx+c)+6B\,a^4\ln(\sec(dx+c)\cdots}{\cdots}$
default	$\frac{a^4A(dx+c)+B\,a^4\sin(dx+c)+4a^4A\ln(\sec(dx+c)+\tan(dx+c))+4B\,a^4(dx+c)+6a^4A\tan(dx+c)+6B\,a^4\ln(\sec(dx+c)\cdots}{\cdots}$
risch	$a^4xA+4a^4Bx-\frac{i\mathrm{e}^{i(dx+c)}B\,a^4}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}B\,a^4}{2d}-\frac{ia^4\left(12A\,\mathrm{e}^{5i(dx+c)}+3B\,\mathrm{e}^{5i(dx+c)}-36A\,\mathrm{e}^{4i(dx+c)}-24B\cdots\right)}{\cdots}$
norman	$\frac{(-a^4A-4B\,a^4)x+(-6a^4A-24B\,a^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(-2a^4A-8B\,a^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(-2a^4A-8B\,a^4)x\tan\left(\frac{dx}{2}\cdots\right.}{\cdots}$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBO
SE)
```

output
```
-a^4*A/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*a^4+4*B*a^4)/d*(d*x+c)+(4*A
*a^4+B*a^4)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+(4
*A*a^4+6*B*a^4)/d*ln(sec(d*x+c)+tan(d*x+c))+(6*A*a^4+4*B*a^4)/d*tan(d*x+c)
+a^4*B/d*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^4(c + dx) \, dx$$
$$= \frac{12\,(A + 4\,B)a^4 dx \cos{(dx + c)}^3 + 3\,(12\,A + 13\,B)a^4 \cos{(dx + c)}^3 \log{(\sin{(dx + c)} + 1)} - 3\,(12\,A + 13\,B}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="f
ricas")
```

output
```
1/12*(12*(A + 4*B)*a^4*d*x*cos(d*x + c)^3 + 3*(12*A + 13*B)*a^4*cos(d*x +
c)^3*log(sin(d*x + c) + 1) - 3*(12*A + 13*B)*a^4*cos(d*x + c)^3*log(-sin(d
*x + c) + 1) + 2*(6*B*a^4*cos(d*x + c)^3 + 8*(5*A + 3*B)*a^4*cos(d*x + c)^
2 + 3*(4*A + B)*a^4*cos(d*x + c) + 2*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^4(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.42

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$

$$= \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+12(dx+c)Aa^4+48(dx+c)Ba^4-12Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\mathrm{s}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="m
axima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 12*(d*x + c)*A*a^4 + 48*
(d*x + c)*B*a^4 - 12*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*B*a^4*(log(sin(d*x + c)
 + 1) - log(sin(d*x + c) - 1)) + 12*B*a^4*sin(d*x + c) + 72*A*a^4*tan(d*x
+ c) + 48*B*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.38

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$

$$= \frac{\frac{12\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(Aa^4+4\,Ba^4\right)(dx+c)+3\left(12\,Aa^4+13\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(12\,A\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="g
iac")
```

output
```
1/6*(12*B*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(A*a^4
 + 4*B*a^4)*(d*x + c) + 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*
(30*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 76*A*
a^4*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*ta
n(1/2*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 41.80 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.54

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$
$$= \frac{B a^4 \sin(c + d x)}{d} + \frac{2 A a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{12 A a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{8 B a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{13 B a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{20 A a^4 \sin(c + d x)}{3 d \cos(c + d x)} + \frac{2 A a^4 \sin(c + d x)}{d \cos(c + d x)^2} + \frac{A a^4 \sin(c + d x)}{3 d \cos(c + d x)^3} + \frac{4 B a^4 \sin(c + d x)}{d \cos(c + d x)} + \frac{B a^4 \sin(c + d x)}{2 d \cos(c + d x)^2}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^4,x)
```

output
```
(B*a^4*sin(c + d*x))/d + (2*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (12*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*B*
a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*B*a^4*atanh(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (20*A*a^4*sin(c + d*x))/(3*d*cos(c
+ d*x)) + (2*A*a^4*sin(c + d*x))/(d*cos(c + d*x)^2) + (A*a^4*sin(c + d*x))
/(3*d*cos(c + d*x)^3) + (4*B*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*s
in(c + d*x))/(2*d*cos(c + d*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.28

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^4(c + dx) \, dx$$
$$= \frac{a^4 \left(-36\cos(dx + c)\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\sin(dx + c)^2 a - 39\cos(dx + c)\log\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\sin(dx\right.}{}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
(a**4*( - 36*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 39
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b + 36*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*a + 39*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*
b + 36*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 39*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 36*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*a - 39*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b + 6*
cos(c + d*x)*sin(c + d*x)**3*b + 6*cos(c + d*x)*sin(c + d*x)**2*a*d*x + 24
*cos(c + d*x)*sin(c + d*x)**2*b*d*x - 12*cos(c + d*x)*sin(c + d*x)*a - 9*c
os(c + d*x)*sin(c + d*x)*b - 6*cos(c + d*x)*a*d*x - 24*cos(c + d*x)*b*d*x
+ 40*sin(c + d*x)**3*a + 24*sin(c + d*x)**3*b - 42*sin(c + d*x)*a - 24*sin
(c + d*x)*b))/(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.35 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 173

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = a^4Bx + \frac{a^4(35A+48B)\text{arctanh}(\sin(c+dx))}{8d} + \frac{5a^4(7A+8B)\tan(c+dx)}{8d} + \frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(7A+4B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^2(c+dx)\tan(c+dx)}{12d} + \frac{aA(a+a\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
a^4*B*x+1/8*a^4*(35*A+48*B)*arctanh(sin(d*x+c))/d+5/8*a^4*(7*A+8*B)*tan(d*
x+c)/d+1/24*(35*A+32*B)*(a^4+a^4*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d+1/12*
(7*A+4*B)*(a^2+a^2*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+a*co
s(d*x+c))^3*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 5.69 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^5(c + dx) \, dx$$
$$= \frac{a^4 (24Bdx + 24(A + 4B)\coth^{-1}(\sin(c + dx)) + (81A + 48B)\text{arctanh}(\sin(c + dx)) + 192A\tan(c + dx) +$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a^4*(24*B*d*x + 24*(A + 4*B)*ArcCoth[Sin[c + d*x]] + (81*A + 48*B)*ArcTan
h[Sin[c + d*x]] + 192*A*Tan[c + d*x] + 168*B*Tan[c + d*x] + 81*A*Sec[c + d
*x]*Tan[c + d*x] + 48*B*Sec[c + d*x]*Tan[c + d*x] + 6*A*Sec[c + d*x]^3*Tan
[c + d*x] + 32*A*Tan[c + d*x]^3 + 8*B*Tan[c + d*x]^3))/(24*d)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {3042, 3454, 3042, 3454, 3042, 3454, 27, 3042, 3447, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c + dx)(a\cos(c + dx) + a)^4 (A + B\cos(c + dx)) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c + dx + \frac{\pi}{2}\right) + a\right)^4 \left(A + B\sin\left(c + dx + \frac{\pi}{2}\right)\right)}{\sin\left(c + dx + \frac{\pi}{2}\right)^5} dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\int (\cos(c + dx)a + a)^3 (a(7A + 4B) + 4aB\cos(c + dx)) \sec^4(c + dx) dx + \frac{aA\tan(c + dx)\sec^3(c + dx)(a\cos(c + dx) + a)^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3\left(a(7A+4B)+4aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\left(\frac{1}{3}\int(\cos(c+dx)a+a)^2\left((35A+32B)a^2+12B\cos(c+dx)a^2\right)\sec^3(c+dx)dx+\frac{(7A+4B)\tan(c+dx)\sec^2}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left((35A+32B)a^2+12B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{(7A+4B)\tan(c+dx)\sec^2(}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int 3(\cos(c+dx)a+a)\left(5(7A+8B)a^3+8B\cos(c+dx)a^3\right)\sec^2(c+dx)dx+\frac{(35A+32B)\tan(c+dx)}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int(\cos(c+dx)a+a)\left(5(7A+8B)a^3+8B\cos(c+dx)a^3\right)\sec^2(c+dx)dx+\frac{(35A+32B)\tan(c+dx)\text{s}}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(5(7A+8B)a^3+8B\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{(35A+32B)\tan(c+dx)\text{se}}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \left(8B\cos^2(c+dx)a^4+5(7A+8B)a^4+\left(8Ba^4+5(7A+8B)a^4\right)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{(35A\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{8B\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+5(7A+8B)a^4+\left(8Ba^4+5(7A+8B)a^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{(35A+3\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\downarrow$ 3500

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\int \left((35A+48B)a^4+8B\cos(c+dx)a^4\right)\sec(c+dx)dx+\frac{5a^4(7A+8B)\tan(c+dx)}{d}\right)+\frac{(35A+32B)\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{(35A+48B)a^4+8B\sin\left(c+dx+\frac{\pi}{2}\right)a^4}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5a^4(7A+8B)\tan(c+dx)}{d}\right)+\frac{(35A+32B)\tan(c-\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\downarrow$ 3214

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(a^4(35A+48B)\int \sec(c+dx)dx+\frac{5a^4(7A+8B)\tan(c+dx)}{d}+8a^4Bx\right)+\frac{(35A+32B)\tan(c+dx)\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(a^4(35A+48B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^4(7A+8B)\tan(c+dx)}{d}+8a^4Bx\right)+\frac{(35A+32B)\tan(c\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$\downarrow$ 4257

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\frac{a^4(35A+48B)\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^4(7A+8B)\tan(c+dx)}{d}+8a^4Bx\right)+\frac{(35A+32B)\tan(c+\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (((7*A +
4*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((35
*A + 32*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (3*
(8*a^4*B*x + (a^4*(35*A + 48*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(7*A + 8
*B)*Tan[c + d*x])/d))/2)/3)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 17.74 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17

method	result
parts	$\frac{a^4 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{(a^4A+4B\,a^4)\ln(\sec(dx+c)+\tan(dx$
parallelrisch	$56a^4\left(-\frac{15\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{48B}{35}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{16}+\frac{15\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{48B}{35}\right)\ln(}{16}\right.$
derivativedivides	$a^4A\ln(\sec(dx+c)+\tan(dx+c))+B\,a^4(dx+c)+4a^4A\tan(dx+c)+4B\,a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4A\left(\frac{\sec(dx+c)\,\mathrm{ta}}{2}\right.$
default	$a^4A\ln(\sec(dx+c)+\tan(dx+c))+B\,a^4(dx+c)+4a^4A\tan(dx+c)+4B\,a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4A\left(\frac{\sec(dx+c)\,\mathrm{ta}}{2}\right.$
risch	$a^4Bx-\frac{ia^4(81A\,\mathrm{e}^{7i(dx+c)}+48B\,\mathrm{e}^{7i(dx+c)}-96A\,\mathrm{e}^{6i(dx+c)}-144B\,\mathrm{e}^{6i(dx+c)}+105A\,\mathrm{e}^{5i(dx+c)}+48B\,\mathrm{e}^{5i(dx+c)}-480A}{}$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBO
SE)
```

output
```
a^4*A/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+(A*a^4+4*B*a^4)/d*ln(sec(d*x+c)+tan(d*x+c))-(4*A*a^4+B*a^4)/d
*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(4*A*a^4+6*B*a^4)/d*tan(d*x+c)+(6*A*a^
4+4*B*a^4)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*a
^4/d*(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{48\,Ba^4dx\cos\left(dx+c\right)^4+3\left(35\,A+48\,B\right)a^4\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(35\,A+48\,B\right)a^4\cos\left(\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="f
ricas")
```

output
```
1/48*(48*B*a^4*d*x*cos(d*x + c)^4 + 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log
(sin(d*x + c) + 1) - 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log(-sin(d*x + c)
+ 1) + 2*(160*(A + B)*a^4*cos(d*x + c)^3 + 3*(27*A + 16*B)*a^4*cos(d*x + c
)^2 + 8*(4*A + B)*a^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c
)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.77

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\sec^5(c + dx)\,dx$$

$$= \frac{64\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa^4+16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba^4+48\left(dx+c\right)Ba^4-3}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="m
axima")
```

output
```
1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 16*(tan(d*x + c)^3 + 3*
tan(d*x + c))*B*a^4 + 48*(d*x + c)*B*a^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*A*a^4*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 48*B*a^4*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*B*a
^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*A*a^4*tan(d*x + c
) + 288*B*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.29

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx$$

$$= \frac{24\,(dx + c)Ba^4 + 3\,(35\,Aa^4 + 48\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(35\,Aa^4 + 48\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="g
iac")
```

output
```
1/24*(24*(d*x + c)*B*a^4 + 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1
/2*c) + 1)) - 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
 2*(105*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7 -
385*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*
A*a^4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*A*a^
4*tan(1/2*d*x + 1/2*c) - 216*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 41.71 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{35\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d} + \frac{2\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{12\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$+ \frac{20\,A\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{27\,A\,a^4\,\sin\left(c+d\,x\right)}{8\,d\,{\cos\left(c+d\,x\right)}^2} + \frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}$$
$$+ \frac{A\,a^4\,\sin\left(c+d\,x\right)}{4\,d\,{\cos\left(c+d\,x\right)}^4} + \frac{20\,B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{2\,B\,a^4\,\sin\left(c+d\,x\right)}{d\,{\cos\left(c+d\,x\right)}^2} + \frac{B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^5,x)
```

output
```
(35*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) + (2*B*a^4*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (20*A*a^4*sin(c + d*x))/(3*d*cos(c + d*x
)) + (27*A*a^4*sin(c + d*x))/(8*d*cos(c + d*x)^2) + (4*A*a^4*sin(c + d*x))
/(3*d*cos(c + d*x)^3) + (A*a^4*sin(c + d*x))/(4*d*cos(c + d*x)^4) + (20*B*
a^4*sin(c + d*x))/(3*d*cos(c + d*x)) + (2*B*a^4*sin(c + d*x))/(d*cos(c + d
*x)^2) + (B*a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.06

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
(a**4*( - 105*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 1
44*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b + 210*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 288*cos(c + d*x)*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)**2*b - 105*cos(c + d*x)*log(tan((c + d*x)
/2) - 1)*a - 144*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 105*cos(c + d*
x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a + 144*cos(c + d*x)*log(tan(
(c + d*x)/2) + 1)*sin(c + d*x)**4*b - 210*cos(c + d*x)*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**2*a - 288*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(
c + d*x)**2*b + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 144*cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*b + 24*cos(c + d*x)*sin(c + d*x)**4*b*d*x
- 81*cos(c + d*x)*sin(c + d*x)**3*a - 48*cos(c + d*x)*sin(c + d*x)**3*b -
48*cos(c + d*x)*sin(c + d*x)**2*b*d*x + 87*cos(c + d*x)*sin(c + d*x)*a + 4
8*cos(c + d*x)*sin(c + d*x)*b + 24*cos(c + d*x)*b*d*x + 160*sin(c + d*x)**
5*a + 160*sin(c + d*x)**5*b - 352*sin(c + d*x)**3*a - 328*sin(c + d*x)**3*
b + 192*sin(c + d*x)*a + 168*sin(c + d*x)*b))/(24*cos(c + d*x)*d*(sin(c +
d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.36 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 198

$$\begin{aligned}
&\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx \\
&= \frac{7a^4(4A+5B)\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{a^4(83A+100B)\tan(c+dx)}{15d} \\
&\quad+\frac{a^4(244A+275B)\sec(c+dx)\tan(c+dx)}{120d} \\
&\quad+\frac{(26A+25B)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{30d} \\
&\quad+\frac{(8A+5B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^3(c+dx)\tan(c+dx)}{20d} \\
&\quad+\frac{aA(a+a\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d}
\end{aligned}$$

output
```
7/8*a^4*(4*A+5*B)*arctanh(sin(d*x+c))/d+1/15*a^4*(83*A+100*B)*tan(d*x+c)/d
+1/120*a^4*(244*A+275*B)*sec(d*x+c)*tan(d*x+c)/d+1/30*(26*A+25*B)*(a^4+a^4
*cos(d*x+c))*sec(d*x+c)^2*tan(d*x+c)/d+1/20*(8*A+5*B)*(a^2+a^2*cos(d*x+c))
^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)^4*tan(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 5.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.17

$$\begin{aligned}\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx &= \frac{a^4B\coth^{-1}(\sin(c+dx))}{d}+\frac{7a^4A\text{arctanh}(\sin(c+dx))}{2d}\\ &+\frac{27a^4B\text{arctanh}(\sin(c+dx))}{8d}+\frac{8a^4A\tan(c+dx)}{d}+\frac{8a^4B\tan(c+dx)}{d}\\ &+\frac{7a^4A\sec(c+dx)\tan(c+dx)}{2d}+\frac{27a^4B\sec(c+dx)\tan(c+dx)}{8d}\\ &+\frac{a^4A\sec^3(c+dx)\tan(c+dx)}{d}+\frac{a^4B\sec^3(c+dx)\tan(c+dx)}{4d}\\ &+\frac{8a^4A\tan^3(c+dx)}{3d}+\frac{4a^4B\tan^3(c+dx)}{3d}+\frac{a^4A\tan^5(c+dx)}{5d}\end{aligned}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(a^4*B*ArcCoth[Sin[c + d*x]])/d + (7*a^4*A*ArcTanh[Sin[c + d*x]])/(2*d) +
(27*a^4*B*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*A*Tan[c + d*x])/d + (8*a^4
*B*Tan[c + d*x])/d + (7*a^4*A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (27*a^4*B
*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*A*Sec[c + d*x]^3*Tan[c + d*x])/d
+ (a^4*B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (8*a^4*A*Tan[c + d*x]^3)/(3*
d) + (4*a^4*B*Tan[c + d*x]^3)/(3*d) + (a^4*A*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 3454, 3042, 3454, 3042, 3454, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a \sin \left(c+dx+\frac{\pi}{2}\right)+a\right)^4 \left(A+B \sin \left(c+dx+\frac{\pi}{2}\right)\right)}{\sin \left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \ 3454$$

$$\frac{1}{5}\int (\cos(c+dx)a+a)^3(a(8A+5B)+a(A+5B)\cos(c+dx))\sec^5(c+dx)dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\int \frac{\left(\sin \left(c+dx+\frac{\pi}{2}\right)a+a\right)^3 \left(a(8A+5B)+a(A+5B)\sin \left(c+dx+\frac{\pi}{2}\right)\right)}{\sin \left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\Big\downarrow \ 3454$$

$$\frac{1}{5}\left(\frac{1}{4}\int (\cos(c+dx)a+a)^2 \left(2(26A+25B)a^2+(12A+25B)\cos(c+dx)a^2\right)\sec^4(c+dx)dx+\frac{(8A+5B)\tan(c}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{1}{4}\int \frac{\left(\sin \left(c+dx+\frac{\pi}{2}\right)a+a\right)^2 \left(2(26A+25B)a^2+(12A+25B)\sin \left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin \left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{(8A+5B)\tan(c-}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\Big\downarrow \ 3454$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int (\cos(c+dx)a+a)\left((244A+275B)a^3+(88A+125B)\cos(c+dx)a^3\right)\sec^3(c+dx)dx+\frac{2(26A+25}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((244A+275B)a^3+(88A+125B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{2(26A+25B\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\downarrow \text{3447}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \left((88A+125B)\cos^2(c+dx)a^4+(244A+275B)a^4+\left((88A+125B)a^4+(244A+275B)a^4\right)\cos(c+\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \frac{(88A+125B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+(244A+275B)a^4+\left((88A+125B)a^4+(244A+275B)a^4\right)\sin}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\downarrow \text{3500}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(8(83A+100B)a^4+105(4A+5B)\cos(c+dx)a^4\right)\sec^2(c+dx)dx+\frac{a^4(244A+275B)\tan(c+d}{2d}\right.\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{8(83A+100B)a^4+105(4A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)a^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^4(244A+275B)\tan(c+dx)\sec(c}{2d}\right.\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(8a^4(83A+100B)\int \sec^2(c+dx)dx+105a^4(4A+5B)\int \sec(c+dx)dx\right)+\frac{a^4(244A+275B)\tan}{2d}\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(105a^4(4A+5B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+8a^4(83A+100B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{a^4(244A\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$\Big\downarrow$ 4254

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(105a^4(4A+5B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{8a^4(83A+100B)\int 1d(-\tan(c+dx))}{d}\right)+\frac{a^4(244A+\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$\Big\downarrow$ 24

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(105a^4(4A+5B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{8a^4(83A+100B)\tan(c+dx)}{d}\right)+\frac{a^4(244A+275B)\tan\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

$\Big\downarrow$ 4257

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{105a^4(4A+5B)\text{arctanh}(\sin(c+dx))}{d}+\frac{8a^4(83A+100B)\tan(c+dx)}{d}\right)+\frac{a^4(244A+275B)\tan(}{2d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^3}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (((8*A +
5*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((2*(
26*A + 25*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) +
 ((a^4*(244*A + 275*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((105*a^4*(4*A +
 5*B)*ArcTanh[Sin[c + d*x]])/d + (8*a^4*(83*A + 100*B)*Tan[c + d*x])/d)/2)
/3)/4)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 17.99 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{70\left(-\frac{3\left(A+\frac{5B}{4}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{3\left(A+\frac{5B}{4}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(\right.}{2}\right.}{3c}$
parts	$-\frac{a^4A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{\left(a^4A+4B\,a^4\right)\tan(dx+c)}{d}+\frac{\left(4a^4A+B\,a^4\right)\left(-\left(-\frac{\sec(dx+c)}{4}\right.\right.}{}$
derivativedivides	$\frac{a^4A\tan(dx+c)+B\,a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4B\,a^4\tan(d}{}$
default	$\frac{a^4A\tan(dx+c)+B\,a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4B\,a^4\tan(d}{}$
risch	$-\frac{ia^4\left(420A\,e^{9i(dx+c)}+405B\,e^{9i(dx+c)}-120A\,e^{8i(dx+c)}-480B\,e^{8i(dx+c)}+1320A\,e^{7i(dx+c)}+930B\,e^{7i(dx+c)}-1920A\,e\right.}{}$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBO
SE)
```

output
```
70/3*(-3/2*(A+5/4*B)*(1/10*cos(5*d*x+5*c)+1/2*cos(3*d*x+3*c)+cos(d*x+c))*l
n(tan(1/2*d*x+1/2*c)-1)+3/2*(A+5/4*B)*(1/10*cos(5*d*x+5*c)+1/2*cos(3*d*x+3
*c)+cos(d*x+c))*ln(tan(1/2*d*x+1/2*c)+1)+(33/35*A+93/140*B)*sin(2*d*x+2*c)
+(11/10*A+38/35*B)*sin(3*d*x+3*c)+(3/10*A+81/280*B)*sin(4*d*x+4*c)+(83/350
*A+2/7*B)*sin(5*d*x+5*c)+sin(d*x+c)*(A+4/5*B))*a^4/d/(cos(5*d*x+5*c)+5*cos
(3*d*x+3*c)+10*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.83

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^6(c + dx)\, dx$$
$$= \frac{105\,(4\,A + 5\,B)a^4 \cos\left(dx + c\right)^5 \log\left(\sin\left(dx + c\right) + 1\right) - 105\,(4\,A + 5\,B)a^4 \cos\left(dx + c\right)^5 \log\left(-\sin\left(dx + c\right)\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="f
ricas")
```

output
```
1/240*(105*(4*A + 5*B)*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*(4*A
 + 5*B)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(83*A + 100*B)*a^
4*cos(d*x + c)^4 + 15*(28*A + 27*B)*a^4*cos(d*x + c)^3 + 16*(17*A + 10*B)*
a^4*cos(d*x + c)^2 + 30*(4*A + B)*a^4*cos(d*x + c) + 24*A*a^4)*sin(d*x + c
))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(186) = 372$.

Time = 0.06 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.90

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^6(c + dx)\, dx$$
$$= \frac{16\left(3\tan\left(dx + c\right)^5 + 10\tan\left(dx + c\right)^3 + 15\tan\left(dx + c\right)\right)Aa^4 + 480\left(\tan\left(dx + c\right)^3 + 3\tan\left(dx + c\right)\right)Aa}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="m
axima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
 480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 320*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a^4 - 60*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 240*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
 + 1) + log(sin(d*x + c) - 1)) - 360*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*a^4*(log(si
n(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 240*A*a^4*tan(d*x + c) + 960*B*
a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.24

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^6(c + dx)\, dx$$
$$= \frac{105\left(4Aa^4 + 5Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 105\left(4Aa^4 + 5Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2(}{}}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="g
iac")
```

output
```
1/120*(105*(4*A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(4
*A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(420*A*a^4*tan(1/
2*d*x + 1/2*c)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 1960*A*a^4*tan(1/2*d
*x + 1/2*c)^7 - 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x
 + 1/2*c)^5 + 4480*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 3160*A*a^4*tan(1/2*d*x +
 1/2*c)^3 - 3950*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*d*x + 1
/2*c) + 1395*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 38.90 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.13

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,A+5\,B\right)}{4\,d}$$
$$-\frac{\left(7\,A\,a^4+\frac{35\,B\,a^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{98\,A\,a^4}{3}-\frac{245\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{896\,A\,a^4}{15}+\frac{224\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\right.}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^6,x)
```

output
```
(7*a^4*atanh(tan(c/2 + (d*x)/2))*(4*A + 5*B))/(4*d) - (tan(c/2 + (d*x)/2)*
(25*A*a^4 + (93*B*a^4)/4) + tan(c/2 + (d*x)/2)^9*(7*A*a^4 + (35*B*a^4)/4)
- tan(c/2 + (d*x)/2)^7*((98*A*a^4)/3 + (245*B*a^4)/6) - tan(c/2 + (d*x)/2)
^3*((158*A*a^4)/3 + (395*B*a^4)/6) + tan(c/2 + (d*x)/2)^5*((896*A*a^4)/15
+ (224*B*a^4)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 1
0*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 -
1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.43

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^6(c + dx)\, dx = \text{Too large to display}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

output
```
(a**4*( - 420*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 5
25*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b + 840*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 1050*cos(c + d*x)*log(t
an((c + d*x)/2) - 1)*sin(c + d*x)**2*b - 420*cos(c + d*x)*log(tan((c + d*x
)/2) - 1)*a - 525*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 420*cos(c + d
*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a + 525*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*sin(c + d*x)**4*b - 840*cos(c + d*x)*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2*a - 1050*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**2*b + 420*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 525*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*b - 420*cos(c + d*x)*sin(c + d*x)**3*a -
 405*cos(c + d*x)*sin(c + d*x)**3*b + 540*cos(c + d*x)*sin(c + d*x)*a + 43
5*cos(c + d*x)*sin(c + d*x)*b + 664*sin(c + d*x)**5*a + 800*sin(c + d*x)**
5*b - 1600*sin(c + d*x)**3*a - 1760*sin(c + d*x)**3*b + 960*sin(c + d*x)*a
 + 960*sin(c + d*x)*b))/(120*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d
*x)**2 + 1))
```

3.37 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 229

$$\begin{aligned}\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx &= \frac{7a^4(7A+8B)\text{arctanh}(\sin(c+dx))}{16d}+\frac{a^4(72A+83B)\tan(c+dx)}{15d}\\ &+\frac{7a^4(7A+8B)\sec(c+dx)\tan(c+dx)}{16d}\\ &+\frac{a^4(159A+176B)\sec^2(c+dx)\tan(c+dx)}{120d}\\ &+\frac{(73A+72B)\left(a^4+a^4\cos(c+dx)\right)\sec^3(c+dx)\tan(c+dx)}{120d}\\ &+\frac{(3A+2B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^4(c+dx)\tan(c+dx)}{10d}\\ &+\frac{aA(a+a\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d}\end{aligned}$$

output
```
7/16*a^4*(7*A+8*B)*arctanh(sin(d*x+c))/d+1/15*a^4*(72*A+83*B)*tan(d*x+c)/d
+7/16*a^4*(7*A+8*B)*sec(d*x+c)*tan(d*x+c)/d+1/120*a^4*(159*A+176*B)*sec(d*
x+c)^2*tan(d*x+c)/d+1/120*(73*A+72*B)*(a^4+a^4*cos(d*x+c))*sec(d*x+c)^3*ta
n(d*x+c)/d+1/10*(3*A+2*B)*(a^2+a^2*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
+1/6*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)^5*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 5.16 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.13

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$$
$$= \frac{49a^4A\text{arctanh}(\sin(c+dx))}{16d}+\frac{7a^4B\text{arctanh}(\sin(c+dx))}{2d}+\frac{8a^4A\tan(c+dx)}{d}+\frac{8a^4B\tan(c+dx)}{d}+\frac{49a^4A\sec(c+dx)\tan(c+dx)}{16d}+\frac{7a^4B\sec(c+dx)\tan(c+dx)}{2d}+\frac{41a^4A\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^4B\sec^3(c+dx)\tan(c+dx)}{d}+\frac{a^4A\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{4a^4A\tan^3(c+dx)}{d}+\frac{8a^4B\tan^3(c+dx)}{3d}+\frac{4a^4A\tan^5(c+dx)}{5d}+\frac{a^4B\tan^5(c+dx)}{5d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

output
```
(49*a^4*A*ArcTanh[Sin[c + d*x]])/(16*d) + (7*a^4*B*ArcTanh[Sin[c + d*x]])/
(2*d) + (8*a^4*A*Tan[c + d*x])/d + (8*a^4*B*Tan[c + d*x])/d + (49*a^4*A*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^4*B*Sec[c + d*x]*Tan[c + d*x])/(2*d
) + (41*a^4*A*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (a^4*B*Sec[c + d*x]^3*
Tan[c + d*x])/d + (a^4*A*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*A*Tan
[c + d*x]^3)/d + (8*a^4*B*Tan[c + d*x]^3)/(3*d) + (4*a^4*A*Tan[c + d*x]^5)
/(5*d) + (a^4*B*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {3042, 3454, 3042, 3454, 3042, 3454, 27, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a\cos(c+dx)+a)^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{6}\int(\cos(c+dx)a+a)^3(3a(3A+2B)+2a(A+3B)\cos(c+dx))\sec^6(c+dx)dx+\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3\left(3a(3A+2B)+2a(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx+\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{6}\left(\frac{1}{5}\int(\cos(c+dx)a+a)^2\left((73A+72B)a^2+14(2A+3B)\cos(c+dx)a^2\right)\sec^5(c+dx)dx+\frac{3(3A+2B)\tan(c}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\int\frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left((73A+72B)a^2+14(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{3(3A+2B)\tan(c-}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\int 3(\cos(c+dx)a+a)\left((159A+176B)a^3+2(43A+52B)\cos(c+dx)a^3\right)\sec^4(c+dx)dx+\frac{(73A+72}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int (\cos(c+dx)a+a)\left((159A+176B)a^3+2(43A+52B)\cos(c+dx)a^3\right)\sec^4(c+dx)dx+\frac{(73A+72B}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((159A+176B)a^3+2(43A+52B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{(73A+72B)}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\downarrow$ 3447

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int \left(2(43A+52B)\cos^2(c+dx)a^4+(159A+176B)a^4+\left(2(43A+52B)a^4+(159A+176B)a^4\right)\cos(c+\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int \frac{2(43A+52B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+(159A+176B)a^4+\left(2(43A+52B)a^4+(159A+176B)a^4\right)\sin}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\downarrow$ 3500

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int \left(105(7A+8B)a^4+8(72A+83B)\cos(c+dx)a^4\right)\sec^3(c+dx)dx+\frac{a^4(159A+176B)\tan(c+dx}{3d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int \frac{105(7A+8B)a^4+8(72A+83B)\sin\left(c+dx+\frac{\pi}{2}\right)a^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^4(159A+176B)\tan(c+dx)\sec^2(c-}{3d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\downarrow$ 3227

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(105a^4(7A+8B)\int \sec^3(c+dx)dx+8a^4(72A+83B)\int \sec^2(c+dx)dx\right)+\frac{a^4(159A+176B)\tan}{3}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(8a^4(72A+83B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+105a^4(7A+8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{a^4(159A}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\big\downarrow$ 4254

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(105a^4(7A+8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{8a^4(72A+83B)\int 1d(-\tan(c+dx))}{d}\right)+\frac{a^4(159A+}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\big\downarrow$ 24

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(105a^4(7A+8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{8a^4(72A+83B)\tan(c+dx)}{d}\right)+\frac{a^4(159A+176B)\tan}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\big\downarrow$ 4255

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(105a^4(7A+8B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{8a^4(72A+83B)\tan(c+dx)}{d}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(105a^4(7A+8B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{8a^4(72A+83B)\tan(c}{d}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

$\big\downarrow$ 4257

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(105a^4(7A+8B)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{8a^4(72A+83B)\tan(c+d}{d}\right.\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a\cos(c+dx)+a)^3}{6d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + ((3*(3*A
+ 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (((
73*A + 72*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (3*((a^4*(159*A + 176*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((8*a^4*(72
*A + 83*B)*Tan[c + d*x])/d + 105*a^4*(7*A + 8*B)*(ArcTanh[Sin[c + d*x]]/(2
*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/3))/4)/5)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 19.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{125a^4\left(-\frac{147\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)+\frac{2}{3}\right)\left(A+\frac{8B}{7}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{100}+\frac{147\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}\right.}{}\right.}{}$
parts	$\frac{a^4A\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{(a^4A+4Ba^4)\left(\frac{\sec(dx\right.}{}}{}$
derivativedivides	$\frac{a^4A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba^4\tan(dx+c)-4a^4A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4Ba^4\left(\right.}{}$
default	$\frac{a^4A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba^4\tan(dx+c)-4a^4A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4Ba^4\left(\right.}{}$
risch	$-\frac{ia^4\left(735A\,e^{11i(dx+c)}+840B\,e^{11i(dx+c)}-240B\,e^{10i(dx+c)}+3845A\,e^{9i(dx+c)}+3480B\,e^{9i(dx+c)}-1920A\,e^{8i(dx+c)}-40\right.}{}$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x,method=_RETURNVERBO
SE)
```

output
```
125/4*a^4*(-147/100*(1/15*cos(6*d*x+6*c)+2/5*cos(4*d*x+4*c)+cos(2*d*x+2*c)
+2/3)*(A+8/7*B)*ln(tan(1/2*d*x+1/2*c)-1)+147/100*(1/15*cos(6*d*x+6*c)+2/5*
cos(4*d*x+4*c)+cos(2*d*x+2*c)+2/3)*(A+8/7*B)*ln(tan(1/2*d*x+1/2*c)+1)+28/1
25*(8*A+7*B)*sin(2*d*x+2*c)+1/125*(116*B+769/6*A)*sin(3*d*x+3*c)+48/625*(1
2*A+13*B)*sin(4*d*x+4*c)+7/125*(4*B+7/2*A)*sin(5*d*x+5*c)+4/625*(24*A+83/3
*B)*sin(6*d*x+6*c)+sin(d*x+c)*(88/125*B+A))/d/(cos(6*d*x+6*c)+6*cos(4*d*x+
4*c)+15*cos(2*d*x+2*c)+10)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.81

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$$

$$=\frac{105\,(7\,A+8\,B)a^4\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-105\,(7\,A+8\,B)a^4\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="f
ricas")
```

output
```
1/480*(105*(7*A + 8*B)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(7*A
 + 8*B)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(72*A + 83*B)*a^
4*cos(d*x + c)^5 + 105*(7*A + 8*B)*a^4*cos(d*x + c)^4 + 32*(18*A + 17*B)*a
^4*cos(d*x + c)^3 + 10*(41*A + 24*B)*a^4*cos(d*x + c)^2 + 48*(4*A + B)*a^4
*cos(d*x + c) + 40*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^7(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(215) = 430$.

Time = 0.05 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.03

$$\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^7(c + dx) \, dx$$

$$= \frac{128 \left(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c)\right) A a^4 + 640 \left(\tan(dx + c)^3 + 3 \tan(dx + c)\right) A}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="m
axima")
```

output
```
1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4
+ 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 32*(3*tan(d*x + c)^5 + 10*
tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 960*(tan(d*x + c)^3 + 3*tan(d*x
+ c))*B*a^4 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d
*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*l
og(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*A*a^4*(2*(3*sin(d*x
 + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(
sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B*a^4*(2*(3*sin(d*x + c
)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 480*B*a
^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 480*B*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.22

$$\int (a + a\cos(c+dx))^4 (A + B\cos(c+dx)) \sec^7(c+dx)\, dx$$

$$= \frac{105\,(7\,Aa^4 + 8\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 105\,(7\,Aa^4 + 8\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2(}{}}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="g
iac")
```

output
```
1/240*(105*(7*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(7
*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(735*A*a^4*tan(1/
2*d*x + 1/2*c)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*A*a^4*tan(1/2
*d*x + 1/2*c)^9 - 4760*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d
*x + 1/2*c)^7 + 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*A*a^4*tan(1/2*d
*x + 1/2*c)^5 - 13488*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*
x + 1/2*c)^3 + 9320*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*A*a^4*tan(1/2*d*x
+ 1/2*c) - 3000*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6
)/d
```

Mupad [B] (verification not implemented)

Time = 44.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.14

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$$
$$=\frac{\left(-\frac{49\,A\,a^4}{8}-7\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{833\,A\,a^4}{24}+\frac{119\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(-\frac{1617\,A\,a^4}{20}-\frac{462\,B\,a^4}{5}\right)\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\right.\right.}$$
$$+\frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(7\,A+8\,B\right)}{8\,d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^7,x)
```

output
```
(tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + 25*B*a^4) - tan(c/2 + (d*x)/2)^11*((4
9*A*a^4)/8 + 7*B*a^4) + tan(c/2 + (d*x)/2)^9*((833*A*a^4)/24 + (119*B*a^4)
/3) - tan(c/2 + (d*x)/2)^3*((1471*A*a^4)/24 + (233*B*a^4)/3) - tan(c/2 + (
d*x)/2)^7*((1617*A*a^4)/20 + (462*B*a^4)/5) + tan(c/2 + (d*x)/2)^5*((1967*
A*a^4)/20 + (562*B*a^4)/5))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x
)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (
d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (7*a^4*atanh(tan(c/2 + (d*x)/2)
)*(7*A + 8*B))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.90

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x)
```

output
```
(a**4*( - 735*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a - 8
40*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*b + 2205*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a + 2520*cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b - 2205*cos(c + d*x)*log(tan((c + d
*x)/2) - 1)*sin(c + d*x)**2*a - 2520*cos(c + d*x)*log(tan((c + d*x)/2) - 1
)*sin(c + d*x)**2*b + 735*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a + 840*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 735*cos(c + d*x)*log(tan((c + d*
x)/2) + 1)*sin(c + d*x)**6*a + 840*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**6*b - 2205*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*
x)**4*a - 2520*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*b +
2205*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2520*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b - 735*cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*a - 840*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b -
735*cos(c + d*x)*sin(c + d*x)**5*a - 840*cos(c + d*x)*sin(c + d*x)**5*b +
1880*cos(c + d*x)*sin(c + d*x)**3*a + 1920*cos(c + d*x)*sin(c + d*x)**3*b
- 1185*cos(c + d*x)*sin(c + d*x)*a - 1080*cos(c + d*x)*sin(c + d*x)*b + 11
52*sin(c + d*x)**7*a + 1328*sin(c + d*x)**7*b - 4032*sin(c + d*x)**5*a - 4
528*sin(c + d*x)**5*b + 4800*sin(c + d*x)**3*a + 5120*sin(c + d*x)**3*b -
1920*sin(c + d*x)*a - 1920*sin(c + d*x)*b))/(240*cos(c + d*x)*d*(sin(c + d
*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.38 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{3(4A-5B)x}{8a}+\frac{4(A-B)\sin(c+dx)}{ad} - \frac{3(4A-5B)\cos(c+dx)\sin(c+dx)}{8ad} - \frac{(4A-5B)\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{4(A-B)\sin^3(c+dx)}{3ad}$$

output
```
-3/8*(4*A-5*B)*x/a+4*(A-B)*sin(d*x+c)/a/d-3/8*(4*A-5*B)*cos(d*x+c)*sin(d*x
+c)/a/d-1/4*(4*A-5*B)*cos(d*x+c)^3*sin(d*x+c)/a/d+(A-B)*cos(d*x+c)^4*sin(d
*x+c)/d/(a+a*cos(d*x+c))-4/3*(A-B)*sin(d*x+c)^3/a/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 311 vs. $2(153) = 306$.

Time = 2.31 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.03

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-72(4A-5B)dx\cos\left(\frac{dx}{2}\right)-72(4A-5B)dx\cos\left(c+\frac{dx}{2}\right)+552A\sin\left(\frac{dx}{2}\right)-55\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(-72*(4*A - 5*B)*d*x*Cos[(d*x)/2] - 72*(4*A - 5
*B)*d*x*Cos[c + (d*x)/2] + 552*A*Sin[(d*x)/2] - 552*B*Sin[(d*x)/2] + 168*A
*Sin[c + (d*x)/2] - 168*B*Sin[c + (d*x)/2] + 144*A*Sin[c + (3*d*x)/2] - 12
0*B*Sin[c + (3*d*x)/2] + 144*A*Sin[2*c + (3*d*x)/2] - 120*B*Sin[2*c + (3*d
*x)/2] - 16*A*Sin[2*c + (5*d*x)/2] + 40*B*Sin[2*c + (5*d*x)/2] - 16*A*Sin[
3*c + (5*d*x)/2] + 40*B*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*d*x)/2] -
5*B*Sin[3*c + (7*d*x)/2] + 8*A*Sin[4*c + (7*d*x)/2] - 5*B*Sin[4*c + (7*d*x
)/2] + 3*B*Sin[4*c + (9*d*x)/2] + 3*B*Sin[5*c + (9*d*x)/2]))/(192*a*d*(1 +
 Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3456, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \cos^3(c+dx)(4a(A-B)-a(4A-5B)\cos(c+dx))dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a(A-B)-a(4A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{4a(A-B)\int \cos^3(c+dx)dx-a(4A-5B)\int \cos^4(c+dx)dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a(A-B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-a(4A-5B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3113}$$

$$\frac{-\frac{4a(A-B)\int(1-\sin^2(c+dx))d(-\sin(c+dx))}{d}-a(4A-5B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a(4A-5B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{4a(A-B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{-a(4A-5B)\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{4a(A-B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-a(4A-5B)\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{4a(A-B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 3115$$

$$\frac{-a(4A-5B)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{4a(A-B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 24$$

$$\frac{-\frac{4a(A-B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-a(4A-5B)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((-4*a*(A
 - B)*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d - a*(4*A - 5*B)*((Cos[c + d*x]
^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))
/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{((-8A+38B)\cos(2dx+2c)+(8A-2B)\cos(3dx+3c)+3B\cos(4dx+4c)+(136A-82B)\cos(dx+c)+248A-221B)\tan\left(\frac{d}{}\right.}{96ad}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{2\left(\left(\frac{25B}{8}-\frac{5A}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+\left(\frac{115B}{24}-\frac{31A}{6}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+\left(\frac{109B}{24}-\frac{25A}{6}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{2\left(\left(\frac{25B}{8}-\frac{5A}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+\left(\frac{115B}{24}-\frac{31A}{6}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+\left(\frac{109B}{24}-\frac{25A}{6}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}}{da}$
risch	$-\frac{3xA}{2a}+\frac{15Bx}{8a}-\frac{7ie^{i(dx+c)}A}{8ad}+\frac{7ie^{i(dx+c)}B}{8ad}+\frac{7ie^{-i(dx+c)}A}{8ad}-\frac{7ie^{-i(dx+c)}B}{8ad}+\frac{2iA}{da(e^{i(dx+c)}+1)}-\frac{}{da(e}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{ad}-\frac{3(4A-5B)x}{8a}+\frac{86(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3ad}-\frac{15(4A-5B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8a}-\frac{15(4A-5B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{4a}-\frac{15(}{}}{}$

input

```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output

```
1/96*(((-8*A+38*B)*cos(2*d*x+2*c)+(8*A-2*B)*cos(3*d*x+3*c)+3*B*cos(4*d*x+4
*c)+(136*A-82*B)*cos(d*x+c)+248*A-221*B)*tan(1/2*d*x+1/2*c)-144*x*(A-5/4*B
)*d)/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx =$$

$$-\frac{9\,(4\,A-5\,B)dx\cos{(dx+c)}+9\,(4\,A-5\,B)dx-\left(6\,B\cos{(dx+c)}^4+2\,(4\,A-B)\cos{(dx+c)}^3-(4\right.}{24\,(ad\cos{(dx+c)}+a}$$

input

```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/24*(9*(4*A - 5*B)*d*x*cos(d*x + c) + 9*(4*A - 5*B)*d*x - (6*B*cos(d*x +
 c)^4 + 2*(4*A - B)*cos(d*x + c)^3 - (4*A - 13*B)*cos(d*x + c)^2 + (28*A -
 19*B)*cos(d*x + c) + 64*A - 64*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(134) = 268$.

Time = 1.91 (sec) , antiderivative size = 1794, normalized size of antiderivative = 11.73

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((-36*A*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*
a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d
*x/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2
)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*t
an(c/2 + d*x/2)**2 + 24*a*d) - 216*A*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c
/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4
+ 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**2/(24
*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 +
d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 36*A*d*x/(24*a*d*tan(c/
2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 +
 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 24*A*tan(c/2 + d*x/2)**9/(24*a*d*t
an(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)
**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 216*A*tan(c/2 + d*x/2)**7/(24
*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 +
d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 392*A*tan(c/2 + d*x/2)*
*5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(
c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 296*A*tan(c/2 + d
*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*
d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 96*A*tan(c/
2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 1...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(145) = 290$.

Time = 0.15 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.58

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{B\left(\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - \frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}\right) - 4A\Big(}{12d}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/12*(B*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 4*A*((9*sin
(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3*sin(d*
x + c)/(a*(cos(d*x + c) + 1))))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{\frac{9(dx+c)(4A-5B)}{a} - \frac{24\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} - \frac{2\left(60A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-75B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+124A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}}{24d}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
-1/24*(9*(d*x + c)*(4*A - 5*B)/a - 24*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*
d*x + 1/2*c))/a - 2*(60*A*tan(1/2*d*x + 1/2*c)^7 - 75*B*tan(1/2*d*x + 1/2*
c)^7 + 124*A*tan(1/2*d*x + 1/2*c)^5 - 115*B*tan(1/2*d*x + 1/2*c)^5 + 100*A
*tan(1/2*d*x + 1/2*c)^3 - 109*B*tan(1/2*d*x + 1/2*c)^3 + 36*A*tan(1/2*d*x
+ 1/2*c) - 21*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/
d
```

Mupad [B] (verification not implemented)

Time = 42.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.11

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{15\,B\,x}{8\,a}-\frac{3\,A\,x}{2\,a}+\frac{7\,A\,\sin\left(c+d\,x\right)}{4\,a\,d}-\frac{7\,B\,\sin\left(c+d\,x\right)}{4\,a\,d}-\frac{A\,\sin\left(2\,c+2\,d\,x\right)}{4\,a\,d}+\frac{A\,\sin\left(3\,c+3\,d\,x\right)}{12\,a\,d}+\frac{A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}+\frac{B\,\sin\left(2\,c+2\,d\,x\right)}{2\,a\,d}-\frac{B\,\sin\left(3\,c+3\,d\,x\right)}{12\,a\,d}+\frac{B\,\sin\left(4\,c+4\,d\,x\right)}{32\,a\,d}-\frac{B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

output
```
(15*B*x)/(8*a) - (3*A*x)/(2*a) + (7*A*sin(c + d*x))/(4*a*d) - (7*B*sin(c +
 d*x))/(4*a*d) - (A*sin(2*c + 2*d*x))/(4*a*d) + (A*sin(3*c + 3*d*x))/(12*a
*d) + (A*tan(c/2 + (d*x)/2))/(a*d) + (B*sin(2*c + 2*d*x))/(2*a*d) - (B*sin
(3*c + 3*d*x))/(12*a*d) + (B*sin(4*c + 4*d*x))/(32*a*d) - (B*tan(c/2 + (d*
x)/2))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{-6\cos(dx+c)\sin(dx+c)^4 b - 12\cos(dx+c)\sin(dx+c)^2 a + 27\cos(dx+c)\sin(dx+c)^2 b - 24\cos(\ldots}{\ldots}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**4*b - 12*cos(c + d*x)*sin(c + d*x)**2*a +
 27*cos(c + d*x)*sin(c + d*x)**2*b - 24*cos(c + d*x)*a + 24*cos(c + d*x)*b
 - 8*sin(c + d*x)**4*a + 8*sin(c + d*x)**4*b + 48*sin(c + d*x)**2*a - 48*s
in(c + d*x)**2*b - 36*sin(c + d*x)*a*d*x + 45*sin(c + d*x)*b*d*x + 24*a -
24*b)/(24*sin(c + d*x)*a*d)
```

3.39 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 122

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{3(A-B)x}{2a} - \frac{(3A-4B)\sin(c+dx)}{ad} + \frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(3A-4B)\sin^3(c+dx)}{3ad}$$

output
```
3/2*(A-B)*x/a-(3*A-4*B)*sin(d*x+c)/a/d+3/2*(A-B)*cos(d*x+c)*sin(d*x+c)/a/d
+(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))+1/3*(3*A-4*B)*sin(d*x+c)
^3/a/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 249 vs. $2(122) = 244$.

Time = 2.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.04

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(36(A-B)dx\cos\left(\frac{dx}{2}\right)+36(A-B)dx\cos\left(c+\frac{dx}{2}\right)-60A\sin\left(\frac{dx}{2}\right)+69B\sin\left(\frac{d}{2}\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(36*(A - B)*d*x*Cos[(d*x)/2] + 36*(A - B)*d*x*C
os[c + (d*x)/2] - 60*A*Sin[(d*x)/2] + 69*B*Sin[(d*x)/2] - 12*A*Sin[c + (d*
x)/2] + 21*B*Sin[c + (d*x)/2] - 9*A*Sin[c + (3*d*x)/2] + 18*B*Sin[c + (3*d
*x)/2] - 9*A*Sin[2*c + (3*d*x)/2] + 18*B*Sin[2*c + (3*d*x)/2] + 3*A*Sin[2*
c + (5*d*x)/2] - 2*B*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2] - 2*B
*Sin[3*c + (5*d*x)/2] + B*Sin[3*c + (7*d*x)/2] + B*Sin[4*c + (7*d*x)/2]))/
(24*a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 3456, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 3456$$

$$\frac{\int \cos^2(c+dx)(3a(A-B)-a(3A-4B)\cos(c+dx))dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a(A-B)-a(3A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textbf{3227}$$

$$\frac{3a(A-B)\int \cos^2(c+dx)dx-a(3A-4B)\int \cos^3(c+dx)dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{3a(A-B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-a(3A-4B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textbf{3113}$$

$$\frac{\frac{a(3A-4B)\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}+3a(A-B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{3a(A-B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{a(3A-4B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{3a(A-B)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a(3A-4B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{\frac{a(3A-4B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+3a(A-B)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + (3*a*(A -
 B)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) + (a*(3*A - 4*B)*(-Sin[c + d
*x] + Sin[c + d*x]^3/3))/d)/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{\left(\left(3A-B\right)\cos\left(2dx+2c\right)+B\cos\left(3dx+3c\right)+\left(-6A+17B\right)\cos\left(dx+c\right)-21A+31B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+18dx\left(A-B\right)}{12ad}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{2\left(-\frac{3A}{2}+\frac{5B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+2\left(\frac{8B}{3}-2A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+2\left(-\frac{A}{2}+\frac{3B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}+3(A}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{2\left(-\frac{3A}{2}+\frac{5B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+2\left(\frac{8B}{3}-2A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+2\left(-\frac{A}{2}+\frac{3B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}+3(A}{da}$
risch	$\frac{3xA}{2a}-\frac{3Bx}{2a}+\frac{i e^{i(dx+c)}A}{2ad}-\frac{7i e^{i(dx+c)}B}{8ad}-\frac{i e^{-i(dx+c)}A}{2ad}+\frac{7i e^{-i(dx+c)}B}{8ad}-\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}+\frac{2iB}{da(e^{i(dx+c}}$
norman	$\frac{\frac{3(A-B)x}{2a}-\frac{2(A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{ad}+\frac{6(A-B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{9(A-B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{a}+\frac{6(A-B)x\tan(}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/12*(((3*A-B)*cos(2*d*x+2*c)+B*cos(3*d*x+3*c)+(-6*A+17*B)*cos(d*x+c)-21*A
+31*B)*tan(1/2*d*x+1/2*c)+18*d*x*(A-B))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{9\,(A-B)dx\cos\left(dx+c\right)+9\,(A-B)dx+\left(2\,B\cos\left(dx+c\right)^3+(3\,A-B)\cos\left(dx+c\right)^2-(3\,A-7\,B)\,c\right.}{6\,(ad\cos\left(dx+c\right)+ad)}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/6*(9*(A - B)*d*x*cos(d*x + c) + 9*(A - B)*d*x + (2*B*cos(d*x + c)^3 + (3
*A - B)*cos(d*x + c)^2 - (3*A - 7*B)*cos(d*x + c) - 12*A + 16*B)*sin(d*x +
 c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(105) = 210$.

Time = 1.22 (sec) , antiderivative size = 1161, normalized size of antiderivative = 9.52

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((9*A*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d
*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(
c/2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(c/2 + d*x/2)**2/(6*a*d*
tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)
**2 + 6*a*d) + 9*A*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2
)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*A*tan(c/2 + d*x/2)**7/(6*a*
d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/
2)**2 + 6*a*d) - 36*A*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*
a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 42*A*tan(c
/2 + d*x/2)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 1
8*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 12*A*tan(c/2 + d*x/2)/(6*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*
a*d) - 9*B*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan
(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2
+ d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a
*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2
+ 6*a*d) - 9*B*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*B*tan(c/2 + d*x/2)**7/(6*a*d...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(116) = 232$.

Time = 0.13 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.54

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\mathrm{s}\ldots}{(\cos\ldots}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\ldots}\ldots\right.}{3d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
1/3*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
)/a + 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*A*((sin(d*x + c)/(cos(d*x
 + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1)))
)/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.24

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\frac{9\,(dx+c)(A-B)}{a} - \frac{6\,\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a} - \frac{2\,\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,B\,\mathrm{t}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right.}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/6*(9*(d*x + c)*(A - B)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1
/2*c))/a - 2*(9*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 + 1
2*A*tan(1/2*d*x + 1/2*c)^3 - 16*B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x
 + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/
d
```

Mupad [B] (verification not implemented)

Time = 42.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{3\,x\,(A-B)}{2\,a} - \frac{(3\,A-5\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(4\,A-\frac{16\,B}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(A-3\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

output
```
(3*x*(A - B))/(2*a) - (tan(c/2 + (d*x)/2)^5*(3*A - 5*B) + tan(c/2 + (d*x)/
2)^3*(4*A - (16*B)/3) + tan(c/2 + (d*x)/2)*(A - 3*B))/(d*(a + 3*a*tan(c/2
+ (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6)) - (tan(
c/2 + (d*x)/2)*(A - B))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{3\cos(dx+c)\sin(dx+c)^2 a-3\cos(dx+c)\sin(dx+c)^2 b+6\cos(dx+c)\,a-6\cos(dx+c)\,b-2\sin(}{6\sin(dx+}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(3*cos(c + d*x)*sin(c + d*x)**2*a - 3*cos(c + d*x)*sin(c + d*x)**2*b + 6*c
os(c + d*x)*a - 6*cos(c + d*x)*b - 2*sin(c + d*x)**4*b - 6*sin(c + d*x)**2
*a + 12*sin(c + d*x)**2*b + 9*sin(c + d*x)*a*d*x - 9*sin(c + d*x)*b*d*x -
6*a + 6*b)/(6*sin(c + d*x)*a*d)
```

3.40 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)x}{a}+\frac{Bx}{2a}+\frac{(A-B)\sin(c+dx)}{ad} + \frac{B\cos(c+dx)\sin(c+dx)}{2ad} + \frac{(A-B)\sin(c+dx)}{ad(1+\cos(c+dx))}$$

output
```
-(A-B)*x/a+1/2*B*x/a+(A-B)*sin(d*x+c)/a/d+1/2*B*cos(d*x+c)*sin(d*x+c)/a/d+
(A-B)*sin(d*x+c)/a/d/(1+cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(90) = 180.

Time = 1.72 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.19

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-4(2A-3B)dx\cos\left(\frac{dx}{2}\right)-4(2A-3B)dx\cos\left(c+\frac{dx}{2}\right)+20A\sin\left(\frac{dx}{2}\right)-20B\,\mathrm{si}\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(-4*(2*A - 3*B)*d*x*Cos[(d*x)/2] - 4*(2*A - 3*B
)*d*x*Cos[c + (d*x)/2] + 20*A*Sin[(d*x)/2] - 20*B*Sin[(d*x)/2] + 4*A*Sin[c
 + (d*x)/2] - 4*B*Sin[c + (d*x)/2] + 4*A*Sin[c + (3*d*x)/2] - 3*B*Sin[c +
(3*d*x)/2] + 4*A*Sin[2*c + (3*d*x)/2] - 3*B*Sin[2*c + (3*d*x)/2] + B*Sin[2
*c + (5*d*x)/2] + B*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \cos(c+dx)(2a(A-B)-a(2A-3B)\cos(c+dx))dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(2a(A-B)-a(2A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\frac{2a(A-B)\sin(c+dx)}{d}-\frac{a(2A-3B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{1}{2}ax(2A-3B)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + (-1/2*(a*
(2*A - 3*B)*x) + (2*a*(A - B)*Sin[c + d*x])/d - (a*(2*A - 3*B)*Cos[c + d*x
]*Sin[c + d*x])/(2*d))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{(B\cos(2dx+2c)+(4A-2B)\cos(dx+c)+8A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4xd\left(-\frac{3B}{2}+A\right)}{4ad}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{2\left(\left(\frac{3B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(\frac{B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}-(2A-3B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{2\left(\left(\frac{3B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(\frac{B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}-(2A-3B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$-\frac{xA}{a}+\frac{3Bx}{2a}-\frac{i e^{i(dx+c)}A}{2ad}+\frac{i e^{i(dx+c)}B}{2ad}+\frac{i e^{-i(dx+c)}A}{2ad}-\frac{i e^{-i(dx+c)}B}{2ad}+\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}-\frac{2iB}{da(e^{i(dx+c)}}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}+\frac{(3A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{(5A-6B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{(2A-3B)x}{2a}+\frac{7(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}-\frac{3(2A-3B)x\tan}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/4*((B*cos(2*d*x+2*c)+(4*A-2*B)*cos(d*x+c)+8*A-7*B)*tan(1/2*d*x+1/2*c)-4*
x*d*(-3/2*B+A))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{(2A-3B)dx\cos(dx+c)+(2A-3B)dx-\left(B\cos(dx+c)^2+(2A-B)\cos(dx+c)+4A-4B\right)}{2(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*((2*A - 3*B)*d*x*cos(d*x + c) + (2*A - 3*B)*d*x - (B*cos(d*x + c)^2 +
 (2*A - B)*cos(d*x + c) + 4*A - 4*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(68) = 136$.

Time = 0.82 (sec) , antiderivative size = 665, normalized size of antiderivative = 7.39

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((-2*A*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d
*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*A*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2
 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*A*d*x/(2*a*d*tan(c/2
 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*A*tan(c/2 + d*x/2)**
5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 8*A*ta
n(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 +
 2*a*d) + 6*A*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2
+ d*x/2)**2 + 2*a*d) + 3*B*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)
**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*B*d*x*tan(c/2 + d*x/2)**2/(2*
a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*B*d*x/(2*
a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*B*tan(c/2
 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*
d) - 10*B*tan(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 +
 d*x/2)**2 + 2*a*d) - 4*B*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*
a*d*tan(c/2 + d*x/2)**2 + 2*a*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(
a*cos(c) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(86) = 172$.

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.50

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx =$$
$$-\frac{B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\cdots}{\left(a+\frac{a\sin\cdots}{(\cos\cdots}\right.}\right.}{d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-(B*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x +
 c)/(a*(cos(d*x + c) + 1))) + A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a - 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x
+ c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx =$$
$$-\frac{\frac{(dx+c)(2A-3B)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
-1/2*((d*x + c)*(2*A - 3*B)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x
+ 1/2*c))/a - 2*(2*A*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c)^3 +
 2*A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)
^2 + 1)^2*a))/d
```

Mupad [B] (verification not implemented)

Time = 41.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(2\,A-3\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(2\,A-B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)} - \frac{x\,(2\,A-3\,B)}{2\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(2*A - 3*B) + tan(c/2 + (d*x)/2)*(2*A - B))/(d*(a +
2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4)) - (x*(2*A - 3*B))/(2*a
) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{\cos(dx+c)^2 bdx+\cos(dx+c)\sin(dx+c)b+\sin(dx+c)^2 bdx+2\sin(dx+c)a-2\sin(dx+c)b+2}{2ad}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(cos(c + d*x)**2*b*d*x + cos(c + d*x)*sin(c + d*x)*b + sin(c + d*x)**2*b*d
*x + 2*sin(c + d*x)*a - 2*sin(c + d*x)*b + 2*tan((c + d*x)/2)*a - 2*tan((c
 + d*x)/2)*b - 2*a*d*x + 2*b*d*x)/(2*a*d)
```

3.41 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 54

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{ad(1+\cos(c+dx))}$$

output
```
(A-B)*x/a+B*sin(d*x+c)/a/d-(A-B)*sin(d*x+c)/a/d/(1+cos(d*x+c))
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.72

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{B\sin(c+dx)}{ad}+(A-B)\left(-\frac{\sin(c+dx)}{ad(1+\cos(c+dx))}-\frac{\arcsin(\cos(c+dx))\sin(c+dx)}{ad\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
(B*Sin[c + d*x])/(a*d) + (A - B)*(-(Sin[c + d*x]/(a*d*(1 + Cos[c + d*x])))
 - (ArcSin[Cos[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[1
+ Cos[c + d*x]]))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int \frac{(A-B)\cos(c+dx)}{\cos(c+dx)+1}dx}{a}+\frac{B\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A-B)\int \frac{\cos(c+dx)}{\cos(c+dx)+1}dx}{a}+\frac{B\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{a}+\frac{B\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{3214}$$

$$\frac{(A-B)\left(x-\int \frac{1}{\cos(c+dx)+1}dx\right)}{a}+\frac{B\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\left(x-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)}{a}+\frac{B\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A-B)\left(x-\frac{\sin(c+dx)}{d(\cos(c+dx)+1)}\right)}{a}+\frac{B\sin(c+dx)}{ad}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
(B*Sin[c + d*x])/(a*d) + ((A - B)*(x - Sin[c + d*x]/(d*(1 + Cos[c + d*x]))
))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(B\cos(dx+c)-A+2B)+dx(A-B)}{ad}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2(A-B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2(A-B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$\frac{xA}{a}-\frac{Bx}{a}-\frac{i e^{i(dx+c)}B}{2ad}+\frac{i e^{-i(dx+c)}B}{2ad}-\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}+\frac{2iB}{da\left(e^{i(dx+c)}+1\right)}$
norman	$\frac{\frac{(A-B)x}{a}+\frac{(A-B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}-\frac{(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2(A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}+\frac{2(A-B)x\tan\left(\frac{dx}{2}\right.}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(tan(1/2*d*x+1/2*c)*(B*cos(d*x+c)-A+2*B)+d*x*(A-B))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)dx\cos(dx+c)+(A-B)dx+(B\cos(dx+c)-A+2B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
((A - B)*d*x*cos(d*x + c) + (A - B)*d*x + (B*cos(d*x + c) - A + 2*B)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(39) = 78.

Time = 0.56 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.89

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \begin{cases} \frac{Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{Adx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{B}{ad\tan^2\left(\right.} \\ \frac{x(A+B\cos(c))\cos(c)}{a\cos(c)+a} \end{cases}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise(((A*d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + A*d
*x/(a*d*tan(c/2 + d*x/2)**2 + a*d) - A*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 +
d*x/2)**2 + a*d) - A*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*
d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*d*x/(a*d*tan(c
/2 + d*x/2)**2 + a*d) + B*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a
*d) + 3*B*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*
(A + B*cos(c))*cos(c)/(a*cos(c) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(54) = 108$.

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.65

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right) - A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
-(B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a
*(cos(d*x + c) + 1))) - A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - s
in(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{\frac{(dx+c)(A-B)}{a} - \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} + \frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a
 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 41.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{x\,(A-B)}{a} + \frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

output
```
(x*(A - B))/a + (2*B*tan(c/2 + (d*x)/2))/(d*(a + a*tan(c/2 + (d*x)/2)^2))
- (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{\sin(dx+c)\,b-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+adx-bdx}{ad}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(sin(c + d*x)*b - tan((c + d*x)/2)*a + tan((c + d*x)/2)*b + a*d*x - b*d*x)
/(a*d)
```

3.42 $\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx=\frac{Bx}{a}+\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
B*x/a+(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(34) = 68.

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\sin(c+dx)\left(B\arcsin(\cos(c+dx))(1+\cos(c+dx))+(-A+B)\sqrt{\sin^2(c+dx)}\right)}{ad\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

output
```
-((Sin[c + d*x]*(B*ArcSin[Cos[c + d*x]]*(1 + Cos[c + d*x]) + (-A + B)*Sqrt
[Sin[c + d*x]^2]))/(a*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(3/2)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{a\cos(c+dx)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\Big\downarrow \text{ 3214}$$
$$(A-B)\int \frac{1}{\cos(c+dx)a+a}dx+\frac{Bx}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$(A-B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{Bx}{a}$$
$$\Big\downarrow \text{ 3127}$$
$$\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Bx}{a}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

output
```
(B*x)/a + ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{dxB+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(A-B)}{ad}$	28
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	45
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	45
risch	$\frac{Bx}{a}+\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}-\frac{2iB}{da\left(e^{i(dx+c)}+1\right)}$	54
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	85

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*(d*x*B+tan(1/2*d*x+1/2*c)*(A-B))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{Bdx\cos(dx+c)+Bdx+(A-B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
(B*d*x*cos(d*x + c) + B*d*x + (A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*
d)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \begin{cases} \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad}+\frac{Bx}{a}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((A*tan(c/2 + d*x/2)/(a*d) + B*x/a - B*tan(c/2 + d*x/2)/(a*d), Ne
(d, 0)), (x*(A + B*cos(c))/(a*cos(c) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(34) = 68.

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
(B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
 + c) + 1))) + A*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{(dx+c)B}{a} + \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)*B/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

Mupad [B] (verification not implemented)

Time = 41.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a} + \frac{B\,d\,x}{a}}{d}$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x)),x)
```

output
```
((tan(c/2 + (d*x)/2)*(A - B))/a + (B*d*x)/a)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+bdx}{ad}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(tan((c + d*x)/2)*a - tan((c + d*x)/2)*b + b*d*x)/(a*d)
```

3.43 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 44

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{A\text{arctanh}(\sin(c+dx))}{ad} - \frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
A*arctanh(sin(d*x+c))/a/d-(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(44) = 88.

Time = 0.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(A\cos\left(\frac{1}{2}(c+dx)\right)\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.\right.}{ad(1+\cos(c+dx))}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

output
```
(2*Cos[(c + d*x)/2]*(A*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-A + B)*Sec[c/2]*S
in[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$
$$\Big\downarrow \text{3457}$$
$$\frac{\int aA\sec(c+dx)dx}{a^2}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$
$$\Big\downarrow \text{27}$$
$$\frac{A\int \sec(c+dx)dx}{a}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$
$$\Big\downarrow \text{4257}$$
$$\frac{A\text{arctanh}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

output
```
(A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c +
 d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

method	result	size
parallelrisch	$\frac{-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(A-B)}{ad}$	54
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	61
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	61
risch	$-\frac{2iA}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{2iB}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}-\frac{A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{ad}$	91
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	106

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(-A*ln(tan(1/2*d*x+1/2*c)-1)+A*ln(tan(1/2*d*x+1/2*c)+1)-tan(1/2*d*x+1/2*c)
*(A-B))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{(A\cos{(dx+c)}+A)\log{(\sin{(dx+c)}+1)}-(A\cos{(dx+c)}+A)\log{(-\sin{(dx+c)}+1)}-2\,(A-B)\,\mathrm{s}}{2\,(ad\cos{(dx+c)}+ad)}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
1/2*((A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - (A*cos(d*x + c) + A)*log
(-sin(d*x + c) + 1) - 2*(A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{A\sec(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

output
```
(Integral(A*sec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*
sec(c + d*x)/(cos(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(44) = 88$.

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right) + \frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
(A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*sin(d*x + c)/
(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
(A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

Mupad [B] (verification not implemented)

Time = 41.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))),x)
```

output
```
(2*A*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{ad}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a - tan((c + d
*x)/2)*a + tan((c + d*x)/2)*b)/(a*d)
```

3.44 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 69

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\text{arctanh}(\sin(c+dx))}{ad} + \frac{(2A-B)\tan(c+dx)}{ad} - \frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
-(A-B)*arctanh(sin(d*x+c))/a/d+(2*A-B)*tan(d*x+c)/a/d-(A-B)*tan(d*x+c)/d/(
a+a*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(69) = 138.

Time = 2.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.91

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

output
```
(2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((A
- B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])))))/(a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int (a(2A-B)-a(A-B)\cos(c+dx))\sec^2(c+dx)dx}{a^2}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(2A-B)-a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a(2A-B)\int \sec^2(c+dx)dx-a(A-B)\int \sec(c+dx)dx}{a^2}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(2A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx - a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2} - \frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{-\frac{a(2A-B)\int 1d(-\tan(c+dx))}{d} - a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2} - \frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{a(2A-B)\tan(c+dx)}{d} - a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2} - \frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{a(2A-B)\tan(c+dx)}{d} - \frac{a(A-B)\text{arctanh}(\sin(c+dx))}{d}}{a^2} - \frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

output
```
-(((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))) + (-((a*(A - B)*ArcTanh
[Sin[c + d*x]])/d) + (a*(2*A - B)*Tan[c + d*x])/d)/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.35

method	result
parallelrisch	$\frac{(A-B)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-(A-B)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(A-\frac{B}{2}\right)\cos(dx+c)\right.}{da\cos(dx+c)}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(-A+B)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(-A+B)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{da}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}-\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}+\frac{(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{(A-B)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{ad}$
risch	$\frac{2i\left(A\,e^{2i(dx+c)}-B\,e^{2i(dx+c)}+A\,e^{i(dx+c)}+2A-B\right)}{da\left(e^{2i(dx+c)}+1\right)\left(e^{i(dx+c)}+1\right)}-\frac{A\ln\left(e^{i(dx+c)}+i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)B}{ad}+\frac{A\ln\left(e^{i(dx+c)}-i\right)}{ad}-$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
((A-B)*cos(d*x+c)*ln(tan(1/2*d*x+1/2*c)-1)-(A-B)*cos(d*x+c)*ln(tan(1/2*d*x
+1/2*c)+1)+2*tan(1/2*d*x+1/2*c)*((A-1/2*B)*cos(d*x+c)+1/2*A))/d/a/cos(d*x+
c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.84

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\left((A-B)\cos\left(dx+c\right)^2+(A-B)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((A-B)\cos\left(dx+c\right)^2+(A}{2\left(ad\cos\left(dx+c\right)^2+ad\,c\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*(((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(sin(d*x + c) + 1
) - ((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1)
 - 2*((2*A - B)*cos(d*x + c) + A)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*
cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{A\sec^2(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

output
```
(Integral(A*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*
x)*sec(c + d*x)**2/(cos(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(69) = 138$.

Time = 0.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.84

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{a + a\cos(c + dx)}\,dx =$$

$$-\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2\,\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right) - B\left(\frac{\log\left(\frac{\sin(dx+\cdots}{\cos(dx+\cdots}\cdots\right.}{a}\cdots\right.}{d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-(A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*
x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(log
(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{a + a\cos(c + dx)}\,dx =$$

$$-\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a\cdots}}{d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
-((A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (A - B)*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a
+ 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 35.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{2A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(A-B)}{a\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(A-B)}{a\,d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))),x)
```

output
```
(2*A*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c
/2 + (d*x)/2))*(A - B))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.51

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)\,a-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)\,b-\mathrm{co}}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x)
```

output
```
(cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a - cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*sin(c + d*x)*b - cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*sin(c + d*x)*a + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*b
 + cos(c + d*x)*a - cos(c + d*x)*b + 2*sin(c + d*x)**2*a - sin(c + d*x)**2
*b - a + b)/(cos(c + d*x)*sin(c + d*x)*a*d)
```

3.45 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 107

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{(3A-2B)\text{arctanh}(\sin(c+dx))}{2ad} - \frac{2(A-B)\tan(c+dx)}{ad} + \frac{(3A-2B)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
1/2*(3*A-2*B)*arctanh(sin(d*x+c))/a/d-2*(A-B)*tan(d*x+c)/a/d+1/2*(3*A-2*B)
*sec(d*x+c)*tan(d*x+c)/a/d-(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(107) = 214.

Time = 4.80 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.70

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(4(-A+B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((-6A+4B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]*(4*(-A + B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-
6*A + 4*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*A*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 4*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
A/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - (4*(A - B)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[
c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])))))/(2*a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

3457

$$\frac{\int (a(3A-2B)-2a(A-B)\cos(c+dx))\sec^3(c+dx)dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

3042

$$\frac{\int \frac{a(3A-2B)-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

3227

$$\frac{a(3A-2B)\int \sec^3(c+dx)dx-2a(A-B)\int \sec^2(c+dx)dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

3042

$$\frac{a(3A-2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-2a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

4254

$$\frac{\frac{2a(A-B)\int 1d(-\tan(c+dx))}{d}+a(3A-2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

24

$$\frac{a(3A-2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2a(A-B)\tan(c+dx)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

4255

$$\frac{a(3A-2B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a(A-B)\tan(c+dx)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

3042

$$\frac{a(3A-2B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a(A-B)\tan(c+dx)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a(3A-2B)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a(A-B)\tan(c+dx)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

output
```
-(((A - B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))) + ((-2*a*(
A - B)*Tan[c + d*x])/d + a*(3*A - 2*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec
[c + d*x]*Tan[c + d*x])/(2*d)))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{-3\left(A-\frac{2B}{3}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\left(A-\frac{2B}{3}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2((2A-2B)\,c}{2da(\cos(2dx+2c)+1)}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-\frac{3A}{2}+B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\left(\frac{3A}{2}-B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-\frac{3A}{2}+B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\left(\frac{3A}{2}-B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{da}$
norman	$\frac{\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}+\frac{(4A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}-\frac{(2A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{2}}-\frac{(3A-2B)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{2ad}$
risch	$-\frac{i\left(3A\,\mathrm{e}^{4i(dx+c)}-2B\,\mathrm{e}^{4i(dx+c)}+3A\,\mathrm{e}^{3i(dx+c)}-2B\,\mathrm{e}^{3i(dx+c)}+5A\,\mathrm{e}^{2i(dx+c)}-6B\,\mathrm{e}^{2i(dx+c)}+A\,\mathrm{e}^{i(dx+c)}-2B\,\mathrm{e}^{i(dx+c)}+\right.}{da\left(\mathrm{e}^{i(dx+c)}+1\right)\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*(-3*(A-2/3*B)*(cos(2*d*x+2*c)+1)*ln(tan(1/2*d*x+1/2*c)-1)+3*(A-2/3*B)*
(cos(2*d*x+2*c)+1)*ln(tan(1/2*d*x+1/2*c)+1)-2*((2*A-2*B)*cos(2*d*x+2*c)+(c
os(d*x+c)+1)*(A-2*B))*tan(1/2*d*x+1/2*c))/d/a/(cos(2*d*x+2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.46

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\left((3\,A-2\,B)\cos\left(dx+c\right)^3+(3\,A-2\,B)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((3\,A-2\,B)\cos\left(dx+c\right.}{4\left(ad\,\mathrm{co}\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/4*(((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log(sin(d*x
 + c) + 1) - ((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log
(-sin(d*x + c) + 1) - 2*(4*(A - B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c)
 - A)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{A\sec^3(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

output
```
(Integral(A*sec(c + d*x)**3/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*
x)*sec(c + d*x)**3/(cos(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(103) = 206$.

Time = 0.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.64

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2B\left(\frac{\log\left(\frac{\sin}{\cos}\right.}{}\right.}{2d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/2*(A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x +
c) + 1))) + 2*B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1
))))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{(3A-2B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(3A-2B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(3A\tan\left(\frac{1}{2}dx+\right.\right.}{}}{2d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/2*((3*A - 2*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (3*A - 2*B)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x
 + 1/2*c))/a + 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3
- A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 - 1)^2*a))/d
```

Mupad [B] (verification not implemented)

Time = 41.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(3A-2B)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(A-2B)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)} + \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\frac{3A}{2}-B\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(A-B)}{a\,d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))),x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(3*A - 2*B) - tan(c/2 + (d*x)/2)*(A - 2*B))/(d*(a -
2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4)) + (2*atanh(tan(c/2 + (
d*x)/2))*((3*A)/2 - B))/(a*d) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.66

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{4\cos(dx+c)\sin(dx+c)^2a-4\cos(dx+c)\sin(dx+c)^2b-2\cos(dx+c)a+2\cos(dx+c)b-3\log(}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x)
```

output

```
(4*cos(c + d*x)*sin(c + d*x)**2*a - 4*cos(c + d*x)*sin(c + d*x)**2*b - 2*c
os(c + d*x)*a + 2*cos(c + d*x)*b - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**3*a + 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*b + 3*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)*a - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*b + 3
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*a - 2*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**3*b - 3*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*a + 2*log(ta
n((c + d*x)/2) + 1)*sin(c + d*x)*b - 3*sin(c + d*x)**2*a + 2*sin(c + d*x)*
*2*b + 2*a - 2*b)/(2*sin(c + d*x)*a*d*(sin(c + d*x)**2 - 1))
```

3.46 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 131

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{3(A-B)\operatorname{arctanh}(\sin(c+dx))}{2ad} + \frac{(4A-3B)\tan(c+dx)}{ad} - \frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(4A-3B)\tan^3(c+dx)}{3ad}$$

output
```
-3/2*(A-B)*arctanh(sin(d*x+c))/a/d+(4*A-3*B)*tan(d*x+c)/a/d-3/2*(A-B)*sec(
d*x+c)*tan(d*x+c)/a/d-(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))+1/3
*(4*A-3*B)*tan(d*x+c)^3/a/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 490 vs. $2(131) = 262$.

Time = 5.85 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.74

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(144(A-B)\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]*(144*(A - B)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec
[c]*Sec[c + d*x]^3*(6*(A + B)*Sin[(d*x)/2] + 3*(13*A - 9*B)*Sin[(3*d*x)/2]
 - 24*A*Sin[c - (d*x)/2] + 12*B*Sin[c - (d*x)/2] - 6*A*Sin[c + (d*x)/2] +
6*B*Sin[c + (d*x)/2] - 24*A*Sin[2*c + (d*x)/2] + 24*B*Sin[2*c + (d*x)/2] +
 21*A*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3*d*x)/
2] - 9*B*Sin[2*c + (3*d*x)/2] - 9*A*Sin[3*c + (3*d*x)/2] + 9*B*Sin[3*c + (
3*d*x)/2] + 7*A*Sin[c + (5*d*x)/2] - 3*B*Sin[c + (5*d*x)/2] + A*Sin[2*c +
(5*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2] - 3*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin
[3*c + (5*d*x)/2] - 9*A*Sin[4*c + (5*d*x)/2] + 9*B*Sin[4*c + (5*d*x)/2] +
16*A*Sin[2*c + (7*d*x)/2] - 12*B*Sin[2*c + (7*d*x)/2] + 10*A*Sin[3*c + (7*
d*x)/2] - 6*B*Sin[3*c + (7*d*x)/2] + 6*A*Sin[4*c + (7*d*x)/2] - 6*B*Sin[4*
c + (7*d*x)/2])))/(48*a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3457, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int (a(4A-3B)-3a(A-B)\cos(c+dx))\sec^4(c+dx)dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(4A-3B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a(4A-3B)\int \sec^4(c+dx)dx-3a(A-B)\int \sec^3(c+dx)dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4A-3B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx-3a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{a(4A-3B)\int (\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}-3a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-3a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a(4A-3B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{-3a(A-B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a(4A-3B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 3042$$

$$\frac{-3a(A-B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a(4A-3B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 4257$$

$$\frac{-3a(A-B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a(4A-3B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

output
```
-(((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))) + (-3*a*
(A - B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
- (a*(4*A - 3*B)*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.30

method	result
parallelrisch	$\frac{27(A-B)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-27(A-B)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-}{6da(3\cos(dx+c)+\cos(3dx+}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{A}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{2A-B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\left(\frac{3A}{2}-\frac{3B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\frac{5A}{2}-}{\tan\left(\frac{dx}{2}+\right.}}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{A}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{2A-B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\left(\frac{3A}{2}-\frac{3B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\frac{5A}{2}-}{\tan\left(\frac{dx}{2}+\right.}}{da}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{ad}+\frac{(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3ad}-\frac{2(2A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{(7A-5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}+\frac{(13A-15B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{3}}$
risch	$\frac{i(9A\,e^{6i(dx+c)}-9B\,e^{6i(dx+c)}+9A\,e^{5i(dx+c)}-9B\,e^{5i(dx+c)}+24A\,e^{4i(dx+c)}-24B\,e^{4i(dx+c)}+24A\,e^{3i(dx+c)}-12B\,e^{3i(dx}}{3ad\left(e^{2i(dx+c)}+1\right)^{3}\left(e^{i(dx+c)}+1\right)}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/6*(27*(A-B)*(cos(d*x+c)+1/3*cos(3*d*x+3*c))*ln(tan(1/2*d*x+1/2*c)-1)-27*
(A-B)*(cos(d*x+c)+1/3*cos(3*d*x+3*c))*ln(tan(1/2*d*x+1/2*c)+1)+44*(1/11*(4
*A-3*B)*cos(3*d*x+3*c)+1/22*(7*A-3*B)*cos(2*d*x+2*c)+(A-6/11*B)*cos(d*x+c)
+1/2*A-3/22*B)*tan(1/2*d*x+1/2*c))/d/a/(3*cos(d*x+c)+cos(3*d*x+3*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.28

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx =$$

$$-\frac{9\left((A-B)\cos\left(dx+c\right)^{4}+(A-B)\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left((A-B)\cos\left(dx+c\right)^{4}+\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/12*(9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(sin(d*x + c
) + 1) - 9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(-sin(d*x
+ c) + 1) - 2*(4*(4*A - 3*B)*cos(d*x + c)^3 + (7*A - 3*B)*cos(d*x + c)^2 -
 (A - 3*B)*cos(d*x + c) + 2*A)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos
(d*x + c)^3)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{A\sec^4(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^4(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

output
```
(Integral(A*sec(c + d*x)**4/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*
x)*sec(c + d*x)**4/(cos(c + d*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(125) = 250$.

Time = 0.04 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.81

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{A\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3}{6d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
1/6*(A*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*
(cos(d*x + c) + 1))) - 3*B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x
+ c)/(a*(cos(d*x + c) + 1))))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.39

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{9\,(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{9\,(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{6\,\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}+\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{6\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
-1/6*(9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(A - B)*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
1/2*c))/a + 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 -
16*A*tan(1/2*d*x + 1/2*c)^3 + 12*B*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/2*d*
x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))
/d
```

Mupad [B] (verification not implemented)

Time = 43.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.16

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{(5A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(4B-\frac{16A}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(3A-B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}$$
$$-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(A-B)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(A-B)}{a\,d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + a*cos(c + d*x))),x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(5*A - 3*B) - tan(c/2 + (d*x)/2)^3*((16*A)/3 - 4*B)
+ tan(c/2 + (d*x)/2)*(3*A - B))/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan
(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6)) - (3*atanh(tan(c/2 + (d*x)/2)
)*(A - B))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.77

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^3a-9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^3b}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x)
```

output
```
(9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a - 9*cos(c + d*
x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*b - 9*cos(c + d*x)*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)*a + 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*
sin(c + d*x)*b - 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*
a + 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*b + 9*cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*a - 9*cos(c + d*x)*log(tan((c
 + d*x)/2) + 1)*sin(c + d*x)*b + 9*cos(c + d*x)*sin(c + d*x)**2*a - 9*cos(
c + d*x)*sin(c + d*x)**2*b - 6*cos(c + d*x)*a + 6*cos(c + d*x)*b + 16*sin(
c + d*x)**4*a - 12*sin(c + d*x)**4*b - 24*sin(c + d*x)**2*a + 18*sin(c + d
*x)**2*b + 6*a - 6*b)/(6*cos(c + d*x)*sin(c + d*x)*a*d*(sin(c + d*x)**2 -
1))
```

3.47 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 170

$$\begin{aligned}\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \frac{(7A-10B)x}{2a^2}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}\\&+\frac{(7A-10B)\cos(c+dx)\sin(c+dx)}{2a^2d}\\&+\frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}\\&+\frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}\\&+\frac{4(2A-3B)\sin^3(c+dx)}{3a^2d}\end{aligned}$$

output
```
1/2*(7*A-10*B)*x/a^2-4*(2*A-3*B)*sin(d*x+c)/a^2/d+1/2*(7*A-10*B)*cos(d*x+c
)*sin(d*x+c)/a^2/d+1/3*(7*A-10*B)*cos(d*x+c)^3*sin(d*x+c)/a^2/d/(1+cos(d*x
+c))+1/3*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+4/3*(2*A-3*B)*
sin(d*x+c)^3/a^2/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(170) = 340.

Time = 2.56 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.17

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(36(7A-10B)dx\cos\left(\frac{dx}{2}\right)+36(7A-10B)dx\cos\left(c+\frac{dx}{2}\right)+84Adx\cos\left(c+\frac{3dx}{2}\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*A - 10*B)*d*x*Cos[(d*x)/2] + 36*(7*A - 1
0*B)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 120*B*d*x*Cos[c
+ (3*d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] - 120*B*d*x*Cos[2*c + (3*d*x)
/2] - 381*A*Sin[(d*x)/2] + 516*B*Sin[(d*x)/2] + 147*A*Sin[c + (d*x)/2] - 1
56*B*Sin[c + (d*x)/2] - 239*A*Sin[c + (3*d*x)/2] + 342*B*Sin[c + (3*d*x)/2
] - 63*A*Sin[2*c + (3*d*x)/2] + 118*B*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c
+ (5*d*x)/2] + 30*B*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*c + (5*d*x)/2] + 30*
B*Sin[3*c + (5*d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] - 3*B*Sin[3*c + (7*d*x)/
2] + 3*A*Sin[4*c + (7*d*x)/2] - 3*B*Sin[4*c + (7*d*x)/2] + B*Sin[4*c + (9*
d*x)/2] + B*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4 \left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$\Big\downarrow$ 3456

$$\frac{\int \frac{\cos^3(c+dx)(4a(A-B)-3a(A-2B)\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a(A-B)-3a(A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3456

$$\frac{\frac{\int 3\cos^2(c+dx)\left(a^2(7A-10B)-4a^2(2A-3B)\cos(c+dx)\right)dx}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \cos^2(c+dx)\left(a^2(7A-10B)-4a^2(2A-3B)\cos(c+dx)\right)dx}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2(7A-10B)-4a^2(2A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3227

$$\frac{\frac{3\left(a^2(7A-10B)\int \cos^2(c+dx)dx-4a^2(2A-3B)\int \cos^3(c+dx)dx\right)}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(a^2(7A-10B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-4a^2(2A-3B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\frac{3\left(\frac{4a^2(2A-3B)\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}+a^2(7A-10B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(a^2(7A-10B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{4a^2(2A-3B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3\left(a^2(7A-10B)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{4a^2(2A-3B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{3\left(\frac{4a^2(2A-3B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+a^2(7A-10B)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (((7*
A - 10*B)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + (3*(a^2*(7
*A - 10*B)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) + (4*a^2*(2*A - 3*B)*
(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d))/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{-163\left(\frac{4(3A-7B)\cos(2dx+2c)}{163}+\frac{(-3A+2B)\cos(3dx+3c)}{163}-\frac{B\cos(4dx+4c)}{163}+\left(A-\frac{258B}{163}\right)\cos(dx+c)+\frac{140A}{163}-\frac{219B}{163}\right)\tan\left(\frac{dx}{2}+\right.}{48a^2d}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{8\left(-\frac{5A}{4}+\frac{5B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+8\left(-2A+\frac{10B}{3}\right)\tan\left(\frac{dx}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}}{2d\,a^2}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{8\left(-\frac{5A}{4}+\frac{5B}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+8\left(-2A+\frac{10B}{3}\right)\tan\left(\frac{dx}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}}{2d\,a^2}$
risch	$\frac{7xA}{2a^2}-\frac{5Bx}{a^2}-\frac{ie^{2i(dx+c)}A}{8a^2d}+\frac{ie^{2i(dx+c)}B}{4a^2d}+\frac{ie^{i(dx+c)}A}{a^2d}-\frac{15ie^{i(dx+c)}B}{8a^2d}-\frac{ie^{-i(dx+c)}A}{a^2d}+\frac{15ie^{-i(dx+c)}B}{8a^2d}$
norman	$\frac{\frac{(7A-10B)x}{2a}+\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{6ad}+\frac{5(7A-10B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}+\frac{5(7A-10B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}+\frac{5(7A-10B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a}+}{}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/48*(-163*(4/163*(3*A-7*B)*cos(2*d*x+2*c)+1/163*(-3*A+2*B)*cos(3*d*x+3*c)
-1/163*B*cos(4*d*x+4*c)+(A-258/163*B)*cos(d*x+c)+140/163*A-219/163*B)*tan(
1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2+168*(A-10/7*B)*x*d)/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{3\,(7\,A-10\,B)dx\cos\left(dx+c\right)^2+6\,(7\,A-10\,B)dx\cos\left(dx+c\right)+3\,(7\,A-10\,B)dx+\left(2\,B\cos\left(dx+c\right)^4\right.}{6\,\left(a^2d\cos\left(dx+c\right)^2\right.}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
1/6*(3*(7*A - 10*B)*d*x*cos(d*x + c)^2 + 6*(7*A - 10*B)*d*x*cos(d*x + c) +
 3*(7*A - 10*B)*d*x + (2*B*cos(d*x + c)^4 + (3*A - 2*B)*cos(d*x + c)^3 - 6
*(A - 2*B)*cos(d*x + c)^2 - (43*A - 66*B)*cos(d*x + c) - 32*A + 48*B)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. 2(155) = 310.

Time = 2.83 (sec) , antiderivative size = 1425, normalized size of antiderivative = 8.38

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((21*A*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) +
63*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan
(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*A*d*x*ta
n(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/
2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*A*d*x/(6*a**2*d*tan
(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x
/2)**2 + 6*a**2*d) + A*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 +
 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
 - 18*A*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(
c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*A*tan(c/2
 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4
 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 110*A*tan(c/2 + d*x/2)**3/(
6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*t
an(c/2 + d*x/2)**2 + 6*a**2*d) - 39*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 +
 d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) - 30*B*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6
+ 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d
) - 90*B*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d
*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(160) = 320$.

Time = 0.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.19

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{B\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{6\left(\frac{3\,\mathrm{si}\ldots}{\cos(\ldots}\right.}{a^2+\frac{2a^2\ldots}{(\cos\ldots}}\right.\ldots}{6\,d}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="m
axima")
```

output

```
1/6*(B*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c)
 + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a^2) - A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(co
s(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{\frac{3\,(dx+c)(7\,A-10\,B)}{a^2} - \frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 30\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 24\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 40\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a^2}}{6\,d}$$

input

```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="g
iac")
```

output

```
1/6*(3*(d*x + c)*(7*A - 10*B)/a^2 - 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 30*B*
tan(1/2*d*x + 1/2*c)^5 + 24*A*tan(1/2*d*x + 1/2*c)^3 - 40*B*tan(1/2*d*x +
1/2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) - 18*B*tan(1/2*d*x + 1/2*c))/((tan(1/2
*d*x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/
2*d*x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x +
1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 41.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{x\,(7\,A-10\,B)}{2\,a^2} - \frac{(5\,A-10\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(8\,A-\frac{40\,B}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(3\,A-6\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2\,(A-B)}{a^2}+\frac{3\,A-5\,B}{2\,a^2}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
(x*(7*A - 10*B))/(2*a^2) - (tan(c/2 + (d*x)/2)^5*(5*A - 10*B) + tan(c/2 +
(d*x)/2)^3*(8*A - (40*B)/3) + tan(c/2 + (d*x)/2)*(3*A - 6*B))/(d*(3*a^2*ta
n(c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6
 + a^2)) - (tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (3*A - 5*B)/(2*a^2)))/d
+ (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.19

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{-2\cos(dx+c)\sin(dx+c)^4 b-9\cos(dx+c)\sin(dx+c)^2 a+18\cos(dx+c)\sin(dx+c)^2 b+21\cos(d}{}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**4*b - 9*cos(c + d*x)*sin(c + d*x)**2*a +
18*cos(c + d*x)*sin(c + d*x)**2*b + 21*cos(c + d*x)*sin(c + d*x)*a*d*x - 3
0*cos(c + d*x)*sin(c + d*x)*b*d*x - 2*cos(c + d*x)*a + 2*cos(c + d*x)*b -
3*sin(c + d*x)**4*a + 4*sin(c + d*x)**4*b - 31*sin(c + d*x)**2*a + 46*sin(
c + d*x)**2*b + 21*sin(c + d*x)*a*d*x - 30*sin(c + d*x)*b*d*x + 2*a - 2*b)
/(6*sin(c + d*x)*a**2*d*(cos(c + d*x) + 1))
```

3.48 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 147

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = -\frac{(4A-7B)x}{2a^2}+\frac{2(5A-8B)\sin(c+dx)}{3a^2d} - \frac{(4A-7B)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{(5A-8B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output

```
-1/2*(4*A-7*B)*x/a^2+2/3*(5*A-8*B)*sin(d*x+c)/a^2/d-1/2*(4*A-7*B)*cos(d*x+
c)*sin(d*x+c)/a^2/d+1/3*(5*A-8*B)*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(1+cos(d*x
+c))+1/3*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 315 vs. $2(147) = 294$.

Time = 2.35 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.14

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-36(4A-7B)dx\cos\left(\frac{dx}{2}\right)-36(4A-7B)dx\cos\left(c+\frac{dx}{2}\right)-48Adx\cos\left(c+\frac{3dx}{2}\right)\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(-36*(4*A - 7*B)*d*x*Cos[(d*x)/2] - 36*(4*A - 7
*B)*d*x*Cos[c + (d*x)/2] - 48*A*d*x*Cos[c + (3*d*x)/2] + 84*B*d*x*Cos[c +
(3*d*x)/2] - 48*A*d*x*Cos[2*c + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2]
 + 264*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] - 120*A*Sin[c + (d*x)/2] + 147*
B*Sin[c + (d*x)/2] + 164*A*Sin[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] +
 36*A*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 12*A*Sin[2*c + (5
*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 12*A*Sin[3*c + (5*d*x)/2] - 15*B*Si
n[3*c + (5*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2]))
/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3456, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(2A-5B)\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a(A-B)-a(2A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \cos(c+dx)(2a^2(5A-8B)-3a^2(4A-7B)\cos(c+dx))dx}{a^2}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(2a^2(5A-8B)-3a^2(4A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\frac{\frac{2a^2(5A-8B)\sin(c+dx)}{d}-\frac{3a^2(4A-7B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3}{2}a^2x(4A-7B)}{a^2}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (((5*
A - 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + ((-3*a^2*(4
*A - 7*B)*x)/2 + (2*a^2*(5*A - 8*B)*Sin[c + d*x])/d - (3*a^2*(4*A - 7*B)*C
os[c + d*x]*Sin[c + d*x])/(2*d))/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\frac{3(A-B)\cos(2dx+2c)}{28}+\frac{3B\cos(3dx+3c)}{112}+\left(A-\frac{163B}{112}\right)\cos(dx+c)+\frac{23A}{28}-\frac{5B}{4}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2-6xd\left(A-\frac{7B}{4}\right.}{3a^2d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{4\left(\left(\frac{5B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{3B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{2d\,a^2}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{4\left(\left(\frac{5B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{3B}{2}-A\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{2d\,a^2}$
risch	$-\frac{2xA}{a^2}+\frac{7Bx}{2a^2}-\frac{ie^{2i(dx+c)}B}{8a^2d}-\frac{ie^{i(dx+c)}A}{2a^2d}+\frac{ie^{i(dx+c)}B}{a^2d}+\frac{ie^{-i(dx+c)}A}{2a^2d}-\frac{ie^{-i(dx+c)}B}{a^2d}+\frac{ie^{-2i(dx+c)}B}{8a^2d}$
norman	$\frac{\frac{(11A-18B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}-\frac{(4A-7B)x}{2a}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{6ad}-\frac{2(4A-7B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{3(4A-7B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}-\frac{2(4A}{}}{}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/3*(7*tan(1/2*d*x+1/2*c)*(3/28*(A-B)*cos(2*d*x+2*c)+3/112*B*cos(3*d*x+3*c
)+(A-163/112*B)*cos(d*x+c)+23/28*A-5/4*B)*sec(1/2*d*x+1/2*c)^2-6*x*d*(A-7/
4*B))/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = -\frac{3\,(4\,A-7\,B)dx\cos\left(dx+c\right)^2+6\,(4\,A-7\,B)dx\cos\left(dx+c\right)+3\,(4\,A-7\,B)dx-\left(3\,B\cos\left(dx+c\right)^3+\cdots\right.}{6\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(a\cdots\right.\right.}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/6*(3*(4*A - 7*B)*d*x*cos(d*x + c)^2 + 6*(4*A - 7*B)*d*x*cos(d*x + c) +
3*(4*A - 7*B)*d*x - (3*B*cos(d*x + c)^3 + 6*(A - B)*cos(d*x + c)^2 + (28*A
 - 43*B)*cos(d*x + c) + 20*A - 32*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. 2(136) = 272.

Time = 1.74 (sec) , antiderivative size = 843, normalized size of antiderivative = 5.73

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((-12*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 1
2*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 24*A*d*x*tan(c/2 + d*x/2)**2/(6
*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) -
12*A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6
*a**2*d) - A*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 13*A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(
c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 41*A*tan(c/2
 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) + 27*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a*
*2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*B*d*x*tan(c/2 + d*x/2)**4/(6*a**
2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*B
*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2
 + d*x/2)**2 + 6*a**2*d) + 21*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**
2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + B*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(
c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*B*tan(c/2
 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) - 71*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12
*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 39*B*tan(c/2 + d*x/2)/(6*a**2*d*
tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0))
, (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(137) = 274$.

Time = 0.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.93

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx}{(\cos(dx+}}{a^2}\right.}{6d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
-1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin
(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^2) - A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(c
os(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 +
 12*sin(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x
 + c) + 1))))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = -\frac{\frac{3\,(dx+c)(4\,A-7\,B)}{a^2} - \frac{6\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2} + \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-}{}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-1/6*(3*(d*x + c)*(4*A - 7*B)/a^2 - 6*(2*A*tan(1/2*d*x + 1/2*c)^3 - 5*B*ta
n(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c)
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B
*a^4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 21*B*a^4*tan
(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 41.91 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3(A-B)}{2a^2}+\frac{2A-4B}{2a^2}\right)}{d}-\frac{x\,(4A-7B)}{2a^2} + \frac{(2A-5B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(A-B)}{6a^2d}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^2) + (2*A - 4*B)/(2*a^2)))/d - (x*(4
*A - 7*B))/(2*a^2) + (tan(c/2 + (d*x)/2)^3*(2*A - 5*B) + tan(c/2 + (d*x)/2
)*(2*A - 3*B))/(d*(2*a^2*tan(c/2 + (d*x)/2)^2 + a^2*tan(c/2 + (d*x)/2)^4 +
 a^2)) - (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{6\cos(dx+c)\sin(dx+c)^2a-9\cos(dx+c)\sin(dx+c)^2b-12\cos(dx+c)\sin(dx+c)adx+21\cos(d}{}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
(6*cos(c + d*x)*sin(c + d*x)**2*a - 9*cos(c + d*x)*sin(c + d*x)**2*b - 12*
cos(c + d*x)*sin(c + d*x)*a*d*x + 21*cos(c + d*x)*sin(c + d*x)*b*d*x + 2*c
os(c + d*x)*a - 2*cos(c + d*x)*b - 3*sin(c + d*x)**4*b + 22*sin(c + d*x)**
2*a - 31*sin(c + d*x)**2*b - 12*sin(c + d*x)*a*d*x + 21*sin(c + d*x)*b*d*x
 - 2*a + 2*b)/(6*sin(c + d*x)*a**2*d*(cos(c + d*x) + 1))
```

3.49 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 99

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-2B)x}{a^2} - \frac{(A-4B)\sin(c+dx)}{3a^2d} - \frac{(A-2B)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
(A-2*B)*x/a^2-1/3*(A-4*B)*sin(d*x+c)/a^2/d-(A-2*B)*sin(d*x+c)/a^2/d/(1+cos
(d*x+c))+1/3*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-2(5A-8B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c\right.\right.}{3a^2d(1+\cos(c+dx))^2}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
(2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] - 2*(5*A - 8*B)*Cos[(c
+ d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((A - 2*B)*d*x +
B*Sin[c + d*x]) + (A - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c
+ d*x])^2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3456, 3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos(c+dx)(2a(A-B)-a(A-4B)\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a(A-B)-a(A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{2a(A-B)\cos(c+dx)-a(A-4B)\cos^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)-a(A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\frac{\int \frac{3a^2(A-2B)\cos(c+dx)}{\cos(c+dx)a+a}dx}{a}-\frac{(A-4B)\sin(c+dx)}{d}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3a(A-2B)\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}dx-\frac{(A-4B)\sin(c+dx)}{d}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a(A-2B)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{(A-4B)\sin(c+dx)}{d}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{3a(A-2B)\left(\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx\right)-\frac{(A-4B)\sin(c+dx)}{d}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a(A-2B)\left(\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)-\frac{(A-4B)\sin(c+dx)}{d}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{3a(A-2B)\left(\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)-\frac{(A-4B)\sin(c+dx)}{d}}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (-(((
A - 4*B)*Sin[c + d*x])/d) + 3*a*(A - 2*B)*(x/a - Sin[c + d*x]/(d*(a + a*Co
s[c + d*x]))))/(3*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-20A+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}(3B\cos(2dx+2c)+28B\cos(dx+c)+2A+23B)\right)+12dx(A-2B)}{12a^{2}d}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{4B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4(A-2B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d\,a^{2}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{4B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4(A-2B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d\,a^{2}}$
risch	$\frac{xA}{a^{2}}-\frac{2Bx}{a^{2}}-\frac{i\,e^{i(dx+c)}B}{2a^{2}d}+\frac{i\,e^{-i(dx+c)}B}{2a^{2}d}-\frac{2i\left(6A\,e^{2i(dx+c)}-9B\,e^{2i(dx+c)}+9A\,e^{i(dx+c)}-15B\,e^{i(dx+c)}+5A-8B\right.}{3d\,a^{2}\left(e^{i(dx+c)}+1\right)^{3}}$
norman	$\frac{\frac{(A-2B)x}{a}+\frac{(A-2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{a}-\frac{3(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{(A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}+\frac{3(A-2B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{3(A-2B)x\tan}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/12*(tan(1/2*d*x+1/2*c)*(-20*A+sec(1/2*d*x+1/2*c)^2*(3*B*cos(2*d*x+2*c)+2
8*B*cos(d*x+c)+2*A+23*B))+12*d*x*(A-2*B))/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{3\,(A-2\,B)dx\cos{(dx+c)}^2 + 6\,(A-2\,B)dx\cos{(dx+c)} + 3\,(A-2\,B)dx + \left(3\,B\cos{(dx+c)}^2 - (5\,A - \right.}{3\,\left(a^2d\cos{(dx+c)}^2 + 2\,a^2d\cos{(dx+c)} + a^2d\right)}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
1/3*(3*(A - 2*B)*d*x*cos(d*x + c)^2 + 6*(A - 2*B)*d*x*cos(d*x + c) + 3*(A
- 2*B)*d*x + (3*B*cos(d*x + c)^2 - (5*A - 14*B)*cos(d*x + c) - 4*A + 10*B)
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(90) = 180.

Time = 1.18 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.15

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{6Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{6Adx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{8A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{9A\tan\left(\frac{c}{2}+\right.}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right.} \\ \frac{x(A+B\cos{(c)})\cos^2{(c)}}{(a\cos{(c)}+a)^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((6*A*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a
**2*d) + 6*A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + A*tan(c/2 + d
*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 8*A*tan(c/2 + d*x/2)*
*3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 9*A*tan(c/2 + d*x/2)/(6*a**
2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x*tan(c/2 + d*x/2)**2/(6*a**2
*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**
2 + 6*a**2*d) - B*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 14*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
 + 27*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d,
0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(95) = 190$.

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.93

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c}{(\cos(dx+c)}}{a^2}\right.}{6d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x
 + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))
) - A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{6\,(dx+c)(A-2\,B)}{a^2}+\frac{12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
1/6*(6*(d*x + c)*(A - 2*B)/a^2 + 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
 1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x +
1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

Mupad [B] (verification not implemented)

Time = 41.98 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{x\,(A-2\,B)}{a^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{A-3\,B}{2\,a^2}\right)}{d} + \frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
(x*(A - 2*B))/a^2 - (tan(c/2 + (d*x)/2)*((A - B)/a^2 + (A - 3*B)/(2*a^2)))
/d + (2*B*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2)) + (tan(
c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.46

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 b-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a+14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 adx-12\,\mathrm{ta}}{6a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
(tan((c + d*x)/2)**5*a - tan((c + d*x)/2)**5*b - 8*tan((c + d*x)/2)**3*a +
 14*tan((c + d*x)/2)**3*b + 6*tan((c + d*x)/2)**2*a*d*x - 12*tan((c + d*x)
/2)**2*b*d*x - 9*tan((c + d*x)/2)*a + 27*tan((c + d*x)/2)*b + 6*a*d*x - 12
*b*d*x)/(6*a**2*d*(tan((c + d*x)/2)**2 + 1))
```

3.50 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 70

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{Bx}{a^2} + \frac{(2A-5B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
B*x/a^2+1/3*(2*A-5*B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)
/d/(a+a*cos(d*x+c))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(70) = 140.

Time = 1.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.19

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(9Bdx\cos\left(\frac{dx}{2}\right)+9Bdx\cos\left(c+\frac{dx}{2}\right)+3Bdx\cos\left(c+\frac{3dx}{2}\right)+3Bdx\cos\left(2c+\frac{3dx}{2}\right)\right.}{}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(9*B*d*x*Cos[(d*x)/2] + 9*B*d*x*Cos[c + (d*x)
/2] + 3*B*d*x*Cos[c + (3*d*x)/2] + 3*B*d*x*Cos[2*c + (3*d*x)/2] + 6*A*Sin[
(d*x)/2] - 18*B*Sin[(d*x)/2] - 6*A*Sin[c + (d*x)/2] + 12*B*Sin[c + (d*x)/2
] + 4*A*Sin[c + (3*d*x)/2] - 10*B*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{2a(A-B)+3aB\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a(A-B)+3aB\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a(A-B)+3aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\downarrow$ 3214

$$\frac{a(2A-5B)\int \frac{1}{\cos(c+dx)a+a}dx+3Bx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\downarrow$ 3042

$$\frac{a(2A-5B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+3Bx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\downarrow$ 3127

$$\frac{\frac{a(2A-5B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+3Bx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + (3*B*x + (a*(2*A
- 5*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{(-A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+(3A-9B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+6dxB}{6a^{2}d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+4B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+4B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
risch	$\frac{Bx}{a^{2}}+\frac{2i\left(3A\,\mathrm{e}^{2i(dx+c)}-6B\,\mathrm{e}^{2i(dx+c)}+3A\,\mathrm{e}^{i(dx+c)}-9B\,\mathrm{e}^{i(dx+c)}+2A-5B\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{a}+\frac{2Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{(A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}+\frac{(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{6ad}+\frac{(5A-17B}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/6*((-A+B)*tan(1/2*d*x+1/2*c)^3+(3*A-9*B)*tan(1/2*d*x+1/2*c)+6*d*x*B)/a^2
/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{3\,Bdx\cos{(dx+c)}^2+6\,Bdx\cos{(dx+c)}+3\,Bdx+((2\,A-5\,B)\cos{(dx+c)}+A-4\,B)\sin{(dx+c)}}{3\,\left(a^2d\cos{(dx+c)}^2+2\,a^2d\cos{(dx+c)}+a^2d\right)}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fri
cas")
```

output
```
1/3*(3*B*d*x*cos(d*x + c)^2 + 6*B*d*x*cos(d*x + c) + 3*B*d*x + ((2*A - 5*B
)*cos(d*x + c) + A - 4*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \begin{cases} -\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}+\frac{Bx}{a^2}+\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}-\frac{3B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos{(c)})\cos{(c)}}{(a\cos{(c)}+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((-A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*
d) + B*x/a**2 + B*tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*B*tan(c/2 + d*x/2)/(2
*a**2*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.71

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = -\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="max
ima")
```

output
```
-1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - A*(3*sin(
d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.23

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{6\,(dx+c)B}{a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gia
c")
```

output
```
1/6*(6*(d*x + c)*B/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
 + 1/2*c)^3 - 3*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*a^4*tan(1/2*d*x + 1/2*c))
/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 41.89 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,B\,d\,x}{6\,a^2\,d}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
(3*A*tan(c/2 + (d*x)/2) - 9*B*tan(c/2 + (d*x)/2) - A*tan(c/2 + (d*x)/2)^3
+ B*tan(c/2 + (d*x)/2)^3 + 6*B*d*x)/(6*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+6bdx}{6a^2d}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
( - tan((c + d*x)/2)**3*a + tan((c + d*x)/2)**3*b + 3*tan((c + d*x)/2)*a -
 9*tan((c + d*x)/2)*b + 6*b*d*x)/(6*a**2*d)
```

3.51 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(A+2B)\sin(c+dx)}{3d\,(a^2+a^2\cos(c+dx))}$$

output
```
1/3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*(A+2*B)*sin(d*x+c)/d/(a^2+a^
2*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(2A+B+(A+2B)\cos(c+dx))\sin(c+dx)}{3a^2d(1+\cos(c+dx))^2}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

output
```
((2*A + B + (A + 2*B)*Cos[c + d*x])*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*
x])^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(A+2B)\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+2B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A+2B)\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((A + 2*B)*Sin[c + d
*x])/(3*a*d*(a + a*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$\frac{\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\left(2A+B+\cos\left(dx+c\right)\left(A+2B\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^{2}d}$	46
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{2d\,a^{2}}$	60
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{2d\,a^{2}}$	60
risch	$\frac{2i\left(3B\,{\mathrm e}^{2i\left(dx+c\right)}+3A\,{\mathrm e}^{i\left(dx+c\right)}+3B\,{\mathrm e}^{i\left(dx+c\right)}+A+2B\right)}{3d\,a^{2}\left({\mathrm e}^{i\left(dx+c\right)}+1\right)^{3}}$	64
norman	$\frac{\frac{\left(A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}+\frac{\left(A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\left(2A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}$	89

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2/d*sec(1/2*d*x+1/2*c)^2*(2*A+B+cos(d*x+c)*(A+2*B))*tan(1/2*d*x+1/2*
c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{((A+2\,B)\cos(dx+c)+2\,A+B)\sin(dx+c)}{3\,\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*((A + 2*B)*cos(d*x + c) + 2*A + B)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2
+ 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \begin{cases} \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d
) - B*tan(c/2 + d*x/2)**3/(6*a**2*d) + B*tan(c/2 + d*x/2)/(2*a**2*d), Ne(d
, 0)), (x*(A + B*cos(c))/(a*cos(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}+\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(A*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/a^2 + B*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(A*tan(1/2*d*x + 1/2*c)^3 - B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x
 + 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 41.59 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(A-B)}{6\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(A+B)}{2\,a^2\,d}$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d) + (tan(c/2 + (d*x)/2)*(A + B))/(2
*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+3a+3b\right)}{6a^2d}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
(tan((c + d*x)/2)*(tan((c + d*x)/2)**2*a - tan((c + d*x)/2)**2*b + 3*a + 3
*b))/(6*a**2*d)
```

3.52 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 79

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{A\text{arctanh}(\sin(c+dx))}{a^2d} - \frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
A*arctanh(sin(d*x+c))/a^2/d-1/3*(4*A-B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/
3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(79) = 158.

Time = 1.02 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.15

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6A\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

output
```
(-2*Cos[(c + d*x)/2]*(6*A*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A - B)*Sec[c/2
]*Sin[(d*x)/2] + 2*(4*A - B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (A
 - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(3aA-a(A-B)\cos(c+dx))\sec(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3aA-a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int 3a^2A\sec(c+dx)dx}{a^2}-\frac{(4A-B)\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3A\int\sec(c+dx)dx-\frac{(4A-B)\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{3A\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(4A-B)\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 4257

$$\frac{\frac{3A\text{arctanh}(\sin(c+dx))}{d}-\frac{(4A-B)\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((3*A*ArcTanh[Sin
[c + d*x]])/d - ((4*A - B)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{-6A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2(A-B)+9A-3B\right)}{6a^2d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}{2d\,a^2}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}{2d\,a^2}$
risch	$-\frac{2i(3A\,e^{2i(dx+c)}+9A\,e^{i(dx+c)}-3B\,e^{i(dx+c)}+4A-B)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}+\frac{A\ln(e^{i(dx+c)}+i)}{a^2d}-\frac{A\ln(e^{i(dx+c)}-i)}{a^2d}$
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{6ad}-\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{(5A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a^2d}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/6*(-6*A*ln(tan(1/2*d*x+1/2*c)-1)+6*A*ln(tan(1/2*d*x+1/2*c)+1)-tan(1/2*d*
x+1/2*c)*(tan(1/2*d*x+1/2*c)^2*(A-B)+9*A-3*B))/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$=\frac{3\left(A\cos\left(dx+c\right)^2+2\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(A\cos\left(dx+c\right)^2+2\,A\cos\left(dx+c\right)\right.}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx\right.\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fri
cas")
```

output
```
1/6*(3*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 3
*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*((4*
A - B)*cos(d*x + c) + 5*A - 2*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + a\cos(c + dx))^2}\,dx = \frac{\int \frac{A\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

output
```
(Integral(A*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + Inte
gral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x
))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + a\cos(c + dx))^2}\,dx = -\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right) - \frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="max
ima")
```

output
```
-1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*(3*sin(d*x + c)/(cos(d*x + c) +
1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} - \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ba^4\,\mathrm{ta}}{a^6}}{6\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="gia
c")
```

output
```
1/6*(6*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*A*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1
/2*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 3*B*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

Mupad [B] (verification not implemented)

Time = 41.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A}{a^2}+\frac{A-B}{2\,a^2}\right)}{d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^2),x)
```

output
```
(2*A*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3*(A - B))/(
6*a^2*d) - (tan(c/2 + (d*x)/2)*(A/a^2 + (A - B)/(2*a^2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{-6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3b-9\tan\left(\frac{dx}{2}\right.}{6a^2d}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x)
```

output
```
( - 6*log(tan((c + d*x)/2) - 1)*a + 6*log(tan((c + d*x)/2) + 1)*a - tan((c
 + d*x)/2)**3*a + tan((c + d*x)/2)**3*b - 9*tan((c + d*x)/2)*a + 3*tan((c
+ d*x)/2)*b)/(6*a**2*d)
```

3.53 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 107

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(2A-B)\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{2(5A-2B)\tan(c+dx)}{3a^2d} - \frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
-(2*A-B)*arctanh(sin(d*x+c))/a^2/d+2/3*(5*A-2*B)*tan(d*x+c)/a^2/d-(2*A-B)*
tan(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. $2(107) = 214$.

Time = 2.66 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.47

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(7A-4B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
(2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + 2*(7*A - 4*B)*Cos[(c
+ d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((2*A - B)*(Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (A
 - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{(a(4A-B)-2a(A-B)\cos(c+dx))\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(4A-B)-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\int (2a^2(5A-2B)-3a^2(2A-B)\cos(c+dx))\sec^2(c+dx)dx}{a^2}-\frac{3(2A-B)\tan(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{2a^2(5A-2B)-3a^2(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{3(2A-B)\tan(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3227

$$\frac{\frac{2a^2(5A-2B)\int \sec^2(c+dx)dx-3a^2(2A-B)\int \sec(c+dx)dx}{a^2}-\frac{3(2A-B)\tan(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2a^2(5A-2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-3a^2(2A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3(2A-B)\tan(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 4254

$$\frac{\frac{-\frac{2a^2(5A-2B)\int 1d(-\tan(c+dx))}{d}-3a^2(2A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3(2A-B)\tan(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 24

$$\frac{\frac{\frac{2a^2(5A-2B)\tan(c+dx)}{d}-3a^2(2A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3(2A-B)\tan(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 4257

$$\frac{\frac{\frac{2a^2(5A-2B)\tan(c+dx)}{d}-\frac{3a^2(2A-B)\text{arctanh}(\sin(c+dx))}{d}}{a^2}-\frac{3(2A-B)\tan(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((-3*(2*A - B)*Ta
n[c + d*x])/(d*(1 + Cos[c + d*x])) + ((-3*a^2*(2*A - B)*ArcTanh[Sin[c + d*
x]])/d + (2*a^2*(5*A - 2*B)*Tan[c + d*x])/d)/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{12\cos(dx+c)\left(A-\frac{B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-12\cos(dx+c)\left(A-\frac{B}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+14\left(\left(\frac{5A}{14}-\frac{B}{7}\right)\cos(2dx+2c)+\right.}{6d\,a^2\cos(dx+c)}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+(2B-4A)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{2d\,a^2}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+(2B-4A)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{2d\,a^2}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6ad}-\frac{3(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{(5A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2ad}-\frac{(13A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)a}+\frac{(2A-B)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{a^2d}$
risch	$\frac{2i\left(6A\,e^{4i(dx+c)}-3B\,e^{4i(dx+c)}+18A\,e^{3i(dx+c)}-9B\,e^{3i(dx+c)}+22A\,e^{2i(dx+c)}-7B\,e^{2i(dx+c)}+24A\,e^{i(dx+c)}-9B\,e^{i(dx+\right.}{3a^2d\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/6*(12*cos(d*x+c)*(A-1/2*B)*ln(tan(1/2*d*x+1/2*c)-1)-12*cos(d*x+c)*(A-1/2
*B)*ln(tan(1/2*d*x+1/2*c)+1)+14*((5/14*A-1/7*B)*cos(2*d*x+2*c)+(A-5/14*B)*
cos(d*x+c)+4/7*A-1/7*B)*sec(1/2*d*x+1/2*c)^2*tan(1/2*d*x+1/2*c))/d/a^2/cos
(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(103) = 206$.

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.93

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{3\left((2A-B)\cos(dx+c)^3+2(2A-B)\cos(dx+c)^2+(2A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/6*(3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A - B)*cos(d*x + c)^2 + (2*A - B)
*cos(d*x + c))*log(sin(d*x + c) + 1) - 3*((2*A - B)*cos(d*x + c)^3 + 2*(2*
A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2
*(2*(5*A - 2*B)*cos(d*x + c)^2 + (14*A - 5*B)*cos(d*x + c) + 3*A)*sin(d*x
+ c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{A\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

output
```
(Integral(A*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + I
ntegral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) +
 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.28

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + a\cos(c + dx))^2}\,dx$$
$$= \frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2} + \frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right) - B}{6\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/
(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) +
1) - 1)/a^2))/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + a\cos(c + dx))^2}\,dx =$$
$$-\frac{\frac{6\,(2\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{6\,(2\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2} - \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\,\mathrm{ta}}{}}{6\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-1/6*(6*(2*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(2*A - B)*log
(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 - 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d
*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) - 9*B*a^4*tan(1/2*d*x + 1/2*
c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 41.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.15

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{3A-B}{2a^2}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(A-B)}{6a^2d} - \frac{2A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(2A-B)}{a^2d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^2),x)
```

output
```
(tan(c/2 + (d*x)/2)*((A - B)/a^2 + (3*A - B)/(2*a^2)))/d + (tan(c/2 + (d*x
)/2)^3*(A - B))/(6*a^2*d) - (2*A*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*
x)/2)^2 - a^2)) - (2*atanh(tan(c/2 + (d*x)/2))*(2*A - B))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.50

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x)
```

output
```
(12*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2*a - 6*log(tan((c + d*x)/
2) - 1)*tan((c + d*x)/2)**2*b - 12*log(tan((c + d*x)/2) - 1)*a + 6*log(tan
((c + d*x)/2) - 1)*b - 12*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*a
+ 6*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*b + 12*log(tan((c + d*x)
/2) + 1)*a - 6*log(tan((c + d*x)/2) + 1)*b + tan((c + d*x)/2)**5*a - tan((
c + d*x)/2)**5*b + 14*tan((c + d*x)/2)**3*a - 8*tan((c + d*x)/2)**3*b - 27
*tan((c + d*x)/2)*a + 9*tan((c + d*x)/2)*b)/(6*a**2*d*(tan((c + d*x)/2)**2
 - 1))
```

3.54 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 152

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(7A-4B)\text{arctanh}(\sin(c+dx))}{2a^2d} - \frac{2(8A-5B)\tan(c+dx)}{3a^2d} + \frac{(7A-4B)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
1/2*(7*A-4*B)*arctanh(sin(d*x+c))/a^2/d-2/3*(8*A-5*B)*tan(d*x+c)/a^2/d+1/2
*(7*A-4*B)*sec(d*x+c)*tan(d*x+c)/a^2/d-1/3*(8*A-5*B)*sec(d*x+c)*tan(d*x+c)
/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 496 vs. $2(152) = 304$.

Time = 4.49 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.26

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{96(7A-4B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/48*(96*(7*A - 4*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Se
c[c/2]*Sec[c]*Sec[c + d*x]^2*(-14*(A - B)*Sin[(d*x)/2] + (97*A - 64*B)*Sin
[(3*d*x)/2] - 126*A*Sin[c - (d*x)/2] + 84*B*Sin[c - (d*x)/2] + 42*A*Sin[c
+ (d*x)/2] - 42*B*Sin[c + (d*x)/2] - 98*A*Sin[2*c + (d*x)/2] + 56*B*Sin[2*
c + (d*x)/2] - 3*A*Sin[c + (3*d*x)/2] + 6*B*Sin[c + (3*d*x)/2] + 37*A*Sin[
2*c + (3*d*x)/2] - 34*B*Sin[2*c + (3*d*x)/2] - 63*A*Sin[3*c + (3*d*x)/2] +
 36*B*Sin[3*c + (3*d*x)/2] + 75*A*Sin[c + (5*d*x)/2] - 48*B*Sin[c + (5*d*x
)/2] + 15*A*Sin[2*c + (5*d*x)/2] - 6*B*Sin[2*c + (5*d*x)/2] + 39*A*Sin[3*c
 + (5*d*x)/2] - 30*B*Sin[3*c + (5*d*x)/2] - 21*A*Sin[4*c + (5*d*x)/2] + 12
*B*Sin[4*c + (5*d*x)/2] + 32*A*Sin[2*c + (7*d*x)/2] - 20*B*Sin[2*c + (7*d*
x)/2] + 12*A*Sin[3*c + (7*d*x)/2] - 6*B*Sin[3*c + (7*d*x)/2] + 20*A*Sin[4*
c + (7*d*x)/2] - 14*B*Sin[4*c + (7*d*x)/2]))/(a^2*d*(1 + Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 3457$$

$$\frac{\int \frac{(a(5A-2B)-3a(A-B)\cos(c+dx))\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(5A-2B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\int (3a^2(7A-4B)-2a^2(8A-5B)\cos(c+dx))\sec^3(c+dx)dx}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{3a^2(7A-4B)-2a^2(8A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{3a^2(7A-4B)\int \sec^3(c+dx)dx-2a^2(8A-5B)\int \sec^2(c+dx)dx}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3a^2(7A-4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-2a^2(8A-5B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{\frac{2a^2(8A-5B)\int 1d(-\tan(c+dx))}{d}+3a^2(7A-4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{3a^2(7A-4B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2a^2(8A-5B)\tan(c+dx)}{d}}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3a^2(7A-4B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2(8A-5B)\tan(c+dx)}{d}}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3a^2(7A-4B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2(8A-5B)\tan(c+dx)}{d}}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{3a^2(7A-4B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2(8A-5B)\tan(c+dx)}{d}}{a^2}-\frac{(8A-5B)\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*((A - B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + (-((
(8*A - 5*B)*Sec[c + d*x]*Tan[c + d*x])/(d*(1 + Cos[c + d*x]))) + ((-2*a^2*
(8*A - 5*B)*Tan[c + d*x])/d + 3*a^2*(7*A - 4*B)*(ArcTanh[Sin[c + d*x]]/(2*
d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{-42\left(A-\frac{4B}{7}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+42\left(A-\frac{4B}{7}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-60\tan\left(\frac{dx}{2}\right.}{12d\,a^2(\cos(2dx+2}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+(-7A+4B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{-5A+2B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2d\,a^2}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+(-7A+4B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{-5A+2B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2d\,a^2}$
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6ad}-\frac{(10A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3ad}-\frac{(13A-9B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{2(13A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3ad}+\frac{(16A-7B)\tan\left(\frac{dx}{2}+\right.}{3ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2a}$
risch	$-\frac{i\left(21A\,e^{6i(dx+c)}-12B\,e^{6i(dx+c)}+63A\,e^{5i(dx+c)}-36B\,e^{5i(dx+c)}+98A\,e^{4i(dx+c)}-56B\,e^{4i(dx+c)}+126A\,e^{3i(dx+c)}-84\right.}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/12*(-42*(A-4/7*B)*(cos(2*d*x+2*c)+1)*ln(tan(1/2*d*x+1/2*c)-1)+42*(A-4/7*
B)*(cos(2*d*x+2*c)+1)*ln(tan(1/2*d*x+1/2*c)+1)-60*tan(1/2*d*x+1/2*c)*((43/
60*A-7/15*B)*cos(2*d*x+2*c)+(4/15*A-1/6*B)*cos(3*d*x+3*c)+(A-7/10*B)*cos(d
*x+c)+37/60*A-7/15*B)*sec(1/2*d*x+1/2*c)^2)/d/a^2/(cos(2*d*x+2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.50

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{3\left((7A-4B)\cos(dx+c)^4+2(7A-4B)\cos(dx+c)^3+(7A-4B)\cos(dx+c)^2\right)\log(\sin(dx+c)-}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
1/12*(3*((7*A - 4*B)*cos(d*x + c)^4 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A
- 4*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((7*A - 4*B)*cos(d*x + c)
^4 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*log(-sin(d
*x + c) + 1) - 2*(4*(8*A - 5*B)*cos(d*x + c)^3 + (43*A - 28*B)*cos(d*x + c
)^2 + 6*(A - B)*cos(d*x + c) - 3*A)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 +
2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{A\sec^3(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

output
```
(Integral(A*sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + I
ntegral(B*cos(c + d*x)*sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) +
 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(142) = 284$.

Time = 0.07 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-}{6}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
-1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin
(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*((15*
sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2
 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(
cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.30

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{3\,(7\,A-4\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{3\,(7\,A-4\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{6\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,A\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{6\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
1/6*(3*(7*A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A - 4*B)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2
*B*tan(1/2*d*x + 1/2*c)^3 - 3*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^
3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^
4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 40.87 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(5A-2B)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3A-2B)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^2\right)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3(A-B)}{2a^2}+\frac{4A-2B}{2a^2}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(A-B)}{6a^2d} + \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(7A-4B)}{a^2d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^2),x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(5*A - 2*B) - tan(c/2 + (d*x)/2)*(3*A - 2*B))/(d*(a^
2*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (tan(c/2 + (
d*x)/2)*((3*(A - B))/(2*a^2) + (4*A - 2*B)/(2*a^2)))/d - (tan(c/2 + (d*x)/
2)^3*(A - B))/(6*a^2*d) + (atanh(tan(c/2 + (d*x)/2))*(7*A - 4*B))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.71

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{-21\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+42\log\left(\tan\left(\frac{dx}{2}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x)
```

output
```
( - 21*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**4*a + 12*log(tan((c + d
*x)/2) - 1)*tan((c + d*x)/2)**4*b + 42*log(tan((c + d*x)/2) - 1)*tan((c +
d*x)/2)**2*a - 24*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2*b - 21*log
(tan((c + d*x)/2) - 1)*a + 12*log(tan((c + d*x)/2) - 1)*b + 21*log(tan((c
+ d*x)/2) + 1)*tan((c + d*x)/2)**4*a - 12*log(tan((c + d*x)/2) + 1)*tan((c
 + d*x)/2)**4*b - 42*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*a + 24*
log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*b + 21*log(tan((c + d*x)/2)
+ 1)*a - 12*log(tan((c + d*x)/2) + 1)*b - tan((c + d*x)/2)**7*a + tan((c +
 d*x)/2)**7*b - 19*tan((c + d*x)/2)**5*a + 13*tan((c + d*x)/2)**5*b + 71*t
an((c + d*x)/2)**3*a - 41*tan((c + d*x)/2)**3*b - 39*tan((c + d*x)/2)*a +
27*tan((c + d*x)/2)*b)/(6*a**2*d*(tan((c + d*x)/2)**4 - 2*tan((c + d*x)/2)
**2 + 1))
```

3.55 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 179

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(10A-7B)\text{arctanh}(\sin(c+dx))}{2a^2d} + \frac{4(3A-2B)\tan(c+dx)}{a^2d} - \frac{(10A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{4(3A-2B)\tan^3(c+dx)}{3a^2d}$$

output
```
-1/2*(10*A-7*B)*arctanh(sin(d*x+c))/a^2/d+4*(3*A-2*B)*tan(d*x+c)/a^2/d-1/2
*(10*A-7*B)*sec(d*x+c)*tan(d*x+c)/a^2/d-1/3*(10*A-7*B)*sec(d*x+c)^2*tan(d*
x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x
+c))^2+4/3*(3*A-2*B)*tan(d*x+c)^3/a^2/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 609 vs. $2(179) = 358$.

Time = 6.43 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.40

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{192(10A-7B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

output
```
(192*(10*A - 7*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/
2]*Sec[c]*Sec[c + d*x]^3*((-6*A + 45*B)*Sin[(d*x)/2] + (310*A - 201*B)*Sin
[(3*d*x)/2] - 306*A*Sin[c - (d*x)/2] + 195*B*Sin[c - (d*x)/2] + 42*A*Sin[c
 + (d*x)/2] - 51*B*Sin[c + (d*x)/2] - 270*A*Sin[2*c + (d*x)/2] + 189*B*Sin
[2*c + (d*x)/2] + 50*A*Sin[c + (3*d*x)/2] - B*Sin[c + (3*d*x)/2] + 90*A*Si
n[2*c + (3*d*x)/2] - 81*B*Sin[2*c + (3*d*x)/2] - 170*A*Sin[3*c + (3*d*x)/2
] + 119*B*Sin[3*c + (3*d*x)/2] + 198*A*Sin[c + (5*d*x)/2] - 129*B*Sin[c +
(5*d*x)/2] + 42*A*Sin[2*c + (5*d*x)/2] - 9*B*Sin[2*c + (5*d*x)/2] + 66*A*S
in[3*c + (5*d*x)/2] - 57*B*Sin[3*c + (5*d*x)/2] - 90*A*Sin[4*c + (5*d*x)/2
] + 63*B*Sin[4*c + (5*d*x)/2] + 114*A*Sin[2*c + (7*d*x)/2] - 75*B*Sin[2*c
+ (7*d*x)/2] + 36*A*Sin[3*c + (7*d*x)/2] - 15*B*Sin[3*c + (7*d*x)/2] + 48*
A*Sin[4*c + (7*d*x)/2] - 39*B*Sin[4*c + (7*d*x)/2] - 30*A*Sin[5*c + (7*d*x
)/2] + 21*B*Sin[5*c + (7*d*x)/2] + 48*A*Sin[3*c + (9*d*x)/2] - 32*B*Sin[3*
c + (9*d*x)/2] + 22*A*Sin[4*c + (9*d*x)/2] - 12*B*Sin[4*c + (9*d*x)/2] + 2
6*A*Sin[5*c + (9*d*x)/2] - 20*B*Sin[5*c + (9*d*x)/2]))/(96*a^2*d*(1 + Cos[
c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(3a(2A-B)-4a(A-B)\cos(c+dx))\sec^4(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a(2A-B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int 3\left(4a^2(3A-2B)-a^2(10A-7B)\cos(c+dx)\right)\sec^4(c+dx)dx}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \left(4a^2(3A-2B)-a^2(10A-7B)\cos(c+dx)\right)\sec^4(c+dx)dx}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\int \frac{4a^2(3A-2B)-a^2(10A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\frac{3\left(4a^2(3A-2B)\int \sec^4(c+dx)dx-a^2(10A-7B)\int \sec^3(c+dx)dx\right)}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3\left(4a^2(3A-2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx-a^2(10A-7B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\frac{3\left(-\frac{4a^2(3A-2B)\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}-\left(a^2(10A-7B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)\right)}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3\left(-a^2(10A-7B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a^2(3A-2B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3\left(-a^2(10A-7B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a^2(3A-2B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(-a^2(10A-7B)\left(\frac{1}{2}\int\csc(c+dx+\frac{\pi}{2})dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a^2(3A-2B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}$$
$$\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 4257

$$\frac{\frac{3\left(-\left(a^2(10A-7B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)\right)-\frac{4a^2(3A-2B)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{(10A-7B)\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}$$
$$\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + (-
(((10*A - 7*B)*Sec[c + d*x]^2*Tan[c + d*x])/(d*(1 + Cos[c + d*x]))) + (3*(
-(a^2*(10*A - 7*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*
x])/(2*d))) - (4*a^2*(3*A - 2*B)*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/a
^2)/(3*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{180\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\left(A-\frac{7B}{10}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-180\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\left(A-\frac{7B}{10}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{6A-2B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+(10A-7B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{6A-2B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+(10A-7B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{6ad}+\frac{(11A-10B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3ad}-\frac{(19A-12B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}-\frac{(21A-13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{(25A-19B)\tan\left(\frac{dx}{2}\right.}{6ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3 a}$
risch	$\frac{i\left(30A\,e^{8i(dx+c)}-21B\,e^{8i(dx+c)}+90A\,e^{7i(dx+c)}-63B\,e^{7i(dx+c)}+170A\,e^{6i(dx+c)}-119B\,e^{6i(dx+c)}+270A\,e^{5i(dx+c)}-18\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/12*(180*(cos(d*x+c)+1/3*cos(3*d*x+3*c))*(A-7/10*B)*ln(tan(1/2*d*x+1/2*c)
-1)-180*(cos(d*x+c)+1/3*cos(3*d*x+3*c))*(A-7/10*B)*ln(tan(1/2*d*x+1/2*c)+1
)+24*sec(1/2*d*x+1/2*c)^2*tan(1/2*d*x+1/2*c)*((11/4*A-43/24*B)*cos(3*d*x+3
*c)+(5*A-19/6*B)*cos(2*d*x+2*c)+(A-2/3*B)*cos(4*d*x+4*c)+(95/12*A-39/8*B)*
cos(d*x+c)+13/3*A-5/2*B))/d/a^2/(3*cos(d*x+c)+cos(3*d*x+3*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.38

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$
$$-\frac{3\left((10A-7B)\cos(dx+c)^5+2(10A-7B)\cos(dx+c)^4+(10A-7B)\cos(dx+c)^3\right)\log(\sin(dx\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/12*(3*((10*A - 7*B)*cos(d*x + c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (1
0*A - 7*B)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((10*A - 7*B)*cos(d*x
 + c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (10*A - 7*B)*cos(d*x + c)^3)*log
(-sin(d*x + c) + 1) - 2*(16*(3*A - 2*B)*cos(d*x + c)^4 + (66*A - 43*B)*cos
(d*x + c)^3 + 6*(2*A - B)*cos(d*x + c)^2 - (2*A - 3*B)*cos(d*x + c) + 2*A)
*sin(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(
d*x + c)^3)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{A\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

output
```
(Integral(A*sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + I
ntegral(B*cos(c + d*x)*sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) +
 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(169) = 338.

Time = 0.05 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.37

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{A\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.}{a^2}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c)
 + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(co
s(d*x + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2
) - B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) +
1) + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.26

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\frac{3(10A-7B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{3(10A-7B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{2\left(30A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-40A\right.}{}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-1/6*(3*(10*A - 7*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(10*A - 7*
B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(30*A*tan(1/2*d*x + 1/2*c)^5
 - 15*B*tan(1/2*d*x + 1/2*c)^5 - 40*A*tan(1/2*d*x + 1/2*c)^3 + 24*B*tan(1/
2*d*x + 1/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^
4*tan(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*tan(1/
2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 41.97 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2(A-B)}{a^2}+\frac{5A-3B}{2a^2}\right)}{d} - \frac{(10A-5B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(8B-\frac{40A}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(6A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(A-B)}{6a^2d} - \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(10A-7B)}{a^2d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + a*cos(c + d*x))^2),x)
```

output
```
(tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (5*A - 3*B)/(2*a^2)))/d - (tan(c/2
+ (d*x)/2)^5*(10*A - 5*B) - tan(c/2 + (d*x)/2)^3*((40*A)/3 - 8*B) + tan(c/
2 + (d*x)/2)*(6*A - 3*B))/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 +
 (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2)) + (tan(c/2 + (d*x)/2)^3*(A
- B))/(6*a^2*d) - (atanh(tan(c/2 + (d*x)/2))*(10*A - 7*B))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.11

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x)
```

output
```
(30*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**6*a - 21*log(tan((c + d*x)
/2) - 1)*tan((c + d*x)/2)**6*b - 90*log(tan((c + d*x)/2) - 1)*tan((c + d*x
)/2)**4*a + 63*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**4*b + 90*log(ta
n((c + d*x)/2) - 1)*tan((c + d*x)/2)**2*a - 63*log(tan((c + d*x)/2) - 1)*t
an((c + d*x)/2)**2*b - 30*log(tan((c + d*x)/2) - 1)*a + 21*log(tan((c + d*
x)/2) - 1)*b - 30*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**6*a + 21*log
(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**6*b + 90*log(tan((c + d*x)/2) + 1
)*tan((c + d*x)/2)**4*a - 63*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**4
*b - 90*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*a + 63*log(tan((c +
d*x)/2) + 1)*tan((c + d*x)/2)**2*b + 30*log(tan((c + d*x)/2) + 1)*a - 21*l
og(tan((c + d*x)/2) + 1)*b + tan((c + d*x)/2)**9*a - tan((c + d*x)/2)**9*b
 + 24*tan((c + d*x)/2)**7*a - 18*tan((c + d*x)/2)**7*b - 138*tan((c + d*x)
/2)**5*a + 90*tan((c + d*x)/2)**5*b + 160*tan((c + d*x)/2)**3*a - 110*tan(
(c + d*x)/2)**3*b - 63*tan((c + d*x)/2)*a + 39*tan((c + d*x)/2)*b)/(6*a**2
*d*(tan((c + d*x)/2)**6 - 3*tan((c + d*x)/2)**4 + 3*tan((c + d*x)/2)**2 -
1))
```

3.56 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 218

$$\begin{aligned}\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx &= \frac{(13A-23B)x}{2a^3}-\frac{4(19A-34B)\sin(c+dx)}{5a^3d}\\ &+\frac{(13A-23B)\cos(c+dx)\sin(c+dx)}{2a^3d}\\ &+\frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}\\ &+\frac{(8A-13B)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}\\ &+\frac{(13A-23B)\cos^3(c+dx)\sin(c+dx)}{3d\,(a^3+a^3\cos(c+dx))}\\ &+\frac{4(19A-34B)\sin^3(c+dx)}{15a^3d}\end{aligned}$$

output

```
1/2*(13*A-23*B)*x/a^3-4/5*(19*A-34*B)*sin(d*x+c)/a^3/d+1/2*(13*A-23*B)*cos
(d*x+c)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+
c))^3+1/15*(8*A-13*B)*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/3*(
13*A-23*B)*cos(d*x+c)^3*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))+4/15*(19*A-34*B)
*sin(d*x+c)^3/a^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 491 vs. $2(218) = 436$.

Time = 3.48 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.25

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(600(13A-23B)dx\cos\left(\frac{dx}{2}\right)+600(13A-23B)dx\cos\left(c+\frac{dx}{2}\right)+3900Adx\cos\left(\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*A - 23*B)*d*x*Cos[(d*x)/2] + 600*(13*A
 - 23*B)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] - 6900*B*d*x
*Cos[c + (3*d*x)/2] + 3900*A*d*x*Cos[2*c + (3*d*x)/2] - 6900*B*d*x*Cos[2*c
 + (3*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] - 1380*B*d*x*Cos[2*c + (5*d
*x)/2] + 780*A*d*x*Cos[3*c + (5*d*x)/2] - 1380*B*d*x*Cos[3*c + (5*d*x)/2]
- 12760*A*Sin[(d*x)/2] + 20410*B*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/2] -
11110*B*Sin[c + (d*x)/2] - 9230*A*Sin[c + (3*d*x)/2] + 15380*B*Sin[c + (3*
d*x)/2] + 930*A*Sin[2*c + (3*d*x)/2] - 380*B*Sin[2*c + (3*d*x)/2] - 2782*A
*Sin[2*c + (5*d*x)/2] + 4777*B*Sin[2*c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d
*x)/2] + 1625*B*Sin[3*c + (5*d*x)/2] - 105*A*Sin[3*c + (7*d*x)/2] + 230*B*
Sin[3*c + (7*d*x)/2] - 105*A*Sin[4*c + (7*d*x)/2] + 230*B*Sin[4*c + (7*d*x
)/2] + 15*A*Sin[4*c + (9*d*x)/2] - 20*B*Sin[4*c + (9*d*x)/2] + 15*A*Sin[5*
c + (9*d*x)/2] - 20*B*Sin[5*c + (9*d*x)/2] + 5*B*Sin[5*c + (11*d*x)/2] + 5
*B*Sin[6*c + (11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3456, 3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(3A-8B)\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(5a(A-B)-a(3A-8B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{\cos^3(c+dx)\left(4a^2(8A-13B)-3a^2(11A-21B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a^2(8A-13B)-3a^2(11A-21B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\frac{\int 3\cos^2(c+dx)\left(5a^3(13A-23B)-4a^3(19A-34B)\cos(c+dx)\right)dx}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \cos^2(c+dx)\left(5a^3(13A-23B)-4a^3(19A-34B)\cos(c+dx)\right)dx}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(5a^3(13A-23B)-4a^3(19A-34B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{3227}$$

$$\frac{\frac{\frac{3\left(5a^3(13A-23B)\int \cos^2(c+dx)dx-4a^3(19A-34B)\int \cos^3(c+dx)dx\right)}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{3\left(5a^3(13A-23B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-4a^3(19A-34B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{3113}$$

$$\frac{\frac{\frac{3\left(\frac{4a^3(19A-34B)\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}+5a^3(13A-23B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{\frac{3\left(5a^3(13A-23B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{4a^3(19A-34B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin(c+dx)\cos}{3d(a\cos(c+dx)+a)}}{5a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3115

$$\frac{\frac{\frac{3\left(5a^3(13A-23B)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{4a^3(19A-34B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}+\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(8A-13B)\sin}{3d(a\cos}}{5a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 24

$$\frac{\frac{\frac{5a^2(13A-23B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3\left(\frac{4a^3(19A-34B)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+5a^3(13A-23B)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}}{3a^2}+\frac{a(8A-13B)\sin(c+}{3d(a\cos(c+}}{5a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^5*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((a*(
8*A - 13*B)*Cos[c + d*x]^4*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((
5*a^2*(13*A - 23*B)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
+ (3*(5*a^3*(13*A - 23*B)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) + (4*a
^3*(19*A - 34*B)*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d))/a^2)/(3*a^2))/(5*
a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{\left(\frac{8(-232A+427B)\cos(2dx+2c)}{15}+\left(-6A+\frac{43B}{3}\right)\cos(3dx+3c)+(A-B)\cos(4dx+4c)+\frac{B\cos(5dx+5c)}{3}+\frac{2\left(-1001A+\frac{5458B}{3}\right)\cos}{5}\right.}{64a^3d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{16\left(-\frac{7}{4}\right.}{}}{4d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{16\left(-\frac{7}{4}\right.}{}}{4d}$
risch	$\frac{13Ax}{2a^3}-\frac{23Bx}{2a^3}-\frac{iB\,e^{3i(dx+c)}}{24a^3d}-\frac{ie^{2i(dx+c)}A}{8a^3d}+\frac{3ie^{2i(dx+c)}B}{8a^3d}+\frac{3ie^{i(dx+c)}A}{2a^3d}-\frac{27ie^{i(dx+c)}B}{8a^3d}-\frac{3ie^{-i(dx+c}}{2a^3d}$
norman	$\frac{\frac{(13A-23B)x}{2a}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}}{20ad}-\frac{(9A-16B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{2ad}+\frac{(11A-16B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}}{30ad}+\frac{3(13A-23B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+}{}$

input
```
int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/64*((8/15*(-232*A+427*B)*cos(2*d*x+2*c)+(-6*A+43/3*B)*cos(3*d*x+3*c)+(A-
B)*cos(4*d*x+4*c)+1/3*B*cos(5*d*x+5*c)+2/5*(-1001*A+5458/3*B)*cos(d*x+c)-4
303/15*A+7783/15*B)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4+416*x*(A-23/13
*B)*d)/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{15\,(13\,A-23\,B)dx\cos{(dx+c)}^3+45\,(13\,A-23\,B)dx\cos{(dx+c)}^2+45\,(13\,A-23\,B)dx\cos{(dx+c)}+}{}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/30*(15*(13*A - 23*B)*d*x*cos(d*x + c)^3 + 45*(13*A - 23*B)*d*x*cos(d*x +
 c)^2 + 45*(13*A - 23*B)*d*x*cos(d*x + c) + 15*(13*A - 23*B)*d*x + (10*B*c
os(d*x + c)^5 + 15*(A - B)*cos(d*x + c)^4 - 5*(9*A - 19*B)*cos(d*x + c)^3
- (479*A - 869*B)*cos(d*x + c)^2 - 3*(239*A - 429*B)*cos(d*x + c) - 304*A
+ 544*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*
a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1584 vs. $2(206) = 412$.

Time = 5.98 (sec) , antiderivative size = 1584, normalized size of antiderivative = 7.27

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((390*A*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 +
180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*
d) + 1170*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a
**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
1170*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d
*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 390*A
*d*x/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180
*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan(c/2 + d*x/2)**11/(60*a*
*3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) + 31*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/
2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/
2)**2 + 60*a**3*d) - 354*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)
**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d) - 1698*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180
*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
- 2075*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*t
an(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*A*t
an(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/
2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 690*B*d*x*tan(c/2 +
d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(204) = 408$.

Time = 0.12 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.89

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{B\left(\frac{20\left(\frac{33\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{51\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{735\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{1380\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right.}{60\,d}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
1/60*(B*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x
+ c) + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)
- A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)
/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.05

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{30\,(dx+c)(13\,A-23\,B)}{a^3} - \frac{20\left(21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-51\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-76\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/60*(30*(d*x + c)*(13*A - 23*B)/a^3 - 20*(21*A*tan(1/2*d*x + 1/2*c)^5 - 5
1*B*tan(1/2*d*x + 1/2*c)^5 + 36*A*tan(1/2*d*x + 1/2*c)^3 - 76*B*tan(1/2*d*
x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) - 33*B*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*
a^12*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 50*B*a^12
*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 735*B*a^12*tan
(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 42.05 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{x\,(13\,A-23\,B)}{2\,a^3} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,(A-B)}{2\,a^3}+\frac{4\,A-6\,B}{a^3}+\frac{5\,A-15\,B}{4\,a^3}\right)}{d}$$
$$- \frac{(7\,A-17\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(12\,A-\frac{76\,B}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(5\,A-11\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$
$$+ \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{3\,a^3}+\frac{4\,A-6\,B}{12\,a^3}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}$$

input
```
int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
(x*(13*A - 23*B))/(2*a^3) - (tan(c/2 + (d*x)/2)*((5*(A - B))/(2*a^3) + (4*
A - 6*B)/a^3 + (5*A - 15*B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(7*A - 17*
B) + tan(c/2 + (d*x)/2)^3*(12*A - (76*B)/3) + tan(c/2 + (d*x)/2)*(5*A - 11
*B))/(d*(3*a^3*tan(c/2 + (d*x)/2)^2 + 3*a^3*tan(c/2 + (d*x)/2)^4 + a^3*tan
(c/2 + (d*x)/2)^6 + a^3)) + (tan(c/2 + (d*x)/2)^3*((A - B)/(3*a^3) + (4*A
- 6*B)/(12*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.23

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{-15\cos(dx+c)\sin(dx+c)^4a+25\cos(dx+c)\sin(dx+c)^4b-404\cos(dx+c)\sin(dx+c)^2a+724}{}$$

input
```
int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
( - 15*cos(c + d*x)*sin(c + d*x)**4*a + 25*cos(c + d*x)*sin(c + d*x)**4*b
- 404*cos(c + d*x)*sin(c + d*x)**2*a + 724*cos(c + d*x)*sin(c + d*x)**2*b
+ 390*cos(c + d*x)*sin(c + d*x)*a*d*x - 690*cos(c + d*x)*sin(c + d*x)*b*d*
x + 6*cos(c + d*x)*a - 6*cos(c + d*x)*b + 10*sin(c + d*x)**6*b + 60*sin(c
+ d*x)**4*a - 140*sin(c + d*x)**4*b - 195*sin(c + d*x)**3*a*d*x + 345*sin(
c + d*x)**3*b*d*x - 358*sin(c + d*x)**2*a + 668*sin(c + d*x)**2*b + 390*si
n(c + d*x)*a*d*x - 690*sin(c + d*x)*b*d*x - 6*a + 6*b)/(30*sin(c + d*x)*a*
*3*d*(2*cos(c + d*x) - sin(c + d*x)**2 + 2))
```

3.57 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 193

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{(6A-13B)x}{2a^3}+\frac{8(9A-19B)\sin(c+dx)}{15a^3d} - \frac{(6A-13B)\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6A-11B)\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{4(9A-19B)\cos^2(c+dx)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output
```
-1/2*(6*A-13*B)*x/a^3+8/15*(9*A-19*B)*sin(d*x+c)/a^3/d-1/2*(6*A-13*B)*cos(
d*x+c)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c
))^3+1/15*(6*A-11*B)*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+4/15*(
9*A-19*B)*cos(d*x+c)^2*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 435 vs. $2(193) = 386$.

Time = 2.96 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.25

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-600(6A-13B)dx\cos\left(\frac{dx}{2}\right)-600(6A-13B)dx\cos\left(c+\frac{dx}{2}\right)-1800Adx\cos\left(\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(-600*(6*A - 13*B)*d*x*Cos[(d*x)/2] - 600*(6*A
- 13*B)*d*x*Cos[c + (d*x)/2] - 1800*A*d*x*Cos[c + (3*d*x)/2] + 3900*B*d*x*
Cos[c + (3*d*x)/2] - 1800*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c
+ (3*d*x)/2] - 360*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x
)/2] - 360*A*d*x*Cos[3*c + (5*d*x)/2] + 780*B*d*x*Cos[3*c + (5*d*x)/2] + 7
020*A*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] - 4500*A*Sin[c + (d*x)/2] + 7560
*B*Sin[c + (d*x)/2] + 4860*A*Sin[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2
] - 900*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] + 1452*A*Sin[2
*c + (5*d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 300*A*Sin[3*c + (5*d*x)/2]
 - 750*B*Sin[3*c + (5*d*x)/2] + 60*A*Sin[3*c + (7*d*x)/2] - 105*B*Sin[3*c
+ (7*d*x)/2] + 60*A*Sin[4*c + (7*d*x)/2] - 105*B*Sin[4*c + (7*d*x)/2] + 15
*B*Sin[4*c + (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[
c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 3456, 3042, 3456, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4 \left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(2A-7B)\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a(A-B)-a(2A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(3a^2(6A-11B)-a^2(18A-43B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{a(6A-11B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a^2(6A-11B)-a^2(18A-43B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{a(6A-11B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\frac{\int \cos(c+dx)\left(8a^3(9A-19B)-15a^3(6A-13B)\cos(c+dx)\right)dx}{a^2}+\frac{4a^2(9A-19B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(6A-11B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(8a^3(9A-19B)-15a^3(6A-13B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{4a^2(9A-19B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{a(6A-11B)\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3213

$$\frac{\frac{\frac{4a^2(9A-19B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{\frac{8a^3(9A-19B)\sin(c+dx)}{d}-\frac{15a^3(6A-13B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{15}{2}a^3x(6A-13B)}{a^2}}{3a^2}+\frac{a(6A-11B)\sin(c+dx)\cos^3(c\cdot}{3d(a\cos(c+dx)+a)^2}}{5a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((a*(
6*A - 11*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((
4*a^2*(9*A - 19*B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) +
 ((-15*a^3*(6*A - 13*B)*x)/2 + (8*a^3*(9*A - 19*B)*Sin[c + d*x])/d - (15*a
^3*(6*A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2)/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
 + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{2916 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\frac{26A}{81}-\frac{464B}{729}\right)\cos(2dx+2c)+\left(\frac{5A}{243}-\frac{5B}{162}\right)\cos(3dx+3c)+\frac{5B\cos(4dx+4c)}{972}+\left(A-\frac{1001B}{486}\right)\cos(dx+c)+\right.}{960a^3d}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 A}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 B}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B}{3}+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{16\left(\left(-\frac{A}{2}\right.\right.}{}}{4d\,a^3}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 A}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 B}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B}{3}+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{16\left(\left(-\frac{A}{2}\right.\right.}{}}{4d\,a^3}$
risch	$-\frac{3Ax}{a^3}+\frac{13Bx}{2a^3}-\frac{i\mathrm{e}^{2i(dx+c)}B}{8a^3d}-\frac{i\mathrm{e}^{i(dx+c)}A}{2a^3d}+\frac{3i\mathrm{e}^{i(dx+c)}B}{2a^3d}+\frac{i\mathrm{e}^{-i(dx+c)}A}{2a^3d}-\frac{3i\mathrm{e}^{-i(dx+c)}B}{2a^3d}+\frac{i\mathrm{e}^{-2i(dx+c}}{8a^3d}$
norman	$\frac{-\frac{(6A-13B)x}{2a}+\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}}{20ad}-\frac{(3A-5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{12ad}-\frac{5(6A-13B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{5(6A-13B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}-}{}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/960*(2916*tan(1/2*d*x+1/2*c)*((26/81*A-464/729*B)*cos(2*d*x+2*c)+(5/243*
A-5/162*B)*cos(3*d*x+3*c)+5/972*B*cos(4*d*x+4*c)+(A-1001/486*B)*cos(d*x+c)
+58/81*A-4303/2916*B)*sec(1/2*d*x+1/2*c)^4-2880*x*d*(A-13/6*B))/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{15\,(6\,A-13\,B)dx\cos{(dx+c)}^3+45\,(6\,A-13\,B)dx\cos{(dx+c)}^2+45\,(6\,A-13\,B)dx\cos{(dx+c)}+}{3}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/30*(15*(6*A - 13*B)*d*x*cos(d*x + c)^3 + 45*(6*A - 13*B)*d*x*cos(d*x +
c)^2 + 45*(6*A - 13*B)*d*x*cos(d*x + c) + 15*(6*A - 13*B)*d*x - (15*B*cos(
d*x + c)^4 + 15*(2*A - 3*B)*cos(d*x + c)^3 + (234*A - 479*B)*cos(d*x + c)^
2 + 3*(114*A - 239*B)*cos(d*x + c) + 144*A - 304*B)*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(178) = 356$.

Time = 3.82 (sec) , antiderivative size = 966, normalized size of antiderivative = 5.01

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((-180*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 360*A*d*x*tan(c/2 + d*x/2)*
*2/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a*
*3*d) - 180*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 3*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)
**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 24*A*tan(c/2 + d*x/2)*
*7/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a*
*3*d) + 198*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**
3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 600*A*tan(c/2 + d*x/2)**3/(60*a**3*
d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*
A*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d
*x/2)**2 + 60*a**3*d) + 390*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 +
 d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*B*d*x*tan(c
/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) + 390*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*
tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*B*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(
c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*B*tan(c
/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) - 388*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)*
*4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 1310*B*tan(c/2 + d*x...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.67

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3A\Bigg(}{60\,d}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) -
40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*A*(40*sin
(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{\frac{30(dx+c)(6A-13B)}{a^3}-\frac{60\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^3}-\frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{60\,c}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/60*(30*(d*x + c)*(6*A - 13*B)/a^3 - 60*(2*A*tan(1/2*d*x + 1/2*c)^3 - 7*
B*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 5*B*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c
)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 +
 40*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 465*
B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 41.98 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3(A-B)}{2a^3}+\frac{3(3A-5B)}{4a^3}+\frac{2A-10B}{4a^3}\right)}{d}-\frac{x\,(6A-13B)}{2a^3}$$
$$+\frac{(2A-7B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2A-5B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3\right)}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{4a^3}+\frac{3A-5B}{12a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(A-B)}{20a^3d}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^3) + (3*(3*A - 5*B))/(4*a^3) + (2*A
- 10*B)/(4*a^3)))/d - (x*(6*A - 13*B))/(2*a^3) + (tan(c/2 + (d*x)/2)^3*(2*
A - 7*B) + tan(c/2 + (d*x)/2)*(2*A - 5*B))/(d*(2*a^3*tan(c/2 + (d*x)/2)^2
+ a^3*tan(c/2 + (d*x)/2)^4 + a^3)) - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3
) + (3*A - 5*B)/(12*a^3)))/d + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.28

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9 a - 3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9 b - 24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 a + 34\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 b + 198\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 a - 3}{}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(3*tan((c + d*x)/2)**9*a - 3*tan((c + d*x)/2)**9*b - 24*tan((c + d*x)/2)**
7*a + 34*tan((c + d*x)/2)**7*b + 198*tan((c + d*x)/2)**5*a - 388*tan((c +
d*x)/2)**5*b - 180*tan((c + d*x)/2)**4*a*d*x + 390*tan((c + d*x)/2)**4*b*d
*x + 600*tan((c + d*x)/2)**3*a - 1310*tan((c + d*x)/2)**3*b - 360*tan((c +
 d*x)/2)**2*a*d*x + 780*tan((c + d*x)/2)**2*b*d*x + 375*tan((c + d*x)/2)*a
 - 765*tan((c + d*x)/2)*b - 180*a*d*x + 390*b*d*x)/(60*a**3*d*(tan((c + d*
x)/2)**4 + 2*tan((c + d*x)/2)**2 + 1))
```

3.58 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 147

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-3B)x}{a^3} - \frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(A-3B)\sin(c+dx)}{d\,(a^3+a^3\cos(c+dx))}$$

output
```
(A-3*B)*x/a^3-1/15*(7*A-27*B)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^3*sin(
d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(4*A-9*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+
a*cos(d*x+c))^2-(A-3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 361 vs. $2(147) = 294$.

Time = 2.64 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.46

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(300(A-3B)dx\cos\left(\frac{dx}{2}\right)+300(A-3B)dx\cos\left(c+\frac{dx}{2}\right)+150Adx\cos\left(c+\frac{3dx}{2}\right)\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(300*(A - 3*B)*d*x*Cos[(d*x)/2] + 300*(A - 3*B)
*d*x*Cos[c + (d*x)/2] + 150*A*d*x*Cos[c + (3*d*x)/2] - 450*B*d*x*Cos[c + (
3*d*x)/2] + 150*A*d*x*Cos[2*c + (3*d*x)/2] - 450*B*d*x*Cos[2*c + (3*d*x)/2
] + 30*A*d*x*Cos[2*c + (5*d*x)/2] - 90*B*d*x*Cos[2*c + (5*d*x)/2] + 30*A*d
*x*Cos[3*c + (5*d*x)/2] - 90*B*d*x*Cos[3*c + (5*d*x)/2] - 740*A*Sin[(d*x)/
2] + 1755*B*Sin[(d*x)/2] + 540*A*Sin[c + (d*x)/2] - 1125*B*Sin[c + (d*x)/2
] - 460*A*Sin[c + (3*d*x)/2] + 1215*B*Sin[c + (3*d*x)/2] + 180*A*Sin[2*c +
 (3*d*x)/2] - 225*B*Sin[2*c + (3*d*x)/2] - 128*A*Sin[2*c + (5*d*x)/2] + 36
3*B*Sin[2*c + (5*d*x)/2] + 75*B*Sin[3*c + (5*d*x)/2] + 15*B*Sin[3*c + (7*d
*x)/2] + 15*B*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3456, 3042, 3456, 3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(A-6B)\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a(A-B)-a(A-6B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\frac{\int \frac{\cos(c+dx)\left(2a^2(4A-9B)-a^2(7A-27B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a^2(4A-9B)-a^2(7A-27B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3447}$$

$$\frac{\frac{\int \frac{2a^2(4A-9B)\cos(c+dx)-a^2(7A-27B)\cos^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{2a^2(4A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)-a^2(7A-27B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\frac{\frac{\int \frac{15a^3(A-3B)\cos(c+dx)}{\cos(c+dx)a+a}dx}{a}-\frac{a(7A-27B)\sin(c+dx)}{d}}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{15a^2(A-3B)\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}dx-\frac{a(7A-27B)\sin(c+dx)}{d}}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{15a^2(A-3B)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{a(7A-27B)\sin(c+dx)}{d}}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{15a^2(A-3B)\left(\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx\right)-\frac{a(7A-27B)\sin(c+dx)}{d}}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{15a^2(A-3B)\left(\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)-\frac{a(7A-27B)\sin(c+dx)}{d}}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\frac{15a^2(A-3B)\left(\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)-\frac{a(7A-27B)\sin(c+dx)}{d}}{3a^2}+\frac{a(4A-9B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((a*(
4*A - 9*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (-(
(a*(7*A - 27*B)*Sin[c + d*x])/d) + 15*a^2*(A - 3*B)*(x/a - Sin[c + d*x]/(d
*(a + a*Cos[c + d*x]))))/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{-204\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\frac{16A}{51}-\frac{39B}{34}\right)\cos(2dx+2c)-\frac{5B\cos(3dx+3c)}{68}+\left(A-\frac{243B}{68}\right)\cos(dx+c)+\frac{38A}{51}-\frac{87B}{34}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2}{240a^3d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{8B\tan(}{1+\tan(}}{4da^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{8B\tan(}{1+\tan(}}{4da^3}$
risch	$\frac{Ax}{a^3}-\frac{3Bx}{a^3}-\frac{ie^{i(dx+c)}B}{2a^3d}+\frac{ie^{-i(dx+c)}B}{2a^3d}-\frac{2i(45A\,e^{4i(dx+c)}-90B\,e^{4i(dx+c)}+135A\,e^{3i(dx+c)}-300B\,e^{3i(dx+c)}+}{15d\,a}$
norman	$\frac{\frac{(A-3B)x}{a}+\frac{(A-3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{a}+\frac{4(A-3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{6(A-3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}+\frac{4(A-3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a}-\frac{(A-B)}{}}{}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/240*(-204*tan(1/2*d*x+1/2*c)*((16/51*A-39/34*B)*cos(2*d*x+2*c)-5/68*B*co
s(3*d*x+3*c)+(A-243/68*B)*cos(d*x+c)+38/51*A-87/34*B)*sec(1/2*d*x+1/2*c)^4
+240*d*x*(A-3*B))/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{15\,(A-3\,B)dx\cos{(dx+c)}^3+45\,(A-3\,B)dx\cos{(dx+c)}^2+45\,(A-3\,B)dx\cos{(dx+c)}+15\,(A-3}{15\,\left(a^3d\cos{(dx+c)}^3+3\,a^3}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(15*(A - 3*B)*d*x*cos(d*x + c)^3 + 45*(A - 3*B)*d*x*cos(d*x + c)^2 +
45*(A - 3*B)*d*x*cos(d*x + c) + 15*(A - 3*B)*d*x + (15*B*cos(d*x + c)^3 -
(32*A - 117*B)*cos(d*x + c)^2 - 3*(17*A - 57*B)*cos(d*x + c) - 22*A + 72*B
)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(134) = 268.

Time = 2.38 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.37

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} \frac{60Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}+\frac{60Adx}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}-\frac{3A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}+\frac{17A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}-\frac{85A}{60a^3d\,\mathrm{ta}} \\ \frac{x(A+B\cos{(c)})\cos^3{(c)}}{(a\cos{(c)}+a)^3} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((60*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 6
0*a**3*d) + 60*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan
(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 17*A*tan(c/
2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 85*A*tan(c/2 +
 d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 105*A*tan(c/2 + d
*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x*tan(c/2 + d*
x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x/(60*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) - 27*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/
2 + d*x/2)**2 + 60*a**3*d) + 225*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2
+ d*x/2)**2 + 60*a**3*d) + 375*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x
/2)**2 + 60*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)
**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.57

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{3B\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right) - A\left(\frac{105}{\cos\cdots}\right.}{60\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
1/60*(3*B*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 12
0*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((105*sin(d*x + c)/(cos
(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{60\,(dx+c)(A-3\,B)}{a^3} + \frac{120\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ba^{12}\tan(}{a^{15}}}{60\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/60*(60*(d*x + c)*(A - 3*B)/a^3 + 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1
/2*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12*tan(1/2*d*x +
 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 41.91 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{A-B}{6\,a^3}+\frac{2\,A-4\,B}{12\,a^3}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,(A-B)}{4\,a^3}-\frac{3\,B}{2\,a^3}+\frac{2\,A-4\,B}{2\,a^3}\right)}{d} + \frac{x\,(A-3\,B)}{a^3} + \frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5(A-B)}{20\,a^3\,d}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (2*A - 4*B)/(12*a^3)))/d - (tan(c
/2 + (d*x)/2)*((3*(A - B))/(4*a^3) - (3*B)/(2*a^3) + (2*A - 4*B)/(2*a^3)))
/d + (x*(A - 3*B))/a^3 + (2*B*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/
2)^2 + a^3)) - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 a+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 b+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 a-27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 b-85\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a+}{6}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
( - 3*tan((c + d*x)/2)**7*a + 3*tan((c + d*x)/2)**7*b + 17*tan((c + d*x)/2
)**5*a - 27*tan((c + d*x)/2)**5*b - 85*tan((c + d*x)/2)**3*a + 225*tan((c
+ d*x)/2)**3*b + 60*tan((c + d*x)/2)**2*a*d*x - 180*tan((c + d*x)/2)**2*b*
d*x - 105*tan((c + d*x)/2)*a + 375*tan((c + d*x)/2)*b + 60*a*d*x - 180*b*d
*x)/(60*a**3*d*(tan((c + d*x)/2)**2 + 1))
```

3.59 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 116

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{Bx}{a^3}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(4A-29B)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output
```
B*x/a^3+1/5*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(2*A-7
*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/15*(4*A-29*B)*sin(d*x+c)/d/(a^3+a^
3*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(116) = 232.

Time = 2.00 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.08

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(150Bdx\cos\left(\frac{dx}{2}\right)+150Bdx\cos\left(c+\frac{dx}{2}\right)+75Bdx\cos\left(c+\frac{3dx}{2}\right)+75Bdx\cos\left(2\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^5*(150*B*d*x*Cos[(d*x)/2] + 150*B*d*x*Cos[c + (
d*x)/2] + 75*B*d*x*Cos[c + (3*d*x)/2] + 75*B*d*x*Cos[2*c + (3*d*x)/2] + 15
*B*d*x*Cos[2*c + (5*d*x)/2] + 15*B*d*x*Cos[3*c + (5*d*x)/2] + 80*A*Sin[(d*
x)/2] - 370*B*Sin[(d*x)/2] - 60*A*Sin[c + (d*x)/2] + 270*B*Sin[c + (d*x)/2
] + 40*A*Sin[c + (3*d*x)/2] - 230*B*Sin[c + (3*d*x)/2] - 30*A*Sin[2*c + (3
*d*x)/2] + 90*B*Sin[2*c + (3*d*x)/2] + 14*A*Sin[2*c + (5*d*x)/2] - 64*B*Si
n[2*c + (5*d*x)/2]))/(480*a^3*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3456, 3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos(c+dx)(2a(A-B)+5aB\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a(A-B)+5aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{5aB\cos^2(c+dx)+2a(A-B)\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5aB\sin\left(c+dx+\frac{\pi}{2}\right)^2+2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3498}$$

$$\frac{-\frac{\int -\frac{2(2A-7B)a^2+15B\cos(c+dx)a^2}{\cos(c+dx)a+a}dx}{3a^2}-\frac{a(2A-7B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2(2A-7B)a^2+15B\cos(c+dx)a^2}{\cos(c+dx)a+a}dx}{3a^2}-\frac{a(2A-7B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2(2A-7B)a^2+15B\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{a(2A-7B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{a^2(4A-29B)\int \frac{1}{\cos(c+dx)a+a}dx+15aBx}{3a^2}-\frac{a(2A-7B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(4A-29B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+15aBx}{3a^2}-\frac{a(2A-7B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\frac{\frac{a^2(4A-29B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+15aBx}{3a^2}-\frac{a(2A-7B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (-1/3
*(a*(2*A - 7*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + (15*a*B*x + (a^
2*(4*A - 29*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
 + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{3(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+10(-A+2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+15(A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+60dxB}{60a^{3}d}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}A}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}B}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+8B\arctan(\mathrm{ta}}{4da^{3}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}A}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}B}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+8B\arctan(\mathrm{ta}}{4da^{3}}$
risch	$\frac{Bx}{a^{3}}+\frac{2i\left(15A\,e^{4i(dx+c)}-45B\,e^{4i(dx+c)}+30A\,e^{3i(dx+c)}-135B\,e^{3i(dx+c)}+40A\,e^{2i(dx+c)}-185B\,e^{2i(dx+c)}+20A\,e^{i(dx+}\right.}{15d\,a^{3}\left(e^{i(dx+c)}+1\right)^{5}}$
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{a}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{a}-\frac{(A-11B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{60ad}+\frac{(A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{(A-B)\tan}{20}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{2}}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/60*(3*(A-B)*tan(1/2*d*x+1/2*c)^5+10*(-A+2*B)*tan(1/2*d*x+1/2*c)^3+15*(A-
7*B)*tan(1/2*d*x+1/2*c)+60*d*x*B)/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{15\,Bdx\cos(dx+c)^3+45\,Bdx\cos(dx+c)^2+45\,Bdx\cos(dx+c)+15\,Bdx+\left((7\,A-32\,B)\cos(dx+\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx\right.}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(15*B*d*x*cos(d*x + c)^3 + 45*B*d*x*cos(d*x + c)^2 + 45*B*d*x*cos(d*x
 + c) + 15*B*d*x + ((7*A - 32*B)*cos(d*x + c)^2 + 3*(2*A - 17*B)*cos(d*x +
 c) + 2*A - 22*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{Bx}{a^3} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d} - \frac{7B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \\ \frac{x(A+B\cos(c))\cos^2(c)}{(a\cos(c)+a)^3} & \text{other} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) - A*tan(c/2 + d*x/2)**3/(6*a*
*3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) + B*x/a**3 - B*tan(c/2 + d*x/2)**5/(
20*a**3*d) + B*tan(c/2 + d*x/2)**3/(3*a**3*d) - 7*B*tan(c/2 + d*x/2)/(4*a*
*3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*(15*sin(d*x + c)/(cos(d*x + c) + 1
) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{\frac{60\,(dx+c)B}{a^3}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{15}}}{60\,d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/60*(60*(d*x + c)*B/a^3 + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan
(1/2*d*x + 1/2*c)^5 - 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*B*a^12*tan(1/2
*d*x + 1/2*c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x
+ 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 41.97 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{B\,x}{a^3} - \frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}\right)-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{7\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)-\frac{A\sin\left(\frac{c}{2}\right.}{2\ldots}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
(B*x)/a^3 - (cos(c/2 + (d*x)/2)^2*((A*sin(c/2 + (d*x)/2)^3)/6 - (B*sin(c/2
 + (d*x)/2)^3)/3) - cos(c/2 + (d*x)/2)^4*((A*sin(c/2 + (d*x)/2))/4 - (7*B*
sin(c/2 + (d*x)/2))/4) - (A*sin(c/2 + (d*x)/2)^5)/20 + (B*sin(c/2 + (d*x)/
2)^5)/20)/(a^3*d*cos(c/2 + (d*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 a-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 b-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right) a-105\ldots}{60a^3d}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(3*tan((c + d*x)/2)**5*a - 3*tan((c + d*x)/2)**5*b - 10*tan((c + d*x)/2)**
3*a + 20*tan((c + d*x)/2)**3*b + 15*tan((c + d*x)/2)*a - 105*tan((c + d*x)
/2)*b + 60*b*d*x)/(60*a**3*d)
```

3.60 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 102

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A+7B)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output
```
-1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(3*A-8*B)*sin(d*x+c)/a/d/(
a+a*cos(d*x+c))^2+1/15*(3*A+7*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(5(3A+8B)\sin\left(\frac{dx}{2}\right)-15(A+2B)\sin\left(c+\frac{dx}{2}\right)+15A\sin\left(c+\frac{3dx}{2}\right)+20B\sin\left(\right.\right.}{30a^3d(1+\cos(c+dx))^3}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 8*B)*Sin[(d*x)/2] - 15*(A + 2*B)*Sin[
c + (d*x)/2] + 15*A*Sin[c + (3*d*x)/2] + 20*B*Sin[c + (3*d*x)/2] - 15*B*Si
n[2*c + (3*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 7*B*Sin[2*c + (5*d*x)/2]))
/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 3447, 3042, 3498, 25, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{3a(A-B)+5aB\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3a(A-B)+5aB\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a(A-B)+5aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3229

$$\frac{\frac{1}{3}(3A+7B)\int \frac{1}{\cos(c+dx)a+a}dx+\frac{a(3A-8B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{3}(3A+7B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a(3A-8B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3127

$$\frac{\frac{(3A+7B)\sin(c+dx)}{3d(a\cos(c+dx)+a)}+\frac{a(3A-8B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + ((a*(3*A - 8*B)*S
in[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((3*A + 7*B)*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])))/(5*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\frac{10B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}-5A-5B\right)}{20a^{3}d}$
derivativedivides	$\frac{\frac{\left(-A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{4d\,a^{3}}$
default	$\frac{\frac{\left(-A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{4d\,a^{3}}$
risch	$\frac{2i\left(15B\,e^{4i(dx+c)}+15A\,e^{3i(dx+c)}+30B\,e^{3i(dx+c)}+15A\,e^{2i(dx+c)}+40B\,e^{2i(dx+c)}+15A\,e^{i(dx+c)}+20B\,e^{i(dx+c)}+3A+7\right.}{15d\,a^{3}\left(e^{i(dx+c)}+1\right)^{5}}$
norman	$\frac{-\frac{\left(A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{20ad}+\frac{\left(A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\left(3A+2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}-\frac{\left(3A+2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{30ad}+\frac{\left(6A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{30ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{2}}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
-1/20*tan(1/2*d*x+1/2*c)*((A-B)*tan(1/2*d*x+1/2*c)^4+10/3*B*tan(1/2*d*x+1/
2*c)^2-5*A-5*B)/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\left((3\,A+7\,B)\cos\left(dx+c\right)^2+3\,(3\,A+2\,B)\cos\left(dx+c\right)+3\,A+2\,B\right)\sin\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
1/15*((3*A + 7*B)*cos(d*x + c)^2 + 3*(3*A + 2*B)*cos(d*x + c) + 3*A + 2*B)
*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} -\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}+\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos\left(c\right))\cos\left(c\right)}{\left(a\cos\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((-A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)/(4*a**3
*d) + B*tan(c/2 + d*x/2)**5/(20*a**3*d) - B*tan(c/2 + d*x/2)**3/(6*a**3*d)
 + B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*c
os(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="max
ima")
```

output
```
1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*A*(5*sin(d*x +
 c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,B}{60\,a^3d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gia
c")
```

output
```
-1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*B*tan(
1/2*d*x + 1/2*c)^3 - 15*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c)
)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 41.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(15\,A+15\,B-3\,A\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-10\,B\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+3\,B\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\right)}{60\,a^3\,d}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(15*A + 15*B - 3*A*tan(c/2 + (d*x)/2)^4 - 10*B*tan(c/2
 + (d*x)/2)^2 + 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+15a+15b\right)}{60a^3d}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(tan((c + d*x)/2)*( - 3*tan((c + d*x)/2)**4*a + 3*tan((c + d*x)/2)**4*b -
10*tan((c + d*x)/2)**2*b + 15*a + 15*b))/(60*a**3*d)
```

3.61 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(2A+3B)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output
```
1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(2*A+3*B)*sin(d*x+c)/a/d/(a
+a*cos(d*x+c))^2+1/15*(2*A+3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(7A+3B+(6A+9B)\cos(c+dx)+(2A+3B)\cos^2(c+dx))\sin(c+dx)}{15a^3d(1+\cos(c+dx))^3}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

output
```
((7*A + 3*B + (6*A + 9*B)*Cos[c + d*x] + (2*A + 3*B)*Cos[c + d*x]^2)*Sin[c
 + d*x])/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(2A+3B)\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2A+3B)\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(2A+3B)\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2A+3B)\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(2A+3B)\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B)*(Sin[c
+ d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[c + d
*x]))))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result	size
parallelrisch	$\frac{\left((A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\frac{10A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}+5A+5B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20a^{3}d}$	56
derivativedivides	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{4d\,a^{3}}$	64
default	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{4d\,a^{3}}$	64
risch	$\frac{2i\left(15B\,e^{3i(dx+c)}+20A\,e^{2i(dx+c)}+15B\,e^{2i(dx+c)}+10A\,e^{i(dx+c)}+15B\,e^{i(dx+c)}+2A+3B\right)}{15d\,a^{3}\left(e^{i(dx+c)}+1\right)^{5}}$	90
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{(5A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{(13A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}}$	117

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*((A-B)*tan(1/2*d*x+1/2*c)^4+10/3*A*tan(1/2*d*x+1/2*c)^2+5*A+5*B)*tan(
1/2*d*x+1/2*c)/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\left((2\,A+3\,B)\cos\left(dx+c\right)^2+3\,(2\,A+3\,B)\cos\left(dx+c\right)+7\,A+3\,B\right)\sin\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*((2*A + 3*B)*cos(d*x + c)^2 + 3*(2*A + 3*B)*cos(d*x + c) + 7*A + 3*B)
*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)**3/(6*a*
*3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) - B*tan(c/2 + d*x/2)**5/(20*a**3*d)
+ B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))/(a*cos(c) +
a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x +
 c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,B\tan}{60\,a^3d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*A*tan(1
/2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c))
/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 41.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(15\,A+15\,B+10\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3\,B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}{60\,a^3\,d}$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(15*A + 15*B + 10*A*tan(c/2 + (d*x)/2)^2 + 3*A*tan(c/2
 + (d*x)/2)^4 - 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+15a+15b\right)}{60a^3d}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(tan((c + d*x)/2)*(3*tan((c + d*x)/2)**4*a - 3*tan((c + d*x)/2)**4*b + 10*
tan((c + d*x)/2)**2*a + 15*a + 15*b))/(60*a**3*d)
```

3.62 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{A\text{arctanh}(\sin(c+dx))}{a^3d} - \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11A-B)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output

```
A*arctanh(sin(d*x+c))/a^3/d-1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15
*(7*A-2*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-2/15*(11*A-B)*sin(d*x+c)/d/(a
^3+a^3*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.68

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{-240A\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

output
```
(-240*A*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-5*(29
*A - 4*B)*Sin[(d*x)/2] + 75*A*Sin[c + (d*x)/2] - 95*A*Sin[c + (3*d*x)/2] +
 10*B*Sin[c + (3*d*x)/2] + 15*A*Sin[2*c + (3*d*x)/2] - 22*A*Sin[2*c + (5*d
*x)/2] + 2*B*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(5aA-2a(A-B)\cos(c+dx))\sec(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5aA-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(15a^2A-a^2(7A-2B)\cos(c+dx)\right)\sec(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{a(7A-2B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{15a^2A-a^2(7A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{a(7A-2B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int 15a^3A\sec(c+dx)dx}{a^2}-\frac{2a^2(11A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(7A-2B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{15aA\int \sec(c+dx)dx-\frac{2a^2(11A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(7A-2B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{15aA\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2a^2(11A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(7A-2B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\frac{15aA\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2(11A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(7A-2B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + (-1/3*(a*(7*A - 2
*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((15*a*A*ArcTanh[Sin[c + d*
x]])/d - (2*a^2*(11*A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2)
)/(5*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{-20A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+20A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left((A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{10(2A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3}\right.}{20a^3d}$
derivativedivides	$\frac{-4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}}{4d\,a^3}$
default	$\frac{-4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}}{4d\,a^3}$
risch	$-\frac{2i\left(15A\,e^{4i(dx+c)}+75A\,e^{3i(dx+c)}+145A\,e^{2i(dx+c)}-20B\,e^{2i(dx+c)}+95A\,e^{i(dx+c)}-10B\,e^{i(dx+c)}+22A-2B\right)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}+\frac{A\ln}{}$
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20ad}-\frac{5(5A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}-\frac{(7A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{(23A-13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{60ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{a^3d}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/20*(-20*A*ln(tan(1/2*d*x+1/2*c)-1)+20*A*ln(tan(1/2*d*x+1/2*c)+1)-tan(1/2
*d*x+1/2*c)*((A-B)*tan(1/2*d*x+1/2*c)^4+10/3*(2*A-B)*tan(1/2*d*x+1/2*c)^2+
35*A-5*B))/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{15\left(A\cos\left(dx+c\right)^3+3\,A\cos\left(dx+c\right)^2+3\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(A\cos\left(dx+c\right.\right.}{30\left(a^3d\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
1/30*(15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*lo
g(sin(d*x + c) + 1) - 15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(
d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*(2*(11*A - B)*cos(d*x + c)^2 + 3*
(17*A - 2*B)*cos(d*x + c) + 32*A - 7*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{A\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

output
```
(Integral(A*sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c +
d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**3 + 3*
cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right) - \frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10}{(\cos\cdots}\right.}{\cdots}}{60\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="max
ima")
```

output
```
-1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1
) - 1)/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="gia
c")
```

output
```
1/60*(60*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*A*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/
2*d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^12*tan(1/2*d*
x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 15*B*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 35.80 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{4\,a^3}+\frac{3\,A+B}{4\,a^3}+\frac{3\,A-B}{4\,a^3}\right)}{d} - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A-B\right)}{20\,a^3\,d} - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{A-B}{12\,a^3}+\frac{3\,A-B}{12\,a^3}\right)}{d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^3),x)
```

output
```
(2*A*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - (tan(c/2 + (d*x)/2)*((A - B)/(4*
a^3) + (3*A + B)/(4*a^3) + (3*A - B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(
A - B))/(20*a^3*d) - (tan(c/2 + (d*x)/2)^3*((A - B)/(12*a^3) + (3*A - B)/(
12*a^3)))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{-60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5a+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5b-20\,\mathrm{t}}{60a^3d}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x)
```

output
```
( - 60*log(tan((c + d*x)/2) - 1)*a + 60*log(tan((c + d*x)/2) + 1)*a - 3*ta
n((c + d*x)/2)**5*a + 3*tan((c + d*x)/2)**5*b - 20*tan((c + d*x)/2)**3*a +
 10*tan((c + d*x)/2)**3*b - 105*tan((c + d*x)/2)*a + 15*tan((c + d*x)/2)*b
)/(60*a**3*d)
```

3.63 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 145

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(3A-B)\text{arctanh}(\sin(c+dx))}{a^3d} + \frac{2(36A-11B)\tan(c+dx)}{15a^3d} - \frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(3A-B)\tan(c+dx)}{d(a^3+a^3\cos(c+dx))}$$

output

```
-(3*A-B)*arctanh(sin(d*x+c))/a^3/d+2/15*(36*A-11*B)*tan(d*x+c)/a^3/d-1/5*(
A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(9*A-4*B)*tan(d*x+c)/a/d/(a+a*co
s(d*x+c))^2-(3*A-B)*tan(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 482 vs. $2(145) = 290$.

Time = 4.48 (sec) , antiderivative size = 482, normalized size of antiderivative = 3.32

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{960(3A-B)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
(960*(3*A - B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]*(-5*(51*A - 32*B)*Sin[(d*x)/2] + (567*A - 167*B)*Sin[(
3*d*x)/2] - 600*A*Sin[c - (d*x)/2] + 170*B*Sin[c - (d*x)/2] + 375*A*Sin[c
+ (d*x)/2] - 170*B*Sin[c + (d*x)/2] - 480*A*Sin[2*c + (d*x)/2] + 160*B*Sin
[2*c + (d*x)/2] - 60*A*Sin[c + (3*d*x)/2] + 75*B*Sin[c + (3*d*x)/2] + 402*
A*Sin[2*c + (3*d*x)/2] - 167*B*Sin[2*c + (3*d*x)/2] - 225*A*Sin[3*c + (3*d
*x)/2] + 75*B*Sin[3*c + (3*d*x)/2] + 315*A*Sin[c + (5*d*x)/2] - 95*B*Sin[c
 + (5*d*x)/2] + 30*A*Sin[2*c + (5*d*x)/2] + 15*B*Sin[2*c + (5*d*x)/2] + 24
0*A*Sin[3*c + (5*d*x)/2] - 95*B*Sin[3*c + (5*d*x)/2] - 45*A*Sin[4*c + (5*d
*x)/2] + 15*B*Sin[4*c + (5*d*x)/2] + 72*A*Sin[2*c + (7*d*x)/2] - 22*B*Sin[
2*c + (7*d*x)/2] + 15*A*Sin[3*c + (7*d*x)/2] + 57*A*Sin[4*c + (7*d*x)/2] -
 22*B*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(a(6A-B)-3a(A-B)\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(6A-B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(a^2(27A-7B)-2a^2(9A-4B)\cos(c+dx)\right)\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(27A-7B)-2a^2(9A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int \left(2a^3(36A-11B)-15a^3(3A-B)\cos(c+dx)\right)\sec^2(c+dx)dx}{a^2}-\frac{15a^2(3A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{2a^3(36A-11B)-15a^3(3A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{15a^2(3A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{\frac{2a^3(36A-11B)\int\sec^2(c+dx)dx-15a^3(3A-B)\int\sec(c+dx)dx}{a^2}-\frac{15a^2(3A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2a^3(36A-11B)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-15a^3(3A-B)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{15a^2(3A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{-\frac{2a^3(36A-11B)\int 1d(-\tan(c+dx))}{d}-15a^3(3A-B)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{15a^2(3A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\frac{\frac{2a^3(36A-11B)\tan(c+dx)}{d}-15a^3(3A-B)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{15a^2(3A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\frac{\frac{2a^3(36A-11B)\tan(c+dx)}{d}-\frac{15a^3(3A-B)\text{arctanh}(\sin(c+dx))}{d}}{a^2}-\frac{15a^2(3A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(9A-4B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + (-1/3*(a*(9*A - 4
*B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((-15*a^2*(3*A - B)*Tan[c +
 d*x])/(d*(a + a*Cos[c + d*x])) + ((-15*a^3*(3*A - B)*ArcTanh[Sin[c + d*x]
])/d + (2*a^3*(36*A - 11*B)*Tan[c + d*x])/d)/a^2)/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{3\cos(dx+c)\left(A-\frac{B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3\cos(dx+c)\left(A-\frac{B}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{57\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\frac{\left(A-\frac{17B}{57}\right)\cos(2}{2}\right.}{}}{a^3 d\cos(dx+c)}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 A}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 B}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B}{3}+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+(-12A+4}{4d\,a^3}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 A}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 B}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B}{3}+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+(-12A+4}{4d\,a^3}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{20ad}+\frac{(3A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6ad}-\frac{(15A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{(25A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{(42A-17B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{10ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)a^2}$
risch	$\frac{2i(45A\,e^{6i(dx+c)}-15B\,e^{6i(dx+c)}+225A\,e^{5i(dx+c)}-75B\,e^{5i(dx+c)}+480A\,e^{4i(dx+c)}-160B\,e^{4i(dx+c)}+600A\,e^{3i(dx+c)}-}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5\left(e^{2i(dx+c}\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
3/20*(20*cos(d*x+c)*(A-1/3*B)*ln(tan(1/2*d*x+1/2*c)-1)-20*cos(d*x+c)*(A-1/
3*B)*ln(tan(1/2*d*x+1/2*c)+1)+19*tan(1/2*d*x+1/2*c)*(1/2*(A-17/57*B)*cos(2
*d*x+2*c)+1/19*(2*A-11/18*B)*cos(3*d*x+3*c)+(A-97/342*B)*cos(d*x+c)+67/114
*A-17/114*B)*sec(1/2*d*x+1/2*c)^4)/d/a^3/cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{15\left((3A-B)\cos(dx+c)^4+3(3A-B)\cos(dx+c)^3+3(3A-B)\cos(dx+c)^2+(3A-B)\cos(\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/30*(15*((3*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A
- B)*cos(d*x + c)^2 + (3*A - B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*(
(3*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A - B)*cos(d*
x + c)^2 + (3*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(36*A - 1
1*B)*cos(d*x + c)^3 + 3*(57*A - 17*B)*cos(d*x + c)^2 + (117*A - 32*B)*cos(
d*x + c) + 15*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\int \frac{A\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

output
```
(Integral(A*sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
 + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**
3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(139) = 278$.

Time = 0.04 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.97

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{3A\left(\frac{40\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(d}{\cos(dx}\right.}{a^3}\right.}{60\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
1/60*(3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60
*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3) - B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.31

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\frac{60\,(3\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,(3\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3}{}}{60\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/60*(60*(3*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(3*A - B)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 120*A*tan(1/2*d*x + 1/2*c)/((tan(
1/2*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12
*tan(1/2*d*x + 1/2*c)^5 + 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*B*a^12*tan
(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2
*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 42.55 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.16

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{6a^3}+\frac{4A-2B}{12a^3}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3A}{2a^3}+\frac{3(A-B)}{4a^3}+\frac{4A-2B}{2a^3}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(A-B)}{20a^3d} - \frac{2A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^3\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(3A-B)}{a^3d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^3),x)
```

output
```
(tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (4*A - 2*B)/(12*a^3)))/d + (tan(c
/2 + (d*x)/2)*((3*A)/(2*a^3) + (3*(A - B))/(4*a^3) + (4*A - 2*B)/(2*a^3)))
/d + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) - (2*A*tan(c/2 + (d*x)/2))/
(d*(a^3*tan(c/2 + (d*x)/2)^2 - a^3)) - (2*atanh(tan(c/2 + (d*x)/2))*(3*A -
 B))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.04

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{180\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-60\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-180\log\left(\tan\left(\frac{dx}{2}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x)
```

output
```
(180*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2*a - 60*log(tan((c + d*x
)/2) - 1)*tan((c + d*x)/2)**2*b - 180*log(tan((c + d*x)/2) - 1)*a + 60*log
(tan((c + d*x)/2) - 1)*b - 180*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)*
*2*a + 60*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*b + 180*log(tan((c
 + d*x)/2) + 1)*a - 60*log(tan((c + d*x)/2) + 1)*b + 3*tan((c + d*x)/2)**7
*a - 3*tan((c + d*x)/2)**7*b + 27*tan((c + d*x)/2)**5*a - 17*tan((c + d*x)
/2)**5*b + 225*tan((c + d*x)/2)**3*a - 85*tan((c + d*x)/2)**3*b - 375*tan(
(c + d*x)/2)*a + 105*tan((c + d*x)/2)*b)/(60*a**3*d*(tan((c + d*x)/2)**2 -
 1))
```

3.64 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 196

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(13A-6B)\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{8(19A-9B)\tan(c+dx)}{15a^3d} + \frac{(13A-6B)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{4(19A-9B)\sec(c+dx)\tan(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output
```
1/2*(13*A-6*B)*arctanh(sin(d*x+c))/a^3/d-8/15*(19*A-9*B)*tan(d*x+c)/a^3/d+
1/2*(13*A-6*B)*sec(d*x+c)*tan(d*x+c)/a^3/d-1/5*(A-B)*sec(d*x+c)*tan(d*x+c)
/d/(a+a*cos(d*x+c))^3-1/15*(11*A-6*B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d
*x+c))^2-4/15*(19*A-9*B)*sec(d*x+c)*tan(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 610 vs. $2(196) = 392$.

Time = 6.51 (sec) , antiderivative size = 610, normalized size of antiderivative = 3.11

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + a\cos(c + dx))^3}\,dx = -\frac{1920(13A - 6B)\cos^6\left(\frac{1}{2}(c + dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \mathrm{s}\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/480*(1920*(13*A - 6*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2
]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*((-1235*A + 870*B)*Sin[(d*x)/2] + 5*(761*
A - 366*B)*Sin[(3*d*x)/2] - 4329*A*Sin[c - (d*x)/2] + 2094*B*Sin[c - (d*x)
/2] + 1989*A*Sin[c + (d*x)/2] - 1314*B*Sin[c + (d*x)/2] - 3575*A*Sin[2*c +
 (d*x)/2] + 1650*B*Sin[2*c + (d*x)/2] - 475*A*Sin[c + (3*d*x)/2] + 450*B*S
in[c + (3*d*x)/2] + 2005*A*Sin[2*c + (3*d*x)/2] - 1230*B*Sin[2*c + (3*d*x)
/2] - 2275*A*Sin[3*c + (3*d*x)/2] + 1050*B*Sin[3*c + (3*d*x)/2] + 2673*A*S
in[c + (5*d*x)/2] - 1278*B*Sin[c + (5*d*x)/2] + 105*A*Sin[2*c + (5*d*x)/2]
 + 90*B*Sin[2*c + (5*d*x)/2] + 1593*A*Sin[3*c + (5*d*x)/2] - 918*B*Sin[3*c
 + (5*d*x)/2] - 975*A*Sin[4*c + (5*d*x)/2] + 450*B*Sin[4*c + (5*d*x)/2] +
1325*A*Sin[2*c + (7*d*x)/2] - 630*B*Sin[2*c + (7*d*x)/2] + 255*A*Sin[3*c +
 (7*d*x)/2] - 60*B*Sin[3*c + (7*d*x)/2] + 875*A*Sin[4*c + (7*d*x)/2] - 480
*B*Sin[4*c + (7*d*x)/2] - 195*A*Sin[5*c + (7*d*x)/2] + 90*B*Sin[5*c + (7*d
*x)/2] + 304*A*Sin[3*c + (9*d*x)/2] - 144*B*Sin[3*c + (9*d*x)/2] + 90*A*Si
n[4*c + (9*d*x)/2] - 30*B*Sin[4*c + (9*d*x)/2] + 214*A*Sin[5*c + (9*d*x)/2
] - 114*B*Sin[5*c + (9*d*x)/2]))/(a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(a(7A-2B)-4a(A-B)\cos(c+dx))\sec^3(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(7A-2B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(a^2(43A-18B)-3a^2(11A-6B)\cos(c+dx)\right)\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(43A-18B)-3a^2(11A-6B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int \left(15a^3(13A-6B)-8a^3(19A-9B)\cos(c+dx)\right)\sec^3(c+dx)dx}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{15a^3(13A-6B)-8a^3(19A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3227

$$\frac{\frac{\frac{15a^3(13A-6B)\int \sec^3(c+dx)dx-8a^3(19A-9B)\int \sec^2(c+dx)dx}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{15a^3(13A-6B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-8a^3(19A-9B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 4254

$$\frac{\frac{\frac{\frac{8a^3(19A-9B)\int 1d(-\tan(c+dx))}{d}+15a^3(13A-6B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 24

$$\frac{\frac{\frac{15a^3(13A-6B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{8a^3(19A-9B)\tan(c+dx)}{d}}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 4255

$$\frac{\frac{\frac{15a^3(13A-6B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{8a^3(19A-9B)\tan(c+dx)}{d}}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\sec(c}{3d(a\cos(c+dx)+a)^2}}{5a^2}$$
$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{15a^3(13A-6B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{8a^3(19A-9B)\tan(c+dx)}{d}}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx)\,s}{3d(a\cos(c+dx)+}}{5a^2}$$
$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 4257

$$\frac{\frac{\frac{15a^3(13A-6B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{8a^3(19A-9B)\tan(c+dx)}{d}}{a^2}-\frac{4a^2(19A-9B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{a(11A-6B)\tan(c+dx}{3d(a\cos(c+dx)}}{5a^2}$$
$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*((A - B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + (-1/
3*(a*(11*A - 6*B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^2) +
((-4*a^2*(19*A - 9*B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
+ ((-8*a^3*(19*A - 9*B)*Tan[c + d*x])/d + 15*a^3*(13*A - 6*B)*(ArcTanh[Sin
[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2)/(3*a^2))/(5*a^
2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

method	result
parallelrisch	$\frac{-1560\left(A-\frac{6B}{13}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+1560\left(A-\frac{6B}{13}\right)(\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-152\left(\left(\right.\right.}{}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 B}{5}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{-14A}{\tan\left(\frac{dx}{2}\right.}}{}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 A}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 B}{5}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 A}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 B-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B-\frac{-14A}{\tan\left(\frac{dx}{2}\right.}}{}$
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{20ad}-\frac{(37A-27B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{60ad}-\frac{(51A-25B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{(109A-45B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}-\frac{(211A-111B)\tan}{30ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2 a^2}$
risch	$-\frac{i\left(195A\,e^{8i(dx+c)}-90B\,e^{8i(dx+c)}+975A\,e^{7i(dx+c)}-450B\,e^{7i(dx+c)}+2275A\,e^{6i(dx+c)}-1050B\,e^{6i(dx+c)}+3575A\,e^{5i(\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/240*(-1560*(A-6/13*B)*(cos(2*d*x+2*c)+1)*ln(tan(1/2*d*x+1/2*c)-1)+1560*(
A-6/13*B)*(cos(2*d*x+2*c)+1)*ln(tan(1/2*d*x+1/2*c)+1)-152*((783/76*A-189/3
8*B)*cos(2*d*x+2*c)+(717/152*A-9/4*B)*cos(3*d*x+3*c)+(A-9/19*B)*cos(4*d*x+
4*c)+(2331/152*A-573/76*B)*cos(d*x+c)+677/76*A-9/2*B)*sec(1/2*d*x+1/2*c)^4
*tan(1/2*d*x+1/2*c))/d/a^3/(cos(2*d*x+2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.51

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{15\left((13\,A-6\,B)\cos\left(dx+c\right)^5+3\,(13\,A-6\,B)\cos\left(dx+c\right)^4+3\,(13\,A-6\,B)\cos\left(dx+c\right)^3+(13\,A-6\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/60*(15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 + 3*
(13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(sin(d*x + c
) + 1) - 15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 +
 3*(13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(-sin(d*x
 + c) + 1) - 2*(16*(19*A - 9*B)*cos(d*x + c)^4 + 3*(239*A - 114*B)*cos(d*x
 + c)^3 + (479*A - 234*B)*cos(d*x + c)^2 + 15*(3*A - 2*B)*cos(d*x + c) - 1
5*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*
d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + a\cos(c + dx))^3}\,dx = \frac{\int \frac{A\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

output
```
(Integral(A*sec(c + d*x)**3/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
 + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)**3/(cos(c + d*x)**
3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(184) = 368$.

Time = 0.04 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.92

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + a\cos(c + dx))^3}\,dx = -\frac{A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1} - \frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3 - \frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1} + \frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^3} + \frac{390\log\left(\frac{\sin}{\cos}\right.}{}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) +
40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin
(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*B*(40*sin(d*x + c)/((a^3 - a^3*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c
)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x +
c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.19

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{30\,(13\,A-6\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(13\,A-6\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{60\left(7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,A\,\text{ta}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/60*(30*(13*A - 6*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(13*A -
6*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*A*tan(1/2*d*x + 1/2*c)
^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d
*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x
+ 1/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^12*tan(1/2*d*x + 1/2
*c)^3 - 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c)
 - 255*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 42.36 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(7A-2B)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(5A-2B)}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3\right)}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3(A-B)}{2a^3}+\frac{3(5A-3B)}{4a^3}+\frac{10A-2B}{4a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{4a^3}+\frac{5A-3B}{12a^3}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(A-B)}{20a^3d}+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(13A-6B)}{a^3d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^3),x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(7*A - 2*B) - tan(c/2 + (d*x)/2)*(5*A - 2*B))/(d*(a^
3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^3)) - (tan(c/2 + (
d*x)/2)*((3*(A - B))/(2*a^3) + (3*(5*A - 3*B))/(4*a^3) + (10*A - 2*B)/(4*a
^3)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) + (5*A - 3*B)/(12*a^3)))/
d - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) + (atanh(tan(c/2 + (d*x)/2))
*(13*A - 6*B))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.25

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x)
```

output
```
( - 390*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**4*a + 180*log(tan((c +
 d*x)/2) - 1)*tan((c + d*x)/2)**4*b + 780*log(tan((c + d*x)/2) - 1)*tan((c
 + d*x)/2)**2*a - 360*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2*b - 39
0*log(tan((c + d*x)/2) - 1)*a + 180*log(tan((c + d*x)/2) - 1)*b + 390*log(
tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**4*a - 180*log(tan((c + d*x)/2) + 1
)*tan((c + d*x)/2)**4*b - 780*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**
2*a + 360*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*b + 390*log(tan((c
 + d*x)/2) + 1)*a - 180*log(tan((c + d*x)/2) + 1)*b - 3*tan((c + d*x)/2)**
9*a + 3*tan((c + d*x)/2)**9*b - 34*tan((c + d*x)/2)**7*a + 24*tan((c + d*x
)/2)**7*b - 388*tan((c + d*x)/2)**5*a + 198*tan((c + d*x)/2)**5*b + 1310*t
an((c + d*x)/2)**3*a - 600*tan((c + d*x)/2)**3*b - 765*tan((c + d*x)/2)*a
+ 375*tan((c + d*x)/2)*b)/(60*a**3*d*(tan((c + d*x)/2)**4 - 2*tan((c + d*x
)/2)**2 + 1))
```

3.65 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 229

$$\begin{aligned}\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx &= -\frac{(8A-21B)x}{2a^4}+\frac{8(83A-216B)\sin(c+dx)}{105a^4d} \\ &\quad -\frac{(8A-21B)\cos(c+dx)\sin(c+dx)}{2a^4d} \\ &\quad +\frac{(52A-129B)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} \\ &\quad +\frac{4(83A-216B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))} \\ &\quad +\frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\ &\quad +\frac{(A-2B)\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3}\end{aligned}$$

output
```
-1/2*(8*A-21*B)*x/a^4+8/105*(83*A-216*B)*sin(d*x+c)/a^4/d-1/2*(8*A-21*B)*c
os(d*x+c)*sin(d*x+c)/a^4/d+1/105*(52*A-129*B)*cos(d*x+c)^3*sin(d*x+c)/a^4/
d/(1+cos(d*x+c))^2+4/105*(83*A-216*B)*cos(d*x+c)^2*sin(d*x+c)/a^4/d/(1+cos
(d*x+c))+1/7*(A-B)*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/5*(A-2*B
)*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 555 vs. $2(229) = 458$.

Time = 7.31 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.42

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-14700(8A-21B)dx\cos\left(\frac{dx}{2}\right)-14700(8A-21B)dx\cos\left(c+\frac{dx}{2}\right)-70560Adx\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(-14700*(8*A - 21*B)*d*x*Cos[(d*x)/2] - 14700*(
8*A - 21*B)*d*x*Cos[c + (d*x)/2] - 70560*A*d*x*Cos[c + (3*d*x)/2] + 185220
*B*d*x*Cos[c + (3*d*x)/2] - 70560*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*B*d*
x*Cos[2*c + (3*d*x)/2] - 23520*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*B*d*x*Co
s[2*c + (5*d*x)/2] - 23520*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*B*d*x*Cos[3*
c + (5*d*x)/2] - 3360*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*B*d*x*Cos[3*c + (7
*d*x)/2] - 3360*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*B*d*x*Cos[4*c + (7*d*x)/
2] + 243320*A*Sin[(d*x)/2] - 539490*B*Sin[(d*x)/2] - 184520*A*Sin[c + (d*x
)/2] + 386190*B*Sin[c + (d*x)/2] + 184464*A*Sin[c + (3*d*x)/2] - 422478*B*
Sin[c + (3*d*x)/2] - 72240*A*Sin[2*c + (3*d*x)/2] + 132930*B*Sin[2*c + (3*
d*x)/2] + 77168*A*Sin[2*c + (5*d*x)/2] - 181461*B*Sin[2*c + (5*d*x)/2] - 8
400*A*Sin[3*c + (5*d*x)/2] + 3675*B*Sin[3*c + (5*d*x)/2] + 15164*A*Sin[3*c
 + (7*d*x)/2] - 36003*B*Sin[3*c + (7*d*x)/2] + 2940*A*Sin[4*c + (7*d*x)/2]
 - 9555*B*Sin[4*c + (7*d*x)/2] + 420*A*Sin[4*c + (9*d*x)/2] - 945*B*Sin[4*
c + (9*d*x)/2] + 420*A*Sin[5*c + (9*d*x)/2] - 945*B*Sin[5*c + (9*d*x)/2] +
 105*B*Sin[5*c + (11*d*x)/2] + 105*B*Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(
1 + Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 3456, 3042, 3456, 3042, 3456, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(2A-9B)\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(5a(A-B)-a(2A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{\cos^3(c+dx)\left(28a^2(A-2B)-a^2(24A-73B)\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{7a(A-2B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(28a^2(A-2B)-a^2(24A-73B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{7a(A-2B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\frac{\int \frac{\cos^2(c+dx)\left(3a^3(52A-129B)-a^3(176A-477B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{7a(A-2B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a^3(52A-129B)-a^3(176A-477B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{7a(A-2B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 3456

$$\frac{\frac{\frac{\frac{\int \cos(c+dx)\left(8a^4(83A-216B)-105a^4(8A-21B)\cos(c+dx)\right)dx}{a^2}+\frac{4a^3(83A-216B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{7a(A-2B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(8a^4(83A-216B)-105a^4(8A-21B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{4a^3(83A-216B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{7a(A-2B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 3213

$$\frac{\frac{\frac{\frac{4a^3(83A-216B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{\frac{8a^4(83A-216B)\sin(c+dx)}{d}-\frac{105a^4(8A-21B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{105}{2}a^4x(8A-21B)}{a^2}}{3a^2}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{7a(A-2B)\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
((A - B)*Cos[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((7*a
*(A - 2*B)*Cos[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (((
52*A - 129*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(1 + Cos[c + d*x])^2) + ((
4*a^3*(83*A - 216*B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
 + ((-105*a^4*(8*A - 21*B)*x)/2 + (8*a^4*(83*A - 216*B)*Sin[c + d*x])/d -
(105*a^4*(8*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2)/(3*a^2))/(5*a
^2))/(7*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

method	result
parallelrisch	$\frac{420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\frac{10964A}{105}-\frac{9376B}{35}\right)\cos(2dx+2c)+\left(\frac{2368A}{105}-\frac{7873B}{140}\right)\cos(3dx+3c)+(A-2B)\cos(4dx+4c)+\frac{B\cos(5dx+}{4}\right.}{26880a^4d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B+49\tan(}{8}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}+13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B+49\tan(}{8}$
risch	$-\frac{4xA}{a^4}+\frac{21Bx}{2a^4}-\frac{iB\,e^{2i(dx+c)}}{8a^4d}-\frac{ie^{i(dx+c)}A}{2a^4d}+\frac{2ie^{i(dx+c)}B}{a^4d}+\frac{ie^{-i(dx+c)}A}{2a^4d}-\frac{2ie^{-i(dx+c)}B}{a^4d}+\frac{iB\,e^{-2i(dx}}{8a^4d}$

input
```
int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/26880*(420*tan(1/2*d*x+1/2*c)*((10964/105*A-9376/35*B)*cos(2*d*x+2*c)+(2
368/105*A-7873/140*B)*cos(3*d*x+3*c)+(A-2*B)*cos(4*d*x+4*c)+1/4*B*cos(5*d*
x+5*c)+(24992/105*A-42881/70*B)*cos(d*x+c)+16171/105*A-13914/35*B)*sec(1/2
*d*x+1/2*c)^6-107520*x*(A-21/8*B)*d)/a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx =$$
$$-\frac{105\,(8\,A-21\,B)dx\cos{(dx+c)}^4+420\,(8\,A-21\,B)dx\cos{(dx+c)}^3+630\,(8\,A-21\,B)dx\cos{(dx+c}}{}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
-1/210*(105*(8*A - 21*B)*d*x*cos(d*x + c)^4 + 420*(8*A - 21*B)*d*x*cos(d*x
 + c)^3 + 630*(8*A - 21*B)*d*x*cos(d*x + c)^2 + 420*(8*A - 21*B)*d*x*cos(d
*x + c) + 105*(8*A - 21*B)*d*x - (105*B*cos(d*x + c)^5 + 210*(A - 2*B)*cos
(d*x + c)^4 + 4*(592*A - 1509*B)*cos(d*x + c)^3 + 4*(1318*A - 3411*B)*cos(
d*x + c)^2 + (4472*A - 11619*B)*cos(d*x + c) + 1328*A - 3456*B)*sin(d*x +
c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^
2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(216) = 432$.

Time = 8.12 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.74

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((-3360*A*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4
 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 6720*A*d*x*tan(c/2 + d*
x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2
+ 840*a**4*d) - 3360*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*t
an(c/2 + d*x/2)**2 + 840*a**4*d) - 15*A*tan(c/2 + d*x/2)**11/(840*a**4*d*t
an(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 117*A
*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2
 + d*x/2)**2 + 840*a**4*d) - 526*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2
 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 3682*A*tan(
c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*
x/2)**2 + 840*a**4*d) + 11165*A*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 +
d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*A*tan(c/2
 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**
2 + 840*a**4*d) + 8820*B*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x
/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 17640*B*d*x*tan(c
/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x
/2)**2 + 840*a**4*d) + 8820*B*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a
**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 15*B*tan(c/2 + d*x/2)**11/(840*a
**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d)
- 159*B*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4...
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.59

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx =$$

$$-\frac{3\,B\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\,\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{a^4}\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
-1/840*(3*B*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) +
1) - 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a^4) - A*(1680*sin(d*x + c)/((a^4 + a^4*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/
(cos(d*x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4
 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.02

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = -\frac{\frac{420\,(dx+c)(8\,A-21\,B)}{a^4} - \frac{840\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-1/840*(420*(d*x + c)*(8*A - 21*B)/a^4 - 840*(2*A*tan(1/2*d*x + 1/2*c)^3 -
 9*B*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 7*B*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x + 1
/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*A*a^24*tan(1/2*d*x + 1/2*
c)^5 + 189*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)
^3 - 1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*A*a^24*tan(1/2*d*x + 1/2*c)
 + 11655*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 42.32 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.13

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5(A-B)}{4a^4}-\frac{5B}{2a^4}+\frac{3(4A-6B)}{4a^4}+\frac{3(5A-15B)}{8a^4}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{4a^4}+\frac{4A-6B}{8a^4}+\frac{5A-15B}{24a^4}\right)}{d}-\frac{x\,(8A-21B)}{2a^4}$$
$$+\frac{(2A-9B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2A-7B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{3(A-B)}{40a^4}+\frac{4A-6B}{40a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(A-B)}{56a^4d}$$

input
```
int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*((5*(A - B))/(4*a^4) - (5*B)/(2*a^4) + (3*(4*A - 6*B))
/(4*a^4) + (3*(5*A - 15*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(
4*a^4) + (4*A - 6*B)/(8*a^4) + (5*A - 15*B)/(24*a^4)))/d - (x*(8*A - 21*B)
)/(2*a^4) + (tan(c/2 + (d*x)/2)^3*(2*A - 9*B) + tan(c/2 + (d*x)/2)*(2*A -
7*B))/(d*(2*a^4*tan(c/2 + (d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 + a^4)) +
(tan(c/2 + (d*x)/2)^5*((3*(A - B))/(40*a^4) + (4*A - 6*B)/(40*a^4)))/d - (
tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}a+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}b+117\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9a-159\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9b-526\tan\left(\frac{dx}{2}+\right.}{}$$

input
```
int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

output
```
( - 15*tan((c + d*x)/2)**11*a + 15*tan((c + d*x)/2)**11*b + 117*tan((c + d
*x)/2)**9*a - 159*tan((c + d*x)/2)**9*b - 526*tan((c + d*x)/2)**7*a + 1002
*tan((c + d*x)/2)**7*b + 3682*tan((c + d*x)/2)**5*a - 9114*tan((c + d*x)/2
)**5*b - 3360*tan((c + d*x)/2)**4*a*d*x + 8820*tan((c + d*x)/2)**4*b*d*x +
 11165*tan((c + d*x)/2)**3*a - 29505*tan((c + d*x)/2)**3*b - 6720*tan((c +
 d*x)/2)**2*a*d*x + 17640*tan((c + d*x)/2)**2*b*d*x + 6825*tan((c + d*x)/2
)*a - 17535*tan((c + d*x)/2)*b - 3360*a*d*x + 8820*b*d*x)/(840*a**4*d*(tan
((c + d*x)/2)**4 + 2*tan((c + d*x)/2)**2 + 1))
```

3.66 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 185

$$\begin{aligned}\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx &= \frac{(A-4B)x}{a^4}-\frac{(55A-244B)\sin(c+dx)}{105a^4d} \\ &\quad+\frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} \\ &\quad-\frac{(A-4B)\sin(c+dx)}{a^4d(1+\cos(c+dx))} \\ &\quad+\frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\ &\quad+\frac{(5A-12B)\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}\end{aligned}$$

output
```
(A-4*B)*x/a^4-1/105*(55*A-244*B)*sin(d*x+c)/a^4/d+1/105*(25*A-88*B)*cos(d*
x+c)^2*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-(A-4*B)*sin(d*x+c)/a^4/d/(1+cos(d
*x+c))+1/7*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(5*A-12
*B)*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 481 vs. $2(185) = 370$.

Time = 6.94 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.60

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(7350(A-4B)dx\cos\left(\frac{dx}{2}\right)+7350(A-4B)dx\cos\left(c+\frac{dx}{2}\right)+4410Adx\cos\left(c+\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(7350*(A - 4*B)*d*x*Cos[(d*x)/2] + 7350*(A - 4*
B)*d*x*Cos[c + (d*x)/2] + 4410*A*d*x*Cos[c + (3*d*x)/2] - 17640*B*d*x*Cos[
c + (3*d*x)/2] + 4410*A*d*x*Cos[2*c + (3*d*x)/2] - 17640*B*d*x*Cos[2*c + (
3*d*x)/2] + 1470*A*d*x*Cos[2*c + (5*d*x)/2] - 5880*B*d*x*Cos[2*c + (5*d*x)
/2] + 1470*A*d*x*Cos[3*c + (5*d*x)/2] - 5880*B*d*x*Cos[3*c + (5*d*x)/2] +
210*A*d*x*Cos[3*c + (7*d*x)/2] - 840*B*d*x*Cos[3*c + (7*d*x)/2] + 210*A*d*
x*Cos[4*c + (7*d*x)/2] - 840*B*d*x*Cos[4*c + (7*d*x)/2] - 19880*A*Sin[(d*x
)/2] + 60830*B*Sin[(d*x)/2] + 16520*A*Sin[c + (d*x)/2] - 46130*B*Sin[c + (
d*x)/2] - 14280*A*Sin[c + (3*d*x)/2] + 46116*B*Sin[c + (3*d*x)/2] + 7560*A
*Sin[2*c + (3*d*x)/2] - 18060*B*Sin[2*c + (3*d*x)/2] - 5600*A*Sin[2*c + (5
*d*x)/2] + 19292*B*Sin[2*c + (5*d*x)/2] + 1680*A*Sin[3*c + (5*d*x)/2] - 21
00*B*Sin[3*c + (5*d*x)/2] - 1040*A*Sin[3*c + (7*d*x)/2] + 3791*B*Sin[3*c +
 (7*d*x)/2] + 735*B*Sin[4*c + (7*d*x)/2] + 105*B*Sin[4*c + (9*d*x)/2] + 10
5*B*Sin[5*c + (9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 3456, 3042, 3456, 3042, 3456, 3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(A-8B)\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a(A-B)-a(A-8B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(3a^2(5A-12B)-2a^2(5A-26B)\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a^2(5A-12B)-2a^2(5A-26B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\frac{\int \frac{\cos(c+dx)\left(2a^3(25A-88B)-a^3(55A-244B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a^3(25A-88B)-a^3(55A-244B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3447

$$\frac{\frac{\frac{\int \frac{2a^3(25A-88B)\cos(c+dx)-a^3(55A-244B)\cos^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{2a^3(25A-88B)\sin\left(c+dx+\frac{\pi}{2}\right)-a^3(55A-244B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3502

$$\frac{\frac{\frac{\frac{\int \frac{105a^4(A-4B)\cos(c+dx)}{\cos(c+dx)a+a}dx}{a}-\frac{a^2(55A-244B)\sin(c+dx)}{d}}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{105a^3(A-4B)\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}dx-\frac{a^2(55A-244B)\sin(c+dx)}{d}}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{105a^3(A-4B)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{a^2(55A-244B)\sin(c+dx)}{d}}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\frac{105a^3(A-4B)\left(\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx\right)-\frac{a^2(55A-244B)\sin(c+dx)}{d}}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{105a^3(A-4B)\left(\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)-\frac{a^2(55A-244B)\sin(c+dx)}{d}}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\frac{\frac{105a^3(A-4B)\left(\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)-\frac{a^2(55A-244B)\sin(c+dx)}{d}}{3a^2}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(5A-12B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((a*(
5*A - 12*B)*Cos[c + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((
(25*A - 88*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(1 + Cos[c + d*x])^2) + (-
((a^2*(55*A - 244*B)*Sin[c + d*x])/d) + 105*a^3*(A - 4*B)*(x/a - Sin[c + d
*x]/(d*(a + a*Cos[c + d*x]))))/(3*a^2))/(5*a^2))/(7*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{-5840\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\frac{31A}{73}-\frac{2741B}{1460}\right)\cos(2dx+2c)+\left(\frac{13A}{146}-\frac{148B}{365}\right)\cos(3dx+3c)-\frac{21B\cos(4dx+4c)}{1168}+\left(A-\frac{1562B}{365}\right)\cos(d\right.}{6720a^4d}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-15\tan\left(\frac{dx}{2}\right.}{8d\,a^4}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A}{3}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-15\tan\left(\frac{dx}{2}\right.}{8d\,a^4}$
risch	$\frac{xA}{a^4}-\frac{4Bx}{a^4}-\frac{i\,\mathrm{e}^{i(dx+c)}B}{2a^4d}+\frac{i\,\mathrm{e}^{-i(dx+c)}B}{2a^4d}-\frac{4i\left(210A\,\mathrm{e}^{6i(dx+c)}-525B\,\mathrm{e}^{6i(dx+c)}+945A\,\mathrm{e}^{5i(dx+c)}-2625B\,\mathrm{e}^{5i(dx+\right.}{}$
norman	$\frac{\frac{(A-4B)x}{a}+\frac{(A-4B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{a}+\frac{(A-22B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{84ad}+\frac{5(A-4B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{10(A-4B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}+\frac{10(A-}{}}{}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/6720*(-5840*tan(1/2*d*x+1/2*c)*((31/73*A-2741/1460*B)*cos(2*d*x+2*c)+(13
/146*A-148/365*B)*cos(3*d*x+3*c)-21/1168*B*cos(4*d*x+4*c)+(A-1562/365*B)*c
os(d*x+c)+47/73*A-16171/5840*B)*sec(1/2*d*x+1/2*c)^6+6720*d*x*(A-4*B))/a^4
/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{105\,(A-4\,B)dx\cos{(dx+c)}^4 + 420\,(A-4\,B)dx\cos{(dx+c)}^3 + 630\,(A-4\,B)dx\cos{(dx+c)}^2 + 420\,(A$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
1/105*(105*(A - 4*B)*d*x*cos(d*x + c)^4 + 420*(A - 4*B)*d*x*cos(d*x + c)^3
 + 630*(A - 4*B)*d*x*cos(d*x + c)^2 + 420*(A - 4*B)*d*x*cos(d*x + c) + 105
*(A - 4*B)*d*x + (105*B*cos(d*x + c)^4 - 4*(65*A - 296*B)*cos(d*x + c)^3 -
 4*(155*A - 659*B)*cos(d*x + c)^2 - (535*A - 2236*B)*cos(d*x + c) - 160*A
+ 664*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*
a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(172) = 344.

Time = 5.21 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.12

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((840*A*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 +
 840*a**4*d) + 840*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 1
5*A*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 90
*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 280
*A*tan(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 119
0*A*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15
75*A*tan(c/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360
*B*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) -
 3360*B*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*B*tan(c/2 +
 d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*B*tan(c/2 +
 d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*B*tan(c/2 +
 d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*B*tan(c/2
+ d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*B*tan(c/2
 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*(A
+ B*cos(c))*cos(c)**4/(a*cos(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.46

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{B\left(\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right.}{840\,d}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
1/840*(B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^4) - 5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^4))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{840\,(dx+c)(A-4\,B)}{a^4} + \frac{1680\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+147\,Ba^{2}}{}}{840\,d}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
1/840*(840*(d*x + c)*(A - 4*B)/a^4 + 1680*B*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*B*a^24*ta
n(1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 805*B*a^24*tan(
1/2*d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*B*a^24*tan(1/
2*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 42.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.09

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{A\,d\,x-4\,B\,d\,x}{a^4\,d} - \frac{\left(\frac{52\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{764\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{143\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}-\frac{16\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 -}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7} + \frac{2\,B\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4\,d}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

output
```
(A*d*x - 4*B*d*x)/(a^4*d) - ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x
)/2))/56 + cos(c/2 + (d*x)/2)^2*((5*A*sin(c/2 + (d*x)/2))/28 - (8*B*sin(c/
2 + (d*x)/2))/35) - cos(c/2 + (d*x)/2)^4*((16*A*sin(c/2 + (d*x)/2))/21 - (
143*B*sin(c/2 + (d*x)/2))/105) + cos(c/2 + (d*x)/2)^6*((52*A*sin(c/2 + (d*
x)/2))/21 - (764*B*sin(c/2 + (d*x)/2))/105))/(a^4*d*cos(c/2 + (d*x)/2)^7)
+ (2*B*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9 a-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9 b-90\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 a+132\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 b+280\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 a}{}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

output
```
(15*tan((c + d*x)/2)**9*a - 15*tan((c + d*x)/2)**9*b - 90*tan((c + d*x)/2)
**7*a + 132*tan((c + d*x)/2)**7*b + 280*tan((c + d*x)/2)**5*a - 658*tan((c
 + d*x)/2)**5*b - 1190*tan((c + d*x)/2)**3*a + 4340*tan((c + d*x)/2)**3*b
+ 840*tan((c + d*x)/2)**2*a*d*x - 3360*tan((c + d*x)/2)**2*b*d*x - 1575*ta
n((c + d*x)/2)*a + 6825*tan((c + d*x)/2)*b + 840*a*d*x - 3360*b*d*x)/(840*
a**4*d*(tan((c + d*x)/2)**2 + 1))
```

3.67 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 154

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{Bx}{a^4} - \frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(12A-215B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)\cos^2(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

output
```
B*x/a^4-1/105*(6*A-55*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+1/105*(12*A-215
*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))+1/7*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a
+a*cos(d*x+c))^4+1/35*(3*A-10*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+
c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(154) = 308.

Time = 6.80 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.14

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(3675Bdx\cos\left(\frac{dx}{2}\right)+3675Bdx\cos\left(c+\frac{dx}{2}\right)+2205Bdx\cos\left(c+\frac{3dx}{2}\right)+2205Bd\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*B*d*x*Cos[(d*x)/2] + 3675*B*d*x*Cos[c +
 (d*x)/2] + 2205*B*d*x*Cos[c + (3*d*x)/2] + 2205*B*d*x*Cos[2*c + (3*d*x)/2
] + 735*B*d*x*Cos[2*c + (5*d*x)/2] + 735*B*d*x*Cos[3*c + (5*d*x)/2] + 105*
B*d*x*Cos[3*c + (7*d*x)/2] + 105*B*d*x*Cos[4*c + (7*d*x)/2] + 1260*A*Sin[(
d*x)/2] - 9940*B*Sin[(d*x)/2] - 1260*A*Sin[c + (d*x)/2] + 8260*B*Sin[c + (
d*x)/2] + 882*A*Sin[c + (3*d*x)/2] - 7140*B*Sin[c + (3*d*x)/2] - 630*A*Sin
[2*c + (3*d*x)/2] + 3780*B*Sin[2*c + (3*d*x)/2] + 294*A*Sin[2*c + (5*d*x)/
2] - 2800*B*Sin[2*c + (5*d*x)/2] - 210*A*Sin[3*c + (5*d*x)/2] + 840*B*Sin[
3*c + (5*d*x)/2] + 72*A*Sin[3*c + (7*d*x)/2] - 520*B*Sin[3*c + (7*d*x)/2])
)/(13440*a^4*d)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3456, 3042, 3456, 3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$\Big\downarrow$ 3456

$$\frac{\int \frac{\cos^2(c+dx)(3a(A-B)+7aB\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a(A-B)+7aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3456

$$\frac{\frac{\int \frac{\cos(c+dx)\left(2(3A-10B)a^2+35B\cos(c+dx)a^2\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2(3A-10B)a^2+35B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3447

$$\frac{\frac{\int \frac{35B\cos^2(c+dx)a^2+2(3A-10B)\cos(c+dx)a^2}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{35B\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+2(3A-10B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3498

$$\frac{\frac{-\frac{\int -\frac{2(6A-55B)a^3+105B\cos(c+dx)a^3}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(6A-55B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\frac{\int \frac{2(6A-55B)a^3+105B\cos(c+dx)a^3}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(6A-55B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{\int \frac{2(6A-55B)a^3+105B\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{(6A-55B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{3214}$$

$$\frac{\frac{\frac{a^3(12A-215B)\int \frac{1}{\cos(c+dx)a+a}dx+105a^2Bx}{3a^2}-\frac{(6A-55B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{a^3(12A-215B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+105a^2Bx}{3a^2}-\frac{(6A-55B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \textbf{3127}$$

$$\frac{\frac{\frac{\frac{a^3(12A-215B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+105a^2Bx}{3a^2}-\frac{(6A-55B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((a*(
3*A - 10*B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (-
1/3*((6*A - 55*B)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^2) + (105*a^2*B*x +
(a^3*(12*A - 215*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(5*a^
2))/(7*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{(-15A+15B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+(63A-105B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+(-105A+385B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+(105A-1575B)\tan\left(\frac{dx}{2}+\right.}{840a^4d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}+\tan\left(\frac{dx}{2}+\right.}{8d\,a^4}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}+\tan\left(\frac{dx}{2}+\right.}{8d\,a^4}$
risch	$\frac{Bx}{a^4}+\frac{2i\left(105A\,e^{6i(dx+c)}-420B\,e^{6i(dx+c)}+315A\,e^{5i(dx+c)}-1890B\,e^{5i(dx+c)}+630A\,e^{4i(dx+c)}-4130B\,e^{4i(dx+c)}+63\right.}{105d\,a^4\left(e\right.}$
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{a}+\frac{4Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{6Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}+\frac{4Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a}+\frac{(A-15B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{(A-15B)\tan\left(\right.}{280a}}{}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/840*((-15*A+15*B)*tan(1/2*d*x+1/2*c)^7+(63*A-105*B)*tan(1/2*d*x+1/2*c)^5
+(-105*A+385*B)*tan(1/2*d*x+1/2*c)^3+(105*A-1575*B)*tan(1/2*d*x+1/2*c)+840
*d*x*B)/a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{105\,Bdx\cos\left(dx+c\right)^4+420\,Bdx\cos\left(dx+c\right)^3+630\,Bdx\cos\left(dx+c\right)^2+420\,Bdx\cos\left(dx+c\right)+105\,B\cdots}{105\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(d\cdots\right.\right.}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
1/105*(105*B*d*x*cos(d*x + c)^4 + 420*B*d*x*cos(d*x + c)^3 + 630*B*d*x*cos
(d*x + c)^2 + 420*B*d*x*cos(d*x + c) + 105*B*d*x + (4*(9*A - 65*B)*cos(d*x
 + c)^3 + (39*A - 620*B)*cos(d*x + c)^2 + (24*A - 535*B)*cos(d*x + c) + 6*
A - 160*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 +
6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}+\frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}-\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{Bx}{a^4}+\frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{11\cdots}{\cdots} \\ \frac{x(A+B\cos\left(c\right))\cos^3\left(c\right)}{\left(a\cos\left(c\right)+a\right)^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(4
0*a**4*d) - A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*
d) + B*x/a**4 + B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/
(8*a**4*d) + 11*B*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*B*tan(c/2 + d*x/2)/
(8*a**4*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**4, True
))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = -\frac{5\,B\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+}{(\cos(dx+c)}\right.}{}}{840\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
-1/840*(5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^4) - 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{840\,(dx+c)B}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,Aa^{24}\tan\left(\frac{1}{2}\,d\right.}{a^{28}}}{840\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
1/840*(840*(d*x + c)*B/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24
*tan(1/2*d*x + 1/2*c)^7 - 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*B*a^24*ta
n(1/2*d*x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(
1/2*d*x + 1/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/2*c) + 1575*B*a^24*tan(1/2
*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 41.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{B\,x}{a^4} + \frac{\left(\frac{12\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}-\frac{52\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{16\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{23\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{70}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

output
```
(B*x)/a^4 + ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)/2))/56 + cos(c
/2 + (d*x)/2)^2*((9*A*sin(c/2 + (d*x)/2))/70 - (5*B*sin(c/2 + (d*x)/2))/28
) + cos(c/2 + (d*x)/2)^6*((12*A*sin(c/2 + (d*x)/2))/35 - (52*B*sin(c/2 + (
d*x)/2))/21) - cos(c/2 + (d*x)/2)^4*((23*A*sin(c/2 + (d*x)/2))/70 - (16*B*
sin(c/2 + (d*x)/2))/21))/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7a+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7b+63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5a-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5b-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{840a^4d}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

output
```
( - 15*tan((c + d*x)/2)**7*a + 15*tan((c + d*x)/2)**7*b + 63*tan((c + d*x)
/2)**5*a - 105*tan((c + d*x)/2)**5*b - 105*tan((c + d*x)/2)**3*a + 385*tan
((c + d*x)/2)**3*b + 105*tan((c + d*x)/2)*a - 1575*tan((c + d*x)/2)*b + 84
0*b*d*x)/(840*a**4*d)
```

3.68 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 136

$$\begin{aligned}\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = &-\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} \\ &+\frac{(13A+36B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))} \\ &+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\ &-\frac{(A-8B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}\end{aligned}$$

output
```
-2/105*(A+27*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+1/105*(13*A+36*B)*sin(d*
x+c)/a^4/d/(1+cos(d*x+c))+1/7*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x
+c))^4-1/35*(A-8*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Mathematica [A] (verified)

Time = 5.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(70(4A+9B)\sin\left(\frac{dx}{2}\right)-35(5A+18B)\sin\left(c+\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+441I\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(70*(4*A + 9*B)*Sin[(d*x)/2] - 35*(5*A + 18*B)*
Sin[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 441*B*Sin[c + (3*d*x)/2] - 1
05*A*Sin[2*c + (3*d*x)/2] - 315*B*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c + (5
*d*x)/2] + 147*B*Sin[2*c + (5*d*x)/2] - 105*B*Sin[3*c + (5*d*x)/2] + 13*A*
Sin[3*c + (7*d*x)/2] + 36*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c +
 d*x])^4)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3456, 3042, 3447, 3042, 3498, 25, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$
$$\Big\downarrow \text{3456}$$
$$\frac{\int \frac{\cos(c+dx)(2a(A-B)+a(A+6B)\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a(A-B)+a(A+6B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{a(A+6B)\cos^2(c+dx)+2a(A-B)\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A+6B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3498}$$

$$\frac{-\frac{\int -\frac{3(A-8B)a^2+5(A+6B)\cos(c+dx)a^2}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{a(A-8B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3(A-8B)a^2+5(A+6B)\cos(c+dx)a^2}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{a(A-8B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3(A-8B)a^2+5(A+6B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{a(A-8B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\frac{\frac{1}{3}a(13A+36B)\int \frac{1}{\cos(c+dx)a+a}dx-\frac{2(A+27B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(A-8B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{1}{3}a(13A+36B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{2(A+27B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(A-8B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\frac{\frac{a(13A+36B)\sin(c+dx)}{3d(a\cos(c+dx)+a)}-\frac{2(A+27B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(A-8B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + (-1/5
*(a*(A - 8*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + ((-2*(A + 27*B)*S
in[c + d*x])/(3*d*(1 + Cos[c + d*x])^2) + (a*(13*A + 36*B)*Sin[c + d*x])/(
3*d*(a + a*Cos[c + d*x])))/(5*a^2))/(7*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left((A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\frac{7(-A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}+7\left(-\frac{A}{3}-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+7A+7B\right)}{56a^{4}d}$
derivativedivides	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{(-A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{(-A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{8d\,a^{4}}$
default	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{(-A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{(-A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{8d\,a^{4}}$
risch	$\frac{2i\left(105B\,e^{6i(dx+c)}+105A\,e^{5i(dx+c)}+315B\,e^{5i(dx+c)}+175A\,e^{4i(dx+c)}+630B\,e^{4i(dx+c)}+280A\,e^{3i(dx+c)}+630B\,e^{3i(dx+}\right.}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{56ad}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3(3A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40ad}+\frac{(4A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}-\frac{(4A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{70ad}+\frac{(4.}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{3}}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/56*tan(1/2*d*x+1/2*c)*((A-B)*tan(1/2*d*x+1/2*c)^6+7/5*(-A+3*B)*tan(1/2*d
*x+1/2*c)^4+7*(-1/3*A-B)*tan(1/2*d*x+1/2*c)^2+7*A+7*B)/a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{((13\,A+36\,B)\cos{(dx+c)}^3+13\,(4\,A+3\,B)\cos{(dx+c)}^2+8\,(4\,A+3\,B)\cos{(dx+c)}+8\,A+6\,B)\sin}{105\,(a^4d\cos{(dx+c)}^4+4\,a^4d\cos{(dx+c)}^3+6\,a^4d\cos{(dx+c)}^2+4\,a^4d\cos{(dx+c)}+a^4d)}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
1/105*((13*A + 36*B)*cos(d*x + c)^3 + 13*(4*A + 3*B)*cos(d*x + c)^2 + 8*(4
*A + 3*B)*cos(d*x + c) + 8*A + 6*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^
4*d)
```

Sympy [A] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.34

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} - \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{d}{}\right.}{8a^4d} \\ \frac{x(A+B\cos{(c)})\cos^2{(c)}}{(a\cos{(c)}+a)^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a
**4*d) - A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
 - B*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*B*tan(c/2 + d*x/2)**5/(40*a**4*d)
 - B*tan(c/2 + d*x/2)**3/(8*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.29

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3B\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right.}{a^4}}{840\,d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/a^4 + 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+63\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,A}{840\,a^4d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 21*A*ta
n(1/2*d*x + 1/2*c)^5 + 63*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/
2*c)^3 - 105*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B
*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 41.86 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = -\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A+3\,B)}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-3\,B)}{40\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}}{d}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

output
```
-((tan(c/2 + (d*x)/2)^3*(A + 3*B))/(24*a^4) + (tan(c/2 + (d*x)/2)^5*(A - 3
*B))/(40*a^4) - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)
/2)*(A + B))/(8*a^4))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6 a-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6 b-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b-35\tan\right.}{840a^4d}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

output
```
(tan((c + d*x)/2)*(15*tan((c + d*x)/2)**6*a - 15*tan((c + d*x)/2)**6*b - 2
1*tan((c + d*x)/2)**4*a + 63*tan((c + d*x)/2)**4*b - 35*tan((c + d*x)/2)**
2*a - 105*tan((c + d*x)/2)**2*b + 105*a + 105*b))/(840*a**4*d)
```

3.69 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} + \frac{(8A+13B)\sin(c+dx)}{105d\left(a^4+a^4\cos(c+dx)\right)}$$

output
```
-1/7*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(4*A-11*B)*sin(d*x+c)/a/d/
(a+a*cos(d*x+c))^3+1/105*(8*A+13*B)*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+1/
105*(8*A+13*B)*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(140(A+2B)\sin\left(\frac{dx}{2}\right)-35(4A+5B)\sin\left(c+\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+168B\right.}{420a}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c/2]*(140*(A + 2*B)*Sin[(d*x)/2] - 35*(4*A + 5*B)*Si
n[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 105
*B*Sin[2*c + (3*d*x)/2] + 56*A*Sin[2*c + (5*d*x)/2] + 91*B*Sin[2*c + (5*d*
x)/2] + 8*A*Sin[3*c + (7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*
(1 + Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 3447, 3042, 3498, 25, 3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$
$$\Big\downarrow \text{3447}$$
$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a\cos(c+dx)+a)^4}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{4a(A-B)+7aB\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4a(A-B)+7aB\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a(A-B)+7aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\frac{1}{5}(8A+13B)\int \frac{1}{(\cos(c+dx)a+a)^2}dx+\frac{a(4A-11B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}(8A+13B)\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx+\frac{a(4A-11B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{\frac{1}{5}(8A+13B)\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)+\frac{a(4A-11B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}(8A+13B)\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)+\frac{a(4A-11B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\frac{a(4A-11B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{1}{5}(8A+13B)\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/7*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^4) + ((a*(4*A - 11*B)*
Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((8*A + 13*B)*(Sin[c + d*x]/(
3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/
5)/(7*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

method	result
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left((A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\frac{7(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}+\frac{7(-A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}-7A-7B\right)}{56a^{4}d}$
derivativedivides	$\frac{\frac{(-A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{(-A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{8d\,a^{4}}$
default	$\frac{\frac{(-A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{(-A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{8d\,a^{4}}$
risch	$\frac{2i\left(105B\,\mathrm{e}^{5i(dx+c)}+140A\,\mathrm{e}^{4i(dx+c)}+175B\,\mathrm{e}^{4i(dx+c)}+140A\,\mathrm{e}^{3i(dx+c)}+280B\,\mathrm{e}^{3i(dx+c)}+168A\,\mathrm{e}^{2i(dx+c)}+168B\,\mathrm{e}^{2i(dx+\cdots}\right.}{105d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{56ad}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{(7A+5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}+\frac{(11A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}-\frac{(11A+31B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{420ad}\cdots}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{3}}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOSE
)
```

output
```
-1/56*tan(1/2*d*x+1/2*c)*((A-B)*tan(1/2*d*x+1/2*c)^6+7/5*(A+B)*tan(1/2*d*x
+1/2*c)^4+7/3*(-A+B)*tan(1/2*d*x+1/2*c)^2-7*A-7*B)/a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{((8A+13B)\cos(dx+c)^3+4(8A+13B)\cos(dx+c)^2+4(13A+8B)\cos(dx+c)+13A+8B)\,\mathrm{s}}{105\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fri
cas")
```

output
```
1/105*((8*A + 13*B)*cos(d*x + c)^3 + 4*(8*A + 13*B)*cos(d*x + c)^2 + 4*(13
*A + 8*B)*cos(d*x + c) + 13*A + 8*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 +
4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a
^4*d)
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.29

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{B\tan^3\left(\frac{c}{2}+\right.}{24a^4d} \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^4} \end{cases}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*
a**4*d) + A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d
) + B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d)
- B*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{A\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{B\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="max
ima")
```

output
```
1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/a^4 + B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = -\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35}{840\,a^4}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gia
c")
```

output
```
-1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 21*A*t
an(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1
/2*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B
*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 41.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.61

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= -\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A+B)}{40\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}}{d}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

output
```
-((tan(c/2 + (d*x)/2)^5*(A + B))/(40*a^4) - (tan(c/2 + (d*x)/2)^3*(A - B))
/(24*a^4) + (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)/2)*
(A + B))/(8*a^4))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6b-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+35\tan\right.}{840a^4d}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

output
```
(tan((c + d*x)/2)*( - 15*tan((c + d*x)/2)**6*a + 15*tan((c + d*x)/2)**6*b
- 21*tan((c + d*x)/2)**4*a - 21*tan((c + d*x)/2)**4*b + 35*tan((c + d*x)/2
)**2*a - 35*tan((c + d*x)/2)**2*b + 105*a + 105*b))/(840*a**4*d)
```

3.70 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^4+a^4\cos(c+dx)\right)}$$

output
```
1/7*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(3*A+4*B)*sin(d*x+c)/a/d/(a
+a*cos(d*x+c))^3+2/105*(3*A+4*B)*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+2/105
*(3*A+4*B)*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(36A+13B+13(3A+4B)\cos(c+dx)+8(3A+4B)\cos^2(c+dx)+(6A+8B)\cos^3(c+dx))\sin(c+}{105a^4d(1+\cos(c+dx))^4}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

output
```
((36*A + 13*B + 13*(3*A + 4*B)*Cos[c + d*x] + 8*(3*A + 4*B)*Cos[c + d*x]^2
 + (6*A + 8*B)*Cos[c + d*x]^3)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
^4)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(3A+4B)\int \frac{1}{(\cos(c+dx)a+a)^3}dx}{7a}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+4B)\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(3A+4B)\left(\frac{2\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+4B)\left(\frac{2\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(3A+4B)\left(\frac{2\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+4B)\left(\frac{2\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(3A+4B)\left(\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}\right)}{7a}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

output
```
((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B)*(Sin[c
+ d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*(Sin[c + d*x]/(3*d*(a + a*Cos[c +
 d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{\left(\left(24A+32B\right)\cos\left(2dx+2c\right)+\left(3A+4B\right)\cos\left(3dx+3c\right)+\left(87A+116B\right)\cos\left(dx+c\right)+96A+58B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{1680a^{4}d}$
derivativedivides	$\frac{\frac{\left(A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{\left(3A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{\left(3A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{8d\,a^{4}}$
default	$\frac{\frac{\left(A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{\left(3A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{\left(3A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{8d\,a^{4}}$
risch	$\frac{4i\left(70B\,\mathrm{e}^{4i(dx+c)}+105A\,\mathrm{e}^{3i(dx+c)}+70B\,\mathrm{e}^{3i(dx+c)}+63A\,\mathrm{e}^{2i(dx+c)}+84B\,\mathrm{e}^{2i(dx+c)}+21A\,\mathrm{e}^{i(dx+c)}+28B\,\mathrm{e}^{i(dx+c)}+3A+\right.}{105d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{\frac{\left(A-B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{56ad}+\frac{\left(A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\left(3A+2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{\left(12A+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}+\frac{\left(13A-6B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{140ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{3}}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/1680*((24*A+32*B)*cos(2*d*x+2*c)+(3*A+4*B)*cos(3*d*x+3*c)+(87*A+116*B)*c
os(d*x+c)+96*A+58*B)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^6/a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(2\,(3\,A+4\,B)\cos\left(dx+c\right)^3+8\,(3\,A+4\,B)\cos\left(dx+c\right)^2+13\,(3\,A+4\,B)\cos\left(dx+c\right)+36\,A+13\,B)\,\text{s}}{105\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/105*(2*(3*A + 4*B)*cos(d*x + c)^3 + 8*(3*A + 4*B)*cos(d*x + c)^2 + 13*(3
*A + 4*B)*cos(d*x + c) + 36*A + 13*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 +
 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) +
a^4*d)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}+\frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}+\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}-\frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}+\frac{B\tan^3\left(\frac{c}{2}+\frac{d}{2}\right.}{24a^4d} \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^4} \end{cases}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40
*a**4*d) + A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d
) - B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d)
+ B*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
 0)), (x*(A + B*cos(c))/(a*cos(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.27

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105}{840\,a^4d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 63*A*ta
n(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1
/2*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B
*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 41.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.63

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(3\,A+B)}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(3\,A-B)}{40\,a^4}}{d}$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^4,x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(3*A + B))/(24*a^4) + (tan(c/2 + (d*x)/2)^7*(A - B)
)/(56*a^4) + (tan(c/2 + (d*x)/2)*(A + B))/(8*a^4) + (tan(c/2 + (d*x)/2)^5*
(3*A - B))/(40*a^4))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6a-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6b+63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+105\,\mathrm{ta}\right.}{840a^4d}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

output
```
(tan((c + d*x)/2)*(15*tan((c + d*x)/2)**6*a - 15*tan((c + d*x)/2)**6*b + 6
3*tan((c + d*x)/2)**4*a - 21*tan((c + d*x)/2)**4*b + 105*tan((c + d*x)/2)*
*2*a + 35*tan((c + d*x)/2)**2*b + 105*a + 105*b))/(840*a**4*d)
```

3.71 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 147

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{A\operatorname{arctanh}(\sin(c+dx))}{a^4d} - \frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{2(80A-3B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

output

```
A*arctanh(sin(d*x+c))/a^4/d-1/105*(55*A-6*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c
))^2-2/105*(80*A-3*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*(A-B)*sin(d*x+c)
/d/(a+a*cos(d*x+c))^4-1/35*(10*A-3*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{-6720A\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

output
```
(-6720*A*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-70*(
49*A - 3*B)*Sin[(d*x)/2] + 2170*A*Sin[c + (d*x)/2] - 2625*A*Sin[c + (3*d*x
)/2] + 126*B*Sin[c + (3*d*x)/2] + 735*A*Sin[2*c + (3*d*x)/2] - 1015*A*Sin[
2*c + (5*d*x)/2] + 42*B*Sin[2*c + (5*d*x)/2] + 105*A*Sin[3*c + (5*d*x)/2]
- 160*A*Sin[3*c + (7*d*x)/2] + 6*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 +
Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$
$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(7aA-3a(A-B)\cos(c+dx))\sec(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{7aA-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\int \frac{\left(35a^2A-2a^2(10A-3B)\cos(c+dx)\right)\sec(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{35a^2A-2a^2(10A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\frac{\int \frac{\left(105a^3A-a^3(55A-6B)\cos(c+dx)\right)\sec(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(55A-6B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{105a^3A-a^3(55A-6B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{(55A-6B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\frac{\frac{\int 105a^4A\sec(c+dx)dx}{a^2}-\frac{2a^3(80A-3B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(55A-6B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{105a^2A\int \sec(c+dx)dx-\frac{2a^3(80A-3B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(55A-6B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{105a^2A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2a^3(80A-3B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(55A-6B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{\frac{\frac{105a^2A\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^3(80A-3B)\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(55A-6B)\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(10A-3B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/7*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^4) + (-1/5*(a*(10*A -
3*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + (-1/3*((55*A - 6*B)*Sin[c
+ d*x])/(d*(1 + Cos[c + d*x])^2) + ((105*a^2*A*ArcTanh[Sin[c + d*x]])/d -
(2*a^3*(80*A - 3*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(5*a^
2))/(7*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{-840A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+840A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left((A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\left(7A-\frac{21B}{5}\right)\tan\left(\frac{dx}{2}\right.\right.}{840a^4d}$
derivativedivides	$\frac{8A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}A}{7}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}B}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}A-\frac{11\tan}{}}{8d\,a^4}$
default	$\frac{8A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}A}{7}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}B}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}A-\frac{11\tan}{}}{8d\,a^4}$
risch	$-\frac{2i\left(105A\,e^{6i(dx+c)}+735A\,e^{5i(dx+c)}+2170A\,e^{4i(dx+c)}+3430A\,e^{3i(dx+c)}-210B\,e^{3i(dx+c)}+2625A\,e^{2i(dx+c)}-126B\,e\right.}{105d\,a^4\left(e^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{56ad}-\frac{(15A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{(20A-13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{140ad}-\frac{(28A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}-\frac{(35A-12B)\tan\left(\frac{dx}{2}+\right.}{60ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^3}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBOSE
)
```

output
```
1/840*(-840*A*ln(tan(1/2*d*x+1/2*c)-1)+840*A*ln(tan(1/2*d*x+1/2*c)+1)-15*t
an(1/2*d*x+1/2*c)*((A-B)*tan(1/2*d*x+1/2*c)^6+(7*A-21/5*B)*tan(1/2*d*x+1/2
*c)^4+(77/3*A-7*B)*tan(1/2*d*x+1/2*c)^2+105*A-7*B))/a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.61

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{105\left(A\cos\left(dx+c\right)^4+4\,A\cos\left(dx+c\right)^3+6\,A\cos\left(dx+c\right)^2+4\,A\cos\left(dx+c\right)+A\right)\log\left(\sin\left(dx+c\right)+1\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fri
cas")
```

output
```
1/210*(105*(A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4
*A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 105*(A*cos(d*x + c)^4 + 4*A*c
os(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*A*cos(d*x + c) + A)*log(-sin(d*x +
c) + 1) - 2*(2*(80*A - 3*B)*cos(d*x + c)^3 + (535*A - 24*B)*cos(d*x + c)^2
 + (620*A - 39*B)*cos(d*x + c) + 260*A - 36*B)*sin(d*x + c))/(a^4*d*cos(d*
x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d
*x + c) + a^4*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\int \frac{A\sec(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

output
```
(Integral(A*sec(c + d*x)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c +
d*x)**2 + 4*cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(
cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) +
 1), x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.55

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$
$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-}{840\,d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="max
ima")
```

output
```
-1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1
)/a^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 3*B*(35*sin(d*
x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)
/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.24

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$=\frac{\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx\right.}{}}{84}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="gia
c")
```

output
```
1/840*(840*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*A*log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*t
an(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 63*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/
2*d*x + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 105*B*a^24*tan(1/2*d
*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 41.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.35

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{2\,A\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^4\,d} - \frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{11\,A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8}-\frac{3\,B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40}\right)+\cos}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^4),x)
```

output
```
(2*A*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^4*d) - (cos(c/2 + (d
*x)/2)^4*((11*A*sin(c/2 + (d*x)/2)^3)/24 - (B*sin(c/2 + (d*x)/2)^3)/8) + c
os(c/2 + (d*x)/2)^2*((A*sin(c/2 + (d*x)/2)^5)/8 - (3*B*sin(c/2 + (d*x)/2)^
5)/40) + cos(c/2 + (d*x)/2)^6*((15*A*sin(c/2 + (d*x)/2))/8 - (B*sin(c/2 +
(d*x)/2))/8) + (A*sin(c/2 + (d*x)/2)^7)/56 - (B*sin(c/2 + (d*x)/2)^7)/56)/
(a^4*d*cos(c/2 + (d*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{-840\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+840\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7a+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7b-}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x)
```

output
```
( - 840*log(tan((c + d*x)/2) - 1)*a + 840*log(tan((c + d*x)/2) + 1)*a - 15
*tan((c + d*x)/2)**7*a + 15*tan((c + d*x)/2)**7*b - 105*tan((c + d*x)/2)**
5*a + 63*tan((c + d*x)/2)**5*b - 385*tan((c + d*x)/2)**3*a + 105*tan((c +
d*x)/2)**3*b - 1575*tan((c + d*x)/2)*a + 105*tan((c + d*x)/2)*b)/(840*a**4
*d)
```

3.72 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 175

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(4A-B)\text{arctanh}(\sin(c+dx))}{a^4d} + \frac{8(83A-20B)\tan(c+dx)}{105a^4d} - \frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(4A-B)\tan(c+dx)}{a^4d(1+\cos(c+dx))} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\tan(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

output
```
-(4*A-B)*arctanh(sin(d*x+c))/a^4/d+8/105*(83*A-20*B)*tan(d*x+c)/a^4/d-1/10
5*(88*A-25*B)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-(4*A-B)*tan(d*x+c)/a^4/d/(
1+cos(d*x+c))-1/7*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^4-1/35*(12*A-5*B)*ta
n(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 595 vs. $2(175) = 350$.

Time = 7.51 (sec) , antiderivative size = 595, normalized size of antiderivative = 3.40

$$\int \frac{(A + B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{26880(4A-B)\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

output
```
(26880*(4*A - B)*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2
]*Sec[c]*Sec[c + d*x]*(-245*(44*A - 17*B)*Sin[(d*x)/2] + 7*(2684*A - 635*B
)*Sin[(3*d*x)/2] - 20524*A*Sin[c - (d*x)/2] + 4795*B*Sin[c - (d*x)/2] + 14
644*A*Sin[c + (d*x)/2] - 4795*B*Sin[c + (d*x)/2] - 16660*A*Sin[2*c + (d*x)
/2] + 4165*B*Sin[2*c + (d*x)/2] - 4690*A*Sin[c + (3*d*x)/2] + 2275*B*Sin[c
 + (3*d*x)/2] + 14378*A*Sin[2*c + (3*d*x)/2] - 4445*B*Sin[2*c + (3*d*x)/2]
 - 9100*A*Sin[3*c + (3*d*x)/2] + 2275*B*Sin[3*c + (3*d*x)/2] + 11668*A*Sin
[c + (5*d*x)/2] - 2785*B*Sin[c + (5*d*x)/2] - 630*A*Sin[2*c + (5*d*x)/2] +
 735*B*Sin[2*c + (5*d*x)/2] + 9358*A*Sin[3*c + (5*d*x)/2] - 2785*B*Sin[3*c
 + (5*d*x)/2] - 2940*A*Sin[4*c + (5*d*x)/2] + 735*B*Sin[4*c + (5*d*x)/2] +
 4228*A*Sin[2*c + (7*d*x)/2] - 1015*B*Sin[2*c + (7*d*x)/2] + 315*A*Sin[3*c
 + (7*d*x)/2] + 105*B*Sin[3*c + (7*d*x)/2] + 3493*A*Sin[4*c + (7*d*x)/2] -
 1015*B*Sin[4*c + (7*d*x)/2] - 420*A*Sin[5*c + (7*d*x)/2] + 105*B*Sin[5*c
+ (7*d*x)/2] + 664*A*Sin[3*c + (9*d*x)/2] - 160*B*Sin[3*c + (9*d*x)/2] + 1
05*A*Sin[4*c + (9*d*x)/2] + 559*A*Sin[5*c + (9*d*x)/2] - 160*B*Sin[5*c + (
9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(a(8A-B)-4a(A-B)\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(8A-B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(2a^2(26A-5B)-3a^2(12A-5B)\cos(c+dx)\right)\sec^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2a^2(26A-5B)-3a^2(12A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int \frac{\left(a^3(244A-55B)-2a^3(88A-25B)\cos(c+dx)\right)\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{a^3(244A-55B)-2a^3(88A-25B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\frac{\frac{\int\left(8a^4(83A-20B)-105a^4(4A-B)\cos(c+dx)\right)\sec^2(c+dx)dx}{a^2}-\frac{105a^3(4A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\int \frac{8a^4(83A-20B)-105a^4(4A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{105a^3(4A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3227

$$\frac{\frac{\frac{\frac{8a^4(83A-20B)\int\sec^2(c+dx)dx-105a^4(4A-B)\int\sec(c+dx)dx}{a^2}-\frac{105a^3(4A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{8a^4(83A-20B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-105a^4(4A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3(4A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 4254

$$\frac{\frac{\frac{\frac{-\frac{8a^4(83A-20B)\int 1d(-\tan(c+dx))}{d}-105a^4(4A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3(4A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 24

$$\frac{\frac{\frac{\frac{\frac{8a^4(83A-20B)\tan(c+dx)}{d}-105a^4(4A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3(4A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 4257

$$\frac{\frac{\frac{\frac{\frac{8a^4(83A-20B)\tan(c+dx)}{d}-\frac{105a^4(4A-B)\text{arctanh}(\sin(c+dx))}{d}}{a^2}-\frac{105a^3(4A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(88A-25B)\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{a(12A-5B)\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/7*((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^4) + (-1/5*(a*(12*A -
5*B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + (-1/3*((88*A - 25*B)*Tan[c
 + d*x])/(d*(1 + Cos[c + d*x])^2) + ((-105*a^3*(4*A - B)*Tan[c + d*x])/(d*
(a + a*Cos[c + d*x])) + ((-105*a^4*(4*A - B)*ArcTanh[Sin[c + d*x]])/d + (8
*a^4*(83*A - 20*B)*Tan[c + d*x])/d)/a^2)/(3*a^2))/(5*a^2))/(7*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{13440\cos(dx+c)\left(A-\frac{B}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-13440\cos(dx+c)\left(A-\frac{B}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+332\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\tan}{}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}A}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}B}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}A}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}B+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+49\tan\left(\frac{dx}{2}\right.}{}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}A}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}B}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}A}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}B+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}A}{3}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}B}{3}+49\tan\left(\frac{dx}{2}\right.}{}$
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{56ad}+\frac{(7A-5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{40ad}-\frac{5(13A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{7(17A-5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}-\frac{(71A-11B)\tan\left(\frac{dx}{2}+\right.}{24ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)a^{3}}$
risch	$\frac{2i\left(420A\,e^{8i(dx+c)}-105B\,e^{8i(dx+c)}+2940A\,e^{7i(dx+c)}-735B\,e^{7i(dx+c)}+9100A\,e^{6i(dx+c)}-2275B\,e^{6i(dx+c)}+16660A\,e\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/3360*(13440*cos(d*x+c)*(A-1/4*B)*ln(tan(1/2*d*x+1/2*c)-1)-13440*cos(d*x+
c)*(A-1/4*B)*ln(tan(1/2*d*x+1/2*c)+1)+332*sec(1/2*d*x+1/2*c)^6*tan(1/2*d*x
+1/2*c)*((1650/83*A-390/83*B)*cos(2*d*x+2*c)+(559/83*A-535/332*B)*cos(3*d*
x+3*c)+(A-20/83*B)*cos(4*d*x+4*c)+(2861/83*A-2645/332*B)*cos(d*x+c)+1672/8
3*A-370/83*B))/d/a^4/cos(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(167) = 334.

Time = 0.09 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.93

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$
$$-\frac{105\left((4A-B)\cos(dx+c)^5+4(4A-B)\cos(dx+c)^4+6(4A-B)\cos(dx+c)^3+4(4A-B)\operatorname{co}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
-1/210*(105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*x + c)^4 + 6*(4*
A - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A - B)*cos(d*x + c
))*log(sin(d*x + c) + 1) - 105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos
(d*x + c)^4 + 6*(4*A - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4
*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(8*(83*A - 20*B)*cos(d*x
+ c)^4 + (2236*A - 535*B)*cos(d*x + c)^3 + 4*(659*A - 155*B)*cos(d*x + c)^
2 + 4*(296*A - 65*B)*cos(d*x + c) + 105*A)*sin(d*x + c))/(a^4*d*cos(d*x +
c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x +
 c)^2 + a^4*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + a\cos(c + dx))^4}\, dx = \frac{\int \frac{A\sec^2(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\, dx + \int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\, dx}{a^4}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

output
```
(Integral(A*sec(c + d*x)**2/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c
 + d*x)**2 + 4*cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x
)**2/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c +
d*x) + 1), x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.86

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{A\left(\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
1/840*(A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4
) - 5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.28

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$
$$-\frac{\frac{840\,(4A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{840\,(4A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{1680\,A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}{}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-1/840*(840*(4*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(4*A -
B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 1680*A*tan(1/2*d*x + 1/2*c)/((
tan(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*
B*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*B*
a^24*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^
24*tan(1/2*d*x + 1/2*c)^3 + 5145*A*a^24*tan(1/2*d*x + 1/2*c) - 1575*B*a^24
*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 41.62 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.35

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{8a^4}+\frac{5A-3B}{12a^4}+\frac{10A-2B}{24a^4}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{A-B}{20a^4}+\frac{5A-3B}{40a^4}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{A-B}{2a^4}+\frac{3(5A-3B)}{8a^4}+\frac{10A-2B}{4a^4}+\frac{10A+2B}{8a^4}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(A-B)}{56a^4d} - \frac{2A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^4\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(4A-B)}{a^4d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^4),x)
```

output
```
(tan(c/2 + (d*x)/2)^3*((A - B)/(8*a^4) + (5*A - 3*B)/(12*a^4) + (10*A - 2*
B)/(24*a^4)))/d + (tan(c/2 + (d*x)/2)^5*((A - B)/(20*a^4) + (5*A - 3*B)/(4
0*a^4)))/d + (tan(c/2 + (d*x)/2)*((A - B)/(2*a^4) + (3*(5*A - 3*B))/(8*a^4
) + (10*A - 2*B)/(4*a^4) + (10*A + 2*B)/(8*a^4)))/d + (tan(c/2 + (d*x)/2)^
7*(A - B))/(56*a^4*d) - (2*A*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2
)^2 - a^4)) - (2*atanh(tan(c/2 + (d*x)/2))*(4*A - B))/(a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{3360\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-840\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-3360\log\left(\tan\left(\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x)
```

output
```
(3360*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2*a - 840*log(tan((c + d
*x)/2) - 1)*tan((c + d*x)/2)**2*b - 3360*log(tan((c + d*x)/2) - 1)*a + 840
*log(tan((c + d*x)/2) - 1)*b - 3360*log(tan((c + d*x)/2) + 1)*tan((c + d*x
)/2)**2*a + 840*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*b + 3360*log
(tan((c + d*x)/2) + 1)*a - 840*log(tan((c + d*x)/2) + 1)*b + 15*tan((c + d
*x)/2)**9*a - 15*tan((c + d*x)/2)**9*b + 132*tan((c + d*x)/2)**7*a - 90*ta
n((c + d*x)/2)**7*b + 658*tan((c + d*x)/2)**5*a - 280*tan((c + d*x)/2)**5*
b + 4340*tan((c + d*x)/2)**3*a - 1190*tan((c + d*x)/2)**3*b - 6825*tan((c
+ d*x)/2)*a + 1575*tan((c + d*x)/2)*b)/(840*a**4*d*(tan((c + d*x)/2)**2 -
1))
```

3.73 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 232

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(21A-8B)\text{arctanh}(\sin(c+dx))}{2a^4d} - \frac{8(216A-83B)\tan(c+dx)}{105a^4d} + \frac{(21A-8B)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{4(216A-83B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2A-B)\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3}$$

output
```
1/2*(21*A-8*B)*arctanh(sin(d*x+c))/a^4/d-8/105*(216*A-83*B)*tan(d*x+c)/a^4
/d+1/2*(21*A-8*B)*sec(d*x+c)*tan(d*x+c)/a^4/d-1/105*(129*A-52*B)*sec(d*x+c
)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-4/105*(216*A-83*B)*sec(d*x+c)*tan(d*x+
c)/a^4/d/(1+cos(d*x+c))-1/7*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))
^4-1/5*(2*A-B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 798 vs. $2(232) = 464$.

Time = 9.02 (sec) , antiderivative size = 798, normalized size of antiderivative = 3.44

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + a\cos(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

output
```
(-8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (
d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^
8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4)
 + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(73206*A*Sin[(d*x)/2
] - 38668*B*Sin[(d*x)/2] - 166668*A*Sin[(3*d*x)/2] + 64384*B*Sin[(3*d*x)/2
] + 183162*A*Sin[c - (d*x)/2] - 70896*B*Sin[c - (d*x)/2] - 100842*A*Sin[c
+ (d*x)/2] + 50316*B*Sin[c + (d*x)/2] + 155526*A*Sin[2*c + (d*x)/2] - 5924
8*B*Sin[2*c + (d*x)/2] + 37380*A*Sin[c + (3*d*x)/2] - 22820*B*Sin[c + (3*d
*x)/2] - 101148*A*Sin[2*c + (3*d*x)/2] + 48004*B*Sin[2*c + (3*d*x)/2] + 10
2900*A*Sin[3*c + (3*d*x)/2] - 39200*B*Sin[3*c + (3*d*x)/2] - 119364*A*Sin[
c + (5*d*x)/2] + 46032*B*Sin[c + (5*d*x)/2] + 8820*A*Sin[2*c + (5*d*x)/2]
- 8750*B*Sin[2*c + (5*d*x)/2] - 78204*A*Sin[3*c + (5*d*x)/2] + 35742*B*Sin
[3*c + (5*d*x)/2] + 49980*A*Sin[4*c + (5*d*x)/2] - 19040*B*Sin[4*c + (5*d*
x)/2] - 64053*A*Sin[2*c + (7*d*x)/2] + 24664*B*Sin[2*c + (7*d*x)/2] - 3885
*A*Sin[3*c + (7*d*x)/2] - 1050*B*Sin[3*c + (7*d*x)/2] - 44733*A*Sin[4*c +
(7*d*x)/2] + 19834*B*Sin[4*c + (7*d*x)/2] + 15435*A*Sin[5*c + (7*d*x)/2] -
 5880*B*Sin[5*c + (7*d*x)/2] - 21987*A*Sin[3*c + (9*d*x)/2] + 8456*B*Sin[3
*c + (9*d*x)/2] - 3675*A*Sin[4*c + (9*d*x)/2] + 630*B*Sin[4*c + (9*d*x)/2]
 - 16107*A*Sin[5*c + (9*d*x)/2] + 6986*B*Sin[5*c + (9*d*x)/2] + 2205*A*Sin
[6*c + (9*d*x)/2] - 840*B*Sin[6*c + (9*d*x)/2] - 3456*A*Sin[4*c + (11*d...
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{(a(9A-2B)-5a(A-B)\cos(c+dx))\sec^3(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a(9A-2B)-5a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\int \frac{\left(a^2(73A-24B)-28a^2(2A-B)\cos(c+dx)\right)\sec^3(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{7a(2A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{a^2(73A-24B)-28a^2(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{7a(2A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\frac{\int \frac{\left(a^3(477A-176B)-3a^3(129A-52B)\cos(c+dx)\right)\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{7a(2A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{a^3(477A-176B)-3a^3(129A-52B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{7a(2A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\frac{\frac{\int \left(105a^4(21A-8B)-8a^4(216A-83B)\cos(c+dx)\right)\sec^3(c+dx)dx}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{7a(2A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\int \frac{105a^4(21A-8B)-8a^4(216A-83B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{7a(2A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3227

$$\frac{\frac{\frac{\frac{105a^4(21A-8B)\int \sec^3(c+dx)dx-8a^4(216A-83B)\int \sec^2(c+dx)dx}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{7a(2A-B)\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{105a^4(21A-8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-8a^4(216A-83B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-7\ldots}{7a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 4254

$$\frac{\frac{\frac{\frac{\frac{8a^4(216A-83B)\int 1d(-\tan(c+dx))}{d}+105a^4(21A-8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-7a\ldots}{7a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 24

$$\frac{\frac{\frac{\frac{105a^4(21A-8B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{8a^4(216A-83B)\tan(c+dx)}{d}}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{7a(2A-B\ldots}{5d(\ldots}}{7a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 4255

$$\frac{\frac{\frac{\frac{105a^4(21A-8B)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{8a^4(216A-83B)\tan(c+dx)}{d}}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx)\sec(\ldots}{3d(\cos(c+dx)+1)^2}}{5a^2}}{7a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{105a^4(21A-8B)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{8a^4(216A-83B)\tan(c+dx)}{d}}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+dx\ldots}{3d(\cos(c+dx)+\ldots}}{5a^2}}{7a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$\downarrow$ 4257

$$\frac{\frac{\frac{\frac{105a^4(21A-8B)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{8a^4(216A-83B)\tan(c+dx)}{d}}{a^2}-\frac{4a^3(216A-83B)\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{(129A-52B)\tan(c+}{3d(\cos(c+dx}}{5a^2}}{7a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/7*((A - B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^4) + ((-7
*a*(2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (-1
/3*((129*A - 52*B)*Sec[c + d*x]*Tan[c + d*x])/(d*(1 + Cos[c + d*x])^2) + (
(-4*a^3*(216*A - 83*B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
 + ((-8*a^4*(216*A - 83*B)*Tan[c + d*x])/d + 105*a^4*(21*A - 8*B)*(ArcTanh
[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2)/(3*a^2))/(
5*a^2))/(7*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{-70560(\cos(2dx+2c)+1)\left(A-\frac{8B}{21}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+70560(\cos(2dx+2c)+1)\left(A-\frac{8B}{21}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-116}{}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}-13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-111\tan}{}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7A}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7B}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5A}{5}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5B}{5}-13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3A+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3B}{3}-111\tan}{}$
norman	$\frac{-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{56ad}-\frac{(29A-22B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{140ad}-\frac{(167A-65B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{(171A-62B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}-\frac{(1161A-643B}{84}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2a^3}$
risch	$-\frac{i\left(2205\,e^{10i(dx+c)}A-840B\,e^{10i(dx+c)}+15435A\,e^{9i(dx+c)}-5880B\,e^{9i(dx+c)}+49980A\,e^{8i(dx+c)}-19040B\,e^{8i(dx+c)}+\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/6720*(-70560*(cos(2*d*x+2*c)+1)*(A-8/21*B)*ln(tan(1/2*d*x+1/2*c)-1)+7056
0*(cos(2*d*x+2*c)+1)*(A-8/21*B)*ln(tan(1/2*d*x+1/2*c)+1)-11619*((23540/387
3*A-3040/1291*B)*cos(2*d*x+2*c)+(3992/1291*A-13864/11619*B)*cos(3*d*x+3*c)
+(A-4472/11619*B)*cos(4*d*x+4*c)+(192/1291*A-664/11619*B)*cos(5*d*x+5*c)+(
34168/3873*A-39952/11619*B)*cos(d*x+c)+19387/3873*A-22888/11619*B)*sec(1/2
*d*x+1/2*c)^6*tan(1/2*d*x+1/2*c))/d/a^4/(cos(2*d*x+2*c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.55

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$=\frac{105\left((21\,A-8\,B)\cos\left(dx+c\right)^6+4\left(21\,A-8\,B\right)\cos\left(dx+c\right)^5+6\left(21\,A-8\,B\right)\cos\left(dx+c\right)^4+4\left(21\,A-\right.\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
1/420*(105*((21*A - 8*B)*cos(d*x + c)^6 + 4*(21*A - 8*B)*cos(d*x + c)^5 +
6*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*A - 8*B)*cos(d*x + c)^3 + (21*A - 8*
B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((21*A - 8*B)*cos(d*x + c)^
6 + 4*(21*A - 8*B)*cos(d*x + c)^5 + 6*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*
A - 8*B)*cos(d*x + c)^3 + (21*A - 8*B)*cos(d*x + c)^2)*log(-sin(d*x + c) +
 1) - 2*(16*(216*A - 83*B)*cos(d*x + c)^5 + (11619*A - 4472*B)*cos(d*x + c
)^4 + 4*(3411*A - 1318*B)*cos(d*x + c)^3 + 4*(1509*A - 592*B)*cos(d*x + c)
^2 + 210*(2*A - B)*cos(d*x + c) - 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)
^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c
)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\int \frac{A\sec^3(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

output
```
(Integral(A*sec(c + d*x)**3/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c
 + d*x)**2 + 4*cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x
)**3/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c +
d*x) + 1), x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.81

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{3A\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+}\right.}{a^4}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
-1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) +
1) + 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a^4) - B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) +
805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1)
 - 1)/a^4))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.15

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
1/840*(420*(21*A - 8*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(21*A
 - 8*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(9*A*tan(1/2*d*x + 1/
2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 7*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24*tan(1/2
*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 189*A*a^24*tan(1/2*d*
x + 1/2*c)^5 - 147*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x
 + 1/2*c)^3 - 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 11655*A*a^24*tan(1/2*d*x
 + 1/2*c) - 5145*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B] (verification not implemented)

Time = 41.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.18

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(9A-2B)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(7A-2B)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4\right)}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5A}{2a^4}+\frac{5(A-B)}{4a^4}+\frac{3(6A-4B)}{4a^4}+\frac{3(15A-5B)}{8a^4}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{4a^4}+\frac{6A-4B}{8a^4}+\frac{15A-5B}{24a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{3(A-B)}{40a^4}+\frac{6A-4B}{40a^4}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(A-B)}{56a^4d}+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(21A-8B)}{a^4d}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^4),x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(9*A - 2*B) - tan(c/2 + (d*x)/2)*(7*A - 2*B))/(d*(a^
4*tan(c/2 + (d*x)/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (tan(c/2 + (
d*x)/2)*((5*A)/(2*a^4) + (5*(A - B))/(4*a^4) + (3*(6*A - 4*B))/(4*a^4) + (
3*(15*A - 5*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^4) + (6*
A - 4*B)/(8*a^4) + (15*A - 5*B)/(24*a^4)))/d - (tan(c/2 + (d*x)/2)^5*((3*(
A - B))/(40*a^4) + (6*A - 4*B)/(40*a^4)))/d - (tan(c/2 + (d*x)/2)^7*(A - B
))/(56*a^4*d) + (atanh(tan(c/2 + (d*x)/2))*(21*A - 8*B))/(a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.02

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x)
```

output
```
( - 8820*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**4*a + 3360*log(tan((c
 + d*x)/2) - 1)*tan((c + d*x)/2)**4*b + 17640*log(tan((c + d*x)/2) - 1)*ta
n((c + d*x)/2)**2*a - 6720*log(tan((c + d*x)/2) - 1)*tan((c + d*x)/2)**2*b
 - 8820*log(tan((c + d*x)/2) - 1)*a + 3360*log(tan((c + d*x)/2) - 1)*b + 8
820*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**4*a - 3360*log(tan((c + d*
x)/2) + 1)*tan((c + d*x)/2)**4*b - 17640*log(tan((c + d*x)/2) + 1)*tan((c
+ d*x)/2)**2*a + 6720*log(tan((c + d*x)/2) + 1)*tan((c + d*x)/2)**2*b + 88
20*log(tan((c + d*x)/2) + 1)*a - 3360*log(tan((c + d*x)/2) + 1)*b - 15*tan
((c + d*x)/2)**11*a + 15*tan((c + d*x)/2)**11*b - 159*tan((c + d*x)/2)**9*
a + 117*tan((c + d*x)/2)**9*b - 1002*tan((c + d*x)/2)**7*a + 526*tan((c +
d*x)/2)**7*b - 9114*tan((c + d*x)/2)**5*a + 3682*tan((c + d*x)/2)**5*b + 2
9505*tan((c + d*x)/2)**3*a - 11165*tan((c + d*x)/2)**3*b - 17535*tan((c +
d*x)/2)*a + 6825*tan((c + d*x)/2)*b)/(840*a**4*d*(tan((c + d*x)/2)**4 - 2*
tan((c + d*x)/2)**2 + 1))
```

3.74 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 187

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{4a(9A+8B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}}+\frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}}-\frac{8(9A+8B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d}+\frac{4(9A+8B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105ad}$$

output
```
4/45*a*(9*A+8*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a*(9*A+8*B)*cos(
d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*a*B*cos(d*x+c)^4*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(1/2)-8/315*(9*A+8*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x
+c)/d+4/105*(9*A+8*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.55

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}(1368A+1321B+94(9A+8B)\cos(c+dx)+4(54A+83B)\cos(2(c+dx))+90}{1260d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*(1368*A + 1321*B + 94*(9*A + 8*B)*Cos[c + d*x]
 + 4*(54*A + 83*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 80*B*Cos[3*(
c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3460, 3042, 3249, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{9}(9A+8B)\int \cos^3(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}(9A+8B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3249

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3238

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\left(\frac{2\int \frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+}{7d\sqrt{a\,\mathrm{co}}}\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 27

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\left(\frac{\int (3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)}{7d\sqrt{a\cos(c}}\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\left(\frac{\int \left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+2a\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3230

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\,\mathrm{si}}{7a}\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\big\downarrow$ 3042

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$\big\downarrow$ 3125

$$\frac{1}{9}(9A+8B)\left(\frac{6}{7}\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\,\mathrm{c}}{7d\sqrt{a\cos(c-}}\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) + ((9*A
 + 8*B)*((2*a*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
+ (6*((2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((14*a^2*Sin[c
 + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]*Si
n[c + d*x])/(3*d))/(5*a)))/7))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.65

method	result
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-360A-1440B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(756A+1512B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(-630A-840B)\right.}{315\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$
parts	$\frac{2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+22\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+9\right)\sqrt{2}}{35\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}+\frac{2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(56\right.}{}$

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*B*sin(1/2*d*x+1/2*c)^8+
(-360*A-1440*B)*sin(1/2*d*x+1/2*c)^6+(756*A+1512*B)*sin(1/2*d*x+1/2*c)^4+(
-630*A-840*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2
*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{2\left(35\,B\cos\left(dx+c\right)^{4}+5\left(9\,A+8\,B\right)\cos\left(dx+c\right)^{3}+6\left(9\,A+8\,B\right)\cos\left(dx+c\right)^{2}+8\left(9\,A+8\,B\right)\cos\left(dx+\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
2/315*(35*B*cos(d*x + c)^4 + 5*(9*A + 8*B)*cos(d*x + c)^3 + 6*(9*A + 8*B)*
cos(d*x + c)^2 + 8*(9*A + 8*B)*cos(d*x + c) + 144*A + 128*B)*sqrt(a*cos(d*
x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{18\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
1/2520*(18*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c
) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*
sqrt(a) + (35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*
c) + 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) +
 1890*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\sqrt{2}\left(35\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
1/2520*sqrt(2)*(35*B*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 45*(
2*A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(7/2*d*x +
 7/2*c) + 126*(A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(cos(1/2*d*x + 1/2*c))
)*sin(5/2*d*x + 5/2*c) + 210*(3*A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(cos(
1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 1890*(A*sgn(cos(1/2*d*x + 1/2*c)
) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \int \cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)a\right)$$

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**4,x)*b + int(sqrt(cos(c
+ d*x) + 1)*cos(c + d*x)**3,x)*a)
```

3.75 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 144

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx &= \frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}\\ &\quad-\frac{4(7A+6B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d}\\ &\quad+\frac{2(7A+6B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35ad}\end{aligned}$$

output
```
2/15*a*(7*A+6*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*B*cos(d*x+c)^3*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-4/105*(7*A+6*B)*(a+a*cos(d*x+c))^(1/2)
*sin(d*x+c)/d+2/35*(7*A+6*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}(266A+228B+(112A+141B)\cos(c+dx)+6(7A+6B)\cos(2(c+dx))+15B\ \cdots}{210d}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*(266*A + 228*B + (112*A + 141*B)*Cos[c + d*x]
+ 6*(7*A + 6*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2]
)/(210*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3460, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 3460$$

$$\frac{1}{7}(7A+6B)\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}(7A+6B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3238}$$

$$\frac{1}{7}(7A+6B)\left(\frac{2\int \frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}(7A+6B)\left(\frac{\int (3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+6B)\left(\frac{\int \left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A+6B)\left(\frac{\frac{7}{3}a\int \sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+6B)\left(\frac{\frac{7}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\downarrow \text{3125}$$

$$\frac{1}{7}(7A+6B)\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2aB\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + ((7*A
 + 6*B)*((2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((14*a^2*Si
n[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]
*Sin[c + d*x])/(3*d))/(5*a)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

method	result
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-120B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(84A+252B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(-140A-210B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+105A+105B\right)\sqrt{2}}{105\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$
parts	$\frac{2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+7\right)\sqrt{2}}{15\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}+\frac{2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-36\cos\right.}{35\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/105*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-120*B*sin(1/2*d*x+1/2*c)^6
+(84*A+252*B)*sin(1/2*d*x+1/2*c)^4+(-140*A-210*B)*sin(1/2*d*x+1/2*c)^2+105
*A+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2\left(15\,B\cos\left(dx+c\right)^3+3\left(7\,A+6\,B\right)\cos\left(dx+c\right)^2+4\left(7\,A+6\,B\right)\cos\left(dx+c\right)+56\,A+48\,B\right)\sqrt{a\cos\left(c\right.}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
2/105*(15*B*cos(d*x + c)^3 + 3*(7*A + 6*B)*cos(d*x + c)^2 + 4*(7*A + 6*B)*
cos(d*x + c) + 56*A + 48*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d
*x + c) + d)
```

Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\cos^2\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*cos(c + d*x)**2,
x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{14\left(3\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+30\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+3\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right.\right.}{420\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
1/420*(14*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
 + 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 3*(5*sqrt(2)*sin(7/2*d*x +
 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c)
 + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\sqrt{2}\left(15\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
1/420*sqrt(2)*(15*B*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 21*(2
*A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x +
5/2*c) + 35*(2*A*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*sgn(cos(1/2*d*x + 1/2*c))
)*sin(3/2*d*x + 3/2*c) + 105*(4*A*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*sgn(cos(
1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^3dx\right)b+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^2dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3,x)*b + int(sqrt(cos(c
+ d*x) + 1)*cos(c + d*x)**2,x)*a)
```

3.76 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 101

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2(5A-2B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}$$

output
```
2/15*a*(5*A+7*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/15*(5*A-2*B)*(a+a*c
os(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{a(1+\cos(c+dx))}(20A+19B+2(5A+4B)\cos(c+dx)+3B\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*(20*A + 19*B + 2*(5*A + 4*B)*Cos[c + d*x] + 3*
B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sqrt{a\cos(c+dx)+a}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}(3aB+a(5A-2B)\cos(c+dx))dx}{5a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{\cos(c+dx)a+a}(3aB+a(5A-2B)\cos(c+dx))dx}{5a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3aB+a(5A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{3}a(5A+7B)\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}a(5A+7B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3125}$$

$$\frac{\frac{2a^2(5A+7B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((2*a^2*(5*A + 7*B
)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(5*A - 2*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
default	$\frac{2 \cos \left(\frac{d x}{2}+\frac{c}{2}\right) a \sin \left(\frac{d x}{2}+\frac{c}{2}\right) \left(12 B \sin \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+\left(-10 A -20 B \right) \sin \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+15 A +15 B \right) \sqrt{2}}{15 \sqrt{a \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\, d}$
parts	$\frac{2 A \cos \left(\frac{d x}{2}+\frac{c}{2}\right) a \sin \left(\frac{d x}{2}+\frac{c}{2}\right) \left(2 \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+1\right) \sqrt{2}}{3 \sqrt{a \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\, d}+\frac{2 B \cos \left(\frac{d x}{2}+\frac{c}{2}\right) a \sin \left(\frac{d x}{2}+\frac{c}{2}\right) \left(12 \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}-4 \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+7\right) \sqrt{}}{15 \sqrt{a \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\, d}$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*B*sin(1/2*d*x+1/2*c)^4+(-
10*A-20*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{2\left(3\,B\cos\left(dx+c\right)^2+\left(5\,A+4\,B\right)\cos\left(dx+c\right)+10\,A+8\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
2/15*(3*B*cos(d*x + c)^2 + (5*A + 4*B)*cos(d*x + c) + 10*A + 8*B)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{10\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(3\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+\right.}{30\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/30*(10*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*A
*sqrt(a) + (3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c
) + 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\sqrt{2}\left(3\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\,\mathrm{s}\right.}{30\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
1/30*sqrt(2)*(3*B*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*(2*A*
sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2
*c) + 30*(A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(1
/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+dx)\ (A+B\cos(c+dx))\ \sqrt{a+a\cos(c+dx)}\,dx$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*a + int(sqrt(cos(c + d
*x) + 1)*cos(c + d*x)**2,x)*b)
```

3.77 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2B\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*a*(3*A+B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*B*(a+a*cos(d*x+c))^(
1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(3A+2B+B\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(3*A + 2*B + B*Cos[c + d*x])*Tan[(c + d*x)/2
])/(3*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{3}(3A+B)\int \sqrt{\cos(c+dx)a+a}dx+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3A+B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*(3*A + B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*B*Sqrt[a
+ a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2 \cos \left(\frac{d x}{2}+\frac{c}{2}\right) a \sin \left(\frac{d x}{2}+\frac{c}{2}\right) \left(2 B \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+3 A+B\right) \sqrt{2}}{3 \sqrt{a \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\, d}$	62
parts	$\frac{2 A a \cos \left(\frac{d x}{2}+\frac{c}{2}\right) \sin \left(\frac{d x}{2}+\frac{c}{2}\right) \sqrt{2}}{\sqrt{a \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\, d}+\frac{2 B \cos \left(\frac{d x}{2}+\frac{c}{2}\right) a \sin \left(\frac{d x}{2}+\frac{c}{2}\right) \left(2 \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+1\right) \sqrt{2}}{3 \sqrt{a \cos \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}\, d}$	103

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(2*B*cos(1/2*d*x+1/2*c)^2+3*A+
B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2\,(B\cos\left(dx+c\right)+3\,A+2\,B)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{3\,(d\cos\left(dx+c\right)+d)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
2/3*(B*cos(d*x + c) + 3*A + 2*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*
cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}(A+B\cos\left(c+dx\right))\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{6\,\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{3\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*(6*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*sin(3/2*d*x + 3/2
*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}\left(B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin\right.}{3\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*(B*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 3*(2*A*sgn
(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)
)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \sqrt{a}\left(\left(\int \sqrt{\cos(dx+c)+1}dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)b\right)$$

input

```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output

```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1),x)*a + int(sqrt(cos(c + d*x) + 1)*cos(
c + d*x),x)*b)
```

3.78 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 66

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2\sqrt{a}A\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a^(1/2)*A*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+2*a*B*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}A\operatorname{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*A*ArcTanh[Sqrt[2]*Si
n[(c + d*x)/2]] + 2*B*Sin[(c + d*x)/2]))/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3460}$$

$$A\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$A\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3252}$$

$$\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2aA\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
 (2*a*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(58) = 116$.

Time = 4.19 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.32

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(A\sqrt{2}\,\ln\left(-\frac{2\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)a+A\sqrt{2}\,\ln\left(\frac{2a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{2}\,\sqrt{}}{2\cos\left(\frac{dx}{2}+\right.}\right.\right.}{2\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$
parts	$\frac{A\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)+\ln\left(\frac{4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\sin\left(\right.}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
1/2/a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*2^(1/2)*l
n(-2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+A*2^(1/2)*ln(2/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+2*a))*a+4*B*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/
sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.92

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{(A\cos(dx+c)+A)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\sqrt{a\cos}}{2(d\cos(dx+c)+d)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
1/2*((A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*B*sin
(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec(c + dx)\,dx$$
$$= \int \sqrt{a\left(\cos\left(c + dx\right) + 1\right)}(A + B\cos\left(c + dx\right))\sec\left(c + dx\right)dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.32

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec(c + dx)\,dx = \frac{2\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
2*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx =$$
$$-\frac{\sqrt{2}\left(\sqrt{2}A\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)-4B\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{2d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*A*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2
*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 4*B*sgn(co
s(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int \frac{(A+B\cos(c+dx))\sqrt{a+a\cos(c+dx)}}{\cos(c+dx)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\sqrt{a}\left(\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)\,dx\right)a\right)\right.$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x),x)*b + int(s
qrt(cos(c + d*x) + 1)*sec(c + d*x),x)*a)
```

3.79 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{\sqrt{a}(A+2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
a^(1/2)*(A+2*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+a*A*t
an(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{2}(A+2B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)\right.}{2d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(A + 2*
B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*A*Sin[(c + d*x)/2]))
/(2*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 3459, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{2}(A+2B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+2B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a(A+2B)\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a}(A+2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(Sqrt[a]*(A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d + (a*A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(60) = 120$.

Time = 4.48 (sec) , antiderivative size = 567, normalized size of antiderivative = 8.34

method	result
parts	$\frac{A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-2a\left(\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)+\ln\left(\frac{4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\sin \cdots}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\cdots}\right.\right.\right.}{\sqrt{a}\left(2\cos\left(\frac{dx}{2}+\cdots\right.\right.}$
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-2a\left(A\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)+A\ln\left(\frac{4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a \cdots}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}\right.\right.\right.}{\cdots}$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
A*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(ln(-4/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)
))*sin(1/2*d*x+1/2*c)^2+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*co
s(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+B*a^(
1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(ln(-2/(2*cos(1/2*d
*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)))/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.25

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \frac{\left((A + 2\,B)\cos\left(dx + c\right)^2 + (A + 2\,B)\cos\left(dx + c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - 4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(d\cos\left(dx + c\right)^2 + d\cos\left(dx + c\right)\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
1/4*(((A + 2*B)*cos(d*x + c)^2 + (A + 2*B)*cos(d*x + c))*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(co
s(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x +
c))
```

Sympy [F]

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \int \sqrt{a\left(\cos\left(c + dx\right) + 1\right)}(A + B\cos\left(c + dx\right))\sec^2\left(c + dx\right)\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x)**2,
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(60) = 120$.

Time = 0.23 (sec) , antiderivative size = 710, normalized size of antiderivative = 10.44

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*
d*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3
/2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log
(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + s...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(60) = 120$.

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.78

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^2(c + dx)\,dx =$$

$$-\frac{\sqrt{2}\left(\sqrt{2}\left(A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right) + 2\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right) + \frac{4\,A\mathrm{sgn}(\cos(\frac{1}{2}}{2\,\sin}}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*(A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(cos(1/2*d*x +
 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*s
in(1/2*d*x + 1/2*c))) + 4*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)
/(2*sin(1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \int \frac{(A + B\,\cos(c + d\,x))\,\sqrt{a + a\,\cos(c + d\,x)}}{\cos(c + d\,x)^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\sqrt{a}\left(\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^2\,dx\right)a\right)\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2,x)*b + in
t(sqrt(cos(c + d*x) + 1)*sec(c + d*x)**2,x)*a)
```

3.80 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 117

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{\sqrt{a}(3A+4B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*a^(1/2)*(3*A+4*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d
+1/4*a*(3*A+4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*sec(d*x+c)*ta
n(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\left(3\sqrt{2}(3A+4B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)+6(2A+(3A+4B)\right.}{24d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*(3*Sqrt[2]*(3*A + 4*B)*ArcTanh[Sqrt[2]*Sin[(c
+ d*x)/2]]*Sec[(c + d*x)/2] + 6*(2*A + (3*A + 4*B)*Cos[c + d*x])*Sec[c + d
*x]^2*Tan[(c + d*x)/2]))/(24*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 3459, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{4}(3A+4B)\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(3A+4B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{4}(3A+4B)\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(3A+4B)\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{4}(3A+4B)\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}(3A+4B)\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + ((3*A + 4
*B)*((Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
+ (a*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(101) = 202.

Time = 4.54 (sec) , antiderivative size = 936, normalized size of antiderivative = 8.00

method	result	size
parts	Expression too large to display	936
default	Expression too large to display	1003

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/2*A*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*a*(ln(-4/(2*co
s(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+
2*a)))*sin(1/2*d*x+1/2*c)^4+(-12*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2
^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2
*a))*a-12*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-12*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+3*ln(-4/(2*cos(1/2*
d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*
a))*a+10*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(1/2)/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c
)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+B*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*a*(ln(-2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(2/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+ln(-2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{\left((3\,A+4\,B)\cos\left(dx+c\right)^3+(3\,A+4\,B)\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(c}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos\left(dx+c\right)^3+d\,\mathrm{co}\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
1/16*(((3*A + 4*B)*cos(d*x + c)^3 + (3*A + 4*B)*cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^
2)) + 4*((3*A + 4*B)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}(A+B\cos\left(c+dx\right))\sec^3\left(c+dx\right)dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x)**3,
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. 2(101) = 202.

Time = 2.98 (sec) , antiderivative size = 3352, normalized size of antiderivative = 28.65

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
1/16*((3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2))*cos(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - l...
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.67

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(\sqrt{2}\left(3\,A\text{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+4\,B\text{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(6\,A\text{sgn}(\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
-1/16*sqrt(2)*(sqrt(2)*(3*A*sgn(cos(1/2*d*x + 1/2*c)) + 4*B*sgn(cos(1/2*d*
x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) +
4*sin(1/2*d*x + 1/2*c))) + 4*(6*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
1/2*c)^3 + 8*B*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 5*A*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*B*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\cos(c+d\,x)^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\sqrt{a}\left(\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^3\,dx\right)a\right)\right.$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3,x)*b + in
t(sqrt(cos(c + d*x) + 1)*sec(c + d*x)**3,x)*a)
```

3.81 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 160

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \frac{\sqrt{a}(5A+6B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} \\ &\quad + \frac{a(5A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
1/8*a^(1/2)*(5*A+6*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d
+1/8*a*(5*A+6*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/12*a*(5*A+6*B)*sec(
d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)
/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.81

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec^3(c+dx)\left(3\sqrt{2}(5A+6B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)\sec\left(\frac{1}{2}(c+\right.\right.}{48d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[c + d*x]^3*(3*Sqrt[2]*(5*A + 6*B)*ArcTanh[
Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3*Sec[(c + d*x)/2] + (31*A + 18*B +
 4*(5*A + 6*B)*Cos[c + d*x] + 3*(5*A + 6*B)*Cos[2*(c + d*x)])*Tan[(c + d*x
)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3459, 3042, 3251, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{6}(5A+6B)\int \sqrt{\cos(c+dx)a+a}\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}(5A+6B)\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3251$$

$$\frac{1}{6}(5A+6B)\left(\frac{3}{4}\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}(5A+6B)\left(\frac{3}{4}\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3251$$

$$\frac{1}{6}(5A+6B)\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}(5A+6B)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3252$$

$$\frac{1}{6}(5A+6B)\left(\frac{3}{4}\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}(5A+6B)\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + ((5*A +
 6*B)*((a*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*(
(Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*
Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(140) = 280$.

Time = 4.47 (sec) , antiderivative size = 1282, normalized size of antiderivative = 8.01

method	result	size
parts	Expression too large to display	1282
default	Expression too large to display	1327

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
1/6*A*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-120*a*(ln(-4/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+2*a)))*sin(1/2*d*x+1/2*c)^6+60*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*a^(1/2)+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+3*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-160*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-90*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)-2*a))*a-90*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x
+1/2*c)^2+66*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+15*ln(-4/(2*co
s(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+15*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+1/4*B*co
s(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*a*2^(1/2)*(ln(-2/(2...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.23

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$

$$= \frac{3\left((5A+6B)\cos(dx+c)^4+(5A+6B)\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{\cdots}}{\cos(dx+c)^3+\cos(dx+c)\cdots}\right.\cdots}{96\left(d\cos\cdots\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
1/96*(3*((5*A + 6*B)*cos(d*x + c)^4 + (5*A + 6*B)*cos(d*x + c)^3)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sq
rt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c
)^2)) + 4*(3*(5*A + 6*B)*cos(d*x + c)^2 + 2*(5*A + 6*B)*cos(d*x + c) + 8*A
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c
)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5021 vs. $2(140) = 280$.

Time = 3.01 (sec) , antiderivative size = 5021, normalized size of antiderivative = 31.38

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
-1/96*((120*(sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*c
os(13/2*d*x + 13/2*c) - 8*(15*sin(11/2*d*x + 11/2*c) + 50*sin(9/2*d*x + 9/
2*c) + 42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x +
3/2*c))*cos(6*d*x + 6*c) + 360*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(1
1/2*d*x + 11/2*c) + 1200*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(9/2*d*x
 + 9/2*c) - 24*(42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3
/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqr
t(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x
 + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*
c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c
) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c))) + 2) + 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c
)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2
)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sq
rt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*co...
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.52

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(5\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+6\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(60\,As\right.}{}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
-1/96*sqrt(2)*(3*sqrt(2)*(5*A*sgn(cos(1/2*d*x + 1/2*c)) + 6*B*sgn(cos(1/2*
d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2)
+ 4*sin(1/2*d*x + 1/2*c))) + 4*(60*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
 + 1/2*c)^5 + 72*B*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 80*A
*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 96*B*sgn(cos(1/2*d*x +
 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 33*A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d
*x + 1/2*c) + 30*B*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(
1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\cos(c+d\,x)^4}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \sqrt{a}\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^4\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**4,x)*b + in
t(sqrt(cos(c + d*x) + 1)*sec(c + d*x)**4,x)*a)
```

3.82 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 234

$$
\begin{aligned}
&\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} \\
&+\frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} \\
&+\frac{2a^2(11A+12B)\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} \\
&-\frac{8a(187A+168B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3465d} \\
&+\frac{2aB\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{11d} \\
&+\frac{4(187A+168B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{1155d}
\end{aligned}
$$

output
```
4/495*a^2*(187*A+168*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/693*a^2*(187
*A+168*B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/99*a^2*(11*A+
12*B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-8/3465*a*(187*A+168
*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/11*a*B*cos(d*x+c)^4*(a+a*cos(d*x
+c))^(1/2)*sin(d*x+c)/d+4/1155*(187*A+168*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*
x+c)/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}(59158A+55482B+(35156A+34734B)\cos(c+dx)+8(150\cdots}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(59158*A + 55482*B + (35156*A + 34734*B)*Cos
[c + d*x] + 8*(1507*A + 1743*B)*Cos[2*(c + d*x)] + 3740*A*Cos[3*(c + d*x)]
 + 4935*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 1470*B*Cos[4*(c + d*
x)] + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3 \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2} \left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{11}\int \frac{1}{2}\cos^3(c+dx)\sqrt{\cos(c+dx)a+a}(a(11A+8B)+a(11A+12B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \cos^3(c+dx)\sqrt{\cos(c+dx)a+a}(a(11A+8B)+a(11A+12B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(11A+8B)+a(11A+12B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\int \cos^3(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c-}{9d\sqrt{a\cos(c+dx)+a}}\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{3249}$$

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(11A+1}{9d}\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3238

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\left(\frac{2\int\frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\left(\frac{\int(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\left(\frac{\int\left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3230

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{11}\left(\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{1}{9}a(187A+168B)\left(\frac{6}{7}\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+}}{3d}}{5a}\right.\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(11*d) + ((2*
a^2*(11*A + 12*B)*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x
]]) + (a*(187*A + 168*B)*((2*a*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a +
a*Cos[c + d*x]]) + (6*((2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d)
 + ((14*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/(5*a)))/7))/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.61

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-5040B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(3080A+18480B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-9900A-27720B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(1247\right.}{3465\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$
parts	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-220\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+114\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4+47\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+94\right)\sqrt{2}}{315\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}+\frac{4B\cos\left(\frac{d}{2}\right.}{}$

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
4/3465*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-5040*B*sin(1/2*d*x+1/2*
c)^10+(3080*A+18480*B)*sin(1/2*d*x+1/2*c)^8+(-9900*A-27720*B)*sin(1/2*d*x+
1/2*c)^6+(12474*A+22176*B)*sin(1/2*d*x+1/2*c)^4+(-8085*A-10395*B)*sin(1/2*
d*x+1/2*c)^2+3465*A+3465*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2\left(315\,Ba\cos\left(dx+c\right)^5+35\,(11\,A+21\,B)a\cos\left(dx+c\right)^4+5\,(187\,A+168\,B)a\cos\left(d\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
2/3465*(315*B*a*cos(d*x + c)^5 + 35*(11*A + 21*B)*a*cos(d*x + c)^4 + 5*(18
7*A + 168*B)*a*cos(d*x + c)^3 + 6*(187*A + 168*B)*a*cos(d*x + c)^2 + 8*(18
7*A + 168*B)*a*cos(d*x + c) + 16*(187*A + 168*B)*a)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{22\left(35\sqrt{2}a\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+135\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+378\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+1}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
1/55440*(22*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x
 + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 21*(15*sqrt(
2)*a*sin(11/2*d*x + 11/2*c) + 55*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 165*sqrt
(2)*a*sin(7/2*d*x + 7/2*c) + 429*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 990*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 3630*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a
))/d
```

Giac [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}\left(315\,Ba\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+385\,\left(2\,Aa\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
1/55440*sqrt(2)*(315*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c)
+ 385*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*
sin(9/2*d*x + 9/2*c) + 495*(6*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 7*B*a*sgn(co
s(1/2*d*x + 1/2*c)))*sin(7/2*d*x + 7/2*c) + 693*(12*A*a*sgn(cos(1/2*d*x +
1/2*c)) + 13*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c) + 2310*(1
0*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 9*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2
*d*x + 3/2*c) + 6930*(12*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 11*B*a*sgn(cos(1/
2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^5\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)a\right)$$

input
```
int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**5,x)*b + int(sqrt(cos(
c + d*x) + 1)*cos(c + d*x)**4,x)*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*
x)**4,x)*b + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3,x)*a)
```

3.83 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 189

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}}-\frac{4a(39A+34B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d}+\frac{2aB\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d}+\frac{2(39A+34B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d}$$

output
```
2/45*a^2*(39*A+34*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a^2*(9*A+10*
B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-4/315*a*(39*A+34*B)*(a
+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/9*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^(1
/2)*sin(d*x+c)/d+2/105*(39*A+34*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}(2964A+2689B+2(759A+799B)\cos(c+dx)+(468A+54$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(2964*A + 2689*B + 2*(759*A + 799*B)*Cos[c +
 d*x] + (468*A + 548*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 170*B*C
os[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{9}\int \frac{1}{2}\cos^2(c+dx)\sqrt{\cos(c+dx)a+a}(3a(3A+2B)+a(9A+10B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}(3a(3A+2B)+a(9A+10B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3a(3A+2B)+a(9A+10B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{9}\left(\frac{3}{7}a(39A+34B)\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{3}{7}a(39A+34B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$$\Big\downarrow \text{3238}$$

$$\frac{1}{9}\left(\frac{3}{7}a(39A+34B)\left(\frac{2\int\frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{3}{7}a(39A+34B)\left(\frac{\int(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a}{}\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{3}{7}a(39A+34B)\left(\frac{\int \left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a}{5ad}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$\big\downarrow$ 3230

$$\frac{1}{9}\left(\frac{3}{7}a(39A+34B)\left(\frac{\frac{7}{3}a\int\sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^3}{5ad}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$\big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{3}{7}a(39A+34B)\left(\frac{\frac{7}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)}{5ad}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

$\big\downarrow$ 3125

$$\frac{1}{9}\left(\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{3}{7}a(39A+34B)\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + ((2*a
^2*(9*A + 10*B)*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]
) + (3*a*(39*A + 34*B)*((2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d
) + ((14*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a +
a*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/(5*a)))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.65

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-180A-900B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(504A+1134B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-525A-735B)\right.}{315\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$
parts	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4+19\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+38\right)\sqrt{2}}{105\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}+\frac{4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\right.}{}$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*B*sin(1/2*d*x+1/2*c)^
8+(-180*A-900*B)*sin(1/2*d*x+1/2*c)^6+(504*A+1134*B)*sin(1/2*d*x+1/2*c)^4+
(-525*A-735*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/
2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.57

$$\int\cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{2\left(35\,Ba\cos\left(dx+c\right)^4+5\,(9\,A+17\,B)a\cos\left(dx+c\right)^3+3\,(39\,A+34\,B)a\cos\left(dx+c\right.\right.}{315\,(d\cos(}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
2/315*(35*B*a*cos(d*x + c)^4 + 5*(9*A + 17*B)*a*cos(d*x + c)^3 + 3*(39*A +
 34*B)*a*cos(d*x + c)^2 + 4*(39*A + 34*B)*a*cos(d*x + c) + 8*(39*A + 34*B)
*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{6\left(15\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+175\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+735\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
1/2520*(6*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x +
5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x +
1/2*c))*A*sqrt(a) + (35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin
(7/2*d*x + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*si
n(3/2*d*x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}\left(35\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
1/2520*sqrt(2)*(35*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 45
*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(7
/2*d*x + 7/2*c) + 378*(A*a*sgn(cos(1/2*d*x + 1/2*c)) + B*a*sgn(cos(1/2*d*x
 + 1/2*c)))*sin(5/2*d*x + 5/2*c) + 1050*(A*a*sgn(cos(1/2*d*x + 1/2*c)) + B
*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 630*(7*A*a*sgn(cos(1/
2*d*x + 1/2*c)) + 6*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*s
qrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos\left(c+d\,x\right)^2\left(A+B\,\cos\left(c+d\,x\right)\right)\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4\,dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**4,x)*b + int(sqrt(cos(
c + d*x) + 1)*cos(c + d*x)**3,x)*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*
x)**3,x)*b + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2,x)*a)
```

3.84 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 138

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(21A+19B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2(7A-2B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}$$

output

```
8/105*a^2*(21*A+19*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/105*a*(21*A+19
*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/35*(7*A-2*B)*(a+a*cos(d*x+c))^(3
/2)*sin(d*x+c)/d+2/7*B*(a+a*cos(d*x+c))^(5/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}(546A+494B+(252A+253B)\cos(c+dx)+6(7A+13B)\cos(2(c+dx))+15B\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{210d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(546*A + 494*B + (252*A + 253*B)*Cos[c + d*x
] + 6*(7*A + 13*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)
/2])/(210*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a\cos(c+dx)+a)^{3/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}(\cos(c+dx)a+a)^{3/2}(5aB+a(7A-2B)\cos(c+dx))dx}{7a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (\cos(c+dx)a+a)^{3/2}(5aB+a(7A-2B)\cos(c+dx))dx}{7a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(5aB+a(7A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{5}a(21A+19B)\int (\cos(c+dx)a+a)^{3/2}dx+\frac{2a(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}a(21A+19B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3126}$$

$$\frac{\frac{1}{5}a(21A+19B)\left(\frac{4}{3}a\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}a(21A+19B)\left(\frac{4}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)}{5d}}{7a}$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3125}$$

$$\frac{\frac{1}{5}a(21A+19B)\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d) + ((2*a*(7*A - 2*B)*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (a*(21*A + 19*B)*((8*a^2*
Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x
]]*Sin[c + d*x])/(3*d)))/5)/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-60B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(42A+168B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-105A-175B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+105A+105B\right)\sqrt{2}}{105\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$
parts	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\right)\sqrt{2}}{5\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}+\frac{4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-12\cos\right.}{105\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-60*B*sin(1/2*d*x+1/2*c)^
6+(42*A+168*B)*sin(1/2*d*x+1/2*c)^4+(-105*A-175*B)*sin(1/2*d*x+1/2*c)^2+10
5*A+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.64

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2\left(15\,Ba\cos\left(dx+c\right)^{3}+3\left(7\,A+13\,B\right)a\cos\left(dx+c\right)^{2}+\left(63\,A+52\,B\right)a\cos\left(dx+c\right)\cdots\right.}{105\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
2/105*(15*B*a*cos(d*x + c)^3 + 3*(7*A + 13*B)*a*cos(d*x + c)^2 + (63*A + 5
2*B)*a*cos(d*x + c) + 2*(63*A + 52*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{42\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+(}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/420*(42*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*
c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (15*sqrt(2)*a*sin(7/2*
d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*d
*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}\left(15\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\right.\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
1/420*sqrt(2)*(15*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 21*
(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/
2*d*x + 5/2*c) + 35*(6*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a*sgn(cos(1/2*d
*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 105*(8*A*a*sgn(cos(1/2*d*x + 1/2*c))
+ 7*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\,(A+B\,\cos(c+dx))\,(a+a\,\cos(c+dx))^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*a + int(sqrt(cos(c +
 d*x) + 1)*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*
*2,x)*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2,x)*b)
```

3.85 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a(5A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
8/15*a^2*(5*A+3*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/15*a*(5*A+3*B)*(a
+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/
d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}(50A+39B+2(5A+9B)\cos(c+dx)+3B\cos(2(c+dx)))\,\mathrm{ta}}{15d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cos[c + d*x] +
3*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A+3B)\int (\cos(c+dx)a+a)^{3/2}dx+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+3B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3125

$$\frac{1}{5}(5A+3B)\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + ((5*A + 3*B)*((8*a^2
*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*
x]]*Sin[c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-5A-15B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+15A+15B\right)\sqrt{2}}{15\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$
parts	$\frac{4A\,a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\right)\sqrt{2}}{3\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}+\frac{4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\right)\sqrt{2}}{5\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/15*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(6*B*sin(1/2*d*x+1/2*c)^4+(
-5*A-15*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{2\left(3\,Ba\cos\left(dx+c\right)^2+(5\,A+9\,B)a\cos\left(dx+c\right)+(25\,A+18\,B)a\right)\sqrt{a\cos\left(dx+c\right)+\cdots}}{15\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
2/15*(3*B*a*cos(d*x + c)^2 + (5*A + 9*B)*a*cos(d*x + c) + (25*A + 18*B)*a)
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\,dx = \int (a(\cos(c + dx) + 1))^{\frac{3}{2}}(A + B\cos(c + dx))\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c+dx))\,dx = \frac{10\left(\sqrt{2}a\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right) + 9\sqrt{2}a\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)A\sqrt{a} + 3\left(\sqrt{2}a\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right) + 5\right.}{30\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c
))*A*sqrt(a) + 3*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x
 + 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c+dx))\,dx = \frac{\sqrt{2}\left(3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx + \frac{5}{2}\,c\right) + 5\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right) + 3\,Ba\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/30*sqrt(2)*(3*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*(2*
A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d
*x + 3/2*c) + 30*(3*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*a*sgn(cos(1/2*d*x
+ 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\,dx = \int (A + B\,\cos(c + d\,x))\,(a + a\,\cos(c + d\,x))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a + a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1),x)*a + int(sqrt(cos(c + d*x) + 1)*co
s(c + d*x),x)*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*b + int(sqrt(
cos(c + d*x) + 1)*cos(c + d*x)**2,x)*b)
```

3.86 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 105

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a^(3/2)*A*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a^2*(
3*A+4*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*B*(a+a*cos(d*x+c))^(1/2
)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}A\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(3A+5B+B\cos$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*A*ArcTanh[Sqrt[2
]*Sin[(c + d*x)/2]] + 2*(3*A + 5*B + B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3
*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}(3aA+a(3A+4B)\cos(c+dx))\sec(c+dx)dx+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{\cos(c+dx)a+a}(3aA+a(3A+4B)\cos(c+dx))\sec(c+dx)dx+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3aA+a(3A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$\big\downarrow$ 3460

$$\frac{1}{3}\left(3aA\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{2a^2(3A+4B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(3aA\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2(3A+4B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$\big\downarrow$ 3252

$$\frac{1}{3}\left(\frac{2a^2(3A+4B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{6a^2A\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{6a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(3A+4B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*a*B*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((6*a^(3/2)*A*ArcTan
h[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*a^2*(3*A + 4*B)
*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(91) = 182$.

Time = 3.88 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.56

method	result
parts	$\frac{A\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{a}+\ln\left(-\frac{4\left(a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)a+\ln\left(\frac{4a}{}\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$
default	$\frac{\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8B\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{a}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+3A\sqrt{2}\ln\left(-\frac{2\left(a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right.\right.}{6\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
A*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(
a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)-2*a))*a+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)/sin(1/2*d*x+1/2*
c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+4/3*B*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)*(cos(1/2*d*x+1/2*c)^2+2)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/
d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec(c + dx)\,dx = \frac{3\,(Aa\cos(dx + c) + Aa)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - 4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{6\,(d\cos(dx + c) + d)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
1/6*(3*(A*a*cos(d*x + c) + A*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x +
 c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(B*a*cos(d*x + c) + (3*A
 + 5*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.37

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{3\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*
sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.33

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=$$
$$-\frac{\sqrt{2}\left(8\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\sqrt{2}Aa\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"giac")
```

output
```
-1/6*sqrt(2)*(8*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 3*s
qrt(2)*A*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*
sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 12*A*a*sgn(cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c) - 24*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*
d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\sqrt{a}\,a\left(\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right)b+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)dx\right)b+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\sec\left(dx+c\right)dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x),x)*a + int
(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(cos(c +
d*x) + 1)*cos(c + d*x)**2*sec(c + d*x),x)*b + int(sqrt(cos(c + d*x) + 1)*s
ec(c + d*x),x)*a)
```

3.87 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 103

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a^{3/2}(3A+2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d}$$

output

```
a^(3/2)*(3*A+2*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d-a^2
*(A-2*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+a*A*(a+a*cos(d*x+c))^(1/2)*ta
n(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{2}(3A+2B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)c\right.}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x
]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(3*A
+ 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 2*B*Cos[c +
 d*x])*Sin[(c + d*x)/2]))/(2*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3454, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3454}$$

$$\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}(a(3A+2B)-a(A-2B)\cos(c+dx))\sec(c+dx)dx+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}(a(3A+2B)-a(A-2B)\cos(c+dx))\sec(c+dx)dx+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(3A+2B)-a(A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \; 3460$$

$$\frac{1}{2}\left(a(3A+2B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-\frac{2a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(a(3A+2B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \; 3252$$

$$\frac{1}{2}\left(-\frac{2a^2(3A+2B)\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{2a^{3/2}(3A+2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
((2*a^(3/2)*(3*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/d - (2*a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/2
 + (a*A*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(93) = 186$.

Time = 4.01 (sec) , antiderivative size = 603, normalized size of antiderivative = 5.85

method	result
parts	$\frac{A\sqrt{a}\,\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{a\sin(\frac{dx}{2}+\frac{c}{2})^2}\,(-6a(\ln(-\frac{4(a\sqrt{2}\,\cos(\frac{dx}{2}+\frac{c}{2})-\sqrt{2}\,\sqrt{a\sin(\frac{dx}{2}+\frac{c}{2})^2}\,\sqrt{a}-2a)}{2\cos(\frac{dx}{2}+\frac{c}{2})-\sqrt{2}})+\ln(\frac{4a\sqrt{2}\,\cos(\frac{dx}{2}+\frac{c}{2})+4\sqrt{2}\,\sqrt{}}{2\cos(\frac{dx}{2}+}}{(2\cos(\frac{dx}{2}}$
default	$\frac{\sqrt{a}\,\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{a\sin(\frac{dx}{2}+\frac{c}{2})^2}\,(((-6A\ln(-\frac{4(a\sqrt{2}\,\cos(\frac{dx}{2}+\frac{c}{2})-\sqrt{2}\,\sqrt{a\sin(\frac{dx}{2}+\frac{c}{2})^2}\,\sqrt{a}-2a)}{2\cos(\frac{dx}{2}+\frac{c}{2})-\sqrt{2}})a-6A\ln(\frac{4a\sqrt{2}\,\cos(\frac{dx}{2}+\frac{c}{2})+4\sqrt{}}{2\cos(\frac{d}{}}}{}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
A*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6*a*(ln(-4/(
2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c
)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+3*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+1
/2*B*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln
(-2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+2^(1/2)*ln(2/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+2*a))*a+4*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1
/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{((3A+2B)a\cos(dx+c)^2+(3A+2B)a\cos(dx+c))\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4(2Ba\cos(dx+c)+Aa)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
1/4*(((3*A + 2*B)*a*cos(d*x + c)^2 + (3*A + 2*B)*a*cos(d*x + c))*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqr
t(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)) + 4*(2*B*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))
/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^2(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(93) = 186$.

Time = 0.24 (sec) , antiderivative size = 1315, normalized size of antiderivative = 12.77

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^2(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
-1/4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(
5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*
sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin
(1/2*d*x + 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin...
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.45

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^2(c+dx)\,dx = \frac{\sqrt{2}\left(8\,Ba\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{4\,Aa\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \sqrt{2}(3\,Aa\operatorname{sgn}(\right.}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
1/4*sqrt(2)*(8*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*A*a*
sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 -
 1) - sqrt(2)*(3*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*a*sgn(cos(1/2*d*x + 1
/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin
(1/2*d*x + 1/2*c))))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^2\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2,x)*a +
int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(sqrt(co
s(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**2,x)*b + int(sqrt(cos(c + d*
x) + 1)*sec(c + d*x)**2,x)*a)
```

3.88 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 119

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a^{3/2}(7A+12B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/4*a^(3/2)*(7*A+12*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/
d+1/4*a^2*(5*A+4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*(a+a*cos(d
*x+c))^(1/2)*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\sqrt{2}(7A+12B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{8d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x
]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(7*
A + 12*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*A + (7*A
 + 4*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{2}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}(a(5A+4B)+a(A+4B)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \sqrt{\cos(c+dx)a+a}(a(5A+4B)+a(A+4B)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(5A+4B)+a(A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{4}\left(\frac{1}{2}a(7A+12B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a^2(5A+4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}a(7A+12B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2(5A+4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{4}\left(\frac{a^2(5A+4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(7A+12B)\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{a^{3/2}(7A+12B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2(5A+4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((a^(3/2)
*(7*A + 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
+ (a^2*(5*A + 4*B)*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(103) = 206$.

Time = 4.16 (sec) , antiderivative size = 938, normalized size of antiderivative = 7.88

method	result	size
parts	Expression too large to display	938
default	Expression too large to display	1003

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/2*A*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*a*(ln(
-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-28*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-28*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-28*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+18*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+7*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/
2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)-2*a))*a+7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c
)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+B*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-6*a*(ln(-2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+
ln(2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(-2/(2*cos(1/2*d*x+1/2*c)...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.53

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{\left((7\,A+12\,B)a\cos\left(dx+c\right)^3+(7\,A+12\,B)a\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a}}{\cos(dx-}\right.}{16\,\left(d\cos\left(dx\right.\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
1/16*(((7*A + 12*B)*a*cos(d*x + c)^3 + (7*A + 12*B)*a*cos(d*x + c)^2)*sqrt
(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x
 + c)^2)) + 4*((7*A + 4*B)*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3339 vs. $2(103) = 206$.

Time = 0.36 (sec) , antiderivative size = 3339, normalized size of antiderivative = 28.06

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
-1/16*((12*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48*a*cos(2*d*x + 2*
c)^2*sin(3/2*d*x + 3/2*c) + 12*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) +
 48*a*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 160*a*cos(7/2*d*x + 7/2*c)
*sin(2*d*x + 2*c) + 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 72*a*cos
(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 24*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3
/2*c) - 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*
c) + 12*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c)
 + 48*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 4
*(12*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 20*a*sin(7/2*d*x + 7/2*c) -
 21*a*sin(5/2*d*x + 5/2*c) - 3*a*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) -
7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)
*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*
sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)
*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*cos(4
*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c
)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2...
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.69

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(\sqrt{2}\left(7\,Aa\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+12\,Ba\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(14\,Aas\right.}{}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
-1/16*sqrt(2)*(sqrt(2)*(7*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 12*B*a*sgn(cos(1
/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(
2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(14*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/
2*d*x + 1/2*c)^3 + 8*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3
- 9*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*B*a*sgn(cos(1/2
*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqr
t(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,{(a+a\,\cos(c+d\,x))}^{3/2}}{{\cos(c+d\,x)}^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^3dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^3dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3,x)*a +
int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(sqrt(co
s(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**3,x)*b + int(sqrt(cos(c + d*
x) + 1)*sec(c + d*x)**3,x)*a)
```

3.89 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 164

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a^{3/2}(11A+14B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(7A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
1/8*a^(3/2)*(11*A+14*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))
/d+1/8*a^2*(11*A+14*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/12*a^2*(7*A+6
*B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*A*(a+a*cos(d*x+c)
)^(1/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(3\sqrt{2}(11A+14B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x
]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*(
11*A + 14*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (7*(7*A +
6*B) + 4*(11*A + 6*B)*Cos[c + d*x] + (33*A + 42*B)*Cos[2*(c + d*x)])*Sin[(
c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}(a(7A+6B)+3a(A+2B)\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \sqrt{\cos(c+dx)a+a}(a(7A+6B)+3a(A+2B)\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(7A+6B)+3a(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{6}\left(\frac{3}{4}a(11A+14B)\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{4}a(11A+14B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{6}\left(\frac{3}{4}a(11A+14B)\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(7A+6B)\tan(c+d}{2d\sqrt{a\cos(c+d}}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{4}a(11A+14B)\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(7A+6B)\tan(c+dx)\sec}{2d\sqrt{a\cos(c+dx)+}}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{6}\left(\frac{3}{4}a(11A+14B)\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2(7A+6B)\tan(c+dx)}{2d\sqrt{a\cos(c+dx)}}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}\left(\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{3}{4}a(11A+14B)\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+}}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((a^2*(
7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*
a*(11*A + 14*B)*((Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
+ d*x]]])/d + (a*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(144) = 288$.

Time = 4.14 (sec) , antiderivative size = 1283, normalized size of antiderivative = 7.82

method	result	size
parts	Expression too large to display	1283
default	Expression too large to display	1326

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
1/6*A*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-264*a*(l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^6+132*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+3*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-22*(16*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+2*a))*a+9*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin
(1/2*d*x+1/2*c)^2+126*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+33*ln
(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+33*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+1/4*B*a
^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*a*2^(1/2)*...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.23

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{3\left((11A+14B)a\cos(dx+c)^4+(11A+14B)a\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\cdots}{\cos\cdots}\right)\cdots}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
1/96*(3*((11*A + 14*B)*a*cos(d*x + c)^4 + (11*A + 14*B)*a*cos(d*x + c)^3)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos
(d*x + c)^2)) + 4*(3*(11*A + 14*B)*a*cos(d*x + c)^2 + 2*(11*A + 6*B)*a*cos
(d*x + c) + 8*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^
4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7567 vs. $2(144) = 288$.

Time = 153.02 (sec) , antiderivative size = 7567, normalized size of antiderivative = 46.14

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^4(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
-1/96*((774*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 162*sqrt(2)*
a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (14*sqrt(2)*a*sin(3/2*d*x + 3/2*
c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(6*d*x + 6*c)
^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2
*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x +
3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1...
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.54

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(11\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+14\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\left(132\right.}{}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
-1/96*sqrt(2)*(3*sqrt(2)*(11*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 14*B*a*sgn(co
s(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sq
rt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(132*A*a*sgn(cos(1/2*d*x + 1/2*c))*si
n(1/2*d*x + 1/2*c)^5 + 168*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2
*c)^5 - 176*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 192*B*a
*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 63*A*a*sgn(cos(1/2*d*x
 + 1/2*c))*sin(1/2*d*x + 1/2*c) + 54*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2
*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^4}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^4\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**4,x)*a +
int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(sqrt(co
s(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**4,x)*b + int(sqrt(cos(c + d*
x) + 1)*sec(c + d*x)**4,x)*a)
```

3.90 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{a^{3/2}(75A+88B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(75A+88B)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(9A+8B)\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{4d}$$

output

```
1/64*a^(3/2)*(75*A+88*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2)
)/d+1/64*a^2*(75*A+88*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/96*a^2*(75*
A+88*B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(9*A+8*B)*
sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*A*(a+a*cos(d*x+c))^
(1/2)*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.72

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(6\sqrt{2}(75A+88B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x
]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*(
75*A + 88*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (492*A + 3
52*B + (1155*A + 1048*B)*Cos[c + d*x] + 4*(75*A + 88*B)*Cos[2*(c + d*x)] +
 225*A*Cos[3*(c + d*x)] + 264*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*
d)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}(a(9A+8B)+a(5A+8B)\cos(c+dx))\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \sqrt{\cos(c+dx)a+a}(a(9A+8B)+a(5A+8B)\cos(c+dx))\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(9A+8B)+a(5A+8B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{8}\left(\frac{1}{6}a(75A+88B)\int \sqrt{\cos(c+dx)a+a}\sec^3(c+dx)dx+\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}a(75A+88B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{8}\left(\frac{1}{6}a(75A+88B)\left(\frac{3}{4}\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(9A+8B)\tan(}{3d\sqrt{a\cos}}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}a(75A+88B)\left(\frac{3}{4}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(9A+8B)\tan(c+dx}{3d\sqrt{a\cos(c+dx}}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$\big\downarrow$ 3251

$$\frac{1}{8}\left(\frac{1}{6}a(75A+88B)\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c}{2d\sqrt{a\cos(c+dx)}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}a(75A+88B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$\big\downarrow$ 3252

$$\frac{1}{8}\left(\frac{1}{6}a(75A+88B)\left(\frac{3}{4}\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\tan(c+dx)\sec(c}{2d\sqrt{a\cos(c+dx)}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$\big\downarrow$ 219

$$\frac{1}{8}\left(\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{1}{6}a(75A+88B)\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+}{d\sqrt{a\cos(c+}}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((a^2*(
9*A + 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*(75*A + 88*B)*((a*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x
]]) + (3*((Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d + (a*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1616 vs. $2(185) = 370$.

Time = 4.18 (sec) , antiderivative size = 1617, normalized size of antiderivative = 7.74

method	result	size
parts	Expression too large to display	1617
default	Expression too large to display	1651

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNV
ERBOSE)
```

output
```
1/8*A*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1200*a*(l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^8-1200*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+2*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+2*ln(-4/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^6+200*(11*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+2*a))*a+9*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin
(1/2*d*x+1/2*c)^4+(-1460*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-60
0*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-600*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+362*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+75*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.05

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{3\left((75\,A+88\,B)a\cos\left(dx+c\right)^5+(75\,A+88\,B)a\cos\left(dx+c\right)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\cdots}{\mathrm{co}\cdots}\right.}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="fricas")
```

output
```
1/768*(3*((75*A + 88*B)*a*cos(d*x + c)^5 + (75*A + 88*B)*a*cos(d*x + c)^4)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c
) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*(3*(75*A + 88*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*c
os(d*x + c)^2 + 8*(15*A + 8*B)*a*cos(d*x + c) + 48*A*a)*sqrt(a*cos(d*x + c
) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10504 vs. $2(185) = 370$.

Time = 164.24 (sec) , antiderivative size = 10504, normalized size of antiderivative = 50.26

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="maxima")
```

output
```
-1/768*(3*(140*a*cos(8*d*x + 8*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*cos(6*d*
x + 6*c)^2*sin(3/2*d*x + 3/2*c) + 5040*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x +
3/2*c) + 2240*a*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 140*a*sin(8*d*x
+ 8*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*sin(6*d*x + 6*c)^2*sin(3/2*d*x + 3/
2*c) + 5040*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 2240*a*sin(2*d*x +
 2*c)^2*sin(3/2*d*x + 3/2*c) + 4064*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c
) + 336*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 240*a*cos(3/2*d*x + 3/2*
c)*sin(2*d*x + 2*c) + 1360*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 36*(a
*sin(8*d*x + 8*c) + 4*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + 4*a*sin(
2*d*x + 2*c))*cos(21/2*d*x + 21/2*c) + 140*(a*sin(8*d*x + 8*c) + 4*a*sin(6
*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + 4*a*sin(2*d*x + 2*c))*cos(19/2*d*x +
19/2*c) + 456*(a*sin(8*d*x + 8*c) + 4*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x +
 4*c) + 4*a*sin(2*d*x + 2*c))*cos(17/2*d*x + 17/2*c) + 4*(280*a*cos(6*d*x
+ 6*c)*sin(3/2*d*x + 3/2*c) + 420*a*cos(4*d*x + 4*c)*sin(3/2*d*x + 3/2*c)
+ 280*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 290*a*sin(15/2*d*x + 15/2*
c) - 596*a*sin(13/2*d*x + 13/2*c) - 780*a*sin(11/2*d*x + 11/2*c) - 750*a*s
in(9/2*d*x + 9/2*c) - 254*a*sin(7/2*d*x + 7/2*c) - 21*a*sin(5/2*d*x + 5/2*
c) + 85*a*sin(3/2*d*x + 3/2*c))*cos(8*d*x + 8*c) + 2320*(2*a*sin(6*d*x + 6
*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(15/2*d*x + 15/2*c)
+ 4768*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2...
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.44

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(75\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+88\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4\,(18\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="giac")
```

output
```
-1/768*sqrt(2)*(3*sqrt(2)*(75*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 88*B*a*sgn(c
os(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*s
qrt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(1800*A*a*sgn(cos(1/2*d*x + 1/2*c))*
sin(1/2*d*x + 1/2*c)^7 + 2112*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
1/2*c)^7 - 3300*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 387
2*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 2190*A*a*sgn(cos(
1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 2416*B*a*sgn(cos(1/2*d*x + 1/2*
c))*sin(1/2*d*x + 1/2*c)^3 - 543*A*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
 + 1/2*c) - 504*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin
(1/2*d*x + 1/2*c)^2 - 1)^4)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx=\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{3/2}}{\cos(c+dx)^5}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^5,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^5, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx=\sqrt{a}\,a\Bigg(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^5dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^5dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^5dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^5dx\right)a\Bigg)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**5,x)*a +
int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**5,x)*b + int(sqrt(co
s(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**5,x)*b + int(sqrt(cos(c + d*
x) + 1)*sec(c + d*x)**5,x)*a)
```

3.91 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 237

$$\begin{aligned}\int \cos^2(c+dx)(a&+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\frac{2a^3(803A+710B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}}\\&+\frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}\\&-\frac{4a^2(803A+710B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3465d}\\&+\frac{2a^2(11A+14B)\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{99d}\\&+\frac{2a(803A+710B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{1155d}\\&+\frac{2aB\cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d}\end{aligned}$$

output
```
2/495*a^3*(803*A+710*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/693*a^3*(209
*A+194*B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-4/3465*a^2*(803
*A+710*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/99*a^2*(11*A+14*B)*cos(d*x
+c)^3*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/1155*a*(803*A+710*B)*(a+a*cos(
d*x+c))^(3/2)*sin(d*x+c)/d+2/11*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*si
n(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(124366A+114640B+(68552A+69890B)\cos(c+dx)+16$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(124366*A + 114640*B + (68552*A + 69890*B)
*Cos[c + d*x] + 16*(1397*A + 1625*B)*Cos[2*(c + d*x)] + 5720*A*Cos[3*(c +
d*x)] + 8675*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 2240*B*Cos[4*(c
 + d*x)] + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\big\downarrow \text{3455}$$

$$\frac{2}{11}\int \frac{1}{2}\cos^2(c+dx)(\cos(c+dx)a+a)^{3/2}(a(11A+6B)+a(11A+14B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{11}\int \cos^2(c+dx)(\cos(c+dx)a+a)^{3/2}(a(11A+6B)+a(11A+14B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\big\downarrow \text{3042}$$

$$\frac{1}{11}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(11A+6B)+a(11A+14B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\big\downarrow \text{3455}$$

$$\frac{1}{11}\left(\frac{2}{9}\int \frac{1}{2}\cos^2(c+dx)\sqrt{\cos(c+dx)a+a}\left(3(55A+46B)a^2+(209A+194B)\cos(c+dx)a^2\right)dx+\frac{2a^2(11A+\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}\left(3(55A+46B)a^2+(209A+194B)\cos(c+dx)a^2\right)dx+\frac{2a^2(11A+14\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3(55A+46B)a^2+(209A+194B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(803A+710B)\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a^3(209A+194B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(803A+710B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^3(209A+194B)\sin(c+dx)}{7d\sqrt{a\cos(c+dx)+}}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{3238}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(803A+710B)\left(\frac{2\int \frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(803A+710B)\left(\frac{\int(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/}}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(803A+710B)\left(\frac{\int\left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c-}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(803A+710B)\left(\frac{\frac{7}{3}a\int\sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+a}{5ad}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(803A+710B)\left(\frac{\frac{7}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{11}\left(\frac{2a^2(11A+14B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}+\frac{1}{9}\left(\frac{2a^3(209A+194B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d) + ((
2*a^2*(11*A + 14*B)*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(9*d) + ((2*a^3*(209*A + 194*B)*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a +
 a*Cos[c + d*x]]) + (3*a^2*(803*A + 710*B)*((2*(a + a*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(5*a*d) + ((14*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x
]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/(5*a)))/7)/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 13.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.60

method	result
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-2520B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(1540A+10780B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-5940A-18810B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(9009\right.}{3465\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$
parts	$\frac{8A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-20\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+39\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4+52\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+104\right)\sqrt{2}}{315\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}+\frac{8B\cos\left(\frac{dx}{2}\right.}{}$

input
```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
8/3465*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-2520*B*sin(1/2*d*x+1/2*
c)^10+(1540*A+10780*B)*sin(1/2*d*x+1/2*c)^8+(-5940*A-18810*B)*sin(1/2*d*x+
1/2*c)^6+(9009*A+17325*B)*sin(1/2*d*x+1/2*c)^4+(-6930*A-9240*B)*sin(1/2*d*
x+1/2*c)^2+3465*A+3465*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.58

$$\int\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\frac{2\left(315\,Ba^2\cos\left(dx+c\right)^5+35\,(11\,A+32\,B)a^2\cos\left(dx+c\right)^4+5\,(286\,A+355\,B)a^2\cos\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
2/3465*(315*B*a^2*cos(d*x + c)^5 + 35*(11*A + 32*B)*a^2*cos(d*x + c)^4 + 5
*(286*A + 355*B)*a^2*cos(d*x + c)^3 + 3*(803*A + 710*B)*a^2*cos(d*x + c)^2
 + 4*(803*A + 710*B)*a^2*cos(d*x + c) + 8*(803*A + 710*B)*a^2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{22\left(35\sqrt{2}a^2\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+225\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+756\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
1/55440*(22*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2
*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*si
n(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*
(63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*x + 9/2
*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5/2*d*x
 + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.08

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}\left(315\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+385\left(2\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
1/55440*sqrt(2)*(315*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c
) + 385*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2
*c)))*sin(9/2*d*x + 9/2*c) + 495*(10*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 13*
B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(7/2*d*x + 7/2*c) + 693*(24*A*a^2*sgn(
cos(1/2*d*x + 1/2*c)) + 25*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x +
5/2*c) + 2310*(20*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 19*B*a^2*sgn(cos(1/2*d
*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 6930*(26*A*a^2*sgn(cos(1/2*d*x + 1/2*
c)) + 23*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int \cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a^2\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^5\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4\,dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4\,dx\right)b + 2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)a\right)$$

input

```
int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output

```
sqrt(a)*a**2*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**5,x)*b + int(sqrt(c
os(c + d*x) + 1)*cos(c + d*x)**4,x)*a + 2*int(sqrt(cos(c + d*x) + 1)*cos(c
 + d*x)**4,x)*b + 2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3,x)*a + int(
sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x) + 1)*c
os(c + d*x)**2,x)*a)
```

3.92 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 175

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(15A+13B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + \frac{2a(15A+13B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} + \frac{2(9A-2B)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2B(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad}$$

output
```
64/315*a^3*(15*A+13*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/315*a^2*(15*
A+13*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/105*a*(15*A+13*B)*(a+a*cos(d
*x+c))^(3/2)*sin(d*x+c)/d+2/63*(9*A-2*B)*(a+a*cos(d*x+c))^(5/2)*sin(d*x+c)
/d+2/9*B*(a+a*cos(d*x+c))^(7/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.60

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(6240A+5653B+(3030A+3116B)\cos(c+dx)+8(90A+$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(6240*A + 5653*B + (3030*A + 3116*B)*Cos[c
 + d*x] + 8*(90*A + 127*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 260*
B*Cos[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a\cos(c+dx)+a)^{5/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{2\int \frac{1}{2}(\cos(c+dx)a+a)^{5/2}(7aB+a(9A-2B)\cos(c+dx))dx}{9a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int(\cos(c+dx)a+a)^{5/2}(7aB+a(9A-2B)\cos(c+dx))dx}{9a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(7aB+a(9A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{\frac{3}{7}a(15A+13B)\int(\cos(c+dx)a+a)^{5/2}dx+\frac{2a(9A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{7}a(15A+13B)\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx+\frac{2a(9A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3126}$$

$$\frac{\frac{3}{7}a(15A+13B)\left(\frac{8}{5}a\int(\cos(c+dx)a+a)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2a(9A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{5/}}{7d}}{9a}$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{7}a(15A+13B)\left(\frac{8}{5}a\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2a(9A-2B)\sin(c+dx)(a\cos(c+a}{7d}}{9a}$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 3126

$$\frac{\frac{3}{7}a(15A+13B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2a}{}}{9a}$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 3042

$$\frac{\frac{3}{7}a(15A+13B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)}{9a}$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$\big\downarrow$ 3125

$$\frac{\frac{3}{7}a(15A+13B)\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2a(9A-2B)\sin(}{}}{9a}$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d) + ((2*a*(9*A - 2*B)*
(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (3*a*(15*A + 13*B)*((2*a*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (8*a*((8*a^2*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(3*d)))/5))/7)/(9*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.70

method	result
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-90A-540B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(315A+819B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-420A-630B)\sin\right.}{315\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$
parts	$\frac{8A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4+4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+8\right)\sqrt{2}}{21\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}+\frac{8B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\right.}{}$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*B*sin(1/2*d*x+1/2*c)^
8+(-90*A-540*B)*sin(1/2*d*x+1/2*c)^6+(315*A+819*B)*sin(1/2*d*x+1/2*c)^4+(-
420*A-630*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*
c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.66

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\frac{2\left(35\,Ba^2\cos\left(dx+c\right)^4+5\left(9\,A+26\,B\right)a^2\cos\left(dx+c\right)^3+3\left(60\,A+73\,B\right)a^2\cos\left(dx\right.\right.}{315\left(d\right.}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
2/315*(35*B*a^2*cos(d*x + c)^4 + 5*(9*A + 26*B)*a^2*cos(d*x + c)^3 + 3*(60
*A + 73*B)*a^2*cos(d*x + c)^2 + (345*A + 292*B)*a^2*cos(d*x + c) + 2*(345*
A + 292*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{30\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/2520*(30*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*
x + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2
*d*x + 1/2*c))*A*sqrt(a) + (35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt
(2)*a^2*sin(7/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100
*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))
*B*sqrt(a))/d
```

Giac [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.22

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}\left(35\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\,\left(2\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
1/2520*sqrt(2)*(35*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) +
45*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))
*sin(7/2*d*x + 7/2*c) + 126*(5*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 6*B*a^2*s
gn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c) + 210*(11*A*a^2*sgn(cos(1/2
*d*x + 1/2*c)) + 10*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)
+ 630*(15*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 13*B*a^2*sgn(cos(1/2*d*x + 1/2
*c)))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a^2\left(\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)b + 2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*a + int(sqrt(cos(
c + d*x) + 1)*cos(c + d*x)**4,x)*b + int(sqrt(cos(c + d*x) + 1)*cos(c + d*
x)**3,x)*a + 2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3,x)*b + 2*int(sqr
t(cos(c + d*x) + 1)*cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d*x) + 1)*cos(
c + d*x)**2,x)*b)
```

3.93 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(7A+5B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2a(7A+5B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
64/105*a^3*(7*A+5*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/105*a^2*(7*A+5
*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/35*a*(7*A+5*B)*(a+a*cos(d*x+c))^
(3/2)*sin(d*x+c)/d+2/7*B*(a+a*cos(d*x+c))^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.60

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}(1246A+1040B+(392A+505B)\cos(c+dx)+6(7A+20}{210d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1246*A + 1040*B + (392*A + 505*B)*Cos[c +
 d*x] + 6*(7*A + 20*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c +
d*x)/2])/(210*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A+5B)\int (\cos(c+dx)a+a)^{5/2}dx+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int (\cos(c+dx)a+a)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+}{5d}\right.$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+}{5d}\right.$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)$$
$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + ((7*A + 5*B)*((2*a*(
a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (8*a*((8*a^2*Sin[c + d*x])
/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*
x])/(3*d)))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

method	result
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-30B\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(21A+105B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(-70A-140B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+105A+105B\right)\sqrt{2}}{105\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$
parts	$\frac{8A\,a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+8\right)\sqrt{2}}{15\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}+\frac{8B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+3\cos\right.}{21\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
8/105*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-30*B*sin(1/2*d*x+1/2*c)^
6+(21*A+105*B)*sin(1/2*d*x+1/2*c)^4+(-70*A-140*B)*sin(1/2*d*x+1/2*c)^2+105
*A+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.69

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2\left(15\,Ba^{2}\cos\left(dx+c\right)^{3}+3\left(7\,A+20\,B\right)a^{2}\cos\left(dx+c\right)^{2}+\left(98\,A+115\,B\right)a^{2}\cos\left(dx+\right.\right.}{105\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
2/105*(15*B*a^2*cos(d*x + c)^3 + 3*(7*A + 20*B)*a^2*cos(d*x + c)^2 + (98*A
 + 115*B)*a^2*cos(d*x + c) + (301*A + 230*B)*a^2)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c+dx))\,dx = \frac{14\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx + \frac{5}{2}\,c\right) + 25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx + \frac{3}{2}\,c\right) + 150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)A$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/420*(14*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(3*sqrt(2)
*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 77*sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt
(a))/d
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.22

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c+dx))\, dx = \frac{\sqrt{2}\left(15\, Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)\right)\sin\left(\frac{7}{2}\, dx + \frac{7}{2}\, c\right) + 21\left(2\, Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)\right) + \right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 2
1*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*
sin(5/2*d*x + 5/2*c) + 35*(10*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 11*B*a^2*s
gn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c) + 525*(4*A*a^2*sgn(cos(1/2*
d*x + 1/2*c)) + 3*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c))*s
qrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\, dx = \int (A + B\,\cos(c + d\,x))\,(a + a\,\cos(c + d\,x))^{5/2}\, dx$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a + a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a^2\left(\left(\int \sqrt{\cos(dx+c)+1}dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(cos(c + d*x) + 1),x)*a + 2*int(sqrt(cos(c + d*x) +
1)*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*b + int(
sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x) + 1)*c
os(c + d*x)**2,x)*a + 2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2,x)*b)
```

3.94 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 142

$$\begin{aligned}\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{2a^{5/2}A\,\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}\\&+\frac{2a^2(5A+8B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}\\&+\frac{2aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}\end{aligned}$$

output
```
2*a^(5/2)*A*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+2/15*a^3*
(35*A+32*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/15*a^2*(5*A+8*B)*(a+a*co
s(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(15\sqrt{2}A\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+(80A+89B+2\right.}{15d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*A*ArcTanh[Sqr
t[2]*Sin[(c + d*x)/2]] + (80*A + 89*B + 2*(5*A + 14*B)*Cos[c + d*x] + 3*B*
Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{5}\int \frac{1}{2}(\cos(c+dx)a+a)^{3/2}(5aA+a(5A+8B)\cos(c+dx))\sec(c+dx)dx+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int (\cos(c+dx)a+a)^{3/2}(5aA+a(5A+8B)\cos(c+dx))\sec(c+dx)dx+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(5aA+a(5A+8B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 3455}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}\left(15Aa^2+(35A+32B)\cos(c+dx)a^2\right)\sec(c+dx)dx+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{}}{3d}\right.$$
$$+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \sqrt{\cos(c+dx)a+a}\left(15Aa^2+(35A+32B)\cos(c+dx)a^2\right)\sec(c+dx)dx+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{c}}{3d}\right.$$
$$+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(15Aa^2+(35A+32B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{c}}{3d}\right.$$
$$+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 3460}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^2A\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{2a^3(35A+32B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(5A+8B)\sin(c+dx)}{3d}\right.$$
$$+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^2A\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^3(35A+32B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\big\downarrow$ 3252

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^3(35A+32B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{30a^3A\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\big\downarrow$ 219

$$\frac{1}{5}\left(\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{1}{3}\left(\frac{30a^{5/2}A\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(35A+32B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)\right)+\frac{2aB\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + ((2*a^2*(5*A + 8*B
)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((30*a^(5/2)*A*ArcTanh[(S
qrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*a^3*(35*A + 32*B)*S
in[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(124) = 248$.

Time = 5.48 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.18

method	result
default	$\frac{a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, \left(48 B \sqrt{a}\, \sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-40 \sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, \sqrt{a}\, (A+4B) \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+15 A \sqrt{\cdots}\right.}{\cdots}$
parts	$\frac{A a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, \left(-4 \sqrt{2}\, \sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, \sqrt{a}\, \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+18 \sqrt{2}\, \sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, \sqrt{a}+3 \ln\left(-\frac{4\left(a \sqrt{2}\, \cos\left(\frac{d\cdots}{2}\right.\right.}{\cdots}\right.\right.}{3 \sin\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a \cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, d\cdots}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
1/30*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*B*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)*(A+4*B)*sin(1/2*d*x+1/2*c)^2+15*A*2^(1/2)*ln(-2/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+15*A*2^(1/2)*ln(2/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+2*a))*a+180*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+240*
B*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)*2^(1/2)/(a*co
s(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.25

$$\int (a + a\cos(c+dx))^{5/2}(A + B\cos(c+dx))\sec(c+dx)\,dx = \frac{15\left(Aa^2\cos(dx+c) + Aa^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+\cdots}{\cos(dx+c)^3+\cos(dx+c)^2}\right.\cdots}{30\,(\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
1/30*(15*(A*a^2*cos(d*x + c) + A*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin
(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*B*a^2*cos(d*x +
 c)^2 + (5*A + 14*B)*a^2*cos(d*x + c) + (40*A + 43*B)*a^2)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.43

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{30\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a)/d
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.42

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{\sqrt{2}\left(48\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 40\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"giac")
```

output
```
1/30*sqrt(2)*(48*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 -
40*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 160*B*a^2*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 15*sqrt(2)*A*a^2*log(abs(-2
*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))
)*sgn(cos(1/2*d*x + 1/2*c)) + 180*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*
d*x + 1/2*c) + 240*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*s
qrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)\,dx\right)a\right)$$

input

```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output

```
sqrt(a)*a**2*(2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x),x)*a
+ int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(cos
(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x),x)*b + int(sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**2*sec(c + d*x),x)*a + 2*int(sqrt(cos(c + d*x) + 1)*cos(c
 + d*x)**2*sec(c + d*x),x)*b + int(sqrt(cos(c + d*x) + 1)*sec(c + d*x),x)*
a)
```

3.95 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 144

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a^{5/2}(5A+2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(3A-2B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{aA(a+a\cos(c+dx))^{3/2}\tan(c+dx)}{d}$$

output
```
a^(5/2)*(5*A+2*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+1/3
*a^3*(3*A+14*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/3*a^2*(3*A-2*B)*(a+a
*cos(d*x+c))^(1/2)*sin(d*x+c)/d+a*A*(a+a*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3\sqrt{2}(5A+2B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{6d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x
]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*(
5*A + 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(3*A + B + 2
*(3*A + 8*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3454, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3454}$$

$$\int \frac{1}{2}(\cos(c+dx)a+a)^{3/2}(a(5A+2B)-a(3A-2B)\cos(c+dx))\sec(c+dx)dx+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int (\cos(c+dx)a+a)^{3/2}(a(5A+2B)-a(3A-2B)\cos(c+dx))\sec(c+dx)dx+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(5A+2B)-a(3A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{2}\left(\frac{2}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}\left(3(5A+2B)a^2+(3A+14B)\cos(c+dx)a^2\right)\sec(c+dx)dx-\frac{2a^2(3A-2B)\sin(c+\cdots}{\cdots}\right.$$
$$\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \sqrt{\cos(c+dx)a+a}\left(3(5A+2B)a^2+(3A+14B)\cos(c+dx)a^2\right)\sec(c+dx)dx-\frac{2a^2(3A-2B)\sin(c+\cdots}{3\cdots}\right.$$
$$\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3(5A+2B)a^2+(3A+14B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2a^2(3A-2B)\sin(c+\cdots}{3\cdots}\right.$$
$$\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(3a^2(5A+2B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{2a^3(3A+14B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)-\frac{2a^2(3A-2B)\sin(\cdots}{\cdots}\right.$$
$$\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(3a^2(5A+2B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^3(3A+14B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)-\frac{2a^2(3A-2B)\sin(c+dx}{3d}\right.$$
$$\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{2a^3(3A+14B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{6a^3(5A+2B)\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)-2a^2(3A-2B)\sin(c\right.$$
$$\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{6a^{5/2}(5A+2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(3A+14B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)-\frac{2a^2(3A-2B)\sin(c+dx)\sqrt{}}{3d}\right.$$
$$\frac{aA\tan(c+dx)(a\cos(c+dx)+a)^{3/2}}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
((-2*a^2*(3*A - 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((6*a^
(5/2)*(5*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (2*a^3*(3*A + 14*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/3)/2
 + (a*A*(a + a*Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(128) = 256$.

Time = 15.78 (sec) , antiderivative size = 683, normalized size of antiderivative = 4.74

method	result
parts	$A a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{a}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-10\ln\left(\frac{4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{a}+}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.$
default	$a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(16B\sqrt{a}\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\left(-30A\ln\left(-\frac{4\left(a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\right.\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right.\right.\right.$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
A*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2-10*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*sin(1/2*d*x+1/2*c)^2*a-10*ln(-4/(2*cos(1/2
*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^2*a+6*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+5*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))
*a+5*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*
x+1/2*c)^2)^(1/2)/d+1/6*B*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-8*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+
3*2^(1/2)*ln(-2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*
c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+3*2^(1/2)*ln(2/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+36*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2))/sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.40

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{3\left((5A+2B)a^2\cos(dx+c)^2+(5A+2B)a^2\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a}\ldots}{\cos(dx\ldots}\right.}{\ldots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
1/12*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sq
rt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) +
 a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*(2*B*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) +
 3*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos
(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^2(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8114 vs. $2(128) = 256$.

Time = 0.48 (sec) , antiderivative size = 8114, normalized size of antiderivative = 56.35

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^2(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
-1/252*(1449*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 1260*sq
rt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sq
rt(2)*a^2)*sin(5/2*d*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*
sin(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2
*c) - 60*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x +
 3/2*c) - 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (25*sqrt
(2)*a^2*cos(3/2*d*x + 3/2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2
*d*x + 2*c))*cos(5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2
*c) - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1
/2*c)^2)*sin(3/2*d*x + 3/2*c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*s
in(3/2*d*x + 3/2*c) + 69*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)
 - 198*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) - 198*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(
2)*a^2*cos(3/2*d*x + 3/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d
*x + 2*c))*sin(5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c
) - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2
*c)^2)*sin(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*
d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1
/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*s...
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(16\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-72\,B\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
-1/12*sqrt(2)*(16*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 -
 24*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 72*B*a^2*sgn(co
s(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) + 12*A*a^2*sgn(cos(1/2*d*x + 1/2*
c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1) + 3*sqrt(2)*(5*A*a
^2*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(
-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c
))))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\begin{aligned}\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \sqrt{a}\,a^2\left(2\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^2dx\right)a\right.\\
&+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^2dx\right)b\\
&+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2dx\right)b\\
&+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2dx\right)a\\
&+2\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2dx\right)b\\
&\left.+\left(\int \sqrt{\cos\left(dx+c\right)+1}\,\sec\left(dx+c\right)^2dx\right)a\right)\end{aligned}$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2,x)
*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(sq
rt(cos(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x)**2,x)*b + int(sqrt(cos(c
 + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**2,x)*a + 2*int(sqrt(cos(c + d*x
) + 1)*cos(c + d*x)**2*sec(c + d*x)**2,x)*b + int(sqrt(cos(c + d*x) + 1)*s
ec(c + d*x)**2,x)*a)
```

3.96 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a^{5/2}(19A+20B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(7A+4B)\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{4d} + \frac{aA(a+a\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/4*a^(5/2)*(19*A+20*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))
/d-1/4*a^3*(9*A-4*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*(7*A+4*B)
*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*
x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\sqrt{2}(19A+20B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{8}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x
]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(
19*A + 20*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*((11*A +
 4*B)*Cos[c + d*x] + 2*(A + 2*B + 2*B*Cos[2*(c + d*x)]))*Sin[(c + d*x)/2])
)/(8*d)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{2}\int \frac{1}{2}(\cos(c+dx)a+a)^{3/2}(a(7A+4B)-a(A-4B)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int (\cos(c+dx)a+a)^{3/2}(a(7A+4B)-a(A-4B)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(7A+4B)-a(A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\left(\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}\left(a^2(19A+20B)-a^2(9A-4B)\cos(c+dx)\right)\sec(c+dx)dx+\frac{a^2(7A+4B)\tan(c+d}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\left(a^2(19A+20B)-a^2(9A-4B)\cos(c+dx)\right)\sec(c+dx)dx+\frac{a^2(7A+4B)\tan(c+}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a^2(19A+20B)-a^2(9A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2(7A+4B)\tan(c+d}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(a^2(19A+20B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-\frac{2a^3(9A-4B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(7A+4B)\tan(c}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(a^2(19A+20B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2a^3(9A-4B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(7A+4B)\tan(c+dx}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$\Big\downarrow$ 3252

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{2a^3(19A+20B)\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2a^3(9A-4B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(7A+4B)\tan}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

$\Big\downarrow$ 219

$$\frac{1}{4}\left(\frac{a^2(7A+4B)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}+\frac{1}{2}\left(\frac{2a^{5/2}(19A+20B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^3(9A-}{d\sqrt{a\cos}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^{3/2}}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((2*a^
(5/2)*(19*A + 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (2*a^3*(9*A - 4*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/2 +
 (a^2*(7*A + 4*B)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(136) = 272$.

Time = 59.41 (sec) , antiderivative size = 995, normalized size of antiderivative = 6.38

method	result	size
parts	Expression too large to display	995
default	Expression too large to display	1028

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/2*A*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(76*a*(ln(
-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-44*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-76*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d
*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-76*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+26*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+19*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)-2*a))*a+19*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)/(2*cos(
1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2
*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+1/2*B*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-10*2^(1/2)*ln(2/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+2*a))*sin(1/2*d*x+1/2*c)^2*a-10*2^(1/2)*ln(-2/(2*cos(1/2*d*x+1/2*c)-2
^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^2*a-16*a^(1/2)*(a*sin(1/2*d*x+1/2*c...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.31

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^3(c + dx)\,dx = \frac{((19\,A + 20\,B)a^2\cos(dx + c)^3 + (19\,A + 20\,B)a^2\cos(dx + c)^2)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - \cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
1/16*(((19*A + 20*B)*a^2*cos(d*x + c)^3 + (19*A + 20*B)*a^2*cos(d*x + c)^2
)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)) + 4*(8*B*a^2*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c
) + 2*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*
cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^3(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11782 vs. $2(136) = 272$.

Time = 3.31 (sec) , antiderivative size = 11782, normalized size of antiderivative = 75.53

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^3(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
-1/1008*(63*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*s
qrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*
d*x + 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/
2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3
/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2))*cos(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sq
rt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)...
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.53

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{\sqrt{2}\left(32\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{2}\left(19\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+20\,Ba^2\mathrm{s}\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
1/16*sqrt(2)*(32*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - sq
rt(2)*(19*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 20*B*a^2*sgn(cos(1/2*d*x + 1/2
*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1
/2*d*x + 1/2*c))) - 4*(22*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/
2*c)^3 + 8*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 13*A*a
^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) - 4*B*a^2*sgn(cos(1/2*d*
x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a
)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$
\begin{aligned}
&\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c \\
&+dx)\,dx = \sqrt{a}\,a^2\Bigg(2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a \\
&+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b \\
&+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)b \\
&+\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a \\
&+2\left(\int \sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b \\
&+\left(\int \sqrt{\cos(dx+c)+1}\,\sec(dx+c)^3\,dx\right)a\Bigg)
\end{aligned}
$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3,x)
*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(sq
rt(cos(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x)**3,x)*b + int(sqrt(cos(c
 + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**3,x)*a + 2*int(sqrt(cos(c + d*x
) + 1)*cos(c + d*x)**2*sec(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x) + 1)*s
ec(c + d*x)**3,x)*a)
```

3.97 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 164

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a^{5/2}(25A+38B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(3A+2B)\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{4d}+\frac{aA(a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
1/8*a^(5/2)*(25*A+38*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))
/d+1/24*a^3*(49*A+54*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*(3*A+2
*B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*(a+a*cos(d*x+c)
)^(3/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.80

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(3\sqrt{2}(25A+38B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x
]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]
*(25*A + 38*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (91*A +
66*B + 4*(17*A + 6*B)*Cos[c + d*x] + (75*A + 66*B)*Cos[2*(c + d*x)])*Sin[(
c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 3454}$$

$$\frac{1}{3}\int \frac{1}{2}(\cos(c+dx)a+a)^{3/2}(3a(3A+2B)+a(A+6B)\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int (\cos(c+dx)a+a)^{3/2}(3a(3A+2B)+a(A+6B)\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(3a(3A+2B)+a(A+6B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}\left((49A+54B)a^2+(13A+30B)\cos(c+dx)a^2\right)\sec^2(c+dx)dx+\frac{3a^2(3A+2B)\tan\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{1}{4}\int \sqrt{\cos(c+dx)a+a}\left((49A+54B)a^2+(13A+30B)\cos(c+dx)a^2\right)\sec^2(c+dx)dx+\frac{3a^2(3A+2B)\tan(\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((49A+54B)a^2+(13A+30B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{3a^2(3A+2B)\tan(c\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}a^2(25A+38B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a^3(49A+54B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{3a^2(3A+2B)\text{t}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}a^2(25A+38B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3(49A+54B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{3a^2(3A+2B)\tan(c+\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$\Big\downarrow$ 3252

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{a^3(49A+54B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{3a^3(25A+38B)\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{3a^2(3A+2B)\text{ta}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

$\Big\downarrow$ 219

$$\frac{1}{6}\left(\frac{3a^2(3A+2B)\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}\left(\frac{3a^{5/2}(25A+38B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((3*a
^2*(3*A + 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) +
 ((3*a^(5/2)*(25*A + 38*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/d + (a^3*(49*A + 54*B)*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]
]))/4)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(144) = 288$.

Time = 175.11 (sec) , antiderivative size = 1282, normalized size of antiderivative = 7.82

method	result	size
parts	Expression too large to display	1282
default	Expression too large to display	1326

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
1/6*A*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-600*a*(l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^6+300*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+3*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-736*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-450*ln(-4/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-2*a))*a-450*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*
sin(1/2*d*x+1/2*c)^2+234*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+75
*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+75*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+1/4*
B*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(76*a*2^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{3\left((25\,A+38\,B)a^2\cos\left(dx+c\right)^4+(25\,A+38\,B)a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2 \cdots}{\cdots}\right)}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
1/96*(3*((25*A + 38*B)*a^2*cos(d*x + c)^4 + (25*A + 38*B)*a^2*cos(d*x + c)
^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)) + 4*(3*(25*A + 22*B)*a^2*cos(d*x + c)^2 + 2*(17*A + 6*B)
*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos
(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^4(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7994 vs. $2(144) = 288$.

Time = 3.33 (sec) , antiderivative size = 7994, normalized size of antiderivative = 48.74

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^4(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
-1/96*((1530*a^2*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*cos(2*
d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*sin(4*d*x + 4*c)^2*sin(3/2*d*
x + 3/2*c) + 1530*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 4176*a^2*c
os(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 2430*a^2*cos(5/2*d*x + 5/2*c)*sin(2
*d*x + 2*c) + 678*a^2*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 342*a^2*cos(
2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 10*(a^2*sin(9/2*d*x + 9/2*c) + 17*a^2*
sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c)^2 + 10*(a^2*sin(9/2*d*x + 9/2*c) +
17*a^2*sin(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c)^2 - 56*a^2*sin(3/2*d*x + 3/2
*c) + 10*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x
+ 2*c))*cos(21/2*d*x + 21/2*c) - 30*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*
x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(19/2*d*x + 19/2*c) - 48*(a^2*sin(6*
d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(17/2*d*x
 + 17/2*c) + 80*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin
(2*d*x + 2*c))*cos(15/2*d*x + 15/2*c) + 396*(a^2*sin(6*d*x + 6*c) + 3*a^2*
sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 6*(170
*a^2*cos(4*d*x + 4*c)*sin(3/2*d*x + 3/2*c) + 170*a^2*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) - 170*a^2*sin(11/2*d*x + 11/2*c) - 232*a^2*sin(7/2*d*x +
7/2*c) - 135*a^2*sin(5/2*d*x + 5/2*c) + 19*a^2*sin(3/2*d*x + 3/2*c) + 10*(
a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c) - 25*a^2)*sin(9/2*d*x + 9/2*c)
)*cos(6*d*x + 6*c) + 3060*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))...
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.63

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(25\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+38\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\frac{4(\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
-1/96*sqrt(2)*(3*sqrt(2)*(25*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 38*B*a^2*sg
n(cos(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(
2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(300*A*a^2*sgn(cos(1/2*d*x + 1/2*
c))*sin(1/2*d*x + 1/2*c)^5 + 264*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d
*x + 1/2*c)^5 - 368*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3
 - 288*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 117*A*a^2*
sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) + 78*B*a^2*sgn(cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/
d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\cos(c+dx)^4}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^4(c + dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^4\,dx\right)a + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^3\sec(dx + c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^2\sec(dx + c)^4\,dx\right)a + 2\left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^2\sec(dx + c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\sec(dx + c)^4\,dx\right)a\right)$$

input

```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output

```
sqrt(a)*a**2*(2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**4,x)
*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(sq
rt(cos(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x)**4,x)*b + int(sqrt(cos(c
 + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**4,x)*a + 2*int(sqrt(cos(c + d*x
) + 1)*cos(c + d*x)**2*sec(c + d*x)**4,x)*b + int(sqrt(cos(c + d*x) + 1)*s
ec(c + d*x)**4,x)*a)
```

3.98 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{a^{5/2}(163A+200B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(95A+104B)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(11A+8B)\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{24d} + \frac{aA(a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
1/64*a^(5/2)*(163*A+200*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/
2))/d+1/64*a^3*(163*A+200*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/96*a^3*
(95*A+104*B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(11*A
+8*B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+a*cos(d*
x+c))^(3/2)*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.73

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(6\sqrt{2}(163A+200B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x
]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]
*(163*A + 200*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (844*A
 + 544*B + (2203*A + 2056*B)*Cos[c + d*x] + (652*A + 544*B)*Cos[2*(c + d*x
)] + 489*A*Cos[3*(c + d*x)] + 600*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(
768*d)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{4}\int \frac{1}{2}(\cos(c+dx)a+a)^{3/2}(a(11A+8B)+a(3A+8B)\cos(c+dx))\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int (\cos(c+dx)a+a)^{3/2}(a(11A+8B)+a(3A+8B)\cos(c+dx))\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(11A+8B)+a(3A+8B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{8}\left(\frac{1}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}\left((95A+104B)a^2+3(17A+24B)\cos(c+dx)a^2\right)\sec^3(c+dx)dx+\frac{a^2(11A+8B)\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \sqrt{\cos(c+dx)a+a}\left((95A+104B)a^2+3(17A+24B)\cos(c+dx)a^2\right)\sec^3(c+dx)dx+\frac{a^2(11A+8B)\tan\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((95A+104B)a^2+3(17A+24B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2(11A+8B)\tan\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(163A+200B)\int\sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a^3(95A+104B)\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(163A+200B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^3(95A+104B)\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(11\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 3251

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(163A+200B)\left(\frac{1}{2}\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^3(95A+104B}{2d\sqrt{a\,c}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(163A+200B)\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^3(95A+104B)\tan(}{2d\sqrt{a\cos(c}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 3252

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(163A+200B)\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^3(95A+104B)}{2d\sqrt{a\,c}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 219

$$\frac{1}{8}\left(\frac{a^2(11A+8B)\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{1}{6}\left(\frac{a^3(95A+104B)\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((a^2
*(11*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
+ ((a^3*(95*A + 104*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c +
d*x]]) + (3*a^2*(163*A + 200*B)*((Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/S
qrt[a + a*Cos[c + d*x]]])/d + (a*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]
)))/4)/6)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. $2(185) = 370$.

Time = 1.30 (sec) , antiderivative size = 1650, normalized size of antiderivative = 7.89

Expression too large to display

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(163*
A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+163*A*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+2*a))+200*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(
1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a
))+200*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^
8-48*(163*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+200*B*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+326*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^
(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)-2*a))*a+326*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+400
*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+400*B*ln(4/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(1793*A*a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2072*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)+1467*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(
1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+1...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.11

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{3\left((163\,A+200\,B)a^2\cos(dx+c)^5+(163\,A+200\,B)a^2\cos(dx+c)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="fricas")
```

output
```
1/768*(3*((163*A + 200*B)*a^2*cos(d*x + c)^5 + (163*A + 200*B)*a^2*cos(d*x
 + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c
)^3 + cos(d*x + c)^2)) + 4*(3*(163*A + 200*B)*a^2*cos(d*x + c)^3 + 2*(163*
A + 136*B)*a^2*cos(d*x + c)^2 + 8*(23*A + 8*B)*a^2*cos(d*x + c) + 48*A*a^2
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c
)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.54

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx =$$
$$-\frac{\sqrt{2}\left(3\sqrt{2}\left(163\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+200\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\log\left(\frac{\left|-2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="giac")
```

output
```
-1/768*sqrt(2)*(3*sqrt(2)*(163*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 200*B*a^2
*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/a
bs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(3912*A*a^2*sgn(cos(1/2*d*x +
1/2*c))*sin(1/2*d*x + 1/2*c)^7 + 4800*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(
1/2*d*x + 1/2*c)^7 - 7172*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/
2*c)^5 - 8288*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 460
6*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 4816*B*a^2*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 1047*A*a^2*sgn(cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c) - 936*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1
/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^4)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^5}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

Reduce [F]

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^5\,dx\right)a + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^5\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^3\sec(dx + c)^5\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^2\sec(dx + c)^5\,dx\right)a + 2\left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^2\sec(dx + c)^5\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\sec(dx + c)^5\,dx\right)a\right)$$

input

```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output

```
sqrt(a)*a**2*(2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**5,x)
*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**5,x)*b + int(sq
rt(cos(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x)**5,x)*b + int(sqrt(cos(c
 + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**5,x)*a + 2*int(sqrt(cos(c + d*x
) + 1)*cos(c + d*x)**2*sec(c + d*x)**5,x)*b + int(sqrt(cos(c + d*x) + 1)*s
ec(c + d*x)**5,x)*a)
```

3.99 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 254

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{a^{5/2}(283A+326B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(283A+326B)\sec(c+dx)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(157A+170B)\sec^2(c+dx)\tan(c+dx)}{240d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(13A+10B)\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{40d} + \frac{aA(a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\tan(c+dx)}{5d}$$

output
```
1/128*a^(5/2)*(283*A+326*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1
/2))/d+1/128*a^3*(283*A+326*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/192*a
^3*(283*A+326*B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/240*a^3*
(157*A+170*B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/40*a^2*(1
3*A+10*B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*A*(a+a*co
s(d*x+c))^(3/2)*sec(d*x+c)^4*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.69

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(60\sqrt{2}(283A+326B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x
]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2
]*(283*A + 326*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (2486
3*A + 22030*B + 36*(781*A + 650*B)*Cos[c + d*x] + 4*(6509*A + 6730*B)*Cos[
2*(c + d*x)] + 5660*A*Cos[3*(c + d*x)] + 6520*B*Cos[3*(c + d*x)] + 4245*A*
Cos[4*(c + d*x)] + 4890*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{ 3454}$$

$$\frac{1}{5}\int \frac{1}{2}(\cos(c+dx)a+a)^{3/2}(a(13A+10B)+5a(A+2B)\cos(c+dx))\sec^5(c+dx)dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\int (\cos(c+dx)a+a)^{3/2}(a(13A+10B)+5a(A+2B)\cos(c+dx))\sec^5(c+dx)dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{10}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(13A+10B)+5a(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \; 3454$$

$$\frac{1}{10}\left(\frac{1}{4}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}\left((157A+170B)a^2+5(21A+26B)\cos(c+dx)a^2\right)\sec^4(c+dx)dx+\frac{a^2(13A+10\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\left(\frac{1}{8}\int \sqrt{\cos(c+dx)a+a}\left((157A+170B)a^2+5(21A+26B)\cos(c+dx)a^2\right)\sec^4(c+dx)dx+\frac{a^2(13A+10B\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{10}\left(\frac{1}{8}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((157A+170B)a^2+5(21A+26B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2(13A+10B)\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \; 3459$$

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(283A+326B)\int\sqrt{\cos(c+dx)a+a}\sec^3(c+dx)dx+\frac{a^3(157A+170B)\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(283A+326B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^3(157A+170B)\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2}{}\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3251

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(283A+326B)\left(\frac{3}{4}\int\sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^3(157A}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(283A+326B)\left(\frac{3}{4}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^3(157A+170}{3d\sqrt{}}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3251

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(283A+326B)\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+d}{2d\sqrt{a\cos}}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(283A+326B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec}{2d\sqrt{a\cos(c+}}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(283A+326B)\left(\frac{3}{4}\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\tan(c+d}{2d\sqrt{a\cos}}\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{10}\left(\frac{a^2(13A+10B)\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{1}{8}\left(\frac{a^3(157A+170B)\tan(c+dx)\sec^2(c+d}{3d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

input

```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output

```
(a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((a^2
*(13*A + 10*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + ((a^3*(157*A + 170*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[
c + d*x]]) + (5*a^2*(283*A + 326*B)*((a*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sq
rt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt
[a + a*Cos[c + d*x]]])/d + (a*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))
/4))/6)/8)/10
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1974 vs. $2(226) = 452$.

Time = 1.34 (sec) , antiderivative size = 1975, normalized size of antiderivative = 7.78

Expression too large to display

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

output
```
1/120*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-480*a*(2
83*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+283*A*ln(4/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+2*a))+326*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-
2*a))+326*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)))*sin(1/2*d*x+1/2*
c)^10+240*(566*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+652*B*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1415*A*ln(-4/(2*cos(1/2*d*x+1/2
*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)-2*a))*a+1415*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)
)*a+1630*B*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2
*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+1630*B*ln(4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^8-80*(3962*A*a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+4564*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+4245*A*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^
(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.99

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^6(c + dx)\,dx = \frac{15\left((283\,A + 326\,B)a^2\cos\left(dx + c\right)^6 + (283\,A + 326\,B)a^2\cos\left(dx + c\right)^5\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorith
m="fricas")
```

output
```
1/7680*(15*((283*A + 326*B)*a^2*cos(d*x + c)^6 + (283*A + 326*B)*a^2*cos(d
*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*c
os(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)) + 4*(15*(283*A + 326*B)*a^2*cos(d*x + c)^4 + 10*(
283*A + 326*B)*a^2*cos(d*x + c)^3 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^2 +
 48*(29*A + 10*B)*a^2*cos(d*x + c) + 384*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^6(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^6(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.48

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorith
m="giac")
```

output
```
-1/7680*sqrt(2)*(15*sqrt(2)*(283*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 326*B*a
^2*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))
/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))) + 4*(67920*A*a^2*sgn(cos(1/2*d*x
 + 1/2*c))*sin(1/2*d*x + 1/2*c)^9 + 78240*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*
sin(1/2*d*x + 1/2*c)^9 - 158480*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*
x + 1/2*c)^7 - 182560*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)
^7 + 144896*A*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 16384
0*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 62780*A*a^2*sgn
(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 67000*B*a^2*sgn(cos(1/2*d*
x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 11115*A*a^2*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c) + 10470*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
+ 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^5)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^6}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^6,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^6, x)
```

Reduce [F]

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^6(c + dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^6\,dx\right)a + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^6\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^3\sec(dx + c)^6\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^2\sec(dx + c)^6\,dx\right)a + 2\left(\int \sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^2\sec(dx + c)^6\,dx\right)b + \left(\int \sqrt{\cos(dx + c) + 1}\,\sec(dx + c)^6\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**6,x)
*a + int(sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**6,x)*b + int(sq
rt(cos(c + d*x) + 1)*cos(c + d*x)**3*sec(c + d*x)**6,x)*b + int(sqrt(cos(c
 + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**6,x)*a + 2*int(sqrt(cos(c + d*x
) + 1)*cos(c + d*x)**2*sec(c + d*x)**6,x)*b + int(sqrt(cos(c + d*x) + 1)*s
ec(c + d*x)**6,x)*a)
```

3.100 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 202

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{2(7A-31B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105ad}$$

output
```
-2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/
2))/a^(1/2)/d+4/105*(49*A-37*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35*(
7*A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*B*cos(d*x+c)^3
*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/105*(7*A-31*B)*(a+a*cos(d*x+c))^(1/
2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.55

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-420(A-B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(406A-178B+(-28A+169B)\cos(c+d}{210d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x
]
```

output
```
(Cos[(c + d*x)/2]*(-420*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*(406*A - 178
*B + (-28*A + 169*B)*Cos[c + d*x] + 6*(7*A - B)*Cos[2*(c + d*x)] + 15*B*Co
s[3*(c + d*x)])*Sin[(c + d*x)/2]))/(210*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {3042, 3462, 27, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3462}$$

$$\frac{2\int \frac{\cos^2(c+dx)(6aB+a(7A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\cos^2(c+dx)(6aB+a(7A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6aB+a(7A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3462$$

$$\frac{\frac{2\int \frac{\cos(c+dx)\left(4a^2(7A-B)-a^2(7A-31B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\cos(c+dx)\left(4a^2(7A-B)-a^2(7A-31B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^2(7A-B)-a^2(7A-31B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3447$$

$$\frac{\frac{\int \frac{4a^2(7A-B)\cos(c+dx)-a^2(7A-31B)\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{4a^2(7A-B)\sin\left(c+dx+\frac{\pi}{2}\right)-a^2(7A-31B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\frac{\frac{2\int -\frac{a^3(7A-31B)-2a^3(49A-37B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{-\frac{\int \frac{a^3(7A-31B)-2a^3(49A-37B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{-\frac{\int \frac{a^3(7A-31B)-2a^3(49A-37B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2a(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{\frac{-\frac{105a^3(A-B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a^3(49A-37B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\frac{105a^3(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^3(49A-37B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3128

$$\frac{-\frac{-\frac{210a^3(A-B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^3(49A-37B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2a(7A-B)\sin(c+a}{5d\sqrt{a\cos(c-}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 219

$$\frac{-\frac{\frac{105\sqrt{2}a^{5/2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^3(49A-37B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2a(7A-B)\sin(c+dx)\cos}{5d\sqrt{a\cos(c+dx)+}}}{7a}+\frac{2B\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*B*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + ((2*a*(
7*A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + ((-
2*a*(7*A - 31*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) - ((105*Sqrt
[2]*a^(5/2)*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/d - (4*a^3*(49*A - 37*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
 d*x]]))/(3*a))/(5*a))/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.39

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-240B\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{a}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+168\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{2}\left(A+2B\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\right.}{}$
parts	$\frac{A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-20\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+30\sqrt{a}\sqrt{a\sin\left(\right.}\right.}{15a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/105*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*B*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+168*a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^4-140*a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^2-105*
2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c)
)*a*A+105*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*
d*x+1/2*c))*a*B+210*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(3
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.91

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{4\left(15\,B\cos\left(dx+c\right)^{3}+3\left(7\,A-B\right)\cos\left(dx+c\right)^{2}-\left(7\,A-31\,B\right)\cos\left(dx+c\right)+91\,A-43\,B\right)\sqrt{a\cos\left(dx\right.}}{210\left(ad\cos\left(dx\right.\right.}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
1/210*(4*(15*B*cos(d*x + c)^3 + 3*(7*A - B)*cos(d*x + c)^2 - (7*A - 31*B)*
cos(d*x + c) + 91*A - 43*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 105*sq
rt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1604723 vs. 2(177) = 354.

Time = 34.22 (sec) , antiderivative size = 1604723, normalized size of antiderivative = 7944.17

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
-1/5040*(3*(84*(sqrt(2)*cos(3/2*d*x + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*co
s(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + sqrt(2)*sin(3/2*d*x
 + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*sin(3/2*d*x + 3/2*c)*sin(d*x + c)*sin
(1/2*d*x + 1/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x
+ 1/2*c)^2)*sin(d*x + c))*cos(7/2*d*x + 7/2*c)^3 - 84*((sqrt(2)*cos(d*x +
c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(d*x + c) + sqrt(2))*si
n(3/2*d*x + 3/2*c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*
x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(
1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 +
 sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)*si
n(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*s
in(7/2*d*x + 7/2*c)^3 - 24*((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)
^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (
sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) +
 sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(
2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2
 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x
+ 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x
 + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**4)/(cos(c + d*x) + 1),
x)*b + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3)/(cos(c + d*x) + 1),x)*
a))/a
```

3.101 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15ad}$$

output
```
2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(1/2)/d-4/15*(5*A-7*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*B*cos(
d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/15*(5*A-B)*(a+a*cos(d*x+c))
^(1/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(15(A-B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+(-10A+29B+2(5A-B)\cos(c+dx)+3B\right.}{15d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x
]
```

output
```
(2*Cos[(c + d*x)/2]*(15*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + (-10*A + 29*B
+ 2*(5*A - B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*
d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3462}$$

$$\frac{2\int \frac{\cos(c+dx)(4aB+a(5A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(c+dx)(4aB+a(5A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4aB+a(5A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{a(5A-B)\cos^2(c+dx)+4aB\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A-B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+4aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{a^2(5A-B)-2a^2(5A-7B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2(5A-B)-2a^2(5A-7B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(5A-B)-2a^2(5A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}+\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{15a^2(A-B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a^2(5A-7B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{15a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^2(5A-7B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{-\frac{30a^2(A-B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^2(5A-7B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{15\sqrt{2}a^{3/2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^2(5A-7B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2B\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*B*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + ((2*(5*
A - B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((15*Sqrt[2]*a^(3/2)
*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])
])/d - (4*a^2*(5*A - 7*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a
))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.51

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24B\sqrt{a}\,\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-20\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{2}\,(A+B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+15\ldots\right.}{15a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\ldots\right.}}$
parts	$\frac{A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-4\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+3\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\right)}{3a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}+\frac{B\cos\left(\ldots\right.}{\ldots}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/15*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*B*a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*a^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*(A+B)*sin(1/2*d*x+1/2*c)^2+15*2^(1/2)*ln(4*
(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a*A-15*2^(1
/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a*
B+30*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin(1/2*d*x
+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{4\left(3\,B\cos\left(dx+c\right)^2+\left(5\,A-B\right)\cos\left(dx+c\right)-5\,A+13\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)-\frac{15\,\sqrt{2}((A-\cdots}{\cdots}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
1/30*(4*(3*B*cos(d*x + c)^2 + (5*A - B)*cos(d*x + c) - 5*A + 13*B)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A -
 B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/s
qrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\left(A+B\cos\left(c+dx\right)\right)\cos^2\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*cos(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 927957 vs. 2(138) = 276.

Time = 17.51 (sec) , antiderivative size = 927957, normalized size of antiderivative = 5836.21

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
1/1680*(28*(20*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 8*(cos(d*x + c)
^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 20*cos(
5/2*d*x + 5/2*c)^3*sin(d*x + c) + 2*(15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2
 + 15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
+ c) + 4*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*
x + 3/2*c) - 20*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(5/2*d*x + 5/2*c)^2 + 30*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1))*sin(d*x + c)^2 + 2*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3)/(cos(c + d*x) + 1),
x)*b + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2)/(cos(c + d*x) + 1),x)*
a))/a
```

3.102 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}$$

output
```
-2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/
2))/a^(1/2)/d+2/3*(3*A-2*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*B*(a+a
*cos(d*x+c))^(1/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-3(A-B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6A\sin\left(\frac{1}{2}(c+dx)\right)-4B\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Cos[(c + d*x)/2]*(-3*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 6*A*Sin[(c + d
*x)/2] - 4*B*Sin[(c + d*x)/2]^3))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{aB+a(3A-2B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{aB+a(3A-2B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aB+a(3A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{2a(3A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-3a(A-B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a(3A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-3a(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{6a(A-B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a(3A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2a(3A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + ((-3*Sqrt[2]*Sqrt[a]
*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])
])/d + (2*a*(3*A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.64

method	result
default	$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4B\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+3A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a-6A\sqrt{a\sin\left(\frac{dx}{2}+\right.}}{3a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$
parts	$\frac{A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\right)}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}+\frac{B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}{}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+3*A*ln(4*(a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-6*A*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)-3*B*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)
/cos(1/2*d*x+1/2*c))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{4\left(B\cos\left(dx+c\right)+3\,A-B\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)-\frac{3\,\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2 \cdots}{\cdots}\right.}{\sqrt{a}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

input

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output

```
1/6*(4*(B*cos(d*x + c) + 3*A - B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) -
3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sq
rt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\cos(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

input

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

output

```
Integral((A + B*cos(c + d*x))*cos(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38386 vs. $2(101) = 202$.

Time = 0.83 (sec) , antiderivative size = 38386, normalized size of antiderivative = 325.31

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
1/60*((20*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 8*(cos(d*x + c)^2 +
sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 20*cos(5/2*d
*x + 5/2*c)^3*sin(d*x + c) + 2*(15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15
*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)
+ 4*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - 20*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*co
s(5/2*d*x + 5/2*c)^2 + 30*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1))*sin(d*x + c)^2 + 2*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{2A\left(2\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|1\right)-\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|1\right)\right)\sqrt{\frac{a+a\cos(c+dx)}{2a}}}{d\sqrt{a+a\cos(c+dx)}}\\&\quad+\frac{2B\sin(c+dx)\sqrt{a+a\cos(c+dx)}}{3ad}\\&\quad-\frac{2B\left(4a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|1\right)-3a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|1\right)\right)\sqrt{\frac{a+a\cos(c+dx)}{2a}}}{3a^2d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*A*(2*ellipticE(c/2 + (d*x)/2, 1) - ellipticF(c/2 + (d*x)/2, 1))*((a + a
*cos(c + d*x))/(2*a))^(1/2))/(d*(a + a*cos(c + d*x))^(1/2)) + (2*B*sin(c +
 d*x)*(a + a*cos(c + d*x))^(1/2))/(3*a*d) - (2*B*(4*a^2*ellipticE(c/2 + (d
*x)/2, 1) - 3*a^2*ellipticF(c/2 + (d*x)/2, 1))*((a + a*cos(c + d*x))/(2*a)
)^(1/2))/(3*a^2*d*(a + a*cos(c + d*x))^(1/2))
```

Reduce [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\cos(dx+c)+1}dx\right)b\right)}{a}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(cos(c + d*x) + 1),x)*
a + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2)/(cos(c + d*x) + 1),x)*b))
/a
```

3.103 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(1/2)/d+2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Cos[(c + d*x)/2]*((A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*B*Sin[(c + d*x)
/2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3230}$$

$$(A-B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2(A-B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
+ d*x]])])/(Sqrt[a]*d) + (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(67) = 134$.

Time = 2.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.05

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a+2B\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$
parts	$\frac{A\sqrt{2}\ \mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},1\right)}{d\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ \mathrm{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\ln\left(\frac{4\sqrt{a}\,\sqrt{a}}{c}\right.\right.}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(4*(a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a+2*B*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)-B*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a
)/cos(1/2*d*x+1/2*c))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.73

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{4\sqrt{a\cos(dx+c)+a}B\sin(dx+c) - \frac{\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{\sqrt{a}}}{2(ad\cos(dx+c)+ad)}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(4*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c) - sqrt(2)*((A - B)*a*cos(d*
x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{A + B\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19040 vs. $2(67) = 134$.

Time = 0.47 (sec) , antiderivative size = 19040, normalized size of antiderivative = 244.10

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(6*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (12*sqrt(2)*cos(3/
2*d*x + 3/2*c)^3*sin(d*x + c) - 12*(sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/
2*d*x + 3/2*c)^3 - 8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 + ((3*sqrt(2)*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*
sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(
d*x + c)^2 + 24*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)
+ 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*
cos(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + a\cos(c + dx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 42.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\cos(c+d\,x))}{a}}+2\,B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\cos(c+d\,x))}{a}}-B\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\cos(c+d\,x}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(A*ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2) + 2*B*el
lipticE(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2) - B*ellipticF
(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*cos(c + d
*x))^(1/2))
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}}{\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)b\right)}{a}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(cos(c + d*x) + 1)/(cos(c + d*x) + 1),x)*a + int((sqrt(c
os(c + d*x) + 1)*cos(c + d*x))/(cos(c + d*x) + 1),x)*b))/a
```

3.104 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 91

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*A*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)*(
A-B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(1/2
)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{2\left((A-B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}A\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(-2*((A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin[(c
+ d*x)/2]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3464}$$

$$\frac{A\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx}{a}-(A-B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2(A-B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{A\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{A\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{3252}$$

$$-\frac{2A\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
 - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos
[c + d*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(76) = 152$.

Time = 3.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.99

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(A\sqrt{2}\,\ln\left(-\frac{2\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)+A\sqrt{2}\,\ln\left(\frac{2a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{2}\,\sqrt{a \cdots}}{2\cos\left(\frac{dx}{2}+\frac{c \cdots}{}\right)}\right.\right.}{2\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2 \cdots}}}$
parts	$-\frac{A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)-\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)-\cdots\right.}{\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*2^(1/2)*ln(-2/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+A*2^(1/2)*ln(2/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+2*a))-2*ln(2*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/
2*d*x+1/2*c))*A+2*ln(2*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*
d*x+1/2*c))*B)/a^(1/2)/sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(76) = 152$.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-A\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)}{}\right.}{2ad}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
-1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) - A*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{\sqrt{a(\cos{(c+dx)}+1)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+s\right.}{2\,\sqrt{a}d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/(sqrt(a)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sec(dx+c)}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x))/(cos(c +
d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sec(c + d*x))/(cos(c + d*x) +
 1),x)*a))/a
```

3.105 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 119

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{(A-2B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
-(A-2*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(1/2)/d+2^(1
/2)*(A-B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a
^(1/2)/d+A*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\mathrm{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}(A-2B)\mathrm{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\,\mathrm{s}\right.}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x
]
```

output
```
(Cos[(c + d*x)/2]*(2*(A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*(A - 2*B)
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sec[c + d*x]*Sin[(c + d*x)/2]))/(
d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \ 3463$$

$$\frac{\int -\frac{(a(A-2B)-aA\cos(c+dx))\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{(a(A-2B)-aA\cos(c+dx))\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a(A-2B)-aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}$$

$$\Big\downarrow \; 3464$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{(A-2B)\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-2a(A-B)\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{(A-2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}$$

$$\Big\downarrow \; 3128$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{4a(A-B)\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+(A-2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}$$

$$\Big\downarrow \; 219$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{(A-2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2\sqrt{2}\sqrt{a}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}$$

$$\Big\downarrow \; 3252$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{2a(A-2B)\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{2}\sqrt{a}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}$$

$$\Big\downarrow \text{219}$$

$$\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\sqrt{a}(A-2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2\sqrt{2}\sqrt{a}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
-1/2*((2*Sqrt[a]*(A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/d - (2*Sqrt[2]*Sqrt[a]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(
Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d)/a + (A*Tan[c + d*x])/(d*Sqrt[a + a*
Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(102) = 204$.

Time = 3.52 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.91

method	result
parts	$\frac{A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-2a\left(2\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)-\ln\left(\frac{4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)\right.\right.}{}$
default	Expression too large to display

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
A*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(2*2^(1/2)*ln(4*
(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))-ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))-ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-
2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-
2*a))*a-ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+2*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/
2)/d+1/2*B*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(-
2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+2^(1/2)*ln(2/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)+2*a))-2*ln(2*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos
(1/2*d*x+1/2*c)))/a^(1/2)/sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)
^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(102) = 204$.

Time = 0.11 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.18

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx =$$

$$-\frac{\left((A-2\,B)\cos\left(dx+c\right)^2+(A-2\,B)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-1/4*(((A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(c
os(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) -
4*sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c) + 2*sqrt(2)*((A - B)*a*cos(d*x +
 c)^2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18436 vs. $2(102) = 204$.

Time = 0.39 (sec) , antiderivative size = 18436, normalized size of antiderivative = 154.92

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
1/4*((2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(d*x + c)^4 + (2*sqrt(2)*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2))*sin(d*x + c)^4 + 4*sqrt(2)*cos(1/2*d*x + 1/2*c)...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^2}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2)/(cos(c
 + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sec(c + d*x)**2)/(cos(c +
d*x) + 1),x)*a))/a
```

3.106 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 165

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{(7A-4B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*(7*A-4*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(1/2)/d
-2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/
2))/a^(1/2)/d-1/4*(A-4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*A*sec(d*
x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-8(A-B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(7A-4B)\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\right.}{4d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x
]
```

output
```
(Cos[(c + d*x)/2]*(-8*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(7*A - 4
*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(-A + 4*B + 2*A*Sec
[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \text{3463}$$
$$\frac{\int -\frac{(a(A-4B)-3aA\cos(c+dx))\sec^2(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 27$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{(a(A-4B)-3aA\cos(c+dx))\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-4B)-3aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}$$

$$\Big\downarrow \ 3463$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{\int -\frac{\left(a^2(7A-4B)-a^2(A-4B)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a}$$

$$\Big\downarrow \ 27$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(a^2(7A-4B)-a^2(A-4B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(7A-4B)-a^2(A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3464$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a(7A-4B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-8a^2(A-B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a(7A-4B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-8a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3128$$

$$\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{16a^2(A-B)\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+a(7A-4B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}}{4a}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a(7A-4B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{8\sqrt{2}a^{3/2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

$$\Big\downarrow \; 3252$$

$$\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{2a^2(7A-4B)\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{2}a^{3/2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a(A-4B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^{3/2}(7A-4B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{8\sqrt{2}a^{3/2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) - (-1/2*((2*a
^(3/2)*(7*A - 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (8*Sqrt[2]*a^(3/2)*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Cos[c + d*x]])])/d)/a + (a*(A - 4*B)*Tan[c + d*x])/(d*Sqrt[a +
a*Cos[c + d*x]]))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(140) = 280$.

Time = 3.46 (sec) , antiderivative size = 1152, normalized size of antiderivative = 6.98

method	result	size
parts	Expression too large to display	1152
default	Expression too large to display	1253

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*A*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(8*2^(1/2)*l
n(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))-7*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))-7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-32*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-4*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)+28*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+28*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+8*2^(
1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a
-7*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-7*ln(4/(2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+2*a))*a-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^
2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d-1/2*B*cos(1/2*d*x+1/
2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(2^(1/2)*ln(-2/(2*cos(1/2*d*x+1/
2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(140) = 280$.

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.72

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx =$$

$$-\frac{\left((7\,A-4\,B)\cos\left(dx+c\right)^{3}+(7\,A-4\,B)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\,\sqrt{a\cos(dx+c)+a}\sqrt{}}{\cos(dx+c)^{3}+\cos(dx+}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-1/16*(((7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqr
t(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)) + 4*((A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c) + 8*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*log(
-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a)
 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*
d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(a*(cos(c + d*x) + 1)),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76209 vs. $2(140) = 280$.

Time = 2.68 (sec) , antiderivative size = 76209, normalized size of antiderivative = 461.87

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
-1/16*((4*sqrt(2)*cos(6*d*x + 6*c)^2*sin(3/2*d*x + 3/2*c) + 16*sqrt(2)*cos
(5*d*x + 5*c)^2*sin(3/2*d*x + 3/2*c) + 36*sqrt(2)*cos(4*d*x + 4*c)^2*sin(3
/2*d*x + 3/2*c) + 64*sqrt(2)*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 36*
sqrt(2)*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*sin(6*d*x + 6*
c)^2*sin(3/2*d*x + 3/2*c) + 16*sqrt(2)*sin(5*d*x + 5*c)^2*sin(3/2*d*x + 3/
2*c) + 36*sqrt(2)*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 64*sqrt(2)*sin
(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2*sin(3
/2*d*x + 3/2*c) - 8*(3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 3*s
qrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 2*sqrt(2)*cos(3/2*d*x + 3/2
*c)*sin(d*x + c) + (sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x +
 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*c
os(6*d*x + 6*c) + 2*(sqrt(2)*sin(9/2*d*x + 9/2*c) + 3*sqrt(2)*sin(7/2*d*x
+ 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) - sqrt(2)*sin(3/2*d*x + 3/2*c))*
cos(5*d*x + 5*c) - (3*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(3*d*x + 3*c
) + 3*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(9/2*d*x + 9/2
*c) + 3*(3*sqrt(2)*sin(7/2*d*x + 7/2*c) - 3*sqrt(2)*sin(5/2*d*x + 5/2*c) -
 sqrt(2)*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 3*(4*sqrt(2)*sin(3*d*x +
 3*c) + 3*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x + c))*cos(7/2*d*x +
 7/2*c) - 4*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + sqrt(2)*sin(3/2*d*x + 3/2*c)
)*cos(3*d*x + 3*c) + 3*(3*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{\sqrt{a + a\cos(c + dx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{A+B\cos(c+dx)}{\cos(c+dx)^3\sqrt{a+a\cos(c+dx)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^3}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3)/(cos(c
 + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sec(c + d*x)**3)/(cos(c +
d*x) + 1),x)*a))/a
```

3.107 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 261

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(15A-19B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} + \frac{(63A-67B)\cos^2(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - \frac{(7A-11B)\cos^3(c+dx)\sin(c+dx)}{14ad\sqrt{a+a\cos(c+dx)}} - \frac{(273A-397B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{210a^2d}$$

output
```
-1/4*(15*A-19*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(
1/2))*2^(1/2)/a^(3/2)/d+1/2*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(3/2)+1/105*(651*A-799*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/70*(6
3*A-67*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/14*(7*A-11*
B)*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/210*(273*A-397*B)*
(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/a^2/d
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{105(15A-19B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)-\frac{1}{2}\text{c}}{}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2)
,x]
```

output
```
(105*(15*A - 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - (Cos[(c
+ d*x)/2]^3*(1974*A - 2161*B + 6*(273*A - 277*B)*Cos[c + d*x] + (-84*A + 2
56*B)*Cos[2*(c + d*x)] + 42*A*Cos[3*(c + d*x)] - 18*B*Cos[3*(c + d*x)] + 1
5*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2])/2)/(105*d*(a*(1 + Cos[c + d*x]))^(
3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3456, 27, 3042, 3462, 27, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^3(c+dx)(8a(A-B)-a(7A-11B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\cos^3(c+dx)(8a(A-B)-a(7A-11B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(8a(A-B)-a(7A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3462$$

$$\frac{\frac{2\int -\frac{\cos^2(c+dx)\left(6a^2(7A-11B)-a^2(63A-67B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{\cos^2(c+dx)\left(6a^2(7A-11B)-a^2(63A-67B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6a^2(7A-11B)-a^2(63A-67B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3462$$

$$\frac{-\frac{\frac{2\int -\frac{\cos(c+dx)\left(4a^3(63A-67B)-a^3(273A-397B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{-\frac{\int \frac{\cos(c+dx)\left(4a^3(63A-67B)-a^3(273A-397B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^3(63A-67B)-a^3(273A-397B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3447$$

$$\frac{-\frac{-\frac{\int \frac{4a^3(63A-67B)\cos(c+dx)-a^3(273A-397B)\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{\int \frac{4a^3(63A-67B)\sin\left(c+dx+\frac{\pi}{2}\right)-a^3(273A-397B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3502$$

$$\frac{-\frac{-\frac{\frac{2\int -\frac{a^4(273A-397B)-2a^4(651A-799B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a^2(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7A-11B)\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{a^4(273A-397B)-2a^4(651A-799B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a^2(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7A-\cdots}{\cdots}}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{-\frac{-\frac{-\frac{\int \frac{a^4(273A-397B)-2a^4(651A-799B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2a^2(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}-\frac{2a(7\cdots}{\cdots}}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3230

$$-\frac{-\frac{-\frac{-\frac{105a^4(15A-19B)\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a^4(651A-799B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a^2(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+\cdots}{5d\sqrt{a\cos(c+dx)+a}}}{7a}\cdots}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{-\frac{-\frac{-\frac{105a^4(15A-19B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^4(651A-799B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a^2(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(63A-67B)\sin(c+dx)\cos\cdots}{5d\sqrt{a\cos(c+dx)+a}}}{7a}\cdots}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3128

$$-\frac{-\frac{-\frac{-\frac{210a^4(15A-19B)\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^4(651A-799B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a^2(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(63\cdots}{\cdots}}{7a}\cdots}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{-\frac{2a^2(63A-67B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{2a^2(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}-\frac{\frac{105\sqrt{2}a^{7/2}(15A-19B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^4(6\ldots}{\ldots}}{3a}}{5a}}{7a}\ldots}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (
(-2*a*(7*A - 11*B)*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*
x]]) - ((-2*a^2*(63*A - 67*B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a
*Cos[c + d*x]]) - ((-2*a^2*(273*A - 397*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(3*d) - ((105*Sqrt[2]*a^(7/2)*(15*A - 19*B)*ArcTanh[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (4*a^4*(651*A - 799*B)*S
in[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a))/(7*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.72

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(960B\sqrt{a}\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8-96\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{2}\left(7A+17B\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+224\sqrt{a}\sqrt{a}\right.}{}$
parts	$\frac{A\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-32\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+32\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+75\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.\right.}{20\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\right.}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/420/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(960*B*a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*(7*A+17*B)*sin(1/2*d*x+1/2*c)^6+224*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*(3*A+8*B)*sin(1/2*d*x+1/2*c)^4+3
5*2^(1/2)*(45*A*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*
x+1/2*c))*a-48*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-57*B*ln(4*(a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a+16*B*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2-1575*2^(1/2)*ln(4*(a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a*A+1995*2^(1/2)*l
n(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a*B+178
5*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-1785*B*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+
1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.92

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{105\sqrt{2}\left((15A-19B)\cos(dx+c)^2+2(15A-19B)\cos(dx+c)+15A-19B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-1/840*(105*sqrt(2)*((15*A - 19*B)*cos(d*x + c)^2 + 2*(15*A - 19*B)*cos(d*
x + c) + 15*A - 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*co
s(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)) - 4*(60*B*cos(d*x + c)^4 + 12*(7*A - 3*B)*cos(
d*x + c)^3 - 28*(3*A - 7*B)*cos(d*x + c)^2 + 12*(63*A - 67*B)*cos(d*x + c)
 + 1029*A - 1201*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^4\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^5}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right.}{a^2}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**5)/(cos(c + d*x)**2 +
2*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**4)/(c
os(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.108 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(11A-15B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a+a\cos(c+dx)}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} + \frac{(35A-39B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{30a^2d}$$

output
```
1/4*(11*A-15*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/a^(3/2)/d+1/2*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(3/2)-1/15*(65*A-93*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/10*(5*A-9
*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/30*(35*A-39*B)*(a
+a*cos(d*x+c))^(1/2)*sin(d*x+c)/a^2/d
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{-15(11A-15B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+\text{co}}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2)
,x]
```

output
```
(-15*(11*A - 15*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + Cos[(c +
 d*x)/2]^3*(85*A - 141*B + 3*(20*A - 39*B)*Cos[c + d*x] + (-10*A + 6*B)*Co
s[2*(c + d*x)] - 3*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/(15*d*(a*(1 + Cos
[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3456, 27, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\int \frac{\cos^2(c+dx)(6a(A-B)-a(5A-9B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\cos^2(c+dx)(6a(A-B)-a(5A-9B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6a(A-B)-a(5A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3462

$$\frac{\frac{2\int -\frac{\cos(c+dx)\left(4a^2(5A-9B)-a^2(35A-39B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{\cos(c+dx)\left(4a^2(5A-9B)-a^2(35A-39B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^2(5A-9B)-a^2(35A-39B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3447

$$\frac{-\frac{\int \frac{4a^2(5A-9B)\cos(c+dx)-a^2(35A-39B)\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{4a^2(5A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)-a^2(35A-39B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3502

$$\frac{-\frac{\frac{2\int -\frac{a^3(35A-39B)-2a^3(65A-93B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{-\frac{-\frac{\int \frac{a^3(35A-39B)-2a^3(65A-93B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{-\frac{-\frac{\int \frac{a^3(35A-39B)-2a^3(65A-93B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2a(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3230

$$\frac{-\frac{-\frac{15a^3(11A-15B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a^3(65A-93B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{-\frac{-\frac{15a^3(11A-15B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^3(65A-93B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a(5A-9B)\sin(c+dx)\cos^2(c+a}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 3128

$$-\frac{-\frac{-\frac{\frac{30a^3(11A-15B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^3(65A-93B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a(5A-9B)}{5d\sqrt{}}}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\big\downarrow$ 219

$$-\frac{-\frac{-\frac{\frac{15\sqrt{2}a^{5/2}(11A-15B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^3(65A-93B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a(5A-9B)\sin(c+}{5d\sqrt{a\cos(c}}}{4a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (
(-2*a*(5*A - 9*B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x
]]) - ((-2*a*(35*A - 39*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) -
((15*Sqrt[2]*a^(5/2)*(11*A - 15*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Cos[c + d*x]])])/d - (4*a^3*(65*A - 93*B)*Sin[c + d*x])/(d*Sqr
t[a + a*Cos[c + d*x]]))/(3*a))/(5*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(189) = 378$.

Time = 2.54 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.06

method	result
default	$\frac{\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-96B\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+80\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}A+96B\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{}$
parts	$\frac{A\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(16\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+8\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-33\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.\right.}{12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/60/cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-96*B*a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+80*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4*A+96*B*a^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)*sin(1/2*d*x+1/2*c)^2-165*A*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+a)/cos(1/2*d*x+1/2*c))*sin(1/2*d*x+1/2*c)^2*a-240*B*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+225*B*ln(4*(a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*sin(1/2*d*x+1/2*c)^2*a-135*A*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+165*A*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a+255*B*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)-225*B*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d
*x+1/2*c))*a)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{15\sqrt{2}\left((11\,A-15\,B)\cos\left(dx+c\right)^{2}+2\,(11\,A-15\,B)\cos\left(dx+c\right)+11\,A-15\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-1/120*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^2 + 2*(11*A - 15*B)*cos(d*x
 + c) + 11*A - 15*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)) - 4*(12*B*cos(d*x + c)^3 + 4*(5*A - 3*B)*cos(d*
x + c)^2 - 12*(5*A - 9*B)*cos(d*x + c) - 95*A + 147*B)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorit
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right.\right.}{a^2}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**4)/(cos(c + d*x)**2 +
2*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3)/(c
os(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.109 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 171

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(7A-11B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}}-\frac{(3A-7B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d}$$

output
```
-1/4*(7*A-11*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/a^(3/2)/d+1/2*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(3/2)+1/3*(9*A-13*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/6*(3*A-7*B)
*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/a^2/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{-3(7A-11B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+(15A}{6ad\sqrt{a(1}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2)
,x]
```

output
```
(-3*(7*A - 11*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (15*A - 17*B
 + 12*(A - B)*Cos[c + d*x] + 2*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(6*a*
d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos(c+dx)(4a(A-B)-a(3A-7B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(c+dx)(4a(A-B)-a(3A-7B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a(A-B)-a(3A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3447

$$\frac{\int \frac{4a(A-B)\cos(c+dx)-a(3A-7B)\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)-a(3A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3502

$$\frac{\frac{2\int -\frac{a^2(3A-7B)-2a^2(9A-13B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{a^2(3A-7B)-2a^2(9A-13B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{a^2(3A-7B)-2a^2(9A-13B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3230

$$\frac{-\frac{3a^2(7A-11B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a^2(9A-13B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{3a^2(7A-11B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^2(9A-13B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3128$$

$$\frac{-\frac{-\frac{6a^2(7A-11B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^2(9A-13B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\frac{3\sqrt{2}a^{3/2}(7A-11B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^2(9A-13B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (
(-2*(3*A - 7*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) - ((3*Sqrt[2]
*a^(3/2)*(7*A - 11*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*C
os[c + d*x]])])/d - (4*a^2*(9*A - 13*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c
+ d*x]]))/(3*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(148) = 296$.

Time = 2.52 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.91

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(16B\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-21A\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+33B\ln\left(\cdots\right.}{\cdots}$
parts	$\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-7\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\sqrt{2}\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\sqrt{2}\sqrt{a\sin\left(\cdots\right.}}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}d}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/12/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*B*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-21*A*ln(2*(2*a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*
d*x+1/2*c)^2*a+33*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/co
s(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+24*A*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-40*B*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+3*A*a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-3*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.20

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((7A-11B)\cos(dx+c)^2+2(7A-11B)\cos(dx+c)+7A-11B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\cdots}{\cos\cdots}\right.}{24\left(a^2d\cos\cdots\right.}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-1/24*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^2 + 2*(7*A - 11*B)*cos(d*x + c
) + 7*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) - 4*(4*B*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) +
15*A - 19*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2
+ 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right.\right.}{a^2}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3)/(cos(c + d*x)**2 +
2*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2)/(c
os(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.110 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(3A-7B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*(3*A-7*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2
))*2^(1/2)/a^(3/2)/d-1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+2*B*sin
(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{-\left((3A-7B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\right)+\cos^3\left(\right.}{d(a(1+\cos(c+dx)))^{3/2}\left(-1\right.}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
(-((3*A - 7*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5) + Cos[(c + d*
x)/2]^3*(A - 5*B - 4*B*Cos[c + d*x])*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c +
d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3447, 3042, 3498, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{3a(A-B)+4aB\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a(A-B)+4aB\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a(A-B)+4aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3230

$$\frac{a(3A-7B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx+\frac{8aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{a(3A-7B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{8aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{8aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a(3A-7B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\sqrt{2}\sqrt{a}(3A-7B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{8aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) + ((Sqrt[2]*Sqr
t[a]*(3*A - 7*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
+ d*x]])])/d + (8*a*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(101) = 202$.

Time = 2.40 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.17

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(3A\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-7B\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+8B}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}\right.}}$
parts	$\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(3\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{a}\right)}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}+\frac{B\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-7\right.}{}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-7*B*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(
1/2)*cos(1/2*d*x+1/2*c)^2*a+8*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)*cos(1/2*d*x+1/2*c)^2-A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.60

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\left((3A-7B)\cos(dx+c)^{2}+2(3A-7B)\cos(dx+c)+3A-7B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx}}{\cos(dx+c}\right.}{8\left(a^{2}d\cos(dx+c)^{2}+2a^{2}d\right.}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
-1/8*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3
*A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1)) - 4*(4*B*cos(d*x + c) - A + 5*B)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\cos(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*cos(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2),
x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(a+a\,\cos(c+d\,x)\right)}^{3/2}} \,d x$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right.\right.\right.}{a^2}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(cos(c + d*x)**2 + 2*c
os(c + d*x) + 1),x)*a + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2)/(cos(
c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b))/a**2
```

3.111 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
1/4*(A+3*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))
*2^(1/2)/a^(3/2)/d+1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{(A+3B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}(A-B)\sin(c+dx}{d(a(1+\cos(c+dx)))^{3/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + ((A - B)*Sin[c +
 d*x])/2)/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \ 3229$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3128$$

$$\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(A+3B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2ad}$$

$$\Big\downarrow \ 219$$

$$\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((A + 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(72) = 144$.

Time = 2.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.53

method	result
default	$\frac{\sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(A \ln\left(\frac{4\sqrt{a}\,\sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a+3B \ln\left(\frac{4\sqrt{a}\,\sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a+A\sqrt{}}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right) a^{\frac{5}{2}} \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a \cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, d}$
parts	$\frac{A\sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a+\sqrt{2}\,\sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}\right)}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a \cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, d} + \frac{B\sqrt{a \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(3\sqrt{2}\right.}{4}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(2*(2*a^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x
+1/2*c)^2*a+3*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/
2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+A*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)-B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(72) = 144$.

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.98

$$\int \frac{A + B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left((A+3\,B)\cos\left(dx+c\right)^{2} + 2\,(A+3\,B)\cos\left(dx+c\right) + A + 3\,B\right)\sqrt{a}\log\left(\right.}{8\,\left(a^{2}d\cos\left(\right.\right.}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A + 3*
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqr
t(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) + 4*sqrt(a*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx = \int \frac{A + B\cos{(c + dx)}}{(a(\cos{(c + dx)} + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62254 vs. $2(72) = 144$.

Time = 2.46 (sec) , antiderivative size = 62254, normalized size of antiderivative = 715.56

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/16*((3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 4*sqrt(2)*sin(5/2*d*x + 5/2*
c))*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x
+ 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (3*(sqr
t(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d
*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*cos(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b\right)}{a^2}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(cos(c + d*x) + 1)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
),x)*a + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(cos(c + d*x)**2 + 2*co
s(c + d*x) + 1),x)*b))/a**2
```

3.112 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 127

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{2A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
2*A*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/4*(5*A-
B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/
a^(3/2)/d-1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(5A-B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right) - 4\sqrt{2}A\text{arct}}{d(a(1+\cos(c}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
((5*A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 4*Sqrt[2]*A*ArcT
anh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (A - B)*Cos[(c + d*x)/2
]^3*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/
2]^2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(4aA-a(A-B)\cos(c+dx))\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4aA-a(A-B)\cos(c+dx))\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4aA-a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3464}$$

$$\frac{4A\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-a(5A-B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{4A\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a(5A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{2a(5A-B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+4A\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{4A\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sqrt{2}\sqrt{a}(5A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3252

$$\frac{-\frac{8aA\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{\sqrt{2}\sqrt{a}(5A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{8\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{\sqrt{2}\sqrt{a}(5A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((8*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
- (Sqrt[2]*Sqrt[a]*(5*A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Cos[c + d*x]])])/d)/(4*a^2) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos
[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(106) = 212$.

Time = 4.39 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.87

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2A\sqrt{2}\,\ln\left(-\frac{2\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2A\sqrt{2}\,\ln\left(\frac{2a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\cdots}{2\cos\left(\cdots\right.}\right.\right.}{\cdots}$
parts	$-\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(5\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-4\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right.\right.}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*2^(1/2)*ln(-2/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a+2*A*2^(1/2)*ln(2/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-5*A*ln(2*(a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^2*a+B*ln
(2*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*cos(1/2*
d*x+1/2*c)^2*a-A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+B*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)*2^(1
/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(106) = 212$.

Time = 0.10 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.21

$$\int \frac{(A + B\cos(c+dx))\sec(c+dx)}{(a + a\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\left((5\,A - B)\cos\left(dx+c\right)^2 + 2\,(5\,A - B)\cos\left(dx+c\right) + 5\,A - B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\,\sqrt{2}\,\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^2+2}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
-1/8*(sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*A -
 B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sq
rt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - 4*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos
(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*
sqrt(a*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx = \int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15722 vs. $2(106) = 212$.

Time = 0.82 (sec) , antiderivative size = 15722, normalized size of antiderivative = 123.80

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*si
n(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)
 + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*
d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*
sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c
)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3
*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x +
c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*
x + 3/2*c) + 96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3
*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sec(d}{\cos(dx+c)^2+2\cos(dx+}\right)\right)}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x))/(cos(c +
d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sec(c + d
*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.113 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 170

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(3A-2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d}+\frac{(9A-5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}$$

output
```
-(3*A-2*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/
4*(9*A-5*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))
*2^(1/2)/a^(3/2)/d-1/2*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/2*(3*A-
B)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.83

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(2(9A-5B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sqrt{2}(3\ldots}{\ldots}\right.}{2d(a(1+\ldots}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2)
,x]
```

output
```
(Cos[(c + d*x)/2]^3*(2*(9*A - 5*B)*ArcTanh[Sin[(c + d*x)/2]] + (4*Sqrt[2]*
(3*A - 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 + 2*(-3*A
 + B - 2*A*Sec[c + d*x])*Sin[(c + d*x)/2])/(-1 + Sin[(c + d*x)/2]^2)))/(2*
d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3457, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 3457$$

$$\frac{\int \frac{(2a(3A-B)-3a(A-B)\cos(c+dx))\sec^2(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(2a(3A-B)-3a(A-B)\cos(c+dx))\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a(3A-B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{\int -\frac{\left(2a^2(3A-2B)-a^2(3A-B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(2a^2(3A-2B)-a^2(3A-B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{2a^2(3A-2B)-a^2(3A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3464}$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a(3A-2B)\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-a^2(9A-5B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a(3A-2B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a^2(9A-5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(9A-5B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+2a(3A-2B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a(3A-2B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sqrt{2}a^{3/2}(9A-5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3252$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{4a^2(3A-2B)\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{\sqrt{2}a^{3/2}(9A-5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{2a(3A-B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{4a^{3/2}(3A-2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{\sqrt{2}a^{3/2}(9A-5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) + (-(((4*a^(3/2
)*(3*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
- (Sqrt[2]*a^(3/2)*(9*A - 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Cos[c + d*x]])])/d)/a) + (2*a*(3*A - B)*Tan[c + d*x])/(d*Sqrt[a +
a*Cos[c + d*x]]))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(145) = 290$.

Time = 3.60 (sec) , antiderivative size = 860, normalized size of antiderivative = 5.06

method	result	size
parts	Expression too large to display	860
default	Expression too large to display	1051

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/2*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-12*l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a-12*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-9*2^(1/2)
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*c
os(1/2*d*x+1/2*c)^2*a+6*cos(1/2*d*x+1/2*c)^2*2^(1/2)*a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+6*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/
2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*
d*x+1/2*c)^2*a+6*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+
1/2*c)^2*a-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+1/4*B/a^(5/2)/cos(1/2*
d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*2^(1/2)*ln(-2/(2*cos(1/2*d*x+
1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a+2*2^(1/2)*ln(2/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a-5*ln(2*(a^(1/2)*(a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(145) = 290$.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.99

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\left((9\,A - 5\,B)\cos\left(dx + c\right)^3 + 2\,(9\,A - 5\,B)\cos\left(dx + c\right)^2 + (9\,A - 5\,B)\cos\left(dx + c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+\cdots}{}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-1/8*(sqrt(2)*((9*A - 5*B)*cos(d*x + c)^3 + 2*(9*A - 5*B)*cos(d*x + c)^2 +
 (9*A - 5*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*((3*A - 2*B)*cos(d*x + c)^3 + 2*(3*A
- 2*B)*cos(d*x + c)^2 + (3*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x +
 c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((3*A
- B)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47933 vs. $2(145) = 290$.

Time = 1.91 (sec) , antiderivative size = 47933, normalized size of antiderivative = 281.96

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
1/4*(128*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^3 + 1152*cos(3/2*d*x + 3/2*
c)*sin(2*d*x + 2*c)^3 - 128*cos(3*d*x + 3*c)^3*sin(3/2*d*x + 3/2*c) - 1152
*cos(2*d*x + 2*c)^3*sin(3/2*d*x + 3/2*c) + 32*(4*cos(3/2*d*x + 3/2*c)*sin(
2*d*x + 2*c) - 9*(3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 28*cos(2*d*x
+ 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(3*d
*x + 3*c)^2 - 96*(11*(3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 9*cos(3/2
*d*x + 3/2*c)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + 32*(28*cos(3/2*d*x + 3/2*
c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 4*cos(3*
d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
+ 27*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x + 3*c)^2 - 288*((3*cos(d
*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 4*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c
) - 11*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2*d*x + 2*c)^2 - 32*(cos(3*d
*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c)
*sin(3/2*d*x + 3/2*c) + 9*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*
d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 9*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2
*c) + 18*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + 2*((3*cos(d*
x + c) + 1)*sin(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
)*cos(3*d*x + 3*c) + 6*(sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + sin(3/2*d*
x + 3/2*c)*sin(d*x + c))*sin(3*d*x + 3*c) + (9*cos(d*x + c)^2 + 9*sin(d*x
+ c)^2 + 6*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c))*cos(5*d*x + 5*c) - 9...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^2\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sec(\cdots}{\cos(dx+c)^2+2\cos(d\cdots}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2)/(cos(c
 + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sec(c
+ d*x)**2)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.114 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 221

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(19A-12B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*(19*A-12*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)
/d-1/4*(13*A-9*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^
(1/2))*2^(1/2)/a^(3/2)/d-1/4*(7*A-6*B)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/
2)-1/2*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/2*(2*A-B)*se
c(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(4(13A-9B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2)
,x]
```

output
```
(Cos[(c + d*x)/2]^3*Sec[c + d*x]^2*(4*(13*A - 9*B)*ArcTanh[Sin[(c + d*x)/2
]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2 - 2*Sqrt[2]*(19*A - 12*B)*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])
^2 + 4*(3*(A - 2*B) + (6*A - 8*B)*Cos[c + d*x] + (7*A - 6*B)*Cos[2*(c + d*
x)])*Sin[(c + d*x)/2]))/(16*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c +
d*x)/2]^2))
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(4a(2A-B)-5a(A-B)\cos(c+dx))\sec^3(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{(4a(2A-B)-5a(A-B)\cos(c+dx))\sec^3(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{4a(2A-B)-5a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3463

$$\frac{\frac{\int -\frac{2\left(a^2(7A-6B)-3a^2(2A-B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(a^2(7A-6B)-3a^2(2A-B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(7A-6B)-3a^2(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3463

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{\int -\frac{\left(a^3(19A-12B)-a^3(7A-6B)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{\left(a^3(19A-12B)-a^3(7A-6B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(19A-12B)-a^3(7A-6B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3464

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(19A-12B)\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-2a^3(13A-9B)\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(19A-12B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a^3(13A-9B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 3128

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(19A-12B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4a^3(13A-9B)\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\downarrow$ 219

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(19A-12B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2\sqrt{2}a^{5/2}(13A-9B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+}}\right.}{d}}{2a}}{a}}{4a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3252

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{2a^3(19A-12B)\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{2}a^{5/2}(13A-9B)\text{arctanh}}{d}}{2a}}{a}}{4a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{2a(2A-B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a^2(7A-6B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^{5/2}(19A-12B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2\sqrt{2}a^{5/2}(13A-9B)\text{arctanh}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{a\cos}}\right.}{d}}{2a}}{a}}{4a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*((A - B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) +
((2*a*(2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) -
(-1/2*((2*a^(5/2)*(19*A - 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/d - (2*Sqrt[2]*a^(5/2)*(13*A - 9*B)*ArcTanh[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d)/a + (a^2*(7*A - 6*B)*Tan[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/a)/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1371 vs. $2(190) = 380$.

Time = 3.60 (sec) , antiderivative size = 1372, normalized size of antiderivative = 6.21

method	result	size
parts	Expression too large to display	1372
default	Expression too large to display	1540

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(104*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-76
*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^6*a-76*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*a-104*2^(
1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c
))*a*cos(1/2*d*x+1/2*c)^4+28*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)*cos(1/2*d*x+1/2*c)^4+76*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*
cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*co
s(1/2*d*x+1/2*c)^4*a+76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1
/2*d*x+1/2*c)^4*a+26*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^2*a-22*cos(1/2*d*x+1/2*c)^2*
2^(1/2)*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-19*ln(-4/(2*cos(1/2*d*x+1/2
*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a-19*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}\left((13A-9B)\cos(dx+c)^4+2(13A-9B)\cos(dx+c)^3+(13A-9B)\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a}{}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-1/16*(2*sqrt(2)*((13*A - 9*B)*cos(d*x + c)^4 + 2*(13*A - 9*B)*cos(d*x + c
)^3 + (13*A - 9*B)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt
(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B)*cos(d*x + c)^4 +
 2*(19*A - 12*B)*cos(d*x + c)^3 + (19*A - 12*B)*cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^
2)) + 4*((7*A - 6*B)*cos(d*x + c)^2 + (3*A - 4*B)*cos(d*x + c) - 2*A)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x
 + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx = \int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(3/2
), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sec(}{\cos(dx+c)^2+2\cos(d}\right.\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x)
```

output

```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3)/(cos(c
 + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sec(c
+ d*x)**3)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.115 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 261

$$\begin{aligned}\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx &= \frac{(163A-283B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\ &+ \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\ &- \frac{(985A-1729B)\sin(c+dx)}{120a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(85A-157B)\cos^2(c+dx)\sin(c+dx)}{80a^2d\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{(475A-787B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{240a^3d}\end{aligned}$$

output
```
1/32*(163*A-283*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))
^(1/2))*2^(1/2)/a^(5/2)/d+1/4*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(5/2)+1/16*(13*A-21*B)*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(
3/2)-1/120*(985*A-1729*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/80*(85
*A-157*B)*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/240*(475*
A-787*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/a^3/d
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.53

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{30(163A-283B)\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(\cdot$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2)
,x]
```

output
```
(30*(163*A - 283*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-1895*
A + 3491*B - 5*(479*A - 887*B)*Cos[c + d*x] + (-400*A + 832*B)*Cos[2*(c +
d*x)] + 40*A*Cos[3*(c + d*x)] - 40*B*Cos[3*(c + d*x)] + 12*B*Cos[4*(c + d*
x)])*Tan[(c + d*x)/2])/(240*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^3(c+dx)(8a(A-B)-a(5A-13B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^3(c+dx)(8a(A-B)-a(5A-13B)\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(8a(A-B)-a(5A-13B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3456

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(6a^2(13A-21B)-a^2(85A-157B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(6a^2(13A-21B)-a^2(85A-157B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6a^2(13A-21B)-a^2(85A-157B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3462

$$\frac{\frac{\frac{2\int -\frac{\cos(c+dx)\left(4a^3(85A-157B)-a^3(475A-787B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{\int \frac{\cos(c+dx)\left(4a^3(85A-157B)-a^3(475A-787B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^3(85A-157B)-a^3(475A-787B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\cdots}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3447

$$\frac{\frac{-\frac{\int \frac{4a^3(85A-157B)\cos(c+dx)-a^3(475A-787B)\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\frac{\int \frac{4a^3(85A-157B)\sin\left(c+dx+\frac{\pi}{2}\right)-a^3(475A-787B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13A-21B)\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\cdots}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3502

$$\frac{\frac{-\frac{\frac{2\int -\frac{a^4(475A-787B)-2a^4(985A-1729B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a^2(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13A-\cdots}{2\cdots}}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{-\frac{\int \frac{a^4(475A-787B)-2a^4(985A-1729B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a^2(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13A-\cdots}{2a\cdots}}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{-\frac{-\frac{\int \frac{a^4(475A-787B)-2a^4(985A-1729B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2a^2(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(13\cdots}{\cdots}}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3230

$$\frac{-\frac{-\frac{-\frac{15a^4(163A-283B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a^4(985A-1729B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a^2(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c\cdots}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}\cdots}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{-\frac{-\frac{15a^4(163A-283B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^4(985A-1729B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a^2(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(85A-157B)\sin(c+dx)\,co\cdots}{5d\sqrt{a\cos(c+dx)+a}}}{4a^2}\cdots}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3128

$$\frac{-\frac{-\frac{-\frac{30a^4(163A-283B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^4(985A-1729B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a^2(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a^2(85\cdots}{\cdots}}{4a^2}\cdots}{8a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{2a^2(85A-157B)\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{2a^2(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}-\frac{\frac{15\sqrt{2}a^{7/2}(163A-283B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^4(9}{}}{3a}}{5a}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (
(a*(13*A - 21*B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3
/2)) + ((-2*a^2*(85*A - 157*B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a +
a*Cos[c + d*x]]) - ((-2*a^2*(475*A - 787*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c
 + d*x])/(3*d) - ((15*Sqrt[2]*a^(7/2)*(163*A - 283*B)*ArcTanh[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (4*a^4*(985*A - 1729*B
)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a))/(4*a^2))/(8*a^
2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(230) = 460$.

Time = 2.87 (sec), antiderivative size = 467, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(768B\sqrt{a}\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+640A\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-2176B\sqrt{2}\sqrt{a\sin\left(\right.}\right.}{}$
parts	$\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(128\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+489\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-512\sqrt{2}\sqrt{}\right.}{96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos}}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/480*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(768*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^8+640*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-2176*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+2445*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a
-4245*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/
2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-2560*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+5248*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-435*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+555*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+30*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-30*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(
1/2*d*x+1/2*c)^3/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)
/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{15\sqrt{2}\big((163\,A-283\,B)\cos{(dx+c)}^3+3\,(163\,A-283\,B)\cos{(dx+c)}^2+3\,(163\,A-283\,B)\cos{(dx+c)}}{}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-1/960*(15*sqrt(2)*((163*A - 283*B)*cos(d*x + c)^3 + 3*(163*A - 283*B)*cos
(d*x + c)^2 + 3*(163*A - 283*B)*cos(d*x + c) + 163*A - 283*B)*sqrt(a)*log(
-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x +
c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(9
6*B*cos(d*x + c)^4 + 160*(A - B)*cos(d*x + c)^3 - 32*(25*A - 49*B)*cos(d*x
 + c)^2 - 5*(503*A - 911*B)*cos(d*x + c) - 1495*A + 2671*B)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^4\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^5}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx}}{\cos(dx+c)^3+3\,c}\right.\right.}{a^3}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**5)/(cos(c + d*x)**3 +
3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)
*cos(c + d*x)**4)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) +
1),x)*a))/a**3
```

3.116 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(75A-163B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(93A-197B)\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(39A-95B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{48a^3d}$$

output

```
-1/32*(75*A-163*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))
^(1/2))*2^(1/2)/a^(5/2)/d+1/4*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(5/2)+1/16*(9*A-17*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3
/2)+1/24*(93*A-197*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/48*(39*A-9
5*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/a^3/d
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.54

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{-6(75A-163B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(1$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2)
,x]
```

output
```
(-6*(75*A - 163*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (195*A -
 379*B + (255*A - 479*B)*Cos[c + d*x] + 16*(3*A - 5*B)*Cos[2*(c + d*x)] +
8*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(48*a*d*(a*(1 + Cos[c + d*x]))^(3/
2))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^2(c+dx)(6a(A-B)-a(3A-11B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^2(c+dx)(6a(A-B)-a(3A-11B)\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6a(A-B)-a(3A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3456

$$\frac{\frac{\int \frac{\cos(c+dx)\left(4a^2(9A-17B)-a^2(39A-95B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\cos(c+dx)\left(4a^2(9A-17B)-a^2(39A-95B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^2(9A-17B)-a^2(39A-95B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3447

$$\frac{\frac{\int \frac{4a^2(9A-17B)\cos(c+dx)-a^2(39A-95B)\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{4a^2(9A-17B)\sin\left(c+dx+\frac{\pi}{2}\right)-a^2(39A-95B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3502

$$\frac{\frac{\frac{2\int -\frac{a^3(39A-95B)-2a^3(93A-197B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{\int \frac{a^3(39A-95B)-2a^3(93A-197B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2a(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\frac{\int \frac{a^3(39A-95B)-2a^3(93A-197B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2a(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3230

$$\frac{\frac{-\frac{3a^3(75A-163B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a^3(93A-197B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{-\frac{3a^3(75A-163B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^3(93A-197B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3128$$

$$\frac{-\frac{-\frac{-\frac{6a^3(75A-163B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^3(93A-197B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{-\frac{\frac{3\sqrt{2}a^{5/2}(75A-163B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^3(93A-197B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2a(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{a(9A-17B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (
(a*(9*A - 17*B)*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/
2)) + ((-2*a*(39*A - 95*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) -
((3*Sqrt[2]*a^(5/2)*(75*A - 163*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Cos[c + d*x]])])/d - (4*a^3*(93*A - 197*B)*Sin[c + d*x])/(d*Sq
rt[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(189) = 378$.

Time = 2.50 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.84

method	result
default	$-\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-128B\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+225A\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-489}{}$
parts	$-\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(75\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4-64\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4-21\cos\left(\frac{dx}{2}\right.}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}d}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/96*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-128*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+225*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-
489*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*
c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-192*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+512*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-63*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+87*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+6*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)-6*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+
1/2*c)^3/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.18

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((75A-163B)\cos(dx+c)^3+3(75A-163B)\cos(dx+c)^2+3(75A-163B)\cos(dx+c)+75\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-1/192*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^3 + 3*(75*A - 163*B)*cos(d*
x + c)^2 + 3*(75*A - 163*B)*cos(d*x + c) + 75*A - 163*B)*sqrt(a)*log(-(a*c
os(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) -
2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*c
os(d*x + c)^3 + 32*(3*A - 5*B)*cos(d*x + c)^2 + (255*A - 503*B)*cos(d*x +
c) + 147*A - 299*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^4}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx}{\cos(dx+c)^3+3\,c}\right.\right.}{a^3}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**4)/(cos(c + d*x)**3 +
3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)
*cos(c + d*x)**3)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) +
1),x)*a))/a**3
```

3.117 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 169

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(19A-75B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(A-9B)\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/32*(19*A-75*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(
1/2))*2^(1/2)/a^(5/2)/d+1/4*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(5/2)-1/16*(5*A-13*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-1/4*(A-9*B)
*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.59

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{2(19A-75B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(-9A}{16ad(a(}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2)
,x]
```

output
```
(2*(19*A - 75*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-9*A + 65
*B + (-13*A + 85*B)*Cos[c + d*x] + 16*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2]
)/(16*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3456, 27, 3042, 3447, 3042, 3498, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\int \frac{\cos(c+dx)(4a(A-B)-a(A-9B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\cos(c+dx)(4a(A-B)-a(A-9B)\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a(A-B)-a(A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{4a(A-B)\cos(c+dx)-a(A-9B)\cos^2(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)-a(A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3498}$$

$$\frac{-\frac{\int -\frac{3a^2(5A-13B)-4a^2(A-9B)\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^2(5A-13B)-4a^2(A-9B)\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3a^2(5A-13B)-4a^2(A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{a^2(19A-75B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{8a^2(A-9B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(19A-75B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8a^2(A-9B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{3128}$$

$$\frac{\frac{-\frac{2a^2(19A-75B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{8a^2(A-9B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{\frac{\sqrt{2}a^{3/2}(19A-75B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{8a^2(A-9B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (
-1/2*(a*(5*A - 13*B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) + ((Sqrt
[2]*a^(3/2)*(19*A - 75*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Cos[c + d*x]])])/d - (8*a^2*(A - 9*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c
 + d*x]]))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(146) = 292.

Time = 2.44 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.93

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(19A\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-75B\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+}{}$
parts	$\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(19\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\sqrt{2}\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+2\sqrt{2}\sqrt{a\sin(}\right.}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(19*A*ln(2*(2*a^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1
/2*d*x+1/2*c)^4*a-75*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+64*B*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-13*A*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+21*B*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.40

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((19\,A-75\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A-75\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A-75\,B)\cos\left(dx+c\right)+19\,A-\right.}{64}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-1/64*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c
)^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos
(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*cos(d*x
+ c)^2 - (13*A - 85*B)*cos(d*x + c) - 9*A + 49*B)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
s(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx}}{\cos(dx+c)^3+3\,c}\right.\right.}{a^3}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3)/(cos(c + d*x)**3 +
3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)
*cos(c + d*x)**2)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) +
1),x)*a))/a**3
```

3.118 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 126

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(5A+19B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
1/32*(5*A+19*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/a^(5/2)/d-1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*
(5*A-13*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+19B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(A-9}{16ad(a(1+\cos(c+dx)}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
(2*(5*A + 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A - 9*B +
(5*A - 13*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x])
)^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3447, 3042, 3498, 27, 3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{5a(A-B)+8aB\cos(c+dx)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a(A-B)+8aB\cos(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a(A-B)+8aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3229

$$\frac{\frac{1}{4}(5A+19B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx+\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{4}(5A+19B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{(5A+19B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2d}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{(5A+19B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}+\frac{a(5A-13B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/4*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) + (((5*A + 19*B
)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*Sqrt[a]*d) + (a*(5*A - 13*B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(107) = 214$.

Time = 2.45 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.32

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(5A\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+19B\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+5}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}$
parts	$\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(5\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^4+5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\sqrt{2}\sqrt{a}\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-2\sqrt{2}\sqrt{a\sin\left(\frac{dx}{2}\right.}\right.}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, d}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*ln(2*(2*a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/
2*d*x+1/2*c)^4*a+19*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/
cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+5*A*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-13*B*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(107) = 214$.

Time = 0.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.77

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left((5\,A+19\,B)\cos\left(dx+c\right)^3+3\,(5\,A+19\,B)\cos\left(dx+c\right)^2+3\right.}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
1/64*(sqrt(2)*((5*A + 19*B)*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^2
 + 3*(5*A + 19*B)*cos(d*x + c) + 5*A + 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^
2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x
+ c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A - 13*B)*cos(d
*x + c) + A - 9*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx\text{-}}}{\cos(dx+c)^3+3\,\text{co}}\right.\right.}{a^3}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(cos(c + d*x)**3 + 3*c
os(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a + int((sqrt(cos(c + d*x) + 1)*co
s(c + d*x)**2)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),
x)*b))/a**3
```

3.119 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(3A+5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
1/32*(3*A+5*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/
2))*2^(1/2)/a^(5/2)/d+1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(
3*A+5*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{4(3A+5B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+(7A+B+(3A+}{16d(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(4*(3*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (7*A + B + (
3*A + 5*B)*Cos[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a\cos(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{(3A+5B)\int \frac{1}{(\cos(c+dx)a+a)^{3/2}}dx}{8a}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(3A+5B)\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{(3A+5B)\left(\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3A+5B)\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*A + 5*B)*(Ar
cTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2
]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(107) = 214$.

Time = 2.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.32

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(3A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+5B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+3A\cdots}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\cdots}$
parts	$\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(3\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\sqrt{2}\,\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+2\sqrt{2}\,\sqrt{a\sin\left(\cdots\right.}\cdots}{32a^{\frac{7}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*ln(2*(2*a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/
2*d*x+1/2*c)^4*a+5*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/c
os(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+3*A*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+5*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))
/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(107) = 214$.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.77

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left((3\,A+5\,B)\cos\left(dx+c\right)^3+3\,(3\,A+5\,B)\cos\left(dx+c\right)^2+3\,(3\,A+5\,B)\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 +
 3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 -
2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c)
 - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A + 5*B)*cos(d*x +
c) + 7*A + B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx}{\cos(dx+c)^3+3\cos(dx+c)^2+3\,c}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(cos(c + d*x) + 1)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2
+ 3*cos(c + d*x) + 1),x)*a + int((sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b))/a**3
```

3.120 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 164

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{2A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
2*A*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/32*(43*
A-3*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1
/2)/a^(5/2)/d-1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(11*A-3*B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.77

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{-2(43A-3B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+64\sqrt{2}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
(-2*(43*A - 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 64*Sqrt[2]
*A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-15*A + 7*B + (
-11*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x])
)^(3/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(8aA-3a(A-B)\cos(c+dx))\sec(c+dx)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(8aA-3a(A-B)\cos(c+dx))\sec(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{8aA-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(32a^2A-a^2(11A-3B)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(32a^2A-a^2(11A-3B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{32a^2A-a^2(11A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3464}$$

$$\frac{\frac{32aA\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-a^2(43A-3B)\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{32aA\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a^2(43A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{\frac{2a^2(43A-3B)\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+32aA\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{32aA\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sqrt{2}a^{3/2}(43A-3B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3252$$

$$\frac{\frac{-\frac{64a^2A\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{\sqrt{2}a^{3/2}(43A-3B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{64a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{\sqrt{2}a^{3/2}(43A-3B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{a(11A-3B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input

```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
-1/4*((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) + (((64*a^(3/2)
*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (Sqrt[2]*
a^(3/2)*(43*A - 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Co
s[c + d*x]])])/d)/(4*a^2) - (a*(11*A - 3*B)*Sin[c + d*x])/(2*d*(a + a*Cos[
c + d*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(139) = 278$.

Time = 4.88 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.62

method	result
default	$\frac{\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(16A\sqrt{2}\,\ln\left(-\frac{2\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-2a\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right)a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+16A\sqrt{2}\,\ln\left(\frac{2a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\cdots}{2\cos\cdots}\right.\right.}{\cdots}$
parts	$-\frac{A\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(43\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-32\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{a}-\cdots\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right.\right.}{32a^{\frac{7}{2}}\cos\cdots}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*A*2^(
1/2)*ln(-2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a*cos(1/2*d*x+1/2*c)^4+
16*A*2^(1/2)*ln(2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a*cos(1/2*d*x+1/
2*c)^4-43*A*ln(2*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/
2*c))*a*cos(1/2*d*x+1/2*c)^4+3*B*ln(2*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-11*A*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+3*B*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+
2*B*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)*2^(1/2)/(a*
cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(139) = 278$.

Time = 0.11 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.07

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}\big((43\,A-3\,B)\cos\left(dx+c\right)^3+3\,(43\,A-3\,B)\cos\left(dx+c\right)^2+3\,(43\,A-3\,B)\cos\left(dx+c\right)+43\,A-3\,B}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
-1/64*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^
2 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)
^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x
 + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 32*(A*cos(d*x + c)^3
 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c)
- 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((11*A - 3
*B)*cos(d*x + c) + 15*A - 7*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3
*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos\left(c+dx\right))\sec\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a*(cos(c + d*x) + 1))**(5/2),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84333 vs. $2(139) = 278$.

Time = 14.56 (sec) , antiderivative size = 84333, normalized size of antiderivative = 514.23

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + a\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c
) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(
5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x +
 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*s
in(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(
5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x
+ 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d
*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*
c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*
sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*
d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5
/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x +
5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx}}{\cos(dx+c)^3+3\cos}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x))/(cos(c +
d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c +
 d*x) + 1)*sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c +
d*x) + 1),x)*a))/a**3
```

3.121 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 207

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(5A-2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A-11B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

output

```
-(5*A-2*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/
32*(115*A-43*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/a^(5/2)/d-1/4*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*
(15*A-7*B)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/16*(35*A-11*B)*tan(d*x+
c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{8(115A-43B)\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)-128}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2)
,x]
```

output
```
(8*(115*A - 43*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 128*Sqrt[
2]*(5*A - 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (67*
A - 11*B + 10*(11*A - 3*B)*Cos[c + d*x] + (35*A - 11*B)*Cos[2*(c + d*x)])*
Tan[c + d*x])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{(2a(5A-B)-5a(A-B)\cos(c+dx))\sec^2(c+dx)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(2a(5A-B)-5a(A-B)\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{2a(5A-B)-5a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3457

$$\frac{\frac{\int \frac{\left(2a^2(35A-11B)-3a^2(15A-7B)\cos(c+dx)\right)\sec^2(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(2a^2(35A-11B)-3a^2(15A-7B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \frac{2a^2(35A-11B)-3a^2(15A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3463

$$\frac{\frac{\frac{\int -\frac{\left(16a^3(5A-2B)-a^3(35A-11B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(16a^3(5A-2B)-a^3(35A-11B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{16a^3(5A-2B)-a^3(35A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3464}$$

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2(5A-2B)\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-a^3(115A-43B)\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2(5A-2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a^3(115A-43B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2(5A-2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^3(115A-43B)\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2(5A-2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sqrt{2}a^{5/2}(115A-43B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{a(15A-7B)\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{32a^3(5A-2B)\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{\sqrt{2}a^{5/2}(115A-43B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{a(15A}{2d(a\,c}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{2a^2(35A-11B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{32a^{5/2}(5A-2B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{\sqrt{2}a^{5/2}(115A-43B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{a(15A-7B)\tan(}{2d(a\cos(c+dx)+}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/4*((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) + (-1/2*(a*(15*
A - 7*B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) + (-(((32*a^(5/2)*(5
*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (S
qrt[2]*a^(5/2)*(115*A - 43*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Cos[c + d*x]])])/d)/a) + (2*a^2*(35*A - 11*B)*Tan[c + d*x])/(d*Sqrt
[a + a*Cos[c + d*x]]))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(178) = 356$.

Time = 3.45 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.47

method	result	size
parts	Expression too large to display	925
default	Expression too large to display	1122

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/16*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-16
0*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^6*a-160
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*a-115*2
^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2
*c))*a*cos(1/2*d*x+1/2*c)^4+70*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)*cos(1/2*d*x+1/2*c)^4+80*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*
cos(1/2*d*x+1/2*c)^4*a+80*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos
(1/2*d*x+1/2*c)^4*a-15*cos(1/2*d*x+1/2*c)^2*2^(1/2)*a^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)
/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d+1/32*B/a^(7/
2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*2^(1/2)*ln(-2/(
2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a+16*2^(1/2)*l
n(2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(178) = 356$.

Time = 0.14 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.95

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + a\cos(c + dx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((115\,A - 43\,B)\cos\left(dx + c\right)^4 + 3\,(115\,A - 43\,B)\cos\left(dx + c\right)^3 + 3\,(115\,A - 43\,B)\cos\left(dx + c\right)^2 + (1\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-1/64*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^4 + 3*(115*A - 43*B)*cos(d*x +
 c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + (115*A - 43*B)*cos(d*x + c))*sqr
t(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
in(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) + 16*((5*A - 2*B)*cos(d*x + c)^4 + 3*(5*A - 2*B)*cos(d*x + c)^3 + 3*(5*
A - 2*B)*cos(d*x + c)^2 + (5*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
 + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((35
*A - 11*B)*cos(d*x + c)^2 + 5*(11*A - 3*B)*cos(d*x + c) + 16*A)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec^2{(c+dx)}}{(a(\cos{(c+dx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(5/2
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)^2\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx}}{\cos(dx+c)^3+3\cos}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**2)/(cos(c
 + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(
c + d*x) + 1)*sec(c + d*x)**2)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*co
s(c + d*x) + 1),x)*a))/a**3
```

3.122 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 264

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(39A-20B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\sec(c+dx)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(31A-15B)\sec(c+dx)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*(39*A-20*B)*arctanh(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)
/d-1/32*(219*A-115*B)*arctanh(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)/a^(5/2)/d-7/16*(9*A-5*B)*tan(d*x+c)/a^2/d/(a+a*cos(d*x+
c))^(1/2)-1/4*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(1
9*A-11*B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/16*(31*A-15*B
)*sec(d*x+c)*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 870 vs. $2(264) = 528$.

Time = 6.21 (sec) , antiderivative size = 870, normalized size of antiderivative = 3.30

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2)
,x]
```

output
```
(-219*A*ArcTanh[Sin[c/2 + (d*x)/2]]*Cos[c/2 + (d*x)/2]^5)/(4*d*(a*(1 + Cos
[c + d*x]))^(5/2)) + (115*B*ArcTanh[Sin[c/2 + (d*x)/2]]*Cos[c/2 + (d*x)/2]
^5)/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) + (39*Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin
[c/2 + (d*x)/2]]*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)) -
(20*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[c/2 + (d*x)/2]]*Cos[c/2 + (d*x)/2]^5)/(d
*(a*(1 + Cos[c + d*x]))^(5/2)) - (A*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos
[c + d*x]))^(5/2)*(1 - Sin[c/2 + (d*x)/2])^2) + (B*Cos[c/2 + (d*x)/2]^5)/(
8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - Sin[c/2 + (d*x)/2])^2) - (27*A*Cos[c
/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - Sin[c/2 + (d*x)/2]
)) + (19*B*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - Si
n[c/2 + (d*x)/2])) + (A*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^
(5/2)*(1 + Sin[c/2 + (d*x)/2])^2) - (B*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 +
Cos[c + d*x]))^(5/2)*(1 + Sin[c/2 + (d*x)/2])^2) + (27*A*Cos[c/2 + (d*x)/2
]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 + Sin[c/2 + (d*x)/2])) - (19*B*C
os[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 + Sin[c/2 + (d*x
)/2])) + (4*A*Cos[c/2 + (d*x)/2]^5*Sin[c/2 + (d*x)/2])/(d*(a*(1 + Cos[c +
d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2) - (18*A*Cos[c/2 + (d*x)/2]^5*
Sin[c/2 + (d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)
/2]^2)) + (8*B*Cos[c/2 + (d*x)/2]^5*Sin[c/2 + (d*x)/2])/(d*(a*(1 + Cos[c +
 d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2))
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.576, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(4a(3A-B)-7a(A-B)\cos(c+dx))\sec^3(c+dx)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4a(3A-B)-7a(A-B)\cos(c+dx))\sec^3(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a(3A-B)-7a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(4a^2(31A-15B)-5a^2(19A-11B)\cos(c+dx)\right)\sec^3(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(19A-11B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(4a^2(31A-15B)-5a^2(19A-11B)\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \frac{4a^2(31A-15B)-5a^2(19A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3463

$$\frac{\frac{\frac{\int -\frac{2\left(14a^3(9A-5B)-3a^3(31A-15B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(14a^3(9A-5B)-3a^3(31A-15B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{14a^3(9A-5B)-3a^3(31A-15B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3463

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{\int -\frac{\left(4a^4(39A-20B)-7a^4(9A-5B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{a}}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\cdots}{2d(a\cos(c+dx)+a\cdots}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(4a^4(39A-20B)-7a^4(9A-5B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{a}}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\,\mathrm{se}\cdots}{2d(a\cos(c+dx)+a)^{3\cdots}}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{4a^4(39A-20B)-7a^4(9A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{a}}{4a^2}-\frac{a(19A-11B)\tan(c+dx)\sec(c+dx\cdots}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3464$$

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{4a^3(39A-20B)\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-a^4(219A-115B)\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{a}}{4a^2}-\cdots}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{4a^3(39A-20B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a^4(219A-115B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{a}}{4a^2}-a}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 3128

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{4a^3(39A-20B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^4(219A-115B)\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\,s}{\sqrt{\cos}}\right.}{d}}{a}}{a}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{4a^3(39A-20B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sqrt{2}a^{7/2}(219A-115B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{a\cos(c+d}}\right.}{d}}{a}}{a}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 3252

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{8a^4(39A-20B)\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{\sqrt{2}a^{7/2}(219A-115B)\text{arcta}}{d}}{a}}{a}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{2a^2(31A-15B)\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^3(9A-5B)\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{8a^{7/2}(39A-20B)\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{\sqrt{2}a^{7/2}(219A-115B)\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a}}\right.}{d}}{a}}{a}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\tan(c+dx)\sec(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/4*((A - B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) +
(-1/2*(a*(19*A - 11*B)*Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^
(3/2)) + ((2*a^2*(31*A - 15*B)*Sec[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + a*Co
s[c + d*x]]) - (-(((8*a^(7/2)*(39*A - 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])
/Sqrt[a + a*Cos[c + d*x]]])/d - (Sqrt[2]*a^(7/2)*(219*A - 115*B)*ArcTanh[(
Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) + (14*a^3
*(9*A - 5*B)*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/a)/(4*a^2))/(8*a^
2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. $2(229) = 458$.

Time = 3.58 (sec) , antiderivative size = 1438, normalized size of antiderivative = 5.45

method	result	size
parts	Expression too large to display	1438
default	Expression too large to display	1610

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/8*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(876*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^8*a-62
4*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^8*a-624
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^8*a-876*2
^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2
*c))*cos(1/2*d*x+1/2*c)^6*a+252*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)*cos(1/2*d*x+1/2*c)^6+624*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(a*2^(1
/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)
)*cos(1/2*d*x+1/2*c)^6*a+624*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a*2^(1/2
)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*
cos(1/2*d*x+1/2*c)^6*a+219*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-188*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-156*ln(-4/(2*cos(1/2
*d*x+1/2*c)-2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a-156*ln(4/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(a*2^(1/2)*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a+19*cos(1/2*d*x+1/2*c)^2
*2^(1/2)*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*2^(1/2)*(a*sin(1/2*d*...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.62

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}\left((219\,A-115\,B)\cos\left(dx+c\right)^{5}+3\,(219\,A-115\,B)\cos\left(dx+c\right)^{4}+3\,(219\,A-115\,B)\cos\left(dx+c\right)^{3}+\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-1/64*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^5 + 3*(219*A - 115*B)*cos(d*x
 + c)^4 + 3*(219*A - 115*B)*cos(d*x + c)^3 + (219*A - 115*B)*cos(d*x + c)^
2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqr
t(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) + 4*((39*A - 20*B)*cos(d*x + c)^5 + 3*(39*A - 20*B)*cos(d*x + c)^
4 + 3*(39*A - 20*B)*cos(d*x + c)^3 + (39*A - 20*B)*cos(d*x + c)^2)*sqrt(a)
*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*s
qrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
c)^2)) + 4*(7*(9*A - 5*B)*cos(d*x + c)^3 + 5*(19*A - 11*B)*cos(d*x + c)^2
+ 4*(5*A - 4*B)*cos(d*x + c) - 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))
/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
 a^3*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos\left(c+dx\right))\sec^3\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(5/2
), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx}}{\cos(dx+c)^3+3\cos}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c + d*x)**3)/(cos(c
 + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(
c + d*x) + 1)*sec(c + d*x)**3)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*co
s(c + d*x) + 1),x)*a))/a**3
```

3.123 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 159

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx &= \frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{10a(A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &+\frac{10a(A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2a(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}\\ &+\frac{2a(A+B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output

```
2/15*a*(9*A+7*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*a*(A+B)*Inv
erseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+10/21*a*(A+B)*cos(d*x+c)^(1/2)*sin(d
*x+c)/d+2/45*a*(9*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*(A+B)*cos(d*x
+c)^(5/2)*sin(d*x+c)/d+2/9*a*B*cos(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.76 (sec) , antiderivative size = 914, normalized size of antiderivative = 5.75

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/15*((9*A
 + 7*B)*Cot[c])/d + (23*(A + B)*Cos[d*x]*Sin[c])/(84*d) + ((18*A + 19*B)*C
os[2*d*x]*Sin[2*c])/(180*d) + ((A + B)*Cos[3*d*x]*Sin[3*c])/(28*d) + (B*Co
s[4*d*x]*Sin[4*c])/(72*d) + (23*(A + B)*Cos[c]*Sin[d*x])/(84*d) + ((18*A +
 19*B)*Cos[2*c]*Sin[2*d*x])/(180*d) + ((A + B)*Cos[3*c]*Sin[3*d*x])/(28*d)
 + (B*Cos[4*c]*Sin[4*d*x])/(72*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x
)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]...
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos^{\frac{5}{2}}(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left((aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2}{9}\int \frac{1}{2}\cos^{\frac{5}{2}}(c+dx)(a(9A+7B)+9a(A+B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \cos^{\frac{5}{2}}(c+dx)(a(9A+7B)+9a(A+B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a(9A+7B)+9a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{9}\left(a(9A+7B)\int \cos^{\frac{5}{2}}(c+dx)dx+9a(A+B)\int \cos^{\frac{7}{2}}(c+dx)dx\right)+\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(a(9A+7B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+9a(A+B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx\right)+\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{9}\left(a(9A+7B)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a(A+B)\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\,\mathrm{s}}{}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(a(9A+7B)\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a(A+B)\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{9}\left(9a(A+B)\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a(9A\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(9a(A+B)\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(9a(A+B)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{9}\left(9a(A+B)\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (a*(9*A + 7*B)*((6*Ellipti
cE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)) + 9
*a*(A + B)*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(142) = 284$.

Time = 19.49 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.58

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(720A+2960B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-1\cdots\right.}{\cdots}$
parts	$-\frac{2(Aa+Ba)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^9-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^7+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3+5\cdots\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\cdots}}$

input
```
int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A+2960*B)*sin(1/2*d*x+1/2*c)
^8*cos(1/2*d*x+1/2*c)+(-1584*A-3152*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+(1344*A+1792*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-366*A-408*B
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
89*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.21

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{-75i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+75i\sqrt{2}(A+B)a\text{weierstras}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/315*(-75*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 75*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*(9*A + 7*B)*a*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2
)*(9*A + 7*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) + 2*(35*B*a*cos(d*x + c)^3 + 45*(A + B)*a*cos(d*x
+ c)^2 + 7*(9*A + 7*B)*a*cos(d*x + c) + 75*(A + B)*a)*sqrt(cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 42.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = -\frac{2\,A\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}} - \frac{2\,A\,a\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}} - \frac{2\,B\,a\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}} - \frac{2\,B\,a\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+d\,x)^2\right)}{11\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
- (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a*cos(c + d*x)^(9/2)*sin(c
 + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/
4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(11
/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(
c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= a\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a\right)$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
a*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*b + int(sqrt(cos(c + d*x))*co
s(c + d*x)**3,x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b + int(sqr
t(cos(c + d*x))*cos(c + d*x)**2,x)*a)
```

3.124 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 132

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx &= \frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\&\quad+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\&\quad+\frac{2a(A+B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
6/5*a*(A+B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(7*A+5*B)*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/21*a*(7*A+5*B)*cos(d*x+c)^(1/2)*sin(
d*x+c)/d+2/5*a*(A+B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*B*cos(d*x+c)^(5/2
)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.38 (sec) , antiderivative size = 872, normalized size of antiderivative = 6.61

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*(A + B)
*Cot[c])/(5*d) + ((28*A + 23*B)*Cos[d*x]*Sin[c])/(84*d) + ((A + B)*Cos[2*d
*x]*Sin[2*c])/(10*d) + (B*Cos[3*d*x]*Sin[3*c])/(28*d) + ((28*A + 23*B)*Cos
[c]*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (B*Cos[3*c]*
Sin[3*d*x])/(28*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(3*d*Sqrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[
c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan
[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec...
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos^{\frac{3}{2}}(c+dx)\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left((aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2}{7}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(a(7A+5B)+7a(A+B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \cos^{\frac{3}{2}}(c+dx)(a(7A+5B)+7a(A+B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a(7A+5B)+7a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{7}\left(a(7A+5B)\int \cos^{\frac{3}{2}}(c+dx)dx+7a(A+B)\int \cos^{\frac{5}{2}}(c+dx)dx\right)+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(a(7A+5B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+7a(A+B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{7}\left(7a(A+B)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a(7A+5B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2}{}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(7a(A+B)\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a(7A+5B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx\right.}}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(a(7A+5B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7a(A+B)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(7a(A+B)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a(7A+5B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (a*(7*A + 5*B)*((2*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 7
*a*(A + B)*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Si
n[c + d*x])/(5*d)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(119) = 238$.

Time = 12.43 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.90

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(-168A-528B\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(308A-}{}$
parts	$-\frac{2(Aa+Ba)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}c}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{d}{}\right.}}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(
-112*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{-5i\sqrt{2}(7A+5B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5B)a\text{weiers}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/105*(-5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt
(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))) + 2*(15*B*a*cos(d*x + c)^2 + 21*(A + B)*a*cos(d*x +
c) + 5*(7*A + 5*B)*a)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 41.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.26

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{2\,A\,a\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d} - \frac{2\,A\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
(2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3
*d) - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, c
os(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*s
in(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4],
 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= a\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3dx\right)b + \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
a*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))*cos(c
 + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a + int(sqrt(c
os(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.125 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 101

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/5*a*(5*A+3*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*Invers
eJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*a*(A+B)*cos(d*x+c)^(1/2)*sin(d*x+c)
/d+2/5*a*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.33 (sec) , antiderivative size = 830, normalized size of antiderivative = 8.22

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/5*((5*A
+ 3*B)*Cot[c])/d + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (B*Cos[2*d*x]*Sin[2*c
])/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (B*Cos[2*c]*Sin[2*d*x])/(10*
d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sq
rt[1 + Cot[c]^2]) - (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]))/(2*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x ...
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sqrt{\cos(c+dx)}\left((aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left((aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\cos(c+dx)}(a(5A+3B)+5a(A+B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\cos(c+dx)}(a(5A+3B)+5a(A+B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a(5A+3B)+5a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(5a(A+B)\int \cos^{\frac{3}{2}}(c+dx)dx+a(5A+3B)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(a(5A+3B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5a(A+B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 3115

$$\frac{1}{5}\left(a(5A+3B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5a(A+B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(a(5A+3B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5a(A+B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 3119

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5a(A+B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + ((2*a*(5*A + 3*B)*Elliptic
E[(c + d*x)/2, 2])/d + 5*a*(A + B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) +
(2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(92) = 184$.

Time = 9.66 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.51

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(20A+44B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-10A-16\right.}{}$
parts	$-\frac{2(Aa+Ba)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{-5i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)a\text{weierstrassP}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*B)*a*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*
A + 3*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) + 2*(3*B*a*cos(d*x + c) + 5*(A + B)*a)*sqrt(cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{2\,A\,a\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,B\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
(2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3
*d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2
)))/(3*d) + (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d - (2*B*a*cos(c + d*x)^(7
/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c
+ d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=a\left(\left(\int\sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)b+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)b\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
a*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a
+ int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))*cos(c
+ d*x)**2,x)*b)
```

3.126 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}+\frac{2a(3A+B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2aB\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a*(A+B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(3*A+B)*InverseJac
obiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*a*B*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.12 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.41

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6(A+B)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c)\sin(d\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x
]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-6*(A + B)*HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan
[c]]] + (9*(A + B)*Cos[c - d*x - ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*A*Cos[c
 + d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*B*Cos[c + d*x + ArcTan[Tan[c]]]
*Csc[c]*Sec[c] - 12*A*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 12*B*Cos[c + d*
x]*Cot[c]*Sqrt[Sec[c]^2] - 4*(3*A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[
Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sqrt[Sec[c]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 4*B*C
os[c + d*x]*Sqrt[Sec[c]^2]*Sin[c + d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]
))/(12*d*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^
2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{2}{3}\int \frac{a(3A+B)+3a(A+B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{a(3A+B)+3a(A+B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{a(3A+B)+3a(A+B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{1}{3}\left(a(3A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a(A+B)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(a(3A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a(A+B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{1}{3}\left(a(3A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{1}{3}\left(\frac{2a(3A+B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

input
```
Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
((6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c +
 d*x)/2, 2])/d)/3 + (2*a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(67) = 134$.

Time = 5.38 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.59

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, a\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\right.\right.}{}$
parts	$\frac{2(Aa+Ba)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}+\frac{2Aa\ \text{Inve}}{}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.03

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\,Ba\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}(3\,A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/3*(2*B*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*(3*A + B)*a*weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(3*A + B)*a
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(
A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) - 3*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= a\left(\int \frac{A}{\sqrt{\cos(c+dx)}}\,dx + \int A\sqrt{\cos(c+dx)}\,dx + \int B\sqrt{\cos(c+dx)}\,dx + \int B\cos^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
a*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(A*sqrt(cos(c + d*x)), x) +
 Integral(B*sqrt(cos(c + d*x)), x) + Integral(B*cos(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 41.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,B\,a\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d} + \frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

output
```
(2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3
*d) + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*(ellipticE(c/2 + (d*x
)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= a\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos(dx+c)}dx\right)b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)),x)*a
+ int(sqrt(cos(c + d*x)),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b)
```

3.127 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 66

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*(A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(A+B)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))/d+2*a*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.93 (sec) , antiderivative size = 783, normalized size of antiderivative = 11.86

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x
]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/2*((-2*A
 + B + B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d)
- (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]
^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + C
ot[c]^2]) + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeome
tricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
2*d) - (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricP
FQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Ar...
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3447}$$

$$\int \frac{(aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3500}$$

$$2\int \frac{a(A+B)-a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{a(A+B)-a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a(A+B)-a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$a(A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-a(A-B)\int \sqrt{\cos(c+dx)}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$a(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(A-B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$a(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2a(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d
*x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(67) = 134$.

Time = 4.72 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.67

method	result
default	$\frac{2a\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(}\right.}{}$
parts	$\frac{2(Aa+Ba)\,\text{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d}-\frac{2Aa\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
2*a*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.62

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}(A+B)a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}(A+B)a\,\mathrm{co}}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*(A - B)*a*cos(d*x + c)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + I*sqrt(2)*(A - B)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*A*a*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)}{\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)}{\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 42.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,B\,a\,\left(\mathrm{E}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|2\right) + \mathrm{F}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|2\right)\right)}{d} + \frac{2\,A\,a\,\sin(c + d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4}, \frac{1}{2};\ \frac{3}{4};\ \cos(c + d\,x)^2\right)}{d\,\sqrt{\cos(c + d\,x)}\,\sqrt{\sin(c + d\,x)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

output
```
(2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*(ellipticE(c/2 + (d*x)/2, 2
) + ellipticF(c/2 + (d*x)/2, 2)))/d + (2*A*a*sin(c + d*x)*hypergeom([-1/4,
 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= a\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+\left(\int \sqrt{\cos(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))/cos(c
 + d*x),x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(cos(
c + d*x)),x)*b)
```

3.128 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*(A+B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+3*B)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*a*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*a*
(A+B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.05 (sec) , antiderivative size = 813, normalized size of antiderivative = 8.56

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x
]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((A + B)*Cs
c[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c
+ d*x]*(A*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c +
d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + C
os[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) + (A*(
1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (B*(1 + C
os[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/...
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{2}{3}\int \frac{3a(A+B)+a(A+3B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a(A+B)+a(A+3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a(A+B)+a(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(3a(A+B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+a(A+3B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3a(A+B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a(A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{3}\left(a(A+3B)\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+3a(A+B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(a(A+3B)\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+3a(A+B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3119

$$\frac{1}{3}\left(a(A+3B)\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+3a(A+B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2a(A+3B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3a(A+B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((2*a*(A + 3*B)*EllipticF[
(c + d*x)/2, 2])/d + 3*a*(A + B)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(90) = 180$.

Time = 5.17 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.20

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}+\frac{A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{6\cdots}\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{2(Aa+Ba)\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{-2\sin\left(\frac{dx}{2}\cdots\right.}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.06

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}(A+3B)a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}(A+3B}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)^2*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(A + B)*a*
cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*
(3*(A + B)*a*cos(d*x + c) + A*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 42.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.58

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

output
```
(2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*A*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d
*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeo
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2
)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = a\left(\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)b+\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^3}dx\right)a + \left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)a+\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/cos(c
 + d*x)**3,x)*a + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(c
os(c + d*x))/cos(c + d*x)**2,x)*b)
```

3.129 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 132

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output

```
-2/5*a*(3*A+5*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/5*a*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)+
2/3*a*(A+B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*a*(3*A+5*B)*sin(d*x+c)/d/cos
(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.31 (sec) , antiderivative size = 865, normalized size of antiderivative = 6.55

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x
]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 5*B
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c
]*Sec[c + d*x]^2*(3*A*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (Sec
[c]*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 9*A*Sin[d*x] + 15*B*Sin[d*x]))
/(15*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt
[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2
}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3
*d*Sqrt[1 + Cot[c]^2]) + (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]
^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]))/(10*d) + (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^...
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(aA+aB)\cos(c+dx)+aA+aB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+aB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{2}{5}\int \frac{5a(A+B)+a(3A+5B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5a(A+B)+a(3A+5B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{5a(A+B)+a(3A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(5a(A+B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+a(3A+5B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a(A+B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(3A+5B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a(3A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}d\right.\right.\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a(3A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c\right.}\right.\right.\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a(3A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+d\right.}{d}\right.\right.\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a(3A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx\right.}{d}\right.\right.\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (5*a*(A + B)*((2*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))) + a*(
3*A + 5*B)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Co
s[c + d*x]])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(119) = 238$.

Time = 6.87 (sec) , antiderivative size = 634, normalized size of antiderivative = 4.80

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(\frac{A\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\cdots\right.}}{\cdots}\right.}{\cdots}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/10*A/(8*
sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B
)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.66

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-5i\sqrt{2}(A+B)a\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}(A+B)}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*
B)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*B)*a*cos(d*x + c)^3*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*(3*(3*A + 5*B)*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c) + 3*A*
a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)}\, dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)}{\cos(dx + c)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 42.81 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2Aa\sin(c+dx)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+dx)^2\right)}{3d\cos(c+dx)^{3/2}\sqrt{\sin(c+dx)^2}} + \frac{2Aa\sin(c+dx)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+dx)^2\right)}{5d\cos(c+dx)^{5/2}\sqrt{\sin(c+dx)^2}} + \frac{2Ba\sin(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+dx)^2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)^2}} + \frac{2Ba\sin(c+dx)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+dx)^2\right)}{3d\cos(c+dx)^{3/2}\sqrt{\sin(c+dx)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

output
```
(2*A*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(
c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a*sin(c + d*x)*hypergeom([-5
/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^
(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/
(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hyperg
eom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*
x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = a\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
a*(int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a + int(sqrt(cos(c + d*x))/co
s(c + d*x)**3,x)*a + int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*b + int(sqr
t(cos(c + d*x))/cos(c + d*x)**2,x)*b)
```

3.130 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 194

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx &= \frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(6A+5B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &\quad+\frac{4a^2(6A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{4a^2(9A+8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}\\ &\quad+\frac{2a^2(9A+11B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}\\ &\quad+\frac{2B\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{9d}\end{aligned}$$

output
```
4/15*a^2*(9*A+8*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(6*A+5
*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/21*a^2*(6*A+5*B)*cos(d*x+c)
^(1/2)*sin(d*x+c)/d+4/45*a^2*(9*A+8*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/63*
a^2*(9*A+11*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*B*cos(d*x+c)^(5/2)*(a^2+a
^2*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.87 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/15*((9*
A + 8*B)*Cot[c])/d + ((51*A + 46*B)*Cos[d*x]*Sin[c])/(168*d) + ((36*A + 37
*B)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((A + 2*B)*Cos[3*d*x]*Sin[3*c])/(56*d)
+ (B*Cos[4*d*x]*Sin[4*c])/(144*d) + ((51*A + 46*B)*Cos[c]*Sin[d*x])/(168*d
) + ((36*A + 37*B)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((A + 2*B)*Cos[3*c]*Sin[
3*d*x])/(56*d) + (B*Cos[4*c]*Sin[4*d*x])/(144*d)) - (2*A*(a + a*Cos[c + d*
x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a +
 a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
rcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2
]) - (3*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^
2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^...
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{9}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)(a(9A+5B)+a(9A+11B)\cos(c+dx))dx+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)(a(9A+5B)+a(9A+11B)\cos(c+dx))dx+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(9A+5B)+a(9A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{9}\int \cos^{\frac{3}{2}}(c+dx)\left((9A+11B)\cos^2(c+dx)a^2+(9A+5B)a^2+\left((9A+5B)a^2+(9A+11B)a^2\right)\cos(c+dx)\right)dx+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left((9A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)^2 a^2+(9A+5B)a^2+\left((9A+5B)a^2+(9A+11B)a^2\right)\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(9(6A+5B)a^2+7(9A+8B)\cos(c+dx)a^2\right)dx+\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx}{7d}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9(6A+5B)a^2+7(9A+8B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx+\frac{2a^2(9A+11B)\sin(c+d}{7d}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{9}\left(\frac{2}{7}\left(9a^2(6A+5B)\int \cos^{\frac{3}{2}}(c+dx)dx+7a^2(9A+8B)\int \cos^{\frac{5}{2}}(c+dx)dx\right)+\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}}{7d}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{2}{7}\left(9a^2(6A+5B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+7a^2(9A+8B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)+\frac{2a^2(9A+11B)\text{s}}{}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{9}\left(\frac{2}{7}\left(7a^2(9A+8B)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a^2(6A+5B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+d}}\right.\right.\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{2}{7}\left(7a^2(9A+8B)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a^2(6A+5B)\left(\frac{1}{3}\int\frac{}{\sqrt{\sin}}\right.\right.\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(\frac{2}{7}\left(9a^2(6A+5B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7a^2(9A+8B)\left(\frac{6E\left(\frac{1}{2}(c+\right.}{5d}\right.\right.\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{9}\left(\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2}{7}\left(7a^2(9A+8B)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+d}{5d}\right.\right.\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d) + ((2
*a^2*(9*A + 11*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*(9*a^2*(6*A
+ 5*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*d)) + 7*a^2*(9*A + 8*B)*((6*EllipticE[(c + d*x)/2, 2])/(5*d) +
(2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d))))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(177) = 354$.

Time = 16.17 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.13

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(360A+1840B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-1\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+1840*B)*sin(1/2*d*x+1/2*c
)^8*cos(1/2*d*x+1/2*c)+(-1044*A-2368*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+(1134*A+1568*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-351*A-387*
B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+90*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.15

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx =$$
$$-\frac{2\left(15i\sqrt{2}(6A+5B)a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}(6A+5B)\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/315*(15*I*sqrt(2)*(6*A + 5*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(6*A + 5*B)*a^2*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(9*A + 8*B)*a^2*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 + 21*I*sqrt(2)*(9*A + 8*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*B*a^2*cos(d*x + c)^3 + 45*(A
 + 2*B)*a^2*cos(d*x + c)^2 + 14*(9*A + 8*B)*a^2*cos(d*x + c) + 30*(6*A + 5
*B)*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 41.99 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.37

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{2\,A\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d} - \frac{4\,A\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,A\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{4\,B\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

output
```
(2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/
(3*d) - (4*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^2*cos(c + d*x)^(
9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c
 + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/
2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*co
s(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))
/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*
hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)
)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= a^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)b\right.$$
$$+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a+2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b$$
$$\left.+2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))*co
s(c + d*x)**4,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*a + 2*int(s
qrt(cos(c + d*x))*cos(c + d*x)**3,x)*b + 2*int(sqrt(cos(c + d*x))*cos(c +
d*x)**2,x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.131 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\begin{aligned}
&\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx \\
&= \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\
&\quad + \frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a^2(7A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} \\
&\quad + \frac{2B\cos^{\frac{3}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d}
\end{aligned}$$

output
```
4/5*a^2*(4*A+3*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(7*A+6*
B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/21*a^2*(7*A+6*B)*cos(d*x+c)^
(1/2)*sin(d*x+c)/d+2/35*a^2*(7*A+9*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*B*
cos(d*x+c)^(3/2)*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.79 (sec) , antiderivative size = 898, normalized size of antiderivative = 5.58

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/5*((4*A
 + 3*B)*Cot[c])/d + ((56*A + 51*B)*Cos[d*x]*Sin[c])/(168*d) + ((A + 2*B)*C
os[2*d*x]*Sin[2*c])/(20*d) + (B*Cos[3*d*x]*Sin[3*c])/(56*d) + ((56*A + 51*
B)*Cos[c]*Sin[d*x])/(168*d) + ((A + 2*B)*Cos[2*c]*Sin[2*d*x])/(20*d) + (B*
Cos[3*c]*Sin[3*d*x])/(56*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]
^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[
c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (2*A*(a + a*Cos[c +
 d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/
4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) - (3*B*(a + a*Cos...
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)(a(7A+3B)+a(7A+9B)\cos(c+dx))dx+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \sqrt{\cos(c+dx)}(\cos(c+dx)a+a)(a(7A+3B)+a(7A+9B)\cos(c+dx))dx+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(7A+3B)+a(7A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{7}\int \sqrt{\cos(c+dx)}\left((7A+9B)\cos^2(c+dx)a^2+(7A+3B)a^2+\left((7A+3B)a^2+(7A+9B)a^2\right)\cos(c+dx)\right)dx+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left((7A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)^2 a^2+(7A+3B)a^2+\left((7A+3B)a^2+(7A+9B)a^2\right)\sin\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{7}\left(\frac{2}{5}\int\sqrt{\cos(c+dx)}\left(7(4A+3B)a^2+5(7A+6B)\cos(c+dx)a^2\right)dx+\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(7(4A+3B)a^2+5(7A+6B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx+\frac{2a^2(7A+9B)\sin(c+dx)}{5d}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(7A+6B)\int\cos^{\frac{3}{2}}(c+dx)dx+7a^2(4A+3B)\int\sqrt{\cos(c+dx)}dx\right)+\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}}{5d}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(7a^2(4A+3B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5a^2(7A+6B)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{2a^2(7A+9B)\sin}{}\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(7a^2(4A+3B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5a^2(7A+6B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c}}{3d}\right.\right.\right.$$
$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{2}{5}\left(7a^2(4A+3B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5a^2(7A+6B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{c}}{3d}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(7A+6B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{14a^2(4A+3B)E\left(\frac{1}{2}(c+d\right.}{d}\right.\right.$$

$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

$$\Big\downarrow \; 3120$$

$$\frac{1}{7}\left(\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2}{5}\left(\frac{14a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5a^2(7A+6B)\left(\frac{2\,\text{EllipticF}}{}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d) + ((2
*a^2*(7*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*((14*a^2*(4*A
 + 3*B)*EllipticE[(c + d*x)/2, 2])/d + 5*a^2*(7*A + 6*B)*((2*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(148) = 296$.

Time = 13.81 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.39

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-84A-348B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(224A-\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-91*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.26

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx =$$
$$-\frac{2\left(5i\sqrt{2}(7A+6B)a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(7A+6B)a^2\cdots\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(4*A + 3*B)*a^2*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 21*I*sqrt(2)*(4*A + 3*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^2*cos(d*x + c)^2 + 21*(A +
 2*B)*a^2*cos(d*x + c) + 10*(7*A + 6*B)*a^2)*sqrt(cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 43.38 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{2\,A\,a^2\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}$$
$$+\frac{2\,B\,a^2\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$
$$-\frac{2\,A\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{4\,B\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,B\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

output
```
(2*A*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)
/2, 2))/3))/d + (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2
+ (d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d - (2*A*a^2
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^
2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x
)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)
) - (2*B*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, c
os(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= a^2\left(\left(\int \sqrt{\cos(dx+c)}dx\right)a+2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a+2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x)),x)*a + 2*int(sqrt(cos(c + d*x))*cos(c + d*x),
x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))*c
os(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a + 2*int(
sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.132 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 126

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(2A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\ &+ \frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} \\ &+ \frac{2B\sqrt{\cos(c+dx)}(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d}\end{aligned}$$

output
```
4/5*a^2*(5*A+4*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+B)*
InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/15*a^2*(5*A+7*B)*cos(d*x+c)^(1/
2)*sin(d*x+c)/d+2/5*B*cos(d*x+c)^(1/2)*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.81 (sec) , antiderivative size = 852, normalized size of antiderivative = 6.76

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]]
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/5*((5*A
 + 4*B)*Cot[c])/d + ((A + 2*B)*Cos[d*x]*Sin[c])/(6*d) + (B*Cos[2*d*x]*Sin[
2*c])/(20*d) + ((A + 2*B)*Cos[c]*Sin[d*x])/(6*d) + (B*Cos[2*c]*Sin[2*d*x])
/(20*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/
2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c
/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT...
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{5}\int \frac{(\cos(c+dx)a+a)(a(5A+B)+a(5A+7B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(\cos(c+dx)a+a)(a(5A+B)+a(5A+7B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(5A+B)+a(5A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{5}\int \frac{(5A+7B)\cos^2(c+dx)a^2+(5A+B)a^2+\left((5A+B)a^2+(5A+7B)a^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{(5A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+(5A+B)a^2+\left((5A+B)a^2+(5A+7B)a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{5(2A+B)a^2+3(5A+4B)\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{5(2A+B)a^2+3(5A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(2A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2(5A+4B)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(2A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2(5A+4B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(2A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2}{3}\left(\frac{10a^2(2A+B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2(5A+4B)E\left(\frac{1}{2}(c\right.}{d}\right.\right.$$
$$\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

input
```
Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d) + ((2
*((6*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^2*(2*A + B)*Elli
pticF[(c + d*x)/2, 2])/d))/3 + (2*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(117) = 234$.

Time = 9.53 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(10A+32B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-5A-13\right.}{}$
parts	$-\frac{2(a^{2}A+2a^{2}B)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.42

$$\int \frac{(a + a\cos(c + dx))^2(A + B\cos(c + dx))}{\sqrt{\cos(c + dx)}}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(2A + B)a^2\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) - 5i\sqrt{2}(2A + B)a^2\text{wei}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 4*B)*a^2*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*s
qrt(2)*(5*A + 4*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) - (3*B*a^2*cos(d*x + c) + 5*(A + 2*B)*a^2)*
sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 44.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.21

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\frac{2\,B\,a^2\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}$$
$$+\frac{2\,A\,a^2\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}$$
$$+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$
$$-\frac{2\,B\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

output
```
(2*B*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)
/2, 2))/3))/d + (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/
2 + (d*x)/2, 2) + 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a^2*ellipti
cE(c/2 + (d*x)/2, 2))/d - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hyperge
om([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= a^2\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+2\left(\int \sqrt{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos(dx+c)}dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)\right.$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + 2*int(sqrt(cos(c + d*x)),
x)*a + int(sqrt(cos(c + d*x)),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x),x
)*a + 2*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))*
cos(c + d*x)**2,x)*b)
```

3.133 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(3A+2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} - \frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output

```
4*a^2*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(3*A+2*B)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))/d-2/3*a^2*(3*A-B)*cos(d*x+c)^(1/2)*sin(d*x+
c)/d+2*A*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.74 (sec) , antiderivative size = 623, normalized size of antiderivative = 5.28

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{(-A+2B+A\cos(2c)+2B\cos(2c))\csc(c)\sec(c)}{4d}+\frac{B\cos(dx)\sin(c)}{6d}+\frac{B\cos(c)\sin(dx)}{6d}+\frac{A\sec(c)\sec(c+dx)\sin(dx)}{2d}\right)$$
$$-\frac{A(a+a\cos(c+dx))^2\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\arctan(\cot$$
$$-\frac{2B(a+a\cos(c+dx))^2\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\arctan(co$$
$$-\frac{B(a+a\cos(c+dx))^2\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arcta}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+ar}}}{2d}$$

input
```
Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2)
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/4*((-A
+ 2*B + A*Cos[2*c] + 2*B*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Cos[d*x]*Sin[c])/
(6*d) + (B*Cos[c]*Sin[d*x])/(6*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d)
) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 +
Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)
/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2
]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(2*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3454, 27, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$2\int \frac{(\cos(c+dx)a+a)(a(3A+B)-a(3A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 27

$$\int \frac{(\cos(c+dx)a+a)(a(3A+B)-a(3A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(3A+B)-a(3A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3447

$$\int \frac{-\left((3A-B)\cos^2(c+dx)a^2\right)+(3A+B)a^2+\left(a^2(3A+B)-a^2(3A-B)\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\int \frac{-\left((3A-B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2\right)+(3A+B)a^2+\left(a^2(3A+B)-a^2(3A-B)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3502

$$\frac{2}{3}\int \frac{(3A+2B)a^2+3B\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}}dx-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{2}{3}\int \frac{(3A+2B)a^2+3B\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{2}{3}\left(a^2(3A+2B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2B\int \sqrt{\cos(c+dx)}dx\right)-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(a^2(3A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2B\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{3}\left(a^2(3A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2}{3}\left(\frac{2a^2(3A+2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(2*((6*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (2*a^2*(3*A + 2*B)*EllipticF[(
c + d*x)/2, 2])/d))/3 - (2*a^2*(3*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(3*d) + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(113) = 226$.

Time = 5.76 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.07

method	result
default	$\frac{4a^2\left(-2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+3A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$
parts	$\frac{2(a^2A+2a^2B)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}+\frac{2(2a^2}{}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
4/3*a^2*(-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.68

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(i\sqrt{2}(3A+2B)a^2\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-i\sqrt{2}(3A\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*a^2*
cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*a^2*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (B*a^2*cos
(d*x + c) + 3*A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 44.84 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,B\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d} + \frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

output
```
(2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2 + (d*x)/2, 2)
+ 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2
, 2))/d + (4*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*
hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c
+ d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a^2\left(2\left(\int\frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)a+\left(\int\frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)b + \left(\int\frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)a+\left(\int\sqrt{\cos\left(dx+c\right)}dx\right)a + 2\left(\int\sqrt{\cos\left(dx+c\right)}dx\right)b+\left(\int\sqrt{\cos\left(dx+c\right)}\cos\left(dx+c\right)dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
a**2*(2*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))/
cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt
(cos(c + d*x)),x)*a + 2*int(sqrt(cos(c + d*x)),x)*b + int(sqrt(cos(c + d*x
))*cos(c + d*x),x)*b)
```

3.134 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 120

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(2A+3B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-4*a^2*A*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+3*B)*Inverse
JacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*a^2*(5*A+3*B)*sin(d*x+c)/d/cos(d*x+c
)^(1/2)+2/3*A*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.53 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.59

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(12A\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2)
,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(12*A*Cos[c + d*x]*Hypergeome
tricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x +
 ArcTan[Tan[c]]] + (Csc[c]*(-6*A*Cos[c + d*x]*(3*Cos[c - d*x - ArcTan[Tan[
c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sec[c] + (12*A*Cos[c] + 2*A*Cos[d*x]
 - 2*A*Cos[2*c + d*x] + 12*A*Cos[c + 2*d*x] + 3*B*Cos[c + 2*d*x] - 3*B*Cos
[3*c + 2*d*x])*Sqrt[Sec[c]^2]) - 8*(2*A + 3*B)*Cos[c + d*x]^2*Sqrt[Cos[d*x
 - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Sec[c]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[
c])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(24*d*Cos[c + d*x]^(3/2)*Sqrt[Sec[
c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3454, 27, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{3}\int \frac{(\cos(c+dx)a+a)(a(5A+3B)-a(A-3B)\cos(c+dx))}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(\cos(c+dx)a+a)(a(5A+3B)-a(A-3B)\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(5A+3B)-a(A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{3}\int \frac{-\left((A-3B)\cos^2(c+dx)a^2\right)+(5A+3B)a^2+\left(a^2(5A+3B)-a^2(A-3B)\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{-\left((A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2\right)+(5A+3B)a^2+\left(a^2(5A+3B)-a^2(A-3B)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\left(2\int \frac{a^2(2A+3B)-3a^2A\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2(5A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2\int \frac{a^2(2A+3B)-3a^2A\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(5A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(2\left(a^2(2A+3B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^2A\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^2(5A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2\left(a^2(2A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^2A\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2(5A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left(2\left(a^2(2A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2(5A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\left(\frac{2a^2(2A+3B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
((-6*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (2*a^2*(2*A + 3*B)*EllipticF[(c
+ d*x)/2, 2])/d) + (2*a^2*(5*A + 3*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(113) = 226$.

Time = 6.10 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.28

method	result
default	$-\frac{4\left(6\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(2A+B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(7A+3B)\sin\left(\frac{dx}{2}\right.\right.}{}$
parts	$\frac{2(a^{2}A+2a^{2}B)\operatorname{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d}-\frac{2(2a^{2}A+a^{2}B)\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\mathrm{s}}}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))*a^2/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3
/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(i\sqrt{2}(2A+3B)a^2\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}(2\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)^2*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*
a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*a^2*cos(d*x + c)^2*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
*(2*A + B)*a^2*cos(d*x + c) + A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*c
os(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(c + dx))^2(A + B\cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + a\cos(c + dx))^2(A + B\cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
x)
```

Mupad [B] (verification not implemented)

Time = 45.60 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.63

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(5/2),x)
```

output
```
(2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticE(c/2 + (d*x)/2
, 2))/d + (4*B*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*A*a^2*sin(c + d*x)*
hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c
+ d*x)^2)^(1/2)) + (2*A*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c
 + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^2*sin
(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/
2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= a^2\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b + \left(\int \sqrt{\cos(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + 2*int(sqrt(cos(c + d*x))/
cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a + 2*int(sq
rt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d*x))/cos(c + d*x
)**2,x)*b + int(sqrt(cos(c + d*x)),x)*b)
```

3.135 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(A+2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(4A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

output

```
-4/5*a^2*(4*A+5*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(A+2*B)
*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/15*a^2*(7*A+5*B)*sin(d*x+c)/d/
cos(d*x+c)^(3/2)+4/5*a^2*(4*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/5*A*(a^
2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.89 (sec) , antiderivative size = 883, normalized size of antiderivative = 5.55

$$\int \frac{(a + a\cos(c + dx))^2 (A + B\cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2)
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((4*A + 5*
B)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec
[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 10*A*Sin[d*x] + 5*B*Sin[d*x]))/(30*d) + (
Sec[c]*Sec[c + d*x]*(10*A*Sin[c] + 5*B*Sin[c] + 24*A*Sin[d*x] + 30*B*Sin[d
*x]))/(30*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(3*d*Sqrt[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (2*A*(a + a*Cos[c + d*x])^2*Csc[c]
*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) + (B*(a + a*Cos[c + d*x])^2*Csc[...
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3454, 27, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{5}\int \frac{(\cos(c+dx)a+a)(a(7A+5B)+a(A+5B)\cos(c+dx))}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(\cos(c+dx)a+a)(a(7A+5B)+a(A+5B)\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(7A+5B)+a(A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{5}\int \frac{(A+5B)\cos^2(c+dx)a^2+(7A+5B)a^2+\left((A+5B)a^2+(7A+5B)a^2\right)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{(A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+(7A+5B)a^2+\left((A+5B)a^2+(7A+5B)a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3(4A+5B)a^2+5(A+2B)\cos(c+dx)a^2}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(7A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3(4A+5B)a^2+5(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2(7A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(3a^2(4A+5B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+5a^2(A+2B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2a^2(7A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(3a^2(4A+5B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+5a^2(A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2(7A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right)+\frac{2a^2}{}\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)\right)\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3119$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)+\frac{2a^2(}{}\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3120$$

$$\frac{1}{5}\left(\frac{2a^2(7A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\left(\frac{10a^2(A+2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3a^2(4A+5B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((2
*a^2*(7*A + 5*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*((10*a^2*(A +
 2*B)*EllipticF[(c + d*x)/2, 2])/d + 3*a^2*(4*A + 5*B)*((-2*EllipticE[(c +
 d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(146) = 292$.

Time = 7.12 (sec) , antiderivative size = 714, normalized size of antiderivative = 4.49

method	result	size
default	Expression too large to display	714
parts	Expression too large to display	801

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/20*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/2*B)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))+(1/2*A+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.50

$$\int \frac{(a + a\cos(c + dx))^2(A + B\cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(A + 2B)a^2\cos(dx + c)^3\,\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) - 5i\sqrt{2}(A\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^3
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(
4*A + 5*B)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(4*A + 5*B)*a^2*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - (6*(4*A + 5*B)*a^2*cos(d*x + c)^2 + 5*(2*A + B)*a^2*cos
(d*x + c) + 3*A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
x)
```

Mupad [B] (verification not implemented)

Time = 42.47 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.44

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{6\,A\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)+20\,A\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \mathrm{c}\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c}} + \frac{2\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,B\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}} + \frac{2\,B\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(7/2),x)
```

output
```
(6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*A*
a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2)
+ 30*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c +
 d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*B*a^2*
ellipticF(c/2 + (d*x)/2, 2))/d + (4*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/
2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) +
(2*B*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = a^2\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+2\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/co
s(c + d*x)**4,x)*a + 2*int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a + int(s
qrt(cos(c + d*x))/cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))/cos(c + d*
x)**2,x)*a + 2*int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*b)
```

3.136 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= -\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &\quad+\frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}\\ &\quad+\frac{4a^2(3A+4B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2A(a^2+a^2\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
-4/5*a^2*(3*A+4*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(6*A+7
*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/35*a^2*(9*A+7*B)*sin(d*x+c)
/d/cos(d*x+c)^(5/2)+4/21*a^2*(6*A+7*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+4/5*a
^2*(3*A+4*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/7*A*(a^2+a^2*cos(d*x+c))*sin(
d*x+c)/d/cos(d*x+c)^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.19 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.77

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2)
,x]
```

output

```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((3*A + 4*
B)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec
[c]*Sec[c + d*x]^3*(5*A*Sin[c] + 14*A*Sin[d*x] + 7*B*Sin[d*x]))/(70*d) + (
Sec[c]*Sec[c + d*x]^2*(42*A*Sin[c] + 21*B*Sin[c] + 60*A*Sin[d*x] + 70*B*Si
n[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(30*A*Sin[c] + 35*B*Sin[c] + 63*A*
Sin[d*x] + 84*B*Sin[d*x]))/(105*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])
^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sq
rt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (3*A*(a + a*
Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sq...
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {3042, 3454, 27, 3042, 3447, 3042, 3500, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{7}\int \frac{(\cos(c+dx)a+a)(a(9A+7B)+a(3A+7B)\cos(c+dx))}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(\cos(c+dx)a+a)(a(9A+7B)+a(3A+7B)\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(9A+7B)+a(3A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{7}\int \frac{(3A+7B)\cos^2(c+dx)a^2+(9A+7B)a^2+\left((3A+7B)a^2+(9A+7B)a^2\right)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \frac{(3A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+(9A+7B)a^2+\left((3A+7B)a^2+(9A+7B)a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3500$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{5(6A+7B)a^2+7(3A+4B)\cos(c+dx)a^2}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2(9A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{5(6A+7B)a^2+7(3A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2(9A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3227$$

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(6A+7B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+7a^2(3A+4B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2a^2(9A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(6A+7B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+7a^2(3A+4B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2a^2(9A+7B)\sin}{5d\cos^{\frac{5}{2}}(c+}\right.$$
$$+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3116$$

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(6A+7B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7a^2(3A+4B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\cos\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(6A+7B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7a^2(3A+4B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3119

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(6A+7B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7a^2(3A+4B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E}{}\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3120

$$\frac{1}{7}\left(\frac{2a^2(9A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\left(5a^2(6A+7B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7a^2(3A+\right.\right.$$
$$\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2
*a^2*(9*A + 7*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(5*a^2*(6*A +
 7*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c +
 d*x]^(3/2))) + 7*a^2*(3*A + 4*B)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*S
in[c + d*x])/(d*Sqrt[Cos[c + d*x]]))))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(177) = 354$.

Time = 9.32 (sec) , antiderivative size = 824, normalized size of antiderivative = 4.25

method	result	size
default	Expression too large to display	824
parts	Expression too large to display	1026

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETURNV
ERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*A*(-
1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)+1/4*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2
)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+1/5*(1/2*A+1/4*B)/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c
)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}(6A+7B)a^2\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(2)*(6*A + 7*B)*a^2*cos(d*x + c)^4*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(6*A + 7*B)*a^2*cos(d*x +
 c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqr
t(2)*(3*A + 4*B)*a^2*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(3*A + 4*B)*a^2
*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - (42*(3*A + 4*B)*a^2*cos(d*x + c)^3 + 10*(6*A + 7
*B)*a^2*cos(d*x + c)^2 + 21*(2*A + B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2),
x)
```

Mupad [B] (verification not implemented)

Time = 37.59 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.21

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{30\,A\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)+84\,A\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4}\right.}{105\,d\cos(c+d\,x)^{7/2}\,\sqrt{1-\cos}}$$
$$+\frac{6\,B\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)+20\,B\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4}\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(9/2),x)
```

output
```
(30*A*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*A
*a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2
) + 70*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c
 + d*x)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (6*B*a
^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*B*a^2*co
s(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*B
*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^
2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a + a\cos(c + dx))^2(A + B\cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)}\,dx$$
$$= a^2\left(\left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^5}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^4}dx\right)a + \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^4}dx\right)b + \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^3}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^3}dx\right)b + \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
a**2*(int(sqrt(cos(c + d*x))/cos(c + d*x)**5,x)*a + 2*int(sqrt(cos(c + d*x
))/cos(c + d*x)**4,x)*a + int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*b + in
t(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a + 2*int(sqrt(cos(c + d*x))/cos(c
 + d*x)**3,x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*b)
```

3.137 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 237

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx
&= \frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(121A+105B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d} \\
&+ \frac{4a^3(121A+105B)\sqrt{\cos(c+dx)}\sin(c+dx)}{231d} \\
&+ \frac{4a^3(17A+15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\
&+ \frac{20a^3(22A+21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} \\
&+ \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} \\
&+ \frac{2(11A+15B)\cos^{\frac{5}{2}}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{99d}
\end{aligned}$$

output
```
4/15*a^3*(17*A+15*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/231*a^3*(12
1*A+105*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/231*a^3*(121*A+105*B
)*cos(d*x+c)^(1/2)*sin(d*x+c)/d+4/45*a^3*(17*A+15*B)*cos(d*x+c)^(3/2)*sin(
d*x+c)/d+20/693*a^3*(22*A+21*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/11*a*B*cos
(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+2/99*(11*A+15*B)*cos(d*x+c)^
(5/2)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.05 (sec) , antiderivative size = 990, normalized size of antiderivative = 4.18

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/30*((17
*A + 15*B)*Cot[c])/d + ((2134*A + 1953*B)*Cos[d*x]*Sin[c])/(7392*d) + ((73
*A + 75*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + (3*(44*A + 63*B)*Cos[3*d*x]*Sin[
3*c])/(4928*d) + ((A + 3*B)*Cos[4*d*x]*Sin[4*c])/(288*d) + (B*Cos[5*d*x]*S
in[5*c])/(704*d) + ((2134*A + 1953*B)*Cos[c]*Sin[d*x])/(7392*d) + ((73*A +
 75*B)*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*(44*A + 63*B)*Cos[3*c]*Sin[3*d*x]
)/(4928*d) + ((A + 3*B)*Cos[4*c]*Sin[4*d*x])/(288*d) + (B*Cos[5*c]*Sin[5*d
*x])/(704*d)) - (11*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(42*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/
2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - A
rcTan[Cot[c]]]])/(22*d*Sqrt[1 + Cot[c]^2]) - (17*A*(a + a*Cos[c + d*x])^3*
Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*...
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3455, 27, 3042, 3455, 3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{11}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^2(a(11A+5B)+a(11A+15B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\big\downarrow$ 27

$$\frac{1}{11}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^2(a(11A+5B)+a(11A+15B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(11A+5B)+a(11A+15B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\big\downarrow$ 3455

$$\frac{1}{11}\left(\frac{2}{9}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)\left((77A+60B)a^2+5(22A+21B)\cos(c+dx)a^2\right)dx+\frac{2(11A+15B)\text{si}}{}\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{2}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((77A+60B)a^2+5(22A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\big\downarrow$ 3447

$$\frac{1}{11}\left(\frac{2}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(5(22A+21B)\cos^2(c+dx)a^3+(77A+60B)a^3+\left(5(22A+21B)a^3+(77A+60B)a^3\right)c\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{2}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5(22A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(77A+60B)a^3+\left(5(22A+21B)a^3+(77\right.\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\Big\downarrow$ 3502

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{2}{7}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\left(9(121A+105B)a^3+77(17A+15B)\cos(c+dx)a^3\right)dx+\frac{10a^3(22A+21B)\sin(c-}{7d}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{1}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(9(121A+105B)a^3+77(17A+15B)\cos(c+dx)a^3\right)dx+\frac{10a^3(22A+21B)\sin(c+}{7d}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9(121A+105B)a^3+77(17A+15B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx+\frac{10a^3(22A+}{}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\Big\downarrow$ 3227

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{1}{7}\left(9a^3(121A+105B)\int \cos^{\frac{3}{2}}(c+dx)dx+77a^3(17A+15B)\int \cos^{\frac{5}{2}}(c+dx)dx\right)+\frac{10a^3(22A+21B)\,\mathrm{s}}{}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{1}{7}\left(9a^3(121A+105B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+77a^3(17A+15B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)+\frac{10a}{}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$\Big\downarrow$ 3115

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{1}{7}\left(77a^3(17A+15B)\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a^3(121A+105B)\left(\frac{1}{3}\int\right.\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{1}{7}\left(77a^3(17A+15B)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a^3(121A+105B\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$$\Big\downarrow \ 3119$$

$$\frac{1}{11}\left(\frac{2}{9}\left(\frac{1}{7}\left(9a^3(121A+105B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+77a^3(17A+15B\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

$$\Big\downarrow \ 3120$$

$$\frac{1}{11}\left(\frac{2(11A+15B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{9d}+\frac{2}{9}\left(\frac{10a^3(22A+21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+\right.\right.}{7d}$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}{11d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(11*d) + ((
2*(11*A + 15*B)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/
(9*d) + (2*((10*a^3*(22*A + 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) +
 (9*a^3*(121*A + 105*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*d)) + 77*a^3*(17*A + 15*B)*((6*EllipticE[(c +
d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/7))/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(216) = 432$.

Time = 27.00 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.86

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(10080B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(-6160A-43680B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080
*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*A-43680*B)*sin(1/2*d*x+
1/2*c)^10*cos(1/2*d*x+1/2*c)+(24200*A+77280*B)*sin(1/2*d*x+1/2*c)^8*cos(1/
2*d*x+1/2*c)+(-37532*A-72240*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(2
9722*A+39270*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8118*A-8820*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1815*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-39
27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+1575*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3465*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx =$$

$$-\frac{2\left(15i\sqrt{2}(121A+105B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}(121A\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/3465*(15*I*sqrt(2)*(121*A + 105*B)*a^3*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(121*A + 105*B)*a^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 231*I*sqrt(2)*(17*A + 15*B)
*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 231*I*sqrt(2)*(17*A + 15*B)*a^3*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (315*B*a^3*cos(d*
x + c)^4 + 385*(A + 3*B)*a^3*cos(d*x + c)^3 + 135*(11*A + 14*B)*a^3*cos(d*
x + c)^2 + 154*(17*A + 15*B)*a^3*cos(d*x + c) + 30*(121*A + 105*B)*a^3)*sq
rt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 42.82 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.52

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{A\,a^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d} - \frac{6\,A\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,A\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,A\,a^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{6\,B\,a^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a^3\cos\left(c+d\,x\right)^{13/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{4};\ \frac{17}{4};\ \cos\left(c+d\,x\right)^2\right)}{13\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

output
```
(A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2
, 2))/3))/d - (6*A*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4
], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c +
d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*
(sin(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hyperg
eom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2
*B*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
 d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c
 + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)
^(1/2)) - (6*B*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4],
 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d
*x)^(13/2)*sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*
d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= a^3\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^5\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)a+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)b + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))*co
s(c + d*x)**5,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*a + 3*int(s
qrt(cos(c + d*x))*cos(c + d*x)**4,x)*b + 3*int(sqrt(cos(c + d*x))*cos(c +
d*x)**3,x)*a + 3*int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b + 3*int(sqrt(
cos(c + d*x))*cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x)**
2,x)*b)
```

3.138 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 204

$$\begin{aligned}&\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx\\&=\frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(13A+11B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\&\quad+\frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\&\quad+\frac{4a^3(24A+23B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d}\\&\quad+\frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d}\\&\quad+\frac{2(9A+13B)\cos^{\frac{3}{2}}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{63d}\end{aligned}$$

output

```
4/15*a^3*(21*A+17*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(13*
A+11*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/21*a^3*(13*A+11*B)*cos(
d*x+c)^(1/2)*sin(d*x+c)/d+4/105*a^3*(24*A+23*B)*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2/9*a*B*cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+2/63*(9*A+13*
B)*cos(d*x+c)^(3/2)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.97 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.63

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/30*((21
*A + 17*B)*Cot[c])/d + ((107*A + 97*B)*Cos[d*x]*Sin[c])/(336*d) + ((54*A +
 73*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + ((A + 3*B)*Cos[3*d*x]*Sin[3*c])/(112
*d) + (B*Cos[4*d*x]*Sin[4*c])/(288*d) + ((107*A + 97*B)*Cos[c]*Sin[d*x])/(
336*d) + ((54*A + 73*B)*Cos[2*c]*Sin[2*d*x])/(720*d) + ((A + 3*B)*Cos[3*c]
*Sin[3*d*x])/(112*d) + (B*Cos[4*c]*Sin[4*d*x])/(288*d)) - (13*A*(a + a*Cos
[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (
11*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 +
 Cot[c]^2]) - (7*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])...
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{9}\int \frac{1}{2}\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2(3a(3A+B)+a(9A+13B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2(3a(3A+B)+a(9A+13B)\cos(c+dx))dx+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(3a(3A+B)+a(9A+13B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{9}\left(\frac{2}{7}\int 3\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)\left(5(3A+2B)a^2+(24A+23B)\cos(c+dx)a^2\right)dx+\frac{2(9A+13B)\sin(\right.$$
$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\Biggl(\frac{6}{7}\int \sqrt{\cos(c+dx)}(\cos(c+dx)a+a)\left(5(3A+2B)a^2+(24A+23B)\cos(c+dx)a^2\right)dx+\frac{2(9A+13B)\sin(c$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\Biggl(\frac{6}{7}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(5(3A+2B)a^2+(24A+23B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \; 3447$$

$$\frac{1}{9}\Biggl(\frac{6}{7}\int \sqrt{\cos(c+dx)}\bigl((24A+23B)\cos^2(c+dx)a^3+5(3A+2B)a^3+\left(5(3A+2B)a^3+(24A+23B)a^3\right)\cos(c$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\Biggl(\frac{6}{7}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\Biggl((24A+23B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+5(3A+2B)a^3+\bigl(5(3A+2B)a^3+(24A+23$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{9}\Biggl(\frac{6}{7}\Biggl(\frac{2}{5}\int \frac{1}{2}\sqrt{\cos(c+dx)}\left(7(21A+17B)a^3+15(13A+11B)\cos(c+dx)a^3\right)dx+\frac{2a^3(24A+23B)\sin(c+dx)}{5d}$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\Biggl(\frac{6}{7}\Biggl(\frac{1}{5}\int \sqrt{\cos(c+dx)}\left(7(21A+17B)a^3+15(13A+11B)\cos(c+dx)a^3\right)dx+\frac{2a^3(24A+23B)\sin(c+dx)\,c}{5d}$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(7(21A+17B)a^3+15(13A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx+\frac{2a^3(24A+23B)}{}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(13A+11B)\int\cos^{\frac{3}{2}}(c+dx)dx+7a^3(21A+17B)\int\sqrt{\cos(c+dx)}dx\right)+\frac{2a^3(24A+23B)\sin(c}{5d}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(7a^3(21A+17B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+15a^3(13A+11B)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{2a^3(24A}{}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(7a^3(21A+17B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+15a^3(13A+11B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx}{}\right.\right.\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(7a^3(21A+17B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+15a^3(13A+11B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c}{}\right.\right.\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(13A+11B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{14a^3(21A+17B)E}{d}\right.\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

$$\Big\downarrow \ 3120$$

$$\frac{1}{9}\left(\frac{2(9A+13B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{7d}+\frac{6}{7}\left(\frac{2a^3(24A+23B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right.\right.$$

$$\frac{2aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}{9d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(2*a*B*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d) + ((2
*(9*A + 13*B)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(7
*d) + (6*((2*a^3*(24*A + 23*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + ((
14*a^3*(21*A + 17*B)*EllipticE[(c + d*x)/2, 2])/d + 15*a^3*(13*A + 11*B)*(
(2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(3*d)))/5))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(187) = 374$.

Time = 19.68 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.02

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{3}\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(360A+2200B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-1\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c
)^8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*
B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-441*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx =$$

$$-\frac{2\left(15i\sqrt{2}(13A+11B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}(13A+1\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/315*(15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(21*A + 17*B)*a^3*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 21*I*sqrt(2)*(21*A + 17*B)*a^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*B*a^3*cos(d*x + c)^3
 + 45*(A + 3*B)*a^3*cos(d*x + c)^2 + 7*(27*A + 34*B)*a^3*cos(d*x + c) + 30
*(13*A + 11*B)*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 41.64 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.58

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}$$
$$+\frac{B\,a^3\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}$$
$$-\frac{6\,A\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,A\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{6\,B\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,B\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,B\,a^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

output
```
(2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + A*a^3*ellipticF(c/2 + (d*x)/2, 2)
+ A*a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (B*a^3*((2*cos(c + d*x)^(1/2
)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (6*A*a^3*cos(c
 + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7
*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hype
rgeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (6
*B*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
 d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c
 + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4],
 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= a^3\Bigg(\left(\int \sqrt{\cos(dx+c)}dx\right)a + 3\left(\int \sqrt{\cos(dx+c)}\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)}\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\cos(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\cos(dx+c)^3\,dx\right)a + 3\left(\int \sqrt{\cos(dx+c)}\cos(dx+c)^3\,dx\right)b + 3\left(\int \sqrt{\cos(dx+c)}\cos(dx+c)^2\,dx\right)a + 3\left(\int \sqrt{\cos(dx+c)}\cos(dx+c)^2\,dx\right)b\Bigg)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
a**3*(int(sqrt(cos(c + d*x)),x)*a + 3*int(sqrt(cos(c + d*x))*cos(c + d*x),
x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))*c
os(c + d*x)**4,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*a + 3*int(
sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b + 3*int(sqrt(cos(c + d*x))*cos(c +
 d*x)**2,x)*a + 3*int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.139 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 171

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(21A+13B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &+\frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d}\\ &+\frac{2aB\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d}\\ &+\frac{2(7A+11B)\sqrt{\cos(c+dx)}(a^3+a^3\cos(c+dx))\sin(c+dx)}{35d}\end{aligned}$$

output
```
4/5*a^3*(9*A+7*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(21*A+1
3*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/105*a^3*(42*A+41*B)*cos(d*
x+c)^(1/2)*sin(d*x+c)/d+2/7*a*B*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2*sin(d*
x+c)/d+2/35*(7*A+11*B)*cos(d*x+c)^(1/2)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.99 (sec) , antiderivative size = 898, normalized size of antiderivative = 5.25

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]]
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/10*((9*
A + 7*B)*Cot[c])/d + ((84*A + 107*B)*Cos[d*x]*Sin[c])/(336*d) + ((A + 3*B)
*Cos[2*d*x]*Sin[2*c])/(40*d) + (B*Cos[3*d*x]*Sin[3*c])/(112*d) + ((84*A +
107*B)*Cos[c]*Sin[d*x])/(336*d) + ((A + 3*B)*Cos[2*c]*Sin[2*d*x])/(40*d) +
 (B*Cos[3*c]*Sin[3*d*x])/(112*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*
x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (13*B*(a + a*Cos[c + d*x])^
3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (9*A*(a + a*
Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (7*B*(a...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3455, 27, 3042, 3455, 3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{7}\int \frac{(\cos(c+dx)a+a)^2(a(7A+B)+a(7A+11B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(\cos(c+dx)a+a)^2(a(7A+B)+a(7A+11B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(7A+B)+a(7A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{(\cos(c+dx)a+a)\left((21A+8B)a^2+(42A+41B)\cos(c+dx)a^2\right)}{\sqrt{\cos(c+dx)}}dx+\frac{2(7A+11B)\sin(c+dx)\sqrt{\cos(c}{5d}\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$
$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((21A+8B)a^2+(42A+41B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(7A+11B)\sin(c+}{}\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$
$$\Big\downarrow \; 3447$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{(42A+41B)\cos^2(c+dx)a^3+(21A+8B)a^3+\left((21A+8B)a^3+(42A+41B)a^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2(7}{}\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$
$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{(42A+41B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(21A+8B)a^3+\left((21A+8B)a^3+(42A+41B)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$
$$\Big\downarrow \; 3502$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{2}{3}\int \frac{5(21A+13B)a^3+21(9A+7B)\cos(c+dx)a^3}{2\sqrt{\cos(c+dx)}}dx+\frac{2a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$
$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{5(21A+13B)a^3+21(9A+7B)\cos(c+dx)a^3}{\sqrt{\cos(c+dx)}}dx+\frac{2a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{5(21A+13B)a^3+21(9A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right.\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$\downarrow$ 3227

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(21A+13B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+21a^3(9A+7B)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^3(42A+41B)\sin(c}{3d}\right.\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(21A+13B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21a^3(9A+7B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^3(42A+}{}\right.\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$\downarrow$ 3119

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(21A+13B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{42a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^3(42A+41B)\sin}{}\right.\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

$\downarrow$ 3120

$$\frac{1}{7}\left(\frac{2(7A+11B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2}{5}\left(\frac{2a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right.\right.$$
$$\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}{7d}$$

input
```
Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) + ((2
*(7*A + 11*B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5
*d) + (2*(((42*a^3*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(21*
A + 13*B)*EllipticF[(c + d*x)/2, 2])/d)/3 + (2*a^3*(42*A + 41*B)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(3*d)))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(158) = 316$.

Time = 13.34 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-84A-432B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(294A-\right.}{}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-126*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\left(5i\sqrt{2}(21A+13B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(21A+13\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(9*A + 7*B)*a^3*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 21*I*sqrt(2)*(9*A + 7*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^3*cos(d*x + c)^2 + 21*
(A + 3*B)*a^3*cos(d*x + c) + 5*(21*A + 26*B)*a^3)*sqrt(cos(d*x + c))*sin(d
*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 41.14 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.49

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d} + \frac{6\,A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d} - \frac{2\,A\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} - \frac{6\,B\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} - \frac{2\,B\,a^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(1/2),x)
```

output
```
(2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + B*a^3*ellipticF(c/2 + (d*x)/2, 2)
+ B*a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (6*A*a^3*ellipticE(c/2 + (d*
x)/2, 2))/d + (4*A*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*cos(c + d
*x)^(1/2)*sin(c + d*x))/d - (2*A*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hyper
geom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (6*
B*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^
(1/2))
```

Reduce [F]

$$\int \frac{(a + a\cos(c + dx))^3 (A + B\cos(c + dx))}{\sqrt{\cos(c + dx)}} \, dx$$
$$= a^3 \left(\left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)} dx \right) a + 3 \left(\int \sqrt{\cos(dx + c)} dx \right) a + \left(\int \sqrt{\cos(dx + c)} dx \right) b + 3 \left(\int \sqrt{\cos(dx + c)} \cos(dx + c) \, dx \right) a + 3 \left(\int \sqrt{\cos(dx + c)} \cos(dx + c) \, dx \right) b + \left(\int \sqrt{\cos(dx + c)} \cos(dx + c)^3 \, dx \right) b + \left(\int \sqrt{\cos(dx + c)} \cos(dx + c)^2 \, dx \right) a + 3 \left(\int \sqrt{\cos(dx + c)} \cos(dx + c)^2 \, dx \right) b \right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + 3*int(sqrt(cos(c + d*x)),
x)*a + int(sqrt(cos(c + d*x)),x)*b + 3*int(sqrt(cos(c + d*x))*cos(c + d*x)
,x)*a + 3*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sqrt(cos(c + d*x)
)*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a + 3*i
nt(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.140 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 169

$$\begin{aligned}
&\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4a^3(5A+3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\
&\quad - \frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&\quad - \frac{2(5A-B)\sqrt{\cos(c+dx)}(a^3+a^3\cos(c+dx))\sin(c+dx)}{5d}
\end{aligned}$$

output
```
4/5*a^3*(5*A+9*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^3*(5*A+3*B
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d-4/15*a^3*(5*A-6*B)*cos(d*x+c)^(
1/2)*sin(d*x+c)/d+2*a*A*(a+a*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2
/5*(5*A-B)*cos(d*x+c)^(1/2)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.14 (sec) , antiderivative size = 888, normalized size of antiderivative = 5.25

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2)
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/40*((5*
A + 18*B + 15*A*Cos[2*c] + 18*B*Cos[2*c])*Csc[c]*Sec[c])/d + ((A + 3*B)*Co
s[d*x]*Sin[c])/(12*d) + (B*Cos[2*d*x]*Sin[2*c])/(40*d) + ((A + 3*B)*Cos[c]
*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(4*d) + (B*Cos[2*c]*S
in[2*d*x])/(40*d)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-
(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^3*Csc
[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
+ ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (9*B*(a + a*Cos[c + d*x])^3...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3454, 27, 3042, 3455, 3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$2\int \frac{(\cos(c+dx)a+a)^2(a(5A+B)-a(5A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(\cos(c+dx)a+a)^2(a(5A+B)-a(5A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(5A+B)-a(5A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{5}\int \frac{(\cos(c+dx)a+a)\left(a^2(10A+3B)-a^2(5A-6B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{2}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a^2(10A+3B)-a^2(5A-6B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3447

$$\frac{2}{5}\int \frac{-\left((5A-6B)\cos^2(c+dx)a^3\right)+(10A+3B)a^3+\left(a^3(10A+3B)-a^3(5A-6B)\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{2}{5}\int \frac{-\left((5A-6B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3\right)+(10A+3B)a^3+\left(a^3(10A+3B)-a^3(5A-6B)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3502

$$\frac{2}{5}\left(\frac{2}{3}\int \frac{5(5A+3B)a^3+3(5A+9B)\cos(c+dx)a^3}{2\sqrt{\cos(c+dx)}}dx-\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{5(5A+3B)a^3+3(5A+9B)\cos(c+dx)a^3}{\sqrt{\cos(c+dx)}}dx-\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{5(5A+3B)a^3+3(5A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{2}{5}\left(\frac{1}{3}\left(5a^3(5A+3B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(5A+9B)\int \sqrt{\cos(c+dx)}dx\right)-\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{c}}{3d}\right.$$
$$-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{1}{3}\left(5a^3(5A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3(5A+9B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{2a^3(5A-6B)\sin}{}\right.$$
$$-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{5}\left(\frac{1}{3}\left(5a^3(5A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{2a^3(5A-6B)\sin(c+dx)}{3d}\right.$$
$$\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}{5d}+$$
$$\frac{2}{5}\left(\frac{1}{3}\left(\frac{10a^3(5A+3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{2a^3(5A-6B)\sin(c+dx)}{3d}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (2*(5
*A - B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5*d) +
(2*(((6*a^3*(5*A + 9*B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(5*A + 3*B)
*EllipticF[(c + d*x)/2, 2])/d)/3 - (2*a^3*(5*A - 6*B)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(158) = 316$.

Time = 9.99 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.99

method	result
default	$-\frac{4a^3\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+10A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+42B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*a^3*(-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+42*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2
0*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-18*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}(5A+3B)a^3\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(5\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(5*A + 3*B)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(5*A + 3*B)*a^3*cos(d*x + c)
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(
5*A + 9*B)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(5*A + 9*B)*a^3*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))) - (3*B*a^3*cos(d*x + c)^2 + 5*(A + 3*B)*a^3*cos(d*x + c) + 15
*A*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + a\cos(c + dx))^3 (A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \frac{(a + a\cos(c + dx))^3 (A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 41.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{A a^3 \left(\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}$$
$$+\frac{6\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{6\,B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}$$
$$+\frac{4\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{2\,B\,a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}$$
$$+\frac{2\,A\,a^3\,\sin(c+dx)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+dx)^2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)^2}}$$
$$-\frac{2\,B\,a^3\cos(c+dx)^{7/2}\,\sin(c+dx)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos(c+dx)^2\right)}{7\,d\sqrt{\sin(c+dx)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(3/2),x)
```

output
```
(A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2
, 2))/3))/d + (6*A*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (6*A*a^3*ellipticF
(c/2 + (d*x)/2, 2))/d + (6*B*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*B*a^3
*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^3*cos(c + d*x)^(1/2)*sin(c + d*x)
)/d + (2*A*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(
d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(7/2)
*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d
*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= a^3\left(3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+3\left(\int \sqrt{\cos(dx+c)}dx\right)a+3\left(\int \sqrt{\cos(dx+c)}dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
a**3*(3*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))/
cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + 3*int(sq
rt(cos(c + d*x)),x)*a + 3*int(sqrt(cos(c + d*x)),x)*b + int(sqrt(cos(c + d
*x))*cos(c + d*x),x)*a + 3*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(
sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.141 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3(A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\\ &\quad -\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\\ &\quad +\frac{2(7A+3B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
-4*a^3*(A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/3*a^3*(A+B)*Invers
eJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d-4/3*a^3*(4*A+B)*cos(d*x+c)^(1/2)*sin(d*
x+c)/d+2/3*a*A*(a+a*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/3*(7*A+3
*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.47 (sec) , antiderivative size = 879, normalized size of antiderivative = 5.46

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2)
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/8*((-5*
A + B + A*Cos[2*c] + 3*B*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Cos[d*x]*Sin[c])/
(12*d) + (B*Cos[c]*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(
12*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(12
*d)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sq
rt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2
+ (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{3}\int \frac{(\cos(c+dx)a+a)^2(a(7A+3B)-3a(A-B)\cos(c+dx))}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(\cos(c+dx)a+a)^2(a(7A+3B)-3a(A-B)\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(7A+3B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{3}\left(2\int \frac{3(\cos(c+dx)a+a)\left(a^2(3A+2B)-a^2(4A+B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx+\frac{2(7A+3B)\sin(c+dx)\left(a^3\cos(c+\right.}{d\sqrt{\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(6\int \frac{(\cos(c+dx)a+a)\left(a^2(3A+2B)-a^2(4A+B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx+\frac{2(7A+3B)\sin(c+dx)\left(a^3\cos(c+dx\right.}{d\sqrt{\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(6\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a^2(3A+2B)-a^2(4A+B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(7A+3B)\sin(c+dx)\left(a\right.}{d\sqrt{\cos(c+}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{3}\left(6\int \frac{-\left((4A+B)\cos^2(c+dx)a^3\right)+(3A+2B)a^3+\left(a^3(3A+2B)-a^3(4A+B)\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2(7A+3}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(6\int \frac{-\left((4A+B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3\right)+(3A+2B)a^3+\left(a^3(3A+2B)-a^3(4A+B)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{3}\left(6\left(\frac{2}{3}\int \frac{5a^3(A+B)-3a^3(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx-\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2(7A+3B)\,\text{s}}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(6\left(\frac{1}{3}\int \frac{5a^3(A+B)-3a^3(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx-\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2(7A+3B)\,\text{s}}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(6\left(\frac{1}{3}\int \frac{5a^3(A+B)-3a^3(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2(7A+}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3227$$

$$\frac{1}{3}\left(6\left(\frac{1}{3}\left(5a^3(A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^3(A-B)\int \sqrt{\cos(c+dx)}dx\right)-\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\cos(}}{3d}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(6\left(\frac{1}{3}\left(5a^3(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3(A-B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{2a^3(4A+B)\sin(c\cdot}{3}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{3}\left(6\left(\frac{1}{3}\left(5a^3(A+B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{2a^3(4A+B)\sin(c+dx)\sqrt{c}}{3d}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2(7A+3B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d\sqrt{\cos(c+dx)}}+6\left(\frac{1}{3}\left(\frac{10a^3(A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^3(A-B)E}{}\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((2
*(7*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
 + 6*(((-6*a^3*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(A + B)*Elli
pticF[(c + d*x)/2, 2])/d)/3 - (2*a^3*(4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*d)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(150) = 300.

Time = 7.92 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.06

method	result
default	$-\frac{4\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(9A+5B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}{}$
parts	Expression too large to display

input

```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output

```
-4/3*(-4*B*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*
B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.39

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}(A+B)a^3\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}(A\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)^2*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A - B
)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(A - B)*a^3*cos(d*x + c)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))) - (B*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
x)
```

Mupad [B] (verification not implemented)

Time = 42.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.56

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$
$$+\frac{B\,a^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$
$$+\frac{6\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(5/2),x)
```

output
```
(2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*A*a^3*ellipticF(c/2 + (d*x)/2, 2
)))/d + (B*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 +
 (d*x)/2, 2))/3))/d + (6*B*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (6*B*a^3*e
llipticF(c/2 + (d*x)/2, 2))/d + (6*A*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2
], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (
2*A*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos
(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom(
[-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^
(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= a^3\left(3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b+\left(\int \sqrt{\cos(dx+c)}dx\right)a+3\left(\int \sqrt{\cos(dx+c)}dx\right)b+\left(\int \sqrt{\cos(dx+c)}\cos(dx+c)\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
a**3*(3*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + 3*int(sqrt(cos(c + d*x)
)/cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a + 3*int(
sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d*x))/cos(c + d
*x)**2,x)*b + int(sqrt(cos(c + d*x)),x)*a + 3*int(sqrt(cos(c + d*x)),x)*b
+ int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b)
```

3.142 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 171

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(3A+5B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}+\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(9A+5B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-4/5*a^3*(9*A+5*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^3*(3*A+5*
B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/15*a^3*(21*A+20*B)*sin(d*x+c
)/d/cos(d*x+c)^(1/2)+2/5*a*A*(a+a*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(5
/2)+2/15*(9*A+5*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.22 (sec) , antiderivative size = 890, normalized size of antiderivative = 5.20

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2)
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/40*((-3
6*A - 25*B + 5*B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^3*Sin
[d*x])/(20*d) + (Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 15*A*Sin[d*x] + 5*B*S
in[d*x]))/(60*d) + (Sec[c]*Sec[c + d*x]*(15*A*Sin[c] + 5*B*Sin[c] + 54*A*S
in[d*x] + 45*B*Sin[d*x]))/(60*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*
x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (9*A*(a + a*Co
s[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(S
qrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
 - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (B*(a + a...
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3454, 27, 3042, 3454, 3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{5}\int \frac{(\cos(c+dx)a+a)^2(a(9A+5B)-a(A-5B)\cos(c+dx))}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(\cos(c+dx)a+a)^2(a(9A+5B)-a(A-5B)\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(9A+5B)-a(A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{(\cos(c+dx)a+a)\left(a^2(21A+20B)-a^2(6A-5B)\cos(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2(9A+5B)\sin(c+dx)\left(a^3\cos(c\right.}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a^2(21A+20B)-a^2(6A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2(9A+5B)\sin(c+dx}{3d\cos^{\frac{3}{2}}(}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3447

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{-\left((6A-5B)\cos^2(c+dx)a^3\right)+(21A+20B)a^3+\left(a^3(21A+20B)-a^3(6A-5B)\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{-\left((6A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3\right)+(21A+20B)a^3+\left(a^3(21A+20B)-a^3(6A-5B)\right)\sin\left(c+dx+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3500

$$\frac{1}{5}\left(\frac{2}{3}\left(2\int \frac{5a^3(3A+5B)-3a^3(9A+5B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2a^3(21A+20B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2(9A+5B)\sin(c\cdot}{3d\,c}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{2}{3}\left(\int \frac{5a^3(3A+5B)-3a^3(9A+5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2a^3(21A+20B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2(9A+5B)\sin(c+}{3d\,co}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2}{3}\left(\int \frac{5a^3(3A+5B)-3a^3(9A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(21A+20B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2(9A+5B)\sin}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3227

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^3(3A+5B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^3(9A+5B)\int \sqrt{\cos(c+dx)}dx+\frac{2a^3(21A+20B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^3(3A+5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3(9A+5B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^3(21A+20B)\sin}{d\sqrt{\cos(c+dx}}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^3(3A+5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3(21A+20B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\left(\frac{10a^3(3A+5B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^3(9A+5B}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((2
*(9*A + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2
)) + (2*((-6*a^3*(9*A + 5*B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(3*A +
 5*B)*EllipticF[(c + d*x)/2, 2])/d + (2*a^3*(21*A + 20*B)*Sin[c + d*x])/(d
*Sqrt[Cos[c + d*x]])))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(158) = 316$.

Time = 9.07 (sec) , antiderivative size = 916, normalized size of antiderivative = 5.36

method	result	size
default	Expression too large to display	916
parts	Expression too large to display	946

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^3*(216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*A*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-108*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^4+180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-60*B*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^4-246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*A*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+108*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*sin(1/2*d*x+1/2*c)^2-190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+100
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+60*B*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1
5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.42

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(3A+5B)a^3\cos(dx+c)^3 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x +
c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(
2)*(9*A + 5*B)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 5*B)*a^3*co
s(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) - (9*(6*A + 5*B)*a^3*cos(d*x + c)^2 + 5*(3*A + B)*a^3
*cos(d*x + c) + 3*A*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
x)
```

Mupad [B] (verification not implemented)

Time = 43.60 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.68

$$\begin{aligned}
\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{2\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} \\
&+\frac{6\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&+\frac{6\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}
\end{aligned}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(7/2),x)
```

output
```
(2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*B*a^3*ellipticF(c/2 + (d*x)/2, 2
)))/d + (2*A*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*A*a^3*sin(c + d*x)*hy
pergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c +
d*x)^2)^(1/2)) + (2*A*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c +
 d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a^3*sin(c +
 d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2
)*(sin(c + d*x)^2)^(1/2)) + (6*B*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*
a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +
 d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= a^3\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b\right.\right.\\ &\quad+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a\\ &\quad+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a\\ &\quad\left.+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b+\left(\int \sqrt{\cos(dx+c)}dx\right)b\right)\end{aligned}$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + 3*int(sqrt(cos(c + d*x))/
cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a + 3*int(sq
rt(cos(c + d*x))/cos(c + d*x)**3,x)*a + int(sqrt(cos(c + d*x))/cos(c + d*x
)**3,x)*b + 3*int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + 3*int(sqrt(cos
(c + d*x))/cos(c + d*x)**2,x)*b + int(sqrt(cos(c + d*x)),x)*b)
```

3.143 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 204

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= -\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(13A+21B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ &+ \frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^3(7A+9B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} \\ &+ \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} \\ &+ \frac{2(11A+7B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output
```
-4/5*a^3*(7*A+9*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(13*A+
21*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/105*a^3*(41*A+42*B)*sin(d
*x+c)/d/cos(d*x+c)^(3/2)+4/5*a^3*(7*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2
/7*a*A*(a+a*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/35*(11*A+7*B)*(a
^3+a^3*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.53

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2)
,x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((7*A + 9*
B)*Csc[c]*Sec[c])/(10*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Se
c[c]*Sec[c + d*x]^3*(5*A*Sin[c] + 21*A*Sin[d*x] + 7*B*Sin[d*x]))/(140*d) +
 (Sec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 21*B*Sin[c] + 130*A*Sin[d*x] + 105*
B*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(130*A*Sin[c] + 105*B*Sin[c] +
 294*A*Sin[d*x] + 378*B*Sin[d*x]))/(420*d)) - (13*A*(a + a*Cos[c + d*x])^3
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos
[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (7
*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
+ Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin...
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3454, 27, 3042, 3454, 3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3454}$$

$$\frac{2}{7}\int \frac{(\cos(c+dx)a+a)^2(a(11A+7B)+a(A+7B)\cos(c+dx))}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\int \frac{(\cos(c+dx)a+a)^2(a(11A+7B)+a(A+7B)\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(11A+7B)+a(A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3454}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{(\cos(c+dx)a+a)\left((41A+42B)a^2+(8A+21B)\cos(c+dx)a^2\right)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2(11A+7B)\sin(c+dx)\left(a^3\cos\right.}{5d\cos^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((41A+42B)a^2+(8A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2(11A+7B)\sin(c+}{5d\cos}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3447

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{(8A+21B)\cos^2(c+dx)a^3+(41A+42B)a^3+\left((8A+21B)a^3+(41A+42B)a^3\right)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2(1}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{(8A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(41A+42B)a^3+\left((8A+21B)a^3+(41A+42B)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3500

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{2}{3}\int \frac{21(7A+9B)a^3+5(13A+21B)\cos(c+dx)a^3}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^3(41A+42B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2(11A+7B)\,\mathrm{s}}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{21(7A+9B)a^3+5(13A+21B)\cos(c+dx)a^3}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^3(41A+42B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2(11A+7B)\,\mathrm{s}}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{21(7A+9B)a^3+5(13A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^3(41A+42B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2(11A+\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(21a^3(7A+9B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+5a^3(13A+21B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2a^3(41A+42B)\sin(c\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(21a^3(7A+9B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+5a^3(13A+21B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^3(41A+}{3d\,c\ldots}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\frac{2(11A+7B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\left(\frac{2a^3(41A+42B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{10a^3(13A+21B)\,\mathrm{E}\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2
*(11*A + 7*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/
2)) + (2*((2*a^3*(41*A + 42*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((
10*a^3*(13*A + 21*B)*EllipticF[(c + d*x)/2, 2])/d + 21*a^3*(7*A + 9*B)*((-
2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))
/3))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(187) = 374$.

Time = 10.37 (sec) , antiderivative size = 902, normalized size of antiderivative = 4.42

method	result	size
default	Expression too large to display	902
parts	Expression too large to display	1056

input

```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETURNV
ERBOSE)
```

output

```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+(1/8*A+3/8*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/5*(3/8*A+1/8*B)/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(
1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(13A+21B)a^3\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d
*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I
*sqrt(2)*(7*A + 9*B)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(7*A + 9*B)
*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c))) - (42*(7*A + 9*B)*a^3*cos(d*x + c)^3 + 5*(26*A
 + 21*B)*a^3*cos(d*x + c)^2 + 21*(3*A + B)*a^3*cos(d*x + c) + 15*A*a^3)*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2),
x)
```

Mupad [B] (verification not implemented)

Time = 42.80 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.50

$$\int \frac{(a + a\cos(c + dx))^3(A + B\cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)}\,dx = \frac{2\,B\,a^3\,\mathrm{F}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|2\right)}{d}$$

$$+ \frac{\frac{2\,A\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)}{7} + \frac{6\,A\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)}{5} + 2\,A\,a^3\cos\left(c +\right.}{d\cos\left(c + d\right.}$$

$$+ \frac{6\,B\,a^3\,\sin\left(c + d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos\left(c + d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c + d\,x\right)}\,\sqrt{\sin\left(c + d\,x\right)^2}}$$

$$+ \frac{2\,B\,a^3\,\sin\left(c + d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos\left(c + d\,x\right)^2\right)}{d\cos\left(c + d\,x\right)^{3/2}\,\sqrt{\sin\left(c + d\,x\right)^2}}$$

$$+ \frac{2\,B\,a^3\,\sin\left(c + d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos\left(c + d\,x\right)^2\right)}{5\,d\cos\left(c + d\,x\right)^{5/2}\,\sqrt{\sin\left(c + d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(9/2),x)
```

output
```
(2*B*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + ((2*A*a^3*sin(c + d*x)*hypergeom
([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*A*a^3*cos(c + d*x)*sin(c + d*x
)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/5 + 2*A*a^3*cos(c + d*x)^2
*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 2*A*a^3*cos(c
+ d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (6*B*a^3*sin(c + d*x)*hyperge
om([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^
2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)
^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)
*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(si
n(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a + a\cos(c + dx))^3 (A + B\cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)}\,dx$$
$$= a^3 \left(\left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)} dx \right) b + \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^5} dx \right) a + 3 \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^4} dx \right) a + \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^4} dx \right) b + 3 \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^3} dx \right) a + 3 \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^3} dx \right) b + \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^2} dx \right) a + 3 \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^2} dx \right) b \right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
a**3*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b + int(sqrt(cos(c + d*x))/co
s(c + d*x)**5,x)*a + 3*int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a + int(s
qrt(cos(c + d*x))/cos(c + d*x)**4,x)*b + 3*int(sqrt(cos(c + d*x))/cos(c +
d*x)**3,x)*a + 3*int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*b + int(sqrt(co
s(c + d*x))/cos(c + d*x)**2,x)*a + 3*int(sqrt(cos(c + d*x))/cos(c + d*x)**
2,x)*b)
```

3.144 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 237

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = -\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+13B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(17A+21B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}+\frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{2(13A+9B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
-4/15*a^3*(17*A+21*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(11
*A+13*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+4/105*a^3*(23*A+24*B)*si
n(d*x+c)/d/cos(d*x+c)^(5/2)+4/21*a^3*(11*A+13*B)*sin(d*x+c)/d/cos(d*x+c)^(
3/2)+4/15*a^3*(17*A+21*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/9*a*A*(a+a*cos(d
*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(9/2)+2/63*(13*A+9*B)*(a^3+a^3*cos(d*x+c)
)*sin(d*x+c)/d/cos(d*x+c)^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.31 (sec) , antiderivative size = 967, normalized size of antiderivative = 4.08

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2
),x]
```

output
```
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((17*A + 2
1*B)*Csc[c]*Sec[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(36*d) + (
Sec[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 27*A*Sin[d*x] + 9*B*Sin[d*x]))/(252*d)
 + (Sec[c]*Sec[c + d*x]*(55*A*Sin[c] + 65*B*Sin[c] + 119*A*Sin[d*x] + 147*
B*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]^3*(135*A*Sin[c] + 45*B*Sin[c]
+ 238*A*Sin[d*x] + 189*B*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]^2*(238
*A*Sin[c] + 189*B*Sin[c] + 330*A*Sin[d*x] + 390*B*Sin[d*x]))/(1260*d)) - (
11*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 +
 Cot[c]^2]) - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(42*d*Sqrt[1 + Cot[c]^2]) + (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[
c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c...
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{9}\int \frac{(\cos(c+dx)a+a)^2(a(13A+9B)+3a(A+3B)\cos(c+dx))}{2\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{(\cos(c+dx)a+a)^2(a(13A+9B)+3a(A+3B)\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(13A+9B)+3a(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{3(\cos(c+dx)a+a)\left((23A+24B)a^2+5(2A+3B)\cos(c+dx)a^2\right)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2(13A+9B)\sin(c+dx)\left(a^3\cos\right.}{7d\cos^{\frac{7}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\left(\frac{6}{7}\int \frac{(\cos(c+dx)a+a)\left((23A+24B)a^2+5(2A+3B)\cos(c+dx)a^2\right)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2(13A+9B)\sin(c+dx)\left(a^3\cos\right.}{7d\cos^{\frac{7}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(\frac{6}{7}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((23A+24B)a^2+5(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2(13A+9B)\sin(c+}{7d\cos}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3447}$$

$$\frac{1}{9}\left(\frac{6}{7}\int \frac{5(2A+3B)\cos^2(c+dx)a^3+(23A+24B)a^3+\left(5(2A+3B)a^3+(23A+24B)a^3\right)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2(1}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(\frac{6}{7}\int \frac{5(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^3+(23A+24B)a^3+\left(5(2A+3B)a^3+(23A+24B)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{2}{5}\int \frac{15(11A+13B)a^3+7(17A+21B)\cos(c+dx)a^3}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^3(23A+24B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2(13A+9B}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\int\frac{15(11A+13B)a^3+7(17A+21B)\cos(c+dx)a^3}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^3(23A+24B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2(13A+9B\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\int\frac{15(11A+13B)a^3+7(17A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^3(23A+24B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2(13A\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3227

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(11A+13B)\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+7a^3(17A+21B)\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2a^3(23A+24B)\sin(c}{5d\cos^{\frac{5}{2}}(c+dx}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(11A+13B)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+7a^3(17A+21B)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2a^3(23A}{5d}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3116

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(11A+13B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7a^3(17A+21B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(11A+13B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7a^3(17A+21B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 3119

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(11A+13B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7a^3(17A+21B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 3120

$$\frac{1}{9}\left(\frac{2(13A+9B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6}{7}\left(\frac{2a^3(23A+24B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(15a^3(11A+13B)\left(\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^2}{9d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((2
*(13*A + 9*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/
2)) + (6*((2*a^3*(23*A + 24*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (1
5*a^3*(11*A + 13*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x]
)/(3*d*Cos[c + d*x]^(3/2))) + 7*a^3*(17*A + 21*B)*((-2*EllipticE[(c + d*x)
/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1150 vs. $2(216) = 432$.

Time = 12.63 (sec) , antiderivative size = 1151, normalized size of antiderivative = 4.86

method	result	size
default	Expression too large to display	1151
parts	Expression too large to display	1421

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RETURN
VERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*A*(
-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1
/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))))+1/8*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/8*A+3/8*B)*(-1/6*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/
2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))+(3/8*A+1/8*B)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5
/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(15i\sqrt{2}(11A+13B)a^3\cos(dx+c)^5\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorit
hm="fricas")
```

output
```
-2/315*(15*I*sqrt(2)*(11*A + 13*B)*a^3*cos(d*x + c)^5*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(11*A + 13*B)*a^3*cos
(d*x + c)^5*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21
*I*sqrt(2)*(17*A + 21*B)*a^3*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(17*A +
 21*B)*a^3*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - (42*(17*A + 21*B)*a^3*cos(d*x + c)^4 +
 30*(11*A + 13*B)*a^3*cos(d*x + c)^3 + 7*(34*A + 27*B)*a^3*cos(d*x + c)^2
+ 45*(3*A + B)*a^3*cos(d*x + c) + 35*A*a^3)*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorit
hm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
 x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$=\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorit
hm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
 x)
```

Mupad [B] (verification not implemented)

Time = 43.78 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.33

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(11/2),x)
```

output
```
(2*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2)*((19*A*a^3*sin(c + d*x))/(c
os(c + d*x)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)) + (9*A*a^3*sin(c + d*x))/(co
s(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (25*B*a^3*sin(c + d*x))/(co
s(c + d*x)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)) + (3*B*a^3*sin(c + d*x))/(cos
(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))))/(21*d) - (8*hypergeom([-1/4,
 1/2], 7/4, cos(c + d*x)^2)*((34*A*a^3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(
1 - cos(c + d*x)^2)^(1/2)) + (5*A*a^3*sin(c + d*x))/(cos(c + d*x)^(5/2)*(1
 - cos(c + d*x)^2)^(1/2)) + (27*B*a^3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(1
 - cos(c + d*x)^2)^(1/2))))/(135*d) + (8*((3*A*a^3*sin(c + d*x))/(cos(c +
d*x)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)) + (B*a^3*sin(c + d*x))/(cos(c + d*x
)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)))*hypergeom([-3/4, 1/2], 5/4, cos(c + d
*x)^2))/(21*d) + (2*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)*((136*A*a^
3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x)^2)^(1/2)) + (39*A*a^
3*sin(c + d*x))/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (5*A*a^3
*sin(c + d*x))/(cos(c + d*x)^(9/2)*(1 - cos(c + d*x)^2)^(1/2)) + (153*B*a^
3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x)^2)^(1/2)) + (27*B*a^
3*sin(c + d*x))/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))))/(45*d)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= a^3\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^6}dx\right)a+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)b+3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + 3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + 3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

output

```
a**3*(int(sqrt(cos(c + d*x))/cos(c + d*x)**6,x)*a + 3*int(sqrt(cos(c + d*x
))/cos(c + d*x)**5,x)*a + int(sqrt(cos(c + d*x))/cos(c + d*x)**5,x)*b + 3*
int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a + 3*int(sqrt(cos(c + d*x))/cos
(c + d*x)**4,x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a + 3*int(sq
rt(cos(c + d*x))/cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x
)**2,x)*b)
```

3.145 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 156

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(A-B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{5(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(5A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
-3/5*(5*A-7*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*(A-B)*Inverse
JacobiAM(1/2*d*x+1/2*c,2^(1/2))/a/d+5/3*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/
a/d-1/5*(5*A-7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d+(A-B)*cos(d*x+c)^(5/2)*s
in(d*x+c)/d/(a+a*cos(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.15 (sec) , antiderivative size = 946, normalized size of antiderivative = 6.06

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x
]
```

output
```
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((2*(5*A - 5*B + 10*A*Cos[c] - 16
*B*Cos[c])*Csc[c])/(5*d) + (4*(A - B)*Cos[d*x]*Sin[c])/(3*d) + (2*B*Cos[2*
d*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*
Sin[(d*x)/2]))/d + (4*(A - B)*Cos[c]*Sin[d*x])/(3*d) + (2*B*Cos[2*c]*Sin[2
*d*x])/(5*d)))/(a + a*Cos[c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (5*B*Cos[c/2 +
 (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c
]^2]) + (3*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-
1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2
))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(a ...
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-a(5A-7B)\cos(c+dx))dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cos^{\frac{3}{2}}(c+dx)(5a(A-B)-a(5A-7B)\cos(c+dx))dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a(A-B)-a(5A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{5a(A-B)\int \cos^{\frac{3}{2}}(c+dx)dx-a(5A-7B)\int \cos^{\frac{5}{2}}(c+dx)dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a(A-B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx - a(5A-7B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3115

$$\frac{5a(A-B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right) - a(5A-7B)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx + \frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{5a(A-B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right) - a(5A-7B)\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx + \frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3119

$$\frac{5a(A-B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right) - a(5A-7B)\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5d} + \frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3120

$$\frac{5a(A-B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right) - a(5A-7B)\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5d} + \frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + (5*a*
(A - B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d)) - a*(5*A - 7*B)*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 7.51 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(25A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+45\right.\right.}{15}$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1
/2*c)^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2
)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.72

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{2\left(6\,B\cos\left(dx+c\right)^{2}+2\,(5\,A-2\,B)\cos\left(dx+c\right)+25\,A-25\,B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25\left(\sqrt{2}(iA\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/30*(2*(6*B*cos(d*x + c)^2 + 2*(5*A - 2*B)*cos(d*x + c) + 25*A - 25*B)*sq
rt(cos(d*x + c))*sin(d*x + c) - 25*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqr
t(2)*(I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
) - 25*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(5*I*A - 7*
I*B)*cos(d*x + c) + sqrt(2)*(5*I*A - 7*I*B))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(sqrt(2)*(-5*I*A
+ 7*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 7*I*B))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x +
c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{\cos(c+d\,x)^{5/2}\,(A+B\,\cos(c+d\,x))}{a+a\,\cos(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)+1}dx\right)a}{a}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x)**3)/(cos(c + d*x) + 1),x)*b + int((s
qrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x) + 1),x)*a)/a
```

3.146 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 123

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(3A-5B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} - \frac{(3A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
3*(A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-1/3*(3*A-5*B)*InverseJac
obiAM(1/2*d*x+1/2*c,2^(1/2))/a/d-1/3*(3*A-5*B)*cos(d*x+c)^(1/2)*sin(d*x+c)
/a/d+(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.69 (sec) , antiderivative size = 893, normalized size of antiderivative = 7.26

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x
]
```

output
```
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*(1 + 2*Cos[c])*Csc[c
])/d + (4*B*Cos[d*x]*Sin[c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin
[(d*x)/2] - B*Sin[(d*x)/2]))/d + (4*B*Cos[c]*Sin[d*x])/(3*d)))/(a + a*Cos[
c + d*x]) + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c
+ d*x])*Sqrt[1 + Cot[c]^2]) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (3*A*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(a + a*Cos[c + d*x])) + (3*B*Cos[c/2
 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}...
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{1}{2}\sqrt{\cos(c+dx)}(3a(A-B)-a(3A-5B)\cos(c+dx))dx}{a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{\cos(c+dx)}(3a(A-B)-a(3A-5B)\cos(c+dx))dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)-a(3A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{3a(A-B)\int \sqrt{\cos(c+dx)}dx-a(3A-5B)\int \cos^{\frac{3}{2}}(c+dx)dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a(A-B)\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx - a(3A-5B)\int \sin \left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3115

$$\frac{3a(A-B)\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx - a(3A-5B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{3a(A-B)\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx - a(3A-5B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3119

$$\frac{\frac{6a(A-B)E\left(\frac{1}{2}(c+dx)|2\right)}{d} - a(3A-5B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$\Big\downarrow$ 3120

$$\frac{\frac{6a(A-B)E\left(\frac{1}{2}(c+dx)|2\right)}{d} - a(3A-5B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2} + \frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

input

```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output

```
((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((6*a
*(A - B)*EllipticE[(c + d*x)/2, 2])/d - a*(3*A - 5*B)*((2*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(118) = 236$.

Time = 5.50 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.13

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(3A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9A\,\mathrm{Elli}\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin}}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(
-3*A+7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.03

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{2\left(2\,B\cos\left(dx+c\right)-3\,A+5\,B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}(3i\,A-5i\,B)\cos\left(dx+c\right)+\sqrt{2}(3i\,A-\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/6*(2*(2*B*cos(d*x + c) - 3*A + 5*B)*sqrt(cos(d*x + c))*sin(d*x + c) + (s
qrt(2)*(3*I*A - 5*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 5*I*B))*weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-3*I*A + 5*I*B)
*cos(d*x + c) + sqrt(2)*(-3*I*A + 5*I*B))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)
*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*
A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{\cos(c+dx)^{3/2}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)+1}dx\right)b}{a}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x) + 1),x)*a + int((sqrt
(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x) + 1),x)*b)/a
```

3.147 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 85

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = -\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
-(A-3*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A-B)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))/a/d+(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x
+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.45 (sec) , antiderivative size = 862, normalized size of antiderivative = 10.14

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x
]
```

output
```
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(-A + B + 2*B*Cos[c])*Csc[c]
)/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d)
)/(a + a*Cos[c + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*
(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (A*Cos[c/2 +
(d*x)/2]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(a + a*Cos[c + d*x])) - (3*B*
Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(S...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3456, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)} \left(A+B\sin \left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin \left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\int \frac{a(A-B)-a(A-3B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a(A-B)-a(A-3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a(A-B)-a(A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{a(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-a(A-3B)\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(A-3B)\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{a(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(A-3B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{2a(A-B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{2a(A-3B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

output
```
((-2*a*(A - 3*B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A - B)*EllipticF[(c
+ d*x)/2, 2])/d)/(2*a^2) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a
 + a*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(86) = 172$.

Time = 4.46 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.87

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+A\,\mathrm{Ell}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1
/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= \frac{2\,(A-B)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}(-i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}(-i\,A+i\,B)\right)\mathrm{weierstrassPIn}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-I*A + I*B)*cos
(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + (sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B
))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-
I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 3*I*B))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(I*
A - 3*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 3*I*B))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c
) + a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{A\sqrt{\cos(c+dx)}}{\cos(c+dx)+1}\,dx + \int \frac{B\cos^{\frac{3}{2}}(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(Integral(A*sqrt(cos(c + d*x))/(cos(c + d*x) + 1), x) + Integral(B*cos(c +
 d*x)**(3/2)/(cos(c + d*x) + 1), x))/a
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{a+a\,\cos(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)b}{a}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x) + 1),x)*a + int((sqrt(cos(c + d*x))*
cos(c + d*x))/(cos(c + d*x) + 1),x)*b)/a
```

3.148 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 83

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
(A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A+B)*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/a/d-(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.04 (sec) , antiderivative size = 858, normalized size of antiderivative = 10.34

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x
]
```

output
```
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*Csc[c])/d - (2*Sec[c
/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d))/(a + a*Cos[c
 + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c +
 d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (A*Cos[c/2 + (d*x)/2]^2*Csc
[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(a + a*Cos[c + d*x])) + (B*Cos[c/2 + (d*x)/
2]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x ...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(A+B)+a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A+B)+a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A+B)+a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a(A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a(A-B)\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a(A-B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2a(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

output
```
((2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d
*x)/2, 2])/d)/(2*a^2) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a +
a*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(84) = 168.

Time = 2.90 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.93

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A\,\mathrm{Elli}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\mathrm{s}}}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2
*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.90

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = -\frac{2\,(A-B)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}(-i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}(-i\,A-i\,B)\right)\mathrm{weierstrassP}}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-I*A - I*B)*co
s(d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - (sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*
B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (sqrt(2)*(
I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - (sqrt(2)*(-I*A +
I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \frac{\int \frac{A}{\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx + \int \frac{B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

output
```
(Integral(A/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x) + Integral(B*co
s(c + d*x)/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x))/a
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2+\cos(dx+c)}dx\right)a}{a}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x) + 1),x)*b + int(sqrt(cos(c + d*x))/(
cos(c + d*x)**2 + cos(c + d*x)),x)*a)/a
```

3.149 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 119

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} + \frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

output
```
-(3*A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-B)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))/a/d+(3*A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)-(A-B)*si
n(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.19 (sec) , antiderivative size = 894, normalized size of antiderivative = 7.51

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x
]
```

output
```
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((2*A + A*Cos[c] - B*Cos[c])*Csc
[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2]
- B*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(a + a*Cos[c
 + d*x]) + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c +
 d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (3*A*Cos[c/2 + (d*x)/2]^2*C
sc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan
[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(a + a*Cos[c + d*x])) - (B*Cos[c/2 + (d*x
)/2]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d...
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(3A-B)-a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(3A-B)-a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(3A-B)-a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a(3A-B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx-a(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(3A-B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-a(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{a(3A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\cos(c+dx)}dx\right)-a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(3A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a(3A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a(3A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-\frac{2a(A-B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

output
```
-(((A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))) + ((
-2*a*(A - B)*EllipticF[(c + d*x)/2, 2])/d + a*(3*A - B)*((-2*EllipticE[(c
+ d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(118) = 236$.

Time = 3.44 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.68

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1
/2*c)^2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.44

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \frac{2\left(\left(3A-B\right)\cos\left(dx+c\right)+2A\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}(iA-iB)\cos\left(dx+c\right)^{2}+\sqrt{2}(iA-i\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/2*(2*((3*A - B)*cos(d*x + c) + 2*A)*sqrt(cos(d*x + c))*sin(d*x + c) + (s
qrt(2)*(I*A - I*B)*cos(d*x + c)^2 + sqrt(2)*(I*A - I*B)*cos(d*x + c))*weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-I*A + I
*B)*cos(d*x + c)^2 + sqrt(2)*(-I*A + I*B)*cos(d*x + c))*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-3*I*A + I*B)*cos(d*x
+ c)^2 + sqrt(2)*(-3*I*A + I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(3*I*A -
I*B)*cos(d*x + c)^2 + sqrt(2)*(3*I*A - I*B)*cos(d*x + c))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*co
s(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{3/2}\,(a+a\cos(c+dx))}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+\cos(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2+\cos(dx+c)}dx\right)b}{a}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**3 + cos(c + d*x)**2),x)*a + int(sqr
t(cos(c + d*x))/(cos(c + d*x)**2 + cos(c + d*x)),x)*b)/a
```

3.150 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 153

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

output
```
3*(A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+1/3*(5*A-3*B)*InverseJac
obiAM(1/2*d*x+1/2*c,2^(1/2))/a/d+1/3*(5*A-3*B)*sin(d*x+c)/a/d/cos(d*x+c)^(
3/2)-3*(A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)-(A-B)*sin(d*x+c)/d/cos(d*x+c)
^(3/2)/(a+a*cos(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.71 (sec) , antiderivative size = 931, normalized size of antiderivative = 6.08

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x
]
```

output
```
(Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(-(((A - B)*(2 + Cos[c])*Csc[c/2]
*Sec[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*
Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*
Sec[c + d*x]*(A*Sin[c] - 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)))/(a + a*Cos[
c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2
}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (3*A*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(a + a*Cos[c + d*x])) + (3*B*Cos[...
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(5A-3B)-3a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(5A-3B)-3a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A-3B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a(5A-3B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx-3a(A-B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(5A-3B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-3a(A-B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{a(5A-3B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a(A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(5A-3B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a(A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a(5A-3B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a(A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a(5A-3B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a(A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

output
```
-(((A - B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))) + (a
*(5*A - 3*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))) - 3*a*(A - B)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(146) = 292$.

Time = 4.56 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.05

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{(-2A+2B)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((-2*A+2*B)/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))+(A-B)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.09

$$\int\frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{2\left(9\,(A-B)\cos\left(dx+c\right)^2+2\,(2\,A-3\,B)\cos\left(dx+c\right)-2\,A\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}(-5i\right.\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/6*(2*(9*(A - B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c) - 2*A)*sqrt
(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-5*I*A + 3*I*B)*cos(d*x + c)^3 + s
qrt(2)*(-5*I*A + 3*I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - (sqrt(2)*(5*I*A - 3*I*B)*cos(d*x + c)^3 + sqrt(2
)*(5*I*A - 3*I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c)^3 + sqrt(2)*(-I*A
 + I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) + 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^3 +
sqrt(2)*(I*A - I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^3 + a*d*cos
(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx=\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{5/2}\,(a+a\,\cos(c+d\,x))}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+\cos(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+\cos(dx+c)^2}dx\right)b}{a}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**4 + cos(c + d*x)**3),x)*a + int(sqr
t(cos(c + d*x))/(cos(c + d*x)**3 + cos(c + d*x)**2),x)*b)/a
```

3.151 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 203

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = -\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} + \frac{5(2A-3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{5(2A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7(5A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{(2A-3B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
-7/5*(5*A-8*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-3*B)*I
nverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d+5/3*(2*A-3*B)*cos(d*x+c)^(1/2)
*sin(d*x+c)/a^2/d-7/15*(5*A-8*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d+(2*A-3*
B)*cos(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*cos(d*x+c)^(
7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.93 (sec) , antiderivative size = 1024, normalized size of antiderivative = 5.04

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2
,x]
```

output
```
(-20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x]
)^2*Sqrt[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqr
t[Cos[c + d*x]]*((4*(15*A - 20*B + 20*A*Cos[c] - 36*B*Cos[c])*Csc[c])/(5*d
) + (8*(A - 2*B)*Cos[d*x]*Sin[c])/(3*d) + (4*B*Cos[2*d*x]*Sin[2*c])/(5*d)
+ (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 4*B*Sin[(d*x)/2]))/d
- (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d
) + (8*(A - 2*B)*Cos[c]*Sin[d*x])/(3*d) + (4*B*Cos[2*c]*Sin[2*d*x])/(5*d)
- (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
 + (7*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos...
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3456, 27, 3042, 3456, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-a(5A-11B)\cos(c+dx))}{2(\cos(c+dx)a+a)}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-a(5A-11B)\cos(c+dx))}{\cos(c+dx)a+a}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(7a(A-B)-a(5A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \cos^{\frac{3}{2}}(c+dx)\left(15a^2(2A-3B)-7a^2(5A-8B)\cos(c+dx)\right)dx}{a^2}+\frac{6(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(15a^2(2A-3B)-7a^2(5A-8B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{6(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3227

$$\frac{\frac{15a^2(2A-3B)\int \cos^{\frac{3}{2}}(c+dx)dx-7a^2(5A-8B)\int \cos^{\frac{5}{2}}(c+dx)dx}{a^2}+\frac{6(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{15a^2(2A-3B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-7a^2(5A-8B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{a^2}+\frac{6(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3115

$$\frac{\frac{15a^2(2A-3B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^2(5A-8B)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{6(2A-3B)\sin(c+}{d(\cos(c+}}{6a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{15a^2(2A-3B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^2(5A-8B)\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{6(2A-3B}{}}{6a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{15a^2(2A-3B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^2(5A-8B)\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{6(2A-3B)\sin(c-}{d(\cos(c+}}{6a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{15a^2(2A-3B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^2(5A-8B)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{6(2A-3B)\sin(c+}{d(\cos(c+}}{6a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (
(6*(2*A - 3*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + (
15*a^2*(2*A - 3*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(3*d)) - 7*a^2*(5*A - 8*B)*((6*EllipticE[(c + d*x)/2,
2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(188) = 376$.

Time = 10.41 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.29

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(96B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+80A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-352B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+60A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+100A\right.}{}$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*B*cos(1/
2*d*x+1/2*c)^10+80*A*cos(1/2*d*x+1/2*c)^8-352*B*cos(1/2*d*x+1/2*c)^8+60*A*
cos(1/2*d*x+1/2*c)^6+100*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3
+210*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+120*B*cos(1/2*d*x+1
/2*c)^6-150*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*B*cos(1/
2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-240*A*cos(1/2*d*x+1/2*c)^4+266*B
*cos(1/2*d*x+1/2*c)^4+105*A*cos(1/2*d*x+1/2*c)^2-135*B*cos(1/2*d*x+1/2*c)^
2-5*A+5*B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.89

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{2\left(6\,B\cos\left(dx+c\right)^{3}+2\left(5\,A-4\,B\right)\cos\left(dx+c\right)^{2}+\left(65\,A-94\,B\right)\cos\left(dx+c\right)+50\,A-75\,B\right)\sqrt{\cos\left(dx\right.}}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/30*(2*(6*B*cos(d*x + c)^3 + 2*(5*A - 4*B)*cos(d*x + c)^2 + (65*A - 94*B)
*cos(d*x + c) + 50*A - 75*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 25*(sqrt(2)
*(2*I*A - 3*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(2*I*A - 3*I*B)*cos(d*x + c) +
 sqrt(2)*(2*I*A - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 25*(sqrt(2)*(-2*I*A + 3*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-2*I*
A + 3*I*B)*cos(d*x + c) + sqrt(2)*(-2*I*A + 3*I*B))*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*(sqrt(2)*(5*I*A - 8*I*B)*cos(d*x
+ c)^2 + 2*sqrt(2)*(5*I*A - 8*I*B)*cos(d*x + c) + sqrt(2)*(5*I*A - 8*I*B))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 21*(sqrt(2)*(-5*I*A + 8*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-5*I*A
 + 8*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 8*I*B))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^{7/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^2}\,dx$$

input
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^4}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a}{a^2}$$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x)**4)/(cos(c + d*x)**2 + 2*cos(c + d*x
) + 1),x)*b + int((sqrt(cos(c + d*x))*cos(c + d*x)**3)/(cos(c + d*x)**2 +
2*cos(c + d*x) + 1),x)*a)/a**2
```

3.152 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 166

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5(A-2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} - \frac{5(A-2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{(4A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output

```
(4*A-7*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*(A-2*B)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d-5/3*(A-2*B)*cos(d*x+c)^(1/2)*sin(d*x+
c)/a^2/d+1/3*(4*A-7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))+1/
3*(A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.44 (sec) , antiderivative size = 980, normalized size of antiderivative = 5.90

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2
,x]
```

output
```
(10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sq
rt[Cos[c + d*x]]*((-4*(2*A - 3*B + 2*A*Cos[c] - 4*B*Cos[c])*Csc[c])/d + (8
*B*Cos[d*x]*Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/
2] - 3*B*Sin[(d*x)/2]))/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/
2] - B*Sin[(d*x)/2]))/(3*d) + (8*B*Cos[c]*Sin[d*x])/(3*d) + (2*(A - B)*Sec
[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2 - (4*A*Cos[c/2
+ (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Co
s[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos...
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-3a(A-3B)\cos(c+dx))}{2(\cos(c+dx)a+a)}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-3a(A-3B)\cos(c+dx))}{\cos(c+dx)a+a}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a(A-B)-3a(A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int 3\sqrt{\cos(c+dx)}\left(a^2(4A-7B)-5a^2(A-2B)\cos(c+dx)\right)dx}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \sqrt{\cos(c+dx)}\left(a^2(4A-7B)-5a^2(A-2B)\cos(c+dx)\right)dx}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^2(4A-7B)-5a^2(A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3227

$$\frac{\frac{3\left(a^2(4A-7B)\int \sqrt{\cos(c+dx)}dx-5a^2(A-2B)\int \cos^{\frac{3}{2}}(c+dx)dx\right)}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(a^2(4A-7B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2(A-2B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3115

$$\frac{\frac{3\left(a^2(4A-7B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2(A-2B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(a^2(4A-7B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2(A-2B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{3\left(\frac{2a^2(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^2(A-2B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{3\left(\frac{2a^2(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^2(A-2B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{2(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (
(2*(4*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + (
3*((2*a^2*(4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/d - 5*a^2*(A - 2*B)*((2*E
llipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
))))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(155) = 310$.

Time = 8.92 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.62

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-16B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{Ellipt}\right.}{}$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2
*d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*co
s(1/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12
*B*cos(1/2*d*x+1/2*c)^6-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)
^3-42*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1
/2*c)^4+48*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d
*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.21

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{2\left(2\,B\cos\left(dx+c\right)^2-(6\,A-13\,B)\cos\left(dx+c\right)-5\,A+10\,B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(\sqrt{2}(-i\,A\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/6*(2*(2*B*cos(d*x + c)^2 - (6*A - 13*B)*cos(d*x + c) - 5*A + 10*B)*sqrt(
cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c)^2 + 2*
sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 2*I*B))*weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(sqrt(2)*(I*A - 2*I*B)*c
os(d*x + c)^2 + 2*sqrt(2)*(I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 2*I*
B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)
*(-4*I*A + 7*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x + c)
 + sqrt(2)*(-4*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(4*I*A - 7*I*B)*cos(d*x
+ c)^2 + 2*sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(4*I*A - 7*I*B))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^{5/2}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a}{a^2}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x)**3)/(cos(c + d*x)**2 + 2*cos(c + d*x
) + 1),x)*b + int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)**2 +
2*cos(c + d*x) + 1),x)*a)/a**2
```

3.153 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{(2A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
-(A-4*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(2*A-5*B)*Inverse
JacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d+1/3*(2*A-5*B)*cos(d*x+c)^(1/2)*sin(d
*x+c)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*co
s(d*x+c))^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.18 (sec) , antiderivative size = 945, normalized size of antiderivative = 6.95

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2
,x]
```

output
```
(-4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sq
rt[Cos[c + d*x]]*((-4*(-A + 2*B + 2*B*Cos[c])*Csc[c])/d + (4*Sec[c/2]*Sec[
c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/
2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) - (2*(A - B)*Sec[c
/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2 + (A*Cos[c/2 + (d
*x)/2]^4*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*
x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(a + a*Cos[c + d*x])^2) - (4*B...
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3042, 3456, 27, 3042, 3456, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)-a(A-7B)\cos(c+dx))}{2(\cos(c+dx)a+a)}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)-a(A-7B)\cos(c+dx))}{\cos(c+dx)a+a}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)-a(A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{a^2(2A-5B)-3a^2(A-4B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{2(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(2A-5B)-3a^2(A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}+\frac{2(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{a^2(2A-5B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^2(A-4B)\int \sqrt{\cos(c+dx)}dx}{a^2}+\frac{2(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(2A-5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^2(A-4B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{2(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{a^2(2A-5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^2(A-4B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}+\frac{2(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2a^2(2A-5B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^2(A-4B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a^2}+\frac{2(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (
((-6*a^2*(A - 4*B)*EllipticE[(c + d*x)/2, 2])/d + (2*a^2*(2*A - 5*B)*Ellip
ticF[(c + d*x)/2, 2])/d)/a^2 + (2*(2*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(d*(1 + Cos[c + d*x])))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(129) = 258.

Time = 5.54 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.10

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}}{}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(
1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B*cos(1/2*d*x+1/2*c)^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2
*c)^4+9*A*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*
d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.59

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{2\left(3\left(A-2B\right)\cos\left(dx+c\right)+2A-5B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}(-2i\,A+5i\,B)\cos\left(dx+c\right)^{2}-\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/6*(2*(3*(A - 2*B)*cos(d*x + c) + 2*A - 5*B)*sqrt(cos(d*x + c))*sin(d*x +
 c) + (sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A - 5*I*B)
*cos(d*x + c) + sqrt(2)*(-2*I*A + 5*I*B))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (sqrt(2)*(2*I*A - 5*I*B)*cos(d*x + c)^2 - 2*sq
rt(2)*(-2*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*(2*I*A - 5*I*B))*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(I*A - 4*I*B)*
cos(d*x + c)^2 + 2*sqrt(2)*(I*A - 4*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 4*I
*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 3*(sqrt(2)*(-I*A + 4*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-I*A
+ 4*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 4*I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^2}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b}{a^2}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) +
 1),x)*a + int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)**2 + 2*c
os(c + d*x) + 1),x)*b)/a**2
```

3.154 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 121

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = -\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A+2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
-B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(A+2*B)*InverseJacobiAM
(1/2*d*x+1/2*c,2^(1/2))/a^2/d+B*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(1+cos(d
*x+c))+1/3*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.78 (sec) , antiderivative size = 694, normalized size of antiderivative = 5.74

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2
,x]
```

output
```
(-2*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqr
t[Cos[c + d*x]]*((4*B*Csc[c])/d + (4*B*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*
x)/2])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/
2]))/(3*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[
c + d*x])^2 + (B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[
1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[
c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(a +
 a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3456, 27, 3042, 3457, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{a(A-B)+a(A+5B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-B)+a(A+5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-B)+a(A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{a^2(A+2B)-3a^2B\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{6B\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(A+2B)-3a^2B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}+\frac{6B\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{a^2(A+2B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^2B\int \sqrt{\cos(c+dx)}dx}{a^2}+\frac{6B\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^2B\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6B\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{a^2(A+2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}+\frac{6B\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2a^2(A+2B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}+\frac{6B\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

output
```
((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (
((-6*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (2*a^2*(A + 2*B)*EllipticF[(c +
d*x)/2, 2])/d)/a^2 + (6*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(1 + Cos[c +
 d*x])))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(116) = 232.

Time = 4.80 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.89

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3+12\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3
*A*cos(1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*
c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{2\left(3\,B\cos\left(dx+c\right)+A+2\,B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}(-i\,A-2i\,B)\cos\left(dx+c\right)^2-2\,\sqrt{2}(i\,A\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/6*(2*(3*B*cos(d*x + c) + A + 2*B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqr
t(2)*(-I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(I*A + 2*I*B)*cos(d*x + c)
+ sqrt(2)*(-I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + (sqrt(2)*(I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*A - 2*I*
B)*cos(d*x + c) + sqrt(2)*(I*A + 2*I*B))*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*B*cos(d*x + c)^2 + 2*I*sqrt(2)*B*c
os(d*x + c) + I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*B*cos(d*x + c)^2 - 2*I*
sqrt(2)*B*cos(d*x + c) - I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sqrt{\cos{(dx+c)}}}{(a\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sqrt{\cos{(dx+c)}}}{(a\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(c+d\,x)}}\,(A+B\,\cos{(c+d\,x)})}{(a+a\,\cos{(c+d\,x)})^2}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b}{a^2}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a + int(
(sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x
)*b)/a**2
```

3.155 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 121

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A+B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} - \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
A*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(2*A+B)*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2))/a^2/d-A*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(1+cos(d*
x+c))-1/3*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.05 (sec) , antiderivative size = 695, normalized size of antiderivative = 5.74

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2)
,x]
```

output
```
(-4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqr
t[Cos[c + d*x]]*((-4*A*Csc[c])/d - (4*A*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d
*x)/2])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)
/2]))/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos
[c + d*x])^2 - (A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((HypergeometricP
FQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
 (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin
[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(a
+ a*Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3042, 3457, 27, 3042, 3457, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(5A+B)-a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(5A+B)-a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A+B)-a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{(2A+B)a^2+3A\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{6A\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(2A+B)a^2+3A\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{6A\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{a^2(2A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2A\int \sqrt{\cos(c+dx)}dx}{a^2}-\frac{6A\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(2A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2A\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{6A\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{a^2(2A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{6A\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2a^2(2A+B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{6A\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

output
```
-1/3*((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2)
+ (((6*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (2*a^2*(2*A + B)*EllipticF[(c
+ d*x)/2, 2])/d)/a^2 - (6*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(1 + Cos[c
 + d*x])))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(116) = 232.

Time = 4.22 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.89

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1
/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^3-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*
A*cos(1/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c
)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.63

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = -\frac{2\left(3A\cos(dx+c)+4A-B\right)\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(-2iA-iB)\cos(dx+c)^{2}-2\sqrt{2}(2\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
-1/6*(2*(3*A*cos(d*x + c) + 4*A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sq
rt(2)*(-2*I*A - I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A + I*B)*cos(d*x + c)
 + sqrt(2)*(-2*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) - (sqrt(2)*(2*I*A + I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-2*I*A - I
*B)*cos(d*x + c) + sqrt(2)*(2*I*A + I*B))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*A*cos(d*x + c)^2 - 2*I*sqrt(2)*A
*cos(d*x + c) - I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*A*cos(d*x + c)^2 + 2*I
*sqrt(2)*A*cos(d*x + c) + I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^2\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b}{a^2}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**3 + 2*cos(c + d*x)**2 + cos(c + d*x
)),x)*a + int(sqrt(cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)
*b)/a**2
```

3.156 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 168

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = &-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}\\ &-\frac{(5A-2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}\\ &+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}\\ &-\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))}\\ &-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\end{aligned}$$

output
```
-(4*A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*(5*A-2*B)*Inverse
JacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d+(4*A-B)*sin(d*x+c)/a^2/d/cos(d*x+c)^
(1/2)-1/3*(5*A-2*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(1+cos(d*x+c))-1/3*(
A-B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.23 (sec) , antiderivative size = 979, normalized size of antiderivative = 5.83

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
,x]
```

output
```
(10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqr
t[Cos[c + d*x]]*((2*(2*A + 2*A*Cos[c] - B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c]
)/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/
(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - B*Sin[(d*x)/2])
)/d + (8*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*(A - B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2 + (4*A*Cos[c/2 + (d*x)/2]^4*Cs
c[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[
Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[T...
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3457, 27, 3042, 3457, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(7A-B)-3a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(7A-B)-3a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(7A-B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{3a^2(4A-B)-a^2(5A-2B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3a^2(4A-B)-a^2(5A-2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{3a^2(4A-B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx-a^2(5A-2B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3a^2(4A-B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-a^2(5A-2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{3a^2(4A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)-a^2(5A-2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3a^2(4A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-a^2(5A-2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3a^2(4A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-a^2(5A-2B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{3a^2(4A-B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{2a^2(5A-2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{2(5A-2B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

output
```
-1/3*((A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2)
+ ((-2*(5*A - 2*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x]))
 + ((-2*a^2*(5*A - 2*B)*EllipticF[(c + d*x)/2, 2])/d + 3*a^2*(4*A - B)*((-
2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))
/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(159) = 318$.

Time = 4.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.94

method	result
default	$-\frac{2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(5A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\right.\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*
x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.42

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{2\left(3\left(4A-B\right)\cos\left(dx+c\right)^2+\left(19A-4B\right)\cos\left(dx+c\right)+6A\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}(5iA-\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/6*(2*(3*(4*A - B)*cos(d*x + c)^2 + (19*A - 4*B)*cos(d*x + c) + 6*A)*sqrt
(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^3 - 2*
sqrt(2)*(-5*I*A + 2*I*B)*cos(d*x + c)^2 + sqrt(2)*(5*I*A - 2*I*B)*cos(d*x
+ c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)
*(-5*I*A + 2*I*B)*cos(d*x + c)^3 - 2*sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^
2 + sqrt(2)*(-5*I*A + 2*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(4*I*A - I*B)*cos(d*x + c)^3 + 2*s
qrt(2)*(4*I*A - I*B)*cos(d*x + c)^2 + sqrt(2)*(4*I*A - I*B)*cos(d*x + c))*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 3*(sqrt(2)*(-4*I*A + I*B)*cos(d*x + c)^3 + 2*sqrt(2)*(-4*I*A + I
*B)*cos(d*x + c)^2 + sqrt(2)*(-4*I*A + I*B)*cos(d*x + c))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*
cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^2}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^2}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+a\,\cos(c+d\,x))^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)}dx\right)b}{a^2}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**4 + 2*cos(c + d*x)**3 + cos(c + d*x
)**2),x)*a + int(sqrt(cos(c + d*x))/(cos(c + d*x)**3 + 2*cos(c + d*x)**2 +
 cos(c + d*x)),x)*b)/a**2
```

3.157 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 201

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{(7A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

output
```
(7*A-4*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-B)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d+5/3*(2*A-B)*sin(d*x+c)/a^2/d/cos(d*x+
c)^(3/2)-(7*A-4*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)-1/3*(7*A-4*B)*sin(d*x
+c)/a^2/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)/d/cos(d*x+c
)^(3/2)/(a+a*cos(d*x+c))^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.85 (sec) , antiderivative size = 1020, normalized size of antiderivative = 5.07

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2)
,x]
```

output
```
(-20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x]
)^2*Sqrt[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*S
qrt[Cos[c + d*x]]*((-2*(4*A - 2*B + 3*A*Cos[c] - 2*B*Cos[c])*Csc[c/2]*Sec[
c/2]*Sec[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 2*B*Si
n[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[
(d*x)/2]))/(3*d) + (8*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*
Sec[c + d*x]*(A*Sin[c] - 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d) - (2*(A - B)*
Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2 - (7*A*Cos[c
/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x...
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{3a(3A-B)-5a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a(3A-B)-5a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a(3A-B)-5a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{3\left(5a^2(2A-B)-a^2(7A-4B)\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{5a^2(2A-B)-a^2(7A-4B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\int \frac{5a^2(2A-B)-a^2(7A-4B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3227

$$\frac{\frac{3\left(5a^2(2A-B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx-a^2(7A-4B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(5a^2(2A-B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-a^2(7A-4B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3116

$$\frac{\frac{3\left(5a^2(2A-B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-a^2(7A-4B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right)}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(5a^2(2A-B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-a^2(7A-4B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)\right)}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+}}{6a^2}$$

$$\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{3\left(5a^2(2A-B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-a^2(7A-4B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-$$

$$\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{3\left(5a^2(2A-B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-a^2(7A-4B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{a^2}-\frac{2(7A-4B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-$$

$$\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

output
```
-1/3*((A - B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
+ ((-2*(7*A - 4*B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x]))
 + (3*(5*a^2*(2*A - B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2))) - a^2*(7*A - 4*B)*((-2*EllipticE[(c + d*x)/
2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(188) = 376.

Time = 5.44 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.60

method	result	size
default	Expression too large to display	723

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*((-8*A+
4*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))+1/3*(A-B)*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(
1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(sin(1/2*d*x+1/2*c)^2-1)+(4*A-2*B)*(cos(1/2*d*x+1/2*c)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.17

$$\int \frac{A + B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx =$$
$$-\frac{2\left(3\,(7\,A-4\,B)\cos\left(dx+c\right)^3+(32\,A-19\,B)\cos\left(dx+c\right)^2+2\,(4\,A-3\,B)\cos\left(dx+c\right)-2\,A\right)\sqrt{\cos}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
-1/6*(2*(3*(7*A - 4*B)*cos(d*x + c)^3 + (32*A - 19*B)*cos(d*x + c)^2 + 2*(
4*A - 3*B)*cos(d*x + c) - 2*A)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2
)*(2*I*A - I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(2*I*A - I*B)*cos(d*x + c)^3 +
sqrt(2)*(2*I*A - I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-2*I*A + I*B)*cos(d*x + c)^4 + 2*sqrt(
2)*(-2*I*A + I*B)*cos(d*x + c)^3 + sqrt(2)*(-2*I*A + I*B)*cos(d*x + c)^2)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-7
*I*A + 4*I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(-7*I*A + 4*I*B)*cos(d*x + c)^3 +
 sqrt(2)*(-7*I*A + 4*I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(7*I*A - 4*
I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(7*I*A - 4*I*B)*cos(d*x + c)^3 + sqrt(2)*(
7*I*A - 4*I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*co
s(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5+2\cos(dx+c)^4+\cos(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2}dx\right)b}{a^2}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**5 + 2*cos(c + d*x)**4 + cos(c + d*x
)**3),x)*a + int(sqrt(cos(c + d*x))/(cos(c + d*x)**4 + 2*cos(c + d*x)**3 +
 cos(c + d*x)**2),x)*b)/a**2
```

3.158 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 254

$$
\begin{aligned}
\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = & -\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} \\
& +\frac{(11A-21B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} \\
& +\frac{(11A-21B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} \\
& -\frac{7(17A-33B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30a^3d} \\
& +\frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
& +\frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
& +\frac{3(11A-21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}
\end{aligned}
$$

output
```
-7/10*(17*A-33*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/2*(11*A-21
*B)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d+1/2*(11*A-21*B)*cos(d*x+c
)^(1/2)*sin(d*x+c)/a^3/d-7/30*(17*A-33*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^3/
d+1/5*(A-B)*cos(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(7*A-12*
B)*cos(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+3/10*(11*A-21*B)*cos
(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.97 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.90

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{\sqrt{\cos(c+dx)}\csc^5(c+dx)\left(-396A+396B-680A\cos(c+dx)+680B\cos(c+dx)+792A\cos^2(c+\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3
,x]
```

output
```
-1/30*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^5*(-396*A + 396*B - 680*A*Cos[c + d
*x] + 680*B*Cos[c + d*x] + 792*A*Cos[c + d*x]^2 - 792*B*Cos[c + d*x]^2 + 6
80*A*Cos[c + d*x]^3 - 680*B*Cos[c + d*x]^3 - 440*A*Cos[c + d*x]^4 + 440*B*
Cos[c + d*x]^4 - 180*A*Cos[c + d*x]^5 + 180*B*Cos[c + d*x]^5 + 20*A*Cos[c
+ d*x]^6 - 20*B*Cos[c + d*x]^6 + 66*A*Sin[c + d*x]^2 + 234*B*Sin[c + d*x]^
2 + 448*B*Cos[c + d*x]*Sin[c + d*x]^2 + 40*B*Cos[c + d*x]^4*Sin[c + d*x]^2
 - 12*B*Cos[c + d*x]^5*Sin[c + d*x]^2 + 165*A*Sin[c + d*x]^4 - 315*B*Sin[c
 + d*x]^4 + 15*(11*A - 21*B)*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]
^2]*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] - 448*B*Cos[c + d*x]*Hypergeometri
c2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] +
680*A*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sin[c
+ d*x]^4*Sqrt[Sin[c + d*x]^2] - 680*B*Cos[c + d*x]*Hypergeometric2F1[3/4,
7/2, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] - 90*B*Sin[2
*(c + d*x)]^2 - 24*B*Csc[c + d*x]*Sin[2*(c + d*x)]^3))/(a^3*d)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3456, 27, 3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)(9a(A-B)-5a(A-3B)\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)(9a(A-B)-5a(A-3B)\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(9a(A-B)-5a(A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(7a^2(7A-12B)-5a^2(10A-21B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{2a(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(7a^2(7A-12B)-5a^2(10A-21B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{2a(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3456

$$\frac{\frac{\frac{\int \frac{5}{2}\cos^{\frac{3}{2}}(c+dx)\left(9a^3(11A-21B)-7a^3(17A-33B)\cos(c+dx)\right)dx}{a^2}+\frac{9a^2(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{5\int \cos^{\frac{3}{2}}(c+dx)\left(9a^3(11A-21B)-7a^3(17A-33B)\cos(c+dx)\right)dx}{2a^2}+\frac{9a^2(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9a^3(11A-21B)-7a^3(17A-33B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{9a^2(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3227

$$\frac{\frac{\frac{5\left(9a^3(11A-21B)\int \cos^{\frac{3}{2}}(c+dx)dx-7a^3(17A-33B)\int \cos^{\frac{5}{2}}(c+dx)dx\right)}{2a^2}+\frac{9a^2(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{5\left(9a^3(11A-21B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-7a^3(17A-33B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{2a^2}+\frac{9a^2(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(7A-12B)\sin(c+dx)\cos^{\frac{7}{2}}}{3d(a\cos(c+dx)+a)^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5\left(9a^3(11A-21B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^3(17A-33B)\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)}{2a^2}+\frac{9a^2(11A-21B)\sin(c+}{d(a\cos(c+dx}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5\left(9a^3(11A-21B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^3(17A-33B)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)}{2a^2}+\frac{9a^2(11A-21}{d(a}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{5\left(9a^3(11A-21B)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^3(17A-33B)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)}{2a^2}+\frac{9a^2(11A-21B)\sin(}{d(a\cos(c}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{9a^2(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\left(9a^3(11A-21B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a^3(17A-33B)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+}{}\right.\right.}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (
(2*a*(7*A - 12*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x
])^2) + ((9*a^2*(11*A - 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*C
os[c + d*x])) + (5*(9*a^3*(11*A - 21*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)) - 7*a^3*(17*A - 33*B)*((6*
EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*
d))))/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(233) = 466.

Time = 18.81 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(192B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+160A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-864B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+468A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\right.}{}$

input
```
int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*B*cos(1
/2*d*x+1/2*c)^12+160*A*cos(1/2*d*x+1/2*c)^10-864*B*cos(1/2*d*x+1/2*c)^10+4
68*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2
*c)^5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-228*B*cos(1/2*
d*x+1/2*c)^8-630*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-1386*B*
cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1/2*c)^
6+1590*B*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-744*B*cos(1/2*d*x
+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2+57*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3
/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.95

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
1/60*(2*(12*B*cos(d*x + c)^4 + 4*(5*A - 6*B)*cos(d*x + c)^3 + 3*(79*A - 14
7*B)*cos(d*x + c)^2 + 2*(188*A - 357*B)*cos(d*x + c) + 165*A - 315*B)*sqrt
(cos(d*x + c))*sin(d*x + c) - 15*(sqrt(2)*(11*I*A - 21*I*B)*cos(d*x + c)^3
 + 3*sqrt(2)*(11*I*A - 21*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(11*I*A - 21*I*B
)*cos(d*x + c) + sqrt(2)*(11*I*A - 21*I*B))*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) - 15*(sqrt(2)*(-11*I*A + 21*I*B)*cos(d*x + c)^
3 + 3*sqrt(2)*(-11*I*A + 21*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-11*I*A + 21*
I*B)*cos(d*x + c) + sqrt(2)*(-11*I*A + 21*I*B))*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 21*(sqrt(2)*(17*I*A - 33*I*B)*cos(d*x +
c)^3 + 3*sqrt(2)*(17*I*A - 33*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(17*I*A - 33
*I*B)*cos(d*x + c) + sqrt(2)*(17*I*A - 33*I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*(sqrt(2)*(-17
*I*A + 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-17*I*A + 33*I*B)*cos(d*x + c)^
2 + 3*sqrt(2)*(-17*I*A + 33*I*B)*cos(d*x + c) + sqrt(2)*(-17*I*A + 33*I*B)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*
x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\cos(c+d\,x)^{9/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^5}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^4}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a}{a^3}$$

input
```
int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x)**5)/(cos(c + d*x)**3 + 3*cos(c + d*x
)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x))*cos(c + d*x)**4)
/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a)/a**3
```

3.159 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 219

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} - \frac{(13A-33B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} + \frac{7(7A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
7/10*(7*A-17*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-33*B
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d-1/6*(13*A-33*B)*cos(d*x+c)^
(1/2)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*
x+c))^3+1/3*(A-2*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+7/3
0*(7*A-17*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.01

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\csc^5(c+dx)\left(156A-156B+280A\cos(c+dx)-280B\cos(c+dx)-312A\cos^2(c+d\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3
,x]
```

output
```
-1/30*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^5*(156*A - 156*B + 280*A*Cos[c + d*
x] - 280*B*Cos[c + d*x] - 312*A*Cos[c + d*x]^2 + 312*B*Cos[c + d*x]^2 - 28
0*A*Cos[c + d*x]^3 + 280*B*Cos[c + d*x]^3 + 180*A*Cos[c + d*x]^4 - 180*B*C
os[c + d*x]^4 + 60*A*Cos[c + d*x]^5 - 60*B*Cos[c + d*x]^5 - 26*A*Sin[c + d
*x]^2 - 174*B*Sin[c + d*x]^2 - 280*B*Cos[c + d*x]*Sin[c + d*x]^2 - 20*B*Co
s[c + d*x]^4*Sin[c + d*x]^2 - 65*A*Sin[c + d*x]^4 + 165*B*Sin[c + d*x]^4 -
 5*(13*A - 33*B)*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sin[c +
d*x]^4*Sqrt[Sin[c + d*x]^2] + 280*B*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/
2, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] - 280*A*Cos[c
+ d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]^4*Sqr
t[Sin[c + d*x]^2] + 280*B*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Co
s[c + d*x]^2]*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] + 60*B*Sin[2*(c + d*x)]^
2 + 15*B*Csc[c + d*x]*Sin[2*(c + d*x)]^3))/(a^3*d)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3456, 27, 3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 3456$$

$$\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-a(3A-13B)\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-a(3A-13B)\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(7a(A-B)-a(3A-13B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3456$$

$$\frac{\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(25a^2(A-2B)-3a^2(8A-23B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{10a(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(25a^2(A-2B)-3a^2(8A-23B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{10a(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3456$$

$$\frac{\frac{\frac{\int \frac{3}{2}\sqrt{\cos(c+dx)}\left(7a^3(7A-17B)-5a^3(13A-33B)\cos(c+dx)\right)dx}{a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{10a(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \sqrt{\cos(c+dx)}\left(7a^3(7A-17B)-5a^3(13A-33B)\cos(c+dx)\right)dx}{2a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{10a(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(7a^3(7A-17B)-5a^3(13A-33B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{10a(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3227

$$\frac{\frac{\frac{3\left(7a^3(7A-17B)\int \sqrt{\cos(c+dx)}dx-5a^3(13A-33B)\int \cos^{\frac{3}{2}}(c+dx)dx\right)}{2a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{10a(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(7a^3(7A-17B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3(13A-33B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)}{2a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{10a(A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3115

$$\frac{\frac{\frac{3\left(7a^3(7A-17B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3(13A-33B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{10a(A-2}{3}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(7a^3(7A-17B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3(13A-33B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+10a}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{3\left(\frac{14a^3(7A-17B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}-5a^3(13A-33B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}+\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+10a(A-}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{7a^2(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3\left(\frac{14a^3(7A-17B)E\left(\frac{1}{2}(c+dx)|2\right)}{d}-5a^3(13A-33B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}}{3a^2}+10a(A-}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (
(10*a*(A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2) + ((7*a^2*(7*A - 17*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[
c + d*x])) + (3*((14*a^3*(7*A - 17*B)*EllipticE[(c + d*x)/2, 2])/d - 5*a^3
*(13*A - 33*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*d))))/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(202) = 404$.

Time = 17.77 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.12

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-160B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+348A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+130A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\right.}{}$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1
/2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-468*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*
x+1/2*c)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos
(1/2*d*x+1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-4
74*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c
)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.18

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
1/60*(2*(20*B*cos(d*x + c)^3 - 3*(29*A - 79*B)*cos(d*x + c)^2 - 2*(73*A -
188*B)*cos(d*x + c) - 65*A + 165*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(s
qrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A + 33*I*B)*co
s(d*x + c)^2 + 3*sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c) + sqrt(2)*(-13*I*
A + 33*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5
*(sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 33*I*B)*c
os(d*x + c)^2 + 3*sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c) + sqrt(2)*(13*I*A
 - 33*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21
*(sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-7*I*A + 17*I*B)*c
os(d*x + c)^2 + 3*sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c) + sqrt(2)*(-7*I*A
 + 17*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) - 21*(sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c)^3 + 3*sqrt
(2)*(7*I*A - 17*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(7*I*A - 17*I*B)*cos(d*x +
 c) + sqrt(2)*(7*I*A - 17*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^4}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a}{a^3}$$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x)**4)/(cos(c + d*x)**3 + 3*cos(c + d*x
)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x))*cos(c + d*x)**3)
/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a)/a**3
```

3.160 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
-1/10*(9*A-49*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A-13*B
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d+1/5*(A-B)*cos(d*x+c)^(5/2)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(3*A-8*B)*cos(d*x+c)^(3/2)*sin(d*x+c)
/a/d/(a+a*cos(d*x+c))^2+1/6*(3*A-13*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+
a^3*cos(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.33 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.13

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{\sqrt{\cos(c+dx)}\csc^5(c+dx)\left(-252A+252B-360A\cos(c+dx)+360B\cos(c+dx)+504A\cos^2(c+\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3
,x]
```

output
```
-1/210*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^5*(-252*A + 252*B - 360*A*Cos[c +
d*x] + 360*B*Cos[c + d*x] + 504*A*Cos[c + d*x]^2 - 504*B*Cos[c + d*x]^2 +
420*A*Cos[c + d*x]^3 - 420*B*Cos[c + d*x]^3 - 420*A*Cos[c + d*x]^4 + 420*B
*Cos[c + d*x]^4 + 42*A*Sin[c + d*x]^2 + 658*B*Sin[c + d*x]^2 - 420*B*Cos[c
 + d*x]^3*Sin[c + d*x]^2 + 105*A*Sin[c + d*x]^4 - 455*B*Sin[c + d*x]^4 + 3
5*(3*A - 13*B)*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sin[c + d*
x]^4*Sqrt[Sin[c + d*x]^2] - 1120*B*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2
, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] + 360*A*Cos[c +
 d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]^4*Sqrt
[Sin[c + d*x]^2] - 360*B*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos
[c + d*x]^2]*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] + 560*B*Sin[c + d*x]*Sin[
2*(c + d*x)] - 210*B*Sin[2*(c + d*x)]^2))/(a^3*d)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3456, 27, 3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-a(A-11B)\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-a(A-11B)\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a(A-B)-a(A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2(3A-8B)-a^2(6A-41B)\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a^2(3A-8B)-a^2(6A-41B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\frac{\int \frac{5a^3(3A-13B)-3a^3(9A-49B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{5a^3(3A-13B)-3a^3(9A-49B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}+\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{5a^3(3A-13B)-3a^3(9A-49B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{\frac{5a^3(3A-13B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^3(9A-49B)\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{5a^3(3A-13B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3(9A-49B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{5a^3(3A-13B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^3(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}+\frac{\frac{10a^3(3A-13B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^3(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{2a(3A-8B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (
(2*a*(3*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x]
)^2) + (((-6*a^3*(9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(3*A
- 13*B)*EllipticF[(c + d*x)/2, 2])/d)/(2*a^2) + (5*a^2*(3*A - 13*B)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(175) = 350$.

Time = 16.82 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.40

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(108A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1
/2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*
cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8
-130*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1
/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/
2*d*x+1/2*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*
cos(1/2*d*x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/
2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.48

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
1/60*(2*(3*(9*A - 29*B)*cos(d*x + c)^2 + 2*(18*A - 73*B)*cos(d*x + c) + 15
*A - 65*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(3*I*A - 13*I*B)*c
os(d*x + c)^3 + 3*sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I
*A - 13*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 13*I*B))*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(sqrt(2)*(-3*I*A + 13*I*B)*cos(d
*x + c)^3 + 3*sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-3*I*A
 + 13*I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + 13*I*B))*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(9*I*A - 49*I*B)*cos(d*x
 + c)^3 + 3*sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(9*I*A - 4
9*I*B)*cos(d*x + c) + sqrt(2)*(9*I*A - 49*I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(-9*I
*A + 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c)^2 +
 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A + 49*I*B))*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c
) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos(c+d\,x)^{5/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a}{a^3}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x)**3)/(cos(c + d*x)**3 + 3*cos(c + d*x
)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x))*cos(c + d*x)**2)
/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a)/a**3
```

3.161 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 180

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(A+9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\,(a^3+a^3\cos(c+dx))}$$

output
```
-1/10*(A+9*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+3*B)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d+1/5*(A-B)*cos(d*x+c)^(3/2)*sin(d*
x+c)/d/(a+a*cos(d*x+c))^3+1/15*(A-6*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+
a*cos(d*x+c))^2+1/10*(A+9*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*
x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.78 (sec) , antiderivative size = 845, normalized size of antiderivative = 4.69

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\csc(c+dx)}{15a^3d(1-\cos(c+dx))(1+\cos(c+dx))} + \frac{4B\sqrt{\cos(c+dx)}\csc(c+dx)}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$+ \frac{4B\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))} - \frac{2B\cos^{\frac{5}{2}}(c+dx)\csc(c+dx)}{a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$+ \frac{2(A-B)\sqrt{\cos(c+dx)}\csc^3(c+dx)}{5a^3d(1-\cos(c+dx))(1+\cos(c+dx))} + \frac{4(A-B)\cos^{\frac{3}{2}}(c+dx)\csc^3(c+dx)}{21a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{6(A-B)\cos^{\frac{5}{2}}(c+dx)\csc^3(c+dx)}{5a^3d(1-\cos(c+dx))(1+\cos(c+dx))} + \frac{2(A-B)\cos^{\frac{7}{2}}(c+dx)\csc^3(c+dx)}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d(1-\cos(c+dx))(1+\cos(c+dx))} - \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{(A-B)\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right)\sin(c+dx)\sqrt{\sin^2(c+dx)}}{6a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{2B\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right)\sin(c+dx)\sqrt{\sin^2(c+dx)}}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{4B\cos^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},\cos^2(c+dx)\right)\sin(c+dx)\sqrt{\sin^2(c+dx)}}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{4(A-B)\cos^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},\cos^2(c+dx)\right)\sin(c+dx)\sqrt{\sin^2(c+dx)}}{21a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3
,x]
```

output
```
-1/15*((A - B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x])/(a^3*d*(1 - Cos[c + d*x])*
(1 + Cos[c + d*x])) + (4*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x])/(3*a^3*d*(1 -
Cos[c + d*x])*(1 + Cos[c + d*x])) + (4*B*Cos[c + d*x]^(3/2)*Csc[c + d*x])/
(3*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - (2*B*Cos[c + d*x]^(5/2)*
Csc[c + d*x])/(a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) + (2*(A - B)*S
qrt[Cos[c + d*x]]*Csc[c + d*x]^3)/(5*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c +
 d*x])) + (4*(A - B)*Cos[c + d*x]^(3/2)*Csc[c + d*x]^3)/(21*a^3*d*(1 - Cos
[c + d*x])*(1 + Cos[c + d*x])) - (6*(A - B)*Cos[c + d*x]^(5/2)*Csc[c + d*x
]^3)/(5*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) + (2*(A - B)*Cos[c +
d*x]^(7/2)*Csc[c + d*x]^3)/(3*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x]))
 - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*a^3*d*(1 - Cos[c + d*x])*(
1 + Cos[c + d*x])) - (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*d*(1 - C
os[c + d*x])*(1 + Cos[c + d*x])) - ((A - B)*Sqrt[Cos[c + d*x]]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sin[c + d*x]*Sqrt[Sin[c + d*x]^2])/(
6*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - (2*B*Sqrt[Cos[c + d*x]]*H
ypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sin[c + d*x]*Sqrt[Sin[c +
d*x]^2])/(3*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - (4*B*Cos[c + d*
x]^(3/2)*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]*Sqr
t[Sin[c + d*x]^2])/(3*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - (4*(A
 - B)*Cos[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^...
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3456, 27, 3042, 3456, 3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+a(A+9B)\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+a(A+9B)\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)+a(A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{(A-6B)a^2+(4A+21B)\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(A-6B)a^2+(4A+21B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int \frac{5a^3(A+3B)-3a^3(A+9B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{3a^2(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{5a^3(A+3B)-3a^3(A+9B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}+\frac{3a^2(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{5a^3(A+3B)-3a^3(A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{3a^2(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\frac{\frac{5a^3(A+3B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^3(A+9B)\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{3a^2(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{5a^3(A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3(A+9B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{3a^2(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\frac{\frac{5a^3(A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^3(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{3a^2(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{\frac{3a^2(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}+\frac{\frac{10a^3(A+3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^3(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{2a(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (
(2*a*(A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2) + (((-6*a^3*(A + 9*B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(A + 3*B)*
EllipticF[(c + d*x)/2, 2])/d)/(2*a^2) + (3*a^2*(A + 9*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(167) = 334$.

Time = 6.32 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.51

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\cdots\right.\right.}{\cdots}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/
2*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+3
0*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c
)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+
1/2*c)^6-24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2
*d*x+1/2*c)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.58

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
1/60*(2*(3*(A + 9*B)*cos(d*x + c)^2 + 2*(7*A + 18*B)*cos(d*x + c) + 5*A +
15*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(I*A + 3*I*B)*cos(d*x +
 c)^3 + 3*sqrt(2)*(I*A + 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + 3*I*B)*c
os(d*x + c) + sqrt(2)*(I*A + 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*(sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2
)*(-I*A - 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c) +
sqrt(2)*(-I*A - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 3*(sqrt(2)*(I*A + 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 9*I*B
)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + 9*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 9
*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 3*(sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*
A - 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c) + sqrt(2
)*(-I*A - 9*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b}{a^3}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(int((sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**
2 + 3*cos(c + d*x) + 1),x)*a + int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(c
os(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b)/a**3
```

3.162 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 178

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
1/10*(A-B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+B)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d+1/5*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/
d/(a+a*cos(d*x+c))^3+1/15*(A+4*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos
(d*x+c))^2-1/10*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.66 (sec) , antiderivative size = 664, normalized size of antiderivative = 3.73

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$
$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\csc(c+dx)}{15a^3d(1-\cos(c+dx))(1+\cos(c+dx))} + \frac{2B\sqrt{\cos(c+dx)}\csc(c+dx)}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{2B\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))} + \frac{2(A-B)\sqrt{\cos(c+dx)}\csc^3(c+dx)}{5a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{6(A-B)\cos^{\frac{3}{2}}(c+dx)\csc^3(c+dx)}{7a^3d(1-\cos(c+dx))(1+\cos(c+dx))} + \frac{2(A-B)\cos^{\frac{5}{2}}(c+dx)\csc^3(c+dx)}{5a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d(1-\cos(c+dx))(1+\cos(c+dx))} - \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{(A-B)\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right)\sin(c+dx)\sqrt{\sin^2(c+dx)}}{6a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$- \frac{B\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right)\sin(c+dx)\sqrt{\sin^2(c+dx)}}{3a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$
$$+ \frac{4(A-B)\cos^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},\cos^2(c+dx)\right)\sin(c+dx)\sqrt{\sin^2(c+dx)}}{21a^3d(1-\cos(c+dx))(1+\cos(c+dx))}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3
,x]
```

output
```
-1/15*((A - B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x])/(a^3*d*(1 - Cos[c + d*x])*
(1 + Cos[c + d*x])) + (2*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x])/(3*a^3*d*(1 -
Cos[c + d*x])*(1 + Cos[c + d*x])) - (2*B*Cos[c + d*x]^(3/2)*Csc[c + d*x])/
(3*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) + (2*(A - B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]^3)/(5*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - (6
*(A - B)*Cos[c + d*x]^(3/2)*Csc[c + d*x]^3)/(7*a^3*d*(1 - Cos[c + d*x])*(1
 + Cos[c + d*x])) + (2*(A - B)*Cos[c + d*x]^(5/2)*Csc[c + d*x]^3)/(5*a^3*d
*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(6*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - (B*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c + d*x])) - ((
A - B)*Sqrt[Cos[c + d*x]]*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]
*Sin[c + d*x]*Sqrt[Sin[c + d*x]^2])/(6*a^3*d*(1 - Cos[c + d*x])*(1 + Cos[c
 + d*x])) - (B*Sqrt[Cos[c + d*x]]*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c +
 d*x]^2]*Sin[c + d*x]*Sqrt[Sin[c + d*x]^2])/(3*a^3*d*(1 - Cos[c + d*x])*(1
 + Cos[c + d*x])) + (4*(A - B)*Cos[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 7
/2, 7/4, Cos[c + d*x]^2]*Sin[c + d*x]*Sqrt[Sin[c + d*x]^2])/(21*a^3*d*(1 -
 Cos[c + d*x])*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3456, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{a(A-B)+a(3A+7B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a(A-B)+a(3A+7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(A-B)+a(3A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\int \frac{(4A+B)a^2+(A+4B)\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{(4A+B)a^2+(A+4B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\frac{\int \frac{5(A+B)a^3+3(A-B)\cos(c+dx)a^3}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{3a^2(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{5(A+B)a^3+3(A-B)\cos(c+dx)a^3}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{3a^2(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{5(A+B)a^3+3(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{3a^2(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3227

$$\frac{\frac{\frac{5a^3(A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(A-B)\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{3a^2(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{5a^3(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3(A-B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{3a^2(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3119

$$\frac{\frac{\frac{5a^3(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{3a^2(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{\frac{10a^3(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{3a^2(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

output
```
((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (
(2*a*(A + 4*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2) + (((6*a^3*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(A + B)*Ellip
ticF[(c + d*x)/2, 2])/d)/(2*a^2) - (3*a^2*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(165) = 330$.

Time = 6.20 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.53

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*
c)^6+6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/
2*c)^2-17*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = -\frac{2\left(3\,(A-B)\cos\left(dx+c\right)^2+2\,(2\,A-7\,B)\cos\left(dx+c\right)-5\,A-5\,B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\,(\sqrt{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
-1/60*(2*(3*(A - B)*cos(d*x + c)^2 + 2*(2*A - 7*B)*cos(d*x + c) - 5*A - 5*
B)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(I*A + I*B)*cos(d*x + c)^3
 + 3*sqrt(2)*(I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + I*B)*cos(d*x +
c) + sqrt(2)*(I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*(sqrt(2)*(-I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - I*B
)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A - I*B)*cos(d*x + c) + sqrt(2)*(-I*A - I
*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2
)*(-I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A + I*B)*cos(d*x + c)^2 + 3*
sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqr
t(2)*(I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A - I*B)*cos(d*x + c)^2 + 3
*sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b}{a^3}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1),x)*a + int((sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x)**3 + 3
*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b)/a**3
```

3.163 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 182

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A+B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(9A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
1/10*(9*A+B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A+B)*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d-1/5*(A-B)*cos(d*x+c)^(1/2)*sin(d*x
+c)/d/(a+a*cos(d*x+c))^3-1/15*(6*A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a
*cos(d*x+c))^2-1/10*(9*A+B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*x
+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.35 (sec) , antiderivative size = 1029, normalized size of antiderivative = 5.65

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3)
,x]
```

output
```
(-2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3
*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[
Cos[c + d*x]]*((-4*(9*A + B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2
]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(6*A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 +
(d*x)/2]*(9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) - (4*(6*A - B)*Sec[c/2
 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])
/(5*d)))/(a + a*Cos[c + d*x])^3 - (9*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c
/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta...
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3457, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(9A+B)-3a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(9A+B)-3a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(9A+B)-3a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{a^2(21A+4B)-a^2(6A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{a^2(21A+4B)-a^2(6A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3457

$$\frac{\frac{\frac{\int \frac{5(3A+B)a^3+3(9A+B)\cos(c+dx)a^3}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{5(3A+B)a^3+3(9A+B)\cos(c+dx)a^3}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{5(3A+B)a^3+3(9A+B)\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3227

$$\frac{\frac{\frac{5a^3(3A+B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(9A+B)\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{5a^3(3A+B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3(9A+B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\frac{5a^3(3A+B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{\frac{10a^3(3A+B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

input

```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

output

```
-1/5*((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3)
+ ((-2*a*(6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2) + (((6*a^3*(9*A + B)*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*(3*A +
B)*EllipticF[(c + d*x)/2, 2])/d)/(2*a^2) - (3*a^2*(9*A + B)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(169) = 338$.

Time = 5.69 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.48

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(108A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*c
os(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-1
0*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)
^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+
1/2*c)^6+24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*
x+1/2*c)^2+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.55

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
-1/60*(2*(3*(9*A + B)*cos(d*x + c)^2 + 2*(33*A + 2*B)*cos(d*x + c) + 45*A
- 5*B)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(3*I*A + I*B)*cos(d*x
+ c)^3 + 3*sqrt(2)*(3*I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I*A + I*B)*
cos(d*x + c) + sqrt(2)*(3*I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-3*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(
2)*(-3*I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-3*I*A - I*B)*cos(d*x + c) +
 sqrt(2)*(-3*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) + 3*(sqrt(2)*(-9*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A -
I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-9*I*A - I*B)*cos(d*x + c) + sqrt(2)*(-9*
I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + 3*(sqrt(2)*(9*I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*
(9*I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(9*I*A + I*B)*cos(d*x + c) + sqrt
(2)*(9*I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b}{a^3}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**4 + 3*cos(c + d*x)**3 + 3*cos(c + d
*x)**2 + cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))/(cos(c + d*x)**3 + 3*
cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b)/a**3
```

3.164 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 221

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx &= -\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} \\ &\quad -\frac{(13A-3B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} \\ &\quad +\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} \\ &\quad -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} \\ &\quad -\frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} \\ &\quad -\frac{(13A-3B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3+a^3\cos(c+dx))}\end{aligned}$$

output
```
-1/10*(49*A-9*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-3*B
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d+1/10*(49*A-9*B)*sin(d*x+c)/
a^3/d/cos(d*x+c)^(1/2)-1/5*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*
x+c))^3-1/15*(8*A-3*B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2-
1/6*(13*A-3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a^3+a^3*cos(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.55 (sec) , antiderivative size = 1069, normalized size of antiderivative = 4.84

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3)
,x]
```

output
```
(26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^3*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[
Cos[c + d*x]]*((2*(20*A + 29*A*Cos[c] - 9*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[
c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] - 9*B*Sin[(d
*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(11*A*Sin[(d*x)/2] - 6*B
*Sin[(d*x)/2]))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2]
- B*Sin[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(11*
A - 6*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x
)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3 + (49*A*Cos[c/2 + (d*x)/2]
^6*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Arc...
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3457, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(11A-B)-5a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(11A-B)-5a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(11A-B)-5a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{a^2(41A-6B)-3a^2(8A-3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(41A-6B)-3a^2(8A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\frac{\int \frac{3a^3(49A-9B)-5a^3(13A-3B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{3a^3(49A-9B)-5a^3(13A-3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{3a^3(49A-9B)-5a^3(13A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3227

$$\frac{\frac{\frac{3a^3(49A-9B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx-5a^3(13A-3B)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3a^3(49A-9B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-5a^3(13A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{\frac{3a^3(49A-9B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)-5a^3(13A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3a^3(49A-9B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-5a^3(13A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\frac{3a^3(49A-9B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a^3(13A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{3a^3(49A-9B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-\frac{10a^3(13A-3B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{5a^2(13A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{2a(8A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

output
```
-1/5*((A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3)
+ ((-2*a*(8*A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c +
d*x])^2) + ((-5*a^2*(13*A - 3*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])) + ((-10*a^3*(13*A - 3*B)*EllipticF[(c + d*x)/2, 2])/d + 3
*a^3*(49*A - 9*B)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]])))/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(204) = 408$.

Time = 5.59 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.10

method	result	size
default	Expression too large to display	685

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(49*A-9*B)*sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*
c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.36

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
1/60*(2*(3*(49*A - 9*B)*cos(d*x + c)^3 + 2*(188*A - 33*B)*cos(d*x + c)^2 +
 5*(59*A - 9*B)*cos(d*x + c) + 60*A)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(
sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos
(d*x + c)^3 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^2 + sqrt(2)*(-13*I*
A + 3*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 5*(sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(13*I*A
- 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^2 + sqrt
(2)*(13*I*A - 3*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*(sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c)^4 + 3*sqrt(2
)*(49*I*A - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c
)^2 + sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(-49*I*A
 + 9*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^3 + 3*
sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^2 + sqrt(2)*(-49*I*A + 9*I*B)*cos(d
*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx=\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^3}\,dx$$
$$= \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)}dx\right)b}{a^3}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**5 + 3*cos(c + d*x)**4 + 3*cos(c + d
*x)**3 + cos(c + d*x)**2),x)*a + int(sqrt(cos(c + d*x))/(cos(c + d*x)**4 +
 3*cos(c + d*x)**3 + 3*cos(c + d*x)**2 + cos(c + d*x)),x)*b)/a**3
```

3.165 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 254

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} + \frac{(33A-13B)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{7(17A-7B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} - \frac{7(17A-7B)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
7/10*(17*A-7*B)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(33*A-13*B
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^3/d+1/6*(33*A-13*B)*sin(d*x+c)/
a^3/d/cos(d*x+c)^(3/2)-7/10*(17*A-7*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)-1
/5*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3-1/3*(2*A-B)*sin(
d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-7/30*(17*A-7*B)*sin(d*x+c)/
d/cos(d*x+c)^(3/2)/(a^3+a^3*cos(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.31 (sec) , antiderivative size = 1110, normalized size of antiderivative = 4.37

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3)
,x]
```

output
```
(-22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^
3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqr
t[Cos[c + d*x]]*((-2*(60*A - 20*B + 59*A*Cos[c] - 29*B*Cos[c])*Csc[c/2]*Se
c[c/2]*Sec[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] -
 29*B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(16*A*Sin[(d
*x)/2] - 11*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*
Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]^2*Sin[d*
x])/(3*d) + (16*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 9*A*Sin[d*x] + 3*B*Sin[d*x
]))/(3*d) - (4*(16*A - 11*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A
 - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3 - (119
*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(S
qrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*...
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3457, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(13A-3B)-7a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a(13A-3B)-7a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(13A-3B)-7a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\int \frac{3a^2(23A-8B)-25a^2(2A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3a^2(23A-8B)-25a^2(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\frac{\int \frac{3\left(5a^3(33A-13B)-7a^3(17A-7B)\cos(c+dx)\right)}{2\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{5a^3(33A-13B)-7a^3(17A-7B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{3\int \frac{5a^3(33A-13B)-7a^3(17A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3227$$

$$\frac{\frac{\frac{3\left(5a^3(33A-13B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx-7a^3(17A-7B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{3\left(5a^3(33A-13B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-7a^3(17A-7B)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3116$$

$$\frac{\frac{\frac{3\left(5a^3(33A-13B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^3(17A-7B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right)}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{10a(2A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(5a^3(33A-13B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^3(17A-7B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)\right)}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\ldots}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3\left(5a^3(33A-13B)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^3(17A-7B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\ldots}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{3\left(5a^3(33A-13B)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^3(17A-7B)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{2a^2}-\frac{7a^2(17A-7B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\ldots}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

output
```
-1/5*((A - B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3)
+ ((-10*a*(2*A - B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d
*x])^2) + ((-7*a^2*(17*A - 7*B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a
*Cos[c + d*x])) + (3*(5*a^3*(33*A - 13*B)*((2*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))) - 7*a^3*(17*A - 7*B)*((-
2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))
)/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(233) = 466$.

Time = 6.78 (sec) , antiderivative size = 876, normalized size of antiderivative = 3.45

method	result	size
default	Expression too large to display	876

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/60*(4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(165*A*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-10*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+14
7*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*
x+1/2*c)+8*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(165*A*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-168*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(17*A-7*B)*sin(1/2*d*x+1/2*
c)^10+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1167*A-48...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.16

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
-1/60*(2*(21*(17*A - 7*B)*cos(d*x + c)^4 + 2*(453*A - 188*B)*cos(d*x + c)^
3 + 5*(139*A - 59*B)*cos(d*x + c)^2 + 60*(2*A - B)*cos(d*x + c) - 20*A)*sq
rt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c)^
5 + 3*sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(33*I*A - 13*I*
B)*cos(d*x + c)^3 + sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c)^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-33*I*A + 13*I
*B)*cos(d*x + c)^5 + 3*sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x + c)^4 + 3*sqrt(
2)*(-33*I*A + 13*I*B)*cos(d*x + c)^3 + sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x
+ c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(sq
rt(2)*(-17*I*A + 7*I*B)*cos(d*x + c)^5 + 3*sqrt(2)*(-17*I*A + 7*I*B)*cos(d
*x + c)^4 + 3*sqrt(2)*(-17*I*A + 7*I*B)*cos(d*x + c)^3 + sqrt(2)*(-17*I*A
+ 7*I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 21*(sqrt(2)*(17*I*A - 7*I*B)*cos(d*x +
c)^5 + 3*sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(17*I*A - 7*I
*B)*cos(d*x + c)^3 + sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c)^2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^
3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3
*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \frac{\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^6+3\cos(dx+c)^5+3\cos(dx+c)^4+\cos(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2}dx\right)b}{a^3}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

output
```
(int(sqrt(cos(c + d*x))/(cos(c + d*x)**6 + 3*cos(c + d*x)**5 + 3*cos(c + d
*x)**4 + cos(c + d*x)**3),x)*a + int(sqrt(cos(c + d*x))/(cos(c + d*x)**5 +
 3*cos(c + d*x)**4 + 3*cos(c + d*x)**3 + cos(c + d*x)**2),x)*b)/a**3
```

3.166 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{5\sqrt{a}(8A+7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{5a(8A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{5a(8A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a(8A+7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}$$

output
```
5/64*a^(1/2)*(8*A+7*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d
+5/64*a*(8*A+7*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/9
6*a*(8*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a*
(8*A+7*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*B*cos
(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.61

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(15\sqrt{2}(8A+7B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(15$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])
,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(8*A + 7*B)*ArcSi
n[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(152*A + 133*B + 2*(40*
A + 53*B)*Cos[c + d*x] + 4*(8*A + 7*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c +
d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3042, 3460, 3042, 3249, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{8}(8A+7B)\int \cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}(8A+7B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3249

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3249

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3249

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 3253

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

$\Big\downarrow$ 223

$$\frac{1}{8}(8A+7B)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin}{3a}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + ((8
*A + 7*B)*((a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x
]]) + (5*((a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]
]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4)
)/6))/8
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(189) = 378$.

Time = 18.46 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.51

method	result
parts	$-\frac{A\sqrt{2}\,\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(15\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-64\,\mathrm{c}\right.\right.}{48d\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sqrt{\frac{2\cos}{(\cos}}}$
default	Expression too large to display

input
```
int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
-1/48*A*2^(1/2)/d*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)+1)/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/
2*c)+1)^2)^(1/2)*(15*sec(1/2*d*x+1/2*c)*2^(1/2)*arctan(1/((2*cos(1/2*d*x+1
/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x
+1/2*c))*2^(1/2))+tan(1/2*d*x+1/2*c)*(-64*cos(1/2*d*x+1/2*c)^5-64*cos(1/2*
d*x+1/2*c)^4+24*cos(1/2*d*x+1/2*c)^3+24*cos(1/2*d*x+1/2*c)^2-26*cos(1/2*d*
x+1/2*c)-26)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2))-
1/384*B*2^(1/2)/d*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)+1)/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/
2*c)+1)^2)^(1/2)*(105*sec(1/2*d*x+1/2*c)*2^(1/2)*arctan(1/((2*cos(1/2*d*x+
1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*
x+1/2*c))*2^(1/2))+tan(1/2*d*x+1/2*c)*(-768*cos(1/2*d*x+1/2*c)^7-768*cos(1
/2*d*x+1/2*c)^6+704*cos(1/2*d*x+1/2*c)^5+704*cos(1/2*d*x+1/2*c)^4-408*cos(
1/2*d*x+1/2*c)^3-408*cos(1/2*d*x+1/2*c)^2-86*cos(1/2*d*x+1/2*c)-86)*((2*co
s(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.77

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$= \frac{\left(48\,B\cos\left(dx+c\right)^{3}+8\,(8\,A+7\,B)\cos\left(dx+c\right)^{2}+10\,(8\,A+7\,B)\cos\left(dx+c\right)+120\,A+105\,B\right)\sqrt{a\cos}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
1/192*((48*B*cos(d*x + c)^3 + 8*(8*A + 7*B)*cos(d*x + c)^2 + 10*(8*A + 7*B
)*cos(d*x + c) + 120*A + 105*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)*sin(d*x + c) + 15*((8*A + 7*B)*cos(d*x + c) + 8*A + 7*B)*sqrt(a)*arctan(
sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 + a*cos(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8220 vs. $2(189) = 378$.

Time = 0.61 (sec) , antiderivative size = 8220, normalized size of antiderivative = 37.19

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
1/768*(8*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a
) + 15*sqrt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(...
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int \cos(c+dx)^{5/2}\,(A+B\,\cos(c+dx))\,\sqrt{a+a\,\cos(c+dx)}\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \sqrt{a}\left(\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a\right)$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*
b + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a)
```

3.167 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 176

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{a}(6A+5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a(6A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(6A+5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/8*a^(1/2)*(6*A+5*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+
1/8*a*(6*A+5*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/12*
a*(6*A+5*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*B*c
os(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.67

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}(6A+5B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(18A\right.}{48d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])
,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*B + 2*(6*A +
5*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{6}(6A+5B)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}(6A+5B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3249

$$\frac{1}{6}(6A+5B)\left(\frac{3}{4}\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{1}{6}(6A+5B)\left(\frac{3}{4}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3249

$$\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3253

$$\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 223$$

$$\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + ((6
*A + 5*B)*((a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x
]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4
))/6
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(150) = 300$.

Time = 14.20 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.85

method	result
parts	$-\frac{A\sqrt{2}\,\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(3\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-8\cos\left(\right.\right.\right.}{8d\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}$
default	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
-1/8*A*2^(1/2)/d*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)+1)/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2
*c)+1)^2)^(1/2)*(3*sec(1/2*d*x+1/2*c)*2^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2
*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1
/2*c))*2^(1/2))+tan(1/2*d*x+1/2*c)*(-8*cos(1/2*d*x+1/2*c)^3-8*cos(1/2*d*x+
1/2*c)^2-2*cos(1/2*d*x+1/2*c)-2)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+
1/2*c)+1)^2)^(1/2))-1/48*B*2^(1/2)/d*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(a*c
os(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)+1)/((2*cos(1/2*d*x+1/2*c)^2
-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(15*sec(1/2*d*x+1/2*c)*2^(1/2)*arctan(
1/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x
+1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))+tan(1/2*d*x+1/2*c)*(-64*cos(1/2*d*x+1
/2*c)^5-64*cos(1/2*d*x+1/2*c)^4+24*cos(1/2*d*x+1/2*c)^3+24*cos(1/2*d*x+1/2
*c)^2-26*cos(1/2*d*x+1/2*c)-26)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1
/2*c)+1)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\left(8\,B\cos\left(dx+c\right)^{2}+2\left(6\,A+5\,B\right)\cos\left(dx+c\right)+18\,A+15\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx\right.}{24\left(d\cos\left(dx+c\right.\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
1/24*((8*B*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 18*A + 15*B)*sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((6*A + 5*B)*cos(
d*x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt
(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*cos(d
*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2981 vs. $2(150) = 300$.

Time = 0.46 (sec) , antiderivative size = 2981, normalized size of antiderivative = 16.94

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))...
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a +
 int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.168 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{a}(4A+3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a(4A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*a^(1/2)*(4*A+3*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+
1/4*a*(4*A+3*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a
*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}(4A+3B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(4A\cdot\right.}{8d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])
,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(4*A + 3*B)*ArcSin[S
qrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d
*x])*Sin[(c + d*x)/2]))/(8*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{4}(4A+3B)\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4A+3B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{1}{4}(4A+3B)\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4A+3B)\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{1}{4}(4A+3B)\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{4}(4A+3B)\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

input

```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output

```
(a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + ((4
*A + 3*B)*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/
4
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(111) = 222$.

Time = 13.16 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.44

method	result
parts	$-\frac{A\sqrt{2}\,\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}\right)+\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}\left(-\cdots\right.\right.}{2d\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}$
default	$\sqrt{2}\left(-\frac{A\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}\right)+\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\cdots}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cdots}}\right.}{2d\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}\right.$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*A*2^(1/2)/d*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)+1)/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2
*c)+1)^2)^(1/2)*(sec(1/2*d*x+1/2*c)*2^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2*c
)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1/2
*c))*2^(1/2))+((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*
(-2*sin(1/2*d*x+1/2*c)-2*tan(1/2*d*x+1/2*c)))-1/8*B*2^(1/2)/d*(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)+1)
/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(3*sec(1/2*d*
x+1/2*c)*2^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+
1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))+tan(1/2*d*x+1
/2*c)*(-8*cos(1/2*d*x+1/2*c)^3-8*cos(1/2*d*x+1/2*c)^2-2*cos(1/2*d*x+1/2*c)
-2)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{(2\,B\cos\left(dx+c\right)+4\,A+3\,B)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+((4\,A+3\,B)\cos\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
1/4*((2*B*cos(d*x + c) + 4*A + 3*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c) + ((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan
(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d
*x + c)^2 + a*cos(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}(A+B\cos\left(c+dx\right))\sqrt{\cos\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sqrt(cos(c + d*x)
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. $2(111) = 222$.

Time = 0.33 (sec) , antiderivative size = 1851, normalized size of antiderivative = 14.13

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)...
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \sqrt{a}\left(\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)a\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b +
 int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*a)
```

3.169 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 78

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{a}(2A+B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{aB\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
a^(1/2)*(2*A+B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+a*B*co
s(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}(2A+B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sqrt{\cos(c+dx)}\sin(\right.}{2d}$$

input
```
Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x
]],x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + B)*ArcSin[Sqr
t[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{2}(2A+B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(2A+B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{(2A+B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{a}(2A+B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(68) = 136$.

Time = 12.50 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.49

method	result
parts	$-\frac{2A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}\right)\left(1+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}}-\frac{B\sqrt{2}\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{}$
default	$\sqrt{2}\left(-\frac{A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}}\right)\left(1+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{d\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}}+\frac{B\sqrt{a\cos\left(\frac{dx}{2}\right.}}{}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*A/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2
*d*x+1/2*c)+1)^2)^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+
1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)*(1+sec(1/2*d*x+1/2*c))-1/2*B*2^(1/2)/d*(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c
)+1)/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(sec(1/2*
d*x+1/2*c)*2^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c
)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))+((2*cos(1/2
*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)-2*
tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{a\cos(dx+c)+a}B\sqrt{\cos(dx+c)}\sin(dx+c)+((2A+B)\cos(dx+c)+2A+B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos}\cdots}{\cdots}\right)}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + B)*c
os(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*cos(
d*x + c) + d)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \int \frac{\sqrt{a(\cos(c+dx)+1)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(cos(c + d*x)
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 939, normalized size of antiderivative = 12.04

$$\int \frac{\sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))}{\sqrt{\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((co...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a
 + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*b)
```

3.170 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 76

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a^(1/2)*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+2*a*A*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}B\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2A\sin\left(\frac{1}{2}(c+\right.\right.}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/
2),x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c +
 d*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{2B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

input
```
Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(66) = 132$.

Time = 12.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.49

method	result
parts	$\frac{2A\sqrt{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{d\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}-\frac{2B\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}}\right)(1+\mathrm{se}}{d\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}$
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{d\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{d\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}-\frac{B\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-4\cos\left(\frac{d}{}\right.\right.\right.}{}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2*A*2^(1/2)/d*tan(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(a*cos(1
/2*d*x+1/2*c)^2)^(1/2)-2*B/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2
*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1
/2*c))*2^(1/2))/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(1+sec(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\sqrt{a\cos(dx+c)+a}A\sqrt{\cos(dx+c)}\sin(dx+c)+\left(B\cos(dx+c)^2+B\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a}}{}\right.\right.}{d\cos(dx+c)^2+d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
2*(sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) + (B*cos(d*x
 + c)^2 + B*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*c
os(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/cos(c + d*x)**(3/
2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(66) = 132$.

Time = 0.22 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{B\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\right.\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
(B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 cos(d*x + c)) + 2*A*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sq
rt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\cos(c+d\,x)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*b
 + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a)
```

3.171 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2/3*a*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a*(2*A+3*
B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(A+(2A+3B)\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/
2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)
/2])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3042, 3459, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{3}(2A+3B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(2A+3B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (
2*a*(2*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 12.99 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.51

method	result
parts	$\frac{2A\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{\frac{3}{2}}}+\frac{2B\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{d\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}}$
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{\frac{3}{2}}}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(-1+\right.}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*A*2^(1/2)/d*tan(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)*(2*cos
(1/2*d*x+1/2*c)+1)*(-1+2*cos(1/2*d*x+1/2*c))*(a*cos(1/2*d*x+1/2*c)^2)^(1/2
)+2*B*2^(1/2)/d*tan(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*(a*cos
(1/2*d*x+1/2*c)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left((2A+3B)\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{3\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a(\cos(c+dx)+1)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/cos(c + d*x)**(5/
2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(73) = 146.

Time = 0.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.40

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(\frac{3B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}+\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{3d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
2/3*(3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a
)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)) + A*(3*sqrt(2)*sqr
t(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) +
 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 43.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a\,(\cos(c+d\,x)+1)}\,(2\,A\,\sin(c+d\,x)+3\,B\,\sin(c+d\,x)+2\,A\,\sin(2\,c+2\,d\,x)+2\,A\,\sin(3\,c+3\,d}{3\,d\,\sqrt{\cos(c+d\,x)}\,(3\,\cos(c+d\,x)+2\,\cos(2\,c+2\,d\,x)+\cos(3\,c+3\,d\,x)+2)}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x
)
```

output
```
(2*(a*(cos(c + d*x) + 1))^(1/2)*(2*A*sin(c + d*x) + 3*B*sin(c + d*x) + 2*A
*sin(2*c + 2*d*x) + 2*A*sin(3*c + 3*d*x) + 3*B*sin(3*c + 3*d*x)))/(3*d*cos
(c + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) +
2))
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{3}}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{2}}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x
)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*b
)
```

3.172 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 130

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2/5*a*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/15*a*(4*A+5
*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/15*a*(4*A+5*B)*
sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(7A+5B+(4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/
2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
 + 5*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{5}(4A+5B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(4A+5B)\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3251$$

$$\frac{1}{5}(4A+5B)\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}(4A+5B)\left(\frac{2}{3}\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3250$$

$$\frac{1}{5}(4A+5B)\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (
(4*A + 5*B)*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
 d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*
x]])))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 14.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.20

method	result
parts	$\frac{2A\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+7\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{15d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{\frac{5}{2}}}+\frac{2B\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(-1+2\right.}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\right.}$
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+7\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{15d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{\frac{5}{2}}}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-24\cos\right.}{15d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15*A*2^(1/2)/d*tan(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(5/2)*(32*c
os(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)^2+7)*(a*cos(1/2*d*x+1/2*c)^2)^(1
/2)+2/3*B*2^(1/2)/d*tan(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)*(2
*cos(1/2*d*x+1/2*c)+1)*(-1+2*cos(1/2*d*x+1/2*c))*(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{2\left(2\left(4A+5B\right)\cos\left(dx+c\right)^{2}+\left(4A+5B\right)\cos\left(dx+c\right)+3A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(d\right.}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*
cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(112) = 224$.

Time = 0.18 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.29

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(\frac{5\,B\left(\frac{3\,\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\,\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{A\left(\frac{15\,\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\,\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\cdots\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\cdots}\right.}{15\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
2/15*(5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*s
qrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)) + A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1)
 - 25*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqr
t(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 46.54 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\left(14\,A\,\sin\left(c+d\,x\right)+10\,B\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(2\,c+2\,d\,x\right)+18\,A\,\sin\left(3\,c+\right.\right.}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\right.}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x
)
```

output
```
(4*(a*(cos(c + d*x) + 1))^(1/2)*(14*A*sin(c + d*x) + 10*B*sin(c + d*x) + 8
*A*sin(2*c + 2*d*x) + 18*A*sin(3*c + 3*d*x) + 4*A*sin(4*c + 4*d*x) + 4*A*s
in(5*c + 5*d*x) + 10*B*sin(2*c + 2*d*x) + 15*B*sin(3*c + 3*d*x) + 5*B*sin(
4*c + 4*d*x) + 5*B*sin(5*c + 5*d*x)))/(15*d*cos(c + d*x)^(1/2)*(10*cos(c +
 d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + cos
(5*c + 5*d*x) + 6))
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x
)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*b
)
```

3.173 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 175

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} \\ &\quad + \frac{8a(6A+7B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
2/7*a*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2/35*a*(6*A+7
*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/105*a*(6*A+7*B)
*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/105*a*(6*A+7*B)*s
in(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{2\sqrt{a(1+\cos(c+dx))}(27A+14B+9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos(3}{105d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/
2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] +
2*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d
*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{7}(6A+7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}(6A+7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3251}$$

$$\frac{1}{7}(6A+7B)\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}(6A+7B)\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3251}$$

$$\frac{1}{7}(6A+7B)\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}(6A+7B)\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c-}}\right.$$
$$\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3250}$$

$$\frac{1}{7}(6A+7B)\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (
(6*A + 7*B)*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c +
 d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]])))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 14.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

method	result
parts	$\frac{2A\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{35d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{\frac{7}{2}}}+\frac{2B\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\right.}{1\ldots}$
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{35d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{\frac{7}{2}}}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(128\,c\right.}{\ldots}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
2/35*A*2^(1/2)/d*tan(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(7/2)*(128*
cos(1/2*d*x+1/2*c)^6-160*cos(1/2*d*x+1/2*c)^4+76*cos(1/2*d*x+1/2*c)^2-9)*(
a*cos(1/2*d*x+1/2*c)^2)^(1/2)+2/15*B*2^(1/2)/d*tan(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(5/2)*(32*cos(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)^2+
7)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{2\left(8\left(6A+7B\right)\cos\left(dx+c\right)^{3}+4\left(6A+7B\right)\cos\left(dx+c\right)^{2}+3\left(6A+7B\right)\cos\left(dx+c\right)+15A\right)\sqrt{a\cos\left(\right.}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*
A + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(151) = 302$.

Time = 0.17 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
2/105*(7*B*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2
)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(co
s(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 3*A*(35*sqrt(2)*sqrt(a)*s
in(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5
8*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 51.75 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x
)
```

output
```
((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((96*A + 1
12*B)*1i)/(105*d) - (B*exp(c*3i + d*x*3i)*8i)/(3*d) + (B*exp(c*4i + d*x*4i
)*8i)/(3*d) - (exp(c*7i + d*x*7i)*(96*A + 112*B)*1i)/(105*d) + (exp(c*2i +
 d*x*2i)*(336*A + 392*B)*1i)/(105*d) - (exp(c*5i + d*x*5i)*(336*A + 392*B)
*1i)/(105*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + ex
p(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3
*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2)
+ 3*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/
2) + 3*exp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^
(1/2) + 3*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2)^(1/2) + exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2)^(1/2) + exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2)^(1/2))
```

Reduce [F]

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \sqrt{a}\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a\right.\right.\\ &\left.\left.+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)b\right)\right.\end{aligned}$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x
)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*b
)
```

3.174 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 227

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{a^{3/2}(88A+75B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(88A+75B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(8A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/64*a^(3/2)*(88*A+75*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))
/d+1/64*a^2*(88*A+75*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+1/96*a^2*(88*A+75*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+1/24*a^2*(8*A+9*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
+1/4*a*B*cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}(88A+75B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(88*A + 75*B)*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(296*A + 285*B + 2*(
88*A + 93*B)*Cos[c + d*x] + 4*(8*A + 15*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(
c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{4}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}(a(8A+5B)+a(8A+9B)\cos(c+dx))dx + \frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{8}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}(a(8A+5B)+a(8A+9B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(8A+5B)+a(8A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3460$$

$$\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3249$$

$$\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(8A+9B)\sin}{3d\sqrt{a\cos}}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(}{}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3249$$

$$\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(}{2d\sqrt{a\cos(c+dx}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$\Big\downarrow$ 3253

$$\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\,c}{2d\sqrt{a\cos(c}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$\Big\downarrow$ 223

$$\frac{1}{8}\left(\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{}}{d\sqrt{a\cos(c-}}\right.\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + ((a
^2*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*
x]]) + (a*(88*A + 75*B)*((a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a +
 a*Cos[c + d*x]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*
Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[
c + d*x]])))/4))/6)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 16.38 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00

method	result
default	$\frac{\left(264A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+225B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(64\cos(dx+c)^2+176\cos(dx+c)+264\right)\sin(d}{192d(\cos(d}$
parts	$\frac{A\left(33\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(8\cos(dx+c)^2+22\cos(dx+c)+33\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{\cos(dx+c)}\sqrt{a\cos\left(\frac{dx}{2}+\right.}}{24d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*(264*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+225*B*
arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(64*cos(d*x+c)^2+176*
cos(d*x+c)+264)*sin(d*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(48*cos(d*x
+c)^3+120*cos(d*x+c)^2+150*cos(d*x+c)+225)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+
c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{(48\,Ba\cos{(dx+c)}^3 + 8\,(8\,A+15\,B)a\cos{(dx+c)}^2 + 2\,(88\,A+75\,B)a\cos{(dx+c)}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
1/192*((48*B*a*cos(d*x + c)^3 + 8*(8*A + 15*B)*a*cos(d*x + c)^2 + 2*(88*A
+ 75*B)*a*cos(d*x + c) + 3*(88*A + 75*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))*sin(d*x + c) + 3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*
B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8904 vs. $2(195) = 390$.

Time = 0.70 (sec) , antiderivative size = 8904, normalized size of antiderivative = 39.22

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos...
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a
 + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b + in
t(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a + int(sqr
t(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.175 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 180

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{a^{3/2}(14A+11B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(6A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/8*a^(3/2)*(14*A+11*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/
d+1/8*a^2*(14*A+11*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
+1/12*a^2*(6*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1
/3*a*B*cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}(14A+11B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(14*A + 11*B)*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(42*A + 37*B + 2*(6*
A + 11*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}(3a(2A+B)+a(6A+7B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}(3a(2A+B)+a(6A+7B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3a(2A+B)+a(6A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3249}$$

$$\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(6A+7B)\sin(c+dx)\cos}{2d\sqrt{a\cos(c+dx)+}}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2(6A+7B)\sin(c+d}{2d\sqrt{a\cos(c+}}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3253}$$

$$\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2(6A+7B)\sin(\ldots}{2d\sqrt{a\cos\ldots}}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{6}\left(\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{3}{4}a(14A+11B)\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos\ldots}}{d\sqrt{a\cos(c+d\ldots}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((a
^2*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*
x]]) + (3*a*(14*A + 11*B)*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a +
 a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*C
os[c + d*x]])))/4)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 17.00 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.15

method	result
default	$\frac{\left(42A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+33B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+(12\cos(dx+c)+42)\sin(dx+c)A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{24d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$
parts	$\frac{A\left(7\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\sin(2dx+2c)+7\sin(dx+c))\right)\sqrt{a(\cos(dx+c)+1)}\,\sqrt{\cos(dx+c)}\,a}{4d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}+\frac{B\left(33\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*(42*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+33*B*arc
tan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(12*cos(d*x+c)+42)*sin(d
*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(8*cos(d*x+c)^2+22*cos(d*x+c)+33
)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*cos(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\left(8\,Ba\cos\left(dx+c\right)^{2}+2\,(6\,A+11\,B)a\cos\left(dx+c\right)+3\,(14\,A+11\,B)a\right)\sqrt{a\cos\left(dx+\right.}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
1/24*((8*B*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 3*(14*A + 11
*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((14*A
 + 11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos
(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3023 vs. $2(154) = 308$.

Time = 0.44 (sec) , antiderivative size = 3023, normalized size of antiderivative = 16.79

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1)
 - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos...
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)a\right)$$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a
 + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(s
qrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b + int(sqrt(c
os(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*a)
```

3.176 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 133

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{a^{3/2}(12A+7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/4*a^(3/2)*(12*A+7*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d
+1/4*a^2*(4*A+5*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/
2*a*B*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}(12A+7B)\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d
*x]],x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(12*A + 7*B)*ArcSi
n[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 7*B + 2*B*Cos[c
+ d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3455}$$

$$\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+B)+a(4A+5B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+B)+a(4A+5B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(4A+B)+a(4A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$\big\downarrow$ 3460

$$\frac{1}{4}\left(\frac{1}{2}a(12A+7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}a(12A+7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$\big\downarrow$ 3253

$$\frac{1}{4}\left(\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{a(12A+7B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$\big\downarrow$ 223

$$\frac{1}{4}\left(\frac{a^{3/2}(12A+7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

input
```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x
]
```

output
```
(a*B*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((a
^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d + (a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos
[c + d*x]]))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.38

method	result
default	$\frac{\left(12A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+7B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+4A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\mathrm{si}}{4d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+1)}$
parts	$\frac{A\sqrt{a(\cos(dx+c)+1)}\left(\frac{\sin(2dx+2c)}{2}+3(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\right)a}{d(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}+\frac{B\left(7\arctan\left(\tan(dx-\right.\right.}{}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*(12*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7*B*arcta
n(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*A*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(sin(2*d*x+2*c)+7
*sin(d*x+c))*B)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*a
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2\,Ba\cos{(dx+c)}+(4\,A+7\,B)a)\sqrt{a\cos{(dx+c)}+a}}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
1/4*((2*B*a*cos(d*x + c) + (4*A + 7*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))*sin(d*x + c) + ((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*
sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x
 + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a(\cos{(c+dx)}+1))^{\frac{3}{2}}\,(A+B\cos{(c+dx)})}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/sqrt(cos(c + d
*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1884 vs. $2(113) = 226$.

Time = 0.34 (sec) , antiderivative size = 1884, normalized size of antiderivative = 14.17

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))}{\sqrt{\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{3/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
*a + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int
(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x) +
1)*sqrt(cos(c + d*x)),x)*b)
```

3.177 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 126

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^{3/2}(2A+3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^2(2A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
a^(3/2)*(2*A+3*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d-a^2*
(2*A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2*a*A*(a+a*co
s(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}(2A+3B)\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + 3*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + B*Cos[c + d*x])*S
in[(c + d*x)/2]))/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3454, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$2\int \frac{\sqrt{\cos(c+dx)a+a}(a(2A+B)-a(2A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{\cos(c+dx)a+a}(a(2A+B)-a(2A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(2A+B)-a(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{2}a(2A+3B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a(2A+3B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3253}$$

$$-\frac{a(2A+3B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{223}$$

$$\frac{a^{3/2}(2A+3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x
]
```

output
```
(a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (a^2*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[
c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

method	result
default	$\frac{\sqrt{a(\cos(dx+c)+1)}\,\left(2A\sin(dx+c)+\frac{B\sin(2dx+2c)}{2}+2(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)A+3(\cos(dx\right.}{d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)}$
parts	$\frac{2A\sqrt{a(\cos(dx+c)+1)}\,\left((\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sin(dx+c)\right)a}{d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)}+\frac{B\sqrt{a(\cos(dx+c)+1)}\,(}{}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/d*(a*(cos(d*x+c)+1))^(1/2)*(2*A*sin(d*x+c)+1/2*B*sin(2*d*x+2*c)+2*(cos(d
*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*A+3*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*B)/cos(d*x+c)^(1/2)/
(cos(d*x+c)+1)*a
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.21

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(Ba\cos{(dx+c)}+2\,Aa)\sqrt{a\cos{(dx+c)}+a}\sqrt{\cos{(dx}}}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
((B*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c) + ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*s
qrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x
+ c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x +
 c))
```

Sympy [F]

$$\int \frac{(a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a(\cos(c + dx) + 1))^{\frac{3}{2}}\,(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/cos(c + d*x)**
(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(112) = 224$.

Time $= 0.33$ (sec) , antiderivative size $= 1801$, normalized size of antiderivative $= 14.29$

$$\int \frac{(a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*b + i
nt((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a + int(
sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*b)
```

3.178 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2*a^(3/2)*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a^2*(4
*A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a*A*(a+a*
cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}B\arcsin\left(\right.\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
5/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*B*ArcSin[Sqrt[2]
*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3*B)*Cos[c + d*x])*S
in[(c + d*x)/2]))/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+3B)+3aB\cos(c+dx))}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+3B)+3aB\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(4A+3B)+3aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{3}\left(3aB\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3aB\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3253}$$

$$\frac{1}{3}\left(\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{6aB\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3}\left(\frac{6a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input

```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x
]
```

output

```
(2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (
(6*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(4*A + 3*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 13.58 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

method	result
default	$\frac{2\left(B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(3\cos(dx+c)^2+3\cos(dx+c)\right)+\left(\frac{5\sin(2dx+2c)}{2}+\sin(dx+c)\right)A+\frac{3B\sin(2dx+2}{2}}{3d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)}$
parts	$\frac{A\sqrt{a(\cos(dx+c)+1)}\,(5\sin(2dx+2c)+2\sin(dx+c))a}{3d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)}+\frac{2B\sqrt{a(\cos(dx+c)+1)}\left((\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan(dx+c)\right.\right.}{d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3/d*(B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*(3*cos(d*x+c)^2+3*cos(d*x+c))+(5/2*sin(2*d*x+2*c)+si
n(d*x+c))*A+3/2*B*sin(2*d*x+2*c))*(a*(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2
)/(cos(d*x+c)+1)*a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.22

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(((5\,A+3\,B)a\cos\left(dx+c\right)+Aa)\sqrt{a\cos\left(dx+c\right)+\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
2/3*(((5*A + 3*B)*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))*sin(d*x + c) + 3*(B*a*cos(d*x + c)^3 + B*a*cos(d*x + c)^2)*sqrt(
a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)
/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^
2)
```

Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/cos(c + d*x)**
(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(107) = 214$.

Time = 0.25 (sec) , antiderivative size = 1124, normalized size of antiderivative = 8.99

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
1/6*(3*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\cos\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)b+\left(\int \frac{\sqrt{\cos\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}}dx\right)a+\left(\int \frac{\sqrt{\cos\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)a+\left(\int \frac{\sqrt{\cos\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*a
+ int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a + i
nt((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*b)
```

3.179 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 134

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
2/15*a^2*(6*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/
15*a^2*(18*A+25*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/
5*a*A*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}(24A+25B+2(9A+5B)\cos(c}{15d\cos^{\frac{5}{2}}}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
7/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] +
(18*A + 25*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2)
)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{5}\int \frac{\sqrt{\cos(c+dx)a+a}(a(6A+5B)+a(2A+5B)\cos(c+dx))}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{\sqrt{\cos(c+dx)a+a}(a(6A+5B)+a(2A+5B)\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(6A+5B)+a(2A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{5}\left(\frac{1}{3}a(18A+25B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(6A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}a(18A+25B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2(6A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3250}$$

$$\frac{1}{5}\left(\frac{2a^2(6A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x
]
```

output
```
(2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
(2*a^2*(6*A + 5*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(18*A + 25*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 12.98 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

method	result	s
default	$\frac{2 \sin(dx+c) \left((12+9\cos(2dx+2c)+9\cos(dx+c))A+\left(25\cos(dx+c)^{2}+5\cos(dx+c)\right)B\right)\sqrt{a(\cos(dx+c)+1)}\, a}{15 d \cos(dx+c)^{\frac{5}{2}} (\cos(dx+c)+1)}$	8
parts	$\frac{2A \sin(dx+c)\left(6\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\sqrt{2}}{5d\cos(dx+c)^{\frac{5}{2}}(\cos(dx+c)+1)}+\frac{B\sqrt{a(\cos(dx+c)+1)}\,(5\sin(2dx+2c)+2\sin(dx+c))a}{3d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)}$	1

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d*sin(d*x+c)*((12+9*cos(2*d*x+2*c)+9*cos(d*x+c))*A+(25*cos(d*x+c)^2+5
*cos(d*x+c))*B)*(a*(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(5/2)/(cos(d*x+c)+1)*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left((18A+25B)a\cos(dx+c)^2+(9A+5B)a\cos(dx\right.}{15\left(d\cos(dx+c\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^
4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(116) = 232$.

Time = 0.18 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.57

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\left(\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)B}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}+\frac{3}{\ }\right.}{\ }$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output

```
4/15*(5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(
3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)) + 3*(5*sqrt(2)*a^(3/2)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)
*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 44.71 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(48\,A\,\sin\left(c+d\,x\right)+50\,B\,\mathrm{si}\right.}{}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x
)
```

output
```
(2*a*(a*(cos(c + d*x) + 1))^(1/2)*(48*A*sin(c + d*x) + 50*B*sin(c + d*x) +
 36*A*sin(2*c + 2*d*x) + 66*A*sin(3*c + 3*d*x) + 18*A*sin(4*c + 4*d*x) + 1
8*A*sin(5*c + 5*d*x) + 20*B*sin(2*c + 2*d*x) + 75*B*sin(3*c + 3*d*x) + 10*
B*sin(4*c + 4*d*x) + 25*B*sin(5*c + 5*d*x)))/(15*d*cos(c + d*x)^(1/2)*(10*
cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x
) + cos(5*c + 5*d*x) + 6))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \sqrt{a}\,a\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4
,x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)
*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*b
+ int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*b)
```

3.180 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(52A+63B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
2/35*a^2*(8*A+7*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/
105*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4
/105*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+
2/7*a*A*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}(82A+63B+3(78A+77B)\cos$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
9/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x]
+ (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c +
 d*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{7}\int \frac{\sqrt{\cos(c+dx)a+a}(a(8A+7B)+a(4A+7B)\cos(c+dx))}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{\sqrt{\cos(c+dx)a+a}(a(8A+7B)+a(4A+7B)\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(8A+7B)+a(4A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3459

$$\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2(8A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2(8A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3251

$$\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(8A+7B)\text{s}}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(8A+}{5d\cos^{\frac{5}{2}}(c+d}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3250

$$\frac{1}{7}\left(\frac{2a^2(8A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{1}{5}a(52A+63B)\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a}{3d\sqrt{\cos(c+}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x
]
```

output
```
(2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (
(2*a^2*(8*A + 7*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (a*(52*A + 63*B)*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 13.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

method	result
default	$\frac{2\sin(dx+c)\left(\left(104\cos(dx+c)^3+52\cos(dx+c)^2+39\cos(dx+c)+15\right)A+\cos(dx+c)\left(126\cos(dx+c)^2+63\cos(dx+c)+21\right)B\right)\sqrt{a\cos(}}{105d\cos(dx+c)^{\frac{7}{2}}(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(104\cos(dx+c)^3+52\cos(dx+c)^2+39\cos(dx+c)+15\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\sqrt{2}}{105d\cos(dx+c)^{\frac{7}{2}}(\cos(dx+c)+1)}+\frac{2B\sin(dx+c)\left(6\cos(dx+c)^2+3\cos(}{5d\cos(dx+c)^{\frac{5}{2}}(c}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105/d*sin(d*x+c)*((104*cos(d*x+c)^3+52*cos(d*x+c)^2+39*cos(d*x+c)+15)*A+
cos(d*x+c)*(126*cos(d*x+c)^2+63*cos(d*x+c)+21)*B)*(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(d*x+c)^(7/2)/(cos(d*x+c)+1)*a*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.59

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(2\,(52\,A+63\,B)a\cos\left(dx+c\right)^3+(52\,A+63\,B)a\cos\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
 3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(157) = 314$.

Time = 0.21 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.66

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
4/105*(21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*
a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ 1)) + (105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)
*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) +
1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 52.84 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(-\frac{8\,a\,\mathrm{e}^{\frac{c\,7\mathrm{i}}{2}+\cdots}}{\cdots}\right.}{6\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{e}^{\frac{c\,7\mathrm{i}}{2}+\frac{d\,x\,7\mathrm{i}}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,\sqrt{\cos\left(c+\cdots\right.}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x
)
```

output
```
((a + a*cos(c + d*x))^(1/2)*((16*a*exp((c*7i)/2 + (d*x*7i)/2)*sin((3*c)/2
+ (3*d*x)/2)*(13*A + 12*B))/(15*d) - (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin(c
/2 + (d*x)/2)*(2*A + 3*B))/(3*d) + (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin((7*
c)/2 + (7*d*x)/2)*(52*A + 63*B))/(105*d)))/(6*cos(c + d*x)^(1/2)*exp((c*7i
)/2 + (d*x*7i)/2)*cos(c/2 + (d*x)/2) + 6*cos(c + d*x)^(1/2)*exp((c*7i)/2 +
 (d*x*7i)/2)*cos((3*c)/2 + (3*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*7i)/2
+ (d*x*7i)/2)*cos((5*c)/2 + (5*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*7i)/2
 + (d*x*7i)/2)*cos((7*c)/2 + (7*d*x)/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \sqrt{a}\,a\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**5
,x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)
*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*b
+ int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*b)
```

3.181 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+39B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{8a^2(34A+39B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}$$

output
```
2/63*a^2*(10*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2
/105*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+
8/315*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)
+16/315*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2)+2/9*a*A*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.54

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}(376A+351B+(374A+324B)}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
11/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x
] + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3
*(c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Tan[(c + d*x)
/2])/(315*d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{9}\int \frac{\sqrt{\cos(c+dx)a+a}(a(10A+9B)+3a(2A+3B)\cos(c+dx))}{2\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{\sqrt{\cos(c+dx)a+a}(a(10A+9B)+3a(2A+3B)\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(10A+9B)+3a(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2(10A+9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2(10A+9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(10A+9B)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a}}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(10A}{7d\cos^{\frac{7}{2}}(c+d}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3251$$

$$\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin}{5d\cos^{\frac{5}{2}}(c+dx)}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5d\cos^{\frac{5}{2}}(c}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3250$$

$$\frac{1}{9}\left(\frac{2a^2(10A+9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3}{7}a(34A+39B)\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\frac{}{3d\cos^{\frac{3}{2}}(c}\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),
x]
```

output
```
(2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (
(2*a^2*(10*A + 9*B)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (3*a*(34*A + 39*B)*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])))/5))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 13.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.57

method	result
default	$\frac{2\sin(dx+c)\left(\left(272\cos(dx+c)^4+136\cos(dx+c)^3+102\cos(dx+c)^2+85\cos(dx+c)+35\right)A+\cos(dx+c)\left(312\cos(dx+c)^3+156\cos(dx+\right.\right.}{315d\cos(dx+c)^{\frac{9}{2}}(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(272\cos(dx+c)^4+136\cos(dx+c)^3+102\cos(dx+c)^2+85\cos(dx+c)+35\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\sqrt{2}}{315d\cos(dx+c)^{\frac{9}{2}}(\cos(dx+c)+1)}+\frac{2B\sin(dx+c)\left(10\right.}{}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/315/d*sin(d*x+c)*((272*cos(d*x+c)^4+136*cos(d*x+c)^3+102*cos(d*x+c)^2+85
*cos(d*x+c)+35)*A+cos(d*x+c)*(312*cos(d*x+c)^3+156*cos(d*x+c)^2+117*cos(d*
x+c)+45)*B)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(d*x+c)^(9/2)/(cos(d*x+c)+1)
*a*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.55

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\left(8\,(34\,A+39\,B)a\cos\left(dx+c\right)^4+4\,(34\,A+39\,B)a\cos\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
 + 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A
*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(198) = 396$.

Time = 0.17 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.51

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output

```
4/315*(3*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2
)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) +
 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 1)) + (315*sqrt(2)*a^(3/2)*sin(d*x + c)/(co
s(d*x + c) + 1) - 840*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 1344*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*
a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x
+ c)^9/(cos(d*x + c) + 1)^9 - 94*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11
/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="giac")
```

output

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 55.01 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.27

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{a+a\,\cos(c+d\,x)}\,\Big(\cdots}{12\,\sqrt{\cos(c+d\,x)}\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,\sqrt{\cos(c+\cdots}}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),
x)
```

output
```
((a + a*cos(c + d*x))^(1/2)*((16*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2
+ (5*d*x)/2)*(34*A + 39*B))/(35*d) - (16*B*a*exp((c*9i)/2 + (d*x*9i)/2)*si
n((3*c)/2 + (3*d*x)/2))/(3*d) + (32*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((9*c)
/2 + (9*d*x)/2)*(34*A + 39*B))/(315*d) + (96*a*exp((c*9i)/2 + (d*x*9i)/2)*
sin(c/2 + (d*x)/2)*(A + B))/(5*d)))/(12*cos(c + d*x)^(1/2)*exp((c*9i)/2 +
(d*x*9i)/2)*cos(c/2 + (d*x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*
9i)/2)*cos((3*c)/2 + (3*d*x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x
*9i)/2)*cos((5*c)/2 + (5*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*
x*9i)/2)*cos((7*c)/2 + (7*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d
*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \sqrt{a}\,a\Bigg(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^6}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)b\Bigg)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**6
,x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)
*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*b
+ int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*b)
```

3.182 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 274

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\frac{a^{5/2}(326A+283B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d}+\frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(326A+283B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}}+\frac{a^3(170A+157B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}}+\frac{a^2(10A+13B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{40d}+\frac{aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
1/128*a^(5/2)*(326*A+283*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/
2))/d+1/128*a^3*(326*A+283*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+1/192*a^3*(326*A+283*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*
x+c))^(1/2)+1/240*a^3*(170*A+157*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos
(d*x+c))^(1/2)+1/40*a^2*(10*A+13*B)*cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2
)*sin(d*x+c)/d+1/5*a*B*cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/
d
```

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.58

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(15\sqrt{2}(326A+283B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+a\right.\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(326*A + 283*
B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(5810*A + 5521*
B + (3620*A + 3874*B)*Cos[c + d*x] + 4*(230*A + 331*B)*Cos[2*(c + d*x)] +
120*A*Cos[3*(c + d*x)] + 348*B*Cos[3*(c + d*x)] + 48*B*Cos[4*(c + d*x)])*S
in[(c + d*x)/2]))/(3840*d)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$\Big\downarrow$ 3042

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\Big\downarrow$ 3455

$$\frac{1}{5}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}(5a(2A+B)+a(10A+13B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{10}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}(5a(2A+B)+a(10A+13B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(5a(2A+B)+a(10A+13B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3455

$$\frac{1}{10}\left(\frac{1}{4}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}\left(5(26A+21B)a^2+(170A+157B)\cos(c+dx)a^2\right)dx+\frac{a^2(10A+1\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{10}\left(\frac{1}{8}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}\left(5(26A+21B)a^2+(170A+157B)\cos(c+dx)a^2\right)dx+\frac{a^2(10A+13I\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}\left(\frac{1}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5(26A+21B)a^2+(170A+157B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\downarrow$ 3460

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a^3(170A+157B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^3(170A+157B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)-}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\downarrow$ 3249

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^3(170A}{}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\downarrow$ 3249

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos}{2d\sqrt{a\cos(c+}}\right.\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+a}{2d\sqrt{a\cos}}\right.\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3253}$$

$$\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(}{2d\sqrt{}}\right.\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{10}\left(\frac{a^2(10A+13B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{1}{8}\left(\frac{a^3(170A+157B)\sin(c+dx)\cos^{\frac{5}{2}}(c+d}{3d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (
(a^2*(10*A + 13*B)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(4*d) + ((a^3*(170*A + 157*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqr
t[a + a*Cos[c + d*x]]) + (5*a^2*(326*A + 283*B)*((a*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*S
in[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6)/8)/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 17.79 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(4890A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+4245B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(480\cos(dx+c)^{3}+1840\cos(dx+c)^{2}+3260\right.}{}$
parts	$\frac{A\left(489\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(48\cos(dx+c)^{3}+184\cos(dx+c)^{2}+326\cos(dx+c)+489\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{\mathrm{co}}}{192d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/1920/d*(4890*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4245
*B*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(480*cos(d*x+c)^3+
1840*cos(d*x+c)^2+3260*cos(d*x+c)+4890)*sin(d*x+c)*A*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)+(384*cos(d*x+c)^4+1392*cos(d*x+c)^3+2264*cos(d*x+c)^2+2830*co
s(d*x+c)+4245)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^
(1/2)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*2^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{(384\,Ba^2\cos(dx+c)^4 + 48\,(10\,A+29\,B)a^2\cos(dx+c)^3 + 8\,(230\,A+283\,B)a^2\cos(dx+c)^2 + \dots}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
1/1920*((384*B*a^2*cos(d*x + c)^4 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^3 +
8*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 10*(326*A + 283*B)*a^2*cos(d*x + c)
 + 15*(326*A + 283*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) + 15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sq
rt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x +
 c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10042 vs. $2(236) = 472$.

Time = 0.80 (sec) , antiderivative size = 10042, normalized size of antiderivative = 36.65

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
1/7680*((10*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(
2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((135*a^2*sin(4/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*sin(3/5*arctan2(sin(5*d*x + 5*c
), cos(5*d*x + 5*c))) + 135*a^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*
x + 5*c))))*cos(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - (135*
a^2*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*cos(3/5*
arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 135*a^2*cos(1/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) - 88*a^2)*sin(3/2*arctan2(sin(2/5*arctan
2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c),
cos(5*d*x + 5*c))) + 1)))*sqrt(a) + 6*(cos(2/5*arctan2(sin(5*d*x + 5*c), c
os(5*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
^2 + 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(8*
(a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2*sin(5*d*x + 5*
c) + a^2*sin(5*d*x + 5*c)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c)))^2 + 2*a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(5*
d*x + 5*c) + a^2*sin(5*d*x + 5*c))*cos(5/2*arctan2(sin(2/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))) + 1)) - 5*(35*a^2*sin(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x...
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a^2\left(\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b + 2\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x
)*a + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*b +
 int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*a + 2*in
t(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b + 2*int(s
qrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a + int(sqrt(c
os(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.183 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 227

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{a^{5/2}(200A+163B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(104A+95B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(8A+11B)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{24d} + \frac{aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d}$$

output

```
1/64*a^(5/2)*(200*A+163*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2
))/d+1/64*a^3*(200*A+163*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))
^(1/2)+1/96*a^3*(104*A+95*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(1/2)+1/24*a^2*(8*A+11*B)*cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*sin(d*
x+c)/d+1/4*a*B*cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}(200A+163B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(200*A + 163*B
)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(632*A + 581*B +
 (272*A + 362*B)*Cos[c + d*x] + 4*(8*A + 23*B)*Cos[2*(c + d*x)] + 12*B*Cos
[3*(c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{4}\int \frac{1}{2}\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}(a(8A+3B)+a(8A+11B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}(a(8A+3B)+a(8A+11B)\cos(c+dx))dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(8A+3B)+a(8A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{8}\left(\frac{1}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}\left(3(24A+17B)a^2+(104A+95B)\cos(c+dx)a^2\right)dx+\frac{a^2(8A+11B}{}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}\left(3(24A+17B)a^2+(104A+95B)\cos(c+dx)a^2\right)dx+\frac{a^2(8A+11B)}{}\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3(24A+17B)a^2+(104A+95B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)d\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a^3(104A+95B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^3(104A+95B)\sin(c+dx)\cos}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 3249

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^3(104A+95B)\sin(}{2d\sqrt{a\cos(c}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^3(104A+95B}{2d\sqrt{a}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 3253

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^3(104A+}{2}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

$\Big\downarrow$ 223

$$\frac{1}{8}\left(\frac{a^2(8A+11B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{1}{6}\left(\frac{a^3(104A+95B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.$$
$$\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}{4d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(a*B*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + (
(a^2*(8*A + 11*B)*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + ((a^3*(104*A + 95*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[
a + a*Cos[c + d*x]]) + (3*a^2*(200*A + 163*B)*((Sqrt[a]*ArcSin[(Sqrt[a]*Si
n[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4)/6)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 17.95 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.01

method	result
default	$\frac{\left(600A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+489B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(64\cos(dx+c)^2+272\cos(dx+c)+600\right)\sin(d}{192d(\cos(}$
parts	$\frac{A\left(75\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(8\cos(dx+c)^2+34\cos(dx+c)+75\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\cos(d}}{24d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*(600*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+489*B*
arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(64*cos(d*x+c)^2+272*
cos(d*x+c)+600)*sin(d*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(48*cos(d*x
+c)^3+184*cos(d*x+c)^2+326*cos(d*x+c)+489)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+
c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{\left(48\,Ba^2\cos\left(dx+c\right)^3+8\left(8\,A+23\,B\right)a^2\cos\left(dx+c\right)^2+2\left(136\,A+163\,B\right)a^2\cos\left(dx\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
1/192*((48*B*a^2*cos(d*x + c)^3 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^2 + 2*(1
36*A + 163*B)*a^2*cos(d*x + c) + 3*(200*A + 163*B)*a^2)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((200*A + 163*B)*a^2*cos(d*x +
c) + (200*A + 163*B)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(d*c
os(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9415 vs. $2(195) = 390$.

Time = 0.75 (sec) , antiderivative size = 9415, normalized size of antiderivative = 41.48

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*
(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x...
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\int \sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)a\right)$$

input

```
int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output

```
sqrt(a)*a**2*(2*int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)
,x)*a + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b +
int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b + int(s
qrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a + 2*int(sqrt
(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b + int(sqrt(cos(
c + d*x) + 1)*sqrt(cos(c + d*x)),x)*a)
```

3.184 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 180

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{a^{5/2}(38A+25B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{aB\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d}$$

output
```
1/8*a^(5/2)*(38*A+25*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/
d+1/24*a^3*(54*A+49*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2
)+1/4*a^2*(2*A+3*B)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+1
/3*a*B*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.67

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}(38A+25\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d
*x]],x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(38*A + 25*B)*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(66*A + 79*B + 2*(
6*A + 17*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(6A+B)+3a(2A+3B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(6A+B)+3a(2A+3B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(6A+B)+3a(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}\left((30A+13B)a^2+(54A+49B)\cos(c+dx)a^2\right)}{2\sqrt{\cos(c+dx)}}dx+\frac{3a^2(2A+3B)\sin(c+dx)\sqrt{\cos}}{2d}\right.$$
$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)a+a}\left((30A+13B)a^2+(54A+49B)\cos(c+dx)a^2\right)}{\sqrt{\cos(c+dx)}}dx+\frac{3a^2(2A+3B)\sin(c+dx)\sqrt{\cos}}{2d}\right.$$
$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((30A+13B)a^2+(54A+49B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a^2(2A+3B)\sin(c}{}\right.$$
$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}a^2(38A+25B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{3a^2(2A+3B}{}\right.$$
$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}a^2(38A+25B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{3a^2(2A\right.$$
$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$\downarrow$ 3253

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{3a^2(38A+25B)\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{3}{}\right.$$
$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

$\downarrow$ 223

$$\frac{1}{6}\left(\frac{3a^2(2A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}\left(\frac{3a^{5/2}(38A+25B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}\right.\right.$$
$$\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}{3d}$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x
]
```

output
```
(a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (
(3*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(2*d) + ((3*a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a
+ a*Cos[c + d*x]]])/d + (a^3*(54*A + 49*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]]))/4)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.68 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(114A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+75B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+(12\cos(dx+c)+66)\sin(dx+c)A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{24d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)}$
parts	$\frac{A\sqrt{a(\cos(dx+c)+1)}\left(\sin(dx+c)\cos(dx+c)(2\cos(dx+c)+11)+19(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.\right.}{4d(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*(114*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+75*B*ar
ctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(12*cos(d*x+c)+66)*sin(
d*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(8*cos(d*x+c)^2+34*cos(d*x+c)+7
5)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*cos
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*
2^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(8\,Ba^2\cos\left(dx+c\right)^2+2\,(6\,A+17\,B)a^2\cos\left(dx+c\right)+\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
1/24*((8*B*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 3*(22*A
+ 25*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*
((38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 + a*cos(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3071 vs. $2(154) = 308$.

Time = 0.48 (sec) , antiderivative size = 3071, normalized size of antiderivative = 17.06

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2
*d*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*
cos(2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) +
 19*(a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \sqrt{a}\,a^2\Bigg(\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a\right.$$
$$+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a$$
$$+2\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b$$
$$+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b$$
$$+2\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)a$$
$$+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)b\Bigg)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)
,x)*a + int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a +
2*int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sq
rt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b + 2*int(sqrt(
cos(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x) + 1)*sqr
t(cos(c + d*x)),x)*b)
```

3.185 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 178

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{a^{5/2}(20A+19B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(4A-B)\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
1/4*a^(5/2)*(20*A+19*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/
d-1/4*a^3*(4*A-9*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1
/2*a^2*(4*A-B)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2*a*A*
(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}(20A+19\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
3/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(20*A + 19*B)*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*A + B + (4*A + 11
*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Cos[c
+ d*x]])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3042, 3454, 27, 3042, 3455, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 3454$$

$$2\int \frac{(\cos(c+dx)a+a)^{3/2}(a(4A+B)-a(4A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(\cos(c+dx)a+a)^{3/2}(a(4A+B)-a(4A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(4A+B)-a(4A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}\left(a^2(12A+5B)-a^2(4A-9B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)}}dx-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)a+a}\left(a^2(12A+5B)-a^2(4A-9B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a^2(12A+5B)-a^2(4A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{4}\left(\frac{1}{2}a^2(20A+19B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}a^2(20A+19B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3253

$$\frac{1}{4}\left(-\frac{a^2(20A+19B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 223

$$-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}\left(\frac{a^{5/2}(20A+19B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x
]
```

output
```
-1/2*(a^2*(4*A - B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*
x])/d + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]) + ((a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Co
s[c + d*x]]])/d - (a^3*(4*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqr
t[a + a*Cos[c + d*x]]))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12

method	result
default	$\frac{\sqrt{a(\cos(dx+c)+1)}\,\left((2\sin(2dx+2c)+8\sin(dx+c))A+\sin(dx+c)\cos(dx+c)(2\cos(dx+c)+11)B+20(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ a}{4d\sqrt{\cos(dx+c)}\,(\cos(dx+}$
parts	$\frac{A\sqrt{a(\cos(dx+c)+1)}\,\left(\frac{\sin(2dx+2c)}{2}+2\sin(dx+c)+5(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\right)a^2}{d(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}+\frac{B\sqrt{a}}{}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*(a*(cos(d*x+c)+1))^(1/2)*((2*sin(2*d*x+2*c)+8*sin(d*x+c))*A+sin(d*x+
c)*cos(d*x+c)*(2*cos(d*x+c)+11)*B+20*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*A+19*(cos
(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*B)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(2\,Ba^2\cos\left(dx+c\right)^2+\left(4\,A+11\,B\right)a^2\cos\left(dx+c\right)+8\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
1/4*((2*B*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + ((20*A + 19*B)*a^
2*cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*c
os(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 + a*cos(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2080 vs. $2(154) = 308$.

Time = 0.38 (sec) , antiderivative size = 2080, normalized size of antiderivative = 11.69

$$\int \frac{(a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*co
s(2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 1
9*(a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arcta...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\cos(c+dx)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)a + 2\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a**2*(2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*
x),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*
b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a +
 int(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sqr
t(cos(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*a + 2*int(sqrt(cos(c + d*x) + 1)
*sqrt(cos(c + d*x)),x)*b)
```

3.186 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 173

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^{5/2}(2A+5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
a^(5/2)*(2*A+5*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d-1/3*
a^3*(14*A+3*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2*a^2*
(2*A+B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a*A*(a+a*
cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}(2A+5B\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
5/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(2*A + 5*B)*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A + 3*B + 4*(8*A +
3*B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d*Cos[c +
d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{3}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(2A+B)-a(2A-3B)\cos(c+dx))}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(2A+B)-a(2A-3B)\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(3a(2A+B)-a(2A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3454$$

$$\frac{1}{3}\left(2\int \frac{\sqrt{\cos(c+dx)a+a}\,(a^2(10A+9B)-a^2(14A+3B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{6a^2(2A+B)\sin(c+dx)\sqrt{a\cos(c}{d\sqrt{\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\int \frac{\sqrt{\cos(c+dx)a+a}\,(a^2(10A+9B)-a^2(14A+3B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{6a^2(2A+B)\sin(c+dx)\sqrt{a\cos(c+}{d\sqrt{\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a^2(10A+9B)-a^2(14A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2(2A+B)\sin(c+dx)}{d\sqrt{\cos(c+}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3460$$

$$\frac{1}{3}\left(\frac{3}{2}a^2(2A+5B)\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2(2A+B)\sin(c+d}{d\sqrt{\cos(}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}a^2(2A+5B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2(2A+B)\sin(}{d\sqrt{}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3253

$$\frac{1}{3}\left(-\frac{3a^2(2A+5B)\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2(2A-}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 223

$$\frac{1}{3}\left(\frac{3a^{5/2}(2A+5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2(2A+B)\sin(c+d}{d\sqrt{\cos(}}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x
]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) +
 ((3*a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/d - (a^3*(14*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a +
 a*Cos[c + d*x]]) + (6*a^2*(2*A + B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.27

method	result
default	$\frac{\sqrt{a(\cos(dx+c)+1)}\left(A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(6\cos(dx+c)^2+6\cos(dx+c)\right)+B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(t\right.\right.}{3d\cos(dx+c}$
parts	$\frac{2A\sqrt{a(\cos(dx+c)+1)}\left(4\sin(2dx+2c)+\sin(dx+c)+\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(3\cos(dx+c)^2+3\cos(dx+c)\right)\right.}{3d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/d*(a*(cos(d*x+c)+1))^(1/2)*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(6*cos(d*x+c)^2+6*cos(d*x+c
))+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*(15*cos(d*x+c)^2+15*cos(d*x+c))+(8*sin(2*d*x+2*c)+2*sin(
d*x+c))*A+sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)+6)*B)/cos(d*x+c)^(3/2)/(cos(
d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(3\,Ba^2\cos\left(dx+c\right)^2+2\,(8\,A+3\,B)a^2\cos\left(dx+c\right)+2\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*((3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^2)*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((2*A + 5*B)*a
^2*cos(d*x + c)^3 + (2*A + 5*B)*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*
cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 + a*cos(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(151) = 302$.

Time $= 0.35$ (sec) , antiderivative size $= 2370$, normalized size of antiderivative $= 13.70$

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
1/12*(3*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\sqrt{a}\,a^2\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a\right.\right.$$
$$+2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b$$
$$+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a$$
$$+2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a$$
$$+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b$$
$$\left.+\left(\int \sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)
,x)*a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*
b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*a +
 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a +
int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*b + int
(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)),x)*b)
```

3.187 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 172

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
2*a^(5/2)*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/d+2/15*a^3*(
32*A+35*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/15*a^2*(
8*A+5*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*a*A*(a+a
*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(30\sqrt{2}B\arcsin\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
7/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sqrt[2]*B*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40*B + 2*(14*A + 5*B)
*Cos[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(30*d*C
os[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3454}$$

$$\frac{2}{5}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(8A+5B)+5aB\cos(c+dx))}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(8A+5B)+5aB\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(8A+5B)+5aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}\left((32A+35B)a^2+15B\cos(c+dx)a^2\right)}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx}}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\sqrt{\cos(c+dx)a+a}\left((32A+35B)a^2+15B\cos(c+dx)a^2\right)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx}}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((32A+35B)a^2+15B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{}}{3d\cos^{\frac{3}{2}}(c+d}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^2B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^3(32A+35B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos}}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^2B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(32A+35B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(8A+5B)\sin(c+dx}{3d\cos^{\frac{3}{2}}(c}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3253

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^3(32A+35B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{30a^2B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2(8A+5B)\sin}{3d}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 223

$$\frac{1}{5}\left(\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{30a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(32A+35B)}{d\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x
]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 ((2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)) + ((30*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/d + (2*a^3*(32*A + 35*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]]))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 13.83 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

method	result
default	$\frac{2\left(B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(15\cos(dx+c)^3+15\cos(dx+c)^2\right)+\left(43\cos(dx+c)^2+14\cos(dx+c)+3\right)\sin(d}{15d\cos(dx+c)^{\frac{5}{2}}(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(43\cos(dx+c)^2+14\cos(dx+c)+3\right)\sqrt{a(\cos(dx+c)+1)}\,a^2}{15d\cos(dx+c)^{\frac{5}{2}}(\cos(dx+c)+1)}+\frac{2B\sqrt{a(\cos(dx+c)+1)}\left(4\sin(2dx+2c)+\sin(dx+c)+\mathrm{arct}\right.}{3d}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d*(B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))*(15*cos(d*x+c)^3+15*cos(d*x+c)^2)+(43*cos(d*x+c)^2+
14*cos(d*x+c)+3)*sin(d*x+c)*A+sin(d*x+c)*cos(d*x+c)*(40*cos(d*x+c)+5)*B)*(
a*(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(5/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{2\left(\left((43\,A+40\,B)a^2\cos\left(dx+c\right)^2+(14\,A+5\,B)a^2\cos\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
2/15*(((43*A + 40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) +
3*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 15*(B*
a^2*cos(d*x + c)^4 + B*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos
(d*x + c))))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1548 vs. $2(148) = 296$.

Time = 0.29 (sec) , antiderivative size = 1548, normalized size of antiderivative = 9.00

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
1/30*(5*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*si
n(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 +
4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2
*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\cos(c+dx)^{7/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)
,x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)
*a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*
a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*b +
 int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a + 2*
int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*b)
```

3.188 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(230A+301B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
2/15*a^3*(10*A+11*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+
2/105*a^3*(230*A+301*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2)+2/35*a^2*(10*A+7*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2
)+2/7*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(290A+196B+(930A+987B}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
9/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d
*x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*C
os[3*(c + d*x)])*Tan[(c + d*x)/2])/(210*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{7}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(10A+7B)+a(2A+7B)\cos(c+dx))}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(10A+7B)+a(2A+7B)\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(10A+7B)+a(2A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{\sqrt{\cos(c+dx)a+a}(7(10A+11B)a^2+(30A+49B)\cos(c+dx)a^2)}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2(10A+7B)\sin(c+dx)\sqrt{\cdots}}{5d\cos^{\frac{5}{2}}(c+dx\cdots}\right.\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{\sqrt{\cos(c+dx)a+a}(7(10A+11B)a^2+(30A+49B)\cos(c+dx)a^2)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2(10A+7B)\sin(c+dx)\sqrt{\cdots}}{5d\cos^{\frac{5}{2}}(c+dx\cdots}\right.\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(7(10A+11B)a^2+(30A+49B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2(10A+7B)\sin\cdots}{5d\cdots}\right.\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}a^2(230A+301B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{14a^3(10A+11B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(10A+7B)\text{s}\cdots}{5\cdots}\right.\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}a^2(230A+301B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{14a^3(10A+11B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(10A+\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3250}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{14a^3(10A+11B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+301B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(10A+7B)\sin(c+d}{5d\cos^{\frac{5}{2}}(c}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x
]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) +
 ((2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c +
d*x]^(5/2)) + ((14*a^3*(10*A + 11*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(230*A + 301*B)*Sin[c + d*x])/(3*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 12.95 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.59

method	result
default	$\frac{2\sin(dx+c)\left(\left(230\cos(dx+c)^3+115\cos(dx+c)^2+60\cos(dx+c)+15\right)A+\cos(dx+c)\left(301\cos(dx+c)^2+98\cos(dx+c)+21\right)B\right)\sqrt{a(\cos}}{105d\cos(dx+c)^{\frac{7}{2}}(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(46\cos(dx+c)^3+23\cos(dx+c)^2+12\cos(dx+c)+3\right)\sqrt{a(\cos(dx+c)+1)}\,a^2}{21d\cos(dx+c)^{\frac{7}{2}}(\cos(dx+c)+1)}+\frac{2B\sin(dx+c)\left(43\cos(dx+c)^2+14\cos(d\right.}{15d\cos(dx+c)^{\frac{5}{2}}(\cos}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105/d*sin(d*x+c)*((230*cos(d*x+c)^3+115*cos(d*x+c)^2+60*cos(d*x+c)+15)*A
+cos(d*x+c)*(301*cos(d*x+c)^2+98*cos(d*x+c)+21)*B)*(a*(cos(d*x+c)+1))^(1/2
)/cos(d*x+c)^(7/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left((230\,A+301\,B)a^2\cos{(dx+c)}^3+(115\,A+98\,B)a^2\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c
)^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(157) = 314$.

Time = 0.16 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.19

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{8\left(\frac{7\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\sqrt{2}a}{(\cos}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
8/105*(7*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(7/2)) + 5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x
 + c) + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*s
qrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 52.10 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.04

$$\int \frac{(a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x
)
```

output
```
((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(230*
A + 301*B)*2i)/(105*d) - (a^2*exp(c*3i + d*x*3i)*(10*A + 17*B)*2i)/(3*d) +
 (a^2*exp(c*4i + d*x*4i)*(10*A + 17*B)*2i)/(3*d) + (a^2*exp(c*2i + d*x*2i)
*(100*A + 113*B)*2i)/(15*d) - (a^2*exp(c*5i + d*x*5i)*(100*A + 113*B)*2i)/
(15*d) - (a^2*exp(c*7i + d*x*7i)*(230*A + 301*B)*2i)/(105*d) - (B*a^2*exp(
c*1i + d*x*1i)*2i)/d + (B*a^2*exp(c*6i + d*x*6i)*2i)/d))/((exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*2i + d*x*2i)*(exp(- c*1i
- d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*3i + d*x*3i)*(exp(- c*
1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*4i + d*x*4i)*(exp(-
 c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*5i + d*x*5i)*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*6i + d*x*6i)*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*7i + d*x*7i)*(
exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b\right)$$

input

```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output

```
sqrt(a)*a**2*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)
**5,x)*a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)*
*4,x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,
x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*
a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*b
 + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*b)
```

3.189 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(292A+345B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}$$

output
```
2/315*a^3*(124*A+135*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/
2)+2/315*a^3*(292*A+345*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^
(1/2)+4/315*a^3*(292*A+345*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2)+2/21*a^2*(4*A+3*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)
^(7/2)+2/9*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.55

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(1454A+1395B+(1396A+1\ldots}{\ldots}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
11/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
 + d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 34
5*B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Ta
n[(c + d*x)/2])/(630*d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{9}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(4A+3B)+a(4A+9B)\cos(c+dx))}{2\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(4A+3B)+a(4A+9B)\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(3a(4A+3B)+a(4A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{\sqrt{\cos(c+dx)a+a}\left((124A+135B)a^2+(76A+99B)\cos(c+dx)a^2\right)}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{6a^2(4A+3B)\sin(c+dx)\sqrt{\text{c}}}{7d\cos^{\frac{7}{2}}(c+d}\right.$$
$$+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{\sqrt{\cos(c+dx)a+a}\left((124A+135B)a^2+(76A+99B)\cos(c+dx)a^2\right)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{6a^2(4A+3B)\sin(c+dx)\sqrt{\text{c}}}{7d\cos^{\frac{7}{2}}(c+d}\right.$$
$$+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((124A+135B)a^2+(76A+99B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{6a^2(4A+3B)\sin}{7d}\right.$$
$$+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^3(124A+135B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{6a^2(4A+3B)\sin}{7a}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^3(124A+135B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{6a^2(4A+3}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^3(124A}{5d\cos^{\frac{5}{2}}(c+}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^3(}{5d\cos^{\frac{5}{2}}}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3250}$$

$$\frac{1}{9}\left(\frac{6a^2(4A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2a^3(124A+135B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3}{5}a^2(292A+345B\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

input

```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),
x]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) +
 ((6*a^2*(4*A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d
*x]^(7/2)) + ((2*a^3*(124*A + 135*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
*Sqrt[a + a*Cos[c + d*x]]) + (3*a^2*(292*A + 345*B)*((2*a*Sin[c + d*x])/(3
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5)/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 12.97 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.55

method	result
default	$\frac{2\sin(dx+c)\left(\left(584\cos(dx+c)^4+292\cos(dx+c)^3+219\cos(dx+c)^2+130\cos(dx+c)+35\right)A+\cos(dx+c)\left(690\cos(dx+c)^3+345\cos(dx+\right.\right.}{315d\cos(dx+c)^{\frac{9}{2}}(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(584\cos(dx+c)^4+292\cos(dx+c)^3+219\cos(dx+c)^2+130\cos(dx+c)+35\right)\sqrt{a(\cos(dx+c)+1)}\,a^2}{315d\cos(dx+c)^{\frac{9}{2}}(\cos(dx+c)+1)}+\frac{2B\sin(dx+c)\left(46\right.}{}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/315/d*sin(d*x+c)*((584*cos(d*x+c)^4+292*cos(d*x+c)^3+219*cos(d*x+c)^2+13
0*cos(d*x+c)+35)*A+cos(d*x+c)*(690*cos(d*x+c)^3+345*cos(d*x+c)^2+180*cos(d
*x+c)+45)*B)*(a*(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(9/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.59

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\left(2\left(292\,A+345\,B\right)a^2\cos\left(dx+c\right)^4+\left(292\,A+345\,B\right)\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x
+ c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x +
 c) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(198) = 396.

Time = 0.16 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
8/315*(15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)
*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + (315*sq
rt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^(5/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
+ 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*sqrt(2)*a^
(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 55.94 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.84

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),
x)
```

output
```
((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(292*
A + 345*B)*4i)/(315*d) - (a^2*exp(c*3i + d*x*3i)*(2*A + 5*B)*4i)/(3*d) + (
a^2*exp(c*6i + d*x*6i)*(2*A + 5*B)*4i)/(3*d) + (a^2*exp(c*4i + d*x*4i)*(24
*A + 25*B)*4i)/(5*d) - (a^2*exp(c*5i + d*x*5i)*(24*A + 25*B)*4i)/(5*d) + (
a^2*exp(c*2i + d*x*2i)*(146*A + 155*B)*4i)/(35*d) - (a^2*exp(c*7i + d*x*7i
)*(146*A + 155*B)*4i)/(35*d) - (a^2*exp(c*9i + d*x*9i)*(292*A + 345*B)*4i)
/(315*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*
1i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*exp
(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*
exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) +
 6*exp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2
) + 6*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(
1/2) + 4*exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2
)^(1/2) + 4*exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2)^(1/2) + exp(c*8i + d*x*8i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2)^(1/2) + exp(c*9i + d*x*9i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^6}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)
**6,x)*a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)*
*5,x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,
x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*
a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*b
 + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*b)
```

3.190 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 275

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx &= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\\
&+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\\
&+\frac{16a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\\
&+\frac{2a^2(14A+11B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)}\\
&+\frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)}\end{aligned}$$

output
```
2/693*a^3*(194*A+209*B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/
2)+2/1155*a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2)+8/3465*a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x
+c))^(1/2)+16/3465*a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2)+2/99*a^2*(14*A+11*B)*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/d/c
os(d*x+c)^(9/2)+2/11*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1
1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(9070A+7678B+(25070A+\cdots}{\cdots}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
13/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos
[c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] +
 10439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*
x)] + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])
/(6930*d*Cos[c + d*x]^(11/2))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{13/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{2}{11}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(14A+11B)+a(6A+11B)\cos(c+dx))}{2\cos^{\frac{11}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(14A+11B)+a(6A+11B)\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(14A+11B)+a(6A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx+\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{1}{11}\left(\frac{2}{9}\int \frac{\sqrt{\cos(c+dx)a+a}\left((194A+209B)a^2+3(46A+55B)\cos(c+dx)a^2\right)}{2\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2(14A+11B)\sin(c+d}{9d\cos^{\frac{9}{2}}(c+}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{\sqrt{\cos(c+dx)a+a}\left((194A+209B)a^2+3(46A+55B)\cos(c+dx)a^2\right)}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2(14A+11B)\sin(c+d}{9d\cos^{\frac{9}{2}}(c+}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((194A+209B)a^2+3(46A+55B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2(14A+11B}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^3(194A+209B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(14A+11B}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^3(194A+209B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2(14A}{}\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^3(194A}{7d\cos^{\frac{7}{2}}(c+}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^3}{7d\cos}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5d\cos^{\frac{5}{2}}(}\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5a}\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3250

$$\frac{1}{11}\left(\frac{2a^2(14A+11B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2a^3(194A+209B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3}{7}a^2(710A+8\right.\right.$$
$$\frac{2aA\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),
x]
```

output
```
(2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
 + ((2*a^2*(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c
 + d*x]^(9/2)) + ((2*a^3*(194*A + 209*B)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(
7/2)*Sqrt[a + a*Cos[c + d*x]]) + (3*a^2*(710*A + 803*B)*((2*a*Sin[c + d*x]
)/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x
])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/
(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/7)/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 12.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.53

method	result
default	$\frac{2\sin(dx+c)\left(\left(5680\cos(dx+c)^{5}+2840\cos(dx+c)^{4}+2130\cos(dx+c)^{3}+1775\cos(dx+c)^{2}+1120\cos(dx+c)+315\right)A+\cos(dx+c)\left(642\right.}{3465d\cos(dx+c)^{\frac{11}{2}}(\cos(dx+c)+}$
parts	$\frac{2A\sin(dx+c)\left(1136\cos(dx+c)^{5}+568\cos(dx+c)^{4}+426\cos(dx+c)^{3}+355\cos(dx+c)^{2}+224\cos(dx+c)+63\right)\sqrt{a(\cos(dx+c)+1)}\,a^{2}}{693d\cos(dx+c)^{\frac{11}{2}}(\cos(dx+c)+1)}+$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/3465/d*sin(d*x+c)*((5680*cos(d*x+c)^5+2840*cos(d*x+c)^4+2130*cos(d*x+c)^
3+1775*cos(d*x+c)^2+1120*cos(d*x+c)+315)*A+cos(d*x+c)*(6424*cos(d*x+c)^4+3
212*cos(d*x+c)^3+2409*cos(d*x+c)^2+1430*cos(d*x+c)+385)*B)*(a*(cos(d*x+c)+
1))^(1/2)/cos(d*x+c)^(11/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.57

$$\int\frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{2\left(8\,(710\,A+803\,B)a^{2}\cos\left(dx+c\right)^{5}+4\,(710\,A+803\,B\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d
*x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*c
os(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*
x + c)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(239) = 478.

Time = 0.18 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.28

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
8/3465*(11*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt
(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)) + 5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x
 + c) + 1) - 2310*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 46
20*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5
/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9 - 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x +
 c) + 1)^11 + 200*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A
*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 +
1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))}{\cos^{\frac{13}{2}}(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 51.01 (sec) , antiderivative size = 773, normalized size of antiderivative = 2.81

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),
x)
```

output
```
((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(710*
A + 803*B)*16i)/(3465*d) - (a^2*exp(c*5i + d*x*5i)*(30*A + 41*B)*8i)/(15*d
) + (a^2*exp(c*6i + d*x*6i)*(30*A + 41*B)*8i)/(15*d) + (a^2*exp(c*4i + d*x
*4i)*(160*A + 157*B)*8i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(160*A + 157*B)*
8i)/(35*d) + (a^2*exp(c*2i + d*x*2i)*(710*A + 803*B)*8i)/(315*d) - (a^2*ex
p(c*9i + d*x*9i)*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*11i + d*x*11i)*(
710*A + 803*B)*16i)/(3465*d) - (B*a^2*exp(c*3i + d*x*3i)*8i)/(3*d) + (B*a^
2*exp(c*8i + d*x*8i)*8i)/(3*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x
*1i)/2)^(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2)^(1/2) + 5*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
 d*x*1i)/2)^(1/2) + 5*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1
i + d*x*1i)/2)^(1/2) + 10*exp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp
(c*1i + d*x*1i)/2)^(1/2) + 10*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 +
 exp(c*1i + d*x*1i)/2)^(1/2) + 10*exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)
/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 10*exp(c*7i + d*x*7i)*(exp(- c*1i - d*x
*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 5*exp(c*8i + d*x*8i)*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 5*exp(c*9i + d*x*9i)*(exp(- c*1i
 - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*10i + d*x*10i)*(exp(- c
*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*11i + d*x*11i)*(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^7}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^6}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^6}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a + 2\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

output
```
sqrt(a)*a**2*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)
**7,x)*a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)*
*6,x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**6,
x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*
a + 2*int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*b
 + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*b)
```

3.191 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 190

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx &= -\frac{(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} \\ &\quad+\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} \\ &\quad+\frac{(4A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-1/4*(4*A-7*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(1/2)/d
+2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))/a^(1/2)/d+1/4*(4*A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a
+a*cos(d*x+c))^(1/2)+1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^
(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.20

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\left((4A-B)\arcsin\left(\sqrt{1-\cos(c+dx)}\right)+8(A-B)\arcsin\left(\sqrt{\cos(c+dx)}\right)-4\sqrt{2}A\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x
]],x]
```

output
```
(((4*A - B)*ArcSin[Sqrt[1 - Cos[c + d*x]]] + 8*(A - B)*ArcSin[Sqrt[Cos[c +
 d*x]]] - 4*Sqrt[2]*A*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]
+ 4*Sqrt[2]*B*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] + 2*B*Sq
rt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 4*A*Sqrt[-((-1 + Cos[c + d*x])*C
os[c + d*x])] - B*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])
/(4*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3462, 27, 3042, 3462, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3462}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3aB+a(4A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3aB+a(4A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3aB+a(4A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3462}$$

$$\frac{\frac{\int \frac{a^2(4A-B)-a^2(4A-7B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2(4A-B)-a^2(4A-7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(4A-B)-a^2(4A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-a(4A-7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-a(4A-7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3253

$$\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2a(4A-7B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 223

$$\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2a^{3/2}(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 3261

$$\frac{\frac{-\frac{16a^3(A-B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2a^{3/2}(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$\downarrow$ 218

$$\frac{\frac{\frac{8\sqrt{2}a^{3/2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^{3/2}(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (((-2
*a^(3/2)*(4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (8*Sqrt[2]*a^(3/2)*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(2*a) + (a*(4*A - B)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 14.79 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.26

method	result
default	$\frac{\left(-4A\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+7B\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+4A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)+\sqrt{2}\,\sqrt{\right.}{8d(\cos(}$
parts	$\frac{A\left(\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)-2\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{\cos(dx+c)}\,\sqrt{2}\,\sqrt{a(c}}{2d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/d*(-4*A*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7
*B*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*A*2^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(sin(2*d*x+2*c)-sin(d*x+c))*B-8*A*arcsin(cot(d*x+c)-csc(d*x
+c))+8*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.16

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{(2\,B\cos(dx+c)+4\,A-B)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-((4\,A-7\,B)\cos(dx+c)}{4}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
1/4*((2*B*cos(d*x + c) + 4*A - B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c) - ((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
 + c)^2 + a*cos(d*x + c))) + 4*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x +
 c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)
+ a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x
)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)+1}dx\right)b\right)}{a}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x) + 1),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(
c + d*x)**2)/(cos(c + d*x) + 1),x)*b))/a
```

3.192 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 141

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{(2A-B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
(2*A-B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2
)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*
x+c))^(1/2))/a^(1/2)/d+B*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\left(B\arcsin\left(\sqrt{1-\cos(c+dx)}\right)-2(A-B)\arcsin\left(\sqrt{\cos(c+dx)}\right)+\sqrt{2}A\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)-\sqrt{}\right.}{d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x
]],x]
```

output
```
((B*ArcSin[Sqrt[1 - Cos[c + d*x]]] - 2*(A - B)*ArcSin[Sqrt[Cos[c + d*x]]]
+ Sqrt[2]*A*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] - Sqrt[2]*
B*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] + B*Sqrt[-((-1 + Cos
[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*
(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3462, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 3462$$

$$\frac{\int \frac{aB+a(2A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{aB+a(2A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aB+a(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{(2A-B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-2a(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2A-B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-2a(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{-2a(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2(2A-B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{2\sqrt{a}(2A-B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-2a(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{\frac{4a^2(A-B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2\sqrt{a}(2A-B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2\sqrt{a}(2A-B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2\sqrt{2}\sqrt{a}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((2*Sqrt[a]*(2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/d - (2*Sqrt[2]*Sqrt[a]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(2*a) + (B*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 16.66 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.39

method	result
default	$\frac{\left(B\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2A\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)-B\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+2A\arcsin(\right.}{2d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}$
parts	$\frac{A\sqrt{\cos(dx+c)}\sqrt{2}\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}+\frac{B\left(\sqrt{2}\sqrt{\frac{\cos(}{\cos(d}}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2/d*(B*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*A*2^(1/2)*
arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-B*2^(1/2)*arctan(tan(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*A*arcsin(cot(d*x+c)-csc(d*x+c)
)-2*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*cos(
d*x+c)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$=\frac{\sqrt{a\cos(dx+c)+a}B\sqrt{\cos(dx+c)}\sin(dx+c)+((2A-B)\cos(dx+c)+2A-B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos}}{}\right.}{ad\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A - B)*c
os(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - sqrt(2
)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*
sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sqrt{\cos{(c+dx)}}}{\sqrt{a\,(\cos{(c+dx)}+1)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found %i
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x
)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(co
s(c + d*x) + 1),x)*a))/a
```

3.193 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 100

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(1/2)/d+2^(1/2)*(A
-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\left(\sqrt{2}B\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+(A-B)\arctan\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
]),x]
```

output
```
(2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3461}$$

$$(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{3253}$$

$$(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{ad}$$

$\Big\downarrow$ 223

$$(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

$\Big\downarrow$ 3261

$$\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2a(A-B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$\Big\downarrow$ 218

$$\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
(2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
+ (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 11.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\sqrt{\cos(dx+c)}\,\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(-B\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+A\arcsin(\cot(dx+c)-\csc(dx+c))-B\arcsin(\cot\right.}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}$
parts	$-\frac{A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\arcsin(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}\,a}+\frac{B\sqrt{\cos(dx+c)}\,\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\sqrt{2}\,\arctan\left(\mathrm{ta}\right.\right.}{d(\cos(dx+c)-}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/d*cos(d*x+c)^(1/2)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*(-B*2^(1/2)*arctan(
tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+A*arcsin(cot(d*x+c)-csc(d*x+
c))-B*arcsin(cot(d*x+c)-csc(d*x+c)))/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}}\right)+2B\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin}{a\cos(dx+c)^2+a\cos(dx+c)}\right.}{ad}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(2)*(A - B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a))) + 2*
B*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(a*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{A+B\cos{(c+dx)}}{\sqrt{a(\cos{(c+dx)}+1)}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x
))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 1221, normalized size of antiderivative = 12.21

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
(sqrt(2)*A*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x
 + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(
I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1
/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (
abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x +
 c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c)
+ 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(
cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c)
+ 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*
x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*ar
ctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e
^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) +
 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a
)*abs(e^(I*d*x + I*c) + 1)))/sqrt(a) - (sqrt(2)*sqrt(a)*arctan2(((abs(2*e^
(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x
+ c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2
 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x +
c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x + c) + 16)^(1/4)*sin(1/2*arctan2(8*(
cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(...
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} \, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} \, dx = \int \frac{A + B\,\cos\left(c + d\,x\right)}{\sqrt{\cos\left(c + d\,x\right)}\,\sqrt{a + a\,\cos\left(c + d\,x\right)}} \, dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} \, dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)+1}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2+\cos(dx+c)}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x) +
1),x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2
 + cos(c + d*x)),x)*a))/a
```

3.194 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))/a^(1/2)/d+2*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d
*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.73 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.05

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)\left(10B\cos(c+dx)-(A-B)\left(-\frac{5}{4}(1+4\cos(c+dx)+\cos(2(c+dx)))\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]
]),x]
```

output
```
(2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*(10*B*Cos[c + d*x] - (A - B)*((-5*(1
+ 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x]
+ ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 -
2*Sec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(
c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Cos[c + d*x]^(3/2)*Sq
rt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3463}$$

$$\frac{2\int -\frac{a(A-B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-(A-B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3261}$$

$$\frac{2a(A-B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
-((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 11.57 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

method	result
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+1)\arcsin(\cot(dx+c)-\csc(dx+c))+\sin(dx+c)\sqrt{2}\right)}{d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)a} - \frac{B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{2}\,\sqrt{}}{}$
default	$\frac{2\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(-\frac{A\left(-\csc(dx+c)^2\arcsin(\cot(dx+c)-\csc(dx+c))(1-\cos(dx+c))^2+2\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\csc(dx+c)-\right.}{(\cos(dx+c)+1)\left(\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)(-\cot(dx+c)+\csc(dx+c)+1)(-}\right.}{d\sqrt{\cos(dx+c)}\,a}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
A/d/cos(d*x+c)^(1/2)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*((cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)+1)*arcsin(cot(d*x+c)-csc(d*x+c))+sin(d*x+c)*2^(
1/2))/(cos(d*x+c)+1)/a-B/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.44

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a\cos(dx+c)+a}A\sqrt{\cos(dx+c)}\sin(dx+c) - \frac{\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+}}{2\left(\cos(dx\right.}\right.}{\sqrt{a}}}{ad\cos(dx+c)^2+ad\cos(dx+c)}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
(2*sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(2)*((
A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos
(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{A+B\cos{(c+dx)}}{\sqrt{a(\cos{(c+dx)}+1)}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3
/2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 1188, normalized size of antiderivative = 12.00

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
(sqrt(2)*B*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x
 + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(
I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1
/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (
abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x +
 c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c)
+ 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(
cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c)
+ 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*
x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*ar
ctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e
^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) +
 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a
)*abs(e^(I*d*x + I*c) + 1)))/sqrt(a) + (2*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 2*(cos(d*x + c) - 1)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(2)*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*
x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - si
n(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d...
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+\cos(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2+\cos(dx+c)}dx\right)b\right)}{a}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3
 + cos(c + d*x)**2),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))
/(cos(c + d*x)**2 + cos(c + d*x)),x)*b))/a
```

3.195 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 142

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(1/2)/d+2/3*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(
d*x+c))^(1/2)-2/3*(A-3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.22 (sec) , antiderivative size = 627, normalized size of antiderivative = 4.42

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{4B\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d\sqrt{a(1+\cos(c+dx))}\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3/2}}$$
$$+ \frac{8B\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d\sqrt{a(1+\cos(c+dx))}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}$$
$$+ \frac{2(A-B)\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]
]),x]
```

output
```
(4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])
]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (8*B*Cos[c/2 + (d*x)/2]*Sin[c/2 +
(d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]
) + (2*(A - B)*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(-12*Cos[(c + d*x)/
2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 -
2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7
/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (
d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-
(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x
)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*
Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2
]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2
))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 -
 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3463}$$

$$\frac{2\int -\frac{a(A-3B)-2aA\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-3B)-2aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-3B)-2aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}$$

$$\Big\downarrow \text{3463}$$

$$\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{3a^2(A-B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a(A-B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}$$

$\Big\downarrow$ 3261

$$\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^2(A-B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}$$

$\Big\downarrow$ 218

$$\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
(2*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-
3*Sqrt[2]*Sqrt[a]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a*(A - 3*B)*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 11.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.33

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\left(3\cos(dx+c)^{2}+3\cos(dx+c)\right)+B\arcsin(\cot(dx+c)-\csc\right.}{3d\cos(dx+c)^{\frac{3}{2}}\,(\cos(dx+c}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\sin(dx+c)(\cos(dx+c)-1)\sqrt{2}+\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\left(3\cos(dx+c)^{2}+3\cos(dx+\right.\right.}{3d\cos(dx+c)^{\frac{3}{2}}\,(\cos(dx+c)+1)a}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/3/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(3*cos(d*x+c)^2+3*cos(d*x+c))+B*arcsin(co
t(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-3*cos(d*x+c)^2-3*
cos(d*x+c))+sin(d*x+c)*(cos(d*x+c)-1)*2^(1/2)*A-3/2*2^(1/2)*sin(2*d*x+2*c)
*B)/cos(d*x+c)^(3/2)/(cos(d*x+c)+1)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx =$$

$$-\frac{2\left((A-3\,B)\cos\left(dx+c\right)-A\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^{3}+(\right.}{}}{3\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
-1/3*(2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))*sin(d*x + c) - 3*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(
d*x + c)^2)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos
(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}}\, dx = \int \frac{A + B\cos{(c + dx)}}{\sqrt{a(\cos{(c + dx)} + 1)}\cos^{\frac{5}{2}}{(c + dx)}}\, dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(5
/2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 1485, normalized size of antiderivative = 10.46

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
1/3*(3*(2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4
 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) +
1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*c
os(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(
1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c)
 + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2
*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d
*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x +
c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d
*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) +
 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*
cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)
^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(
d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a
))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)) + (3*(sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + s...
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+\cos(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+\cos(dx+c)^2}dx\right)b\right)}{a}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4
 + cos(c + d*x)**3),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))
/(cos(c + d*x)**3 + cos(c + d*x)**2),x)*b))/a
```

3.196 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 187

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))/a^(1/2)/d+2/5*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos
(d*x+c))^(1/2)-2/15*(A-5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
^(1/2)+2/15*(13*A-5*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.61 (sec) , antiderivative size = 1759, normalized size of antiderivative = 9.41

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]
]),x]
```

output
```
(4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(5*d*Sqrt[a*(1 + Cos[c + d*x])
]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (16*B*Cos[c/2 + (d*x)/2]*Sin[c/2 +
 (d*x)/2])/(15*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(
3/2)) + (32*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(15*d*Sqrt[a*(1 + Cos
[c + d*x])]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - (2*(A - B)*Cot[c/2 + (d*x)
/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x
)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 65581
2*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c
+ d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c
/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 +
 (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x
)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2...
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3463}$$

$$\frac{2\int -\frac{a(A-5B)-4aA\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-5B)-4aA\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-5B)-4aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}$$

$$\Big\downarrow \text{3463}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{a^2(13A-5B)-2a^2(A-5B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(13A-5B)-2a^2(A-5B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(13A-5B)-2a^2(A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \textbf{3463}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{15a^3(A-B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^2(13A-5B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{5a}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(13A-5B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^2(A-B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(13A-5B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^2(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \textbf{3261}$$

$$\frac{\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-}{\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{30a^3(A-B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^2(13A-5B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{5a}}$$

$\downarrow$ 218

$$\frac{\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-}{\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(13A-5B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{15\sqrt{2}a^{3/2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}}{5a}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
(2*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2
*a*(A - 5*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]
]) - ((-15*Sqrt[2]*a^(3/2)*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^2*(13*A - 5*B)*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 11.66 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.16

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\left(15\cos(dx+c)^{3}+15\cos(dx+c)^{2}\right)+B\arcsin(\cot(dx+c)-\mathrm{cs}}{15}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\sin(dx+c)\left(13\cos(dx+c)^{2}-\cos(dx+c)+3\right)\sqrt{2}+\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\left(15\cos(dx\right.\right.}{15d\cos(dx+c)^{\frac{5}{2}}(\cos(dx+c)+1)a}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/15/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(15*cos(d*x+c)^3+15*cos(d*x+c)^2)+B*arcsi
n(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-15*cos(d*x+c)
^3-15*cos(d*x+c)^2)+sin(d*x+c)*(13*cos(d*x+c)^2-cos(d*x+c)+3)*2^(1/2)*A+si
n(d*x+c)*cos(d*x+c)*(-5*cos(d*x+c)+5)*2^(1/2)*B)/cos(d*x+c)^(5/2)/(cos(d*
x+c)+1)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}}\,dx$$

$$= \frac{2\left((13\,A - 5\,B)\cos\left(dx + c\right)^{2} - (A - 5\,B)\cos\left(dx + c\right) + 3\,A\right)\sqrt{a\cos\left(dx + c\right) + a}\sqrt{\cos\left(dx + c\right)}\sin\left(dx\right.}{15\,\left(ad\cos\left(dx + c\right)^{4} + ad\cos\left(d\right.\right.}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
1/15*(2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*((A - B)*
a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x
+ c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 1826, normalized size of antiderivative = 9.76

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
1/15*(5*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + c
os(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*co
s(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x
 + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*
x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^
(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d
*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c
))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4
 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) +
1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*c
os(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(
1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*
x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x +
c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x
+ c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 2*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 3)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*ar...
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(7/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{7/2}\sqrt{a+a\cos(c+dx)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5+\cos(dx+c)^4}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+\cos(dx+c)^3}dx\right)b\right)}{a}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**5
 + cos(c + d*x)**4),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))
/(cos(c + d*x)**4 + cos(c + d*x)**3),x)*b))/a
```

3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 197

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(2A-3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-9B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}$$

output

```
(2*A-3*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/4*
(5*A-9*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(
d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+1/2*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(
a+a*cos(d*x+c))^(3/2)-1/2*(A-3*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos
(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.29

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\left(-4(A-3B)\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)-20\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
3/2),x]
```

output
```
((-4*(A - 3*B)*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c + d*x)/2]^2 - 20*A*Ar
cSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^2 + 36*B*ArcSin[Sqrt[Cos[c + d*x
]]]*Cos[(c + d*x)/2]^2 + 2*Sqrt[2]*(5*A - 9*B)*ArcTan[Sqrt[Cos[c + d*x]]/S
qrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^2 + 4*B*Sqrt[1 - Cos[c + d*x]]*C
os[c + d*x]^(3/2) - 2*A*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])] + 6*B*Sq
rt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c
 + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)-2a(A-3B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)-2a(A-3B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)-2a(A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3462}$$

$$\frac{\frac{\int -\frac{a^2(A-3B)-2a^2(2A-3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{a^2(A-3B)-2a^2(2A-3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{a^2(A-3B)-2a^2(2A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{-\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-2a(2A-3B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a(2A-3B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3253$$

$$-\frac{\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a(2A-3B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}(2A-3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3261$$

$$-\frac{\frac{-\frac{2a^3(5A-9B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^{3/2}(2A-3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{-\frac{\sqrt{2}a^{3/2}(5A-9B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}(2A-3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
 + (-(((-4*a^(3/2)*(2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Co
s[c + d*x]]])/d + (Sqrt[2]*a^(3/2)*(5*A - 9*B)*ArcTan[(Sqrt[a]*Sin[c + d*x
])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) - (2*a*(A
 - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*
a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 16.86 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.45

method	result
default	$\frac{\left(-A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)+\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\sin(2dx+2c)+3\sin(dx+c))B+(4\cos(dx+c)+4)\sqrt{2}\,A\arctan\left(\tan(dx+c)\sqrt{\right.}}{}$
parts	$-\frac{A\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)\left(\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)^2\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(-4\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.\right.}{16d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^2}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*(-A*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(sin(2*d*x+2*c)+3*sin(d*x+c))*B+(4*cos(d*x+
c)+4)*2^(1/2)*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-(6*co
s(d*x+c)+6)*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*B
+(5*cos(d*x+c)+5)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(-9*cos(d*x+c)-9)*B*arcs
in(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/
2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.40

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((5A-9B)\cos(dx+c)^2+2(5A-9B)\cos(dx+c)+5A-9B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{c}}{2\left(a\cos(dx+c)^2+a\right.}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5
*A - 9*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt
(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*(2*B*
cos(d*x + c) - A + 3*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*
x + c) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c) + 2*A
- 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x
)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{}}{\cos(dx+c)^2}\right.\right.}{a^2}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a + int((sqrt(cos(c + d*x) + 1)*sqr
t(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)
*b))/a**2
```

3.198 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 145

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/4*(A-5*B
)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^
(1/2))*2^(1/2)/a^(3/2)/d+1/2*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(
d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\left(B\arcsin\left(\sqrt{1-\cos(c+dx)}\right)(1+\cos(c+dx)) + 5B\arcsin\left(\sqrt{\cos(c+dx)}\right)(1+\cos(c+dx)) + \sqrt{2}\left(\right.\right.}{2d\sqrt{1-\cos(c+dx}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
3/2),x]
```

output
```
-1/2*((B*ArcSin[Sqrt[1 - Cos[c + d*x]]]*(1 + Cos[c + d*x]) + 5*B*ArcSin[Sq
rt[Cos[c + d*x]]]*(1 + Cos[c + d*x]) + Sqrt[2]*((A - 5*B)*ArcTan[Sqrt[Cos[
c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^2 + (-A + B)*Sqrt[Cos
[c + d*x]*Sin[(c + d*x)/2]^2]))*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*(a
*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{a(A-B)+4aB\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-B)+4aB\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-B)+4aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{a(A-5B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+4B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A-5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+4B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{a(A-5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{a(A-5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{8\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{\frac{8\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^2(A-5B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt{2}\sqrt{a}(A-5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{8\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
((8*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
 (Sqrt[2]*Sqrt[a]*(A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(4*a^2) + ((A - B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(120) = 240$.

Time = 8.95 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sqrt{\cos(dx+c)}\left(A\left(\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)\left(\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\csc(dx+c)-\cot(dx+c))-\arcsin(\cot(\right.\right.}{}$
parts	$\frac{A\sqrt{\cos(dx+c)}\left(\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\csc(dx+c)-\cot(dx+c))-\arcsin(\cot(d\right.}{8d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^2}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^(1/2)*(A*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(2^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(csc(d*x+c)-cot(d*x+c))-arcsin(cot(d*x+c)-csc(d*x+c)
))-1/2*B*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*(cos(d*x+c)+1)*(-4*2^(1/2)*ar
ctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(csc(d*x+c)-cot(d*x+c))-5*arcsin(cot(d*x+c)-csc(d*x+c))
))/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(120) = 240$.

Time = 2.64 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left((A-5\,B)\cos\left(dx+c\right)^2+2\,(A-5\,B)\cos\left(dx+c\right)+A-5\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*
B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d
*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) + 8*(B*cos(d*x + c)^2
 + 2*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(a^2*
d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(
3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x
)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{}}{\cos(dx+c)^2+2\,\mathrm{co}}\right.\right.}{a^2}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sqr
t(cos(c + d*x)))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.199 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 107

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx=\frac{(3A+B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
1/4*(3*A+B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-1/2*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/d
/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(2B\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\cos^3\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3
/2)),x]
```

output
```
(Sqrt[Cos[(c + d*x)/2]^2]*(2*B*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/
2]^2]]*Cos[(c + d*x)/2]^3 - 3*Sqrt[2]*A*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c
 + d*x)/2]^2)]]*Cos[(c + d*x)/2]^2*Sqrt[-1 + Cos[c + d*x]]*Cot[(c + d*x)/2
] + (-A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(Sqrt[2]*d*Sqrt[1 + Cos[c +
 d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(3A+B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3261}$$

$$-\frac{(3A+B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(3A+B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x
]
```

output
```
((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 12.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

method	result
default	$-\frac{\left((3\cos(dx+c)+3)A\arcsin(\cot(dx+c)-\csc(dx+c))+A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+(\cos(dx+c)+1)B\arcsin(\cot(dx+c)-\csc(d\right.}{4d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c)+1)^2a^2}$
parts	$-\frac{A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\csc(dx+c)-\cot(dx+c))+3\arcsin(\cot(dx+c)-\csc(dx+c))\right)}{4d\sqrt{\cos(dx+c)}\,a^2}+\frac{B\sqrt{}}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/d*((3*cos(d*x+c)+3)*A*arcsin(cot(d*x+c)-csc(d*x+c))+A*2^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+(cos(d*x+c)+1)*B*arcsin(cot(d*x+c)-c
sc(d*x+c))-B*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x
+c)^(1/2)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)/(cos(d*x+c)+1)^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left((3\,A+B)\cos\left(dx+c\right)^2+2\,(3\,A+B)\cos\left(dx+c\right)+3\,A\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d
*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c +
d*x))), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)+1}}{\cos(dx+c)^2+2\,\mathrm{c}}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3
 + 2*cos(c + d*x)**2 + cos(c + d*x)),x)*a + int((sqrt(cos(c + d*x) + 1)*sq
rt(cos(c + d*x)))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b))/a**2
```

3.200 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 156

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(7A-3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/4*(7*A-3*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(1
/2)/(a+a*cos(d*x+c))^(3/2)+1/2*(5*A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.41 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.71

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(30(A-B)\arctan\left(\frac{1-2\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)-30(A\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3
/2)),x]
```

output
```
(Cos[(c + d*x)/2]^3*(30*(A - B)*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c
 + d*x]]] - 30*(A - B)*ArcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]]
 - (20*(A - B)*Sqrt[Cos[c + d*x]])/(-1 + Sin[(c + d*x)/2]) - (20*(A - B)*S
qrt[Cos[c + d*x]])/(1 + Sin[(c + d*x)/2]) + (5*(A - B)*(-1 + 2*Sin[(c + d*
x)/2]))/(Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2) - (5*
(A - B)*(1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(-1 + Sin[(c + d*x)/
2])) + ((A + 3*B)*Csc[(c + d*x)/2]^3*(5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d
*x)])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]
]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2*Hypergeometric2F1[2, 5/2, 7/2
, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*Sin[c + d*x]*Tan[
c + d*x]))/(2*Cos[c + d*x]^(3/2))))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{a(5A-B)-2a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a(5A-B)-2a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a(5A-B)-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3463}$$

$$\frac{\frac{2\int -\frac{a^2(7A-3B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a(7A-3B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a(7A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3261}$$

$$\frac{\frac{2a^2(7A-3B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{a}(7A-3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x
]
```

output
```
-1/2*((A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/
2)) + (-((Sqrt[2]*Sqrt[a]*(7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d) + (2*a*(5*A - B)*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 12.62 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.24

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\frac{7(3+\cos(2dx+2c)+4\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,A\arcsin(\cot(dx+c)-\csc(dx+c))}{2}-\frac{3(3+\cos(2dx+2c)+4\cos(dx+c))\sqrt{\cdots}}{\cdots}\cdots\right.}{4d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)^2a^2}\cdots$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sin(dx+c)(5\cos(dx+c)+4)\sqrt{2}+\frac{7(3+\cos(2dx+2c)+4\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arcsin(\cot(dx+c)-\csc(dx+c))}{2}\right)}{4d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)^2a^2}\cdots$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*(7/2*(3+cos(2*d*x+2*c)+4*cos(d*x+c)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*A*arcsin(cot(d*x+c)-csc(d*x+c))-3/2*(3
+cos(2*d*x+2*c)+4*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*B*arcsin(c
ot(d*x+c)-csc(d*x+c))+sin(d*x+c)*(5*cos(d*x+c)+4)*2^(1/2)*A-1/2*2^(1/2)*si
n(2*d*x+2*c)*B)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((7\,A-3\,B)\cos\left(dx+c\right)^{3}+2\,(7\,A-3\,B)\cos\left(dx+c\right)^{2}+(7\,A-3\,B)\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}}{}\right.}{4\left(a^{2}d\cos\left(dx+c\right)^{3}+2\,a^{2}\right.}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^3 + 2*(7*A - 3*B)*cos(d*x + c)^2 +
 (7*A - 3*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x
 + c))) - 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2
+ a^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{3/2}}\,dx = \int \frac{A + B\cos(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}\cos^{\frac{3}{2}}(c + dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x)*
*(3/2)), x)
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{3/2}}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^{\frac{3}{2}}\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3
/2)), x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{3/2}}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^{\frac{3}{2}}\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c}}{\cos(dx+c)^3+2\,\mathrm{co}}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4
 + 2*cos(c + d*x)**3 + cos(c + d*x)**2),x)*a + int((sqrt(cos(c + d*x) + 1)
*sqrt(cos(c + d*x)))/(cos(c + d*x)**3 + 2*cos(c + d*x)**2 + cos(c + d*x)),
x)*b))/a**2
```

3.201 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 203

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(11A-7B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{(19A-15B)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*(11*A-7*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3
/2)/(a+a*cos(d*x+c))^(3/2)+1/6*(7*A-3*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(
a+a*cos(d*x+c))^(1/2)-1/6*(19*A-15*B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.94 (sec) , antiderivative size = 1054, normalized size of antiderivative = 5.19

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3
/2)),x]
```

output
```
-1/6*((A - B)*Cos[c/2 + (d*x)/2]^3*(1 - 2*Sin[c/2 + (d*x)/2]))/(d*(a*(1 +
Cos[c + d*x]))^(3/2)*(1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)
^(3/2)) + ((A - B)*Cos[c/2 + (d*x)/2]^3*(1 + 2*Sin[c/2 + (d*x)/2]))/(6*d*(
a*(1 + Cos[c + d*x]))^(3/2)*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x
)/2]^2)^(3/2)) - ((A - B)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 - 2*Sin[c/2 +
(d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 + Sin[c/2 + (d*x)/2])/((1
 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*S
in[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))
^(3/2)) + ((A - B)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2
])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[
c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2
+ (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
 + ((A + 3*B)*Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^2*(-12*Cos[(c + d*x)
/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 -
 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2,
7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 +
(d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[
-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*
x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7
*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(7A-3B)-4a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(7A-3B)-4a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(7A-3B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{2\int -\frac{a^2(19A-15B)-2a^2(7A-3B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(19A-15B)-2a^2(7A-3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(19A-15B)-2a^2(7A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3463

$$\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{3a^3(11A-7B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^2(11A-7B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^2(11A-7B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3261

$$\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^3(11A-7B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}a^{3/2}(11A-7B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x
]
```

output
```
-1/2*((A - B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)) + ((2*a*(7*A - 3*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*C
os[c + d*x]]) - ((-3*Sqrt[2]*a^(3/2)*(11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c +
d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^2*(
19*A - 15*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
)/(3*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 12.90 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(33\cos(dx+c)^{3}+66\cos(dx+c)^{2}+33\cos(dx+c)\right)+B\arcs$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sin(dx+c)\left(19\cos(dx+c)^{2}+12\cos(dx+c)-4\right)\sqrt{2}+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arcsin(\cot(dx+c)-\csc(dx+c))\left(33\cos\right.\right.}{12d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)^{2}a^{2}}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/12/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*(A*arcsin(cot(d*x+c)-csc(d*x+c))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(33*cos(d*x+c)^3+66*cos(d*x+c)^2+33*cos(
d*x+c))+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(-21*cos(d*x+c)^3-42*cos(d*x+c)^2-21*cos(d*x+c))+sin(d*x+c)*(19*cos(d*x+c)
^2+12*cos(d*x+c)-4)*2^(1/2)*A+sin(d*x+c)*cos(d*x+c)*(-15*cos(d*x+c)-12)*2^
(1/2)*B)/cos(d*x+c)^(3/2)/(cos(d*x+c)+1)^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \frac{3\sqrt{2}\left((11\,A-7\,B)\cos\left(dx+c\right)^{4}+2\,(11\,A-7\,B)\cos\left(dx+c\right)^{3}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^4 + 2*(11*A - 7*B)*cos(d*x + c)
^3 + (11*A - 7*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*
cos(d*x + c))) - 2*((19*A - 15*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c)
 - 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))^{3/2}}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^{\frac{3}{2}}\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{5/2}(a+a\cos(c+dx))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5+2\cos(dx+c)^4+\cos(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c}}{\cos(dx+c)^4+2\cos}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**5
 + 2*cos(c + d*x)**4 + cos(c + d*x)**3),x)*a + int((sqrt(cos(c + d*x) + 1)
*sqrt(cos(c + d*x)))/(cos(c + d*x)**4 + 2*cos(c + d*x)**3 + cos(c + d*x)**
2),x)*b))/a**2
```

3.202 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 246

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(2A-5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-115B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(11A-35B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

output
```
(2*A-5*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/32
*(43*A-115*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+1/4*(A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)
/d/(a+a*cos(d*x+c))^(5/2)+1/16*(7*A-15*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/
(a+a*cos(d*x+c))^(3/2)-1/16*(11*A-35*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/
(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.72 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\left(-8(11A-35B)\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right) -\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
5/2),x]
```

output
```
((-8*(11*A - 35*B)*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 - 344
*A*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 + 920*B*ArcSin[Sqrt[Cos[c
 + d*x]]]*Cos[(c + d*x)/2]^4 + 4*Sqrt[2]*(43*A - 115*B)*ArcTan[Sqrt[Cos[c
+ d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 - 30*A*Sqrt[1 - Cos[c
 + d*x]]*Cos[c + d*x]^(3/2) + 110*B*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3
/2) + 32*B*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2) - 22*A*Sqrt[-((-1 + C
os[c + d*x])*Cos[c + d*x])] + 70*B*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x]
)])*Sin[c + d*x])/(32*d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2
))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-2a(A-5B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-2a(A-5B)\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a(A-B)-2a(A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2(7A-15B)-2a^2(11A-35B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2(7A-15B)-2a^2(11A-35B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a^2(7A-15B)-2a^2(11A-35B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3462}$$

$$\frac{\frac{\frac{\int -\frac{a^3(11A-35B)-16a^3(2A-5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{-\frac{\int \frac{a^3(11A-35B)-16a^3(2A-5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{\int \frac{a^3(11A-35B)-16a^3(2A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3461$$

$$\frac{\frac{-\frac{a^3(43A-115B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-16a^2(2A-5B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cdots}{2d(a\cos(c+dx)+\cdots}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{a^3(43A-115B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2(2A-5B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B\cdots}{2d(a\cdots}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3253$$

$$\frac{\frac{-\frac{a^3(43A-115B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2(2A-5B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+\cdots}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}\cdots}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 223

$$\frac{\frac{-\frac{a^3(43A-115B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}(2A-5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-\cdots}{2\cdots}}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 3261

$$\frac{\frac{-\frac{-\frac{2a^4(43A-115B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{32a^{5/2}(2A-5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a^2(11A-35B)\sin(c+\cdots}{d\sqrt{a\cos(c+d\cdots}}}{4a^2}\cdots}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 218

$$\frac{\frac{-\frac{\frac{\sqrt{2}a^{5/2}(43A-115B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}(2A-5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(\cdots}{\cdots}}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2))
 + ((a*(7*A - 15*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)) + (-(((-32*a^(5/2)*(2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/S
qrt[a + a*Cos[c + d*x]]])/d + (Sqrt[2]*a^(5/2)*(43*A - 115*B)*ArcTan[(Sqrt
[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/
d)/a) - (2*a^2*(11*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a +
a*Cos[c + d*x]]))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 17.31 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.44

method	result
default	$\frac{\left(\sin(dx+c)(-15\cos(dx+c)-11)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)(43+8\cos(2dx+2c)+55\cos(dx+c))\sqrt{2}\,B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+(48+16\,\text{c}\right.}{}$
parts	$\frac{A\left((48+16\cos(2dx+2c)+64\cos(dx+c))\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sin(dx+c)(-15\cos(dx+c)-11)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{32d\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)\right.}$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/32/d*(sin(d*x+c)*(-15*cos(d*x+c)-11)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+sin(d*x+c)*(43+8*cos(2*d*x+2*c)+55*cos(d*x+c))*2^(1/2)*B*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)+(48+16*cos(2*d*x+2*c)+64*cos(d*x+c))*2^(1/2)*A*a
rctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-40*(3+cos(2*d*x+2*c)+4
*cos(d*x+c))*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
B+(43*cos(d*x+c)^2+86*cos(d*x+c)+43)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(-115
*cos(d*x+c)^2-230*cos(d*x+c)-115)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)
*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*
cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.39

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((43\,A-115\,B)\cos\left(dx+c\right)^3+3\,(43\,A-115\,B)\cos\left(dx+c\right)^2+3\,(43\,A-115\,B)\cos\left(dx+c\right)+43\,A\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x +
 c)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(1/2*s
qrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*(16*B*cos(d*x + c)^2 - 5*(3*A - 11*
B)*cos(d*x + c) - 11*A + 35*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
*sin(d*x + c) - 32*((2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c
)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*
cos(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x
)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)+}}{\cos(dx+c)^3+3\,c}\right.\right.\right.}{a^3}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3)/
(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqr
t(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)**3 +
 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a))/a**3
```

3.203 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 194

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/32*(3*A-
43*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)/a^(5/2)/d+1/4*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*
cos(d*x+c))^(5/2)+1/16*(3*A-11*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos
(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.06 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.30

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(88B\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+344B\arcsin\left(\sqrt{\cos(c+dx)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+4\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
5/2),x]
```

output
```
-1/32*((88*B*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 + 344*B*Arc
Sin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 + 4*Sqrt[2]*(3*A - 43*B)*ArcTan
[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 - 14*A*Sq
rt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 30*B*Sqrt[1 - Cos[c + d*x]]*Cos[
c + d*x]^(3/2) - 6*A*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])] + 22*B*Sqrt
[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d
*x]]*(a*(1 + Cos[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+8aB\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+8aB\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)+8aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{(3A-11B)a^2+32B\cos(c+dx)a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(3A-11B)a^2+32B\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(3A-11B)a^2+32B\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+32aB\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+32aB\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3253

$$\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64aB\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 223

$$\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 3261

$$\frac{\frac{\frac{64a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^3(3A-43B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\sqrt{2}a^{3/2}(3A-43B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{64a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2))
 + (((64*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (Sqrt[2]*a^(3/2)*(3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(4*a^2) + (a*(3*A - 11*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a
^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 8.60 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.50

method	result
default	$\frac{\left((48+16\cos(2dx+2c)+64\cos(dx+c))\sqrt{2}\,B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sin(dx+c)(7\cos(dx+c)+3)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\mathrm{si}\right.}{}$
parts	$\frac{A\left(\sin(dx+c)(7\cos(dx+c)+3)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(-3\cos(dx+c)^{2}-6\cos(dx+c)-3\right)\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{\cos(dx+c)}}{32d\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^{3}}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/32/d*((48+16*cos(2*d*x+2*c)+64*cos(d*x+c))*2^(1/2)*B*arctan(tan(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*(7*cos(d*x+c)+3)*2^(1/2)*A*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(-15*cos(d*x+c)-11)*2^(1/2)*B*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*A*arcsin
(cot(d*x+c)-csc(d*x+c))+(43*cos(d*x+c)^2+86*cos(d*x+c)+43)*B*arcsin(cot(d*
x+c)-csc(d*x+c)))*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/2)/(cos(d
*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a
^3
```

Fricas [A] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.58

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left((3\,A-43\,B)\cos\left(dx+c\right)^3+3\,(3\,A-43\,B)\cos\left(dx+c\right)^2+\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2
 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(1/2*sqrt(2)*sq
rt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 + a*cos(d*x + c))) + 2*((7*A - 15*B)*cos(d*x + c) + 3*A - 11*B)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 64*(B*cos(d*x + c)
^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 +
a*cos(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x
)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)+}}{\cos(dx+c)^3+3\,c}\right.\right.}{a^3}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a + int((sqrt(c
os(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)**3 + 3*
cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b))/a**3
```

3.204 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(5A+3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
1/32*(5*A+3*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+1/4*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+c
)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(A+7*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a
+a*cos(d*x+c))^(3/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 491 vs. $2(154) = 308$.

Time = 6.42 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{3B\arcsin\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d(a(1+\cos(c+dx)))^{5/2}}$$
$$+\frac{5B\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}{4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(a(1+\cos(c+dx)))^{5/2}}$$
$$-\frac{B\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sin^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}{2d\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}(a(1+\cos(c+dx)))^{5/2}}$$
$$+\frac{A\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(3-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-5\operatorname{arctanh}\left(\sqrt{-\frac{}{1}}\right.\right.}{4d(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
5/2),x]
```

output
```
(3*B*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Cos[c/2 + (d*x)/2
]^5)/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) + (5*B*Cos[c/2 + (d*x)/2]^5*Sin[c/
2 + (d*x)/2]*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(4*d*Sqrt[
Cos[(c + d*x)/2]^2]*(a*(1 + Cos[c + d*x]))^(5/2)) - (B*Cos[c/2 + (d*x)/2]^
5*Sin[c/2 + (d*x)/2]^3*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/
(2*d*(Cos[(c + d*x)/2]^2)^(3/2)*(a*(1 + Cos[c + d*x]))^(5/2)) + (A*Cos[c/2
 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*Sqrt[1 - 2*Sin[c/2 + (
d*x)/2]^2]*(3 - Sin[c/2 + (d*x)/2]^2 - 5*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]
^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*Cos[(c + d*x)/2]^4*Csc[c/2 + (d*x)/2]^2
*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]))/(4*d*(a*(1 +
 Cos[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{a(A-B)+2a(A+3B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-B)+2a(A+3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-B)+2a(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{a^2(5A+3B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}(5A+3B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+$$
$$\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{4}(5A+3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3261$$

$$\frac{\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{a(5A+3B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{(5A+3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2))
 + (((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) + (a*(A + 7*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.53

method	result
default	$\frac{\left(\sin(dx+c)(\cos(dx+c)+5)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)(7\cos(dx+c)+3)\sqrt{2}\,B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(-5\cos(dx+c)^2-10\cos(dx+c)-\right.\right.}{32d\left(\cos(dx+c)^3+3\cos(dx+\right.}$
parts	$\frac{A\left(\sqrt{2}\,\sin(dx+c)(\cos(dx+c)+5)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(-5\cos(dx+c)^2-10\cos(dx+c)-5\right)\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{\cos(dx+c)}}{32d\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^3}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/32/d*(sin(d*x+c)*(cos(d*x+c)+5)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+sin(d*x+c)*(7*cos(d*x+c)+3)*2^(1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+(-5*cos(d*x+c)^2-10*cos(d*x+c)-5)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(-3*co
s(d*x+c)^2-6*cos(d*x+c)-3)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x
+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left((5\,A+3\,B)\cos\left(dx+c\right)^3+3\,(5\,A+3\,B)\cos\left(dx+c\right)^2+3\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 + a*cos(d*x + c))) + 2*((A + 7*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(
5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x
)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\cos(}}{\cos(dx+c)^3+3}\right.\right.}{a^3}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

output

```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(c
os(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2
+ 3*cos(c + d*x) + 1),x)*a))/a**3
```

3.205 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 156

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(19A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
1/32*(19*A+5*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-1/4*(A-B)*cos(d*x+c)^(1/2)*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(9*A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/(
a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.93 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(4(19A+5B)\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5
/2)),x]
```

output
```
(Sec[(c + d*x)/2]^2*(4*(19*A + 5*B)*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d
*x)/2]^2)]]*Cos[(c + d*x)/2]^4 + Cos[c + d*x]*(-13*A + 5*B + (-9*A + B)*Co
s[c + d*x])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(32*Sqrt[2]*a^2*d*
Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(7A+B)-2a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(7A+B)-2a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(7A+B)-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{a^2(19A+5B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}(19A+5B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}(19A+5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{-\frac{a(19A+5B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(19A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x
]
```

output
```
-1/4*((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/
2)) + (((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) - (a*(9*A - B)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 12.79 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.53

method	result
default	$-\frac{\left(\sin(dx+c)(9\cos(dx+c)+13)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)(-\cos(dx+c)-5)\sqrt{2}\,B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(19\cos(dx+c)^{2}+38\cos(dx\right.\right.}{32d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\cos\right.}$
parts	$-\frac{A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\sqrt{a(\cos(dx+c)+1)}\left(2\csc(dx+c)^{3}\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(1-\cos(dx+c))^{3}+11\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\csc(dx+c)-\cot\right.}{32d\sqrt{\cos(dx+c)}\,a^{3}}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/32/d*(sin(d*x+c)*(9*cos(d*x+c)+13)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)+sin(d*x+c)*(-cos(d*x+c)-5)*2^(1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+(19*cos(d*x+c)^2+38*cos(d*x+c)+19)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(5
*cos(d*x+c)^2+10*cos(d*x+c)+5)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*cos(d*x+c)
^(1/2)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/
(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.39

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(1/2*sqrt(2)*sq
rt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 + a*cos(d*x + c))) - 2*((9*A - B)*cos(d*x + c) + 13*A - 5*B)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\cos{(c+dx)}}{(a(\cos{(c+dx)}+1))^{\frac{5}{2}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(5/2)*sqrt(cos(c +
d*x))), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos{(dx+c)}+A}{(a\cos{(dx+c)}+a)^{\frac{5}{2}}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)}dx\right)a+\left(\int \cdots\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4
 + 3*cos(c + d*x)**3 + 3*cos(c + d*x)**2 + cos(c + d*x)),x)*a + int((sqrt(
cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2
 + 3*cos(c + d*x) + 1),x)*b))/a**3
```

3.206 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 203

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(75A-19B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/32*(75*A-19*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)
^(1/2)/(a+a*cos(d*x+c))^(5/2)-1/16*(13*A-5*B)*sin(d*x+c)/a/d/cos(d*x+c)^(1
/2)/(a+a*cos(d*x+c))^(3/2)+1/16*(49*A-9*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.41 (sec) , antiderivative size = 728, normalized size of antiderivative = 3.59

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5
/2)),x]
```

output
```
-1/4*(B*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*(11 - 3
1*Sin[c/2 + (d*x)/2]^2 + 18*Sin[c/2 + (d*x)/2]^4 - (19*ArcTanh[Sqrt[-(Sin[
c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*Cos[(c + d*x)/2]^4)/Sqrt[
-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]))/(d*(a*(1 + Cos[c +
 d*x]))^(5/2)*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (2*A*Cos[c/2 + (d*x)/2]^
5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((8*Cos[(c + d*x)/2]^6*Hypergeomet
ricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) +
 (Csc[c/2 + (d*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-343 + 1465*Sin[c/2
+ (d*x)/2]^2 - 2021*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d*x)/2]^6) + Sqr
t[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145 + 33980*Sin[c
/2 + (d*x)/2]^2 - 87764*Sin[c/2 + (d*x)/2]^4 + 109737*Sin[c/2 + (d*x)/2]^6
 - 66122*Sin[c/2 + (d*x)/2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))/120))/(d*(a
*(1 + Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 3457$$

$$\frac{\int \frac{a(9A-B)-4a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a(9A-B)-4a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(9A-B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\int \frac{a^2(49A-9B)-2a^2(13A-5B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{a^2(49A-9B)-2a^2(13A-5B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{a^2(49A-9B)-2a^2(13A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 3463

$$\frac{\frac{\frac{2\int -\frac{a^3(75A-19B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a^2(75A-19B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a^2(75A-19B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 3261

$$\frac{\frac{\frac{2a^3(75A-19B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$\downarrow$ 218

$$\frac{\frac{\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}a^{3/2}(75A-19B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x
]
```

output
```
-1/4*((A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/
2)) + (-1/2*(a*(13*A - 5*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos
[c + d*x])^(3/2)) + (-((Sqrt[2]*a^(3/2)*(75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[
c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d) + (2*
a^2*(49*A - 9*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*
x]]))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 12.72 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

method	result
default	$\frac{\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\left(75\cos(dx+c)^{3}+225\cos(dx+c)^{2}+225\cos(dx+c)+75\right)A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(-19\,\mathrm{c}\right.\right.}{}$
parts	$\frac{A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\sin(dx+c)\left(49\cos(dx+c)^{2}+85\cos(dx+c)+32\right)\sqrt{2}+\left(75\cos(dx+c)^{3}+225\cos(dx+c)^{2}+225\cos(dx+c)+75\right)\mathrm{arc}\right.}{16d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)\left(\cos(dx+c)^{2}+2\cos(dx+c)+1\right)a^{3}}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/16/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*((75*cos(d*x+c)^3+225*cos(d*x+c)^2+2
25*cos(d*x+c)+75)*A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)+(-19*cos(d*x+c)^3-57*cos(d*x+c)^2-57*cos(d*x+c)-19)*B*arcsin(cot
(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(49*cos(d
*x+c)^2+85*cos(d*x+c)+32)*2^(1/2)*A+sin(d*x+c)*cos(d*x+c)*(-9*cos(d*x+c)-1
3)*2^(1/2)*B)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)^2+2*cos(d*x+c)+1
)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\big((75\,A-19\,B)\cos\left(dx+c\right)^4+3\,(75\,A-19\,B)\cos\left(dx+c\right)^3+3\,(75\,A-19\,B)\cos\left(dx+c\right)^2+(75\,A}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^4 + 3*(75*A - 19*B)*cos(d*x + c
)^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + (75*A - 19*B)*cos(d*x + c))*sqrt(a)
*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*si
n(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((49*A - 9*B)*cos(d*x
+ c)^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2}dx\right)a+\left(\int \frac{}{co}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**5
 + 3*cos(c + d*x)**4 + 3*cos(c + d*x)**3 + cos(c + d*x)**2),x)*a + int((sq
rt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4 + 3*cos(c + d*x)
**3 + 3*cos(c + d*x)**2 + cos(c + d*x)),x)*b))/a**3
```

3.207 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 250

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(163A-75B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(95A-39B)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output

```
1/32*(163*A-75*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)
^(3/2)/(a+a*cos(d*x+c))^(5/2)-1/16*(17*A-9*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3
/2)/(a+a*cos(d*x+c))^(3/2)+1/48*(95*A-39*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3
/2)/(a+a*cos(d*x+c))^(1/2)-1/48*(299*A-147*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^
(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{3i(163A-75B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\text{arctan}}{\sqrt{1+e^{2i(c+dx)}}}\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5
/2)),x]
```

output
```
(Cos[(c + d*x)/2]^5*(((3*I)*(163*A - 75*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E
^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((878*A
 - 510*B + (1537*A - 825*B)*Cos[c + d*x] + 2*(503*A - 255*B)*Cos[2*(c + d*
x)] + 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^3*
Tan[(c + d*x)/2])/(8*Cos[c + d*x]^(3/2))))/(12*d*(a*(1 + Cos[c + d*x]))^(5
/2))
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(11A-3B)-6a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(11A-3B)-6a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(11A-3B)-6a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{a^2(95A-39B)-4a^2(17A-9B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2(95A-39B)-4a^2(17A-9B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(95A-39B)-4a^2(17A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{\frac{2\int -\frac{a^3(299A-147B)-2a^3(95A-39B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^3(299A-147B)-2a^3(95A-39B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^3(299A-147B)-2a^3(95A-39B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{3a^4(163A-75B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^3(163A-75B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^3(163A-75B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{a(17A-9B)\sin(c+\ldots}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+d\ldots}}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3261$$

$$\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^4(163A-75B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{\ldots}{2d\ldots}}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}a^{5/2}(163A-75B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{a(17A-9B)\mathrm{si}\ldots}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos\ldots}}{8a^2}$$

$$\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x
]
```

output
```
-1/4*((A - B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/
2)) + (-1/2*(a*(17*A - 9*B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos
[c + d*x])^(3/2)) + ((2*a^2*(95*A - 39*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-3*Sqrt[2]*a^(5/2)*(163*A - 75*B)*ArcT
an[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]])])/d + (2*a^3*(299*A - 147*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 12.73 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(489\cos(dx+c)^{4}+1467\cos(dx+c)^{3}+1467\cos(dx+c)^{2}+489\,c\right.\right.}{}$
parts	$-\frac{A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\sin(dx+c)\left(299\cos(dx+c)^{3}+503\cos(dx+c)^{2}+160\cos(dx+c)-32\right)\sqrt{2}+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arcsin(\cot(dx+c)-c\right.}{48d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)\left(\cos(dx+c)^{2}+2\cos(dx\right.}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/48/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(489*cos(d*x+c)^4+1467*cos(d*x+c)^3+1467*c
os(d*x+c)^2+489*cos(d*x+c))+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(-225*cos(d*x+c)^4-675*cos(d*x+c)^3-675*cos(d*x+c)^2-2
25*cos(d*x+c))+sin(d*x+c)*(299*cos(d*x+c)^3+503*cos(d*x+c)^2+160*cos(d*x+c
)-32)*2^(1/2)*A+sin(d*x+c)*cos(d*x+c)*(-147*cos(d*x+c)^2-255*cos(d*x+c)-96
)*2^(1/2)*B)/cos(d*x+c)^(3/2)/(cos(d*x+c)+1)/(cos(d*x+c)^2+2*cos(d*x+c)+1)
/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.08

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))^{5/2}}\,dx = \frac{3\sqrt{2}\big((163\,A - 75\,B)\cos(dx + c)^5 + 3\,(163\,A - 75\,B)\cos(dx \cdots}{\cdots}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^5 + 3*(163*A - 75*B)*cos(d*x
+ c)^4 + 3*(163*A - 75*B)*cos(d*x + c)^3 + (163*A - 75*B)*cos(d*x + c)^2)*
sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((299*A - 147*B
)*cos(d*x + c)^3 + (503*A - 255*B)*cos(d*x + c)^2 + 32*(5*A - 3*B)*cos(d*x
 + c) - 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a
^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^
3*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))^{5/2}}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^{\frac{5}{2}}\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a\cos(c + dx))^{5/2}}\,dx = \int \frac{A + B\cos(c + d\,x)}{\cos(c + d\,x)^{5/2}\,(a + a\cos(c + d\,x))^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^6+3\cos(dx+c)^5+3\cos(dx+c)^4+\cos(dx+c)^3}dx\right)a+\left(\int \cdots\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**6
 + 3*cos(c + d*x)**5 + 3*cos(c + d*x)**4 + cos(c + d*x)**3),x)*a + int((sq
rt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**5 + 3*cos(c + d*x)
**4 + 3*cos(c + d*x)**3 + cos(c + d*x)**2),x)*b))/a**3
```

3.208 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 293

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(2A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} - \frac{(177A-637B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{(79A-259B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} - \frac{7(7A-27B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}}$$

output
```
(2*A-7*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d-1/12
8*(177*A-637*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))*2^(1/2)/a^(7/2)/d+1/6*(A-B)*cos(d*x+c)^(7/2)*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(7/2)+1/16*(3*A-7*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^(5/2)+1/192*(79*A-259*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2
/d/(a+a*cos(d*x+c))^(3/2)-7/64*(7*A-27*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^3/
d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.13 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.29

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\left(-336(7A-27B)\arcsin\left(\sqrt{1-\cos(c+d\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
7/2),x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*(-336*(7*A - 27*B)*ArcSin[Sqrt[1 - Cos[c + d*x
]]]*Cos[(c + d*x)/2]^6 - 8496*A*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2
]^6 + 30576*B*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^6 + 24*Sqrt[2]*(
177*A - 637*B)*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c
+ d*x)/2]^6 - 724*A*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 2884*B*Sqr
t[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) - 494*A*Sqrt[1 - Cos[c + d*x]]*Cos[
c + d*x]^(5/2) + 2198*B*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2) + 384*B*
Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(7/2) - 294*A*Sqrt[-((-1 + Cos[c + d*x
])*Cos[c + d*x])] + 1134*B*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[
c + d*x])/(384*a^4*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{7/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \textbf{3456}$$

$$\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-2a(A-7B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-2a(A-7B)\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(7a(A-B)-2a(A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \textbf{3456}$$

$$\frac{\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2(3A-7B)-2a^2(17A-77B)\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2(3A-7B)-2a^2(17A-77B)\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(15a^2(3A-7B)-2a^2(17A-77B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \textbf{3456}$$

$$\frac{\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left(a^3(79A-259B)-14a^3(7A-27B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{\sqrt{\cos(c+dx)}\left(a^3(79A-259B)-14a^3(7A-27B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^3(79A-259B)-14a^3(7A-27B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3462$$

$$\frac{\frac{3\left(\frac{\int -\frac{7a^4(7A-27B)-64a^4(2A-7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(-\frac{\int \frac{7a^4(7A-27B)-64a^4(2A-7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A-7B)\sin(c+\cdots}{4d(a\cos(c+\cdots}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{\int \frac{7a^4(7A-27B)-64a^4(2A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A-7B)\sin\cdots}{4d(a\cos\cdots}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3461

$$\frac{\frac{3\left(-\frac{a^4(177A-637B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-64a^3(2A-7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-259B)\sin(c+\cdots}{2d(a\cos(c+d\cdots}}{8a^2}\cdots}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{a^4(177A-637B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-64a^3(2A-7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-\cdots}{\cdots}}{8a^2}\cdots}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3253

$$\frac{\frac{\frac{3\left(-\frac{a^4(177A-637B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{128a^3(2A-7B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{c}}{d\sqrt{a\cos(c+dx)+a}}\right.}{4a^2}}{8a^2}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 223

$$\frac{\frac{\frac{3\left(-\frac{a^4(177A-637B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{128a^{7/2}(2A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(}{}}{8a^2}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3261

$$\frac{\frac{\frac{3\left(-\frac{-\frac{2a^5(177A-637B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{128a^{7/2}(2A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{14a^3(7A-27B)\sin}{d\sqrt{a\cos}}\right.}{4a^2}}{8a^2}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 218

$$\frac{\frac{\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\left(-\frac{\frac{\sqrt{2}a^{7/2}(177A-637B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{128a^{7/2}(2A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\right.}{4a^2}}{8a^2}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x
]
```

output
```
((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2))
 + ((3*a*(3*A - 7*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((a^2*(79*A - 259*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) + (3*(-(((-128*a^(7/2)*(2*A - 7*B)*ArcSin[(Sqr
t[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (Sqrt[2]*a^(7/2)*(177*A
- 637*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])])/d)/a) - (14*a^3*(7*A - 27*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/(4*a^2))/(8*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 17.03 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.44

method	result
default	$\frac{\left(\left(384\cos(dx+c)^3+1152\cos(dx+c)^2+1152\cos(dx+c)+384\right)\sqrt{2}\,A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(-1344\cos(dx+c)^3-4032\right.\right.}{}$
parts	$\frac{A\left(\left(384\cos(dx+c)^3+1152\cos(dx+c)^2+1152\cos(dx+c)+384\right)\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sin(dx+c)\left(-247\cos(dx+c\right.\right.}{192d\left(\cos(dx+c)^4+4\,\mathrm{co}\right.}$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*((384*cos(d*x+c)^3+1152*cos(d*x+c)^2+1152*cos(d*x+c)+384)*2^(1/2)*
A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(-1344*cos(d*x+c)^3
-4032*cos(d*x+c)^2-4032*cos(d*x+c)-1344)*2^(1/2)*B*arctan(tan(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*(-247*cos(d*x+c)^2-362*cos(d*x+c)
-147)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(192*cos(d*x+
c)^3+1099*cos(d*x+c)^2+1442*cos(d*x+c)+567)*2^(1/2)*B*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+(531*cos(d*x+c)^3+1593*cos(d*x+c)^2+1593*cos(d*x+c)+531)*A*a
rcsin(cot(d*x+c)-csc(d*x+c))+(-1911*cos(d*x+c)^3-5733*cos(d*x+c)^2-5733*co
s(d*x+c)-1911)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*cos(1/2*d*x+1/2*c)^2)^(
1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*cos(d*x+c)^2+4*cos(d*
x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 12.63 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.39

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((177\,A-637\,B)\cos\left(dx+c\right)^{4}+4\,(177\,A-637\,B)\cos\left(dx+c\right)^{3}+6\,(177\,A-637\,B)\cos\left(dx+c\right)^{2}\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(
d*x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x
+ c) + 177*A - 637*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*
sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c)
)) - 2*(192*B*cos(d*x + c)^3 - (247*A - 1099*B)*cos(d*x + c)^2 - 2*(181*A
- 721*B)*cos(d*x + c) - 147*A + 567*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c) - 384*((2*A - 7*B)*cos(d*x + c)^4 + 4*(2*A - 7*B)*co
s(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 + 4*(2*A - 7*B)*cos(d*x + c) +
 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x +
c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos(c+d\,x)^{7/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x
)
```

output
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^4}{\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}dx\right)b+\left(\int \frac{}{c}\right.\right.}{a^4}$$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**4)/
(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x)
+ 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3
)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x
) + 1),x)*a))/a**4
```

3.209 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 241

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} + \frac{(5A-177B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(5A-49B)\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d+1/128*(5*A
-177*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*
x+c))^(1/2))*2^(1/2)/a^(7/2)/d+1/6*(A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+
a*cos(d*x+c))^(7/2)+1/48*(5*A-17*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*c
os(d*x+c))^(5/2)+1/64*(5*A-49*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*co
s(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.57 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.32

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx =$$
$$-\frac{\sqrt{a(1+\cos(c+dx))}\left(1176B\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)+4248B\arcsin\left(\sqrt{\cos(c+dx}\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
7/2),x]
```

output
```
-1/192*(Sqrt[a*(1 + Cos[c + d*x])]*(1176*B*ArcSin[Sqrt[1 - Cos[c + d*x]]]*
Cos[(c + d*x)/2]^6 + 4248*B*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^6
+ 12*Sqrt[2]*(5*A - 177*B)*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]
^2]]*Cos[(c + d*x)/2]^6 - 50*A*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) +
 362*B*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) - 67*A*Sqrt[1 - Cos[c + d
*x]]*Cos[c + d*x]^(5/2) + 247*B*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)
- 15*A*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])] + 147*B*Sqrt[-((-1 + Cos[
c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(a^4*d*Sqrt[1 - Cos[c + d*x]]*(1 +
 Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$\downarrow$ 3456

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)+12aB\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)+12aB\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a(A-B)+12aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3456

$$\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left((5A-17B)a^2+32B\cos(c+dx)a^2\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{\sqrt{\cos(c+dx)}\left((5A-17B)a^2+32B\cos(c+dx)a^2\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left((5A-17B)a^2+32B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3456

$$\frac{\frac{3\left(\frac{\int \frac{(5A-49B)a^3+128B\cos(c+dx)a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\left(\frac{\int \frac{(5A-49B)a^3+128B\cos(c+dx)a^3}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(\frac{\int \frac{(5A-49B)a^3+128B\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{\frac{3\left(\frac{a^3(5A-177B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+128a^2B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}}{4d(a\cos(c+dx)+a)^{5/}}}{12a^2}$$
$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(\frac{a^3(5A-177B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+128a^2B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(}{4d(a\cos(}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3253

$$\frac{\frac{3\left(\frac{a^3(5A-177B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{256a^2B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 223

$$\frac{\frac{3\left(\frac{a^3(5A-177B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{256a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)}{4d(a\,c}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3261

$$\frac{\frac{3\left(\frac{\frac{256a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^4(5A-177B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c}}{2d(a\cos(c+dx)+a)^{3/2}}\right.}{8a^2}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{\frac{\sqrt{2}a^{5/2}(5A-177B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{256a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}\right)}{8a^2}+\frac{a(5A-1}{4}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x
]
```

output
```
((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2))
 + ((a*(5*A - 17*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d
*x])^(5/2)) + (3*(((256*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a
*Cos[c + d*x]]])/d + (Sqrt[2]*a^(5/2)*(5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(4*a^2)
 + (a^2*(5*A - 49*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c +
d*x])^(3/2))))/(8*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 8.67 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.45

method	result
default	$-\frac{\left(\left(-384\cos(dx+c)^3-1152\cos(dx+c)^2-1152\cos(dx+c)-384\right)\sqrt{2}\,B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sin(dx+c)\left(-67\cos(dx\right.\right.}{}$
parts	$\frac{A\left(\sin(dx+c)\left(67\cos(dx+c)^2+50\cos(dx+c)+15\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(-15\cos(dx+c)^3-45\cos(dx+c)^2-45\cos(dx+c)-15\right)\arcsin\right.}{384d\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx}{\cos(dx+}}}$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/192/d*((-384*cos(d*x+c)^3-1152*cos(d*x+c)^2-1152*cos(d*x+c)-384)*2^(1/2
)*B*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*(-67*c
os(d*x+c)^2-50*cos(d*x+c)-15)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
sin(d*x+c)*(247*cos(d*x+c)^2+362*cos(d*x+c)+147)*2^(1/2)*B*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+(15*cos(d*x+c)^3+45*cos(d*x+c)^2+45*cos(d*x+c)+15)*A*ar
csin(cot(d*x+c)-csc(d*x+c))+(-531*cos(d*x+c)^3-1593*cos(d*x+c)^2-1593*cos(
d*x+c)-531)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*cos(1/2*d*x+1/2*c)^2)^(1/2
)*cos(d*x+c)^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*cos(d*x+c)^2+4*cos(d*x+c
)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.52

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{3\sqrt{2}\left((5\,A-177\,B)\cos\left(dx+c\right)^4+4\,(5\,A-177\,B)\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x +
 c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5*
A - 177*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*((67
*A - 247*B)*cos(d*x + c)^2 + 2*(25*A - 181*B)*cos(d*x + c) + 15*A - 147*B)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) + 768*(B*cos(d*x
 + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*
sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x
 + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*
d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x
)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}dx\right)b+\left(\int \right.\right.}{a^4}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**3)/
(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x)
+ 1),x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2
)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x
) + 1),x)*a))/a**4
```

3.210 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 201

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(7A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{(17A+67B)\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

output

```
1/128*(7*A+5*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))*2^(1/2)/a^(7/2)/d+1/6*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(7/2)+1/48*(A-13*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/
(a+a*cos(d*x+c))^(5/2)+1/192*(17*A+67*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d
/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 613 vs. $2(201) = 402$.

Time = 6.61 (sec) , antiderivative size = 613, normalized size of antiderivative = 3.05

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{5B\arcsin\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{7/2}}$$
$$+\frac{11B\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}{8d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(a(1+\cos(c+dx)))^{7/2}}$$
$$-\frac{13B\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sin^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}{12d\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}(a(1+\cos(c+dx)))^{7/2}}$$
$$+\frac{B\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sin^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}{3d\cos^2\left(\frac{1}{2}(c+dx)\right)^{5/2}(a(1+\cos(c+dx)))^{7/2}}$$
$$+\frac{A\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(27-106\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)+121\sin^4\left(\frac{c}{2}+\frac{dx}{2}\right)-34\sin^6\left(\frac{c}{2}+\right.\right.}{24d(a(1+\cos(c+dx)))^{7/2}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
7/2),x]
```

output
```
(5*B*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Cos[c/2 + (d*x)/2
]^7)/(8*d*(a*(1 + Cos[c + d*x]))^(7/2)) + (11*B*Cos[c/2 + (d*x)/2]^7*Sin[c
/2 + (d*x)/2]*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(8*d*Sqrt
[Cos[(c + d*x)/2]^2]*(a*(1 + Cos[c + d*x]))^(7/2)) - (13*B*Cos[c/2 + (d*x)
/2]^7*Sin[c/2 + (d*x)/2]^3*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^
2])/(12*d*(Cos[(c + d*x)/2]^2)^(3/2)*(a*(1 + Cos[c + d*x]))^(7/2)) + (B*Co
s[c/2 + (d*x)/2]^7*Sin[c/2 + (d*x)/2]^5*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/
2 + (d*x)/2]^2])/(3*d*(Cos[(c + d*x)/2]^2)^(5/2)*(a*(1 + Cos[c + d*x]))^(7
/2)) + (A*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*(27 -
 106*Sin[c/2 + (d*x)/2]^2 + 121*Sin[c/2 + (d*x)/2]^4 - 34*Sin[c/2 + (d*x)/
2]^6 + (21*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2
))]]*Cos[(c + d*x)/2]^6)/Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x
)/2]^2))]))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2)*Sqrt[1 - 2*Sin[c/2 + (d*x)/
2]^2])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+2a(A+5B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+2a(A+5B)\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)+2a(A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3456$$

$$\frac{\frac{\int \frac{(A-13B)a^2+18(A+3B)\cos(c+dx)a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(A-13B)a^2+18(A+3B)\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{(A-13B)a^2+18(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\frac{\int \frac{3a^3(7A+5B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{4}a(7A+5B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3}{4}a(7A+5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\Big\downarrow$ 3261

$$\frac{\frac{\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{3a^2(7A+5B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\sqrt{a}(7A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x
]
```

output
```
((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2))
 + ((a*(A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x
])^(5/2)) + ((3*Sqrt[a]*(7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) + (a^2*(17*A
 + 67*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
)/(8*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 7.92 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(\sin(dx+c)\left(17\cos(dx+c)^2+70\cos(dx+c)+21\right)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)\left(67\cos(dx+c)^2+50\cos(dx+c)+15\right)\sqrt{2}\,B\sqrt{\frac{\cos(d}{\cos(dx}}}{}$
parts	$\frac{A\left(\sin(dx+c)\left(17\cos(dx+c)^2+70\cos(dx+c)+21\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(-21\cos(dx+c)^3-63\cos(dx+c)^2-63\cos(dx+c)-21\right)\arcsin}{384d\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx}{\cos(dx+}}}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
1/384/d*(sin(d*x+c)*(17*cos(d*x+c)^2+70*cos(d*x+c)+21)*2^(1/2)*A*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(67*cos(d*x+c)^2+50*cos(d*x+c)+15)*2^(
1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-21*cos(d*x+c)^3-63*cos(d*x+c)^2
-63*cos(d*x+c)-21)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(-15*cos(d*x+c)^3-45*co
s(d*x+c)^2-45*cos(d*x+c)-15)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*cos(d
*x+c)^2+4*cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.32

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{3\sqrt{2}\big((7A+5B)\cos(dx+c)^4+4(7A+5B)\cos(dx+c)^3+6\cdots}{\cdots}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)^
3 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B)
*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x
+ c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*((17*A + 67*B)
*cos(d*x + c)^2 + 10*(7*A + 5*B)*cos(d*x + c) + 21*A + 15*B)*sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4
*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x
)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}dx\right)a+\left(\int \right.\right.}{a^4}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1
),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(
cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) +
 1),x)*b))/a**4
```

3.211 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 201

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(13A+7B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{(5A-17B)\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

output
```
1/128*(13*A+7*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a
+a*cos(d*x+c))^(1/2))*2^(1/2)/a^(7/2)/d+1/6*(A-B)*cos(d*x+c)^(1/2)*sin(d*x
+c)/d/(a+a*cos(d*x+c))^(7/2)+1/16*(A+3*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d/
(a+a*cos(d*x+c))^(5/2)-1/192*(5*A-17*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/
(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(48(13A+7B)\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\right.\right.}{}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(
7/2),x]
```

output
```
(Sec[(c + d*x)/2]^4*(48*(13*A + 7*B)*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c +
d*x)/2]^2)]]*Cos[(c + d*x)/2]^6 + Cos[c + d*x]*(73*A + 59*B + 4*(A + 35*B)
*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*
Tan[(c + d*x)/2])/(1536*Sqrt[2]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 +
Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3456, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{a(A-B)+4a(A+2B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-B)+4a(A+2B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-B)+4a(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{(11A+B)a^2+6(A+3B)\cos(c+dx)a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(11A+B)a^2+6(A+3B)\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(11A+B)a^2+6(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int \frac{3a^3(13A+7B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3}{4}a(13A+7B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3}{4}a(13A+7B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\Big\downarrow$ 3261

$$\frac{\frac{-\frac{3a^2(13A+7B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{3\sqrt{a}(13A+7B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x
]
```

output
```
((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2))
 + ((3*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*
x])^(5/2)) + ((3*Sqrt[a]*(13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (a^2*(5*
A - 17*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)
))/(8*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 8.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\left(\sin(dx+c)\left(5\cos(dx+c)^2-2\cos(dx+c)-39\right)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)\left(-17\cos(dx+c)^2-70\cos(dx+c)-21\right)\sqrt{2}\,B\sqrt{\frac{\cos}{\cos(}}}{}$
parts	$-\frac{A\left(\sin(dx+c)\left(5\cos(dx+c)^2-2\cos(dx+c)-39\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(39\cos(dx+c)^3+117\cos(dx+c)^2+117\cos(dx+c)+39\right)\operatorname{arcsi}}{384d\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{\frac{\cos(d}{\cos(dx}}}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/384/d*(sin(d*x+c)*(5*cos(d*x+c)^2-2*cos(d*x+c)-39)*2^(1/2)*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(-17*cos(d*x+c)^2-70*cos(d*x+c)-21)*2^(
1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(39*cos(d*x+c)^3+117*cos(d*x+c)^2
+117*cos(d*x+c)+39)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(21*cos(d*x+c)^3+63*co
s(d*x+c)^2+63*cos(d*x+c)+21)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*cos(d
*x+c)^2+4*cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{3\sqrt{2}((13\,A+7\,B)\cos{(dx+c)}^4+4\,(13\,A+7\,B)\cos{(dx+c)}^3}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x + c
)^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A +
 7*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos
(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((5*A - 1
7*B)*cos(d*x + c)^2 - 2*(A + 35*B)*cos(d*x + c) - 39*A - 21*B)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x
)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}dx\right)b+\left(\int\right.\right.}{a^4}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(co
s(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1
),x)*b + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4
+ 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1),x)*a))/a**4
```

3.212 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 203

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(63A+13B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{(103A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

output
```
1/128*(63*A+13*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2))*2^(1/2)/a^(7/2)/d-1/6*(A-B)*cos(d*x+c)^(1/2)*sin(d*
x+c)/d/(a+a*cos(d*x+c))^(7/2)-1/16*(5*A-B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^(5/2)-1/192*(103*A+5*B)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/
d/(a+a*cos(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx =$$
$$-\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-48(63A+13B)\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)+\cos(c}{1536\sqrt{2}a^3d\sqrt{-1+}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7
/2)),x]
```

output
```
-1/1536*(Sec[(c + d*x)/2]^4*(-48*(63*A + 13*B)*ArcTanh[Sqrt[-(Sec[c + d*x]
*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^6 + Cos[c + d*x]*(493*A - 73*B + (
532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(c + d*x)])*Sqrt[2 - 2*Sec
[c + d*x]])*Tan[(c + d*x)/2])/(Sqrt[2]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[
a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{a(11A+B)-4a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{a(11A+B)-4a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{a(11A+B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3457

$$\frac{\frac{\int \frac{a^2(73A+11B)-6a^2(5A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{a^2(73A+11B)-6a^2(5A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{a^2(73A+11B)-6a^2(5A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3457

$$\frac{\frac{\frac{\int \frac{3a^3(63A+13B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3}{4}a(63A+13B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3}{4}a(63A+13B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3261

$$\frac{\frac{-\frac{3a^2(63A+13B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(}}{4d(a\cos(c+dx)+a)^{5/}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 218

$$\frac{\frac{\frac{3\sqrt{a}(63A+13B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x
]
```

output
```
-1/6*((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(7/
2)) + ((-3*a*(5*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) + ((3*Sqrt[a]*(63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(
Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (a^
2*(103*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])
^(3/2)))/(8*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 13.00 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.42

method	result
default	$-\frac{\left(\sin(dx+c)\left(103\cos(dx+c)^2+266\cos(dx+c)+195\right)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)\left(5\cos(dx+c)^2-2\cos(dx+c)-39\right)\sqrt{2}\,B\sqrt{\frac{c}{co}}\right.}{}$
parts	$-\frac{A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\sin(dx+c)\cos(dx+c)\sqrt{2}\left(103\cos(dx+c)^2+266\cos(dx+c)+195\right)+\left(189\cos(dx+c)^4+756\cos(dx+c)^3+1134\right.\right.}{192d(\cos(dx+c)+1)\sqrt{\cos(dx+c)}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\right.}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/384/d*(sin(d*x+c)*(103*cos(d*x+c)^2+266*cos(d*x+c)+195)*2^(1/2)*A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(5*cos(d*x+c)^2-2*cos(d*x+c)-39)*2
^(1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(189*cos(d*x+c)^3+567*cos(d*x+c
)^2+567*cos(d*x+c)+189)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(39*cos(d*x+c)^3+1
17*cos(d*x+c)^2+117*cos(d*x+c)+39)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*cos(d*
x+c)^(1/2)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*cos(d*x+c)^2+4*cos(d*x+c)+1)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.31

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \frac{3\sqrt{2}\left((63\,A+13\,B)\cos\left(dx+c\right)^4+4\,(63\,A+13\,B)\cos\left(dx+\right.\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x +
 c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 63
*A + 13*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((10
3*A + 5*B)*cos(d*x + c)^2 + 2*(133*A - B)*cos(d*x + c) + 195*A - 39*B)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)
^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{7/2}}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^{\frac{7}{2}}\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x +
c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5+4\cos(dx+c)^4+6\cos(dx+c)^3+4\cos(dx+c)^2+\cos(dx+c)}dx\right.\right.}{a^4}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**5
 + 4*cos(c + d*x)**4 + 6*cos(c + d*x)**3 + 4*cos(c + d*x)**2 + cos(c + d*x
)),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4
 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1),x)*b))/a**4
```

3.213 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 250

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = &-\frac{3(121A-21B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} \\ &-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} \\ &-\frac{(19A-7B)\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} \\ &-\frac{(199A-43B)\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\ &+\frac{(691A-103B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-3/128*(121*A-21*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)
/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/a^(7/2)/d-1/6*(A-B)*sin(d*x+c)/d/cos(d*x+
c)^(1/2)/(a+a*cos(d*x+c))^(7/2)-1/48*(19*A-7*B)*sin(d*x+c)/a/d/cos(d*x+c)^
(1/2)/(a+a*cos(d*x+c))^(5/2)-1/192*(199*A-43*B)*sin(d*x+c)/a^2/d/cos(d*x+c
)^(1/2)/(a+a*cos(d*x+c))^(3/2)+1/192*(691*A-103*B)*sin(d*x+c)/a^3/d/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.69 (sec) , antiderivative size = 798, normalized size of antiderivative = 3.19

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7
/2)),x]
```

output
```
-1/24*(B*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*(141 -
 518*Sin[c/2 + (d*x)/2]^2 + 575*Sin[c/2 + (d*x)/2]^4 - 206*Sin[c/2 + (d*x)
/2]^6 - (189*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]
^2))]]*Cos[(c + d*x)/2]^6)/Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d
*x)/2]^2))]))/(d*(a*(1 + Cos[c + d*x]))^(7/2)*Sqrt[1 - 2*Sin[c/2 + (d*x)/2
]^2]) + (2*A*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((
16*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 13/2}
, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2
)/(3465*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin
[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]*(105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]
*Cos[(c + d*x)/2]^6*(2187 - 12908*Sin[c/2 + (d*x)/2]^2 + 27986*Sin[c/2 + (
d*x)/2]^4 - 26380*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[c/2 + (d*x)/2]^8) + Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-229635 + 2120790*Si
n[c/2 + (d*x)/2]^2 - 8267707*Sin[c/2 + (d*x)/2]^4 + 17646926*Sin[c/2 + (d*
x)/2]^6 - 22251094*Sin[c/2 + (d*x)/2]^8 + 16548816*Sin[c/2 + (d*x)/2]^10 -
 6712984*Sin[c/2 + (d*x)/2]^12 + 1144608*Sin[c/2 + (d*x)/2]^14)))/1680))/(
d*(a*(1 + Cos[c + d*x]))^(7/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{a(13A-B)-6a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(13A-B)-6a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(13A-B)-6a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{3a^2(41A-5B)-4a^2(19A-7B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^2(41A-5B)-4a^2(19A-7B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3a^2(41A-5B)-4a^2(19A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\frac{\int \frac{a^3(691A-103B)-2a^3(199A-43B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{a^3(691A-103B)-2a^3(199A-43B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{a^3(691A-103B)-2a^3(199A-43B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3463$$

$$\frac{\frac{\frac{\frac{2\int -\frac{9a^4(121A-21B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-9a^3(121A-21B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-9a^3(121A-21B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(}{4d\sqrt{\cos(c+dx)}(a\cos(c}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{\frac{\frac{\frac{18a^4(121A-21B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{9\sqrt{2}a^{5/2}(121A-21B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B}{4d\sqrt{\cos(c+dx)}(a}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

input

```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x
]
```

output
```
-1/6*((A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/
2)) + (-1/4*(a*(19*A - 7*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos
[c + d*x])^(5/2)) + (-1/2*(a^2*(199*A - 43*B)*Sin[c + d*x])/(d*Sqrt[Cos[c
+ d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((-9*Sqrt[2]*a^(5/2)*(121*A - 21*B)*
ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])])/d + (2*a^3*(691*A - 103*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2))/(8*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 11.83 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.22

method	result
default	$\sqrt{a \cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\left(1089\cos(dx+c)^{4}+4356\cos(dx+c)^{3}+6534\cos(dx+c)^{2}+4356\cos(dx+c)+1089\right)A\arcsin(\cot(dx+c)-\csc(dx+c$
parts	$\frac{A\sqrt{a \cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\sin(dx+c)\left(691\cos(dx+c)^{3}+1874\cos(dx+c)^{2}+1599\cos(dx+c)+384\right)\sqrt{2}+\left(1089\cos(dx+c)^{4}+4356\cos(dx+c\right.\right.}{192d\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)\left(\cos(dx+c)^{3}+3\cos(dx+c)\right.}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*((1089*cos(d*x+c)^4+4356*cos(d*x+c)
^3+6534*cos(d*x+c)^2+4356*cos(d*x+c)+1089)*A*arcsin(cot(d*x+c)-csc(d*x+c))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-189*cos(d*x+c)^4-756*cos(d*x+c)^3-113
4*cos(d*x+c)^2-756*cos(d*x+c)-189)*B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(691*cos(d*x+c)^3+1874*cos(d*x+c)^2+
1599*cos(d*x+c)+384)*2^(1/2)*A+sin(d*x+c)*cos(d*x+c)*(-103*cos(d*x+c)^2-26
6*cos(d*x+c)-195)*2^(1/2)*B)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)^3
+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.19

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{7/2}}\,dx =$$

$$-\frac{9\sqrt{2}\left((121\,A - 21\,B)\cos\left(dx + c\right)^5 + 4\,(121\,A - 21\,B)\cos\left(dx + c\right)^4 + 6\,(121\,A - 21\,B)\cos\left(dx + c\right)^3 + 4\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
-1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^5 + 4*(121*A - 21*B)*cos(d*
x + c)^4 + 6*(121*A - 21*B)*cos(d*x + c)^3 + 4*(121*A - 21*B)*cos(d*x + c)
^2 + (121*A - 21*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*
cos(d*x + c))) - 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(937*A - 133*B)*cos
(d*x + c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*
x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x
 + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{3/2}\,(a+a\cos(c+dx))^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2}dx\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**6
 + 4*cos(c + d*x)**5 + 6*cos(c + d*x)**4 + 4*cos(c + d*x)**3 + cos(c + d*x
)**2),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)
**5 + 4*cos(c + d*x)**4 + 6*cos(c + d*x)**3 + 4*cos(c + d*x)**2 + cos(c +
d*x)),x)*b))/a**4
```

3.214 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 297

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx &= \frac{(1015A-363B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} \\ &- \frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} \\ &- \frac{(109A-41B)\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} \\ &+ \frac{(579A-199B)\sin(c+dx)}{192a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{(1887A-691B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
1/128*(1015*A-363*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2
)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/a^(7/2)/d-1/6*(A-B)*sin(d*x+c)/d/cos(d*x
+c)^(3/2)/(a+a*cos(d*x+c))^(7/2)-1/48*(23*A-11*B)*sin(d*x+c)/a/d/cos(d*x+c
)^(3/2)/(a+a*cos(d*x+c))^(5/2)-1/64*(109*A-41*B)*sin(d*x+c)/a^2/d/cos(d*x+
c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1/192*(579*A-199*B)*sin(d*x+c)/a^3/d/cos(d
*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-1/192*(1887*A-691*B)*sin(d*x+c)/a^3/d/c
os(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.88

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{3i(1015A-363B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}\text{arct}}{\sqrt{1+e^{2i(c+dx)}}}\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7
/2)),x]
```

output
```
(Cos[(c + d*x)/2]^7*(((3*I)*(1015*A - 363*B)*E^((I/2)*(c + d*x))*Sqrt[(1 +
 E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((216
41*A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos
[2*(c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*
A*Cos[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d
*x)/2])/(32*Cos[c + d*x]^(3/2))))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.09, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{3a(5A-B)-8a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3a(5A-B)-8a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{3a(5A-B)-8a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\int \frac{3\left(a^2(63A-19B)-2a^2(23A-11B)\cos(c+dx)\right)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \frac{a^2(63A-19B)-2a^2(23A-11B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3\int \frac{a^2(63A-19B)-2a^2(23A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{3\left(\frac{\int \frac{a^3(579A-199B)-4a^3(109A-41B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\left(\frac{\int \frac{a^3(579A-199B)-4a^3(109A-41B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3\left(\frac{\int \frac{a^3(579A-199B)-4a^3(109A-41B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 3463}$$

$$\frac{\frac{3\left(\frac{\frac{2\int -\frac{a^4(1887A-691B)-2a^4(579A-199B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11B}{4d\cos^{\frac{3}{2}}(c+dx)(a\,c}}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^4(1887A-691B)-2a^4(579A-199B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11B)\sin}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^4(1887A-691B)-2a^4(579A-199B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11B}{4d\cos^{\frac{3}{2}}(c+dx)(a}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3463

$$\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{3a^5(1015A-363B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{}{4d\,\mathrm{co}}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 27

$$\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^4(1015A-363B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/}}\right.}{8a^2}$$

$$12a^2$$

$$\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^4(1015A-363B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{a^2(109A-41B)\sin(}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+}\right.}{8a^2}$$

$$12a^2$$

$$\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 3261

$$\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^5(1015A-363B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\right.}{8a^2}$$

$$12a^2$$

$$\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$\downarrow$ 218

$$\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}a^{7/2}(1015A-363B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{a^2(109A-41B}{2d\cos^{\frac{3}{2}}(c+dx)(a\,c}\right.}{8a^2}}{12a^2}$$

$$\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x
]
```

output
```
-1/6*((A - B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/
2)) + (-1/4*(a*(23*A - 11*B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Co
s[c + d*x])^(5/2)) + (3*(-1/2*(a^2*(109*A - 41*B)*Sin[c + d*x])/(d*Cos[c +
 d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((2*a^3*(579*A - 199*B)*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-3*Sqrt[2]*a^(
7/2)*(1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^4*(1887*A - 691*B)*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2)))/(8*a^2
))/(12*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 12.43 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(3045\cos(dx+c)^{5}+12180\cos(dx+c)^{4}+18270\cos(dx+c)^{3}+1\right.\right.}{}$
parts	$-\frac{A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\sin(dx+c)\left(1887\cos(dx+c)^{4}+5082\cos(dx+c)^{3}+4251\cos(dx+c)^{2}+896\cos(dx+c)-128\right)\sqrt{2}+\arcsin(\cot(dx\right.}{192d\cos(dx+c)^{\frac{3}{2}}(\cos(dx+c)+1)\left(\cos(d\right.}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/192/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(3045*cos(d*x+c)^5+12180*cos(d*x+c)^4+182
70*cos(d*x+c)^3+12180*cos(d*x+c)^2+3045*cos(d*x+c))+B*arcsin(cot(d*x+c)-cs
c(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-1089*cos(d*x+c)^5-4356*cos(d
*x+c)^4-6534*cos(d*x+c)^3-4356*cos(d*x+c)^2-1089*cos(d*x+c))+sin(d*x+c)*(1
887*cos(d*x+c)^4+5082*cos(d*x+c)^3+4251*cos(d*x+c)^2+896*cos(d*x+c)-128)*2
^(1/2)*A+sin(d*x+c)*cos(d*x+c)*(-691*cos(d*x+c)^3-1874*cos(d*x+c)^2-1599*c
os(d*x+c)-384)*2^(1/2)*B)/cos(d*x+c)^(3/2)/(cos(d*x+c)+1)/(cos(d*x+c)^3+3*
cos(d*x+c)^2+3*cos(d*x+c)+1)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.07

$$\int \frac{A + B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \frac{3\sqrt{2}\left((1015\,A - 363\,B)\cos\left(dx+c\right)^6 + 4\,(1015\,A - 363\,B)\cos\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^6 + 4*(1015*A - 363*B)*cos
(d*x + c)^5 + 6*(1015*A - 363*B)*cos(d*x + c)^4 + 4*(1015*A - 363*B)*cos(d
*x + c)^3 + (1015*A - 363*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sq
rt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 + a*cos(d*x + c))) - 2*((1887*A - 691*B)*cos(d*x + c)^4 + 2*(2541*A
 - 937*B)*cos(d*x + c)^3 + 39*(109*A - 41*B)*cos(d*x + c)^2 + 128*(7*A - 3
*B)*cos(d*x + c) - 128*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x
 + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{7}+4\cos(dx+c)^{6}+6\cos(dx+c)^{5}+4\cos(dx+c)^{4}+\cos(dx+c)^{3}}dx\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**7
 + 4*cos(c + d*x)**6 + 6*cos(c + d*x)**5 + 4*cos(c + d*x)**4 + cos(c + d*x
)**3),x)*a + int((sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)
**6 + 4*cos(c + d*x)**5 + 6*cos(c + d*x)**4 + 4*cos(c + d*x)**3 + cos(c +
d*x)**2),x)*b))/a**4
```

3.215 $\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 105

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{1}{8}(4aA+3bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{(4aA+3bB)\cos(c+dx)\sin(c+dx)}{8d}+\frac{bB\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{(Ab+aB)\sin^3(c+dx)}{3d}$$

output
```
1/8*(4*A*a+3*B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/8*(4*A*a+3*B*b)*cos(d*x+c)*si
n(d*x+c)/d+1/4*b*B*cos(d*x+c)^3*sin(d*x+c)/d-1/3*(A*b+B*a)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{48aAc+36bBc+48aAdx+36bBdx+96(Ab+aB)\sin(c+dx)-32(Ab+aB)\sin^3(c+dx)+24(aA+}{96d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(48*a*A*c + 36*b*B*c + 48*a*A*d*x + 36*b*B*d*x + 96*(A*b + a*B)*Sin[c + d*
x] - 32*(A*b + a*B)*Sin[c + d*x]^3 + 24*(a*A + b*B)*Sin[2*(c + d*x)] + 3*b
*B*Sin[4*(c + d*x)])/(96*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 3447, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos^2(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left((aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{4}\int \cos^2(c+dx)(4aA+3bB+4(Ab+aB)\cos(c+dx))dx+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(4aA+3bB+4(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(4(aB+Ab)\int \cos^3(c+dx)dx+(4aA+3bB)\int \cos^2(c+dx)dx\right)+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left((4aA+3bB)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4(aB+Ab)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{4}\left((4aA+3bB)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4(aB+Ab)\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left((4aA+3bB)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4(aB+Ab)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left((4aA+3bB)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4(aB+Ab)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left((4aA+3bB)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4(aB+Ab)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{bB\sin(c+dx)\cos^3(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(b*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*a*A + 3*b*B)*(x/2 + (Cos[c +
 d*x]*Sin[c + d*x])/(2*d)) - (4*(A*b + a*B)*(-Sin[c + d*x] + Sin[c + d*x]^
3/3))/d)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{(24Aa+24Bb)\sin(2dx+2c)+(8Ab+8Ba)\sin(3dx+3c)+3Bb\sin(4dx+4c)+(72Ab+72Ba)\sin(dx+c)+48xd\left(Aa+\frac{3Bb}{4}\right)}{96d}$
parts	$\frac{(Ab+Ba)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}+\frac{Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{Bb\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}\right.}{d}$
derivativedivides	$\frac{Bb\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+\frac{Ba\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Aa(}{d}$
default	$\frac{Bb\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+\frac{Ba\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+Aa(}{d}$
risch	$\frac{axA}{2}+\frac{3bBx}{8}+\frac{3\sin(dx+c)Ab}{4d}+\frac{3aB\sin(dx+c)}{4d}+\frac{Bb\sin(4dx+4c)}{32d}+\frac{\sin(3dx+3c)Ab}{12d}+\frac{\sin(3dx+3c)Ba}{12d}+$
norman	$\frac{\left(\frac{Aa}{2}+\frac{3Bb}{8}\right)x+\left(2Aa+\frac{3Bb}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(2Aa+\frac{3Bb}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(3Aa+\frac{9Bb}{4}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{Aa}{2}+\frac{3Bb}{8}\right.}{}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/96*((24*A*a+24*B*b)*sin(2*d*x+2*c)+(8*A*b+8*B*a)*sin(3*d*x+3*c)+3*B*b*si
n(4*d*x+4*c)+(72*A*b+72*B*a)*sin(d*x+c)+48*x*d*(A*a+3/4*B*b))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{3(4Aa+3Bb)dx + \left(6Bb\cos(dx+c)^3 + 8(Ba+Ab)\cos(dx+c)^2 + 16Ba + 16Ab + 3(4Aa+3Bb)\right.}{24d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/24*(3*(4*A*a + 3*B*b)*d*x + (6*B*b*cos(d*x + c)^3 + 8*(B*a + A*b)*cos(d*
x + c)^2 + 16*B*a + 16*A*b + 3*(4*A*a + 3*B*b)*cos(d*x + c))*sin(d*x + c))
/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(97) = 194.

Time = 0.20 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.40

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Ab\sin^3(c+dx)}{3d} + \frac{Ab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{2Ba\sin^3(c+dx)}{3d} \\ x(A+B\cos(c))(a+b\cos(c))\cos^2(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
 d*x)*cos(c + d*x)/(2*d) + 2*A*b*sin(c + d*x)**3/(3*d) + A*b*sin(c + d*x)*
cos(c + d*x)**2/d + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(c +
 d*x)**2/d + 3*B*b*x*sin(c + d*x)**4/8 + 3*B*b*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*B*b*x*cos(c + d*x)**4/8 + 3*B*b*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*B*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos
(c))*(a + b*cos(c))*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa-32\,\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba-32\,\left(\sin(dx+c)^3-3\,\sin(\right.}{96\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 32*(sin(d*x + c)^3 - 3*sin
(d*x + c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b + 3*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{1}{8}\,(4\,Aa+3\,Bb)x+\frac{Bb\sin(4\,dx+4\,c)}{32\,d}+\frac{(Ba+Ab)\sin(3\,dx+3\,c)}{12\,d}$$
$$+\frac{(Aa+Bb)\sin(2\,dx+2\,c)}{4\,d}+\frac{3\,(Ba+Ab)\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/8*(4*A*a + 3*B*b)*x + 1/32*B*b*sin(4*d*x + 4*c)/d + 1/12*(B*a + A*b)*sin
(3*d*x + 3*c)/d + 1/4*(A*a + B*b)*sin(2*d*x + 2*c)/d + 3/4*(B*a + A*b)*sin
(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 41.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{A\,a\,x}{2}+\frac{3\,B\,b\,x}{8}+\frac{3\,A\,b\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,B\,a\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,b\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

output
```
(A*a*x)/2 + (3*B*b*x)/8 + (3*A*b*sin(c + d*x))/(4*d) + (3*B*a*sin(c + d*x)
)/(4*d) + (A*a*sin(2*c + 2*d*x))/(4*d) + (A*b*sin(3*c + 3*d*x))/(12*d) + (
B*a*sin(3*c + 3*d*x))/(12*d) + (B*b*sin(2*c + 2*d*x))/(4*d) + (B*b*sin(4*c
 + 4*d*x))/(32*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{-6\cos(dx+c)\sin(dx+c)^3 b^2+12\cos(dx+c)\sin(dx+c)\,a^2+15\cos(dx+c)\sin(dx+c)\,b^2-16\sin}{24d}$$

input
```
int(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**3*b**2 + 12*cos(c + d*x)*sin(c + d*x)*a**
2 + 15*cos(c + d*x)*sin(c + d*x)*b**2 - 16*sin(c + d*x)**3*a*b + 48*sin(c
+ d*x)*a*b + 12*a**2*d*x + 9*b**2*d*x)/(24*d)
```

3.216 $\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 84

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{1}{2}(Ab+aB)x+\frac{(3aA+2bB)\sin(c+dx)}{3d} + \frac{(Ab+aB)\cos(c+dx)\sin(c+dx)}{2d}+\frac{bB\cos^2(c+dx)\sin(c+dx)}{3d}$$

output
```
1/2*(A*b+B*a)*x+1/3*(3*A*a+2*B*b)*sin(d*x+c)/d+1/2*(A*b+B*a)*cos(d*x+c)*si
n(d*x+c)/d+1/3*b*B*cos(d*x+c)^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{6Abc+6aBc+6Abdx+6aBdx+3(4aA+3bB)\sin(c+dx)+3(Ab+aB)\sin(2(c+dx))+bB\sin(3(c}{12d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 3*(4*a*A + 3*b*B)*Sin[c + d*x
] + 3*(A*b + a*B)*Sin[2*(c + d*x)] + b*B*Sin[3*(c + d*x)])/(12*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3447, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left((aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{3}\int \cos(c+dx)(3aA+2bB+3(Ab+aB)\cos(c+dx))dx+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(3aA+2bB+3(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\frac{(3aA+2bB)\sin(c+dx)}{d}+\frac{3(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}x(aB+Ab)\right)+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(b*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*(A*b + a*B)*x)/2 + ((3*a*A +
 2*b*B)*Sin[c + d*x])/d + (3*(A*b + a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))
/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{3(Ab+Ba)\sin(2dx+2c)+Bb\sin(3dx+3c)+3(4Aa+3Bb)\sin(dx+c)+6(Ab+Ba)xd}{12d}$
parts	$\frac{(Ab+Ba)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{\sin(dx+c)Aa}{d}+\frac{Bb\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3d}$
derivativedivides	$\frac{\frac{Bb\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3}+Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)}{d}$
default	$\frac{\frac{Bb\left(\cos(dx+c)^{2}+2\right)\sin(dx+c)}{3}+Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)}{d}$
risch	$\frac{xAb}{2}+\frac{aBx}{2}+\frac{\sin(dx+c)Aa}{d}+\frac{3bB\sin(dx+c)}{4d}+\frac{Bb\sin(3dx+3c)}{12d}+\frac{\sin(2dx+2c)Ab}{4d}+\frac{\sin(2dx+2c)Ba}{4d}$
norman	$\frac{\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\left(\frac{3Ab}{2}+\frac{3Ba}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(\frac{3Ab}{2}+\frac{3Ba}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\frac{(2Aa-Ab-Ba}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$
orering	Expression too large to display

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*(A*b+B*a)*sin(2*d*x+2*c)+B*b*sin(3*d*x+3*c)+3*(4*A*a+3*B*b)*sin(d*
x+c)+6*(A*b+B*a)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int\cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{3\,(Ba+Ab)dx+\left(2\,Bb\cos\left(dx+c\right)^{2}+6\,Aa+4\,Bb+3\,(Ba+Ab)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
1/6*(3*(B*a + A*b)*d*x + (2*B*b*cos(d*x + c)^2 + 6*A*a + 4*B*b + 3*(B*a +
A*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(76) = 152$.

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.00

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{Aa\sin(c+dx)}{d} + \frac{Abx\sin^2(c+dx)}{2} + \frac{Abx\cos^2(c+dx)}{2} + \frac{Ab\sin(c+dx)\cos(c+dx)}{2d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{Ba\sin(\ldots}{} \\ x(A+B\cos(c))(a+b\cos(c))\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a*sin(c + d*x)/d + A*b*x*sin(c + d*x)**2/2 + A*b*x*cos(c + d*
x)**2/2 + A*b*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*x*sin(c + d*x)**2/2 +
B*a*x*cos(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*b*sin(
c + d*x)**3/(3*d) + B*b*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A +
 B*cos(c))*(a + b*cos(c))*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba + 3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab - 4\left(\sin(dx+c)^3 - 3\,\sin(dx+\ldots\right.}{12\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*b - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b + 12*A*a*sin(d*x + c
))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{1}{2}(Ba+Ab)x+\frac{Bb\sin(3\,dx+3\,c)}{12\,d}+\frac{(Ba+Ab)\sin(2\,dx+2\,c)}{4\,d}+\frac{(4\,Aa+3\,Bb)\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
1/2*(B*a + A*b)*x + 1/12*B*b*sin(3*d*x + 3*c)/d + 1/4*(B*a + A*b)*sin(2*d*
x + 2*c)/d + 1/4*(4*A*a + 3*B*b)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 41.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{A\,b\,x}{2}+\frac{B\,a\,x}{2}+\frac{A\,a\,\sin(c+d\,x)}{d}+\frac{3\,B\,b\,\sin(c+d\,x)}{4\,d}+\frac{A\,b\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,a\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,b\,\sin(3\,c+3\,d\,x)}{12\,d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

output
```
(A*b*x)/2 + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (3*B*b*sin(c + d*x))/(4*d)
+ (A*b*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*c + 2*d*x))/(4*d) + (B*b*sin(3
*c + 3*d*x))/(12*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{3\cos(dx+c)\sin(dx+c)ab-\sin(dx+c)^3b^2+3\sin(dx+c)a^2+3\sin(dx+c)b^2+3abdx}{3d}$$

input
```
int(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
(3*cos(c + d*x)*sin(c + d*x)*a*b - sin(c + d*x)**3*b**2 + 3*sin(c + d*x)*a
**2 + 3*sin(c + d*x)*b**2 + 3*a*b*d*x)/(3*d)
```

3.217 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{1}{2}(2aA+bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{bB\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*(2*A*a+B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/2*b*B*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{2bBc+4aAdx+2bBdx+4(Ab+aB)\sin(c+dx)+bB\sin(2(c+dx))}{4d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*c + 4*a*A*d*x + 2*b*B*d*x + 4*(A*b + a*B)*Sin[c + d*x] + b*B*Sin[2*
(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3213}$$

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(2aA+bB)+\frac{bB\sin(c+dx)\cos(c+dx)}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
((2*a*A + b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (b*B*Cos[c + d*x]*Sin
[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result
risch	$axA+\frac{bBx}{2}+\frac{\sin(dx+c)Ab}{d}+\frac{aB\sin(dx+c)}{d}+\frac{\sin(2dx+2c)Bb}{4d}$
parallelrisch	$\frac{4Aadx+2Bbdx+4A\sin(dx+c)b+4B\sin(dx+c)a+B\sin(2dx+2c)b}{4d}$
parts	$axA+\frac{(Ab+Ba)\sin(dx+c)}{d}+\frac{Bb\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
derivativedivides	$\frac{Bb\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+A\sin(dx+c)b+B\sin(dx+c)a+Aa(dx+c)}{d}$
default	$\frac{Bb\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+A\sin(dx+c)b+B\sin(dx+c)a+Aa(dx+c)}{d}$
norman	$\frac{\left(Aa+\frac{Bb}{2}\right)x+\left(Aa+\frac{Bb}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(2Aa+Bb)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{(2Ab+2Ba-Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{(2Ab+2Ba+Bb)\,\mathrm{ta}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
orering	$x(a+\cos(dx+c)\,b)\,(A+B\cos(dx+c))-\frac{5(-d\sin(dx+c)b(A+B\cos(dx+c))-(a+\cos(dx+c)b)Bc}{4d^2}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*x*A+1/2*b*B*x+sin(d*x+c)/d*A*b+a*B*sin(d*x+c)/d+1/4/d*sin(2*d*x+2*c)*B*b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{(2\,Aa+Bb)dx+(Bb\cos(dx+c)+2\,Ba+2\,Ab)\sin(dx+c)}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((2*A*a + B*b)*d*x + (B*b*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/
d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(44) = 88$.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.81

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} Aax + \frac{Ab\sin{(c+dx)}}{d} + \frac{Ba\sin{(c+dx)}}{d} + \frac{Bbx\sin^2{(c+dx)}}{2} + \frac{Bbx\cos^2{(c+dx)}}{2} + \frac{Bb\sin{(c+dx)}\cos{(c+dx)}}{2d} & \text{for } d \neq 0 \\ x(A+B\cos{(c)})\,(a+b\cos{(c)}) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a*x + A*b*sin(c + d*x)/d + B*a*sin(c + d*x)/d + B*b*x*sin(c +
 d*x)**2/2 + B*b*x*cos(c + d*x)**2/2 + B*b*sin(c + d*x)*cos(c + d*x)/(2*d)
, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{4\,(dx+c)Aa + (2\,dx+2\,c+\sin{(2\,dx+2\,c)})Bb + 4\,Ba\sin{(dx+c)} + 4\,Ab\sin{(dx+c)}}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*b + 4*B*a*sin(d*
x + c) + 4*A*b*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{1}{2}\,(2\,Aa+Bb)x+\frac{Bb\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(Ba+Ab)\sin\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*A*a + B*b)*x + 1/4*B*b*sin(2*d*x + 2*c)/d + (B*a + A*b)*sin(d*x + c
)/d
```

Mupad [B] (verification not implemented)

Time = 41.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=A\,a\,x+\frac{B\,b\,x}{2}+\frac{A\,b\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,\sin\left(c+d\,x\right)}{d}+\frac{B\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

output
```
A*a*x + (B*b*x)/2 + (A*b*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (B*b*sin
(2*c + 2*d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{\cos(dx+c)\sin(dx+c)\,b^2+4\sin(dx+c)\,ab+2a^2dx+b^2dx}{2d}$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*b**2 + 4*sin(c + d*x)*a*b + 2*a**2*d*x + b**2*d
*x)/(2*d)
```

3.218 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 35

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = (Ab+aB)x+\frac{aA\text{arctanh}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}$$

output
```
(A*b+B*a)*x+a*A*arctanh(sin(d*x+c))/d+b*B*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = Abx+aBx+\frac{aA\coth^{-1}(\sin(c+dx))}{d}+\frac{bB\cos(dx)\sin(c)}{d}+\frac{bB\cos(c)\sin(dx)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
A*b*x + a*B*x + (a*A*ArcCoth[Sin[c + d*x]])/d + (b*B*Cos[d*x]*Sin[c])/d +
(b*B*Cos[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sec(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3502}$$

$$\int (aA+(Ab+aB)\cos(c+dx))\sec(c+dx)dx+\frac{bB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA+(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{bB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3214}$$

$$aA\int \sec(c+dx)dx+x(aB+Ab)+\frac{bB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$aA\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+x(aB+Ab)+\frac{bB\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{aA\text{arctanh}(\sin(c+dx))}{d}+x(aB+Ab)+\frac{bB\sin(c+dx)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(A*b + a*B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba(dx+c)+Ab(dx+c)+B\sin(dx+c)b}{d}$
default	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba(dx+c)+Ab(dx+c)+B\sin(dx+c)b}{d}$
parts	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(Ab+Ba)(dx+c)}{d}+\frac{bB\sin(dx+c)}{d}$
parallelrisch	$\frac{-Aa\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+Aa\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+B\sin(dx+c)b+(Ab+Ba)xd}{d}$
risch	$xAb+aBx-\frac{iBb\,e^{i(dx+c)}}{2d}+\frac{iBb\,e^{-i(dx+c)}}{2d}+\frac{Aa\ln(e^{i(dx+c)}+i)}{d}-\frac{Aa\ln(e^{i(dx+c)}-i)}{d}$
norman	$\frac{(Ab+Ba)x+(Ab+Ba)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(2Ab+2Ba)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{Aa\ln(\mathrm{ta}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(A*a*ln(sec(d*x+c)+tan(d*x+c))+B*a*(d*x+c)+A*b*(d*x+c)+B*sin(d*x+c)*b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{2\,(Ba+Ab)dx+Aa\log\left(\sin\left(dx+c\right)+1\right)-Aa\log\left(-\sin\left(dx+c\right)+1\right)+2\,Bb\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="frica
s")
```

output
```
1/2*(2*(B*a + A*b)*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c)
 + 1) + 2*B*b*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})\sec{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{(dx+c)Ba+(dx+c)Ab+Aa\log{(\sec{(dx+c)}+\tan{(dx+c)})}+Bb\sin{(dx+c)}}{d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxim
a")
```

output
```
((d*x + c)*B*a + (d*x + c)*A*b + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*
b*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.26

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Ba+Ab)(dx+c)+\frac{2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac"
)
```

output
```
(A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) + (B*a + A*b)*(d*x + c) + 2*B*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x +
 1/2*c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 41.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.86

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{B\,b\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
(B*b*sin(c + d*x))/d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/d + (2*A*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^{2}+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a^{2}+\sin\left(dx+c\right)b^{2}+2abdx}{d}$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*a**2 + log(tan((c + d*x)/2) + 1)*a**2 + sin(
c + d*x)*b**2 + 2*a*b*d*x)/d
```

3.219 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 35

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = bBx+\frac{(Ab+aB)\text{arctanh}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}$$

output
```
b*B*x+(A*b+B*a)*arctanh(sin(d*x+c))/d+a*A*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = bBx+\frac{Ab\coth^{-1}(\sin(c+dx))}{d}+\frac{aB\coth^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b*B*x + (A*b*ArcCoth[Sin[c + d*x]])/d + (a*B*ArcCoth[Sin[c + d*x]])/d + (a
*A*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3042, 3447, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sec^2(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3500}$$

$$\int (Ab+B\cos(c+dx)b+aB)\sec(c+dx)dx+\frac{aA\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+B\sin\left(c+dx+\frac{\pi}{2}\right)b+aB}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{aA\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3214}$$

$$(aB+Ab)\int \sec(c+dx)dx+\frac{aA\tan(c+dx)}{d}+bBx$$

$$\Big\downarrow \text{3042}$$

$$(aB+Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{aA\tan(c+dx)}{d}+bBx$$
$$\Big\downarrow \; 4257$$
$$\frac{(aB+Ab)\text{arctanh}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+bBx$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b*B*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

method	result
parts	$\frac{aA\tan(dx+c)}{d}+\frac{(Ab+Ba)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bb(dx+c)}{d}$
derivativedivides	$\frac{A\tan(dx+c)a+Ba\ln(\sec(dx+c)+\tan(dx+c))+Ab\ln(\sec(dx+c)+\tan(dx+c))+Bb(dx+c)}{d}$
default	$\frac{A\tan(dx+c)a+Ba\ln(\sec(dx+c)+\tan(dx+c))+Ab\ln(\sec(dx+c)+\tan(dx+c))+Bb(dx+c)}{d}$
parallelrisch	$\frac{-(Ab+Ba)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(Ab+Ba)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+Bbdx\cos(dx+c)+Aa\sin(dx+}{\cos(dx+c)d}$
risch	$bBx+\frac{2iAa}{d\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)Ab}{d}+\frac{\ln\left(e^{i(dx+c)}+i\right)Ba}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)Ab}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)Ba}{d}$
norman	$\frac{bBx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+bBx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-bBx-\frac{2Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}-\frac{2Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}-bBx\tan\left(\frac{dx}{2}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOSE
)
```

output
```
a*A*tan(d*x+c)/d+(A*b+B*a)/d*ln(sec(d*x+c)+tan(d*x+c))+B*b/d*(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(35) = 70.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{2\,Bbdx\cos\left(dx+c\right)+\left(Ba+Ab\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(Ba+Ab\right)\cos\left(dx+c\right)\log\left(-\sin\left(\right.\right.}{2\,d\cos\left(dx+c\right)}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fri
cas")
```

output
```
1/2*(2*B*b*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x + c) +
1) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))
/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \int (A+B\cos(c+dx))(a+b\cos(c+dx))\sec^2(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{2(dx+c)Bb + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Ab(\log(\sin(dx+c)+1) - \log(\sin}{2d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="max
ima")
```

output
```
1/2*(2*(d*x + c)*B*b + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + A*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c
))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.40

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \frac{(dx + c)Bb + (Ba + Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Ba + Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\,Aa\tan\left(\frac{1}{2}\,dx\right.}{\tan\left(\frac{1}{2}\,dx\right.}}{d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gia
c")
```

output
```
((d*x + c)*B*b + (B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + A
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1
/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 41.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.26

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^2(c + dx)\,dx$$
$$= \frac{2\,B\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{A\,a\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$
$$- \frac{A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2\mathrm{i}}{d} - \frac{B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2\mathrm{i}}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
(2*B*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (B*a*atan((sin(c/2
 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/d - (A*b*atan((sin(c/2 + (d*x)/2)*
1i)/cos(c/2 + (d*x)/2))*2i)/d + (A*a*sin(c + d*x))/(d*cos(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.26

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{-2\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)ab+2\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)ab+\cos(dx+c)\,b^2dx+}{\cos(dx+c)\,d}$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
( - 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b + 2*cos(c + d*x)*log(tan(
(c + d*x)/2) + 1)*a*b + cos(c + d*x)*b**2*d*x + sin(c + d*x)*a**2)/(cos(c
+ d*x)*d)
```

3.220 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 61

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{(aA+2bB)\text{arctanh}(\sin(c+dx))}{2d} + \frac{(Ab+aB)\tan(c+dx)}{d} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*(A*a+2*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/2*a*A*sec(d
*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{bB\coth^{-1}(\sin(c+dx))}{d} + \frac{aA\text{arctanh}(\sin(c+dx))}{2d} + \frac{Ab\tan(c+dx)}{d} + \frac{aB\tan(c+dx)}{d} + \frac{aA\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(b*B*ArcCoth[Sin[c + d*x]])/d + (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (A*b*T
an[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2
*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \ 3447$$

$$\int \sec^3(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \ 3500$$

$$\frac{1}{2}\int (2(Ab+aB)+(aA+2bB)\cos(c+dx))\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{2(Ab+aB)+(aA+2bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \; 3227$$

$$\frac{1}{2}\left(2(aB+Ab)\int \sec^2(c+dx)dx+(aA+2bB)\int \sec(c+dx)dx\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left((aA+2bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2(aB+Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \; 4254$$

$$\frac{1}{2}\left((aA+2bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2(aB+Ab)\int 1d(-\tan(c+dx))}{d}\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{2}\left((aA+2bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2(aB+Ab)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{2}\left(\frac{(aA+2bB)\text{arctanh}(\sin(c+dx))}{d}+\frac{2(aB+Ab)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((a*A + 2*b*B)*ArcTanh[Sin[c + d*
x]])/d + (2*(A*b + a*B)*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba\tan(dx+c)+Ab\tan(dx+c)+Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ba\tan(dx+c)+Ab\tan(dx+c)+Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parts	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{(Ab+Ba)\tan(dx+c)}{d}+\frac{Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{-(\cos(2dx+2c)+1)(Aa+2Bb)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(\cos(2dx+2c)+1)(Aa+2Bb)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(2Ab+2Ba)\,\mathrm{s}}{2d(\cos(2dx+2c)+1)}$
risch	$-\frac{i\left(Aa\,\mathrm{e}^{3i(dx+c)}-2Ab\,\mathrm{e}^{2i(dx+c)}-2Ba\,\mathrm{e}^{2i(dx+c)}-Aa\,\mathrm{e}^{i(dx+c)}-2Ab-2Ba\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{Aa\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$
norman	$\frac{\frac{(Aa-2Ab-2Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{d}+\frac{(Aa+2Ab+2Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{(3Aa-2Ab-2Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}+\frac{(3Aa+2Ab+2Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{2}}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(A*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*a*tan
(d*x+c)+A*b*tan(d*x+c)+B*b*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{(Aa+2\,Bb)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(Aa+2\,Bb)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fri
cas")
```

output
```
1/4*((A*a + 2*B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a + 2*B*b)*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a + 2*(B*a + A*b)*cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})\sec^3{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx =$$
$$-\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2\,Bb(\log(\sin(dx+c)+1)-\log(}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="max
ima")
```

output
```
-1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 2*B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) - 4*B*a*tan(d*x + c) - 4*A*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(57) = 114$.

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.48

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{(Aa+2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\right.}{}}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gia
c")
```

output
```
1/2*((A*a + 2*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*b)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan
(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2
*c) + 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 41.66 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(A\,a+2\,A\,b+2\,B\,a)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,(2\,A\,b-A\,a+2\,B\,a)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(A\,a+2\,B\,b)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(A*a + 2*A*b + 2*B*a) - tan(c/2 + (d*x)/2)^3*(2*A*b -
A*a + 2*B*a))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a
tanh(tan(c/2 + (d*x)/2))*(A*a + 2*B*b))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.57

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a^2-2\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2}{}$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2 - 2*cos(c
+ d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**2 + cos(c + d*x)*log(t
an((c + d*x)/2) - 1)*a**2 + 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b**2
+ cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2 + 2*cos(c +
d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 - cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*a**2 - 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b**2 -
cos(c + d*x)*sin(c + d*x)*a**2 + 4*sin(c + d*x)**3*a*b - 4*sin(c + d*x)*a*
b)/(2*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.221 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 93

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{(Ab+aB)\text{arctanh}(\sin(c+dx))}{2d} + \frac{(2aA+3bB)\tan(c+dx)}{3d} + \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
1/2*(A*b+B*a)*arctanh(sin(d*x+c))/d+1/3*(2*A*a+3*B*b)*tan(d*x+c)/d+1/2*(A*
b+B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{3(Ab+aB)\text{arctanh}(\sin(c+dx)) + \tan(c+dx)\left(6aA+6bB+3(Ab+aB)\sec(c+dx)+2aA\tan^2(c+\right.}{6d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(3*(A*b + a*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*A + 6*b*B + 3*(A*
b + a*B)*Sec[c + d*x] + 2*a*A*Tan[c + d*x]^2))/(6*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sec^4(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\int (3(Ab+aB)+(2aA+3bB)\cos(c+dx))\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3(Ab+aB)+(2aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(3(aB+Ab)\int \sec^3(c+dx)dx+(2aA+3bB)\int \sec^2(c+dx)dx\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left((2aA+3bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3(aB+Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 4254

$$\frac{1}{3}\left(3(aB+Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{(2aA+3bB)\int 1d(-\tan(c+dx))}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 24

$$\frac{1}{3}\left(3(aB+Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{(2aA+3bB)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 4255

$$\frac{1}{3}\left(3(aB+Ab)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2aA+3bB)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(3(aB+Ab)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2aA+3bB)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 4257

$$\frac{1}{3}\left(3(aB+Ab)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2aA+3bB)\tan(c+dx)}{d}\right)+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((2*a*A + 3*b*B)*Tan[c + d*x])/
d + 3*(A*b + a*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x
])/(2*d)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 6.84 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result
parts	$-\frac{Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}+\frac{(Ab+Ba)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{Bb\tan(dx+c)}{d}$
derivativedivides	$\frac{-Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln}{}\right.}{d}$
default	$\frac{-Aa\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln}{}\right.}{d}$
parallelrisch	$\frac{-9(Ab+Ba)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9(Ab+Ba)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{6d(3\cos(dx+c)+\cos(3dx+3c))}$
risch	$-\frac{i\left(3Ab\,e^{5i(dx+c)}+3Ba\,e^{5i(dx+c)}-6Bb\,e^{4i(dx+c)}-12Aa\,e^{2i(dx+c)}-12Bb\,e^{2i(dx+c)}-3Ab\,e^{i(dx+c)}-3Ba\,e^{i(dx+c)}-4A\right.}{3d\left(e^{2i(dx+c)}+1\right)^{3}}$
norman	$\frac{-\frac{4(Aa-3Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3d}-\frac{2(4Aa-3Ab-3Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3d}-\frac{2(4Aa+3Ab+3Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}-\frac{(2Aa-Ab-Ba+2Bb)\tan\left(\frac{d}{2}\right.}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{3}}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOSE
)
```

output
```
-A*a/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*b+B*a)/d*(1/2*sec(d*x+c)*tan(
d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*b/d*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.24

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{3\,(Ba+Ab)\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,(Ba+Ab)\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1)}+2}{12\,d\cos{(dx+c)}^3}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fri
cas")
```

output
```
1/12*(3*(B*a + A*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a + A*b)*c
os(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A*a + 3*B*b)*cos(d*x + c)^2
 + 2*A*a + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})\sec^4{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.37

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^4(c + dx)\, dx$$
$$= \frac{4\left(\tan\left(dx + c\right)^3 + 3\tan\left(dx + c\right)\right)Aa - 3\,Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx + c\right) + 1\right) + \log\left(\sin\left(dx + c\right) - 1\right)\right)}{12\,d}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="max
ima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*B*a*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) + 12*B*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(85) = 170.

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.26

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^4(c + dx)\, dx$$
$$= \frac{3\left(Ba + Ab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(Ba + Ab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gia
c")
```

output
```
1/6*(3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*b)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*t
an(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x +
 1/2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 +
 6*A*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d
*x + 1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/
d
```

Mupad [B] (verification not implemented)

Time = 42.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.56

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(A\,b+B\,a)}{d} - \frac{(2\,A\,a-A\,b-B\,a+2\,B\,b)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{4\,A\,a}{3}-4\,B\,b\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2\,A\,a+A\,b+B\,a+2\,B\,b)\ldots}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^4,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(A*b + B*a))/d - (tan(c/2 + (d*x)/2)*(2*A*a + A
*b + B*a + 2*B*b) - tan(c/2 + (d*x)/2)^3*((4*A*a)/3 + 4*B*b) + tan(c/2 + (
d*x)/2)^5*(2*A*a - A*b - B*a + 2*B*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(
c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.09

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 ab+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)ab+3\cos(\ldots}{\ldots}$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b + 3*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*a*b + 3*cos(c + d*x)*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**2*a*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b
- 3*cos(c + d*x)*sin(c + d*x)*a*b + 2*sin(c + d*x)**3*a**2 + 3*sin(c + d*x
)**3*b**2 - 3*sin(c + d*x)*a**2 - 3*sin(c + d*x)*b**2)/(3*cos(c + d*x)*d*(
sin(c + d*x)**2 - 1))
```

3.222 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\begin{aligned}\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \frac{(3aA+4bB)\text{arctanh}(\sin(c+dx))}{8d}+\frac{(Ab+aB)\tan(c+dx)}{d}\\ &\quad+\frac{(3aA+4bB)\sec(c+dx)\tan(c+dx)}{8d}\\ &\quad+\frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{(Ab+aB)\tan^3(c+dx)}{3d}\end{aligned}$$

output
```
1/8*(3*A*a+4*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/8*(3*A*a+
4*B*b)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3*(A*b+
B*a)*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{3(3aA+4bB)\text{arctanh}(\sin(c+dx))+\sec(c+dx)\left(9aA+12bB+8(Ab+aB)(2+\cos(2(c+dx)))\sec(c\ldots\right.}{24d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(3*(3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*a*A + 12*b*B +
8*(A*b + a*B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*a*A*Sec[c + d*x]^2)*
Tan[c + d*x])/(24*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.414$, Rules used = {3042, 3447, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$
$$\Big\downarrow \; 3447$$
$$\int \sec^5(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{4}\int (4(Ab+aB)+(3aA+4bB)\cos(c+dx))\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{4(Ab+aB)+(3aA+4bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(4(aB+Ab)\int \sec^4(c+dx)dx+(3aA+4bB)\int \sec^3(c+dx)dx\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left((3aA+4bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4(aB+Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}\left((3aA+4bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4(aB+Ab)\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left((3aA+4bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4(aB+Ab)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left((3aA+4bB)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4(aB+Ab)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left((3aA+4bB)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4(aB+Ab)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 4257

$$\frac{1}{4}\left((3aA+4bB)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4(aB+Ab)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+\right.}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*a*A + 4*b*B)*(ArcTanh[Sin[c
+ d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*(A*b + a*B)*(-Tan[
c + d*x] - Tan[c + d*x]^3/3))/d)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

method	result
parts	$\frac{Aa\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\frac{(Ab+Ba)\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx\text{-}}{d}$
derivativedivides	$\frac{Aa\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-Ba\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-Ab(}{d}$
default	$\frac{Aa\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-Ba\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-Ab(}{d}$
parallelrisch	$\frac{-18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(Aa+\frac{4Bb}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(Aa+\frac{4Bb}{3}\right.}{12d(\cos(4dx\text{-}}$
risch	$-\frac{i(9Aa\,e^{7i(dx+c)}+12Bb\,e^{7i(dx+c)}+33Aa\,e^{5i(dx+c)}+12Bb\,e^{5i(dx+c)}-48Ab\,e^{4i(dx+c)}-48Ba\,e^{4i(dx+c)}-33Aa\,e^{3i(dx+}}{12d\left(e^{2i(dx+c)}+1\right)}$
norman	$\frac{\frac{(5Aa-8Ab-8Ba+4Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{4d}+\frac{(5Aa+8Ab+8Ba+4Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(21Aa-8Ab-8Ba-12Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{6d}+\frac{(21Aa\text{-}}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\right.}$

input

```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOSE
)
```

output

```
A*a/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))-(A*b+B*a)/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*b/d*(1/2*sec(d
*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.19

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{3\,(3\,Aa+4\,Bb)\cos{(dx+c)}^4\log{(\sin{(dx+c)}+1)}-3\,(3\,Aa+4\,Bb)\cos{(dx+c)}^4\log{(-\sin{(dx+c)}+}}{48\,d}$$

input

```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fri
cas")
```

output
```
1/48*(3*(3*A*a + 4*B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a +
4*B*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B*a + A*b)*cos(d*x +
 c)^3 + 3*(3*A*a + 4*B*b)*cos(d*x + c)^2 + 6*A*a + 8*(B*a + A*b)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))\sec^5(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.43

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ab-3\,Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin\ldots}{\sin(dx+c)^4-2\sin(\ldots}\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="max
ima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*ta
n(d*x + c))*A*b - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 12*B*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(106) = 212$.

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.67

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{3\,(3\,Aa+4\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa+4\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gia
c")
```

output
```
1/24*(3*(3*A*a + 4*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*
B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7
 - 24*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*
tan(1/2*d*x + 1/2*c)^7 + 9*A*a*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*tan(1/2*d*x
 + 1/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*b*tan(1/2*d*x + 1/2*c)^
5 + 9*A*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*
tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 15*A*a*tan(1/2*d*
x + 1/2*c) + 24*B*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c) + 1
2*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 43.99 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.70

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{\left(\frac{5\,A\,a}{4}-2\,A\,b-2\,B\,a+B\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,A\,a}{4}+\frac{10\,A\,b}{3}+\frac{10\,B\,a}{3}-B\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,A\,a}{4}-\frac{10\,A\,b}{3}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$
$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,A\,a}{4}+B\,b\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*((5*A*a)/4 + 2*A*b + 2*B*a + B*b) + tan(c/2 + (d*x)/2)
^7*((5*A*a)/4 - 2*A*b - 2*B*a + B*b) - tan(c/2 + (d*x)/2)^3*((10*A*b)/3 -
(3*A*a)/4 + (10*B*a)/3 + B*b) + tan(c/2 + (d*x)/2)^5*((3*A*a)/4 + (10*A*b)
/3 + (10*B*a)/3 - B*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^
2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan(c/2 +
 (d*x)/2))*((3*A*a)/4 + B*b))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.23

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{-9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4a^2-12\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)}{}$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
( - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2 - 12*cos
(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b**2 + 18*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2 + 24*cos(c + d*x)*log(tan(
(c + d*x)/2) - 1)*sin(c + d*x)**2*b**2 - 9*cos(c + d*x)*log(tan((c + d*x)/
2) - 1)*a**2 - 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b**2 + 9*cos(c +
d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**2 + 12*cos(c + d*x)*log(
tan((c + d*x)/2) + 1)*sin(c + d*x)**4*b**2 - 18*cos(c + d*x)*log(tan((c +
d*x)/2) + 1)*sin(c + d*x)**2*a**2 - 24*cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*sin(c + d*x)**2*b**2 + 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**2 +
 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b**2 - 9*cos(c + d*x)*sin(c + d
*x)**3*a**2 - 12*cos(c + d*x)*sin(c + d*x)**3*b**2 + 15*cos(c + d*x)*sin(c
 + d*x)*a**2 + 12*cos(c + d*x)*sin(c + d*x)*b**2 + 32*sin(c + d*x)**5*a*b
- 80*sin(c + d*x)**3*a*b + 48*sin(c + d*x)*a*b)/(24*cos(c + d*x)*d*(sin(c
+ d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.223 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 189

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx &= \frac{1}{8}\left(4a^2A+3Ab^2+6abB\right)x+\frac{(4b^2B+5a(2Ab+aB))\sin(c+dx)}{5d}\\&\quad+\frac{(4a^2A+3Ab^2+6abB)\cos(c+dx)\sin(c+dx)}{8d}\\&\quad+\frac{b(5Ab+6aB)\cos^3(c+dx)\sin(c+dx)}{20d}\\&\quad+\frac{bB\cos^3(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{5d}\\&\quad-\frac{(4b^2B+5a(2Ab+aB))\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*x+1/5*(4*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)/d
+1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*cos(d*x+c)*sin(d*x+c)/d+1/20*b*(5*A*b+6*B*a
)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*b*B*cos(d*x+c)^3*(a+b*cos(d*x+c))*sin(d*x+
c)/d-1/15*(4*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\frac{60(4a^2A+3Ab^2+6abB)(c+dx)+60(12aAb+6a^2B+5b^2B)\sin(c+dx)+120(a^2A+Ab^2+2abB)\,\mathrm{s}}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(60*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*(c + d*x) + 60*(12*a*A*b + 6*a^2*B + 5*b
^2*B)*Sin[c + d*x] + 120*(a^2*A + A*b^2 + 2*a*b*B)*Sin[2*(c + d*x)] + 10*(
8*a*A*b + 4*a^2*B + 5*b^2*B)*Sin[3*(c + d*x)] + 15*b*(A*b + 2*a*B)*Sin[4*(
c + d*x)] + 6*b^2*B*Sin[5*(c + d*x)])/(480*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3469, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 3469$$

$$\frac{1}{5}\int \cos^2(c+dx)\left(b(5Ab+6aB)\cos^2(c+dx)+\left(4Bb^2+5a(2Ab+aB)\right)\cos(c+dx)+a(5aA+3bB)\right)dx+\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b(5Ab+6aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(4Bb^2+5a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5aA\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \ 3502$$

$$\frac{1}{5}\left(\frac{1}{4}\int \cos^2(c+dx)\left(5\left(4Aa^2+6bBa+3Ab^2\right)+4\left(4Bb^2+5a(2Ab+aB)\right)\cos(c+dx)\right)dx+\frac{b(6aB+5Ab)\sin(}{}\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(5\left(4Aa^2+6bBa+3Ab^2\right)+4\left(4Bb^2+5a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b(6aB}{}\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \ 3227$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5\left(4a^2A+6abB+3Ab^2\right)\int \cos^2(c+dx)dx+4\left(5a(aB+2Ab)+4b^2B\right)\int \cos^3(c+dx)dx\right)+\frac{b(6aB+5A}{}\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5\left(4a^2A+6abB+3Ab^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4\left(5a(aB+2Ab)+4b^2B\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \ 3113$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5\left(4a^2A+6abB+3Ab^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4\left(5a(aB+2Ab)+4b^2B\right)\int\left(1-\sin^2(c+dx)\right)d(-\mathrm{si}}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5\left(4a^2A+6abB+3Ab^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4\left(5a(aB+2Ab)+4b^2B\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+d\right.}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$\Big\downarrow$ 3115

$$\frac{1}{5}\left(\frac{1}{4}\left(5\left(4a^2A+6abB+3Ab^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4\left(5a(aB+2Ab)+4b^2B\right)\left(\frac{1}{3}\sin^3(c+dx\right.}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

$\Big\downarrow$ 24

$$\frac{1}{5}\left(\frac{1}{4}\left(5\left(4a^2A+6abB+3Ab^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4\left(5a(aB+2Ab)+4b^2B\right)\left(\frac{1}{3}\sin^3(c+dx)-\text{si}\right.}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))}{5d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(b*B*Cos[c + d*x]^3*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) + ((b*(5*A*b
+ 6*a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (5*(4*a^2*A + 3*A*b^2 + 6*a*
b*B)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*(4*b^2*B + 5*a*(2*A*b
+ a*B))*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d)/4)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 31.69 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{120\left(a^{2} A+A b^{2}+2 B a b\right) \sin\left(2 d x+2 c\right)+10\left(8 A a b+4 a^{2} B+5 B b^{2}\right) \sin\left(3 d x+3 c\right)+15\left(A b^{2}+2 B a b\right) \sin\left(4 d x+4 c\right)+6 B b^{2} \sin(}{480 d}$
parts	$\frac{\left(A b^{2}+2 B a b\right)\left(\frac{\left(\cos\left(d x+c\right)^{3}+\frac{3 \cos\left(d x+c\right)}{2}\right) \sin\left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}+\frac{\left(2 A a b+a^{2} B\right)\left(\cos\left(d x+c\right)^{2}+2\right) \sin\left(d x+c\right)}{3 d}+\frac{B b}{}$
derivativedivides	$\frac{a^{2} A\left(\frac{\cos\left(d x+c\right) \sin\left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{a^{2} B\left(\cos\left(d x+c\right)^{2}+2\right) \sin\left(d x+c\right)}{3}+\frac{2 A a b\left(\cos\left(d x+c\right)^{2}+2\right) \sin\left(d x+c\right)}{3}+2 B a b\left(\frac{\left(\cos\left(d x+c\right)^{3}\right.}{}\right.}{}$
default	$\frac{a^{2} A\left(\frac{\cos\left(d x+c\right) \sin\left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+\frac{a^{2} B\left(\cos\left(d x+c\right)^{2}+2\right) \sin\left(d x+c\right)}{3}+\frac{2 A a b\left(\cos\left(d x+c\right)^{2}+2\right) \sin\left(d x+c\right)}{3}+2 B a b\left(\frac{\left(\cos\left(d x+c\right)^{3}\right.}{}\right.}{}$
risch	$\frac{x a^{2} A}{2}+\frac{3 x A b^{2}}{8}+\frac{3 x B a b}{4}+\frac{3 \sin\left(d x+c\right) A a b}{2 d}+\frac{3 \sin\left(d x+c\right) a^{2} B}{4 d}+\frac{5 b^{2} B \sin\left(d x+c\right)}{8 d}+\frac{B b^{2} \sin\left(5 d x+5 c\right)}{80 d}+$
norman	$\frac{\left(\frac{1}{2} a^{2} A+\frac{3}{8} A b^{2}+\frac{3}{4} B a b\right) x+\left(5 a^{2} A+\frac{15}{4} A b^{2}+\frac{15}{2} B a b\right) x \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+\left(5 a^{2} A+\frac{15}{4} A b^{2}+\frac{15}{2} B a b\right) x \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}+\left(\frac{1}{2} a^{2}\right.}{}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/480*(120*(A*a^2+A*b^2+2*B*a*b)*sin(2*d*x+2*c)+10*(8*A*a*b+4*B*a^2+5*B*b^
2)*sin(3*d*x+3*c)+15*(A*b^2+2*B*a*b)*sin(4*d*x+4*c)+6*B*b^2*sin(5*d*x+5*c)
+60*(12*A*a*b+6*B*a^2+5*B*b^2)*sin(d*x+c)+240*x*d*(a^2*A+3/4*A*b^2+3/2*B*a
*b))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.75

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$=\frac{15\,(4\,Aa^2+6\,Bab+3\,Ab^2)dx+\left(24\,Bb^2\cos\left(dx+c\right)^4+30\,(2\,Bab+Ab^2)\cos\left(dx+c\right)^3+80\,Ba^2+160\,\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/120*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*d*x + (24*B*b^2*cos(d*x + c)^4 + 3
0*(2*B*a*b + A*b^2)*cos(d*x + c)^3 + 80*B*a^2 + 160*A*a*b + 64*B*b^2 + 8*(
5*B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^2 + 15*(4*A*a^2 + 6*B*a*b + 3*A
*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(184) = 368$.

Time = 0.31 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.43

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$= \begin{cases} \frac{Aa^2x\sin^2(c+dx)}{2} + \frac{Aa^2x\cos^2(c+dx)}{2} + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{4Aab\sin^3(c+dx)}{3d} + \frac{2Aab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Ab^2x\sin}{8} \\ x(A+B\cos(c))\,(a+b\cos(c))^2\cos^2(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**
2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*A*a*b*sin(c + d*x)**3/(3*d) + 2*A*a*
b*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*b**2*x*sin(c + d*x)**4/8 + 3*A*b**2
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*b**2*x*cos(c + d*x)**4/8 + 3*A*
b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*b**2*sin(c + d*x)*cos(c + d*
x)**3/(8*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x)*cos(c +
 d*x)**2/d + 3*B*a*b*x*sin(c + d*x)**4/4 + 3*B*a*b*x*sin(c + d*x)**2*cos(c
 + d*x)**2/2 + 3*B*a*b*x*cos(c + d*x)**4/4 + 3*B*a*b*sin(c + d*x)**3*cos(c
 + d*x)/(4*d) + 5*B*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*B*b**2*sin(
c + d*x)**5/(15*d) + 4*B*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b*
*2*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos
(c))**2*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2-160\,\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2-320\,\left(\sin(dx+c)^3-3\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 - 160*(sin(d*x + c)^3 -
3*sin(d*x + c))*B*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b + 30*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b + 15*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^2 + 32*(3*sin(d*x + c)
^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{Bb^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(4\,Aa^2+6\,Bab+3\,Ab^2\right)x + \frac{(2\,Bab+Ab^2)\sin(4\,dx+4\,c)}{32\,d}+\frac{(4\,Ba^2+8\,Aab+5\,Bb^2)\sin(3\,dx+3\,c)}{48\,d} + \frac{(Aa^2+2\,Bab+Ab^2)\sin(2\,dx+2\,c)}{4\,d}+\frac{(6\,Ba^2+12\,Aab+5\,Bb^2)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/80*B*b^2*sin(5*d*x + 5*c)/d + 1/8*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*x + 1/32
*(2*B*a*b + A*b^2)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^2 + 8*A*a*b + 5*B*b^2)
*sin(3*d*x + 3*c)/d + 1/4*(A*a^2 + 2*B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1
/8*(6*B*a^2 + 12*A*a*b + 5*B*b^2)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 38.08 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.62

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{x\left(A\,a^2+\frac{3\,B\,a\,b}{2}+\frac{3\,A\,b^2}{4}\right)}{2} + \frac{\left(2\,B\,a^2-\frac{5\,A\,b^2}{4}-A\,a^2+2\,B\,b^2+4\,A\,a\,b-\frac{5\,B\,a\,b}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{16\,B\,a^2}{3}-\frac{A\,b^2}{2}-2\,A\,a^2+\frac{8\,B\,b^2}{3}+\cdots\right.}{}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

output
```
(x*(A*a^2 + (3*A*b^2)/4 + (3*B*a*b)/2))/2 + (tan(c/2 + (d*x)/2)^5*((20*B*a
^2)/3 + (116*B*b^2)/15 + (40*A*a*b)/3) - tan(c/2 + (d*x)/2)^9*(A*a^2 + (5*
A*b^2)/4 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + (5*B*a*b)/2) + tan(c/2 + (d*x)/2)
^3*(2*A*a^2 + (A*b^2)/2 + (16*B*a^2)/3 + (8*B*b^2)/3 + (32*A*a*b)/3 + B*a*
b) - tan(c/2 + (d*x)/2)^7*(2*A*a^2 + (A*b^2)/2 - (16*B*a^2)/3 - (8*B*b^2)/
3 - (32*A*a*b)/3 + B*a*b) + tan(c/2 + (d*x)/2)*(A*a^2 + (5*A*b^2)/4 + 2*B*
a^2 + 2*B*b^2 + 4*A*a*b + (5*B*a*b)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*ta
n(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + ta
n(c/2 + (d*x)/2)^10 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.74

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{-90\cos(dx+c)\sin(dx+c)^3\,a\,b^2+60\cos(dx+c)\sin(dx+c)\,a^3+225\cos(dx+c)\sin(dx+c)\,a\,b^2+\cdots}{}$$

input
```
int(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
( - 90*cos(c + d*x)*sin(c + d*x)**3*a*b**2 + 60*cos(c + d*x)*sin(c + d*x)*
a**3 + 225*cos(c + d*x)*sin(c + d*x)*a*b**2 + 24*sin(c + d*x)**5*b**3 - 12
0*sin(c + d*x)**3*a**2*b - 80*sin(c + d*x)**3*b**3 + 360*sin(c + d*x)*a**2
*b + 120*sin(c + d*x)*b**3 + 60*a**3*d*x + 135*a*b**2*d*x)/(120*d)
```

3.224 $\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 170

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx\\ &= \frac{1}{8}\left(8aAb+4a^2B+3b^2B\right)x+\frac{(4a^2Ab+4Ab^3-a^3B+8ab^2B)\sin(c+dx)}{6bd}\\ &\quad+\frac{(8aAb-2a^2B+9b^2B)\cos(c+dx)\sin(c+dx)}{24d}\\ &\quad+\frac{(4Ab-aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12bd}+\frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4bd}\end{aligned}$$

output
```
1/8*(8*A*a*b+4*B*a^2+3*B*b^2)*x+1/6*(4*A*a^2*b+4*A*b^3-B*a^3+8*B*a*b^2)*si
n(d*x+c)/b/d+1/24*(8*A*a*b-2*B*a^2+9*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*(
4*A*b-B*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/b/d+1/4*B*(a+b*cos(d*x+c))^3*sin(
d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{12(8aAb+4a^2B+3b^2B)\,(c+dx)+24(4a^2A+3Ab^2+6abB)\sin(c+dx)+24(2aAb+a^2B+b^2B)\sin}{96d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(12*(8*a*A*b + 4*a^2*B + 3*b^2*B)*(c + d*x) + 24*(4*a^2*A + 3*A*b^2 + 6*a*
b*B)*Sin[c + d*x] + 24*(2*a*A*b + a^2*B + b^2*B)*Sin[2*(c + d*x)] + 8*b*(A
*b + 2*a*B)*Sin[3*(c + d*x)] + 3*b^2*B*Sin[4*(c + d*x)])/(96*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 3447, 3042, 3502, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a+b\cos(c+dx))^2\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \ 3502$$

$$\frac{\int (a+b\cos(c+dx))^2(3bB+(4Ab-aB)\cos(c+dx))dx}{4b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(3bB+(4Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{4b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4bd}$$

$$\Big\downarrow \ 3232$$

$$\frac{\frac{1}{3}\int (a+b\cos(c+dx))\left(b(8Ab+7aB)+\left(-2Ba^2+8Aba+9b^2B\right)\cos(c+dx)\right)dx+\frac{(4Ab-aB)\sin(c+dx)(a+b\cos(c+d}{3d}}{4b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b(8Ab+7aB)+\left(-2Ba^2+8Aba+9b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{(4Ab-aB)\sin(c+dx)}{3d}}{4b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4bd}$$

$$\Big\downarrow \ 3213$$

$$\frac{\frac{1}{3}\left(\frac{b(-2a^2B+8aAb+9b^2B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}bx\left(4a^2B+8aAb+3b^2B\right)+\frac{2(a^3(-B)+4a^2Ab+8ab^2B+4Ab^3)\sin(c+dx)}{d}\right)+}{4b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4bd}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d) + (((4*A*b - a*B)*(a + b*C
os[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*b*(8*a*A*b + 4*a^2*B + 3*b^2*B)*x
)/2 + (2*(4*a^2*A*b + 4*A*b^3 - a^3*B + 8*a*b^2*B)*Sin[c + d*x])/d + (b*(8
*a*A*b - 2*a^2*B + 9*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3)/(4*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 12.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{24(2Aab+a^2B+B\,b^2)\sin(2dx+2c)+8(A\,b^2+2Bab)\sin(3dx+3c)+3B\,b^2\sin(4dx+4c)+24(4a^2A+3A\,b^2+6Bab)\sin(d}{96d}$
parts	$\frac{(A\,b^2+2Bab)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}+\frac{(2Aab+a^2B)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{B\,b^2\left(\left(\cos(dx+c)^3+\frac{3\,c}{}\right.\right.}{}$
derivativedivides	$\frac{a^2A\sin(dx+c)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2Bab\left(\cos(dx+c)^2+2\right)\sin(dx+}{3}}{d}$
default	$\frac{a^2A\sin(dx+c)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2Bab\left(\cos(dx+c)^2+2\right)\sin(dx+}{3}}{d}$
risch	$xAab+\frac{a^2Bx}{2}+\frac{3b^2Bx}{8}+\frac{\sin(dx+c)a^2A}{d}+\frac{3\sin(dx+c)A\,b^2}{4d}+\frac{3\sin(dx+c)Bab}{2d}+\frac{B\,b^2\sin(4dx+4c)}{32d}+\sin$
norman	$\frac{(Aab+\frac{1}{2}a^2B+\frac{3}{8}B\,b^2)x+(Aab+\frac{1}{2}a^2B+\frac{3}{8}B\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(4Aab+2a^2B+\frac{3}{2}B\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(4Aab+2a^2B}{}$
orering	Expression too large to display

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/96*(24*(2*A*a*b+B*a^2+B*b^2)*sin(2*d*x+2*c)+8*(A*b^2+2*B*a*b)*sin(3*d*x+
3*c)+3*B*b^2*sin(4*d*x+4*c)+24*(4*A*a^2+3*A*b^2+6*B*a*b)*sin(d*x+c)+96*x*d
*(A*a*b+1/2*a^2*B+3/8*B*b^2))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\frac{3\left(4\,Ba^2+8\,Aab+3\,Bb^2\right)dx+\left(6\,Bb^2\cos\left(dx+c\right)^3+24\,Aa^2+32\,Bab+16\,Ab^2+8\left(2\,Bab+Ab^2\right)\cos\left(\right.\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/24*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*d*x + (6*B*b^2*cos(d*x + c)^3 + 24*A
*a^2 + 32*B*a*b + 16*A*b^2 + 8*(2*B*a*b + A*b^2)*cos(d*x + c)^2 + 3*(4*B*a
^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(162) = 324$.

Time = 0.20 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.99

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{Aa^2\sin(c+dx)}{d} + Aabx\sin^2(c+dx) + Aabx\cos^2(c+dx) + \frac{Aab\sin(c+dx)\cos(c+dx)}{d} + \frac{2Ab^2\sin^3(c+dx)}{3d} + \frac{Ab^2\sin(\\ x(A+B\cos(c))\,(a+b\cos(c))^2\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**2*sin(c + d*x)/d + A*a*b*x*sin(c + d*x)**2 + A*a*b*x*cos(c
 + d*x)**2 + A*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*A*b**2*sin(c + d*x)**3/
(3*d) + A*b**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*x*sin(c + d*x)**2/2
 + B*a**2*x*cos(c + d*x)**2/2 + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4
*B*a*b*sin(c + d*x)**3/(3*d) + 2*B*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*
B*b**2*x*sin(c + d*x)**4/8 + 3*B*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4
+ 3*B*b**2*x*cos(c + d*x)**4/8 + 3*B*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*
d) + 5*B*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos
(c))*(a + b*cos(c))**2*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+48\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab-64\,\left(\sin(dx+c)^3-3\,\sin(\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 48*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a*b - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b - 32*(sin(d
*x + c)^3 - 3*sin(d*x + c))*A*b^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) +
8*sin(2*d*x + 2*c))*B*b^2 + 96*A*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{Bb^2\sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\,(4\,Ba^2+8\,Aab+3\,Bb^2)x+\frac{(2\,Bab+Ab^2)\sin(3\,dx+3\,c)}{12\,d}$$
$$+\frac{(Ba^2+2\,Aab+Bb^2)\sin(2\,dx+2\,c)}{4\,d}+\frac{(4\,Aa^2+6\,Bab+3\,Ab^2)\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/32*B*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*x + 1/12
*(2*B*a*b + A*b^2)*sin(3*d*x + 3*c)/d + 1/4*(B*a^2 + 2*A*a*b + B*b^2)*sin(
2*d*x + 2*c)/d + 1/4*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 40.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B\,a^2\,x}{2}+\frac{3\,B\,b^2\,x}{8}+\frac{A\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{3\,A\,b^2\,\sin\left(c+d\,x\right)}{4\,d}+A\,a\,b\,x+\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{3\,B\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{A\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{B\,a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

output
```
(B*a^2*x)/2 + (3*B*b^2*x)/8 + (A*a^2*sin(c + d*x))/d + (3*A*b^2*sin(c + d*
x))/(4*d) + A*a*b*x + (B*a^2*sin(2*c + 2*d*x))/(4*d) + (A*b^2*sin(3*c + 3*
d*x))/(12*d) + (B*b^2*sin(2*c + 2*d*x))/(4*d) + (B*b^2*sin(4*c + 4*d*x))/(
32*d) + (3*B*a*b*sin(c + d*x))/(2*d) + (A*a*b*sin(2*c + 2*d*x))/(2*d) + (B
*a*b*sin(3*c + 3*d*x))/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3 b^3+12\cos\left(dx+c\right)\sin\left(dx+c\right)a^2b+5\cos\left(dx+c\right)\sin\left(dx+c\right)b^3-8\sin(}{8d}$$

input
```
int(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*b**3 + 12*cos(c + d*x)*sin(c + d*x)*a**
2*b + 5*cos(c + d*x)*sin(c + d*x)*b**3 - 8*sin(c + d*x)**3*a*b**2 + 8*sin(
c + d*x)*a**3 + 24*sin(c + d*x)*a*b**2 + 12*a**2*b*d*x + 3*b**3*d*x)/(8*d)
```

3.225 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\begin{aligned}\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx &= \frac{1}{2}\left(2a^2A+Ab^2+2abB\right)x+\frac{2(3aAb+a^2B+b^2B)\sin(c+dx)}{3d}\\&\quad+\frac{b(3Ab+2aB)\cos(c+dx)\sin(c+dx)}{6d}+\frac{B(a+b\cos(c+dx))^2\sin(c+dx)}{3d}\end{aligned}$$

output
```
1/2*(2*A*a^2+A*b^2+2*B*a*b)*x+2/3*(3*A*a*b+B*a^2+B*b^2)*sin(d*x+c)/d+1/6*b
*(3*A*b+2*B*a)*cos(d*x+c)*sin(d*x+c)/d+1/3*B*(a+b*cos(d*x+c))^2*sin(d*x+c)
/d
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{6\left(2a^2A+Ab^2+2abB\right)(c+dx)+3(8aAb+4a^2B+3b^2B)\sin(c+dx)+3b(Ab+2aB)\sin(2(c+dx))}{12d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(6*(2*a^2*A + A*b^2 + 2*a*b*B)*(c + d*x) + 3*(8*a*A*b + 4*a^2*B + 3*b^2*B)
*Sin[c + d*x] + 3*b*(A*b + 2*a*B)*Sin[2*(c + d*x)] + b^2*B*Sin[3*(c + d*x)
])/(12*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{3}\int (a+b\cos(c+dx))(3aA+2bB+(3Ab+2aB)\cos(c+dx))dx+\frac{B\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(3aA+2bB+(3Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{B\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\frac{2(a^2B+3aAb+b^2B)\sin(c+dx)}{d}+\frac{3}{2}x(2a^2A+2abB+Ab^2)+\frac{b(2aB+3Ab)\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{B\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*(2*a^2*A + A*b^2 + 2*a
*b*B)*x)/2 + (2*(3*a*A*b + a^2*B + b^2*B)*Sin[c + d*x])/d + (b*(3*A*b + 2*
a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\frac{a^2A(dx+c)+a^2B\sin(dx+c)+2A\sin(dx+c)ab+2Bab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab^2\left(\frac{\cos(dx+c)\,\mathrm{si}}{2}\right)}{d}$$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(a^2*A*(d*x+c)+a^2*B*sin(d*x+c)+2*A*sin(d*x+c)*a*b+2*B*a*b*(1/2*cos(d*
x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/
2*c)+1/3*B*b^2*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int (a + b\cos(c + dx))^2(A + B\cos(c + dx))\,dx$$
$$= \frac{3\,(2\,Aa^2 + 2\,Bab + Ab^2)dx + \left(2\,Bb^2\cos\left(dx + c\right)^2 + 6\,Ba^2 + 12\,Aab + 4\,Bb^2 + 3\,(2\,Bab + Ab^2)\cos\left(dx \right.\right.}{6\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*d*x + (2*B*b^2*cos(d*x + c)^2 + 6*B*a^2
 + 12*A*a*b + 4*B*b^2 + 3*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

$$\int (a + b\cos(c + dx))^2(A + B\cos(c + dx))\,dx$$
$$= \begin{cases} Aa^2x + \frac{2Aab\sin\left(c+dx\right)}{d} + \frac{Ab^2x\sin^2\left(c+dx\right)}{2} + \frac{Ab^2x\cos^2\left(c+dx\right)}{2} + \frac{Ab^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} + \frac{Ba^2\sin\left(c+dx\right)}{d} + Babx\sin^2 \\ x(A + B\cos\left(c\right))\left(a + b\cos\left(c\right)\right)^2 \end{cases}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**2*x + 2*A*a*b*sin(c + d*x)/d + A*b**2*x*sin(c + d*x)**2/2
+ A*b**2*x*cos(c + d*x)**2/2 + A*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + B*
a**2*sin(c + d*x)/d + B*a*b*x*sin(c + d*x)**2 + B*a*b*x*cos(c + d*x)**2 +
B*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*B*b**2*sin(c + d*x)**3/(3*d) + B*b**
2*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(
c))**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{12\,(dx+c)Aa^2+6\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Bab+3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Ab^2-4\,\big(\sin{(dx}}{12\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(12*(d*x + c)*A*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b + 3*(2
*d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))
*B*b^2 + 12*B*a^2*sin(d*x + c) + 24*A*a*b*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{Bb^2\sin{(3\,dx+3\,c)}}{12\,d}+\frac{1}{2}\,\big(2\,Aa^2+2\,Bab+Ab^2\big)x$$
$$+\frac{(2\,Bab+Ab^2)\sin{(2\,dx+2\,c)}}{4\,d}+\frac{(4\,Ba^2+8\,Aab+3\,Bb^2)\sin{(dx+c)}}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/12*B*b^2*sin(3*d*x + 3*c)/d + 1/2*(2*A*a^2 + 2*B*a*b + A*b^2)*x + 1/4*(2
*B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*sin
(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 40.95 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= A\,a^2\,x+\frac{A\,b^2\,x}{2}+\frac{B\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,b^2\,\sin\left(c+d\,x\right)}{4\,d} + B\,a\,b\,x+\frac{A\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d} + \frac{2\,A\,a\,b\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

output
```
A*a^2*x + (A*b^2*x)/2 + (B*a^2*sin(c + d*x))/d + (3*B*b^2*sin(c + d*x))/(4
*d) + B*a*b*x + (A*b^2*sin(2*c + 2*d*x))/(4*d) + (B*b^2*sin(3*c + 3*d*x))/
(12*d) + (2*A*a*b*sin(c + d*x))/d + (B*a*b*sin(2*c + 2*d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.70

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{9\cos\left(dx+c\right)\sin\left(dx+c\right)a\,b^2-2\sin\left(dx+c\right)^3b^3+18\sin\left(dx+c\right)a^2b+6\sin\left(dx+c\right)b^3+6a^3dx+9a\,b}{6d}$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
(9*cos(c + d*x)*sin(c + d*x)*a*b**2 - 2*sin(c + d*x)**3*b**3 + 18*sin(c +
d*x)*a**2*b + 6*sin(c + d*x)*b**3 + 6*a**3*d*x + 9*a*b**2*d*x)/(6*d)
```

3.226 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 86

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{a^2A\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{b(2Ab+3aB)\sin(c+dx)}{2d}+\frac{bB(a+b\cos(c+dx))\sin(c+dx)}{2d}$$

output
```
1/2*(4*A*a*b+2*B*a^2+B*b^2)*x+a^2*A*arctanh(sin(d*x+c))/d+1/2*b*(2*A*b+3*B
*a)*sin(d*x+c)/d+1/2*b*B*(a+b*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.40

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2(4aAb+2a^2B+b^2B)(c+dx)-4a^2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^2A\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{4d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*(4*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) - 4*a^2*A*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + 4*a^2*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*b
*(A*b + 2*a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 3469, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{2}\int \left(2Aa^2+b(2Ab+3aB)\cos^2(c+dx)+\left(2Ba^2+4Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2Aa^2+b(2Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(2Ba^2+4Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(2Aa^2+\left(2Ba^2+4Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{b(3aB+2Ab)\sin(c+dx)}{d}\right)+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{2Aa^2+\left(2Ba^2+4Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b(3aB+2Ab)\sin(c+dx)}{d}\right)+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(2a^2A\int \sec(c+dx)dx+x\left(2a^2B+4aAb+b^2B\right)+\frac{b(3aB+2Ab)\sin(c+dx)}{d}\right)+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2a^2A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+x\left(2a^2B+4aAb+b^2B\right)+\frac{b(3aB+2Ab)\sin(c+dx)}{d}\right)+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{2a^2A\text{arctanh}(\sin(c+dx))}{d}+x\left(2a^2B+4aAb+b^2B\right)+\frac{b(3aB+2Ab)\sin(c+dx)}{d}\right)+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(b*B*(a + b*Cos[c + d*x])*Sin[c + d*x])/(2*d) + ((4*a*A*b + 2*a^2*B + b^2*
B)*x + (2*a^2*A*ArcTanh[Sin[c + d*x]])/d + (b*(2*A*b + 3*a*B)*Sin[c + d*x]
)/d)/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))+a^2 B(dx+c)+2Aab(dx+c)+2B\sin(dx+c)ab+A\sin(dx+c)b^2+B\,b^2\left(\frac{\cos(dx+c)\sin(dx+}{2}\right.}{d}$
default	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))+a^2 B(dx+c)+2Aab(dx+c)+2B\sin(dx+c)ab+A\sin(dx+c)b^2+B\,b^2\left(\frac{\cos(dx+c)\sin(dx+}{2}\right.}{d}$
parts	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(A\,b^2+2Bab)\sin(dx+c)}{d}+\frac{(2Aab+a^2B)(dx+c)}{d}+\frac{B\,b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\right.}{d}$
parallelrisch	$\frac{-4a^2 A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4a^2 A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+B\sin(2dx+2c)b^2+(4A\,b^2+8Bab)\sin(dx+c)+8xd\left(Aab+\frac{1}{2}a\right.}{4d}$
risch	$2xAab+a^2Bx+\frac{b^2Bx}{2}-\frac{i\mathrm{e}^{i(dx+c)}A\,b^2}{2d}-\frac{i\mathrm{e}^{i(dx+c)}Bab}{d}+\frac{i\mathrm{e}^{-i(dx+c)}A\,b^2}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}Bab}{d}+\frac{a^2A\ln(}{}$
norman	$\frac{(2Aab+a^2B+\frac{1}{2}B\,b^2)x+(2Aab+a^2B+\frac{1}{2}B\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(6Aab+3a^2B+\frac{3}{2}B\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(6Aab+3a^2B}{\left(1+\tan\left(\frac{dx}{2}+\right.\right.}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*(d*x+c)+2*A*a*b*(d*x+c)+2*B*sin
(d*x+c)*a*b+A*sin(d*x+c)*b^2+B*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*
c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{Aa^2\log(\sin(dx+c)+1)-Aa^2\log(-\sin(dx+c)+1)+(2\,Ba^2+4\,Aab+Bb^2)dx+(Bb^2\cos(dx+c}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fri
cas")
```

output
```
1/2*(A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*x + c) + 1) + (2*B*a^2
 + 4*A*a*b + B*b^2)*d*x + (B*b^2*cos(d*x + c) + 4*B*a*b + 2*A*b^2)*sin(d*x
 + c))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})(a+b\cos{(c+dx)})^2\sec{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{4\,(dx+c)Ba^2+8\,(dx+c)Aab+(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Bb^2+4\,Aa^2\log{(\sec{(dx+c)}+\tan{(dx+}}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="max
ima")
```

output
```
1/4*(4*(d*x + c)*B*a^2 + 8*(d*x + c)*A*a*b + (2*d*x + 2*c + sin(2*d*x + 2*
c))*B*b^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*B*a*b*sin(d*x + c
) + 4*A*b^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(80) = 160$.

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.07

$$\int (a + b\cos(c + dx))^2(A + B\cos(c + dx))\sec(c + dx)\,dx$$
$$= \frac{2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + (2\,Ba^2 + 4\,Aab + Bb^2)(dx + c)}{2}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gia
c")
```

output
```
1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) + (2*B*a^2 + 4*A*a*b + B*b^2)*(d*x + c) + 2*(4*B*a*b*ta
n(1/2*d*x + 1/2*c)^3 + 2*A*b^2*tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 4*B*a*b*tan(1/2*d*x + 1/2*c) + 2*A*b^2*tan(1/2*d*x + 1/2*c) +
 B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 41.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.97

$$\int (a + b\cos(c + dx))^2(A + B\cos(c + dx))\sec(c + dx)\,dx$$
$$= \frac{A\,b^2\,\sin(c + d\,x)}{d} + \frac{2\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d}$$
$$+ \frac{B\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{B\,b^2\,\sin(2\,c + 2\,d\,x)}{4\,d}$$
$$+ \frac{2\,B\,a\,b\,\sin(c + d\,x)}{d} + \frac{4\,A\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x),x)
```

output
```
(A*b^2*sin(c + d*x))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)))/d + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*b^2
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*b^2*sin(2*c + 2*d*x))
/(4*d) + (2*B*a*b*sin(c + d*x))/d + (4*A*a*b*atan(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{\cos(dx+c)\sin(dx+c)\,b^3 - 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^3 + 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a^3 + 6\sin(dx+c)\,a\,b}{2d}$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*b**3 - 2*log(tan((c + d*x)/2) - 1)*a**3 + 2*log
(tan((c + d*x)/2) + 1)*a**3 + 6*sin(c + d*x)*a*b**2 + 6*a**2*b*d*x + b**3*
d*x)/(2*d)
```

3.227 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 60

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b(Ab+2aB)x+\frac{a(2Ab+aB)\text{arctanh}(\sin(c+dx))}{d} + \frac{b^2B\sin(c+dx)}{d}+\frac{a^2A\tan(c+dx)}{d}$$

output
```
b*(A*b+2*B*a)*x+a*(2*A*b+B*a)*arctanh(sin(d*x+c))/d+b^2*B*sin(d*x+c)/d+a^2
*A*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.82

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{b(Ab+2aB)(c+dx)-a(2Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(2Ab+aB)\log\left(\cos\left(\frac{1}{2}\right.\right.}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(b*(A*b + 2*a*B)*(c + d*x) - a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b
^2*B*Sin[c + d*x] + a^2*A*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 3467, 25, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{a^2A\tan(c+dx)}{d}-\int -\left(\left(b^2B\cos^2(c+dx)+b(Ab+2aB)\cos(c+dx)+a(2Ab+aB)\right)\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{25}$$

$$\int \left(b^2B\cos^2(c+dx)+b(Ab+2aB)\cos(c+dx)+a(2Ab+aB)\right)\sec(c+dx)dx+\frac{a^2A\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(2Ab+aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2A\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3502}$$

$$\int (a(2Ab+aB)+b(Ab+2aB)\cos(c+dx))\sec(c+dx)dx+\frac{a^2A\tan(c+dx)}{d}+\frac{b^2B\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a(2Ab+aB)+b(Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2A\tan(c+dx)}{d}+\frac{b^2B\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3214}$$

$$a(aB+2Ab)\int \sec(c+dx)dx+\frac{a^2A\tan(c+dx)}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a(aB+2Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2A\tan(c+dx)}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2A\tan(c+dx)}{d}+\frac{a(aB+2Ab)\text{arctanh}(\sin(c+dx))}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b*(A*b + 2*a*B)*x + (a*(2*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b^2*B*Sin
[c + d*x])/d + (a^2*A*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32

method	result
parts	$\frac{a^{2} A \tan \left(d x +c \right)}{d}+\frac{\left(A \,b^{2}+2 B a b \right) \left(d x +c \right)}{d}+\frac{\left(2 A a b +a^{2} B \right) \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{d}+\frac{b^{2} B \sin \left(d x +c \right)}{d}$
derivativedivides	$\frac{A \tan \left(d x +c \right) a^{2}+a^{2} B \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)+2 A a b \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)+2 B a b \left(d x +c \right)+A \,b^{2} \left(d x +c \right)+B \sin}{d}$
default	$\frac{A \tan \left(d x +c \right) a^{2}+a^{2} B \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)+2 A a b \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)+2 B a b \left(d x +c \right)+A \,b^{2} \left(d x +c \right)+B \sin}{d}$
parallelrisch	$\frac{\left(-4 A a b -2 a^{2} B \right) \cos \left(d x +c \right) \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)+\left(4 A a b +2 a^{2} B \right) \cos \left(d x +c \right) \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)+B \sin \left(2 d x +2 c \right) b^{2}+}{2 \cos \left(d x +c \right) d}$
risch	$x A \,b^{2}+2 x B a b -\frac{i B \,b^{2} {\mathrm e}^{i \left(d x +c \right)}}{2 d}+\frac{i B \,b^{2} {\mathrm e}^{-i \left(d x +c \right)}}{2 d}+\frac{2 i a^{2} A}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}-\frac{2 a \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right) A b}{d}-\frac{a^{2} \ln \left({\mathrm e}^{i (}\right.}{}$
norman	$\frac{\left(-A \,b^{2}-2 B a b \right) x +\left(-2 A \,b^{2}-4 B a b \right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+\left(A \,b^{2}+2 B a b \right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{8}+\left(2 A \,b^{2}+4 B a b \right) x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}-}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{3} \left(\mathrm{t}\right.}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBO
SE)
```

output
```
a^2*A*tan(d*x+c)/d+(A*b^2+2*B*a*b)/d*(d*x+c)+(2*A*a*b+B*a^2)/d*ln(sec(d*x+
c)+tan(d*x+c))+b^2*B*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{2\,(2\,Bab+Ab^2)dx\cos{(dx+c)}+(Ba^2+2\,Aab)\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-(Ba^2+2\,Aab)\cos{(}}{2\,d\cos{(dx+c)}}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="f
ricas")
```

output
```
1/2*(2*(2*B*a*b + A*b^2)*d*x*cos(d*x + c) + (B*a^2 + 2*A*a*b)*cos(d*x + c)
*log(sin(d*x + c) + 1) - (B*a^2 + 2*A*a*b)*cos(d*x + c)*log(-sin(d*x + c)
+ 1) + 2*(B*b^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^2\sec^2{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.72

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{4\,(dx+c)Bab+2\,(dx+c)Ab^2+Ba^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,Aab(\log(\sin(d}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="m
axima")
```

output
```
1/2*(4*(d*x + c)*B*a*b + 2*(d*x + c)*A*b^2 + B*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*A*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 2*B*b^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(60) = 120.

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.53

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{(2\,Bab+Ab^2)(dx+c)+(Ba^2+2\,Aab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ba^2+2\,Aab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{d}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="g
iac")
```

output
```
((2*B*a*b + A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^
2*tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*
x + 1/2*c) + B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 42.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.82

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{A\,a^2\tan(c+d\,x)}{d} + \frac{2\,A\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{B\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\cos\left(c+d\,x\right)} + \frac{4\,B\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2\mathrm{i}}{d} - \frac{A\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,4\mathrm{i}}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

output
```
(A*a^2*tan(c + d*x))/d + (2*A*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d - (B*a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/d + (
B*b^2*sin(2*c + 2*d*x))/(2*d*cos(c + d*x)) - (A*a*b*atan((sin(c/2 + (d*x)/
2)*1i)/cos(c/2 + (d*x)/2))*4i)/d + (4*B*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2b + 3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a^2b + \cos(dx+c)\sin(}{\cos(dx+c)\,d}$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**2*b + 3*cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*a**2*b + cos(c + d*x)*sin(c + d*x)*b**3 + 3*cos(c + d
*x)*a*b**2*d*x + sin(c + d*x)*a**3)/(cos(c + d*x)*d)
```

3.228 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^2Bx+\frac{(a^2A+2Ab^2+4abB)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a(2Ab+aB)\tan(c+dx)}{d}+\frac{a^2A\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
b^2*B*x+1/2*(A*a^2+2*A*b^2+4*B*a*b)*arctanh(sin(d*x+c))/d+a*(2*A*b+B*a)*ta
n(d*x+c)/d+1/2*a^2*A*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^2Bx+\frac{b(Ab+2aB)\coth^{-1}(\sin(c+dx))}{d}+\frac{a^2A\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a(2Ab+aB)\tan(c+dx)}{d}+\frac{a^2A\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
b^2*B*x + (b*(A*b + 2*a*B)*ArcCoth[Sin[c + d*x]])/d + (a^2*A*ArcTanh[Sin[c
 + d*x]])/(2*d) + (a*(2*A*b + a*B)*Tan[c + d*x])/d + (a^2*A*Sec[c + d*x]*T
an[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3042, 3467, 25, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int -\left(\left(2b^2B\cos^2(c+dx)+\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+2a(2Ab+aB)\right)\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \left(2b^2B\cos^2(c+dx)+\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+2a(2Ab+aB)\right)\sec^2(c+dx)dx+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+4bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(2Ab+aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{2}\left(\int \left(Aa^2+4bBa+2Ab^2+2b^2B\cos(c+dx)\right)\sec(c+dx)dx+\frac{2a(aB+2Ab)\tan(c+dx)}{d}\right)+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{Aa^2+4bBa+2Ab^2+2b^2B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a(aB+2Ab)\tan(c+dx)}{d}\right)+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(\left(a^2A+4abB+2Ab^2\right)\int \sec(c+dx)dx+\frac{2a(aB+2Ab)\tan(c+dx)}{d}+2b^2Bx\right)+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(a^2A+4abB+2Ab^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2a(aB+2Ab)\tan(c+dx)}{d}+2b^2Bx\right)+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{\left(a^2A+4abB+2Ab^2\right)\text{arctanh}(\sin(c+dx))}{d}+\frac{2a(aB+2Ab)\tan(c+dx)}{d}+2b^2Bx\right)+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a^2*A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (2*b^2*B*x + ((a^2*A + 2*A*b^2 +
 4*a*b*B)*ArcTanh[Sin[c + d*x]])/d + (2*a*(2*A*b + a*B)*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

method	result
parts	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{(A\,b^2+2Bab)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(2Aab+a^2B)\,\mathrm{ta}}{d}$
derivativedivides	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2B\tan(dx+c)+2Aab\tan(dx+c)+2Bab\ln(\sec(dx+c)+\tan(dx-}{d}$
default	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2B\tan(dx+c)+2Aab\tan(dx+c)+2Bab\ln(\sec(dx+c)+\tan(dx-}{d}$
parallelrisch	$\frac{-(\cos(2dx+2c)+1)(a^2A+2A\,b^2+4Bab)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(\cos(2dx+2c)+1)(a^2A+2A\,b^2+4Bab)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2d(\cos(2dx+2c)+1)}$
risch	$b^2Bx-\frac{ia\left(Aa\,e^{3i(dx+c)}-4Ab\,e^{2i(dx+c)}-2Ba\,e^{2i(dx+c)}-Aa\,e^{i(dx+c)}-4Ab-2Ba\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a^2A\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{\ln(\mathrm{e}}{}$
norman	$\frac{b^2Bx+\frac{a(Aa-4Ab-2Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}+\frac{a(Aa+4Ab+2Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+b^2Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+b^2Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+b^2Bx}{\left(1+\tan\right.}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBO
SE)
```

output
```
a^2*A/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+(A*b^2+2
*B*a*b)/d*ln(sec(d*x+c)+tan(d*x+c))+(2*A*a*b+B*a^2)/d*tan(d*x+c)+B*b^2/d*(
d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{4\,Bb^2dx\cos(dx+c)^2+(Aa^2+4\,Bab+2\,Ab^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa^2+4\,Bab+2\,A}{4\,d\cos(dx+}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="f
ricas")
```

output
```
1/4*(4*B*b^2*d*x*cos(d*x + c)^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^2*log(
-sin(d*x + c) + 1) + 2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^2\sec^3{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.75

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{4\,(dx+c)Bb^2 - Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log{(\sin{(dx+c)}+1)} + \log{(\sin{(dx+c)}-1)}\right) + 4\,Bab(\log{(\sin{(dx}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="m
axima")
```

output
```
1/4*(4*(d*x + c)*B*b^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(
sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a*b*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 4*B*a^2*tan(d*x + c) + 8*A*a*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(76) = 152$.

Time = 0.18 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.38

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{2\,(dx+c)Bb^2 + (Aa^2+4\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (Aa^2+4\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\right.\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="g
iac")
```

output
```
1/2*(2*(d*x + c)*B*b^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) + 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 -
4*A*a*b*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x + 1/2*c) + 2*B*a^2*tan(
1/2*d*x + 1/2*c) + 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 42.40 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.20

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{2\left(\frac{A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2} + A\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) + B\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) + 2\,B\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right.}{d}$$
$$+ \frac{\frac{B\,a^2\,\sin(2\,c+2\,d\,x)}{2} + \frac{A\,a^2\,\sin(c+d\,x)}{2} + A\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

output
```
(2*((A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + A*b^2*atanh(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*b^2*atan(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)) + 2*B*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d
 + ((B*a^2*sin(2*c + 2*d*x))/2 + (A*a^2*sin(c + d*x))/2 + A*a*b*sin(2*c +
2*d*x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.01

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2a^3-6\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2}{}$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 - 6*cos(c
+ d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**2 + cos(c + d*x)*log
(tan((c + d*x)/2) - 1)*a**3 + 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b
**2 + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3 + 6*cos(
c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**2 - cos(c + d*x)*l
og(tan((c + d*x)/2) + 1)*a**3 - 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a
*b**2 + 2*cos(c + d*x)*sin(c + d*x)**2*b**3*d*x - cos(c + d*x)*sin(c + d*x
)*a**3 - 2*cos(c + d*x)*b**3*d*x + 6*sin(c + d*x)**3*a**2*b - 6*sin(c + d*
x)*a**2*b)/(2*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.229 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 116

$$\begin{aligned}\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \frac{(2aAb+a^2B+2b^2B)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{(2a^2A+3Ab^2+6abB)\tan(c+dx)}{3d}\\ &\quad+\frac{a(2Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^2A\sec^2(c+dx)\tan(c+dx)}{3d}\end{aligned}$$

output
```
1/2*(2*A*a*b+B*a^2+2*B*b^2)*arctanh(sin(d*x+c))/d+1/3*(2*A*a^2+3*A*b^2+6*B
*a*b)*tan(d*x+c)/d+1/2*a*(2*A*b+B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*A*sec
(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{6b^2B\coth^{-1}(\sin(c+dx))+3a(2Ab+aB)\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3a(2Ab+aB)\sec(c+d\right.}{6d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(6*b^2*B*ArcCoth[Sin[c + d*x]] + 3*a*(2*A*b + a*B)*ArcTanh[Sin[c + d*x]] +
 Tan[c + d*x]*(3*a*(2*A*b + a*B)*Sec[c + d*x] + 2*(3*a^2*A + 3*A*b^2 + 6*a
*b*B + a^2*A*Tan[c + d*x]^2)))/(6*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3467, 25, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \ 3467$$

$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}-$$
$$\frac{1}{3}\int -\left(\left(3b^2B\cos^2(c+dx)+\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(2Ab+aB)\right)\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}\int \left(3b^2B\cos^2(c+dx)+\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(2Ab+aB)\right)\sec^3(c+dx)dx+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\int \frac{3b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(2Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(2Ab+aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 3500

$$\frac{1}{3}\left(\frac{1}{2}\int \left(2\left(2Aa^2+6bBa+3Ab^2\right)+3\left(Ba^2+2Aba+2b^2B\right)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{3a(aB+2Ab)\tan(c}{2d}\right.$$
$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{2\left(2Aa^2+6bBa+3Ab^2\right)+3\left(Ba^2+2Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{3a(aB+2Ab)\tan(c+dx)\sec}{2d}\right.$$
$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 3227

$$\frac{1}{3}\left(\frac{1}{2}\left(2\left(2a^2A+6abB+3Ab^2\right)\int \sec^2(c+dx)dx+3\left(a^2B+2aAb+2b^2B\right)\int \sec(c+dx)dx\right)+\frac{3a(aB+2Ab)}{}\right.$$
$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(a^2B+2aAb+2b^2B\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2\left(2a^2A+6abB+3Ab^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{3}{}\right.$$
$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$\big\downarrow$ 4254

$$\frac{1}{3}\left(\frac{1}{2}\left(3(a^2B+2aAb+2b^2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2(2a^2A+6abB+3Ab^2)\int 1d(-\tan(c+dx))}{d}\right)+\frac{3a(a\right.$$
$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3(a^2B+2aAb+2b^2B)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2(2a^2A+6abB+3Ab^2)\tan(c+dx)}{d}\right)+\frac{3a(aB+2Ab\right.$$
$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3(a^2B+2aAb+2b^2B)\,\text{arctanh}(\sin(c+dx))}{d}+\frac{2(2a^2A+6abB+3Ab^2)\tan(c+dx)}{d}\right)+\frac{3a(aB+2Ab)\right.$$
$$\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((3*a*(2*A*b + a*B)*Sec[c + d*
x]*Tan[c + d*x])/(2*d) + ((3*(2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d
*x]])/d + (2*(2*a^2*A + 3*A*b^2 + 6*a*b*B)*Tan[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.83 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

method	result
parts	$-\frac{a^2 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{(A b^2+2Bab)\tan(dx+c)}{d}+\frac{(2Aab+a^2B)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+}{}\right.}{d}$
derivativedivides	$\frac{-a^2 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^2B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2Aab\left(\frac{\sec(dx+c)\tan(dx+c}{2}\right.}{d}$
default	$\frac{-a^2 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^2B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2Aab\left(\frac{\sec(dx+c)\tan(dx+c}{2}\right.}{d}$
parallelrisch	$\frac{-9\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\left(Aab+\frac{1}{2}a^2B+Bb^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\left(Aab+\frac{1}{2}a^2B+B\right.}{3d(3\cos(dx}$
risch	$-\frac{i\left(6Aab\,e^{5i(dx+c)}+3B\,a^2e^{5i(dx+c)}-6A\,b^2e^{4i(dx+c)}-12Bab\,e^{4i(dx+c)}-12A\,a^2e^{2i(dx+c)}-12A\,b^2e^{2i(dx+c)}-24Bab\,e^{2}\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{-\frac{2\left(2a^2A-2Aab-2A\,b^2-a^2B-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{\left(2a^2A-2Aab+2A\,b^2-a^2B+4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{d}-\frac{2\left(2a^2A+2Aab-2A\,b\right.}{}}{}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBO
SE)
```

output
```
-a^2*A/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*b^2+2*B*a*b)/d*tan(d*x+c)+(
2*A*a*b+B*a^2)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))
+B*b^2/d*ln(sec(d*x+c)+tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log}{12}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="f
ricas")
```

output
```
1/12*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
 3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(
2*A*a^2 + 2*(2*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^2 + 3*(B*a^2 + 2*A*
a*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^2\sec^4(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.48

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2-3\,Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="m
axima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 3*B*a^2*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*
A*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(s
in(d*x + c) - 1)) + 6*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 24*B*a*b*tan(d*x + c) + 12*A*b^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(108) = 216$.

Time = 0.17 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.53

$$\int (a + b\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$

$$= \frac{3\left(Ba^2 + 2\,Aab + 2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(Ba^2 + 2\,Aab + 2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \right.\right.}{}$$

input

```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="g
iac")
```

output

```
1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*
(B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^
2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/
2*d*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x +
 1/2*c)^5 - 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)
^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*
a^2*tan(1/2*d*x + 1/2*c) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2
*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3)/d
```

Mupad [B] (verification not implemented)

Time = 45.01 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

$$\int (a + b\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^4(c + dx)\, dx$$

$$= \frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{B\,a^2}{2}+A\,a\,b+B\,b^2\right)}{2\,B\,a^2+4\,A\,a\,b+4\,B\,b^2}\right)\left(B\,a^2 + 2\,A\,a\,b + 2\,B\,b^2\right)}{d} - \frac{\left(2\,A\,a^2 + 2\,A\,b^2 - B\,a^2 - 2\,A\,a\,b + 4\,B\,a\,b\right)\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 + \left(-\frac{4\,A\,a^2}{3} - 8\,B\,a\,b - 4\,A\,b^2\right)\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2} + \frac{d}{}\right.\right.}$$

input

```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^4,x)
```

output
```
(atanh((4*tan(c/2 + (d*x)/2)*((B*a^2)/2 + B*b^2 + A*a*b))/(2*B*a^2 + 4*B*b
^2 + 4*A*a*b))*(B*a^2 + 2*B*b^2 + 2*A*a*b))/d - (tan(c/2 + (d*x)/2)*(2*A*a
^2 + 2*A*b^2 + B*a^2 + 2*A*a*b + 4*B*a*b) - tan(c/2 + (d*x)/2)^3*((4*A*a^2
)/3 + 4*A*b^2 + 8*B*a*b) + tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 2*A*b^2 - B*a^2
 - 2*A*a*b + 4*B*a*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4
 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.71

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{-9\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 a^2 b - 6\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)}{}$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
( - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b - 6*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**3 + 9*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*a**2*b + 6*cos(c + d*x)*log(tan((c + d*x)/2) -
1)*b**3 + 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b
+ 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**3 - 9*cos(c
+ d*x)*log(tan((c + d*x)/2) + 1)*a**2*b - 6*cos(c + d*x)*log(tan((c + d*x)
/2) + 1)*b**3 - 9*cos(c + d*x)*sin(c + d*x)*a**2*b + 4*sin(c + d*x)**3*a**
3 + 18*sin(c + d*x)**3*a*b**2 - 6*sin(c + d*x)*a**3 - 18*sin(c + d*x)*a*b*
*2)/(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.230 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 156

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{(3a^2A+4Ab^2+8abB)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{(4aAb+2a^2B+3b^2B)\tan(c+dx)}{3d} + \frac{(3a^2A+4Ab^2+8abB)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a(2Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2A\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*arctanh(sin(d*x+c))/d+1/3*(4*A*a*b+2*B*a^2+3
*B*b^2)*tan(d*x+c)/d+1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*sec(d*x+c)*tan(d*x+c)/d
+1/3*a*(2*A*b+B*a)*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a^2*A*sec(d*x+c)^3*tan(d*
x+c)/d
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.77

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{3(3a^2A+4Ab^2+8abB)\,\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(24(2aAb+a^2B+b^2B)+3(3a^2A+4Ab^2}{24d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*
(2*a*A*b + a^2*B + b^2*B) + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[c + d*x] +
 6*a^2*A*Sec[c + d*x]^3 + 8*a*(2*A*b + a*B)*Tan[c + d*x]^2))/(24*d)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3467, 25, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{1}{4}\int -\left(\left(4b^2B\cos^2(c+dx)+\left(3Aa^2+8bBa+4Ab^2\right)\cos(c+dx)+4a(2Ab+aB)\right)\sec^4(c+dx)\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \left(4b^2B\cos^2(c+dx)+\left(3Aa^2+8bBa+4Ab^2\right)\cos(c+dx)+4a(2Ab+aB)\right)\sec^4(c+dx)dx+\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{4b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+8bBa+4Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+4a(2Ab+aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 3500

$$\frac{1}{4}\left(\frac{1}{3}\int \left(3\left(3Aa^2+8bBa+4Ab^2\right)+4\left(2Ba^2+4Aba+3b^2B\right)\cos(c+dx)\right)\sec^3(c+dx)dx+\frac{4a(aB+2Ab)\tan(c}{3}\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{3\left(3Aa^2+8bBa+4Ab^2\right)+4\left(2Ba^2+4Aba+3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{4a(aB+2Ab)\tan(c+dx)\text{se}}{3d}\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 3227

$$\frac{1}{4}\left(\frac{1}{3}\left(3\left(3a^2A+8abB+4Ab^2\right)\int \sec^3(c+dx)dx+4\left(2a^2B+4aAb+3b^2B\right)\int \sec^2(c+dx)dx\right)+\frac{4a(aB+2A}{}\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(4\left(2a^2B+4aAb+3b^2B\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3\left(3a^2A+8abB+4Ab^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 4254

$$\frac{1}{4}\left(\frac{1}{3}\left(3\left(3a^2A+8abB+4Ab^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4\left(2a^2B+4aAb+3b^2B\right)\int 1d(-\tan(c+dx))}{d}\right)+\frac{4a\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 24

$$\frac{1}{4}\left(\frac{1}{3}\left(3\left(3a^2A+8abB+4Ab^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{4\left(2a^2B+4aAb+3b^2B\right)\tan(c+dx)}{d}\right)+\frac{4a(aB+2\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 4255

$$\frac{1}{4}\left(\frac{1}{3}\left(3\left(3a^2A+8abB+4Ab^2\right)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4\left(2a^2B+4aAb+3b^2B\right)\tan\right.\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(3\left(3a^2A+8abB+4Ab^2\right)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4\left(2a^2B+4aAb+3b^2\right.}{d}\right.\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\downarrow$ 4257

$$\frac{1}{4}\left(\frac{1}{3}\left(3\left(3a^2A+8abB+4Ab^2\right)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4\left(2a^2B+4aAb+3b^2B\right)}{d}\right.\right.$$
$$\frac{a^2A\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((4*a*(2*A*b + a*B)*Sec[c + d*
x]^2*Tan[c + d*x])/(3*d) + ((4*(4*a*A*b + 2*a^2*B + 3*b^2*B)*Tan[c + d*x])
/d + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c
 + d*x]*Tan[c + d*x])/(2*d)))/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 8.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

method	result
parts	$\frac{a^2 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{(A\,b^2+2Bab)\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\right.}{d}$
derivativedivides	$\frac{a^2 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^2 B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-2A}{}$
default	$\frac{a^2 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^2 B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-2A}{}$
parallelrisch	$\frac{-36\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^2A+\frac{4}{3}A\,b^2+\frac{8}{3}Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+36\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)(}{}$
norman	$\frac{\left(7a^2A-4A\,b^2-8Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{\left(27a^2A-32Aab+12A\,b^2-16a^2B+24Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{6d}+\frac{\left(27a^2A+32Aab+12A\,b^2+16a^2\right.}{6d}$
risch	$-\frac{i\left(9A\,a^2\mathrm{e}^{7i(dx+c)}+12A\,b^2\mathrm{e}^{7i(dx+c)}+24Bab\,\mathrm{e}^{7i(dx+c)}-24B\,b^2\mathrm{e}^{6i(dx+c)}+33A\,a^2\mathrm{e}^{5i(dx+c)}+12A\,b^2\mathrm{e}^{5i(dx+c)}+24Bab\right.}{}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBO
SE)
```

output
```
a^2*A/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+(A*b^2+2*B*a*b)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c
)+tan(d*x+c)))-(2*A*a*b+B*a^2)/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*b^2/
d*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.15

$$\int (a + b\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^5(c + dx) \, dx$$
$$= \frac{3\,(3\,Aa^2 + 8\,Bab + 4\,Ab^2)\cos{(dx + c)}^4 \log{(\sin{(dx + c)} + 1)} - 3\,(3\,Aa^2 + 8\,Bab + 4\,Ab^2)\cos{(dx + c)}^4 \mathrm{l}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="f
ricas")
```

output
```
1/48*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
 - 3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) +
 2*(8*(2*B*a^2 + 4*A*a*b + 3*B*b^2)*cos(d*x + c)^3 + 6*A*a^2 + 3*(3*A*a^2
+ 8*B*a*b + 4*A*b^2)*cos(d*x + c)^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^5(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.46

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2+32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aab-3Aa^2\left(\frac{2\left(3\sin(dx+c)^3\right.}{\sin(dx+c)^4-2}\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="m
axima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 32*(tan(d*x + c)^3 + 3*
tan(d*x + c))*A*a*b - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 24*B*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*b^2*tan
(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(146) = 292.

Time = 0.18 (sec) , antiderivative size = 478, normalized size of antiderivative = 3.06

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="g
iac")
```

output
```
1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(1
5*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*
b*tan(1/2*d*x + 1/2*c)^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(
1/2*d*x + 1/2*c)^7 - 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^2*tan(1/2*d*x
 + 1/2*c)^5 + 40*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 80*A*a*b*tan(1/2*d*x + 1/2
*c)^5 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^5
+ 72*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*
a^2*tan(1/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*ta
n(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*
d*x + 1/2*c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 24*B*a^2*tan(1/2*d*x + 1/
2*c) + 48*A*a*b*tan(1/2*d*x + 1/2*c) + 24*B*a*b*tan(1/2*d*x + 1/2*c) + 12*
A*b^2*tan(1/2*d*x + 1/2*c) + 24*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 45.42 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.01

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,A\,a^2}{8}+B\,a\,b+\frac{A\,b^2}{2}\right)}{\frac{3\,A\,a^2}{2}+4\,B\,a\,b+2\,A\,b^2}\right)\left(\frac{3\,A\,a^2}{4}+2\,B\,a\,b+A\,b^2\right)}{d}$$
$$+ \frac{\left(\frac{5\,A\,a^2}{4}+A\,b^2-2\,B\,a^2-2\,B\,b^2-4\,A\,a\,b+2\,B\,a\,b\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,A\,a^2}{4}-A\,b^2+\frac{10\,B\,a^2}{3}+6\,B\,b^2+\cdots\right.}{d\,\left(\mathrm{ta}\cdots\right.}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^5,x)
```

output
```
(atanh((4*tan(c/2 + (d*x)/2)*((3*A*a^2)/8 + (A*b^2)/2 + B*a*b))/((3*A*a^2)
/2 + 2*A*b^2 + 4*B*a*b))*((3*A*a^2)/4 + A*b^2 + 2*B*a*b))/d + (tan(c/2 + (
d*x)/2)^7*((5*A*a^2)/4 + A*b^2 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + 2*B*a*b) -
tan(c/2 + (d*x)/2)^3*(A*b^2 - (3*A*a^2)/4 + (10*B*a^2)/3 + 6*B*b^2 + (20*A
*a*b)/3 + 2*B*a*b) + tan(c/2 + (d*x)/2)^5*((3*A*a^2)/4 - A*b^2 + (10*B*a^2
)/3 + 6*B*b^2 + (20*A*a*b)/3 - 2*B*a*b) + tan(c/2 + (d*x)/2)*((5*A*a^2)/4
+ A*b^2 + 2*B*a^2 + 2*B*b^2 + 4*A*a*b + 2*B*a*b))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^
8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 533, normalized size of antiderivative = 3.42

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3 - 12*cos
(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**2 + 6*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 + 24*cos(c + d*x)*log(tan
((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**2 - 3*cos(c + d*x)*log(tan((c + d*
x)/2) - 1)*a**3 - 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2 + 3*cos
(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**3 + 12*cos(c + d*x)
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*b**2 - 6*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3 - 24*cos(c + d*x)*log(tan((c + d*x
)/2) + 1)*sin(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1
)*a**3 + 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**2 - 3*cos(c + d*x)
*sin(c + d*x)**3*a**3 - 12*cos(c + d*x)*sin(c + d*x)**3*a*b**2 + 5*cos(c +
 d*x)*sin(c + d*x)*a**3 + 12*cos(c + d*x)*sin(c + d*x)*a*b**2 + 16*sin(c +
 d*x)**5*a**2*b + 8*sin(c + d*x)**5*b**3 - 40*sin(c + d*x)**3*a**2*b - 16*
sin(c + d*x)**3*b**3 + 24*sin(c + d*x)*a**2*b + 8*sin(c + d*x)*b**3)/(8*co
s(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.231 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 269

$$\begin{aligned}
&\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx \\
&= \frac{1}{16}\left(8a^3A+18aAb^2+18a^2bB+5b^3B\right)x \\
&\quad+\frac{(15a^2Ab+4Ab^3+5a^3B+12ab^2B)\sin(c+dx)}{5d} \\
&\quad+\frac{(8a^3A+18aAb^2+18a^2bB+5b^3B)\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad+\frac{b(18aAb+14a^2B+5b^2B)\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad+\frac{b^2(3Ab+4aB)\cos^4(c+dx)\sin(c+dx)}{15d} \\
&\quad+\frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{6d} \\
&\quad-\frac{(15a^2Ab+4Ab^3+5a^3B+12ab^2B)\sin^3(c+dx)}{15d}
\end{aligned}$$

output
```
1/16*(8*A*a^3+18*A*a*b^2+18*B*a^2*b+5*B*b^3)*x+1/5*(15*A*a^2*b+4*A*b^3+5*B
*a^3+12*B*a*b^2)*sin(d*x+c)/d+1/16*(8*A*a^3+18*A*a*b^2+18*B*a^2*b+5*B*b^3)
*cos(d*x+c)*sin(d*x+c)/d+1/24*b*(18*A*a*b+14*B*a^2+5*B*b^2)*cos(d*x+c)^3*s
in(d*x+c)/d+1/15*b^2*(3*A*b+4*B*a)*cos(d*x+c)^4*sin(d*x+c)/d+1/6*b*B*cos(d
*x+c)^3*(a+b*cos(d*x+c))^2*sin(d*x+c)/d-1/15*(15*A*a^2*b+4*A*b^3+5*B*a^3+1
2*B*a*b^2)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{480a^3Ac+1080aAb^2c+1080a^2bBc+300b^3Bc+480a^3Adx+1080aAb^2dx+1080a^2bBdx+300b^3Bdx}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(480*a^3*A*c + 1080*a*A*b^2*c + 1080*a^2*b*B*c + 300*b^3*B*c + 480*a^3*A*d
*x + 1080*a*A*b^2*d*x + 1080*a^2*b*B*d*x + 300*b^3*B*d*x + 120*(18*a^2*A*b
 + 5*A*b^3 + 6*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 15*(16*a^3*A + 48*a*A*b^
2 + 48*a^2*b*B + 15*b^3*B)*Sin[2*(c + d*x)] + 240*a^2*A*b*Sin[3*(c + d*x)]
 + 100*A*b^3*Sin[3*(c + d*x)] + 80*a^3*B*Sin[3*(c + d*x)] + 300*a*b^2*B*Si
n[3*(c + d*x)] + 90*a*A*b^2*Sin[4*(c + d*x)] + 90*a^2*b*B*Sin[4*(c + d*x)]
 + 45*b^3*B*Sin[4*(c + d*x)] + 12*A*b^3*Sin[5*(c + d*x)] + 36*a*b^2*B*Sin[
5*(c + d*x)] + 5*b^3*B*Sin[6*(c + d*x)])/(960*d)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.86, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3469, 3042, 3512, 3042, 3502, 27, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{6}\int \cos^2(c+dx)(a+b\cos(c+dx))\left(2b(3Ab+4aB)\cos^2(c+dx)+\left(5Bb^2+6a(2Ab+aB)\right)\cos(c+dx)+3a(2aA+bB)\right)dx+\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(2b(3Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Bb^2+6a(2Ab+aB)\right)\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3512}$$

$$\frac{1}{6}\left(\frac{1}{5}\int \cos^2(c+dx)\left(15(2aA+bB)a^2+5b\left(14Ba^2+18Aba+5b^2B\right)\cos^2(c+dx)+6\left(5Ba^3+15Aba^2+12b^2B\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(15(2aA+bB)a^2+5b\left(14Ba^2+18Aba+5b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+6\left(5Ba^3+15A\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\int 3\cos^2(c+dx)\left(5\left(8Aa^3+18bBa^2+18Ab^2a+5b^3B\right)+8\left(5Ba^3+15Aba^2+12b^2Ba+4Ab^3\right)\cos(c+\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int \cos^2(c+dx)\left(5\left(8Aa^3+18bBa^2+18Ab^2a+5b^3B\right)+8\left(5Ba^3+15Aba^2+12b^2Ba+4Ab^3\right)\cos(c+d\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(5\left(8Aa^3+18bBa^2+18Ab^2a+5b^3B\right)+8\left(5Ba^3+15Aba^2+12b^2Ba+4Ab^3\right)\sin\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$\downarrow$ 3227

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(8\left(5a^3B+15a^2Ab+12ab^2B+4Ab^3\right)\int \cos^3(c+dx)dx+5\left(8a^3A+18a^2bB+18aAb^2+5b^3B\right)\int \cos\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(5\left(8a^3A+18a^2bB+18aAb^2+5b^3B\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+8\left(5a^3B+15a^2Ab+12ab^2B+4Ab^3\right)\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$\downarrow$ 3113

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(5\left(8a^3A+18a^2bB+18aAb^2+5b^3B\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8\left(5a^3B+15a^2Ab+12ab^2B+4Ab^3\right)}{}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$\downarrow$ 2009

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(5\left(8a^3A+18a^2bB+18aAb^2+5b^3B\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8\left(5a^3B+15a^2Ab+12ab^2B+4Ab^3\right)}{d}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$\downarrow$ 3115

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(5\left(8a^3A+18a^2bB+18aAb^2+5b^3B\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{8\left(5a^3B+15a^2Ab+12a\right.}{}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{5b\left(14a^2B+18aAb+5b^2B\right)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(5\left(8a^3A+18a^2bB+18aAb^2+5b^3B\right)\left(\frac{\sin(c\right.\right.\right.\right.}{}$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^2}{6d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(b*B*Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + ((2*b^2*(
3*A*b + 4*a*B)*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + ((5*b*(18*a*A*b + 14*a
^2*B + 5*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(5*(8*a^3*A + 18*a
*A*b^2 + 18*a^2*b*B + 5*b^3*B)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) -
 (8*(15*a^2*A*b + 4*A*b^3 + 5*a^3*B + 12*a*b^2*B)*(-Sin[c + d*x] + Sin[c +
 d*x]^3/3))/d))/4)/5)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 147.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.79

method	result
parts	$\frac{(A b^3+3Ba b^2)\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}+\frac{(3Aa b^2+3B a^2 b)\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c}{4}\right.}{d}$
parallelrisch	$(240a^3A+720Aa b^2+720B a^2b+225b^3B)\sin(2dx+2c)+(240A a^2b+100A b^3+80a^3B+300Ba b^2)\sin(3dx+3c)+(90A$
derivativedivides	$a^3A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3B\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+A a^2b\left(\cos(dx+c)^2+2\right)\sin(dx+c)+3B a^2b\left(\left(\cos(d\right.\right.$
default	$a^3A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3B\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+A a^2b\left(\cos(dx+c)^2+2\right)\sin(dx+c)+3B a^2b\left(\left(\cos(d\right.\right.$
risch	$\frac{a^3Ax}{2}+\frac{9xAa b^2}{8}+\frac{9xB a^2b}{8}+\frac{5b^3Bx}{16}+\frac{9\sin(dx+c)A a^2b}{4d}+\frac{5\sin(dx+c)A b^3}{8d}+\frac{3a^3B\sin(dx+c)}{4d}+\frac{15\sin(}{}$
norman	$(\frac{1}{2}a^3A+\frac{9}{8}Aa b^2+\frac{9}{8}B a^2b+\frac{5}{16}b^3B)x+(3a^3A+\frac{27}{4}Aa b^2+\frac{27}{4}B a^2b+\frac{15}{8}b^3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(3a^3A+\frac{27}{4}Aa b^2+\frac{27}{4}B a^2$
orering	Expression too large to display

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/5*(A*b^3+3*B*a*b^2)/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+(3*
A*a*b^2+3*B*a^2*b)/d*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x
+3/8*c)+1/3*(3*A*a^2*b+B*a^3)/d*(cos(d*x+c)^2+2)*sin(d*x+c)+a^3*A/d*(1/2*c
os(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^3*B/d*(1/6*(cos(d*x+c)^5+5/4*cos(d*x
+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{15\,(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3)dx+\left(40\,Bb^3\cos\left(dx+c\right)^5+48\,(3\,Bab^2+Ab^3)\cos\left(dx+c\right)^4+\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/240*(15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*d*x + (40*B*b^3*co
s(d*x + c)^5 + 48*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^4 + 160*B*a^3 + 480*A*a
^2*b + 384*B*a*b^2 + 128*A*b^3 + 10*(18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*co
s(d*x + c)^3 + 16*(5*B*a^3 + 15*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x +
c)^2 + 15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(277) = 554$.

Time = 0.45 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.68

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**3*x*sin(c + d*x)**2/2 + A*a**3*x*cos(c + d*x)**2/2 + A*a**
3*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*b*sin(c + d*x)**3/d + 3*A*a**
2*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a*b**2*x*sin(c + d*x)**4/8 + 9*A*
a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*A*a*b**2*x*cos(c + d*x)**4/
8 + 9*A*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*A*a*b**2*sin(c + d*
x)*cos(c + d*x)**3/(8*d) + 8*A*b**3*sin(c + d*x)**5/(15*d) + 4*A*b**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x)**4/d
+ 2*B*a**3*sin(c + d*x)**3/(3*d) + B*a**3*sin(c + d*x)*cos(c + d*x)**2/d +
 9*B*a**2*b*x*sin(c + d*x)**4/8 + 9*B*a**2*b*x*sin(c + d*x)**2*cos(c + d*x
)**2/4 + 9*B*a**2*b*x*cos(c + d*x)**4/8 + 9*B*a**2*b*sin(c + d*x)**3*cos(c
 + d*x)/(8*d) + 15*B*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*a*b**
2*sin(c + d*x)**5/(5*d) + 4*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3
*B*a*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*b**3*x*sin(c + d*x)**6/16 +
 15*B*b**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**3*x*sin(c + d*x)
**2*cos(c + d*x)**4/16 + 5*B*b**3*x*cos(c + d*x)**6/16 + 5*B*b**3*sin(c +
d*x)**5*cos(c + d*x)/(16*d) + 5*B*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*
d) + 11*B*b**3*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*c
os(c))*(a + b*cos(c))**3*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3 - 320\,\left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)Ba^3 - 960\,\left(\sin(dx+c)^3 - 3\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 -
3*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b + 90
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b + 90*(12*
d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^2 + 192*(3*sin(d
*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a*b^2 + 64*(3*sin(d*x +
 c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*b^3 - 5*(4*sin(2*d*x + 2*c)
^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.86

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{Bb^3\sin(6\,dx+6\,c)}{192\,d} + \frac{1}{16}\left(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3\right)x + \frac{(3\,Bab^2+Ab^3)\sin(5\,dx+5\,c)}{80\,d} + \frac{3\,(2\,Ba^2b+2\,Aab^2+Bb^3)\sin(4\,dx+4\,c)}{64\,d} + \frac{(4\,Ba^3+12\,Aa^2b+15\,Bab^2+5\,Ab^3)\sin(3\,dx+3\,c)}{48\,d} + \frac{(16\,Aa^3+48\,Ba^2b+48\,Aab^2+15\,Bb^3)\sin(2\,dx+2\,c)}{64\,d} + \frac{(6\,Ba^3+18\,Aa^2b+15\,Bab^2+5\,Ab^3)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/192*B*b^3*sin(6*d*x + 6*c)/d + 1/16*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 +
 5*B*b^3)*x + 1/80*(3*B*a*b^2 + A*b^3)*sin(5*d*x + 5*c)/d + 3/64*(2*B*a^2*
b + 2*A*a*b^2 + B*b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^3 + 12*A*a^2*b + 1
5*B*a*b^2 + 5*A*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*A*a^3 + 48*B*a^2*b + 48
*A*a*b^2 + 15*B*b^3)*sin(2*d*x + 2*c)/d + 1/8*(6*B*a^3 + 18*A*a^2*b + 15*B
*a*b^2 + 5*A*b^3)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.94 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.31

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{A a^3 x}{2} + \frac{5 B b^3 x}{16} + \frac{9 A a b^2 x}{8} + \frac{9 B a^2 b x}{8} + \frac{5 A b^3 \sin(c+dx)}{8 d} + \frac{3 B a^3 \sin(c+dx)}{4 d} + \frac{A a^3 \sin(2c+2dx)}{4 d} + \frac{5 A b^3 \sin(3c+3dx)}{48 d} + \frac{B a^3 \sin(3c+3dx)}{12 d} + \frac{A b^3 \sin(5c+5dx)}{80 d} + \frac{15 B b^3 \sin(2c+2dx)}{64 d} + \frac{3 B b^3 \sin(4c+4dx)}{64 d} + \frac{B b^3 \sin(6c+6dx)}{192 d} + \frac{3 A a b^2 \sin(2c+2dx)}{4 d} + \frac{A a^2 b \sin(3c+3dx)}{4 d} + \frac{3 A a b^2 \sin(4c+4dx)}{32 d} + \frac{3 B a^2 b \sin(2c+2dx)}{4 d} + \frac{5 B a b^2 \sin(3c+3dx)}{16 d} + \frac{3 B a^2 b \sin(4c+4dx)}{32 d} + \frac{3 B a b^2 \sin(5c+5dx)}{80 d} + \frac{9 A a^2 b \sin(c+dx)}{4 d} + \frac{15 B a b^2 \sin(c+dx)}{8 d}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

output
```
(A*a^3*x)/2 + (5*B*b^3*x)/16 + (9*A*a*b^2*x)/8 + (9*B*a^2*b*x)/8 + (5*A*b^
3*sin(c + d*x))/(8*d) + (3*B*a^3*sin(c + d*x))/(4*d) + (A*a^3*sin(2*c + 2*
d*x))/(4*d) + (5*A*b^3*sin(3*c + 3*d*x))/(48*d) + (B*a^3*sin(3*c + 3*d*x))
/(12*d) + (A*b^3*sin(5*c + 5*d*x))/(80*d) + (15*B*b^3*sin(2*c + 2*d*x))/(6
4*d) + (3*B*b^3*sin(4*c + 4*d*x))/(64*d) + (B*b^3*sin(6*c + 6*d*x))/(192*d
) + (3*A*a*b^2*sin(2*c + 2*d*x))/(4*d) + (A*a^2*b*sin(3*c + 3*d*x))/(4*d)
+ (3*A*a*b^2*sin(4*c + 4*d*x))/(32*d) + (3*B*a^2*b*sin(2*c + 2*d*x))/(4*d)
 + (5*B*a*b^2*sin(3*c + 3*d*x))/(16*d) + (3*B*a^2*b*sin(4*c + 4*d*x))/(32*
d) + (3*B*a*b^2*sin(5*c + 5*d*x))/(80*d) + (9*A*a^2*b*sin(c + d*x))/(4*d)
+ (15*B*a*b^2*sin(c + d*x))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{40\cos(dx+c)\sin(dx+c)^5 b^4 - 360\cos(dx+c)\sin(dx+c)^3 a^2b^2 - 130\cos(dx+c)\sin(dx+c)^3 b^4 +}{}$$

input
```
int(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
(40*cos(c + d*x)*sin(c + d*x)**5*b**4 - 360*cos(c + d*x)*sin(c + d*x)**3*a
**2*b**2 - 130*cos(c + d*x)*sin(c + d*x)**3*b**4 + 120*cos(c + d*x)*sin(c
+ d*x)*a**4 + 900*cos(c + d*x)*sin(c + d*x)*a**2*b**2 + 165*cos(c + d*x)*s
in(c + d*x)*b**4 + 192*sin(c + d*x)**5*a*b**3 - 320*sin(c + d*x)**3*a**3*b
 - 640*sin(c + d*x)**3*a*b**3 + 960*sin(c + d*x)*a**3*b + 960*sin(c + d*x)
*a*b**3 + 120*a**4*d*x + 540*a**2*b**2*d*x + 75*b**4*d*x)/(240*d)
```

3.232 $\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 243

$$\begin{aligned}&\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx\\&=\frac{1}{8}\left(12a^2Ab+3Ab^3+4a^3B+9ab^2B\right)x\\&+\frac{(15a^3Ab+60aAb^3-3a^4B+52a^2b^2B+16b^4B)\sin(c+dx)}{30bd}\\&+\frac{(30a^2Ab+45Ab^3-6a^3B+71ab^2B)\cos(c+dx)\sin(c+dx)}{120d}\\&+\frac{(15aAb-3a^2B+16b^2B)\,(a+b\cos(c+dx))^2\sin(c+dx)}{60bd}\\&+\frac{(5Ab-aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20bd}+\frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5bd}\end{aligned}$$

output
```
1/8*(12*A*a^2*b+3*A*b^3+4*B*a^3+9*B*a*b^2)*x+1/30*(15*A*a^3*b+60*A*a*b^3-3
*B*a^4+52*B*a^2*b^2+16*B*b^4)*sin(d*x+c)/b/d+1/120*(30*A*a^2*b+45*A*b^3-6*
B*a^3+71*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/60*(15*A*a*b-3*B*a^2+16*B*b^2)
*(a+b*cos(d*x+c))^2*sin(d*x+c)/b/d+1/20*(5*A*b-B*a)*(a+b*cos(d*x+c))^3*sin
(d*x+c)/b/d+1/5*B*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.72

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{60(12a^2Ab+3Ab^3+4a^3B+9ab^2B)(c+dx)+60(8a^3A+18aAb^2+18a^2bB+5b^3B)\sin(c+dx)+12\ldots}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(60*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*(c + d*x) + 60*(8*a^3*A +
 18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*Sin[c + d*x] + 120*(3*a^2*A*b + A*b^3
+ a^3*B + 3*a*b^2*B)*Sin[2*(c + d*x)] + 10*b*(12*a*A*b + 12*a^2*B + 5*b^2*
B)*Sin[3*(c + d*x)] + 15*b^2*(A*b + 3*a*B)*Sin[4*(c + d*x)] + 6*b^3*B*Sin[
5*(c + d*x)])/(480*d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 3447, 3042, 3502, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a+b\cos(c+dx))^3\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$\downarrow$ 3502

$$\frac{\int (a+b\cos(c+dx))^3(4bB+(5Ab-aB)\cos(c+dx))dx}{5b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$\downarrow$ 3042

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(4bB+(5Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$\downarrow$ 3232

$$\frac{\frac{1}{4}\int (a+b\cos(c+dx))^2\left(b(15Ab+13aB)+\left(-3Ba^2+15Aba+16b^2B\right)\cos(c+dx)\right)dx+\frac{(5Ab-aB)\sin(c+dx)(a+b\cos}{4d}}{5b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(b(15Ab+13aB)+\left(-3Ba^2+15Aba+16b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{(5Ab-aB)\sin}{}}{5b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$\downarrow$ 3232

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(b\left(33Ba^2+75Aba+32b^2B\right)+\left(-6Ba^3+30Aba^2+71b^2Ba+45Ab^3\right)\cos(c+dx)\right)dx\right.}{5b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(33Ba^2+75Aba+32b^2B\right)+\left(-6Ba^3+30Aba^2+71b^2Ba+45Ab^3\right)\sin\left(c+d\right.\right.\right.}{5b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$\downarrow$ 3213

$$\frac{\frac{1}{4}\left(\frac{(-3a^2B+15aAb+16b^2B)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{b(-6a^3B+30a^2Ab+71ab^2B+45Ab^3)\sin(c+dx)\cos(c+dx)}{2d}+\frac{15}{2}bx\left(4a^3B\right.\right.\right.}{5}$$

$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d) + (((5*A*b - a*B)*(a + b*C
os[c + d*x])^3*Sin[c + d*x])/(4*d) + (((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a
+ b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((15*b*(12*a^2*A*b + 3*A*b^3 + 4
*a^3*B + 9*a*b^2*B)*x)/2 + (2*(15*a^3*A*b + 60*a*A*b^3 - 3*a^4*B + 52*a^2*
b^2*B + 16*b^4*B)*Sin[c + d*x])/d + (b*(30*a^2*A*b + 45*A*b^3 - 6*a^3*B +
71*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3)/4)/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 42.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.72

method	result
parts	$\frac{(A\,b^3+3B a\,b^2)\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{(3Aa\,b^2+3B\,a^2b)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}$
parallelrisch	$\frac{120(3A\,a^2b+A\,b^3+a^3B+3Ba\,b^2)\sin(2dx+2c)+10(12Aa\,b^2+12B\,a^2b+5b^3B)\sin(3dx+3c)+15(A\,b^3+3Ba\,b^2)\sin(4dx}{480}$
derivativedivides	$\frac{a^3A\sin(dx+c)+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+B\,a^2b\left(\cos(dx+c)^2+2\right)\sin}{}$
default	$\frac{a^3A\sin(dx+c)+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+B\,a^2b\left(\cos(dx+c)^2+2\right)\sin}{}$
risch	$\frac{3xA\,a^2b}{2}+\frac{3xA\,b^3}{8}+\frac{a^3Bx}{2}+\frac{9xBa\,b^2}{8}+\frac{a^3A\sin(dx+c)}{d}+\frac{9\sin(dx+c)Aa\,b^2}{4d}+\frac{9\sin(dx+c)B\,a^2b}{4d}+\frac{5\sin(}{}$
norman	$\frac{(\frac{3}{2}A\,a^2b+\frac{3}{8}A\,b^3+\frac{1}{2}a^3B+\frac{9}{8}Ba\,b^2)x+(15A\,a^2b+\frac{15}{4}A\,b^3+5a^3B+\frac{45}{4}Ba\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(15A\,a^2b+\frac{15}{4}A\,b^3+5a^3B}{}$
orering	Expression too large to display

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
(A*b^3+3*B*a*b^2)/d*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+1/3*(3*A*a*b^2+3*B*a^2*b)/d*(cos(d*x+c)^2+2)*sin(d*x+c)+(3*A*a^2*b+
B*a^3)/d*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*A*sin(d*x+c)/d+1/5*
b^3*B/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.72

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{15\,(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3)dx+\left(24\,Bb^3\cos\left(dx+c\right)^4+120\,Aa^3+240\,Ba^2b+240\,Aab^2+6\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/120*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*d*x + (24*B*b^3*cos
(d*x + c)^4 + 120*A*a^3 + 240*B*a^2*b + 240*A*a*b^2 + 64*B*b^3 + 30*(3*B*a
*b^2 + A*b^3)*cos(d*x + c)^3 + 8*(15*B*a^2*b + 15*A*a*b^2 + 4*B*b^3)*cos(d
*x + c)^2 + 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c))*
sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(241) = 482$.

Time = 0.31 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.27

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{Aa^3\sin(c+dx)}{d} + \frac{3Aa^2bx\sin^2(c+dx)}{2} + \frac{3Aa^2bx\cos^2(c+dx)}{2} + \frac{3Aa^2b\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Aab^2\sin^3(c+dx)}{d} + \frac{3Aab^2\sin(c+d}{c} \\ x(A+B\cos(c))\,(a+b\cos(c))^3\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**3*sin(c + d*x)/d + 3*A*a**2*b*x*sin(c + d*x)**2/2 + 3*A*a*
*2*b*x*cos(c + d*x)**2/2 + 3*A*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*
A*a*b**2*sin(c + d*x)**3/d + 3*A*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3
*A*b**3*x*sin(c + d*x)**4/8 + 3*A*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4
 + 3*A*b**3*x*cos(c + d*x)**4/8 + 3*A*b**3*sin(c + d*x)**3*cos(c + d*x)/(8
*d) + 5*A*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*x*sin(c + d*x)*
*2/2 + B*a**3*x*cos(c + d*x)**2/2 + B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d)
 + 2*B*a**2*b*sin(c + d*x)**3/d + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)**2/
d + 9*B*a*b**2*x*sin(c + d*x)**4/8 + 9*B*a*b**2*x*sin(c + d*x)**2*cos(c +
d*x)**2/4 + 9*B*a*b**2*x*cos(c + d*x)**4/8 + 9*B*a*b**2*sin(c + d*x)**3*co
s(c + d*x)/(8*d) + 15*B*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*b*
*3*sin(c + d*x)**5/(15*d) + 4*B*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d)
 + B*b**3*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a
+ b*cos(c))**3*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3+360\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b-480\,\left(\sin(dx+c)^3-3\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin
(2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b - 4
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^2 + 45*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^2 + 15*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*A*b^3 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^
3 + 15*sin(d*x + c))*B*b^3 + 480*A*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{Bb^3\sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\left(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3\right)x + \frac{(3\,Bab^2+Ab^3)\sin(4\,dx+4\,c)}{32\,d} + \frac{(12\,Ba^2b+12\,Aab^2+5\,Bb^3)\sin(3\,dx+3\,c)}{48\,d} + \frac{(Ba^3+3\,Aa^2b+3\,Bab^2+Ab^3)\sin(2\,dx+2\,c)}{4\,d} + \frac{(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/80*B*b^3*sin(5*d*x + 5*c)/d + 1/8*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*
A*b^3)*x + 1/32*(3*B*a*b^2 + A*b^3)*sin(4*d*x + 4*c)/d + 1/48*(12*B*a^2*b
+ 12*A*a*b^2 + 5*B*b^3)*sin(3*d*x + 3*c)/d + 1/4*(B*a^3 + 3*A*a^2*b + 3*B*
a*b^2 + A*b^3)*sin(2*d*x + 2*c)/d + 1/8*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2
 + 5*B*b^3)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.56 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.14

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{3\,A\,b^3\,x}{8} + \frac{B\,a^3\,x}{2} + \frac{3\,A\,a^2\,b\,x}{2} + \frac{9\,B\,a\,b^2\,x}{8} + \frac{A\,a^3\,\sin(c+d\,x)}{d} + \frac{5\,B\,b^3\,\sin(c+d\,x)}{8\,d} + \frac{A\,b^3\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{B\,a^3\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{A\,b^3\,\sin(4\,c+4\,d\,x)}{32\,d} + \frac{5\,B\,b^3\,\sin(3\,c+3\,d\,x)}{48\,d} + \frac{B\,b^3\,\sin(5\,c+5\,d\,x)}{80\,d} + \frac{3\,A\,a^2\,b\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{A\,a\,b^2\,\sin(3\,c+3\,d\,x)}{4\,d} + \frac{3\,B\,a\,b^2\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{B\,a^2\,b\,\sin(3\,c+3\,d\,x)}{4\,d} + \frac{3\,B\,a\,b^2\,\sin(4\,c+4\,d\,x)}{32\,d} + \frac{9\,A\,a\,b^2\,\sin(c+d\,x)}{4\,d} + \frac{9\,B\,a^2\,b\,\sin(c+d\,x)}{4\,d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

output
```
(3*A*b^3*x)/8 + (B*a^3*x)/2 + (3*A*a^2*b*x)/2 + (9*B*a*b^2*x)/8 + (A*a^3*s
in(c + d*x))/d + (5*B*b^3*sin(c + d*x))/(8*d) + (A*b^3*sin(2*c + 2*d*x))/(
4*d) + (B*a^3*sin(2*c + 2*d*x))/(4*d) + (A*b^3*sin(4*c + 4*d*x))/(32*d) +
(5*B*b^3*sin(3*c + 3*d*x))/(48*d) + (B*b^3*sin(5*c + 5*d*x))/(80*d) + (3*A
*a^2*b*sin(2*c + 2*d*x))/(4*d) + (A*a*b^2*sin(3*c + 3*d*x))/(4*d) + (3*B*a
*b^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*b*sin(3*c + 3*d*x))/(4*d) + (3*B*a*b
^2*sin(4*c + 4*d*x))/(32*d) + (9*A*a*b^2*sin(c + d*x))/(4*d) + (9*B*a^2*b*
sin(c + d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{-30\cos(dx+c)\sin(dx+c)^3\,a\,b^3+60\cos(dx+c)\sin(dx+c)\,a^3b+75\cos(dx+c)\sin(dx+c)\,a\,b^3+6}{}$$

input
```
int(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
( - 30*cos(c + d*x)*sin(c + d*x)**3*a*b**3 + 60*cos(c + d*x)*sin(c + d*x)*
a**3*b + 75*cos(c + d*x)*sin(c + d*x)*a*b**3 + 6*sin(c + d*x)**5*b**4 - 60
*sin(c + d*x)**3*a**2*b**2 - 20*sin(c + d*x)**3*b**4 + 30*sin(c + d*x)*a**
4 + 180*sin(c + d*x)*a**2*b**2 + 30*sin(c + d*x)*b**4 + 60*a**3*b*d*x + 45
*a*b**3*d*x)/(30*d)
```

3.233 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$\begin{aligned}\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx &= \frac{1}{8}\left(8a^3A+12aAb^2+12a^2bB+3b^3B\right)x \\ &+ \frac{(16a^2Ab+4Ab^3+3a^3B+12ab^2B)\sin(c+dx)}{6d} \\ &+ \frac{b(20aAb+6a^2B+9b^2B)\cos(c+dx)\sin(c+dx)}{24d} \\ &+ \frac{(4Ab+3aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4d}\end{aligned}$$

output
```
1/8*(8*A*a^3+12*A*a*b^2+12*B*a^2*b+3*B*b^3)*x+1/6*(16*A*a^2*b+4*A*b^3+3*B*
a^3+12*B*a*b^2)*sin(d*x+c)/d+1/24*b*(20*A*a*b+6*B*a^2+9*B*b^2)*cos(d*x+c)*
sin(d*x+c)/d+1/12*(4*A*b+3*B*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*B*(a+b
*cos(d*x+c))^3*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{12(8a^3A+12aAb^2+12a^2bB+3b^3B)(c+dx)+24(12a^2Ab+3Ab^3+4a^3B+9ab^2B)\sin(c+dx)+24}{96d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(12*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*(c + d*x) + 24*(12*a^2*A
*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x] + 24*b*(3*a*A*b + 3*a^2*B
 + b^2*B)*Sin[2*(c + d*x)] + 8*b^2*(A*b + 3*a*B)*Sin[3*(c + d*x)] + 3*b^3*
B*Sin[4*(c + d*x)])/(96*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{4}\int (a+b\cos(c+dx))^2(4aA+3bB+(4Ab+3aB)\cos(c+dx))dx+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4aA+3bB+(4Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3232

$$\frac{1}{4}\left(\frac{1}{3}\int(a+b\cos(c+dx))\left(12Aa^2+15bBa+8Ab^2+\left(6Ba^2+20Aba+9b^2B\right)\cos(c+dx)\right)dx+\frac{(3aB+4Ab)\text{s}}{}\right.$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(12Aa^2+15bBa+8Ab^2+\left(6Ba^2+20Aba+9b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\right.$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3213

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{b\left(6a^2B+20aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{2\left(3a^3B+16a^2Ab+12ab^2B+4Ab^3\right)\sin(c+dx)}{d}+\right.\right.$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (((4*A*b + 3*a*B)*(a + b*C
os[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*
B + 3*b^3*B)*x)/2 + (2*(16*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sin[c
 + d*x])/d + (b*(20*a*A*b + 6*a^2*B + 9*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/
(2*d))/3)/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05

$$a^3A(dx+c)+a^3B\sin(dx+c)+3A\sin(dx+c)\,a^2b+3B\,a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3Aa\,b^2\left(\frac{\cos(d\right.$$

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(a^3*A*(d*x+c)+a^3*B*sin(d*x+c)+3*A*sin(d*x+c)*a^2*b+3*B*a^2*b*(1/2*co
s(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*A*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/
2*d*x+1/2*c)+B*a*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+1/3*A*b^3*(cos(d*x+c)^2+2
)*sin(d*x+c)+b^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3
/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{3\,(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3)dx+\left(6\,Bb^3\cos\left(dx+c\right)^3+24\,Ba^3+72\,Aa^2b+48\,Bab^2+16\,Ab\right.}{24\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*d*x + (6*B*b^3*cos(d
*x + c)^3 + 24*B*a^3 + 72*A*a^2*b + 48*B*a*b^2 + 16*A*b^3 + 8*(3*B*a*b^2 +
 A*b^3)*cos(d*x + c)^2 + 9*(4*B*a^2*b + 4*A*a*b^2 + B*b^3)*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(170) = 340.

Time = 0.21 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.26

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} Aa^3x + \frac{3Aa^2b\sin(c+dx)}{d} + \frac{3Aab^2x\sin^2(c+dx)}{2} + \frac{3Aab^2x\cos^2(c+dx)}{2} + \frac{3Aab^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Ab^3\sin^3(c+dx)}{3d} + \frac{Ab}{} \\ x(A+B\cos(c))\,(a+b\cos(c))^3 \end{cases}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**3*x + 3*A*a**2*b*sin(c + d*x)/d + 3*A*a*b**2*x*sin(c + d*x
)**2/2 + 3*A*a*b**2*x*cos(c + d*x)**2/2 + 3*A*a*b**2*sin(c + d*x)*cos(c +
d*x)/(2*d) + 2*A*b**3*sin(c + d*x)**3/(3*d) + A*b**3*sin(c + d*x)*cos(c +
d*x)**2/d + B*a**3*sin(c + d*x)/d + 3*B*a**2*b*x*sin(c + d*x)**2/2 + 3*B*a
**2*b*x*cos(c + d*x)**2/2 + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2
*B*a*b**2*sin(c + d*x)**3/d + 3*B*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d +
3*B*b**3*x*sin(c + d*x)**4/8 + 3*B*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*B*b**3*x*cos(c + d*x)**4/8 + 3*B*b**3*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 5*B*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*c
os(c))*(a + b*cos(c))**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{96\,(dx+c)Aa^3+72\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Ba^2b+72\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Aab^2-96\,(\sin}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/96*(96*(d*x + c)*A*a^3 + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b + 7
2*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^2 - 96*(sin(d*x + c)^3 - 3*sin(d*
x + c))*B*a*b^2 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^3 + 3*(12*d*x +
 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b^3 + 96*B*a^3*sin(d*x +
c) + 288*A*a^2*b*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{Bb^3\sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\left(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3\right)x + \frac{(3\,Bab^2+Ab^3)\sin(3\,dx+3\,c)}{12\,d}+\frac{(3\,Ba^2b+3\,Aab^2+Bb^3)\sin(2\,dx+2\,c)}{4\,d} + \frac{(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3)\sin(dx+c)}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/32*B*b^3*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3
*B*b^3)*x + 1/12*(3*B*a*b^2 + A*b^3)*sin(3*d*x + 3*c)/d + 1/4*(3*B*a^2*b +
 3*A*a*b^2 + B*b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^3 + 12*A*a^2*b + 9*B*a
*b^2 + 3*A*b^3)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.18

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = A\,a^3\,x+\frac{3\,B\,b^3\,x}{8}+\frac{3\,A\,a\,b^2\,x}{2}+\frac{3\,B\,a^2\,b\,x}{2}+\frac{3\,A\,b^3\,\sin(c+d\,x)}{4\,d} + \frac{B\,a^3\,\sin(c+d\,x)}{d}+\frac{A\,b^3\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{B\,b^3\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{B\,b^3\,\sin(4\,c+4\,d\,x)}{32\,d}+\frac{3\,A\,a\,b^2\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{3\,B\,a^2\,b\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{B\,a\,b^2\,\sin(3\,c+3\,d\,x)}{4\,d}+\frac{3\,A\,a^2\,b\,\sin(c+d\,x)}{d}+\frac{9\,B\,a\,b^2\,\sin(c+d\,x)}{4\,d}$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

output
```
A*a^3*x + (3*B*b^3*x)/8 + (3*A*a*b^2*x)/2 + (3*B*a^2*b*x)/2 + (3*A*b^3*sin
(c + d*x))/(4*d) + (B*a^3*sin(c + d*x))/d + (A*b^3*sin(3*c + 3*d*x))/(12*d
) + (B*b^3*sin(2*c + 2*d*x))/(4*d) + (B*b^3*sin(4*c + 4*d*x))/(32*d) + (3*
A*a*b^2*sin(2*c + 2*d*x))/(4*d) + (3*B*a^2*b*sin(2*c + 2*d*x))/(4*d) + (B*
a*b^2*sin(3*c + 3*d*x))/(4*d) + (3*A*a^2*b*sin(c + d*x))/d + (9*B*a*b^2*si
n(c + d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{-6\cos(dx+c)\sin(dx+c)^3 b^4 + 72\cos(dx+c)\sin(dx+c)\,a^2b^2 + 15\cos(dx+c)\sin(dx+c)\,b^4 - 32\,\mathrm{s}}{24d}$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**3*b**4 + 72*cos(c + d*x)*sin(c + d*x)*a**
2*b**2 + 15*cos(c + d*x)*sin(c + d*x)*b**4 - 32*sin(c + d*x)**3*a*b**3 + 9
6*sin(c + d*x)*a**3*b + 96*sin(c + d*x)*a*b**3 + 24*a**4*d*x + 72*a**2*b**
2*d*x + 9*b**4*d*x)/(24*d)
```

3.234 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 137

$$\begin{aligned}\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x+\frac{a^3A\text{arctanh}(\sin(c+dx))}{d}\\ &+\frac{b(9aAb+8a^2B+2b^2B)\sin(c+dx)}{3d}\\ &+\frac{b^2(3Ab+5aB)\cos(c+dx)\sin(c+dx)}{6d}+\frac{bB(a+b\cos(c+dx))^2\sin(c+dx)}{3d}\end{aligned}$$

output
```
1/2*(6*A*a^2*b+A*b^3+2*B*a^3+3*B*a*b^2)*x+a^3*A*arctanh(sin(d*x+c))/d+1/3*
b*(9*A*a*b+8*B*a^2+2*B*b^2)*sin(d*x+c)/d+1/6*b^2*(3*A*b+5*B*a)*cos(d*x+c)*
sin(d*x+c)/d+1/3*b*B*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.16

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{6(6a^2Ab+Ab^3+2a^3B+3ab^2B)(c+dx)-12a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3A\log}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(6*(6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*(c + d*x) - 12*a^3*A*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*A*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + 9*b*(4*a*A*b + 4*a^2*B + b^2*B)*Sin[c + d*x] + 3*b^2*(A*b + 3
*a*B)*Sin[2*(c + d*x)] + b^3*B*Sin[3*(c + d*x)])/(12*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 3469, 3042, 3512, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{3}\int (a+b\cos(c+dx))\left(3Aa^2+b(3Ab+5aB)\cos^2(c+dx)+\left(3Ba^2+6Aba+2b^2B\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(3Aa^2+b(3Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Ba^2+6Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3512$$

$$\frac{1}{3}\left(\frac{1}{2}\int \left(6Aa^3+2b\left(8Ba^2+9Aba+2b^2B\right)\cos^2(c+dx)+3\left(2Ba^3+6Aba^2+3b^2Ba+Ab^3\right)\cos(c+dx)\right)\sec(c\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{6Aa^3+2b\left(8Ba^2+9Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+3\left(2Ba^3+6Aba^2+3b^2Ba+Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(2Aa^3+\left(2Ba^3+6Aba^2+3b^2Ba+Ab^3\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{2b\left(8a^2B+9aAb+2b^2B\right)\sin}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int \left(2Aa^3+\left(2Ba^3+6Aba^2+3b^2Ba+Ab^3\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{2b\left(8a^2B+9aAb+2b^2B\right)\sin}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int \frac{2Aa^3+\left(2Ba^3+6Aba^2+3b^2Ba+Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\left(8a^2B+9aAb+2b^2B\right)\sin(c+dx)}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(2a^3A\int\sec(c+dx)dx+x\left(2a^3B+6a^2Ab+3ab^2B+Ab^3\right)\right)+\frac{2b(8a^2B+9aAb+2b^2B)\sin(c+dx)}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(2a^3A\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+x\left(2a^3B+6a^2Ab+3ab^2B+Ab^3\right)\right)+\frac{2b(8a^2B+9aAb+2b^2B)\sin(c+dx)}{d}\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{2b(8a^2B+9aAb+2b^2B)\sin(c+dx)}{d}+3\left(\frac{2a^3A\operatorname{arctanh}(\sin(c+dx))}{d}+x\left(2a^3B+6a^2Ab+3ab^2B+Ab^3\right)\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(b*B*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((b^2*(3*A*b + 5*a*B)*Co
s[c + d*x]*Sin[c + d*x])/(2*d) + (3*((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^
2*B)*x + (2*a^3*A*ArcTanh[Sin[c + d*x]])/d) + (2*b*(9*a*A*b + 8*a^2*B + 2*
b^2*B)*Sin[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a^3 A \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{(A b^3+3 B a b^2)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{(3 A a b^2+3 B a^2 b)\sin(dx+c)}{d} +$
parallelrisch	$\frac{-12 a^3 A \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+12 a^3 A \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+3(A b^3+3 B a b^2)\sin(2dx+2c)+B\sin(3dx+3c) b^3+9(4 A a}{12d}$
derivativedivides	$\frac{a^3 A \ln(\sec(dx+c)+\tan(dx+c))+a^3 B(dx+c)+3 A a^2 b(dx+c)+3 B\sin(dx+c) a^2 b+3 A\sin(dx+c) a b^2+3 B a b^2\left(\frac{\cos(dx+c}{}\right.}{d}$
default	$\frac{a^3 A \ln(\sec(dx+c)+\tan(dx+c))+a^3 B(dx+c)+3 A a^2 b(dx+c)+3 B\sin(dx+c) a^2 b+3 A\sin(dx+c) a b^2+3 B a b^2\left(\frac{\cos(dx+c}{}\right.}{d}$
risch	$3 x A a^2 b + \frac{x A b^3}{2} + a^3 B x + \frac{3 x B a b^2}{2} - \frac{3 i e^{i(dx+c)} A a b^2}{2d} - \frac{3 i e^{i(dx+c)} B a^2 b}{2d} - \frac{3 i e^{i(dx+c)} b^3 B}{8d} + \frac{3 i e^{-i(d}}{}$
norman	$\frac{(3 A a^2 b+\frac{1}{2} A b^3+a^3 B+\frac{3}{2} B a b^2)x+(3 A a^2 b+\frac{1}{2} A b^3+a^3 B+\frac{3}{2} B a b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(12 A a^2 b+2 A b^3+4 a^3 B+6 B a b^2}{}$

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
a^3*A/d*ln(sec(d*x+c)+tan(d*x+c))+(A*b^3+3*B*a*b^2)/d*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c)+(3*A*a*b^2+3*B*a^2*b)/d*sin(d*x+c)+(3*A*a^2*b+B*a^3)
/d*(d*x+c)+1/3*b^3*B/d*(cos(d*x+c)^2+2)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int (a+b\cos(c+dx))^3 (A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{3\,Aa^3\log(\sin(dx+c)+1) - 3\,Aa^3\log(-\sin(dx+c)+1) + 3\,(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3)dx + (}{6\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fri
cas")
```

output
```
1/6*(3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-sin(d*x + c) + 1) + 3*(2
*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*d*x + (2*B*b^3*cos(d*x + c)^2 + 18
*B*a^2*b + 18*A*a*b^2 + 4*B*b^3 + 3*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^3\sec{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{12\,(dx+c)Ba^3+36\,(dx+c)Aa^2b+9\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Bab^2+3\,(2\,dx+2\,c+\sin{(2\,dx+2}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="max
ima")
```

output
```
1/12*(12*(d*x + c)*B*a^3 + 36*(d*x + c)*A*a^2*b + 9*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a*b^2 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^3 - 4*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*B*b^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)
) + 36*B*a^2*b*sin(d*x + c) + 36*A*a*b^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(129) = 258$.

Time = 0.19 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.29

$$\int (a + b\cos(c + dx))^3 (A + B\cos(c + dx)) \sec(c + dx)\, dx$$

$$= \frac{6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + 3\left(2\,Ba^3 + 6\,Aa^2b + 3\,Bab^2 + A\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gia
c")
```

output
```
1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*(d*x + c)
 + 2*(18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^
5 - 9*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*
B*b^3*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*
b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*B*a^2*b*t
an(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c) + 9*B*a*b^2*tan(1/2*
d*x + 1/2*c) + 3*A*b^3*tan(1/2*d*x + 1/2*c) + 6*B*b^3*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 42.98 (sec) , antiderivative size = 1924, normalized size of antiderivative = 14.04

$$\int (a + b\cos(c + dx))^3 (A + B\cos(c + dx)) \sec(c + dx)\, dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x),x)
```

output
```
(tan(c/2 + (d*x)/2)*(A*b^3 + 2*B*b^3 + 6*A*a*b^2 + 3*B*a*b^2 + 6*B*a^2*b)
+ tan(c/2 + (d*x)/2)^3*((4*B*b^3)/3 + 12*A*a*b^2 + 12*B*a^2*b) + tan(c/2 +
 (d*x)/2)^5*(2*B*b^3 - A*b^3 + 6*A*a*b^2 - 3*B*a*b^2 + 6*B*a^2*b))/(d*(3*t
an(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1))
+ (atan((((((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(32*A*a^
3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2) + tan(c/2 + (d*x)/2)*(3
2*A^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72
*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3
*b^3))*((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*1i - (((A*b
^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 16*A*b^3 +
32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2) - tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A
^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 +
96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3))*((A*b^3*
1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*1i)/((((A*b^3*1i)/2 + B*a^
3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A
*a^2*b + 48*B*a*b^2) + tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2
*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2
+ 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3))*((A*b^3*1i)/2 + B*a^3*1
i + A*a^2*b*3i + (B*a*b^2*3i)/2) + (((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i
+ (B*a*b^2*3i)/2)*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{6\cos(dx+c)\sin(dx+c)\,a\,b^3 - 3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^4 + 3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a^4 - \sin(dx+c)^3}{3d}$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
(6*cos(c + d*x)*sin(c + d*x)*a*b**3 - 3*log(tan((c + d*x)/2) - 1)*a**4 + 3
*log(tan((c + d*x)/2) + 1)*a**4 - sin(c + d*x)**3*b**4 + 18*sin(c + d*x)*a
**2*b**2 + 3*sin(c + d*x)*b**4 + 12*a**3*b*d*x + 6*a*b**3*d*x)/(3*d)
```

3.235 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 131

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{1}{2}b\left(6aAb+6a^2B+b^2B\right)x+\frac{a^2(3Ab+aB)\text{arctanh}(\sin(c+dx))}{d} - \frac{b(2a^2A-Ab^2-3abB)\sin(c+dx)}{d} - \frac{b^2(2aA-bB)\cos(c+dx)\sin(c+dx)}{2d} + \frac{aA(a+b\cos(c+dx))^2\tan(c+dx)}{d}$$

output

```
1/2*b*(6*A*a*b+6*B*a^2+B*b^2)*x+a^2*(3*A*b+B*a)*arctanh(sin(d*x+c))/d-b*(2
*A*a^2-A*b^2-3*B*a*b)*sin(d*x+c)/d-1/2*b^2*(2*A*a-B*b)*cos(d*x+c)*sin(d*x+
c)/d+a*A*(a+b*cos(d*x+c))^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.66

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{2b(6aAb+6a^2B+b^2B)(c+dx)-4a^2(3Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^2(3Ab+}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(2*b*(6*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - 4*a^2*(3*A*b + a*B)*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^2*(3*A*b + a*B)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]) + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]) + 4*b^2*(A*b + 3*a*B)*Sin[c + d*x] + b^3*B*Sin[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3468, 3042, 3512, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{3468}$$

$$\int (a+b\cos(c+dx))\left(-b(2aA-bB)\cos^2(c+dx)+b(Ab+2aB)\cos(c+dx)+a(3Ab+aB)\right)\sec(c+dx)dx+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$\Big\downarrow$ 3042

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-b(2aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(3Ab+aB)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}d\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$\Big\downarrow$ 3512

$$\frac{1}{2}\int \left(2(3Ab+aB)a^2-2b\left(2Aa^2-3bBa-Ab^2\right)\cos^2(c+dx)+b\left(6Ba^2+6Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\int \frac{2(3Ab+aB)a^2-2b\left(2Aa^2-3bBa-Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(6Ba^2+6Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$\Big\downarrow$ 3502

$$\frac{1}{2}\left(\int \left(2(3Ab+aB)a^2+b\left(6Ba^2+6Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{2b\left(2a^2A-3abB-Ab^2\right)\sin(c+dx)}{d}\right)-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{2(3Ab+aB)a^2+b\left(6Ba^2+6Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b\left(2a^2A-3abB-Ab^2\right)\sin(c+dx)}{d}\right)-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$\Big\downarrow$ 3214

$$\frac{1}{2}\left(2a^2(aB+3Ab)\int \sec(c+dx)dx-\frac{2b(2a^2A-3abB-Ab^2)\sin(c+dx)}{d}+bx(6a^2B+6aAb+b^2B)\right)-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2a^2(aB+3Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2b(2a^2A-3abB-Ab^2)\sin(c+dx)}{d}+bx(6a^2B+6aAb+b^2B)\right)-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{2a^2(aB+3Ab)\text{arctanh}(\sin(c+dx))}{d}-\frac{2b(2a^2A-3abB-Ab^2)\sin(c+dx)}{d}+bx(6a^2B+6aAb+b^2B)\right)-\frac{b^2(2aA-bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
-1/2*(b^2*(2*a*A - b*B)*Cos[c + d*x]*Sin[c + d*x])/d + (b*(6*a*A*b + 6*a^2
*B + b^2*B)*x + (2*a^2*(3*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d - (2*b*(2*a^
2*A - A*b^2 - 3*a*b*B)*Sin[c + d*x])/d)/2 + (a*A*(a + b*Cos[c + d*x])^2*Ta
n[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 5.93 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

method	result
parts	$\frac{a^3 A\tan(dx+c)}{d}+\frac{(A b^3+3Ba b^2)\sin(dx+c)}{d}+\frac{(3Aa b^2+3B a^2b)(dx+c)}{d}+\frac{(3A a^2b+a^3B)\ln(\sec(dx+c)+\tan(dx$
derivativedivides	$\frac{A\tan(dx+c)a^3+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A a^2b\ln(\sec(dx+c)+\tan(dx+c))+3B a^2b(dx+c)+3Aa b^2(dx+c)+}{d}$
default	$\frac{A\tan(dx+c)a^3+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A a^2b\ln(\sec(dx+c)+\tan(dx+c))+3B a^2b(dx+c)+3Aa b^2(dx+c)+}{d}$
parallelrisch	$\frac{8(-3A a^2b-a^3B)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8(3A a^2b+a^3B)\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4(A b^3+3Ba b^2)}{8\cos(dx+c)d}$
risch	$3xAa b^2+3xB a^2b+\frac{b^3Bx}{2}-\frac{ib^3B\,e^{2i(dx+c)}}{8d}-\frac{ie^{i(dx+c)}A b^3}{2d}-\frac{3ie^{i(dx+c)}Ba b^2}{2d}+\frac{ie^{-i(dx+c)}A b^3}{2d}+$
norman	$\frac{(-3Aa b^2-3B a^2b-\frac{1}{2}b^3B)x+(-9Aa b^2-9B a^2b-\frac{3}{2}b^3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(3Aa b^2+3B a^2b+\frac{1}{2}b^3B)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-}{}$

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBO
SE)
```

output
```
a^3*A/d*tan(d*x+c)+(A*b^3+3*B*a*b^2)/d*sin(d*x+c)+(3*A*a*b^2+3*B*a^2*b)/d*
(d*x+c)+(3*A*a^2*b+B*a^3)/d*ln(sec(d*x+c)+tan(d*x+c))+b^3*B/d*(1/2*cos(d*x
+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.16

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{(6\,Ba^2b+6\,Aab^2+Bb^3)dx\cos(dx+c)+(Ba^3+3\,Aa^2b)\cos(dx+c)\log(\sin(dx+c)+1)-(Ba^3+3}{2}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="f
ricas")
```

output
```
1/2*((6*B*a^2*b + 6*A*a*b^2 + B*b^3)*d*x*cos(d*x + c) + (B*a^3 + 3*A*a^2*b
)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^3 + 3*A*a^2*b)*cos(d*x + c)*lo
g(-sin(d*x + c) + 1) + (B*b^3*cos(d*x + c)^2 + 2*A*a^3 + 2*(3*B*a*b^2 + A*
b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{12\,(dx+c)Ba^2b+12\,(dx+c)Aab^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^3+2\,Ba^3(\log(\sin(dx+c)+1)-}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="m
axima")
```

output
```
1/4*(12*(d*x + c)*B*a^2*b + 12*(d*x + c)*A*a*b^2 + (2*d*x + 2*c + sin(2*d*
x + 2*c))*B*b^3 + 2*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 6*A*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*a*b^2*s
in(d*x + c) + 4*A*b^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.79

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx =$$

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-(6\,Ba^2b+6\,Aab^2+Bb^3)(dx+c)-2\,(Ba^3+3\,Aa^2b)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="g
iac")
```

output
```
-1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*B*a^2
*b + 6*A*a*b^2 + B*b^3)*(d*x + c) - 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) + 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(6*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^3
 - B*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3
*tan(1/2*d*x + 1/2*c) + B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 42.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.80

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$

$$=\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,A\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,B\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

$$+\frac{\frac{A\,b^3\,\sin(2\,c+2\,d\,x)}{2}+\frac{B\,b^3\,\sin(3\,c+3\,d\,x)}{8}+A\,a^3\,\sin\left(c+d\,x\right)+\frac{B\,b^3\,\sin(c+d\,x)}{8}+\frac{3\,B\,a\,b^2\,\sin(2\,c+2\,d\,x)}{2}}{d\,\cos\left(c+d\,x\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

output
```
(B*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - B*a^3*atan((sin(c/2 +
 (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i + 6*A*a*b^2*atan(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2)) - A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/
2))*6i + 6*B*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + ((A*b^
3*sin(2*c + 2*d*x))/2 + (B*b^3*sin(3*c + 3*d*x))/8 + A*a^3*sin(c + d*x) +
(B*b^3*sin(c + d*x))/8 + (3*B*a*b^2*sin(2*c + 2*d*x))/2)/(d*cos(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{\cos\left(dx+c\right)^2\sin\left(dx+c\right)b^4-8\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^3b+8\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2\cos\left(d\right.}$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
(cos(c + d*x)**2*sin(c + d*x)*b**4 - 8*cos(c + d*x)*log(tan((c + d*x)/2) -
 1)*a**3*b + 8*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**3*b + 8*cos(c + d
*x)*sin(c + d*x)*a*b**3 + 12*cos(c + d*x)*a**2*b**2*d*x + cos(c + d*x)*b**
4*d*x + 2*sin(c + d*x)*a**4)/(2*cos(c + d*x)*d)
```

3.236 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 124

$$\begin{aligned}\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= b^2(Ab+3aB)x+\frac{a(a^2A+6Ab^2+6abB)\,\mathrm{arctanh}(\sin(c+dx))}{2d}\\ &\quad -\frac{b^2(aA-2bB)\sin(c+dx)}{2d}+\frac{a^2(2Ab+aB)\tan(c+dx)}{d}\\ &\quad +\frac{aA(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
b^2*(A*b+3*B*a)*x+1/2*a*(A*a^2+6*A*b^2+6*B*a*b)*arctanh(sin(d*x+c))/d-1/2*
b^2*(A*a-2*B*b)*sin(d*x+c)/d+a^2*(2*A*b+B*a)*tan(d*x+c)/d+1/2*a*A*(a+b*cos
(d*x+c))^2*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 277 vs. $2(124) = 248$.

Time = 4.92 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.23

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{4b^2(Ab+3aB)(c+dx) - 2a(a^2A+6Ab^2+6abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2a(a^2A+ \cdots}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(4*b^2*(A*b + 3*a*B)*(c + d*x) - 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*A)/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^2 + (4*a^2*(3*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]) - (a^3*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a^2*(
3*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*b
^3*B*Sin[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3468, 3042, 3510, 25, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 3468$$

$$\frac{1}{2}\int (a+b\cos(c+dx))\left(-b(aA-2bB)\cos^2(c+dx)+\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+2a(2Ab+aB)\right)\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-b(aA-2bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+4bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(2\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$\downarrow$ 3510

$$\frac{1}{2}\left(\frac{2a^2(aB+2Ab)\tan(c+dx)}{d}-\int -((-((aA-2bB)\cos^2(c+dx)b^2)+2(Ab+3aB)\cos(c+dx)b^2+a\left(Aa^2+\right.\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$\downarrow$ 25

$$\frac{1}{2}\left(\int \left(-((aA-2bB)\cos^2(c+dx)b^2)+2(Ab+3aB)\cos(c+dx)b^2+a\left(Aa^2+6bBa+6Ab^2\right)\right)\sec(c+dx)dx+\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{-\left((aA-2bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2\right)+2(Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(Aa^2+6bBa+6Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$\downarrow$ 3502

$$\frac{1}{2}\left(\int \left(2(Ab+3aB)\cos(c+dx)b^2+a\left(Aa^2+6bBa+6Ab^2\right)\right)\sec(c+dx)dx+\frac{2a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(}{}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{2(Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(Aa^2+6bBa+6Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2b\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3214$$

$$\frac{1}{2}\left(a\left(a^2A+6abB+6Ab^2\right)\int \sec(c+dx)dx+\frac{2a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{d}+2b^2x(\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(a\left(a^2A+6abB+6Ab^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{d}+\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{2}\left(\frac{a\left(a^2A+6abB+6Ab^2\right)\text{arctanh}(\sin(c+dx))}{d}+\frac{2a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{d}+2\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (2*b^2*(A*b
 + 3*a*B)*x + (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])/d - (b
^2*(a*A - 2*b*B)*Sin[c + d*x])/d + (2*a^2*(2*A*b + a*B)*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

method	result
parts	$\frac{a^3 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{(A\,b^3+3B\,a\,b^2)(dx+c)}{d}+\frac{(3A\,a\,b^2+3B\,a^2b)\ln(\sec(dx+c)+t}{d}$
derivativedivides	$\frac{a^3 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^3B\tan(dx+c)+3A\,a^2b\tan(dx+c)+3B\,a^2b\ln(\sec(dx+c)+\tan(}{d}$
default	$\frac{a^3 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^3B\tan(dx+c)+3A\,a^2b\tan(dx+c)+3B\,a^2b\ln(\sec(dx+c)+\tan(}{d}$
parallelrisch	$\frac{-a(\cos(2dx+2c)+1)(a^2A+6A\,b^2+6Bab)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a(\cos(2dx+2c)+1)(a^2A+6A\,b^2+6Bab)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{2c}$
risch	$xA\,b^3+3xBa\,b^2-\frac{ie^{i(dx+c)}b^3B}{2d}+\frac{ie^{-i(dx+c)}b^3B}{2d}-\frac{ia^2(Aa\,e^{3i(dx+c)}-6Ab\,e^{2i(dx+c)}-2Ba\,e^{2i(dx+c)}-Aa}{d(e^{2i(dx+c)}+1)^2}$
norman	$\frac{(A\,b^3+3Ba\,b^2)x+(-4A\,b^3-12Ba\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-A\,b^3-3Ba\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-A\,b^3-3Ba\,b^2)x\tan\left(\frac{dx}{2}+\right.}{}$

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBO
SE)
```

output
```
a^3*A/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+(A*b^3+3
*B*a*b^2)/d*(d*x+c)+(3*A*a*b^2+3*B*a^2*b)/d*ln(sec(d*x+c)+tan(d*x+c))+(3*A
*a^2*b+B*a^3)/d*tan(d*x+c)+1/d*sin(d*x+c)*b^3*B
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{4\,(3\,Bab^2+Ab^3)dx\cos{(dx+c)}^2+(Aa^3+6\,Ba^2b+6\,Aab^2)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-(Aa^3$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="f
ricas")
```

output
```
1/4*(4*(3*B*a*b^2 + A*b^3)*d*x*cos(d*x + c)^2 + (A*a^3 + 6*B*a^2*b + 6*A*a
*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^
2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*B*b^3*cos(d*x + c)^2 + A*a
^3 + 2*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^3\sec^3{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{12\,(dx+c)Bab^2+4\,(dx+c)Ab^3-Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="m
axima")
```

output
```
1/4*(12*(d*x + c)*B*a*b^2 + 4*(d*x + c)*A*b^3 - A*a^3*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^
2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*A*a*b^2*(log(sin(d
*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*b^3*sin(d*x + c) + 4*B*a^3*tan
(d*x + c) + 12*A*a^2*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(118) = 236.

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.93

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{\frac{4\,Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\,(3\,Bab^2+Ab^3)(dx+c)+(Aa^3+6\,Ba^2b+6\,Aab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="g
iac")
```

output
```
1/2*(4*B*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*B*a*
b^2 + A*b^3)*(d*x + c) + (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d
*x + 1/2*c) + 1)) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x + 1/2
*c)^3 - 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a^3*tan(1/2*d*x + 1/2*c) + 2*
B*a^3*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 42.01 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.01

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{\frac{B\,a^3\,\sin(2\,c+2\,d\,x)}{2}+\frac{B\,b^3\,\sin(3\,c+3\,d\,x)}{4}+\frac{A\,a^3\,\sin(c+d\,x)}{2}+\frac{B\,b^3\,\sin(c+d\,x)}{4}+\frac{3\,A\,a^2\,b\,\sin(2\,c+2\,d\,x)}{2}}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$
$$-\frac{2\left(\frac{A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,1\mathrm{i}}{2}-A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+A\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3\mathrm{i}-3\,B\,a\,b^2\,\mathrm{atan}\left(\frac{\sin}{\cos}\right.\right.}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^3,x)
```

output
```
((B*a^3*sin(2*c + 2*d*x))/2 + (B*b^3*sin(3*c + 3*d*x))/4 + (A*a^3*sin(c +
d*x))/2 + (B*b^3*sin(c + d*x))/4 + (3*A*a^2*b*sin(2*c + 2*d*x))/2)/(d*(cos
(2*c + 2*d*x)/2 + 1/2)) - (2*((A*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2
+ (d*x)/2))*1i)/2 - A*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + A*
a*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i - 3*B*a*b^2*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*a^2*b*atan((sin(c/2 + (d*x)/2)
*1i)/cos(c/2 + (d*x)/2))*3i))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.96

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a^4 - 12\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)}{}$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4 - 12*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**2 + cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*a**4 + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1
)*a**2*b**2 + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4
+ 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 - co
s(c + d*x)*log(tan((c + d*x)/2) + 1)*a**4 - 12*cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*a**2*b**2 + 2*cos(c + d*x)*sin(c + d*x)**3*b**4 + 8*cos(c + d*
x)*sin(c + d*x)**2*a*b**3*d*x - cos(c + d*x)*sin(c + d*x)*a**4 - 2*cos(c +
 d*x)*sin(c + d*x)*b**4 - 8*cos(c + d*x)*a*b**3*d*x + 8*sin(c + d*x)**3*a*
*3*b - 8*sin(c + d*x)*a**3*b)/(2*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.237 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 145

$$\begin{aligned}\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= b^3Bx+\frac{(3a^2Ab+2Ab^3+a^3B+6ab^2B)\,\text{arctanh}(\sin(c+dx))}{2d}\\ &\quad+\frac{a(2a^2A+8Ab^2+9abB)\tan(c+dx)}{3d}+\frac{a^2(5Ab+3aB)\sec(c+dx)\tan(c+dx)}{6d}\\ &\quad+\frac{aA(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}\end{aligned}$$

output
```
b^3*B*x+1/2*(3*A*a^2*b+2*A*b^3+B*a^3+6*B*a*b^2)*arctanh(sin(d*x+c))/d+1/3*
a*(2*A*a^2+8*A*b^2+9*B*a*b)*tan(d*x+c)/d+1/6*a^2*(5*A*b+3*B*a)*sec(d*x+c)*
tan(d*x+c)/d+1/3*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{6b^3Bdx+6b^2(Ab+3aB)\coth^{-1}(\sin(c+dx))+3a^2(3Ab+aB)\text{arctanh}(\sin(c+dx))+18ab(Ab+aB)\cdots}{6d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(6*b^3*B*d*x + 6*b^2*(A*b + 3*a*B)*ArcCoth[Sin[c + d*x]] + 3*a^2*(3*A*b +
a*B)*ArcTanh[Sin[c + d*x]] + 18*a*b*(A*b + a*B)*Tan[c + d*x] + 3*a^2*(3*A*
b + a*B)*Sec[c + d*x]*Tan[c + d*x] + 2*a^3*A*Tan[c + d*x]*(3 + Tan[c + d*x
]^2))/(6*d)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3468, 3042, 3510, 25, 3042, 3500, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{3}\int (a+b\cos(c+dx))\left(3b^2B\cos^2(c+dx)+\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(5Ab+3aB)\right)\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(3b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(2Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5Ab+3aI\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3510}$$

$$\frac{1}{3}\left(\frac{a^2(3aB+5Ab)\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int -\left(\left(6B\cos^2(c+dx)b^3+2a\left(2Aa^2+9bBa+8Ab^2\right)+3\left(Ba^3+\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \left(6B\cos^2(c+dx)b^3+2a\left(2Aa^2+9bBa+8Ab^2\right)+3\left(Ba^3+3Aba^2+6b^2Ba+2Ab^3\right)\cos(c+dx)\right)\sec^2(c\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{6B\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+2a\left(2Aa^2+9bBa+8Ab^2\right)+3\left(Ba^3+3Aba^2+6b^2Ba+2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(Ba^3+3Aba^2+6b^2Ba+2Ab^3+2b^3B\cos(c+dx)\right)\sec(c+dx)dx+\frac{2a\left(2a^2A+9abB+8Ab^2\right)\tan(c}{d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int \left(Ba^3+3Aba^2+6b^2Ba+2Ab^3+2b^3B\cos(c+dx)\right)\sec(c+dx)dx+\frac{2a\left(2a^2A+9abB+8Ab^2\right)\tan(c}{d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int\frac{Ba^3+3Aba^2+6b^2Ba+2Ab^3+2b^3B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a\left(2a^2A+9abB+8Ab^2\right)\tan(c+dx)}{d}\right)\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$\downarrow$ 3214

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(\left(a^3B+3a^2Ab+6ab^2B+2Ab^3\right)\int\sec(c+dx)dx+2b^3Bx\right)+\frac{2a\left(2a^2A+9abB+8Ab^2\right)\tan(c+dx)}{d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(\left(a^3B+3a^2Ab+6ab^2B+2Ab^3\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+2b^3Bx\right)+\frac{2a\left(2a^2A+9abB+8Ab^2\right)\tan(c\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$\downarrow$ 4257

$$\frac{1}{3}\left(\frac{a^2(3aB+5Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(\frac{2a\left(2a^2A+9abB+8Ab^2\right)\tan(c+dx)}{d}+3\left(\frac{\left(a^3B+3a^2Ab+6\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((a^2*(5*
A*b + 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (3*(2*b^3*B*x + ((3*a^2*A*
b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c + d*x]])/d) + (2*a*(2*a^2*A
 + 8*A*b^2 + 9*a*b*B)*Tan[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 9.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result
parts	$-\frac{a^3 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{(A\,b^3+3Ba\,b^2)\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{(3Aa\,b^2+3B\,a^2b)\tan(dx+c)}{d}$
derivativedivides	$-a^3 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^3B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3A\,a^2b\left(\frac{\sec(dx+c)\tan(dx+}{2}\right.$
default	$-a^3 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^3B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3A\,a^2b\left(\frac{\sec(dx+c)\tan(dx+}{2}\right.$
parallelrisch	$-27\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\left(A\,a^2b+\frac{2}{3}A\,b^3+\frac{1}{3}a^3B+2Ba\,b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+27\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)(A$
risch	$b^3Bx-\frac{ia\left(9Aab\,e^{5i(dx+c)}+3B\,a^2e^{5i(dx+c)}-18A\,b^2e^{4i(dx+c)}-18Bab\,e^{4i(dx+c)}-12A\,a^2e^{2i(dx+c)}-36A\,b^2e^{2i(dx+c)}\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$b^3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+b^3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-b^3Bx-\frac{8a\left(a^2A-3A\,b^2-3Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{a\left(2a^2A-3Aab+6A\,b^2-a^2B+6\right.}{d}$

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBO
SE)
```

output
```
-a^3*A/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*b^3+3*B*a*b^2)/d*ln(sec(d*x
+c)+tan(d*x+c))+(3*A*a*b^2+3*B*a^2*b)/d*tan(d*x+c)+(3*A*a^2*b+B*a^3)/d*(1/
2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+b^3*B/d*(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.30

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{12\,Bb^3dx\cos\left(dx+c\right)^3+3\left(Ba^3+3\,Aa^2b+6\,Bab^2+2\,Ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="f
ricas")
```

output
```
1/12*(12*B*b^3*d*x*cos(d*x + c)^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A
*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*
b^2 + 2*A*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*A*a^3 + 2*(2*A
*a^3 + 9*B*a^2*b + 9*A*a*b^2)*cos(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.49

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$

$$= \frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+12\left(dx+c\right)Bb^3-3\,Ba^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="m
axima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*b^3 - 3*B
*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(si
n(d*x + c) - 1)) - 9*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 18*B*a*b^2*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1)) + 6*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 36*B*a^2*b*tan(d*x + c) + 36*A*a*b^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(137) = 274.

Time = 0.20 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.32

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$

$$= \frac{6\left(dx+c\right)Bb^3+3\left(Ba^3+3\,Aa^2b+6\,Bab^2+2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ba^3+3\,Aa^2b+6\,\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="g
iac")
```

output
```
1/6*(6*(d*x + c)*B*b^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*log(a
bs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3
)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 -
 3*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*
a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^3
*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*t
an(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 3*B*a^3*tan(1/2*d*x
 + 1/2*c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a^2*b*tan(1/2*d*x + 1/2*
c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 36.73 (sec) , antiderivative size = 526, normalized size of antiderivative = 3.63

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^4,x)
```

output
```
((A*a^3*sin(3*c + 3*d*x))/6 + (B*a^3*sin(2*c + 2*d*x))/4 + (A*a^3*sin(c +
d*x))/2 + (3*A*a*b^2*sin(c + d*x))/4 + (3*B*a^2*b*sin(c + d*x))/4 - (A*b^3
*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/2 - (B*
a^3*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/4 +
(3*B*b^3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (3*
A*a^2*b*sin(2*c + 2*d*x))/4 + (3*A*a*b^2*sin(3*c + 3*d*x))/4 + (3*B*a^2*b*
sin(3*c + 3*d*x))/4 - (A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/
2))*cos(3*c + 3*d*x)*1i)/2 - (B*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
 (d*x)/2))*cos(3*c + 3*d*x)*1i)/4 + (B*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2))*cos(3*c + 3*d*x))/2 - (A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/c
os(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i)/4 - (B*a*b^2*atan((sin(c/2 + (d*x)
/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i)/2 - (A*a^2*b*cos(c + d*x)
*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/4 - (B*a*b^2*cos(c +
 d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/2)/(d*((3*cos(c
 + d*x))/4 + cos(3*c + 3*d*x)/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.46

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{-6\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a^3b-12\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+}{}$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
( - 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b - 12*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**3 + 6*cos(c + d
*x)*log(tan((c + d*x)/2) - 1)*a**3*b + 12*cos(c + d*x)*log(tan((c + d*x)/2
) - 1)*a*b**3 + 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a
**3*b + 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**3 -
 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**3*b - 12*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*a*b**3 + 3*cos(c + d*x)*sin(c + d*x)**2*b**4*d*x - 6*co
s(c + d*x)*sin(c + d*x)*a**3*b - 3*cos(c + d*x)*b**4*d*x + 2*sin(c + d*x)*
*3*a**4 + 18*sin(c + d*x)**3*a**2*b**2 - 3*sin(c + d*x)*a**4 - 18*sin(c +
d*x)*a**2*b**2)/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.238 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 188

$$\begin{aligned}\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= \frac{(3a^3A+12aAb^2+12a^2bB+8b^3B)\operatorname{arctanh}(\sin(c+dx))}{8d} \\ &+ \frac{(6a^2Ab+3Ab^3+2a^3B+9ab^2B)\tan(c+dx)}{3d} \\ &+ \frac{a(3a^2A+10Ab^2+12abB)\sec(c+dx)\tan(c+dx)}{8d} \\ &+ \frac{a^2(3Ab+2aB)\sec^2(c+dx)\tan(c+dx)}{6d} \\ &+ \frac{aA(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

output

```
1/8*(3*A*a^3+12*A*a*b^2+12*B*a^2*b+8*B*b^3)*arctanh(sin(d*x+c))/d+1/3*(6*A
*a^2*b+3*A*b^3+2*B*a^3+9*B*a*b^2)*tan(d*x+c)/d+1/8*a*(3*A*a^2+10*A*b^2+12*
B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1/6*a^2*(3*A*b+2*B*a)*sec(d*x+c)^2*tan(d*x+
c)/d+1/4*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{24b^3B\coth^{-1}(\sin(c+dx))+9a(a^2A+4Ab^2+4abB)\,\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(24(3a^2Ab+\ldots}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(24*b^3*B*ArcCoth[Sin[c + d*x]] + 9*a*(a^2*A + 4*A*b^2 + 4*a*b*B)*ArcTanh[
Sin[c + d*x]] + Tan[c + d*x]*(24*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B) +
 9*a*(a^2*A + 4*A*b^2 + 4*a*b*B)*Sec[c + d*x] + 6*a^3*A*Sec[c + d*x]^3 + 8
*a^2*(3*A*b + a*B)*Tan[c + d*x]^2))/(24*d)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3468, 3042, 3510, 25, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{4}\int (a+b\cos(c+dx))\left(b(aA+4bB)\cos^2(c+dx)+\left(3Aa^2+8bBa+4Ab^2\right)\cos(c+dx)+2a(3Ab+2aB)\right)\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b(aA+4bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+8bBa+4Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(3}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \text{3510}$$

$$\frac{1}{4}\left(\frac{2a^2(2aB+3Ab)\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{1}{3}\int -\left(\left(3b^2(aA+4bB)\cos^2(c+dx)+4\left(2Ba^3+6Aba^2+9b^2B\right.\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \left(3b^2(aA+4bB)\cos^2(c+dx)+4\left(2Ba^3+6Aba^2+9b^2Ba+3Ab^3\right)\cos(c+dx)+3a\left(3Aa^2+12bBa+1\right.\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{3b^2(aA+4bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+4\left(2Ba^3+6Aba^2+9b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\left(3Aa^2+1\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(8\left(2Ba^3+6Aba^2+9b^2Ba+3Ab^3\right)+3\left(3Aa^3+12bBa^2+12Ab^2a+8b^3B\right)\cos(c+dx)\right)\sec^2(c+dx\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{8\left(2Ba^3+6Aba^2+9b^2Ba+3Ab^3\right)+3\left(3Aa^3+12bBa^2+12Ab^2a+8b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{3a}{}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(8\left(2a^3B+6a^2Ab+9ab^2B+3Ab^3\right)\int\sec^2(c+dx)dx+3\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\int\sec(c\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+8\left(2a^3B+6a^2Ab+9ab^2B+3Ab^3\right)\int\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$\big\downarrow$ 4254

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{8\left(2a^3B+6a^2Ab+9ab^2B+3Ab^3\right)\int}{d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$\big\downarrow$ 24

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{8\left(2a^3B+6a^2Ab+9ab^2B+3Ab^3\right)\tan}{d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$\big\downarrow$ 4257

$$\frac{1}{4}\left(\frac{2a^2(2aB+3Ab)\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{1}{3}\left(\frac{3a\left(3a^2A+12abB+10Ab^2\right)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(3\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((2*a^2*(
3*A*b + 2*a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((3*a*(3*a^2*A + 10*A*
b^2 + 12*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((3*(3*a^3*A + 12*a*A*b
^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]])/d + (8*(6*a^2*A*b + 3*A*
b^3 + 2*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/d)/2)/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 9.25 (sec), antiderivative size = 185, normalized size of antiderivative = 0.98

method	result
parts	$\frac{a^3A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{(A\,b^3+3Ba\,b^2)\tan(dx+c)}{d}+\frac{(3Aa$
derivativedivides	$\frac{a^3A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^3B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-3A}{}$
default	$\frac{a^3A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^3B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-3A}{}$
parallelrisch	$\frac{-18(a^3A+4Aa\,b^2+4B\,a^2b+\frac{8}{3}b^3B)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18(a^3A+4Aa\,b^2+4B\,a^2b+}{}$
risch	$-\frac{i(-16a^3B-48A\,a^2b-72Ba\,b^2-36Aa\,b^2e^{i(dx+c)}-36B\,a^2b\,e^{i(dx+c)}+36B\,a^2b\,e^{5i(dx+c)}-144A\,a^2b\,e^{4i(dx+c)}-24A\,b^3}{}$
norman	$\frac{\left(5a^3A-24A\,a^2b+12Aa\,b^2-8A\,b^3-8a^3B+12B\,a^2b-24Ba\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}}{4d}+\frac{\left(5a^3A+24A\,a^2b+12Aa\,b^2+8A\,b^3+8a^3B+12B\,a^2b}{4d}$

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBO
SE)
```

output
```
a^3*A/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+(A*b^3+3*B*a*b^2)/d*tan(d*x+c)+(3*A*a*b^2+3*B*a^2*b)/d*(1/2*s
ec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-(3*A*a^2*b+B*a^3)/d*(-
2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^3*B/d*ln(sec(d*x+c)+tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec), antiderivative size = 211, normalized size of antiderivative = 1.12

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{3\left(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Aa^3+12\,Ba^2b+12\,Aa\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="f
ricas")
```

output
```
1/48*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x + c)^4*log(s
in(d*x + c) + 1) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x
 + c)^4*log(-sin(d*x + c) + 1) + 2*(6*A*a^3 + 8*(2*B*a^3 + 6*A*a^2*b + 9*B
*a*b^2 + 3*A*b^3)*cos(d*x + c)^3 + 9*(A*a^3 + 4*B*a^2*b + 4*A*a*b^2)*cos(d
*x + c)^2 + 8*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.45

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba^3+48\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa^2b-3\,Aa^3\left(\frac{2\left(3\sin(dx+c)\cdots\right)}{\sin(dx+c)^4-2\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="m
axima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 48*(tan(d*x + c)^3 + 3*
tan(d*x + c))*A*a^2*b - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(si
n(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin
(d*x + c) - 1)) - 36*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 36*A*a*b^2*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*b
^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 144*B*a*b^2*tan(d*x +
 c) + 48*A*b^3*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(178) = 356$.

Time = 0.17 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.12

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="g
iac")
```

output
```
1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*
tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*ta
n(1/2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(
1/2*d*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^3*tan(1/2*d*x
 + 1/2*c)^5 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 120*A*a^2*b*tan(1/2*d*x +
1/2*c)^5 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*A*a*b^2*tan(1/2*d*x + 1/
2*c)^5 + 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c
)^5 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 1
20*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36
*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*
A*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^3*tan(1/2*d*x + 1/2*c) + 24*B*a^3*ta
n(1/2*d*x + 1/2*c) + 72*A*a^2*b*tan(1/2*d*x + 1/2*c) + 36*B*a^2*b*tan(1/2*
d*x + 1/2*c) + 36*A*a*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a*b^2*tan(1/2*d*x +
1/2*c) + 24*A*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 45.66 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.10

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Aa^3}{8}+\frac{3Ba^2b}{2}+\frac{3Aab^2}{2}+Bb^3\right)}{\frac{3Aa^3}{2}+6Ba^2b+6Aab^2+4Bb^3}\right)\left(\frac{3Aa^3}{4}+3Ba^2b+3Aab^2+2Bb^3\right)}{d} - \frac{\left(2Ab^3-\frac{5Aa^3}{4}+2Ba^3-3Aab^2+6Aa^2b+6Bab^2-3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(3Aab^2-6Ab^3\right.}{}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^5,x)
```

output
```
(atanh((4*tan(c/2 + (d*x)/2)*((3*A*a^3)/8 + B*b^3 + (3*A*a*b^2)/2 + (3*B*a
^2*b)/2))/((3*A*a^3)/2 + 4*B*b^3 + 6*A*a*b^2 + 6*B*a^2*b))*((3*A*a^3)/4 +
2*B*b^3 + 3*A*a*b^2 + 3*B*a^2*b))/d - (tan(c/2 + (d*x)/2)^7*(2*A*b^3 - (5*
A*a^3)/4 + 2*B*a^3 - 3*A*a*b^2 + 6*A*a^2*b + 6*B*a*b^2 - 3*B*a^2*b) + tan(
c/2 + (d*x)/2)^3*(6*A*b^3 - (3*A*a^3)/4 + (10*B*a^3)/3 + 3*A*a*b^2 + 10*A*
a^2*b + 18*B*a*b^2 + 3*B*a^2*b) - tan(c/2 + (d*x)/2)^5*((3*A*a^3)/4 + 6*A*
b^3 + (10*B*a^3)/3 - 3*A*a*b^2 + 10*A*a^2*b + 18*B*a*b^2 - 3*B*a^2*b) - ta
n(c/2 + (d*x)/2)*((5*A*a^3)/4 + 2*A*b^3 + 2*B*a^3 + 3*A*a*b^2 + 6*A*a^2*b
+ 6*B*a*b^2 + 3*B*a^2*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 722, normalized size of antiderivative = 3.84

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
( - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**4 - 72*cos
(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2*b**2 - 24*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b**4 + 18*cos(c + d*x)*log
(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4 + 144*cos(c + d*x)*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**2 + 48*cos(c + d*x)*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)**2*b**4 - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*a**4 - 72*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**2*b**2 - 24*cos(c + d
*x)*log(tan((c + d*x)/2) - 1)*b**4 + 9*cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*sin(c + d*x)**4*a**4 + 72*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c
 + d*x)**4*a**2*b**2 + 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d
*x)**4*b**4 - 18*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*
*4 - 144*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2
- 48*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**4 + 9*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*a**4 + 72*cos(c + d*x)*log(tan((c + d*x)
/2) + 1)*a**2*b**2 + 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b**4 - 9*co
s(c + d*x)*sin(c + d*x)**3*a**4 - 72*cos(c + d*x)*sin(c + d*x)**3*a**2*b**
2 + 15*cos(c + d*x)*sin(c + d*x)*a**4 + 72*cos(c + d*x)*sin(c + d*x)*a**2*
b**2 + 64*sin(c + d*x)**5*a**3*b + 96*sin(c + d*x)**5*a*b**3 - 160*sin(c +
 d*x)**3*a**3*b - 192*sin(c + d*x)**3*a*b**3 + 96*sin(c + d*x)*a**3*b + 96
*sin(c + d*x)*a*b**3)/(24*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d...
```

3.239 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 236

$$\begin{aligned}\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx &= \frac{(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\operatorname{arctanh}(\sin(c+dx))}{8d}\\ &+\frac{(8a^3A+30aAb^2+30a^2bB+15b^3B)\tan(c+dx)}{15d}\\ &+\frac{(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\sec(c+dx)\tan(c+dx)}{8d}\\ &+\frac{a(4a^2A+12Ab^2+15abB)\sec^2(c+dx)\tan(c+dx)}{15d}\\ &+\frac{a^2(7Ab+5aB)\sec^3(c+dx)\tan(c+dx)}{20d}\\ &+\frac{aA(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}\end{aligned}$$

output
```
1/8*(9*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*arctanh(sin(d*x+c))/d+1/15*(8*A
*a^3+30*A*a*b^2+30*B*a^2*b+15*B*b^3)*tan(d*x+c)/d+1/8*(9*A*a^2*b+4*A*b^3+3
*B*a^3+12*B*a*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*a*(4*A*a^2+12*A*b^2+15*B*a
*b)*sec(d*x+c)^2*tan(d*x+c)/d+1/20*a^2*(7*A*b+5*B*a)*sec(d*x+c)^3*tan(d*x+
c)/d+1/5*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.77

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{15(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15(9a^2Ab+4Ab^3+3a^3B+1\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]] + T
an[c + d*x]*(15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x]
+ 30*a^2*(3*A*b + a*B)*Sec[c + d*x]^3 + 8*(15*(a^3*A + 3*a*A*b^2 + 3*a^2*b
*B + b^3*B) + 5*a*(2*a^2*A + 3*A*b^2 + 3*a*b*B)*Tan[c + d*x]^2 + 3*a^3*A*T
an[c + d*x]^4)))/(120*d)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 3468, 3042, 3510, 25, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{5}\int (a+b\cos(c+dx))\left(b(2aA+5bB)\cos^2(c+dx)+\left(4Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(7Ab+5aB)\right)\sec^5(c+dx)dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b(2aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(4Aa^2+10bBa+5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$\downarrow$ 3510

$$\frac{1}{5}\left(\frac{a^2(5aB+7Ab)\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{1}{4}\int -\left(\left(4b^2(2aA+5bB)\cos^2(c+dx)+5\left(3Ba^3+9Aba^2+12b^2\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$\downarrow$ 25

$$\frac{1}{5}\left(\frac{1}{4}\int \left(4b^2(2aA+5bB)\cos^2(c+dx)+5\left(3Ba^3+9Aba^2+12b^2Ba+4Ab^3\right)\cos(c+dx)+4a\left(4Aa^2+15bBa+\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\int \frac{4b^2(2aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+5\left(3Ba^3+9Aba^2+12b^2Ba+4Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+4a\left(4Aa^2+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$\downarrow$ 3500

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \left(15\left(3Ba^3+9Aba^2+12b^2Ba+4Ab^3\right)+4\left(8Aa^3+30bBa^2+30Ab^2a+15b^3B\right)\cos(c+dx)\right)\sec^3(c-\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int\frac{15\left(3Ba^3+9Aba^2+12b^2Ba+4Ab^3\right)+4\left(8Aa^3+30bBa^2+30Ab^2a+15b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right)\int\sec^3(c+dx)dx+4\left(8a^3A+30a^2bB+30aAb^2+15b^3B\right)\int se\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(4\left(8a^3A+30a^2bB+30aAb^2+15b^3B\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+15\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4\left(8a^3A+30a^2bB+30aAb^2+15b^3B\right.}{d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{4\left(8a^3A+30a^2bB+30aAb^2+15b^3B\right.}{d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4\left(8a^3A+3\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(15\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4(8a^3\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{1}{5}\left(\frac{a^2(5aB+7Ab)\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{1}{4}\left(\frac{4a\left(4a^2A+15abB+12Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{1}{3}\left(15\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((a^2*(7*
A*b + 5*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((4*a*(4*a^2*A + 12*A*b^
2 + 15*a*b*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((4*(8*a^3*A + 30*a*A*b
^2 + 30*a^2*b*B + 15*b^3*B)*Tan[c + d*x])/d + 15*(9*a^2*A*b + 4*A*b^3 + 3*
a^3*B + 12*a*b^2*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/3)/4)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 12.79 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{a^{3} A \left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}+\frac{(A\,b^{3}+3B\,a\,b^{2})\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c)}{2}\right.}{d}$
derivativedivides	$\frac{-a^{3} A \left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)+a^{3} B\left(-\left(-\frac{\sec(dx+c)^{3}}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+}{8}\right.}{}$
default	$\frac{-a^{3} A \left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)+a^{3} B\left(-\left(-\frac{\sec(dx+c)^{3}}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+}{8}\right.}{}$
parallelrisch	$\frac{-135(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos(dx+c))\left(A\,a^{2} b+\frac{4}{9} A\,b^{3}+\frac{1}{3} a^{3} B+\frac{4}{3} B a\,b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+135(\cos(5d}{}$
risch	$-\frac{i\left(-64a^{3} A-240 A a\,b^{2}-240 B\,a^{2} b-120 b^{3} B+60 A\,b^{3} e^{9i(dx+c)}+210 B\,a^{3} e^{7i(dx+c)}-480 b^{3} B\,e^{2i(dx+c)}-60 A\,b^{3} e^{i(dx+c)}\right.}{}$

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBO
SE)
```

output
```
-a^3*A/d*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+(A*b^3+3*B*
a*b^2)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-(3*A*a*
b^2+3*B*a^2*b)/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(3*A*a^2*b+B*a^3)/d*(-
(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)
))+b^3*B/d*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.06

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{15\,(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3)\cos{(dx+c)}^5\log{(\sin{(dx+c)}+1)}-15\,(3\,Ba^3+9\,Aa^2b+12\,Ba}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="f
ricas")
```

output
```
1/240*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c)^5*log(
sin(d*x + c) + 1) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*
x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(8*A*a^3 + 30*B*a^2*b + 30*A*a*b^2
+ 15*B*b^3)*cos(d*x + c)^4 + 24*A*a^3 + 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b
^2 + 4*A*b^3)*cos(d*x + c)^3 + 8*(4*A*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d
*x + c)^2 + 30*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.44

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$

$$= \frac{16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Aa^3+240\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="m
axima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
 240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b + 240*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a*b^2 - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(si
n(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin
(d*x + c) - 1)) - 45*A*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 180*B*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*A*b^3*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*b^3*
tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(224) = 448.

Time = 0.19 (sec) , antiderivative size = 722, normalized size of antiderivative = 3.06

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="g
iac")
```

output
```
1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^
3*tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*
b*tan(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^
2*tan(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan
(1/2*d*x + 1/2*c)^9 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^3*tan(1/2*
d*x + 1/2*c)^7 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*B*a^2*b*tan(1/2*d
*x + 1/2*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 360*B*a*b^2*tan(1/2*d
*x + 1/2*c)^7 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*B*b^3*tan(1/2*d*x +
 1/2*c)^7 + 464*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x +
1/2*c)^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*B*b^3*tan(1/2*d*x + 1
/2*c)^5 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^3*tan(1/2*d*x + 1/2*c)
^3 - 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^
3 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^
3 - 120*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^3 +
120*A*a^3*tan(1/2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d*x + 1/2*c) + 225*A*a^2
*b*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*t
an(1/2*d*x + 1/2*c) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c) + 60*A*b^3*tan(1/2*
d*x + 1/2*c) + 120*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 ...
```

Mupad [B] (verification not implemented)

Time = 45.59 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.99

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Ba^3}{8}+\frac{9Aa^2b}{8}+\frac{3Bab^2}{2}+\frac{Ab^3}{2}\right)}{\frac{3Ba^3}{2}+\frac{9Aa^2b}{2}+6Bab^2+2Ab^3}\right)\left(\frac{3Ba^3}{4}+\frac{9Aa^2b}{4}+3Bab^2+Ab^3\right)}{d}$$
$$-\frac{\left(2Aa^3-Ab^3-\frac{5Ba^3}{4}+2Bb^3+6Aab^2-\frac{15Aa^2b}{4}-3Bab^2+6Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(2Ab^3-\ldots\right.}{}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^6,x)
```

output
```
(atanh((4*tan(c/2 + (d*x)/2)*((A*b^3)/2 + (3*B*a^3)/8 + (9*A*a^2*b)/8 + (3
*B*a*b^2)/2))/(2*A*b^3 + (3*B*a^3)/2 + (9*A*a^2*b)/2 + 6*B*a*b^2))*(A*b^3
+ (3*B*a^3)/4 + (9*A*a^2*b)/4 + 3*B*a*b^2))/d - (tan(c/2 + (d*x)/2)*(2*A*a
^3 + A*b^3 + (5*B*a^3)/4 + 2*B*b^3 + 6*A*a*b^2 + (15*A*a^2*b)/4 + 3*B*a*b^
2 + 6*B*a^2*b) + tan(c/2 + (d*x)/2)^5*((116*A*a^3)/15 + 12*B*b^3 + 20*A*a*
b^2 + 20*B*a^2*b) + tan(c/2 + (d*x)/2)^9*(2*A*a^3 - A*b^3 - (5*B*a^3)/4 +
2*B*b^3 + 6*A*a*b^2 - (15*A*a^2*b)/4 - 3*B*a*b^2 + 6*B*a^2*b) - tan(c/2 +
(d*x)/2)^3*((8*A*a^3)/3 + 2*A*b^3 + (B*a^3)/2 + 8*B*b^3 + 16*A*a*b^2 + (3*
A*a^2*b)/2 + 6*B*a*b^2 + 16*B*a^2*b) - tan(c/2 + (d*x)/2)^7*((8*A*a^3)/3 -
 2*A*b^3 - (B*a^3)/2 + 8*B*b^3 + 16*A*a*b^2 - (3*A*a^2*b)/2 - 6*B*a*b^2 +
16*B*a^2*b))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan
(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.47

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

output
```
( - 45*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3*b - 60*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**3 + 90*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b + 120*cos(c + d*x)*
log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**3 - 45*cos(c + d*x)*log(tan
((c + d*x)/2) - 1)*a**3*b - 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b*
*3 + 45*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**3*b + 60
*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*b**3 - 90*cos(c
+ d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3*b - 120*cos(c + d*x)
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**3 + 45*cos(c + d*x)*log(ta
n((c + d*x)/2) + 1)*a**3*b + 60*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b
**3 - 45*cos(c + d*x)*sin(c + d*x)**3*a**3*b - 60*cos(c + d*x)*sin(c + d*x
)**3*a*b**3 + 75*cos(c + d*x)*sin(c + d*x)*a**3*b + 60*cos(c + d*x)*sin(c
+ d*x)*a*b**3 + 16*sin(c + d*x)**5*a**4 + 120*sin(c + d*x)**5*a**2*b**2 +
30*sin(c + d*x)**5*b**4 - 40*sin(c + d*x)**3*a**4 - 300*sin(c + d*x)**3*a*
*2*b**2 - 60*sin(c + d*x)**3*b**4 + 30*sin(c + d*x)*a**4 + 180*sin(c + d*x
)*a**2*b**2 + 30*sin(c + d*x)*b**4)/(30*cos(c + d*x)*d*(sin(c + d*x)**4 -
2*sin(c + d*x)**2 + 1))
```

3.240 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 366

$$\begin{aligned}
&\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx \\
&= \frac{1}{16}\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B\right)x \\
&\quad+\frac{(140a^3Ab+112aAb^3+35a^4B+168a^2b^2B+24b^4B)\sin(c+dx)}{35d} \\
&\quad+\frac{(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B)\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad+\frac{b(224a^2Ab+35Ab^3+104a^3B+140ab^2B)\cos^3(c+dx)\sin(c+dx)}{168d} \\
&\quad+\frac{b^2(49aAb+31a^2B+18b^2B)\cos^4(c+dx)\sin(c+dx)}{105d} \\
&\quad+\frac{b(7Ab+10aB)\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{42d} \\
&\quad+\frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{7d} \\
&\quad-\frac{(140a^3Ab+112aAb^3+35a^4B+168a^2b^2B+24b^4B)\sin^3(c+dx)}{105d}
\end{aligned}$$

output
```
1/16*(8*A*a^4+36*A*a^2*b^2+5*A*b^4+24*B*a^3*b+20*B*a*b^3)*x+1/35*(140*A*a^
3*b+112*A*a*b^3+35*B*a^4+168*B*a^2*b^2+24*B*b^4)*sin(d*x+c)/d+1/16*(8*A*a^
4+36*A*a^2*b^2+5*A*b^4+24*B*a^3*b+20*B*a*b^3)*cos(d*x+c)*sin(d*x+c)/d+1/16
8*b*(224*A*a^2*b+35*A*b^3+104*B*a^3+140*B*a*b^2)*cos(d*x+c)^3*sin(d*x+c)/d
+1/105*b^2*(49*A*a*b+31*B*a^2+18*B*b^2)*cos(d*x+c)^4*sin(d*x+c)/d+1/42*b*(
7*A*b+10*B*a)*cos(d*x+c)^3*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/7*b*B*cos(d*x
+c)^3*(a+b*cos(d*x+c))^3*sin(d*x+c)/d-1/105*(140*A*a^3*b+112*A*a*b^3+35*B*
a^4+168*B*a^2*b^2+24*B*b^4)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.11

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{3360a^4Ac+15120a^2Ab^2c+2100Ab^4c+10080a^3bBc+8400ab^3Bc+3360a^4Adx+15120a^2Ab^2dx+210}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(3360*a^4*A*c + 15120*a^2*A*b^2*c + 2100*A*b^4*c + 10080*a^3*b*B*c + 8400*
a*b^3*B*c + 3360*a^4*A*d*x + 15120*a^2*A*b^2*d*x + 2100*A*b^4*d*x + 10080*
a^3*b*B*d*x + 8400*a*b^3*B*d*x + 105*(192*a^3*A*b + 160*a*A*b^3 + 48*a^4*B
 + 240*a^2*b^2*B + 35*b^4*B)*Sin[c + d*x] + 105*(16*a^4*A + 96*a^2*A*b^2 +
 15*A*b^4 + 64*a^3*b*B + 60*a*b^3*B)*Sin[2*(c + d*x)] + 2240*a^3*A*b*Sin[3
*(c + d*x)] + 2800*a*A*b^3*Sin[3*(c + d*x)] + 560*a^4*B*Sin[3*(c + d*x)] +
 4200*a^2*b^2*B*Sin[3*(c + d*x)] + 735*b^4*B*Sin[3*(c + d*x)] + 1260*a^2*A
*b^2*Sin[4*(c + d*x)] + 315*A*b^4*Sin[4*(c + d*x)] + 840*a^3*b*B*Sin[4*(c
+ d*x)] + 1260*a*b^3*B*Sin[4*(c + d*x)] + 336*a*A*b^3*Sin[5*(c + d*x)] + 5
04*a^2*b^2*B*Sin[5*(c + d*x)] + 147*b^4*B*Sin[5*(c + d*x)] + 35*A*b^4*Sin[
6*(c + d*x)] + 140*a*b^3*B*Sin[6*(c + d*x)] + 15*b^4*B*Sin[7*(c + d*x)])/(
6720*d)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.86, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {3042, 3469, 3042, 3528, 3042, 3512, 3042, 3502, 27, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{7}\int \cos^2(c+dx)(a+b\cos(c+ dx))^2\left(b(7Ab+10aB)\cos^2(c+dx)+\left(6Bb^2+7a(2Ab+aB)\right)\cos(c+dx)+a(7aA+3bB)\right)dx+\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(b(7Ab+10aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(6Bb^2+7a(2Ab+aB\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{1}{7}\left(\frac{1}{6}\int \cos^2(c+dx)(a+b\cos(c+dx))\left(2b\left(31Ba^2+49Aba+18b^2B\right)\cos^2(c+dx)+\left(42Ba^3+126Aba^2+104b\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(2b\left(31Ba^2+49Aba+18b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(42Ba\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \; 3512$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\int \cos^2(c+dx)\left(15\left(14Aa^2+16bBa+7Ab^2\right)a^2+5b\left(104Ba^3+224Aba^2+140b^2Ba+35Ab^3\right)\cos^2(c+\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(15\left(14Aa^2+16bBa+7Ab^2\right)a^2+5b\left(104Ba^3+224Aba^2+140b^2Ba+35Ab^3\right)\text{si}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\int 3\cos^2(c+dx)\left(35\left(8Aa^4+24bBa^3+36Ab^2a^2+20b^3Ba+5Ab^4\right)+8\left(35Ba^4+140Aba^3+168b^2\right.\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int \cos^2(c+dx)\left(35\left(8Aa^4+24bBa^3+36Ab^2a^2+20b^3Ba+5Ab^4\right)+8\left(35Ba^4+140Aba^3+168b^2B\right.\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(35\left(8Aa^4+24bBa^3+36Ab^2a^2+20b^3Ba+5Ab^4\right)+8\left(35Ba^4+140Aba^3+1\right.\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$$\Big\downarrow \; 3227$$

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(8\left(35a^4B+140a^3Ab+168a^2b^2B+112aAb^3+24b^4B\right)\int \cos^3(c+dx)dx+35\left(8a^4A+24a^3bB+3\right.\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(35\left(8a^4A+24a^3bB+36a^2Ab^2+20ab^3B+5Ab^4\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+8\left(35a^4B+140a^3Ab\right.\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$\big\downarrow$ 3113

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(35\left(8a^4A+24a^3bB+36a^2Ab^2+20ab^3B+5Ab^4\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8\left(35a^4B+140a^3Ab\right.}{}\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$\big\downarrow$ 2009

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(35\left(8a^4A+24a^3bB+36a^2Ab^2+20ab^3B+5Ab^4\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{8\left(35a^4B+140a^3Ab\right.}{}\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$\big\downarrow$ 3115

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\left(35\left(8a^4A+24a^3bB+36a^2Ab^2+20ab^3B+5Ab^4\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{8\left(35a^4B\right.}{}\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

$\big\downarrow$ 24

$$\frac{1}{7}\left(\frac{1}{6}\left(\frac{2b^2\left(31a^2B+49aAb+18b^2B\right)\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{1}{5}\left(\frac{5b\left(104a^3B+224a^2Ab+140ab^2B+35Ab^3\right.}{4d}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^3(c+dx)(a+b\cos(c+dx))^3}{7d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(b*B*Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) + ((b*(7*A*
b + 10*a*B)*Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(6*d) + ((
2*b^2*(49*a*A*b + 31*a^2*B + 18*b^2*B)*Cos[c + d*x]^4*Sin[c + d*x])/(5*d)
+ ((5*b*(224*a^2*A*b + 35*A*b^3 + 104*a^3*B + 140*a*b^2*B)*Cos[c + d*x]^3*
Sin[c + d*x])/(4*d) + (3*(35*(8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*
B + 20*a*b^3*B)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (8*(140*a^3*A*
b + 112*a*A*b^3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*(-Sin[c + d*x] + Si
n[c + d*x]^3/3))/d))/4)/5)/6)/7
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [A] (verified)

Time = 5.77 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.01

$$a^4A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{B\,a^4\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+\frac{4A\,a^3b\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3}+4B\,a^3b\left(\frac{\left(\cos(dx+\right.}{}\right.$$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(a^4*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*B*a^4*(cos(d*x+c)
^2+2)*sin(d*x+c)+4/3*A*a^3*b*(cos(d*x+c)^2+2)*sin(d*x+c)+4*B*a^3*b*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*a^2*b^2*(1/4*(co
s(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*B*a^2*b^2*(8/3+co
s(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/5*A*a*b^3*(8/3+cos(d*x+c)^4+4/3*
cos(d*x+c)^2)*sin(d*x+c)+4*B*a*b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/
8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+A*b^4*(1/6*(cos(d*x+c)^5+5/4*cos
(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/7*B*b^4*(16/5+cos
(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{105\,(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4)dx+\left(240\,Bb^4\cos\left(dx+c\right)^6+280\,(4\,Bab^3+Ab^4)\,c\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/1680*(105*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*d
*x + (240*B*b^4*cos(d*x + c)^6 + 280*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^5 +
1120*B*a^4 + 4480*A*a^3*b + 5376*B*a^2*b^2 + 3584*A*a*b^3 + 768*B*b^4 + 96
*(21*B*a^2*b^2 + 14*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4 + 70*(24*B*a^3*b + 3
6*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c)^3 + 16*(35*B*a^4 + 140*A*
a^3*b + 168*B*a^2*b^2 + 112*A*a*b^3 + 24*B*b^4)*cos(d*x + c)^2 + 105*(8*A*
a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. 2(391) = 782.

Time = 0.62 (sec) , antiderivative size = 1017, normalized size of antiderivative = 2.78

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**4*x*sin(c + d*x)**2/2 + A*a**4*x*cos(c + d*x)**2/2 + A*a**
4*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*A*a**3*b*sin(c + d*x)**3/(3*d) + 4*A
*a**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a**2*b**2*x*sin(c + d*x)**4/4
 + 9*A*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*A*a**2*b**2*x*cos
(c + d*x)**4/4 + 9*A*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*A*a
**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*A*a*b**3*sin(c + d*x)**5/
(15*d) + 16*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*A*a*b**3*si
n(c + d*x)*cos(c + d*x)**4/d + 5*A*b**4*x*sin(c + d*x)**6/16 + 15*A*b**4*x
*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*A*b**4*x*sin(c + d*x)**2*cos(c +
d*x)**4/16 + 5*A*b**4*x*cos(c + d*x)**6/16 + 5*A*b**4*sin(c + d*x)**5*cos(
c + d*x)/(16*d) + 5*A*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*A*b*
*4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) +
B*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a**3*b*x*sin(c + d*x)**4/2 + 3
*B*a**3*b*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*B*a**3*b*x*cos(c + d*x)**4
/2 + 3*B*a**3*b*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a**3*b*sin(c + d*
x)*cos(c + d*x)**3/(2*d) + 16*B*a**2*b**2*sin(c + d*x)**5/(5*d) + 8*B*a**2
*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 6*B*a**2*b**2*sin(c + d*x)*cos(c
 + d*x)**4/d + 5*B*a*b**3*x*sin(c + d*x)**6/4 + 15*B*a*b**3*x*sin(c + d*x)
**4*cos(c + d*x)**2/4 + 15*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/4 +
5*B*a*b**3*x*cos(c + d*x)**6/4 + 5*B*a*b**3*sin(c + d*x)**5*cos(c + d*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{1680\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^4-2240\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^4-8960\,\left(\sin\left(dx+c\right)^3\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
1/6720*(1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 - 2240*(sin(d*x + c)^3
 - 3*sin(d*x + c))*B*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3*b
+ 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3*b + 12
60*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b^2 + 268
8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^2*b^2 + 179
2*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a*b^3 - 140*(
4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*B*a*b^3 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x +
4*c) - 48*sin(2*d*x + 2*c))*A*b^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c
)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*B*b^4)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.86

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{Bb^4\sin(7\,dx+7\,c)}{448\,d} + \frac{1}{16}\left(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4\right)x$$
$$+ \frac{(4\,Bab^3+Ab^4)\sin(6\,dx+6\,c)}{192\,d} + \frac{(24\,Ba^2b^2+16\,Aab^3+7\,Bb^4)\sin(5\,dx+5\,c)}{320\,d}$$
$$+ \frac{(8\,Ba^3b+12\,Aa^2b^2+12\,Bab^3+3\,Ab^4)\sin(4\,dx+4\,c)}{64\,d}$$
$$+ \frac{(16\,Ba^4+64\,Aa^3b+120\,Ba^2b^2+80\,Aab^3+21\,Bb^4)\sin(3\,dx+3\,c)}{192\,d}$$
$$+ \frac{(16\,Aa^4+64\,Ba^3b+96\,Aa^2b^2+60\,Bab^3+15\,Ab^4)\sin(2\,dx+2\,c)}{64\,d}$$
$$+ \frac{(48\,Ba^4+192\,Aa^3b+240\,Ba^2b^2+160\,Aab^3+35\,Bb^4)\sin(dx+c)}{64\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/448*B*b^4*sin(7*d*x + 7*c)/d + 1/16*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
 + 20*B*a*b^3 + 5*A*b^4)*x + 1/192*(4*B*a*b^3 + A*b^4)*sin(6*d*x + 6*c)/d
+ 1/320*(24*B*a^2*b^2 + 16*A*a*b^3 + 7*B*b^4)*sin(5*d*x + 5*c)/d + 1/64*(8
*B*a^3*b + 12*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*sin(4*d*x + 4*c)/d + 1/192
*(16*B*a^4 + 64*A*a^3*b + 120*B*a^2*b^2 + 80*A*a*b^3 + 21*B*b^4)*sin(3*d*x
 + 3*c)/d + 1/64*(16*A*a^4 + 64*B*a^3*b + 96*A*a^2*b^2 + 60*B*a*b^3 + 15*A
*b^4)*sin(2*d*x + 2*c)/d + 1/64*(48*B*a^4 + 192*A*a^3*b + 240*B*a^2*b^2 +
160*A*a*b^3 + 35*B*b^4)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 45.59 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.19

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{420\,A\,a^4\,\sin\left(2\,c+2\,d\,x\right)+\frac{1575\,A\,b^4\,\sin(2\,c+2\,d\,x)}{4}+140\,B\,a^4\,\sin\left(3\,c+3\,d\,x\right)+\frac{315\,A\,b^4\,\sin(4\,c+4\,d\,x)}{4}+\frac{35\,A\,b^4\,\sin(}{4}}{}$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

output
```
(420*A*a^4*sin(2*c + 2*d*x) + (1575*A*b^4*sin(2*c + 2*d*x))/4 + 140*B*a^4*
sin(3*c + 3*d*x) + (315*A*b^4*sin(4*c + 4*d*x))/4 + (35*A*b^4*sin(6*c + 6*
d*x))/4 + (735*B*b^4*sin(3*c + 3*d*x))/4 + (147*B*b^4*sin(5*c + 5*d*x))/4
+ (15*B*b^4*sin(7*c + 7*d*x))/4 + 1260*B*a^4*sin(c + d*x) + (3675*B*b^4*si
n(c + d*x))/4 + 4200*A*a*b^3*sin(c + d*x) + 5040*A*a^3*b*sin(c + d*x) + 84
0*A*a^4*d*x + 525*A*b^4*d*x + 700*A*a*b^3*sin(3*c + 3*d*x) + 560*A*a^3*b*s
in(3*c + 3*d*x) + 84*A*a*b^3*sin(5*c + 5*d*x) + 1575*B*a*b^3*sin(2*c + 2*d
*x) + 1680*B*a^3*b*sin(2*c + 2*d*x) + 315*B*a*b^3*sin(4*c + 4*d*x) + 210*B
*a^3*b*sin(4*c + 4*d*x) + 35*B*a*b^3*sin(6*c + 6*d*x) + 6300*B*a^2*b^2*sin
(c + d*x) + 2520*A*a^2*b^2*sin(2*c + 2*d*x) + 315*A*a^2*b^2*sin(4*c + 4*d*
x) + 1050*B*a^2*b^2*sin(3*c + 3*d*x) + 126*B*a^2*b^2*sin(5*c + 5*d*x) + 21
00*B*a*b^3*d*x + 2520*B*a^3*b*d*x + 3780*A*a^2*b^2*d*x)/(1680*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.74

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{1400\cos(dx+c)\sin(dx+c)^5\,a\,b^4 - 4200\cos(dx+c)\sin(dx+c)^3\,a^3b^2 - 4550\cos(dx+c)\sin(dx+c)}{}$$

input
```
int(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
(1400*cos(c + d*x)*sin(c + d*x)**5*a*b**4 - 4200*cos(c + d*x)*sin(c + d*x)
**3*a**3*b**2 - 4550*cos(c + d*x)*sin(c + d*x)**3*a*b**4 + 840*cos(c + d*x
)*sin(c + d*x)*a**5 + 10500*cos(c + d*x)*sin(c + d*x)*a**3*b**2 + 5775*cos
(c + d*x)*sin(c + d*x)*a*b**4 - 240*sin(c + d*x)**7*b**5 + 3360*sin(c + d*
x)**5*a**2*b**3 + 1008*sin(c + d*x)**5*b**5 - 2800*sin(c + d*x)**3*a**4*b
- 11200*sin(c + d*x)**3*a**2*b**3 - 1680*sin(c + d*x)**3*b**5 + 8400*sin(c
 + d*x)*a**4*b + 16800*sin(c + d*x)*a**2*b**3 + 1680*sin(c + d*x)*b**5 + 8
40*a**5*d*x + 6300*a**3*b**2*d*x + 2625*a*b**4*d*x)/(1680*d)
```

3.241 $\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 325

$$\begin{aligned}
&\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx\\
&=\frac{1}{16}\left(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B\right)x\\
&+\frac{(24a^4Ab+224a^2Ab^3+32Ab^5-4a^5B+121a^3b^2B+128ab^4B)\sin(c+dx)}{60bd}\\
&+\frac{(48a^3Ab+232aAb^3-8a^4B+178a^2b^2B+75b^4B)\cos(c+dx)\sin(c+dx)}{240d}\\
&+\frac{(24a^2Ab+32Ab^3-4a^3B+53ab^2B)\,(a+b\cos(c+dx))^2\sin(c+dx)}{120bd}\\
&+\frac{(24aAb-4a^2B+25b^2B)\,(a+b\cos(c+dx))^3\sin(c+dx)}{120bd}\\
&+\frac{(6Ab-aB)(a+b\cos(c+dx))^4\sin(c+dx)}{30bd}+\frac{B(a+b\cos(c+dx))^5\sin(c+dx)}{6bd}
\end{aligned}$$

output
```
1/16*(32*A*a^3*b+24*A*a*b^3+8*B*a^4+36*B*a^2*b^2+5*B*b^4)*x+1/60*(24*A*a^4
*b+224*A*a^2*b^3+32*A*b^5-4*B*a^5+121*B*a^3*b^2+128*B*a*b^4)*sin(d*x+c)/b/
d+1/240*(48*A*a^3*b+232*A*a*b^3-8*B*a^4+178*B*a^2*b^2+75*B*b^4)*cos(d*x+c)
*sin(d*x+c)/d+1/120*(24*A*a^2*b+32*A*b^3-4*B*a^3+53*B*a*b^2)*(a+b*cos(d*x+
c))^2*sin(d*x+c)/b/d+1/120*(24*A*a*b-4*B*a^2+25*B*b^2)*(a+b*cos(d*x+c))^3*
sin(d*x+c)/b/d+1/30*(6*A*b-B*a)*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d+1/6*B*(a
+b*cos(d*x+c))^5*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 4.21 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.02

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{1920a^3Abc + 1440aAb^3c + 480a^4Bc + 2160a^2b^2Bc + 300b^4Bc + 1920a^3Abdx + 1440aAb^3dx + 480a^4Bc}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(1920*a^3*A*b*c + 1440*a*A*b^3*c + 480*a^4*B*c + 2160*a^2*b^2*B*c + 300*b^
4*B*c + 1920*a^3*A*b*d*x + 1440*a*A*b^3*d*x + 480*a^4*B*d*x + 2160*a^2*b^2
*B*d*x + 300*b^4*B*d*x + 120*(8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*
B + 20*a*b^3*B)*Sin[c + d*x] + 15*(64*a^3*A*b + 64*a*A*b^3 + 16*a^4*B + 96
*a^2*b^2*B + 15*b^4*B)*Sin[2*(c + d*x)] + 480*a^2*A*b^2*Sin[3*(c + d*x)] +
 100*A*b^4*Sin[3*(c + d*x)] + 320*a^3*b*B*Sin[3*(c + d*x)] + 400*a*b^3*B*S
in[3*(c + d*x)] + 120*a*A*b^3*Sin[4*(c + d*x)] + 180*a^2*b^2*B*Sin[4*(c +
d*x)] + 45*b^4*B*Sin[4*(c + d*x)] + 12*A*b^4*Sin[5*(c + d*x)] + 48*a*b^3*B
*Sin[5*(c + d*x)] + 5*b^4*B*Sin[6*(c + d*x)])/(960*d)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {3042, 3447, 3042, 3502, 3042, 3232, 3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a+b\cos(c+dx))^4\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (a+b\cos(c+dx))^4(5bB+(6Ab-aB)\cos(c+dx))dx}{6b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(5bB+(6Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{6b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{5}\int (a+b\cos(c+dx))^3\left(3b(8Ab+7aB)+\left(-4Ba^2+24Aba+25b^2B\right)\cos(c+dx)\right)dx+\frac{(6Ab-aB)\sin(c+dx)(a+b\,\text{co}}{5d}}{6b}+\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(3b(8Ab+7aB)+\left(-4Ba^2+24Aba+25b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{(6Ab-aB)\sin(}{}}{6b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$\downarrow$ 3232

$$\frac{\frac{1}{5}\left(\frac{1}{4}\int 3(a+b\cos(c+dx))^2\left(b\left(24Ba^2+56Aba+25b^2B\right)+\left(-4Ba^3+24Aba^2+53b^2Ba+32Ab^3\right)\cos(c+dx)\right)}{6b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$\downarrow$ 27

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int (a+b\cos(c+dx))^2\left(b\left(24Ba^2+56Aba+25b^2B\right)+\left(-4Ba^3+24Aba^2+53b^2Ba+32Ab^3\right)\cos(c+dx)\right)d}{6b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(b\left(24Ba^2+56Aba+25b^2B\right)+\left(-4Ba^3+24Aba^2+53b^2Ba+32Ab^3\right)\sin\left(c+\right.\right.\right.}{6b}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$\downarrow$ 3232

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(b\left(64Ba^3+216Aba^2+181b^2Ba+64Ab^3\right)+\left(-8Ba^4+48Aba^3+178b^2Ba^2+232\right.\right.\right.\right.}{}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(64Ba^3+216Aba^2+181b^2Ba+64Ab^3\right)+\left(-8Ba^4+48Aba^3+178b^2Ba^2\right.\right.\right.\right.}{}$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$\downarrow$ 3213

$$\frac{\frac{1}{5}\left(\frac{(-4a^2B+24aAb+25b^2B)\sin(c+dx)(a+b\cos(c+dx))^3}{4d}+\frac{3}{4}\left(\frac{(-4a^3B+24a^2Ab+53ab^2B+32Ab^3)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{b(-8\right.\right.\right.}{}$$

$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d) + (((6*A*b - a*B)*(a + b*C
os[c + d*x])^4*Sin[c + d*x])/(5*d) + (((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a
+ b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (3*(((24*a^2*A*b + 32*A*b^3 - 4*
a^3*B + 53*a*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((15*b*(3
2*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*x)/2 + (2*(24*a
^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*a^3*b^2*B + 128*a*b^4*B)
*Sin[c + d*x])/d + (b*(48*a^3*A*b + 232*a*A*b^3 - 8*a^4*B + 178*a^2*b^2*B
+ 75*b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3))/4)/5)/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 212.17 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.74

method	result
parts	$\frac{(A\,b^4+4Ba\,b^3)\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}+\frac{(4Aa\,b^3+6B\,a^2b^2)\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+}{4}\right.}{d}$
parallelrisch	$\frac{(960A\,a^3b+960Aa\,b^3+240B\,a^4+1440B\,a^2b^2+225B\,b^4)\sin(2dx+2c)+(480A\,a^2b^2+100A\,b^4+320B\,a^3b+400Ba\,b^3)\sin}{}$
derivativedivides	$\frac{a^4A\sin(dx+c)+B\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4A\,a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4B\,a^3b\left(\cos(dx+c)^2+2\right)\sin(}{3}}{}$
default	$\frac{a^4A\sin(dx+c)+B\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4A\,a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4B\,a^3b\left(\cos(dx+c)^2+2\right)\sin(}{3}}{}$
risch	$\frac{5b^4Bx}{16}+\frac{\sin(2dx+2c)B\,a^4}{4d}+2xA\,a^3b+\frac{3xAa\,b^3}{2}+\frac{9xB\,a^2b^2}{4}+\frac{\sin(dx+c)a^4A}{d}+\frac{5\sin(dx+c)A\,b^4}{8d}+\sin$
norman	$\frac{(2A\,a^3b+\frac{3}{2}Aa\,b^3+\frac{1}{2}B\,a^4+\frac{9}{4}B\,a^2b^2+\frac{5}{16}B\,b^4)x+(2A\,a^3b+\frac{3}{2}Aa\,b^3+\frac{1}{2}B\,a^4+\frac{9}{4}B\,a^2b^2+\frac{5}{16}B\,b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(12A}{}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/5*(A*b^4+4*B*a*b^3)/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+(4*
A*a*b^3+6*B*a^2*b^2)/d*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+1/3*(6*A*a^2*b^2+4*B*a^3*b)/d*(cos(d*x+c)^2+2)*sin(d*x+c)+(4*A*a
^3*b+B*a^4)/d*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b^4/d*(1/6*(cos(
d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/d
*sin(d*x+c)*a^4*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.75

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$=\frac{15\,(8\,Ba^4+32\,Aa^3b+36\,Ba^2b^2+24\,Aab^3+5\,Bb^4)dx+\left(40\,Bb^4\cos\left(dx+c\right)^5+240\,Aa^4+640\,Ba^3b+\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/240*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*d*x
 + (40*B*b^4*cos(d*x + c)^5 + 240*A*a^4 + 640*B*a^3*b + 960*A*a^2*b^2 + 51
2*B*a*b^3 + 128*A*b^4 + 48*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 10*(36*B*a
^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^3 + 32*(10*B*a^3*b + 15*A*a^2*
b^2 + 8*B*a*b^3 + 2*A*b^4)*cos(d*x + c)^2 + 15*(8*B*a^4 + 32*A*a^3*b + 36*
B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(335) = 670$.

Time = 0.46 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.50

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**4*sin(c + d*x)/d + 2*A*a**3*b*x*sin(c + d*x)**2 + 2*A*a**3
*b*x*cos(c + d*x)**2 + 2*A*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*A*a**2*b
**2*sin(c + d*x)**3/d + 6*A*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A
*a*b**3*x*sin(c + d*x)**4/2 + 3*A*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2
 + 3*A*a*b**3*x*cos(c + d*x)**4/2 + 3*A*a*b**3*sin(c + d*x)**3*cos(c + d*x
)/(2*d) + 5*A*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*A*b**4*sin(c +
 d*x)**5/(15*d) + 4*A*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*b**4*
sin(c + d*x)*cos(c + d*x)**4/d + B*a**4*x*sin(c + d*x)**2/2 + B*a**4*x*cos
(c + d*x)**2/2 + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*B*a**3*b*sin(c
 + d*x)**3/(3*d) + 4*B*a**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**2*b*
*2*x*sin(c + d*x)**4/4 + 9*B*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2
 + 9*B*a**2*b**2*x*cos(c + d*x)**4/4 + 9*B*a**2*b**2*sin(c + d*x)**3*cos(c
 + d*x)/(4*d) + 15*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*B*a
*b**3*sin(c + d*x)**5/(15*d) + 16*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + 4*B*a*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*b**4*x*sin(c + d*
x)**6/16 + 15*B*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**4*x*si
n(c + d*x)**2*cos(c + d*x)**4/16 + 5*B*b**4*x*cos(c + d*x)**6/16 + 5*B*b**
4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*B*b**4*sin(c + d*x)**3*cos(c + d
*x)**3/(6*d) + 11*B*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (
x*(A + B*cos(c))*(a + b*cos(c))**4*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.94

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{240\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^4+960\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^3b-1280\,\left(\sin\left(dx+c\right)^3-3\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2*c + sin
(2*d*x + 2*c))*A*a^3*b - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3*b -
1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b^2 + 180*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b^2 + 120*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^3 + 256*(3*sin(d*x + c)^5 - 10*si
n(d*x + c)^3 + 15*sin(d*x + c))*B*a*b^3 + 64*(3*sin(d*x + c)^5 - 10*sin(d*
x + c)^3 + 15*sin(d*x + c))*A*b^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*
c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^4 + 960*A*a^4*sin(d*x +
c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{Bb^4\sin(6\,dx+6\,c)}{192\,d} + \frac{1}{16}\left(8\,Ba^4+32\,Aa^3b+36\,Ba^2b^2+24\,Aab^3+5\,Bb^4\right)x + \frac{(4\,Bab^3+Ab^4)\sin(5\,dx+5\,c)}{80\,d} + \frac{(12\,Ba^2b^2+8\,Aab^3+3\,Bb^4)\sin(4\,dx+4\,c)}{64\,d} + \frac{(16\,Ba^3b+24\,Aa^2b^2+20\,Bab^3+5\,Ab^4)\sin(3\,dx+3\,c)}{48\,d} + \frac{(16\,Ba^4+64\,Aa^3b+96\,Ba^2b^2+64\,Aab^3+15\,Bb^4)\sin(2\,dx+2\,c)}{64\,d} + \frac{(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/192*B*b^4*sin(6*d*x + 6*c)/d + 1/16*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2
 + 24*A*a*b^3 + 5*B*b^4)*x + 1/80*(4*B*a*b^3 + A*b^4)*sin(5*d*x + 5*c)/d +
 1/64*(12*B*a^2*b^2 + 8*A*a*b^3 + 3*B*b^4)*sin(4*d*x + 4*c)/d + 1/48*(16*B
*a^3*b + 24*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*sin(3*d*x + 3*c)/d + 1/64*(1
6*B*a^4 + 64*A*a^3*b + 96*B*a^2*b^2 + 64*A*a*b^3 + 15*B*b^4)*sin(2*d*x + 2
*c)/d + 1/8*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*s
in(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.90 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.24

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{B\,a^4\,x}{2} + \frac{5\,B\,b^4\,x}{16} + \frac{3\,A\,a\,b^3\,x}{2} + 2\,A\,a^3\,b\,x + \frac{A\,a^4\,\sin(c+d\,x)}{d} + \frac{5\,A\,b^4\,\sin(c+d\,x)}{8\,d} + \frac{9\,B\,a^2\,b^2\,x}{4} + \frac{B\,a^4\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{5\,A\,b^4\,\sin(3\,c+3\,d\,x)}{48\,d} + \frac{A\,b^4\,\sin(5\,c+5\,d\,x)}{80\,d} + \frac{15\,B\,b^4\,\sin(2\,c+2\,d\,x)}{64\,d} + \frac{3\,B\,b^4\,\sin(4\,c+4\,d\,x)}{64\,d} + \frac{B\,b^4\,\sin(6\,c+6\,d\,x)}{192\,d} + \frac{A\,a\,b^3\,\sin(2\,c+2\,d\,x)}{d} + \frac{A\,a^3\,b\,\sin(2\,c+2\,d\,x)}{d} + \frac{A\,a\,b^3\,\sin(4\,c+4\,d\,x)}{8\,d} + \frac{9\,A\,a^2\,b^2\,\sin(c+d\,x)}{2\,d} + \frac{5\,B\,a\,b^3\,\sin(3\,c+3\,d\,x)}{12\,d} + \frac{B\,a^3\,b\,\sin(3\,c+3\,d\,x)}{3\,d} + \frac{B\,a\,b^3\,\sin(5\,c+5\,d\,x)}{20\,d} + \frac{A\,a^2\,b^2\,\sin(3\,c+3\,d\,x)}{2\,d} + \frac{3\,B\,a^2\,b^2\,\sin(2\,c+2\,d\,x)}{2\,d} + \frac{3\,B\,a^2\,b^2\,\sin(4\,c+4\,d\,x)}{16\,d} + \frac{5\,B\,a\,b^3\,\sin(c+d\,x)}{2\,d} + \frac{3\,B\,a^3\,b\,\sin(c+d\,x)}{d}$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

output
```
(B*a^4*x)/2 + (5*B*b^4*x)/16 + (3*A*a*b^3*x)/2 + 2*A*a^3*b*x + (A*a^4*sin(
c + d*x))/d + (5*A*b^4*sin(c + d*x))/(8*d) + (9*B*a^2*b^2*x)/4 + (B*a^4*si
n(2*c + 2*d*x))/(4*d) + (5*A*b^4*sin(3*c + 3*d*x))/(48*d) + (A*b^4*sin(5*c
 + 5*d*x))/(80*d) + (15*B*b^4*sin(2*c + 2*d*x))/(64*d) + (3*B*b^4*sin(4*c
+ 4*d*x))/(64*d) + (B*b^4*sin(6*c + 6*d*x))/(192*d) + (A*a*b^3*sin(2*c + 2
*d*x))/d + (A*a^3*b*sin(2*c + 2*d*x))/d + (A*a*b^3*sin(4*c + 4*d*x))/(8*d)
 + (9*A*a^2*b^2*sin(c + d*x))/(2*d) + (5*B*a*b^3*sin(3*c + 3*d*x))/(12*d)
+ (B*a^3*b*sin(3*c + 3*d*x))/(3*d) + (B*a*b^3*sin(5*c + 5*d*x))/(20*d) + (
A*a^2*b^2*sin(3*c + 3*d*x))/(2*d) + (3*B*a^2*b^2*sin(2*c + 2*d*x))/(2*d) +
 (3*B*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (5*B*a*b^3*sin(c + d*x))/(2*d) +
(3*B*a^3*b*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.70

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{8\cos(dx+c)\sin(dx+c)^5 b^5 - 120\cos(dx+c)\sin(dx+c)^3 a^2b^3 - 26\cos(dx+c)\sin(dx+c)^3 b^5 + 12}{}$$

input
```
int(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
(8*cos(c + d*x)*sin(c + d*x)**5*b**5 - 120*cos(c + d*x)*sin(c + d*x)**3*a*
*2*b**3 - 26*cos(c + d*x)*sin(c + d*x)**3*b**5 + 120*cos(c + d*x)*sin(c +
d*x)*a**4*b + 300*cos(c + d*x)*sin(c + d*x)*a**2*b**3 + 33*cos(c + d*x)*si
n(c + d*x)*b**5 + 48*sin(c + d*x)**5*a*b**4 - 160*sin(c + d*x)**3*a**3*b**
2 - 160*sin(c + d*x)**3*a*b**4 + 48*sin(c + d*x)*a**5 + 480*sin(c + d*x)*a
**3*b**2 + 240*sin(c + d*x)*a*b**4 + 120*a**4*b*d*x + 180*a**2*b**3*d*x +
15*b**5*d*x)/(48*d)
```

3.242 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 241

$$
\begin{aligned}
&\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx \\
&= \frac{1}{8}\left(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B\right)x \\
&\quad+\frac{(95a^3Ab+80aAb^3+12a^4B+112a^2b^2B+16b^4B)\sin(c+dx)}{30d} \\
&\quad+\frac{b(130a^2Ab+45Ab^3+24a^3B+116ab^2B)\cos(c+dx)\sin(c+dx)}{120d} \\
&\quad+\frac{(35aAb+12a^2B+16b^2B)\,(a+b\cos(c+dx))^2\sin(c+dx)}{60d} \\
&\quad+\frac{(5Ab+4aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20d}+\frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5d}
\end{aligned}
$$

output
```
1/8*(8*A*a^4+24*A*a^2*b^2+3*A*b^4+16*B*a^3*b+12*B*a*b^3)*x+1/30*(95*A*a^3*
b+80*A*a*b^3+12*B*a^4+112*B*a^2*b^2+16*B*b^4)*sin(d*x+c)/d+1/120*b*(130*A*
a^2*b+45*A*b^3+24*B*a^3+116*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/60*(35*A*a*
b+12*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/20*(5*A*b+4*B*a)*(a
+b*cos(d*x+c))^3*sin(d*x+c)/d+1/5*B*(a+b*cos(d*x+c))^4*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.09

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{480a^4Ac+1440a^2Ab^2c+180Ab^4c+960a^3bBc+720ab^3Bc+480a^4Adx+1440a^2Ab^2dx+180Ab^4dx+}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(480*a^4*A*c + 1440*a^2*A*b^2*c + 180*A*b^4*c + 960*a^3*b*B*c + 720*a*b^3*
B*c + 480*a^4*A*d*x + 1440*a^2*A*b^2*d*x + 180*A*b^4*d*x + 960*a^3*b*B*d*x
 + 720*a*b^3*B*d*x + 60*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B
+ 5*b^4*B)*Sin[c + d*x] + 120*b*(6*a^2*A*b + A*b^3 + 4*a^3*B + 4*a*b^2*B)*
Sin[2*(c + d*x)] + 160*a*A*b^3*Sin[3*(c + d*x)] + 240*a^2*b^2*B*Sin[3*(c +
 d*x)] + 50*b^4*B*Sin[3*(c + d*x)] + 15*A*b^4*Sin[4*(c + d*x)] + 60*a*b^3*
B*Sin[4*(c + d*x)] + 6*b^4*B*Sin[5*(c + d*x)])/(480*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3232, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{5}\int (a+b\cos(c+dx))^3(5aA+4bB+(5Ab+4aB)\cos(c+dx))dx+\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(5aA+4bB+(5Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$\Big\downarrow$ 3232

$$\frac{1}{5}\left(\frac{1}{4}\int (a+b\cos(c+dx))^2\left(20Aa^2+28bBa+15Ab^2+\left(12Ba^2+35Aba+16b^2B\right)\cos(c+dx)\right)dx+\frac{(4aB+5}{}\right.$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(20Aa^2+28bBa+15Ab^2+\left(12Ba^2+35Aba+16b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)d\right.$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$\Big\downarrow$ 3232

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(60Aa^3+108bBa^2+115Ab^2a+32b^3B+\left(24Ba^3+130Aba^2+116b^2Ba+45Ab\right.\right.\right.\right.$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(60Aa^3+108bBa^2+115Ab^2a+32b^3B+\left(24Ba^3+130Aba^2+116b^2Ba\right.\right.\right.\right.$$
$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$\Big\downarrow$ 3213

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{(12a^2B+35aAb+16b^2B)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{b(24a^3B+130a^2Ab+116ab^2B+45Ab\right.\right.\right.$$

$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

output
```
(B*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + (((5*A*b + 4*a*B)*(a + b*C
os[c + d*x])^3*Sin[c + d*x])/(4*d) + (((35*a*A*b + 12*a^2*B + 16*b^2*B)*(a
 + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((15*(8*a^4*A + 24*a^2*A*b^2 +
3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*x)/2 + (2*(95*a^3*A*b + 80*a*A*b^3 + 12
*a^4*B + 112*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/d + (b*(130*a^2*A*b + 45*
A*b^3 + 24*a^3*B + 116*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3)/4)/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.07

$$\frac{a^4 A(dx+c) + B\,a^4 \sin(dx+c) + 4A\sin(dx+c)\,a^3 b + 4B\,a^3 b\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 6A\,a^2 b^2\left(\frac{\cos(}{}\right.}{}$$

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c)),x)
```

output
```
1/d*(a^4*A*(d*x+c)+B*a^4*sin(d*x+c)+4*A*sin(d*x+c)*a^3*b+4*B*a^3*b*(1/2*co
s(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*A*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+
1/2*d*x+1/2*c)+2*B*a^2*b^2*(cos(d*x+c)^2+2)*sin(d*x+c)+4/3*A*a*b^3*(cos(d*
x+c)^2+2)*sin(d*x+c)+4*B*a*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)+3/8*d*x+3/8*c)+A*b^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*
d*x+3/8*c)+1/5*B*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.82

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{15\,(8\,Aa^4 + 16\,Ba^3b + 24\,Aa^2b^2 + 12\,Bab^3 + 3\,Ab^4)dx + \left(24\,Bb^4\cos(dx+c)^4 + 120\,Ba^4 + 480\,Aa^3b + \right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/120*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*d*x
 + (24*B*b^4*cos(d*x + c)^4 + 120*B*a^4 + 480*A*a^3*b + 480*B*a^2*b^2 + 32
0*A*a*b^3 + 64*B*b^4 + 30*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^3 + 16*(15*B*a^
2*b^2 + 10*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 15*(16*B*a^3*b + 24*A*a^2*b
^2 + 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(248) = 496.

Time = 0.32 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.41

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} Aa^4x + \frac{4Aa^3b\sin(c+dx)}{d} + 3Aa^2b^2x\sin^2(c+dx) + 3Aa^2b^2x\cos^2(c+dx) + \frac{3Aa^2b^2\sin(c+dx)\cos(c+dx)}{d} + \frac{8Aab}{} \\ x(A+B\cos(c))\,(a+b\cos(c))^4 \end{cases}$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((A*a**4*x + 4*A*a**3*b*sin(c + d*x)/d + 3*A*a**2*b**2*x*sin(c +
d*x)**2 + 3*A*a**2*b**2*x*cos(c + d*x)**2 + 3*A*a**2*b**2*sin(c + d*x)*cos
(c + d*x)/d + 8*A*a*b**3*sin(c + d*x)**3/(3*d) + 4*A*a*b**3*sin(c + d*x)*c
os(c + d*x)**2/d + 3*A*b**4*x*sin(c + d*x)**4/8 + 3*A*b**4*x*sin(c + d*x)*
*2*cos(c + d*x)**2/4 + 3*A*b**4*x*cos(c + d*x)**4/8 + 3*A*b**4*sin(c + d*x
)**3*cos(c + d*x)/(8*d) + 5*A*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*
a**4*sin(c + d*x)/d + 2*B*a**3*b*x*sin(c + d*x)**2 + 2*B*a**3*b*x*cos(c +
d*x)**2 + 2*B*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*B*a**2*b**2*sin(c + d
*x)**3/d + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*b**3*x*sin
(c + d*x)**4/2 + 3*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*B*a*b**3
*x*cos(c + d*x)**4/2 + 3*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B
*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*B*b**4*sin(c + d*x)**5/(15*
d) + 4*B*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**4*sin(c + d*x)*
cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{480\,(dx+c)Aa^4 + 480\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3b + 720\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b^2 - 9}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/480*(480*(d*x + c)*A*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3*b
+ 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b^2 - 960*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a^2*b^2 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^3 +
 60*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^3 + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^4 + 32*(3*sin(d
*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^4 + 480*B*a^4*sin(d*x
 + c) + 1920*A*a^3*b*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{Bb^4\sin(5\,dx+5\,c)}{80\,d} + \frac{1}{8}\left(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4\right)x + \frac{(4\,Bab^3+Ab^4)\sin(4\,dx+4\,c)}{32\,d} + \frac{(24\,Ba^2b^2+16\,Aab^3+5\,Bb^4)\sin(3\,dx+3\,c)}{48\,d} + \frac{(4\,Ba^3b+6\,Aa^2b^2+4\,Bab^3+Ab^4)\sin(2\,dx+2\,c)}{4\,d} + \frac{(8\,Ba^4+32\,Aa^3b+36\,Ba^2b^2+24\,Aab^3+5\,Bb^4)\sin(dx+c)}{8\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/80*B*b^4*sin(5*d*x + 5*c)/d + 1/8*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 +
 12*B*a*b^3 + 3*A*b^4)*x + 1/32*(4*B*a*b^3 + A*b^4)*sin(4*d*x + 4*c)/d + 1
/48*(24*B*a^2*b^2 + 16*A*a*b^3 + 5*B*b^4)*sin(3*d*x + 3*c)/d + 1/4*(4*B*a^
3*b + 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sin(2*d*x + 2*c)/d + 1/8*(8*B*a^4 +
 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 42.40 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.27

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= A\,a^4\,x+\frac{3\,A\,b^4\,x}{8}+\frac{3\,B\,a\,b^3\,x}{2}+2\,B\,a^3\,b\,x+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{5\,B\,b^4\,\sin\left(c+d\,x\right)}{8\,d}+3\,A\,a^2\,b^2\,x+\frac{A\,b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,b^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{5\,B\,b^4\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{B\,b^4\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{A\,a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{B\,a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{B\,a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{B\,a\,b^3\,\sin\left(4\,c+4\,d\,x\right)}{8\,d}+\frac{9\,B\,a^2\,b^2\,\sin\left(c+d\,x\right)}{2\,d}+\frac{3\,A\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{B\,a^2\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{2\,d}+\frac{3\,A\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{4\,A\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

output
```
A*a^4*x + (3*A*b^4*x)/8 + (3*B*a*b^3*x)/2 + 2*B*a^3*b*x + (B*a^4*sin(c + d
*x))/d + (5*B*b^4*sin(c + d*x))/(8*d) + 3*A*a^2*b^2*x + (A*b^4*sin(2*c + 2
*d*x))/(4*d) + (A*b^4*sin(4*c + 4*d*x))/(32*d) + (5*B*b^4*sin(3*c + 3*d*x)
)/(48*d) + (B*b^4*sin(5*c + 5*d*x))/(80*d) + (A*a*b^3*sin(3*c + 3*d*x))/(3
*d) + (B*a*b^3*sin(2*c + 2*d*x))/d + (B*a^3*b*sin(2*c + 2*d*x))/d + (B*a*b
^3*sin(4*c + 4*d*x))/(8*d) + (9*B*a^2*b^2*sin(c + d*x))/(2*d) + (3*A*a^2*b
^2*sin(2*c + 2*d*x))/(2*d) + (B*a^2*b^2*sin(3*c + 3*d*x))/(2*d) + (3*A*a*b
^3*sin(c + d*x))/d + (4*A*a^3*b*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.70

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$
$$= \frac{-150\cos(dx+c)\sin(dx+c)^3\,a\,b^4+600\cos(dx+c)\sin(dx+c)\,a^3b^2+375\cos(dx+c)\sin(dx+c)\,a\,b}{}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

output
```
( - 150*cos(c + d*x)*sin(c + d*x)**3*a*b**4 + 600*cos(c + d*x)*sin(c + d*x
)*a**3*b**2 + 375*cos(c + d*x)*sin(c + d*x)*a*b**4 + 24*sin(c + d*x)**5*b*
*5 - 400*sin(c + d*x)**3*a**2*b**3 - 80*sin(c + d*x)**3*b**5 + 600*sin(c +
 d*x)*a**4*b + 1200*sin(c + d*x)*a**2*b**3 + 120*sin(c + d*x)*b**5 + 120*a
**5*d*x + 600*a**3*b**2*d*x + 225*a*b**4*d*x)/(120*d)
```

3.243 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 200

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{1}{8}\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)x+\frac{a^4A\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{b(34a^2Ab+4Ab^3+19a^3B+16ab^2B)\sin(c+dx)}{6d} + \frac{b^2(32aAb+26a^2B+9b^2B)\cos(c+dx)\sin(c+dx)}{24d} + \frac{b(4Ab+7aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{bB(a+b\cos(c+dx))^3\sin(c+dx)}{4d}$$

output
```
1/8*(32*A*a^3*b+16*A*a*b^3+8*B*a^4+24*B*a^2*b^2+3*B*b^4)*x+a^4*A*arctanh(s
in(d*x+c))/d+1/6*b*(34*A*a^2*b+4*A*b^3+19*B*a^3+16*B*a*b^2)*sin(d*x+c)/d+1
/24*b^2*(32*A*a*b+26*B*a^2+9*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*b*(4*A*b+
7*B*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*b*B*(a+b*cos(d*x+c))^3*sin(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{12(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B)(c+dx)-96a^4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(12*(32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*(c + d*x)
 - 96*a^4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*a^4*A*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 24*b*(24*a^2*A*b + 3*A*b^3 + 16*a^3*B + 1
2*a*b^2*B)*Sin[c + d*x] + 24*b^2*(4*a*A*b + 6*a^2*B + b^2*B)*Sin[2*(c + d*
x)] + 8*b^3*(A*b + 4*a*B)*Sin[3*(c + d*x)] + 3*b^4*B*Sin[4*(c + d*x)])/(96
*d)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 3469, 3042, 3528, 3042, 3512, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{4}\int (a+b\cos(c+dx))^2 \left(4Aa^2+b(4Ab+7aB)\cos^2(c+dx)+\left(4Ba^2+8Aba+3b^2B\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4Aa^2+b(4Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(4Ba^2+8Aba+3b^2B\right)\sin\left(c+dx+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3528

$$\frac{1}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(12Aa^3+b\left(26Ba^2+32Aba+9b^2B\right)\cos^2(c+dx)+\left(12Ba^3+36Aba^2+23b^2Ba+8\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(12Aa^3+b\left(26Ba^2+32Aba+9b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(12Ba^3+36Aba^2+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3512

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(24Aa^4+4b\left(19Ba^3+34Aba^2+16b^2Ba+4Ab^3\right)\cos^2(c+dx)+3\left(8Ba^4+32Aba^3+24b^2Ba^2+16\right.\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{24Aa^4+4b\left(19Ba^3+34Aba^2+16b^2Ba+4Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+3\left(8Ba^4+32Aba^3+24b^2Ba^2-\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3502

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(8Aa^4+\left(8Ba^4+32Aba^3+24b^2Ba^2+16Ab^3a+3b^4B\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{4b\left(19a^3B\right.}{}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 27

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\int\left(8Aa^4+\left(8Ba^4+32Aba^3+24b^2Ba^2+16Ab^3a+3b^4B\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{4b\left(19a^3B\right.}{}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\int\frac{8Aa^4+\left(8Ba^4+32Aba^3+24b^2Ba^2+16Ab^3a+3b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4b\left(19a^3B+34a^2A\right.}{}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3214

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(8a^4A\int\sec(c+dx)dx+x\left(8a^4B+32a^3Ab+24a^2b^2B+16aAb^3+3b^4B\right)\right)+\frac{4b\left(19a^3B+34a^2Ab\right.}{}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(8a^4A\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+x\left(8a^4B+32a^3Ab+24a^2b^2B+16aAb^3+3b^4B\right)\right)+\frac{4b\left(19a^3B+3\right.}{}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\big\downarrow$ 4257

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{b^2\left(26a^2B+32aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}\left(\frac{4b\left(19a^3B+34a^2Ab+16ab^2B+4Ab^3\right)\sin(c+}{d}\right.\right.\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(b*B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + ((b*(4*A*b + 7*a*B)*(a +
 b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((b^2*(32*a*A*b + 26*a^2*B + 9*b^
2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*((32*a^3*A*b + 16*a*A*b^3 + 8*a
^4*B + 24*a^2*b^2*B + 3*b^4*B)*x + (8*a^4*A*ArcTanh[Sin[c + d*x]])/d) + (4
*b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*B)*Sin[c + d*x])/d)/2)/3)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 15.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

method	result
parts	$\frac{a^4 A \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{(A b^4+4Ba b^3)\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d} + \frac{(4Aa b^3+6B a^2b^2)\left(\frac{\cos(dx+c)\sin(}{2}\right.}{d}$
parallelrisch	$\frac{-96a^4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+96a^4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+24(4Aab^3+6B a^2b^2+B b^4)\sin(2dx+2c)+8(A b^4+4Ba b^3}{}$
derivativedivides	$\frac{a^4 A \ln(\sec(dx+c)+\tan(dx+c))+B a^4(dx+c)+4A a^3b(dx+c)+4B\sin(dx+c)a^3b+6A\sin(dx+c)a^2b^2+6B a^2b^2\left(\frac{\cos(dx}{}\right.}{}$
default	$\frac{a^4 A \ln(\sec(dx+c)+\tan(dx+c))+B a^4(dx+c)+4A a^3b(dx+c)+4B\sin(dx+c)a^3b+6A\sin(dx+c)a^2b^2+6B a^2b^2\left(\frac{\cos(dx}{}\right.}{}$
risch	$4xA a^3b+3xB a^2b^2+2xAa b^3+\frac{3b^4Bx}{8}+a^4Bx-\frac{3ie^{i(dx+c)}A b^4}{8d}+\frac{3ie^{-i(dx+c)}A b^4}{8d}-\frac{2ie^{i(dx+}}{}$
norman	$\frac{(4A a^3b+3B a^2b^2+2Aa b^3+\frac{3}{8}B b^4+B a^4)x+(4A a^3b+3B a^2b^2+2Aa b^3+\frac{3}{8}B b^4+B a^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(20A a^3b+1}{}$

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
a^4*A/d*ln(sec(d*x+c)+tan(d*x+c))+1/3*(A*b^4+4*B*a*b^3)/d*(cos(d*x+c)^2+2)
*sin(d*x+c)+(4*A*a*b^3+6*B*a^2*b^2)/d*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c)+(6*A*a^2*b^2+4*B*a^3*b)/d*sin(d*x+c)+(4*A*a^3*b+B*a^4)/d*(d*x+c)+B*b
^4/d*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{12\,Aa^4\log(\sin(dx+c)+1)-12\,Aa^4\log(-\sin(dx+c)+1)+3\,(8\,Ba^4+32\,Aa^3b+24\,Ba^2b^2+16\,Aa}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fri
cas")
```

output
```
1/24*(12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*log(-sin(d*x + c) + 1) + 3
*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*d*x + (6*B*b
^4*cos(d*x + c)^3 + 96*B*a^3*b + 144*A*a^2*b^2 + 64*B*a*b^3 + 16*A*b^4 + 8
*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 3*(24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b
^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^4\sec{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**4*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{96\,(dx+c)Ba^4+384\,(dx+c)Aa^3b+144\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Ba^2b^2+96\,(2\,dx+2\,c+\sin{(2\,d}}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="max
ima")
```

output
```
1/96*(96*(d*x + c)*B*a^4 + 384*(d*x + c)*A*a^3*b + 144*(2*d*x + 2*c + sin(
2*d*x + 2*c))*B*a^2*b^2 + 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^3 - 12
8*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b^3 - 32*(sin(d*x + c)^3 - 3*sin(d
*x + c))*A*b^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*B*b^4 + 96*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 384*B*a^3*b*sin(d*x +
 c) + 576*A*a^2*b^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(190) = 380$.

Time = 0.21 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.02

$$\int (a + b\cos(c + dx))^4 (A + B\cos(c + dx)) \sec(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gia
c")
```

output
```
1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^
3 + 3*B*b^4)*(d*x + c) + 2*(96*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 144*A*a^2*
b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*
b^3*tan(1/2*d*x + 1/2*c)^7 + 96*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^4*
tan(1/2*d*x + 1/2*c)^7 - 15*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 288*B*a^3*b*tan
(1/2*d*x + 1/2*c)^5 + 432*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*a^2*b^2*
tan(1/2*d*x + 1/2*c)^5 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*B*a*b^3*t
an(1/2*d*x + 1/2*c)^5 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 9*B*b^4*tan(1/2*
d*x + 1/2*c)^5 + 288*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 432*A*a^2*b^2*tan(1/
2*d*x + 1/2*c)^3 + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 48*A*a*b^3*tan(1/
2*d*x + 1/2*c)^3 + 160*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*b^4*tan(1/2*d
*x + 1/2*c)^3 - 9*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 96*B*a^3*b*tan(1/2*d*x +
1/2*c) + 144*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a^2*b^2*tan(1/2*d*x + 1
/2*c) + 48*A*a*b^3*tan(1/2*d*x + 1/2*c) + 96*B*a*b^3*tan(1/2*d*x + 1/2*c)
+ 24*A*b^4*tan(1/2*d*x + 1/2*c) + 15*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 43.20 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.84

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{3\,A\,b^4\,\sin\left(c+d\,x\right)}{4\,d} + \frac{2\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,B\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d} + \frac{A\,b^4\,\sin\left(3\,c+3\,d\,x\right)}{12\,d} + \frac{B\,b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} + \frac{B\,b^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d} + \frac{4\,A\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{8\,A\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{A\,a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d} + \frac{6\,A\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d} + \frac{B\,a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{3\,d} + \frac{6\,B\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,B\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d} + \frac{3\,B\,a\,b^3\,\sin\left(c+d\,x\right)}{d} + \frac{4\,B\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

input

```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x),x)
```

output

```
(3*A*b^4*sin(c + d*x))/(4*d) + (2*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/d + (2*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d +
(3*B*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) + (A*b^4*sin(3
*c + 3*d*x))/(12*d) + (B*b^4*sin(2*c + 2*d*x))/(4*d) + (B*b^4*sin(4*c + 4*
d*x))/(32*d) + (4*A*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d +
 (8*A*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a*b^3*sin(
2*c + 2*d*x))/d + (6*A*a^2*b^2*sin(c + d*x))/d + (B*a*b^3*sin(3*c + 3*d*x)
)/(3*d) + (6*B*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3
*B*a^2*b^2*sin(2*c + 2*d*x))/(2*d) + (3*B*a*b^3*sin(c + d*x))/d + (4*B*a^3
*b*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{-6\cos{(dx+c)}\sin{(dx+c)}^3 b^5 + 120\cos{(dx+c)}\sin{(dx+c)}\,a^2b^3 + 15\cos{(dx+c)}\sin{(dx+c)}\,b^5 - 24}{}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**3*b**5 + 120*cos(c + d*x)*sin(c + d*x)*a*
*2*b**3 + 15*cos(c + d*x)*sin(c + d*x)*b**5 - 24*log(tan((c + d*x)/2) - 1)
*a**5 + 24*log(tan((c + d*x)/2) + 1)*a**5 - 40*sin(c + d*x)**3*a*b**4 + 24
0*sin(c + d*x)*a**3*b**2 + 120*sin(c + d*x)*a*b**4 + 120*a**4*b*d*x + 120*
a**2*b**3*d*x + 9*b**5*d*x)/(24*d)
```

3.244 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 195

$$\begin{aligned}\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x+\frac{a^3(4Ab+aB)\text{arctanh}(\sin(c+dx))}{d}\\ &\quad-\frac{b(6a^3A-12aAb^2-17a^2bB-2b^3B)\sin(c+dx)}{3d}\\ &\quad-\frac{b^2(6a^2A-3Ab^2-8abB)\cos(c+dx)\sin(c+dx)}{6d}\\ &\quad-\frac{b(3aA-bB)(a+b\cos(c+dx))^2\sin(c+dx)}{3d}\\ &\quad+\frac{aA(a+b\cos(c+dx))^3\tan(c+dx)}{d}\end{aligned}$$

output
```
1/2*b*(12*A*a^2*b+A*b^3+8*B*a^3+4*B*a*b^2)*x+a^3*(4*A*b+B*a)*arctanh(sin(d
*x+c))/d-1/3*b*(6*A*a^3-12*A*a*b^2-17*B*a^2*b-2*B*b^3)*sin(d*x+c)/d-1/6*b^
2*(6*A*a^2-3*A*b^2-8*B*a*b)*cos(d*x+c)*sin(d*x+c)/d-1/3*b*(3*A*a-B*b)*(a+b
*cos(d*x+c))^2*sin(d*x+c)/d+a*A*(a+b*cos(d*x+c))^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.32

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{6b(12a^2Ab+Ab^3+8a^3B+4ab^2B)(c+dx)-12a^3(4Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(6*b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*(c + d*x) - 12*a^3*(4*A*b
+ a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*(4*A*b + a*B)*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]) + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 3*b^2*(16*a*A*b + 24*a^2*B + 3*b^2*B)*Sin[c +
d*x] + 3*b^3*(A*b + 4*a*B)*Sin[2*(c + d*x)] + b^4*B*Sin[3*(c + d*x)])/(12*
d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3468, 3042, 3528, 3042, 3512, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3468}$$

$$\int (a+b\cos(c+dx))^2 \left(-b(3aA-bB)\cos^2(c+dx)+b(Ab+2aB)\cos(c+dx)+a(4Ab+aB)\right)\sec(c+dx)dx+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\big\downarrow$ 3042

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-b(3aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(4Ab+aB)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}$$
$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\big\downarrow$ 3528

$$\frac{1}{3}\int (a+b\cos(c+dx))\left(3(4Ab+aB)a^2-b\left(6Aa^2-8bBa-3Ab^2\right)\cos^2(c+dx)+b\left(9Ba^2+9Aba+2b^2B\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(3(4Ab+aB)a^2-b\left(6Aa^2-8bBa-3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(9Ba^2+9Aba+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\big\downarrow$ 3512

$$\frac{1}{3}\left(\frac{1}{2}\int \left(6(4Ab+aB)a^3-2b\left(6Aa^3-17bBa^2-12Ab^2a-2b^3B\right)\cos^2(c+dx)+3b\left(8Ba^3+12Aba^2+4b^2Ba+\right.\right.\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{6(4Ab+aB)a^3-2b\left(6Aa^3-17bBa^2-12Ab^2a-2b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(8Ba^3+12Aba^2+4b^2B\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\big\downarrow$ 3502

$$\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(2(4Ab+aB)a^3+b\left(8Ba^3+12Aba^2+4b^2Ba+Ab^3\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{2b\left(6a^3A-17a^2b\right.}{}\right.\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int \left(2(4Ab+aB)a^3+b\left(8Ba^3+12Aba^2+4b^2Ba+Ab^3\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{2b\left(6a^3A-17a^2b\right.}{}\right.\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int \frac{2(4Ab+aB)a^3+b\left(8Ba^3+12Aba^2+4b^2Ba+Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b\left(6a^3A-17a^2bB-12a\right.}{d}\right.\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\Big\downarrow$ 3214

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(2a^3(aB+4Ab)\int \sec(c+dx)dx+bx\left(8a^3B+12a^2Ab+4ab^2B+Ab^3\right)\right)-\frac{2b\left(6a^3A-17a^2bB-12aA\right.}{d}\right.\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(2a^3(aB+4Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+bx\left(8a^3B+12a^2Ab+4ab^2B+Ab^3\right)\right)-\frac{2b\left(6a^3A-17a^2bB\right.}{}\right.\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(\frac{2a^3(aB+4Ab)\text{arctanh}(\sin(c+dx))}{d}+bx\left(8a^3B+12a^2Ab+4ab^2B+Ab^3\right)\right)-\frac{2b\left(6a^3A-17a^2bB-\right.}{}\right.\right.$$
$$\frac{b(3aA-bB)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^3}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
-1/3*(b*(3*a*A - b*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/d + (-1/2*(b^2*
(6*a^2*A - 3*A*b^2 - 8*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/d + (3*(b*(12*a^2
*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*x + (2*a^3*(4*A*b + a*B)*ArcTanh[Sin[c
 + d*x]])/d) - (2*b*(6*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Sin[c +
d*x])/d)/2)/3 + (a*A*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 16.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.84

method	result
parts	$\frac{a^4 A \tan(dx+c)}{d} + \frac{(A b^4+4Ba b^3)\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{(4Aa b^3+6B a^2b^2)\sin(dx+c)}{d} + \frac{(6A a^2b^2+4B}{d}$
derivativedivides	$\frac{a^4 A\tan(dx+c)+B a^4 \ln(\sec(dx+c)+\tan(dx+c))+4A a^3 b \ln(\sec(dx+c)+\tan(dx+c))+4B a^3 b(dx+c)+6A a^2b^2(dx+c)}{}$
default	$\frac{a^4 A\tan(dx+c)+B a^4 \ln(\sec(dx+c)+\tan(dx+c))+4A a^3 b \ln(\sec(dx+c)+\tan(dx+c))+4B a^3 b(dx+c)+6A a^2b^2(dx+c)}{}$
parallelrisch	$\frac{-96\cos(dx+c)\left(Ab+\frac{Ba}{4}\right)a^3 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+96\cos(dx+c)\left(Ab+\frac{Ba}{4}\right)a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(48Aa b^3+72B a^2}{}$
risch	$6A a^2b^2x + \frac{A b^4 x}{2} + 4B a^3bx + 2Ba b^3x + \frac{i e^{-2i(dx+c)} Ba b^3}{2d} + \frac{2i e^{-i(dx+c)} Aa b^3}{d} + \frac{3i e^{-i(dx+c)} B a^2}{d}$
norman	$\frac{(-6A a^2b^2-\frac{1}{2}A b^4-4B a^3 b-2Ba b^3)x+(-30A a^2b^2-\frac{5}{2}A b^4-20B a^3b-10Ba b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(6A a^2b^2+\frac{1}{2}A b^4+}{}$

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBO
SE)
```

output
```
a^4*A/d*tan(d*x+c)+(A*b^4+4*B*a*b^3)/d*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+
1/2*c)+(4*A*a*b^3+6*B*a^2*b^2)/d*sin(d*x+c)+(6*A*a^2*b^2+4*B*a^3*b)/d*(d*x
+c)+(4*A*a^3*b+B*a^4)/d*ln(sec(d*x+c)+tan(d*x+c))+1/3*B*b^4/d*(cos(d*x+c)^
2+2)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{3\,(8\,Ba^3b+12\,Aa^2b^2+4\,Bab^3+Ab^4)dx\cos(dx+c)+3\,(Ba^4+4\,Aa^3b)\cos(dx+c)\log(\sin(dx+c)+}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="f
ricas")
```

output
```
1/6*(3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c) + 3
*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(B*a^4 + 4*A*a
^3*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*B*b^4*cos(d*x + c)^3 + 6*A*
a^4 + 3*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 4*(9*B*a^2*b^2 + 6*A*a*b^3 +
B*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int(A+B\cos(c+dx))\,(a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**4*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{48\,(dx+c)Ba^3b+72\,(dx+c)Aa^2b^2+12\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab^3+3\,(2\,dx+2\,c+\sin(2\,dx}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="m
axima")
```

output
```
1/12*(48*(d*x + c)*B*a^3*b + 72*(d*x + c)*A*a^2*b^2 + 12*(2*d*x + 2*c + si
n(2*d*x + 2*c))*B*a*b^3 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*b^4 + 6*B*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 24*A*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 72*B*a^2*b^2*sin(d*x + c) + 48*A*a*b^3*sin(d*x + c) + 12*A*a^4*t
an(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.90

$$\int (a + b\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx =$$

$$-\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\,(8\,Ba^3b + 12\,Aa^2b^2 + 4\,Bab^3 + Ab^4)(dx + c) - 6\,(Ba^4 + 4\,Aa^3b)\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="g
iac")
```

output
```
-1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(8*B*
a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - 6*(B*a^4 + 4*A*a^3*b
)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(B*a^4 + 4*A*a^3*b)*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) - 2*(36*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*a*b^
3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^4*tan
(1/2*d*x + 1/2*c)^5 + 6*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*b^2*tan(1/
2*d*x + 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*B*b^4*tan(1/2*d*x
 + 1/2*c)^3 + 36*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*A*a*b^3*tan(1/2*d*x +
 1/2*c) + 12*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3*A*b^4*tan(1/2*d*x + 1/2*c) +
 6*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 44.09 (sec) , antiderivative size = 2522, normalized size of antiderivative = 12.93

$$\int (a + b\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^2(c + dx)\, dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

output
```
(tan(c/2 + (d*x)/2)*(2*A*a^4 + A*b^4 + 2*B*b^4 + 12*B*a^2*b^2 + 8*A*a*b^3
+ 4*B*a*b^3) + tan(c/2 + (d*x)/2)^7*(2*A*a^4 + A*b^4 - 2*B*b^4 - 12*B*a^2*
b^2 - 8*A*a*b^3 + 4*B*a*b^3) + tan(c/2 + (d*x)/2)^3*(6*A*a^4 - A*b^4 - (2*
B*b^4)/3 + 12*B*a^2*b^2 + 8*A*a*b^3 - 4*B*a*b^3) - tan(c/2 + (d*x)/2)^5*(A
*b^4 - 6*A*a^4 - (2*B*b^4)/3 + 12*B*a^2*b^2 + 8*A*a*b^3 + 4*B*a*b^3))/(d*(
2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1
)) - (atan(((B*a^4 + 4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4
+ 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) + tan(c/2 + (d*x
)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^
2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a
*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3))*1i - (B*a^4 +
4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128
*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) - tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*
B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a
^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b
+ 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3))*1i)/((B*a^4 + 4*A*a^3*b)*((B*a^4 +
4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3
 + 128*B*a^3*b) + tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2
*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*
b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{15\cos(dx+c)^2\sin(dx+c)\,a\,b^4-30\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^4b+30\cos(dx+c)\log\left(\tan\left(\frac{d}{2}\right.\right.}{}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
(15*cos(c + d*x)**2*sin(c + d*x)*a*b**4 - 30*cos(c + d*x)*log(tan((c + d*x
)/2) - 1)*a**4*b + 30*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**4*b - 2*co
s(c + d*x)*sin(c + d*x)**3*b**5 + 60*cos(c + d*x)*sin(c + d*x)*a**2*b**3 +
 6*cos(c + d*x)*sin(c + d*x)*b**5 + 60*cos(c + d*x)*a**3*b**2*d*x + 15*cos
(c + d*x)*a*b**4*d*x + 6*sin(c + d*x)*a**5)/(6*cos(c + d*x)*d)
```

3.245 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 209

$$\begin{aligned}&\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx\\&=\frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x+\frac{a^2(a^2A+12Ab^2+8abB)\operatorname{arctanh}(\sin(c+dx))}{2d}\\&-\frac{b(13a^2Ab-2Ab^3+4a^3B-8ab^2B)\sin(c+dx)}{2d}\\&-\frac{b^2(6aAb+2a^2B-b^2B)\cos(c+dx)\sin(c+dx)}{2d}\\&+\frac{a(5Ab+2aB)(a+b\cos(c+dx))^2\tan(c+dx)}{2d}\\&+\frac{aA(a+b\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output

```
1/2*b^2*(8*A*a*b+12*B*a^2+B*b^2)*x+1/2*a^2*(A*a^2+12*A*b^2+8*B*a*b)*arctan
h(sin(d*x+c))/d-1/2*b*(13*A*a^2*b-2*A*b^3+4*B*a^3-8*B*a*b^2)*sin(d*x+c)/d-
1/2*b^2*(6*A*a*b+2*B*a^2-B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/2*a*(5*A*b+2*B*a
)*(a+b*cos(d*x+c))^2*tan(d*x+c)/d+1/2*a*A*(a+b*cos(d*x+c))^3*sec(d*x+c)*ta
n(d*x+c)/d
```

Mathematica [A] (verified)

Time = 5.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.48

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{2b^2(8aAb+12a^2B+b^2B)\,(c+dx)-2a^2(a^2A+12Ab^2+8abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(2*b^2*(8*a*A*b + 12*a^2*B + b^2*B)*(c + d*x) - 2*a^2*(a^2*A + 12*A*b^2 +
8*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*(a^2*A + 12*A*b^
2 + 8*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^4*A)/(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^4*A)/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]) + 4*b^3*(A*b + 4*a*B)*Sin[c + d*x] + b^4*B*Sin[2*(c + d*x)
])/(4*d)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3468, 3042, 3526, 3042, 3512, 27, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{2}\int (a+b\cos(c+dx))^2\left(-2b(aA-bB)\cos^2(c+dx)+\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+a(5Ab+2aB)\right)\sec^2(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-2b(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+4bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\downarrow$ 3526

$$\frac{1}{2}\left(\int (a+b\cos(c+dx))\left(-2b\left(2Ba^2+6Aba-b^2B\right)\cos^2(c+dx)-b\left(Aa^2-6bBa-2Ab^2\right)\cos(c+dx)+a\left(Aa^2\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-2b\left(2Ba^2+6Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(Aa^2-6bBa-2Ab^2\right)\sin\left(c+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\downarrow$ 3512

$$\frac{1}{2}\left(\frac{1}{2}\int 2\left(\left(Aa^2+8bBa+12Ab^2\right)a^2-b\left(4Ba^3+13Aba^2-8b^2Ba-2Ab^3\right)\cos^2(c+dx)+b^2\left(12Ba^2+8Aba+\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\int \left(\left(Aa^2+8bBa+12Ab^2\right)a^2-b\left(4Ba^3+13Aba^2-8b^2Ba-2Ab^3\right)\cos^2(c+dx)+b^2\left(12Ba^2+8Aba+b^2B\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{(Aa^2+8bBa+12Ab^2)\,a^2-b(4Ba^3+13Aba^2-8b^2Ba-2Ab^3)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b^2(12Ba^2+8Aba+\right.$$
$$\sin\left(c+dx+\frac{\pi}{2}\right)$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\big\downarrow$ 3502

$$\frac{1}{2}\left(\int \left((Aa^2+8bBa+12Ab^2)\,a^2+b^2(12Ba^2+8Aba+b^2B)\cos(c+dx)\right)\sec(c+dx)dx-\frac{b^2(2a^2B+6aAb-\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{(Aa^2+8bBa+12Ab^2)\,a^2+b^2(12Ba^2+8Aba+b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2(2a^2B+6aAb-b^2B)\sin(}{d}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\big\downarrow$ 3214

$$\frac{1}{2}\left(a^2(a^2A+8abB+12Ab^2)\int\sec(c+dx)dx-\frac{b^2(2a^2B+6aAb-b^2B)\sin(c+dx)\cos(c+dx)}{d}+b^2x(12a^2B\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(a^2(a^2A+8abB+12Ab^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b^2(2a^2B+6aAb-b^2B)\sin(c+dx)\cos(c+dx)}{d}+b^2x(1\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

$\big\downarrow$ 4257

$$\frac{1}{2}\left(\frac{a^2(a^2A+8abB+12Ab^2)\,\text{arctanh}(\sin(c+dx))}{d}-\frac{b^2(2a^2B+6aAb-b^2B)\sin(c+dx)\cos(c+dx)}{d}+b^2x(12\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^3}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b^2*(8*a*A
*b + 12*a^2*B + b^2*B)*x + (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c
 + d*x]])/d - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2*B)*Sin[c + d*x]
)/d - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Cos[c + d*x]*Sin[c + d*x])/d + (a*(
5*A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 17.94 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

method	result
parts	$\frac{a^4 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{(A\,b^4+4B a\,b^3)\sin(dx+c)}{d}+\frac{(4Aa\,b^3+6B\,a^2b^2)(dx+c)}{d}+$
derivativedivides	$\frac{a^4 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+B\,a^4\tan(dx+c)+4A\,a^3b\tan(dx+c)+4B\,a^3b\ln(\sec(dx+c)+\tan(}{}$
default	$\frac{a^4 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+B\,a^4\tan(dx+c)+4A\,a^3b\tan(dx+c)+4B\,a^3b\ln(\sec(dx+c)+\tan(}{}$
parallelrisch	$\frac{-4a^2(\cos(2dx+2c)+1)(a^2A+12A\,b^2+8Bab)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4a^2(\cos(2dx+2c)+1)(a^2A+12A\,b^2+8Bab)\ln\left(\tan\right.}{}$
risch	$4xAa\,b^3+6xB\,a^2b^2+\frac{b^4Bx}{2}+\frac{2i\mathrm{e}^{-i(dx+c)}Ba\,b^3}{d}-\frac{i\mathrm{e}^{i(dx+c)}A\,b^4}{2d}-\frac{iB\,b^4\mathrm{e}^{2i(dx+c)}}{8d}+\frac{i\mathrm{e}^{-i(dx+c)}A\,b^4}{2d}$
norman	$\frac{(4Aa\,b^3+6B\,a^2b^2+\frac{1}{2}B\,b^4)x+(-20Aa\,b^3-30B\,a^2b^2-\frac{5}{2}B\,b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-20Aa\,b^3-30B\,a^2b^2-\frac{5}{2}B\,b^4)x\tan\left(\frac{d}{2}\right.}{}$

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBO
SE)
```

output
```
a^4*A/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+(A*b^4+4
*B*a*b^3)/d*sin(d*x+c)+(4*A*a*b^3+6*B*a^2*b^2)/d*(d*x+c)+(6*A*a^2*b^2+4*B*
a^3*b)/d*ln(sec(d*x+c)+tan(d*x+c))+(4*A*a^3*b+B*a^4)/d*tan(d*x+c)+B*b^4/d*
(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{2\left(12\,Ba^2b^2+8\,Aab^3+Bb^4\right)dx\cos\left(dx+c\right)^2+\left(Aa^4+8\,Ba^3b+12\,Aa^2b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="f
ricas")
```

output
```
1/4*(2*(12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*d*x*cos(d*x + c)^2 + (A*a^4 + 8*
B*a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^4 + 8*
B*a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*b^4*c
os(d*x + c)^3 + A*a^4 + 2*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 2*(B*a^4 +
4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{24\,(dx+c)Ba^2b^2 + 16\,(dx+c)Aab^3 + (2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^4 - Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(c\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="m
axima")
```

output
```
1/4*(24*(d*x + c)*B*a^2*b^2 + 16*(d*x + c)*A*a*b^3 + (2*d*x + 2*c + sin(2*
d*x + 2*c))*B*b^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 8*B*a^3*b*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 12*A*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*
x + c) - 1)) + 16*B*a*b^3*sin(d*x + c) + 4*A*b^4*sin(d*x + c) + 4*B*a^4*ta
n(d*x + c) + 16*A*a^3*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(197) = 394$.

Time = 0.19 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.52

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="g
iac")
```

output
```
1/2*((12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*(d*x + c) + (A*a^4 + 8*B*a^3*b + 1
2*A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a^4 + 8*B*a^3*b + 12*
A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^4*tan(1/2*d*x + 1/2
*c)^7 - 2*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^7
+ 8*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^7 - B*b^
4*tan(1/2*d*x + 1/2*c)^7 + 3*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 2*B*a^4*tan(1/
2*d*x + 1/2*c)^5 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 8*B*a*b^3*tan(1/2*d*
x + 1/2*c)^5 - 2*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^4*tan(1/2*d*x + 1/2*
c)^5 + 3*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 8
*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 8*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*b
^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^3 + A*a^4*tan(1/2
*d*x + 1/2*c) + 2*B*a^4*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b*tan(1/2*d*x + 1/2
*c) + 8*B*a*b^3*tan(1/2*d*x + 1/2*c) + 2*A*b^4*tan(1/2*d*x + 1/2*c) + B*b^
4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 43.80 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.58

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$

$$= \frac{2\left(\frac{A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{B\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+4\,A\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+4\,B\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right.}{d}$$

$$+\frac{\frac{B\,a^4\,\sin(2\,c+2\,d\,x)}{2}+\frac{A\,b^4\,\sin(3\,c+3\,d\,x)}{4}+\frac{B\,b^4\,\sin(2\,c+2\,d\,x)}{8}+\frac{B\,b^4\,\sin(4\,c+4\,d\,x)}{16}+\frac{A\,a^4\,\sin(c+d\,x)}{2}+\frac{A\,b^4\,\sin(c+d\,x)}{4}+B\,a}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^3,x)
```

output
```
(2*((A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (B*b^4*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + 4*A*a*b^3*atan(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)) + 4*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)) + 6*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 6*B*a^2
*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d + ((B*a^4*sin(2*c + 2
*d*x))/2 + (A*b^4*sin(3*c + 3*d*x))/4 + (B*b^4*sin(2*c + 2*d*x))/8 + (B*b^
4*sin(4*c + 4*d*x))/16 + (A*a^4*sin(c + d*x))/2 + (A*b^4*sin(c + d*x))/4 +
 B*a*b^3*sin(c + d*x) + 2*A*a^3*b*sin(2*c + 2*d*x) + B*a*b^3*sin(3*c + 3*d
*x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.13

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{\cos(dx+c)^2\sin(dx+c)^3b^5-\cos(dx+c)^2\sin(dx+c)b^5-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)}{}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
(cos(c + d*x)**2*sin(c + d*x)**3*b**5 - cos(c + d*x)**2*sin(c + d*x)*b**5
- cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**5 - 20*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b**2 + cos(c + d*x)*l
og(tan((c + d*x)/2) - 1)*a**5 + 20*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*
a**3*b**2 + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**5 +
20*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3*b**2 - cos(
c + d*x)*log(tan((c + d*x)/2) + 1)*a**5 - 20*cos(c + d*x)*log(tan((c + d*x
)/2) + 1)*a**3*b**2 + 10*cos(c + d*x)*sin(c + d*x)**3*a*b**4 + 20*cos(c +
d*x)*sin(c + d*x)**2*a**2*b**3*d*x + cos(c + d*x)*sin(c + d*x)**2*b**5*d*x
 - cos(c + d*x)*sin(c + d*x)*a**5 - 10*cos(c + d*x)*sin(c + d*x)*a*b**4 -
20*cos(c + d*x)*a**2*b**3*d*x - cos(c + d*x)*b**5*d*x + 10*sin(c + d*x)**3
*a**4*b - 10*sin(c + d*x)*a**4*b)/(2*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.246 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 198

$$\begin{aligned}
&\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx \\
&= b^3(Ab+4aB)x+\frac{a(4a^2Ab+8Ab^3+a^3B+12ab^2B)\,\text{arctanh}(\sin(c+dx))}{2d} \\
&\quad-\frac{b^2(8aAb+3a^2B-6b^2B)\sin(c+dx)}{6d}+\frac{a^2(2a^2A+9Ab^2+9abB)\tan(c+dx)}{3d} \\
&\quad+\frac{a(2Ab+aB)(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad+\frac{aA(a+b\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}$$

output

```
b^3*(A*b+4*B*a)*x+1/2*a*(4*A*a^2*b+8*A*b^3+B*a^3+12*B*a*b^2)*arctanh(sin(d
*x+c))/d-1/6*b^2*(8*A*a*b+3*B*a^2-6*B*b^2)*sin(d*x+c)/d+1/3*a^2*(2*A*a^2+9
*A*b^2+9*B*a*b)*tan(d*x+c)/d+1/2*a*(2*A*b+B*a)*(a+b*cos(d*x+c))^2*sec(d*x+
c)*tan(d*x+c)/d+1/3*a*A*(a+b*cos(d*x+c))^3*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 831 vs. $2(198) = 396$.

Time = 10.29 (sec) , antiderivative size = 831, normalized size of antiderivative = 4.20

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(b^3*(A*b + 4*a*B)*(c + d*x)*Cos[c + d*x]^4*(b + a*Sec[c + d*x])^4)/(d*(a
+ b*Cos[c + d*x])^4) + ((-4*a^3*A*b - 8*a*A*b^3 - a^4*B - 12*a^2*b^2*B)*Co
s[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(b + a*Sec[c + d*x])
^4)/(2*d*(a + b*Cos[c + d*x])^4) + ((4*a^3*A*b + 8*a*A*b^3 + a^4*B + 12*a^
2*b^2*B)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(b + a*Se
c[c + d*x])^4)/(2*d*(a + b*Cos[c + d*x])^4) + ((a^4*A + 12*a^3*A*b + 3*a^4
*B)*Cos[c + d*x]^4*(b + a*Sec[c + d*x])^4)/(12*d*(a + b*Cos[c + d*x])^4*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (a^4*A*Cos[c + d*x]^4*(b + a*Sec[
c + d*x])^4*Sin[(c + d*x)/2])/(6*d*(a + b*Cos[c + d*x])^4*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^3) + (a^4*A*Cos[c + d*x]^4*(b + a*Sec[c + d*x])^4*Si
n[(c + d*x)/2])/(6*d*(a + b*Cos[c + d*x])^4*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3) + ((-(a^4*A) - 12*a^3*A*b - 3*a^4*B)*Cos[c + d*x]^4*(b + a*Sec[
c + d*x])^4)/(12*d*(a + b*Cos[c + d*x])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2) + (2*Cos[c + d*x]^4*(b + a*Sec[c + d*x])^4*(a^4*A*Sin[(c + d*x)/2
] + 9*a^2*A*b^2*Sin[(c + d*x)/2] + 6*a^3*b*B*Sin[(c + d*x)/2]))/(3*d*(a +
b*Cos[c + d*x])^4*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*Cos[c + d*x]
^4*(b + a*Sec[c + d*x])^4*(a^4*A*Sin[(c + d*x)/2] + 9*a^2*A*b^2*Sin[(c + d
*x)/2] + 6*a^3*b*B*Sin[(c + d*x)/2]))/(3*d*(a + b*Cos[c + d*x])^4*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])) + (b^4*B*Cos[c + d*x]^4*(b + a*Sec[c + d*x]
)^4*Sin[c + d*x])/(d*(a + b*Cos[c + d*x])^4)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3468, 3042, 3526, 3042, 3510, 25, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{3}\int (a+b\cos(c+dx))^2\left(-b(aA-3bB)\cos^2(c+dx)+\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(2Ab+aB)\right)\sec^3(c+dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-b(aA-3bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(2Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3526}$$

$$\frac{1}{3}\left(\frac{1}{2}\int (a+b\cos(c+dx))\left(-b\left(3Ba^2+8Aba-6b^2B\right)\cos^2(c+dx)+\left(3Ba^3+8Aba^2+18b^2Ba+6Ab^3\right)\cos(c+\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-b\left(3Ba^2+8Aba-6b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Ba^3+8Aba^2+18b^2Ba+6\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 3510

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{2a^2\left(2a^2A+9abB+9Ab^2\right)\tan(c+dx)}{d}-\int -\left(\left(6(Ab+4aB)\cos(c+dx)b^3-\left(3Ba^2+8Aba-6b^2B\right)\text{co}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{2}\left(\int \left(6(Ab+4aB)\cos(c+dx)b^3-\left(3Ba^2+8Aba-6b^2B\right)\cos^2(c+dx)b^2+3a\left(Ba^3+4Aba^2+12b^2Ba+\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(\int \frac{6(Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)b^3-\left(3Ba^2+8Aba-6b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+3a\left(Ba^3+4Aba^2+1\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 3502

$$\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(2(Ab+4aB)\cos(c+dx)b^3+a\left(Ba^3+4Aba^2+12b^2Ba+8Ab^3\right)\right)\sec(c+dx)dx-\frac{b^2\left(3a^2B+8aAb\right.}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int \left(2(Ab+4aB)\cos(c+dx)b^3+a\left(Ba^3+4Aba^2+12b^2Ba+8Ab^3\right)\right)\sec(c+dx)dx-\frac{b^2\left(3a^2B+8aAb\right.}{}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\int \frac{2(Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)b^3+a\left(Ba^3+4Aba^2+12b^2Ba+8Ab^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2\left(3a^2B+8aAb-6b^2B\right.}{d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 3214

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(a\left(a^3B+4a^2Ab+12ab^2B+8Ab^3\right)\int\sec(c+dx)dx+2b^3x(4aB+Ab)\right)-\frac{b^2\left(3a^2B+8aAb-6b^2B\right)}{d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(a\left(a^3B+4a^2Ab+12ab^2B+8Ab^3\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+2b^3x(4aB+Ab)\right)-\frac{b^2\left(3a^2B+8aAb-\right.}{d}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

$\big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{b^2\left(3a^2B+8aAb-6b^2B\right)\sin(c+dx)}{d}+\frac{2a^2\left(2a^2A+9abB+9Ab^2\right)\tan(c+dx)}{d}+3\left(\frac{a\left(a^3B+4a^2Ab-\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^3}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((3*a*(2*
A*b + a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (3*(2
*b^3*(A*b + 4*a*B)*x + (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*ArcTa
nh[Sin[c + d*x]])/d) - (b^2*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Sin[c + d*x])/d
+ (2*a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Tan[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 19.66 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

method	result
parts	$-\frac{a^4 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{(A\,b^4+4Ba\,b^3)(dx+c)}{d}+\frac{(4Aa\,b^3+6B\,a^2b^2)\ln(\sec(dx+c)+\tan(dx+c))}{d}-$
derivativedivides	$\frac{-a^4 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+B\,a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4A\,a^3b\left(\frac{\sec(dx+c)\tan(dx-}{2}\right.}{}$
default	$\frac{-a^4 A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+B\,a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4A\,a^3b\left(\frac{\sec(dx+c)\tan(dx-}{2}\right.}{}$
parallelrisch	$\frac{-36a\left(A\,a^2b+2A\,b^3+\frac{1}{4}a^3B+3Ba\,b^2\right)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+36a\left(A\,a^2b+2A\,b^3+\frac{1}{4}a^3B+3Ba\right.}{}$
risch	$A\,b^4x+4Ba\,b^3x-\frac{iB\,b^4\mathrm{e}^{i(dx+c)}}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}B\,b^4}{2d}-\frac{ia^2\left(12Aab\,\mathrm{e}^{5i(dx+c)}+3B\,a^2\mathrm{e}^{5i(dx+c)}-36A\,b^2\mathrm{e}^{4i(dx-}\right.}{}$
norman	$\frac{(-A\,b^4-4Ba\,b^3)x+(-6A\,b^4-24Ba\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(-2A\,b^4-8Ba\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(-2A\,b^4-8Ba\,b^3)x\tan}{}$

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBO
SE)
```

output
```
-a^4*A/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+(A*b^4+4*B*a*b^3)/d*(d*x+c)+(4
*A*a*b^3+6*B*a^2*b^2)/d*ln(sec(d*x+c)+tan(d*x+c))+(6*A*a^2*b^2+4*B*a^3*b)/
d*tan(d*x+c)+(4*A*a^3*b+B*a^4)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x
+c)+tan(d*x+c)))+B*b^4/d*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{12\,(4\,Bab^3+Ab^4)dx\cos{(dx+c)}^3+3\,(Ba^4+4\,Aa^3b+12\,Ba^2b^2+8\,Aab^3)\cos{(dx+c)}^3\log{(\sin{(dx+c}}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="f
ricas")
```

output
```
1/12*(12*(4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c)^3 + 3*(B*a^4 + 4*A*a^3*b + 1
2*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^4 +
 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) +
1) + 2*(6*B*b^4*cos(d*x + c)^3 + 2*A*a^4 + 4*(A*a^4 + 6*B*a^3*b + 9*A*a^2*
b^2)*cos(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.24

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+48\,(dx+c)Bab^3+12\,(dx+c)Ab^4-3\,Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(s\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="m
axima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 48*(d*x + c)*B*a*b^3 + 1
2*(d*x + c)*A*b^4 - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin
(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*a^3*b*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*B*a^
2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*A*a*b^3*(log(si
n(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*b^4*sin(d*x + c) + 48*B*a^
3*b*tan(d*x + c) + 72*A*a^2*b^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(188) = 376$.

Time = 0.25 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.95

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$

$$= \frac{\frac{12\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 6\,(4\,Bab^3 + Ab^4)(dx+c) + 3\,(Ba^4 + 4\,Aa^3b + 12\,Ba^2b^2 + 8\,Aab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \right.\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="g
iac")
```

output
```
1/6*(12*B*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(4*B*a
*b^3 + A*b^4)*(d*x + c) + 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)
*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2
+ 8*A*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^4*tan(1/2*d*x +
 1/2*c)^5 - 3*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*b*tan(1/2*d*x + 1/2*
c)^5 + 24*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*
c)^5 - 4*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3
- 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B
*a^4*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b*t
an(1/2*d*x + 1/2*c) + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 44.90 (sec) , antiderivative size = 636, normalized size of antiderivative = 3.21

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^4,x)
```

output
```
((A*a^4*sin(3*c + 3*d*x))/6 + (B*a^4*sin(2*c + 2*d*x))/4 + (B*b^4*sin(2*c
+ 2*d*x))/4 + (B*b^4*sin(4*c + 4*d*x))/8 + (A*a^4*sin(c + d*x))/2 + B*a^3*
b*sin(c + d*x) + (3*A*b^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/2 - (B*a^4*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*3i)/4 + A*a^3*b*sin(2*c + 2*d*x) + (3*A*a^2*b^2*sin(c + d*x))/2 +
 B*a^3*b*sin(3*c + 3*d*x) + (A*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2))*cos(3*c + 3*d*x))/2 - (B*a^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*cos(3*c + 3*d*x)*1i)/4 + (3*A*a^2*b^2*sin(3*c + 3*d*x))/2 - A*a*b
^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*2i -
A*a^3*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*
1i + 2*B*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x
) - B*a^2*b^2*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)
)*9i - B*a^2*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c
+ 3*d*x)*3i - A*a*b^3*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
(d*x)/2))*6i - A*a^3*b*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
 (d*x)/2))*3i + 6*B*a*b^3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/(d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.03

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{-15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a^4b - 60\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)}{}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4*b - 60*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**3 + 15*cos(
c + d*x)*log(tan((c + d*x)/2) - 1)*a**4*b + 60*cos(c + d*x)*log(tan((c + d
*x)/2) - 1)*a**2*b**3 + 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c +
d*x)**2*a**4*b + 60*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
*a**2*b**3 - 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**4*b - 60*cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*a**2*b**3 + 6*cos(c + d*x)*sin(c + d*x)**3
*b**5 + 30*cos(c + d*x)*sin(c + d*x)**2*a*b**4*d*x - 15*cos(c + d*x)*sin(c
 + d*x)*a**4*b - 6*cos(c + d*x)*sin(c + d*x)*b**5 - 30*cos(c + d*x)*a*b**4
*d*x + 4*sin(c + d*x)**3*a**5 + 60*sin(c + d*x)**3*a**3*b**2 - 6*sin(c + d
*x)*a**5 - 60*sin(c + d*x)*a**3*b**2)/(6*cos(c + d*x)*d*(sin(c + d*x)**2 -
 1))
```

3.247 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 216

$$\begin{aligned}\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx &= b^4Bx+\frac{(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B)\operatorname{arctanh}(\sin(c+dx))}{8d}\\&+\frac{a(16a^2Ab+19Ab^3+4a^3B+34ab^2B)\tan(c+dx)}{6d}\\&+\frac{a^2(9a^2A+26Ab^2+32abB)\sec(c+dx)\tan(c+dx)}{24d}\\&+\frac{a(7Ab+4aB)(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{12d}\\&+\frac{aA(a+b\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

output
```
b^4*B*x+1/8*(3*A*a^4+24*A*a^2*b^2+8*A*b^4+16*B*a^3*b+32*B*a*b^3)*arctanh(s
in(d*x+c))/d+1/6*a*(16*A*a^2*b+19*A*b^3+4*B*a^3+34*B*a*b^2)*tan(d*x+c)/d+1
/24*a^2*(9*A*a^2+26*A*b^2+32*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1/12*a*(7*A*b+
4*B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+b*cos(d*x+c
))^3*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.86

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{24b^4Bdx+24b^3(Ab+4aB)\coth^{-1}(\sin(c+dx))+9a^4A\text{arctanh}(\sin(c+dx))+24a^2b(3Ab+2aB)\text{arcta}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(24*b^4*B*d*x + 24*b^3*(A*b + 4*a*B)*ArcCoth[Sin[c + d*x]] + 9*a^4*A*ArcTa
nh[Sin[c + d*x]] + 24*a^2*b*(3*A*b + 2*a*B)*ArcTanh[Sin[c + d*x]] + 48*a*b
^2*(2*A*b + 3*a*B)*Tan[c + d*x] + 9*a^4*A*Sec[c + d*x]*Tan[c + d*x] + 24*a
^2*b*(3*A*b + 2*a*B)*Sec[c + d*x]*Tan[c + d*x] + 6*a^4*A*Sec[c + d*x]^3*Ta
n[c + d*x] + 8*a^3*(4*A*b + a*B)*Tan[c + d*x]*(3 + Tan[c + d*x]^2))/(24*d)
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3468, 3042, 3526, 3042, 3510, 25, 3042, 3500, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{4}\int (a+b\cos(c+dx))^2\left(4b^2B\cos^2(c+dx)+\left(3Aa^2+8bBa+4Ab^2\right)\cos(c+dx)+a(7Ab+4aB)\right)\sec^4(c+dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+8bBa+4Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(7Ab+4a\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3526

$$\frac{1}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(12B\cos^2(c+dx)b^3+a\left(9Aa^2+32bBa+26Ab^2\right)+\left(8Ba^3+23Aba^2+36b^2Ba+12\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(12B\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+a\left(9Aa^2+32bBa+26Ab^2\right)+\left(8Ba^3+23Aba^2+3\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3510

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{a^2\left(9a^2A+32abB+26Ab^2\right)\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int -\left(\left(24B\cos^2(c+dx)b^4+4a\left(4Ba^3+16Aba^2\right.\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 25

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(24B\cos^2(c+dx)b^4+4a\left(4Ba^3+16Aba^2+34b^2Ba+19Ab^3\right)+3\left(3Aa^4+16bBa^3+24Ab^2a^2+32\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{24B\sin\left(c+dx+\frac{\pi}{2}\right)^2b^4+4a\left(4Ba^3+16Aba^2+34b^2Ba+19Ab^3\right)+3\left(3Aa^4+16bBa^3+24Ab^2a^2\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3500

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(3Aa^4+16bBa^3+24Ab^2a^2+32b^3Ba+8Ab^4+8b^4B\cos(c+dx)\right)\sec(c+dx)dx+\frac{4a\left(4a^3B+1\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\int\left(3Aa^4+16bBa^3+24Ab^2a^2+32b^3Ba+8Ab^4+8b^4B\cos(c+dx)\right)\sec(c+dx)dx+\frac{4a\left(4a^3B+1\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\int\frac{3Aa^4+16bBa^3+24Ab^2a^2+32b^3Ba+8Ab^4+8b^4B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4a\left(4a^3B+16a^2Ab+\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3214

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(\left(3a^4A+16a^3bB+24a^2Ab^2+32ab^3B+8Ab^4\right)\int\sec(c+dx)dx+8b^4Bx\right)+\frac{4a\left(4a^3B+16a^2Ab\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(\left(3a^4A+16a^3bB+24a^2Ab^2+32ab^3B+8Ab^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+8b^4Bx\right)+\frac{4a\left(4a^3B+16\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$\downarrow$ 4257

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{a^2(9a^2A+32abB+26Ab^2)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(\frac{4a(4a^3B+16a^2Ab+34ab^2B+19Ab^3)\tan(c\right.\right.\right.}{d}$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^3}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((a*(7*A*
b + 4*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((a
^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (3*(
8*b^4*B*x + ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*
ArcTanh[Sin[c + d*x]])/d) + (4*a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b
^2*B)*Tan[c + d*x])/d)/2)/3)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 19.53 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a^4 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{(A b^4+4Bab^3)\ln(\sec(dx+c)+\tan(d}{d}$
derivativedivides	$\frac{a^4 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-B a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4A}{}$
default	$\frac{a^4 A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-B a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4A}{}$
parallelrisch	$\frac{-36\left(a^4A+8Aa^2b^2+\frac{8}{3}Ab^4+\frac{16}{3}Ba^3b+\frac{32}{3}Bab^3\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+36\left(a^4A+8A\right.}{}$
risch	$b^4Bx-\frac{ia\left(-16a^3B-64Aa^2b-144Bab^2-72Aab^2e^{i(dx+c)}-48Ba^2b\,e^{i(dx+c)}+48Ba^2b\,e^{5i(dx+c)}-192Aa^2b\,e^{4i(dx\right.}}{}$

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBO
SE)
```

output
```
a^4*A/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+(A*b^4+4*B*a*b^3)/d*ln(sec(d*x+c)+tan(d*x+c))+(4*A*a*b^3+6*B*
a^2*b^2)/d*tan(d*x+c)+(6*A*a^2*b^2+4*B*a^3*b)/d*(1/2*sec(d*x+c)*tan(d*x+c)
+1/2*ln(sec(d*x+c)+tan(d*x+c)))-(4*A*a^3*b+B*a^4)/d*(-2/3-1/3*sec(d*x+c)^2
)*tan(d*x+c)+B*b^4/d*(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.16

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{48\,Bb^4dx\cos{(dx+c)}^4+3\,(3\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+32\,Bab^3+8\,Ab^4)\cos{(dx+c)}^4\log{(\sin{(dx+}}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="f
ricas")
```

output
```
1/48*(48*B*b^4*d*x*cos(d*x + c)^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a^4
 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(6*A*a^4 + 16*(B*a^4 + 4*A*a^3*b + 9*B*a^2*b^2 + 6*A*
a*b^3)*cos(d*x + c)^3 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*cos(d*x +
c)^2 + 8*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.47

$$\int (a + b\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx$$

$$= \frac{16\left(\tan(dx + c)^3 + 3\tan(dx + c)\right)Ba^4 + 64\left(\tan(dx + c)^3 + 3\tan(dx + c)\right)Aa^3b + 48(dx + c)Bb^4 - 3}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="m
axima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 64*(tan(d*x + c)^3 + 3*
tan(d*x + c))*A*a^3*b + 48*(d*x + c)*B*b^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 48*B*a^3*b*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 72*A*a^2*b
^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 96*B*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 24*A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 288*B*a^2*b
^2*tan(d*x + c) + 192*A*a*b^3*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(206) = 412$.

Time = 0.21 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.94

$$\int (a + b\cos(c + dx))^4 (A + B\cos(c + dx)) \sec^5(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="g
iac")
```

output
```
1/24*(24*(d*x + c)*B*b^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a
*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^4 + 16*B*a^3
*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) + 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/2*d*x + 1/2*c)^7
- 96*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 +
72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7
 - 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 40
*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 48*B*
a^3*b*tan(1/2*d*x + 1/2*c)^5 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 432*B
*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A
*a^4*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^
2*tan(1/2*d*x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*A*a*
b^3*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) + 24*B*a^4*tan(
1/2*d*x + 1/2*c) + 96*A*a^3*b*tan(1/2*d*x + 1/2*c) + 48*B*a^3*b*tan(1/2*d*
x + 1/2*c) + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 144*B*a^2*b^2*tan(1/2*d*x
 + 1/2*c) + 96*A*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
4)/d
```

Mupad [B] (verification not implemented)

Time = 44.20 (sec) , antiderivative size = 1969, normalized size of antiderivative = 9.12

$$\int (a + b\cos(c + dx))^4(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^5,x)
```

output
```
((27*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + 9*A*b^4*atanh
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (9*A*a^4*sin(3*c + 3*d*x))/8 + 4
*B*a^4*sin(2*c + 2*d*x) + B*a^4*sin(4*c + 4*d*x) + 9*B*b^4*atan((9*A^2*a^8
*sin(c/2 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2) + 64*B^2*b^8*sin(c/2 +
 (d*x)/2) + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*A^2*a^4*b^4*sin(c/2 +
 (d*x)/2) + 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1024*B^2*a^2*b^6*sin(c/2
+ (d*x)/2) + 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 256*B^2*a^6*b^2*sin(c/2
 + (d*x)/2) + 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 960*A*B*a^5*b^3*sin(c/
2 + (d*x)/2) + 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96*A*B*a^7*b*sin(c/2 + (
d*x)/2))/(cos(c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*b^8 + 64*B^2*b^8 + 384*A^
2*a^2*b^6 + 624*A^2*a^4*b^4 + 144*A^2*a^6*b^2 + 1024*B^2*a^2*b^6 + 1024*B^
2*a^4*b^4 + 256*B^2*a^6*b^2 + 512*A*B*a*b^7 + 96*A*B*a^7*b + 1792*A*B*a^3*
b^5 + 960*A*B*a^5*b^3))) + (33*A*a^4*sin(c + d*x))/8 + 12*B*b^4*cos(2*c +
2*d*x)*atan((9*A^2*a^8*sin(c/2 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2)
+ 64*B^2*b^8*sin(c/2 + (d*x)/2) + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624
*A^2*a^4*b^4*sin(c/2 + (d*x)/2) + 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 102
4*B^2*a^2*b^6*sin(c/2 + (d*x)/2) + 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 2
56*B^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) +
960*A*B*a^5*b^3*sin(c/2 + (d*x)/2) + 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96
*A*B*a^7*b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.65

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
( - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**5 - 120*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3*b**2 - 120*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**4 + 18*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**5 + 240*cos(c + d*x)*log(tan
((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b**2 + 240*cos(c + d*x)*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**2*a*b**4 - 9*cos(c + d*x)*log(tan((c + d*x)/
2) - 1)*a**5 - 120*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**3*b**2 - 120*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**4 + 9*cos(c + d*x)*log(tan((c
+ d*x)/2) + 1)*sin(c + d*x)**4*a**5 + 120*cos(c + d*x)*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**4*a**3*b**2 + 120*cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*sin(c + d*x)**4*a*b**4 - 18*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*a**5 - 240*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x
)**2*a**3*b**2 - 240*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**
2*a*b**4 + 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**5 + 120*cos(c + d*x
)*log(tan((c + d*x)/2) + 1)*a**3*b**2 + 120*cos(c + d*x)*log(tan((c + d*x)
/2) + 1)*a*b**4 + 24*cos(c + d*x)*sin(c + d*x)**4*b**5*d*x - 9*cos(c + d*x
)*sin(c + d*x)**3*a**5 - 120*cos(c + d*x)*sin(c + d*x)**3*a**3*b**2 - 48*c
os(c + d*x)*sin(c + d*x)**2*b**5*d*x + 15*cos(c + d*x)*sin(c + d*x)*a**5 +
 120*cos(c + d*x)*sin(c + d*x)*a**3*b**2 + 24*cos(c + d*x)*b**5*d*x + 80*s
in(c + d*x)**5*a**4*b + 240*sin(c + d*x)**5*a**2*b**3 - 200*sin(c + d*x...
```

3.248 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 267

$$\begin{aligned}\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx &= \frac{(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B)\operatorname{arctanh}(\sin(c+dx))}{8d}\\ &+\frac{(8a^4A+60a^2Ab^2+15Ab^4+40a^3bB+60ab^3B)\tan(c+dx)}{15d}\\ &+\frac{a(60a^2Ab+56Ab^3+15a^3B+110ab^2B)\sec(c+dx)\tan(c+dx)}{40d}\\ &+\frac{a^2(8a^2A+18Ab^2+25abB)\sec^2(c+dx)\tan(c+dx)}{30d}\\ &+\frac{a(8Ab+5aB)(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{20d}\\ &+\frac{aA(a+b\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d}\end{aligned}$$

output
```
1/8*(12*A*a^3*b+16*A*a*b^3+3*B*a^4+24*B*a^2*b^2+8*B*b^4)*arctanh(sin(d*x+c
))/d+1/15*(8*A*a^4+60*A*a^2*b^2+15*A*b^4+40*B*a^3*b+60*B*a*b^3)*tan(d*x+c)
/d+1/40*a*(60*A*a^2*b+56*A*b^3+15*B*a^3+110*B*a*b^2)*sec(d*x+c)*tan(d*x+c)
/d+1/30*a^2*(8*A*a^2+18*A*b^2+25*B*a*b)*sec(d*x+c)^2*tan(d*x+c)/d+1/20*a*(
8*A*b+5*B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*A*(a+b*cos
(d*x+c))^3*sec(d*x+c)^4*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.95

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{b^4B\coth^{-1}(\sin(c+dx))}{d} + \frac{3a^3(4Ab+aB)\text{arctanh}(\sin(c+dx))}{8d} + \frac{ab^2(2Ab+3aB)\text{arctanh}(\sin(c+dx))}{d} + \frac{b^3(Ab+4aB)\tan(c+dx)}{d} + \frac{3a^3(4Ab+aB)\sec(c+dx)\tan(c+dx)}{8d} + \frac{ab^2(2Ab+3aB)\sec(c+dx)\tan(c+dx)}{d} + \frac{a^3(4Ab+aB)\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{2a^2b(3Ab+2aB)\left(3\tan(c+dx)+\tan^3(c+dx)\right)}{3d} + \frac{a^4A(15\tan(c+dx)+10\tan^3(c+dx)+3\tan^5(c+dx))}{15d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(b^4*B*ArcCoth[Sin[c + d*x]])/d + (3*a^3*(4*A*b + a*B)*ArcTanh[Sin[c + d*x
]])/(8*d) + (a*b^2*(2*A*b + 3*a*B)*ArcTanh[Sin[c + d*x]])/d + (b^3*(A*b +
4*a*B)*Tan[c + d*x])/d + (3*a^3*(4*A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(
8*d) + (a*b^2*(2*A*b + 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/d + (a^3*(4*A*b +
 a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*a^2*b*(3*A*b + 2*a*B)*(3*Tan
[c + d*x] + Tan[c + d*x]^3))/(3*d) + (a^4*A*(15*Tan[c + d*x] + 10*Tan[c +
d*x]^3 + 3*Tan[c + d*x]^5))/(15*d)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 3468, 3042, 3526, 3042, 3510, 25, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{5}\int (a+b\cos(c+dx))^2\left(b(aA+5bB)\cos^2(c+dx)+\left(4Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(8Ab+5aB)\right)\sec^5(c+dx)dx+\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(b(aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(4Aa^2+10bBa+5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

$$\Big\downarrow \text{3526}$$

$$\frac{1}{5}\left(\frac{1}{4}\int (a+b\cos(c+dx))\left(b\left(5Ba^2+12Aba+20b^2B\right)\cos^2(c+dx)+\left(15Ba^3+44Aba^2+60b^2Ba+20Ab^3\right)\cos\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(5Ba^2+12Aba+20b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(15Ba^3+44Aba^2+60b^2Ba+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$
$$\Big\downarrow \text{3510}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{2a^2\left(8a^2A+25abB+18Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{1}{3}\int -\left(\left(3b^2\left(5Ba^2+12Aba+20b^2B\right)\cos^2(c+\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \left(3b^2\left(5Ba^2+12Aba+20b^2B\right)\cos^2(c+dx)+4\left(8Aa^4+40bBa^3+60Ab^2a^2+60b^3Ba+15Ab^4\right)\cos(\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \frac{3b^2\left(5Ba^2+12Aba+20b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+4\left(8Aa^4+40bBa^3+60Ab^2a^2+60b^3Ba+15Ab^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$
$$\Big\downarrow \text{3500}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(8\left(8Aa^4+40bBa^3+60Ab^2a^2+60b^3Ba+15Ab^4\right)+15\left(3Ba^4+12Aba^3+24b^2Ba^2+16Ab^3a+\right.\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{8\left(8Aa^4+40bBa^3+60Ab^2a^2+60b^3Ba+15Ab^4\right)+15\left(3Ba^4+12Aba^3+24b^2Ba^2+16Ab^3a+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$
$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(8\left(8a^4A+40a^3bB+60a^2Ab^2+60ab^3B+15Ab^4\right)\int \sec^2(c+dx)dx+15\left(3a^4B+12a^3Ab+24a^2b\right.\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15\left(3a^4B+12a^3Ab+24a^2b^2B+16aAb^3+8b^4B\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+8\left(8a^4A+40a^3bB+6\right.\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

$\downarrow$ 4254

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15\left(3a^4B+12a^3Ab+24a^2b^2B+16aAb^3+8b^4B\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{8\left(8a^4A+40a^3bB+6\right.}{}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

$\downarrow$ 24

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15\left(3a^4B+12a^3Ab+24a^2b^2B+16aAb^3+8b^4B\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{8\left(8a^4A+40a^3bB+6\right.}{}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

$\downarrow$ 4257

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{2a^2\left(8a^2A+25abB+18Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{1}{3}\left(\frac{3a\left(15a^3B+60a^2Ab+110ab^2B+56Ab^3\right)\tan}{2d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^3}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((a*(8*A*
b + 5*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((2
*a^2*(8*a^2*A + 18*A*b^2 + 25*a*b*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) +
((3*a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b^2*B)*Sec[c + d*x]*Tan[c
+ d*x])/(2*d) + ((15*(12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8
*b^4*B)*ArcTanh[Sin[c + d*x]])/d + (8*(8*a^4*A + 60*a^2*A*b^2 + 15*A*b^4 +
 40*a^3*b*B + 60*a*b^3*B)*Tan[c + d*x])/d)/2)/3)/4)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 21.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{a^4 A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{(A\,b^4+4Ba\,b^3)\tan(dx+c)}{d}+\frac{(4Aa\,b^3+6B\,a^2b^2)\left(\frac{\sec(dx+c)}{}\right.}{}$
derivativedivides	$\frac{-a^4 A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+B\,a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)}{8}\right.}{}$
default	$\frac{-a^4 A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+B\,a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)}{8}\right.}{}$
parallelrisch	$\frac{-180(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos(dx+c))\left(A\,a^3b+\frac{4}{3}Aa\,b^3+\frac{1}{4}B\,a^4+2B\,a^2b^2+\frac{2}{3}B\,b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+}{}$
risch	$\frac{i\left(480Ba\,b^3+480A\,a^2b^2+320B\,a^3b+45B\,a^4\mathrm{e}^{i(dx+c)}+640A\,a^4\mathrm{e}^{4i(dx+c)}+480A\,b^4\mathrm{e}^{6i(dx+c)}+320A\,a^4\mathrm{e}^{2i(dx+c)}+480A\,b\right.}{}$

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x,method=_RETURNVERBO
SE)
```

output
```
-a^4*A/d*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+(A*b^4+4*B*
a*b^3)/d*tan(d*x+c)+(4*A*a*b^3+6*B*a^2*b^2)/d*(1/2*sec(d*x+c)*tan(d*x+c)+1
/2*ln(sec(d*x+c)+tan(d*x+c)))-(6*A*a^2*b^2+4*B*a^3*b)/d*(-2/3-1/3*sec(d*x+
c)^2)*tan(d*x+c)+(4*A*a^3*b+B*a^4)/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*
tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+B*b^4/d*ln(sec(d*x+c)+tan(d*x+c)
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{15\,(3\,Ba^4+12\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+8\,Bb^4)\cos{(dx+c)}^5\log{(\sin{(dx+c)}+1)}-15\,(3\,Ba^4+12$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="f
ricas")
```

output
```
1/240*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*cos
(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^
2 + 16*A*a*b^3 + 8*B*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*A*
a^4 + 8*(8*A*a^4 + 40*B*a^3*b + 60*A*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4)*cos(
d*x + c)^4 + 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3)*cos(d*x
 + c)^3 + 16*(2*A*a^4 + 10*B*a^3*b + 15*A*a^2*b^2)*cos(d*x + c)^2 + 30*(B*
a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.45

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$

$$= \frac{16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Aa^4+320\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="m
axima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
 320*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3*b + 480*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^2*b^2 - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(
sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(s
in(d*x + c) - 1)) - 60*A*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) - 360*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(
sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 240*A*a*b^3*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120
*B*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 960*B*a*b^3*tan(d
*x + c) + 240*A*b^4*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. 2(255) = 510.

Time = 0.20 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.18

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="g
iac")
```

output
```
1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*log
(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2
+ 16*A*a*b^3 + 8*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^4*
tan(1/2*d*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 300*A*a^3*b*tan
(1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*t
an(1/2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*a*b^3
*tan(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^4*t
an(1/2*d*x + 1/2*c)^9 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^4*tan(1/
2*d*x + 1/2*c)^7 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 1280*B*a^3*b*tan(1
/2*d*x + 1/2*c)^7 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^2*b^2*
tan(1/2*d*x + 1/2*c)^7 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 1920*B*a*b^3
*tan(1/2*d*x + 1/2*c)^7 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^4*tan
(1/2*d*x + 1/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 2400*A*a^2*b^2
*tan(1/2*d*x + 1/2*c)^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 720*A*b^4*
tan(1/2*d*x + 1/2*c)^5 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^4*tan(1
/2*d*x + 1/2*c)^3 - 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1280*B*a^3*b*tan(
1/2*d*x + 1/2*c)^3 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*B*a^2*b^2
*tan(1/2*d*x + 1/2*c)^3 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1920*B*a*b^
3*tan(1/2*d*x + 1/2*c)^3 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^4*ta
n(1/2*d*x + 1/2*c) + 75*B*a^4*tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1/...
```

Mupad [B] (verification not implemented)

Time = 45.21 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.08

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Ba^4}{8}+\frac{3Aa^3b}{2}+3Ba^2b^2+2Aab^3+Bb^4\right)}{\frac{3Ba^4}{2}+6Aa^3b+12Ba^2b^2+8Aab^3+4Bb^4}\right)\left(\frac{3Ba^4}{4}+3Aa^3b+6Ba^2b^2+4Aab^3+2Bb^4\right)}{d}$$
$$-\frac{\left(2Aa^4+2Ab^4-\frac{5Ba^4}{4}+12Aa^2b^2-6Ba^2b^2-4Aab^3-5Aa^3b+8Bab^3+8Ba^3b\right)\tan\left(\frac{c}{2}+\frac{d}{}\right.}{}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^6,x)
```

output
```
(atanh((4*tan(c/2 + (d*x)/2)*((3*B*a^4)/8 + B*b^4 + 3*B*a^2*b^2 + 2*A*a*b^
3 + (3*A*a^3*b)/2))/((3*B*a^4)/2 + 4*B*b^4 + 12*B*a^2*b^2 + 8*A*a*b^3 + 6*
A*a^3*b))*((3*B*a^4)/4 + 2*B*b^4 + 6*B*a^2*b^2 + 4*A*a*b^3 + 3*A*a^3*b))/d
 - (tan(c/2 + (d*x)/2)*(2*A*a^4 + 2*A*b^4 + (5*B*a^4)/4 + 12*A*a^2*b^2 + 6
*B*a^2*b^2 + 4*A*a*b^3 + 5*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) + tan(c/2 + (d
*x)/2)^5*((116*A*a^4)/15 + 12*A*b^4 + 40*A*a^2*b^2 + 48*B*a*b^3 + (80*B*a^
3*b)/3) + tan(c/2 + (d*x)/2)^9*(2*A*a^4 + 2*A*b^4 - (5*B*a^4)/4 + 12*A*a^2
*b^2 - 6*B*a^2*b^2 - 4*A*a*b^3 - 5*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) - tan(
c/2 + (d*x)/2)^3*((8*A*a^4)/3 + 8*A*b^4 + (B*a^4)/2 + 32*A*a^2*b^2 + 12*B*
a^2*b^2 + 8*A*a*b^3 + 2*A*a^3*b + 32*B*a*b^3 + (64*B*a^3*b)/3) - tan(c/2 +
 (d*x)/2)^7*((8*A*a^4)/3 + 8*A*b^4 - (B*a^4)/2 + 32*A*a^2*b^2 - 12*B*a^2*b
^2 - 8*A*a*b^3 - 2*A*a^3*b + 32*B*a*b^3 + (64*B*a^3*b)/3))/(d*(5*tan(c/2 +
 (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/
2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 773, normalized size of antiderivative = 2.90

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

output
```
( - 225*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**4*b - 60
0*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2*b**3 - 120*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b**5 + 450*cos(c + d
*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4*b + 1200*cos(c + d*x)*l
og(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**3 + 240*cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**5 - 225*cos(c + d*x)*log(tan((c +
 d*x)/2) - 1)*a**4*b - 600*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**2*b**
3 - 120*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b**5 + 225*cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**4*b + 600*cos(c + d*x)*log(tan((
c + d*x)/2) + 1)*sin(c + d*x)**4*a**2*b**3 + 120*cos(c + d*x)*log(tan((c +
 d*x)/2) + 1)*sin(c + d*x)**4*b**5 - 450*cos(c + d*x)*log(tan((c + d*x)/2)
 + 1)*sin(c + d*x)**2*a**4*b - 1200*cos(c + d*x)*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**2*a**2*b**3 - 240*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**2*b**5 + 225*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**4*b + 6
00*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**2*b**3 + 120*cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*b**5 - 225*cos(c + d*x)*sin(c + d*x)**3*a**4*b - 60
0*cos(c + d*x)*sin(c + d*x)**3*a**2*b**3 + 375*cos(c + d*x)*sin(c + d*x)*a
**4*b + 600*cos(c + d*x)*sin(c + d*x)*a**2*b**3 + 64*sin(c + d*x)**5*a**5
+ 800*sin(c + d*x)**5*a**3*b**2 + 600*sin(c + d*x)**5*a*b**4 - 160*sin(c +
 d*x)**3*a**5 - 2000*sin(c + d*x)**3*a**3*b**2 - 1200*sin(c + d*x)**3*a...
```

3.249 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 324

$$\begin{aligned}
&\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx\\
&=\frac{(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)\operatorname{arctanh}(\sin(c+dx))}{16d}\\
&\quad+\frac{(32a^3Ab+40aAb^3+8a^4B+60a^2b^2B+15b^4B)\tan(c+dx)}{15d}\\
&\quad+\frac{(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)\sec(c+dx)\tan(c+dx)}{16d}\\
&\quad+\frac{a(16a^2Ab+13Ab^3+4a^3B+27ab^2B)\sec^2(c+dx)\tan(c+dx)}{15d}\\
&\quad+\frac{a^2(25a^2A+48Ab^2+72abB)\sec^3(c+dx)\tan(c+dx)}{120d}\\
&\quad+\frac{a(3Ab+2aB)(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{10d}\\
&\quad+\frac{aA(a+b\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d}
\end{aligned}$$

output
```
1/16*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*arctanh(sin(d*x+
c))/d+1/15*(32*A*a^3*b+40*A*a*b^3+8*B*a^4+60*B*a^2*b^2+15*B*b^4)*tan(d*x+c
)/d+1/16*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*sec(d*x+c)*t
an(d*x+c)/d+1/15*a*(16*A*a^2*b+13*A*b^3+4*B*a^3+27*B*a*b^2)*sec(d*x+c)^2*t
an(d*x+c)/d+1/120*a^2*(25*A*a^2+48*A*b^2+72*B*a*b)*sec(d*x+c)^3*tan(d*x+c)
/d+1/10*a*(3*A*b+2*B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d+1/6*a
*A*(a+b*cos(d*x+c))^3*sec(d*x+c)^5*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$$
$$= \frac{15(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\,(240(4a^3Ab+4aA}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

output
```
(15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[S
in[c + d*x]] + Tan[c + d*x]*(240*(4*a^3*A*b + 4*a*A*b^3 + a^4*B + 6*a^2*b^
2*B + b^4*B) + 15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^
3*B)*Sec[c + d*x] + 10*a^2*(5*a^2*A + 36*A*b^2 + 24*a*b*B)*Sec[c + d*x]^3
+ 40*a^4*A*Sec[c + d*x]^5 + 160*a*(4*a^2*A*b + 2*A*b^3 + a^3*B + 3*a*b^2*B
)*Tan[c + d*x]^2 + 48*a^3*(4*A*b + a*B)*Tan[c + d*x]^4))/(240*d)
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 3468, 3042, 3526, 3042, 3510, 25, 3042, 3500, 27, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$$

$\downarrow$ 3042

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$\downarrow$ 3468

$$\frac{1}{6}\int (a+b\cos(c+dx))^2\left(2b(aA+3bB)\cos^2(c+dx)+\left(5Aa^2+12bBa+6Ab^2\right)\cos(c+dx)+3a(3Ab+2aB)\right)\sec^6(c+dx)dx+\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(2b(aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Aa^2+12bBa+6Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\downarrow$ 3526

$$\frac{1}{6}\left(\frac{1}{5}\int (a+b\cos(c+dx))\left(2b\left(6Ba^2+14Aba+15b^2B\right)\cos^2(c+dx)+\left(24Ba^3+71Aba^2+90b^2Ba+30Ab^3\right)c\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(2b\left(6Ba^2+14Aba+15b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(24Ba^3+71Aba^2+90b^2Ba\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\downarrow$ 3510

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{a^2\left(25a^2A+72abB+48Ab^2\right)\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{1}{4}\int -\left(\left(8b^2\left(6Ba^2+14Aba+15b^2B\right)\cos^2(c+\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\Big\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\int\left(8b^2\left(6Ba^2+14Aba+15b^2B\right)\cos^2(c+dx)+15\left(5Aa^4+24bBa^3+36Ab^2a^2+32b^3Ba+8Ab^4\right)\cos(\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\int\frac{8b^2\left(6Ba^2+14Aba+15b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+15\left(5Aa^4+24bBa^3+36Ab^2a^2+32b^3Ba+8Ab^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\Big\downarrow$ 3500

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int3\left(15\left(5Aa^4+24bBa^3+36Ab^2a^2+32b^3Ba+8Ab^4\right)+8\left(8Ba^4+32Aba^3+60b^2Ba^2+40Ab^3a+\right.\right.\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\Big\downarrow$ 27

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(\int\left(15\left(5Aa^4+24bBa^3+36Ab^2a^2+32b^3Ba+8Ab^4\right)+8\left(8Ba^4+32Aba^3+60b^2Ba^2+40Ab^3a+15b\right.\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(\int\frac{15\left(5Aa^4+24bBa^3+36Ab^2a^2+32b^3Ba+8Ab^4\right)+8\left(8Ba^4+32Aba^3+60b^2Ba^2+40Ab^3a+15b\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$\Big\downarrow$ 3227

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(15\left(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4\right)\int\sec^3(c+dx)dx+8\left(8a^4B+32a^3Ab+60a^2b^2B+\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(8\left(8a^4B+32a^3Ab+60a^2b^2B+40aAb^3+15b^4B\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+15\left(5a^4A+24a^3bB+36\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{8\left(8a^4B+32a^3Ab+60a^2b^2B+40aAb^3+15b^4B\right)\int 1d(-\tan(c+dx))}{d}+15\left(5a^4A+24a^3bB+36a^{2}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(15\left(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{8a\left(4a^3B+16a^2Ab+27\right.}{}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(15\left(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4\right)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\left(15\left(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4\right)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+d}{2d}\right.\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{a^2\left(25a^2A+72abB+48Ab^2\right)\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{1}{4}\left(\frac{8a\left(4a^3B+16a^2Ab+27ab^2B+13Ab^3\right)\tan(}{d}\right.\right.\right.$$
$$\frac{aA\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^3}{6d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + ((3*a*(3*
A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (
(a^2*(25*a^2*A + 48*A*b^2 + 72*a*b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 ((8*(32*a^3*A*b + 40*a*A*b^3 + 8*a^4*B + 60*a^2*b^2*B + 15*b^4*B)*Tan[c +
 d*x])/d + (8*a*(16*a^2*A*b + 13*A*b^3 + 4*a^3*B + 27*a*b^2*B)*Sec[c + d*x
]^2*Tan[c + d*x])/d + 15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B +
32*a*b^3*B)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*
d)))/4)/5)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 23.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.85

method	result
parts	$\frac{a^4 A\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{(A b^4+4Ba b^3)\left(\frac{\sec(dx}{}\right.}{}$
derivativedivides	$\frac{a^4 A\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-B a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\right.}{}$
default	$\frac{a^4 A\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-B a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\right.}{}$
parallelrisch	$\frac{-75\left(a^4A+\frac{36}{5}Aa^2b^2+\frac{8}{5}Ab^4+\frac{24}{5}Ba^3b+\frac{32}{5}Bab^3\right)(\cos(6dx+6c)+6\cos(4dx+4c)+15\cos(2dx+2c)+10)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
risch	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x,method=_RETURNVERBO
SE)
```

output
```
a^4*A/d*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+(A*b^4+4*B*a*b^3)/d*(1/2*sec(d*x+c)*tan(d
*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-(4*A*a*b^3+6*B*a^2*b^2)/d*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c)+(6*A*a^2*b^2+4*B*a^3*b)/d*(-(-1/4*sec(d*x+c)^3-3/8
*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-(4*A*a^3*b+B*a^4)/d
*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+B*b^4/d*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$$
$$=\frac{15\left(5Aa^4+24Ba^3b+36Aa^2b^2+32Bab^3+8Ab^4\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left(5Aa^4+24\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="f
ricas")
```

output
```
1/480*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos
(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^
2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(8
*B*a^4 + 32*A*a^3*b + 60*B*a^2*b^2 + 40*A*a*b^3 + 15*B*b^4)*cos(d*x + c)^5
 + 40*A*a^4 + 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b
^4)*cos(d*x + c)^4 + 32*(2*B*a^4 + 8*A*a^3*b + 15*B*a^2*b^2 + 10*A*a*b^3)*
cos(d*x + c)^3 + 10*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2)*cos(d*x + c)^2 +
 48*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.46

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$$

$$= \frac{32\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^4+128\left(3\tan(dx+c)^5+10\tan(dx+c)^3\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="m
axima")
```

output
```
1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 +
 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3*b + 96
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b^2 + 640*(tan(d*x + c)^3 + 3*ta
n(d*x + c))*A*a*b^3 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 +
33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1
) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 120*B*a^3*b*(2*
(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1
) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*A*a^2*b^2*(2*
(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1
) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 480*B*a*b^3*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) - 120*A*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
 + 1) + log(sin(d*x + c) - 1)) + 480*B*b^4*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1186 vs. $2(310) = 620$.

Time = 0.25 (sec) , antiderivative size = 1186, normalized size of antiderivative = 3.66

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="g
iac")
```

output
```
1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log
(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
+ 32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*A*a^4*
tan(1/2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*A*a^3*b*
tan(1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*A*a^2*
b^2*tan(1/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960
*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 1
20*A*b^4*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x + 1/2*c)^11 + 25*
A*a^4*tan(1/2*d*x + 1/2*c)^9 + 560*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*A*a
^3*b*tan(1/2*d*x + 1/2*c)^9 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1260*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3
520*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 -
 360*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 1200*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 45
0*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*
A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*
A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 -
 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^7
+ 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 4
50*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992
*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + ...
```

Mupad [B] (verification not implemented)

Time = 44.83 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.18

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \text{Too large to display}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^7,x)
```

output
```
(atanh((4*tan(c/2 + (d*x)/2)*((5*A*a^4)/16 + (A*b^4)/2 + (9*A*a^2*b^2)/4 +
 2*B*a*b^3 + (3*B*a^3*b)/2))/((5*A*a^4)/4 + 2*A*b^4 + 9*A*a^2*b^2 + 8*B*a*
b^3 + 6*B*a^3*b))*((5*A*a^4)/8 + A*b^4 + (9*A*a^2*b^2)/2 + 4*B*a*b^3 + 3*B
*a^3*b))/d + (tan(c/2 + (d*x)/2)*((11*A*a^4)/8 + A*b^4 + 2*B*a^4 + 2*B*b^4
 + (15*A*a^2*b^2)/2 + 12*B*a^2*b^2 + 8*A*a*b^3 + 8*A*a^3*b + 4*B*a*b^3 + 5
*B*a^3*b) + tan(c/2 + (d*x)/2)^11*((11*A*a^4)/8 + A*b^4 - 2*B*a^4 - 2*B*b^
4 + (15*A*a^2*b^2)/2 - 12*B*a^2*b^2 - 8*A*a*b^3 - 8*A*a^3*b + 4*B*a*b^3 +
5*B*a^3*b) - tan(c/2 + (d*x)/2)^3*(3*A*b^4 - (5*A*a^4)/24 + (14*B*a^4)/3 +
 10*B*b^4 + (21*A*a^2*b^2)/2 + 44*B*a^2*b^2 + (88*A*a*b^3)/3 + (56*A*a^3*b
)/3 + 12*B*a*b^3 + 7*B*a^3*b) + tan(c/2 + (d*x)/2)^9*((5*A*a^4)/24 - 3*A*b
^4 + (14*B*a^4)/3 + 10*B*b^4 - (21*A*a^2*b^2)/2 + 44*B*a^2*b^2 + (88*A*a*b
^3)/3 + (56*A*a^3*b)/3 - 12*B*a*b^3 - 7*B*a^3*b) + tan(c/2 + (d*x)/2)^5*((
15*A*a^4)/4 + 2*A*b^4 + (52*B*a^4)/5 + 20*B*b^4 + 3*A*a^2*b^2 + 72*B*a^2*b
^2 + 48*A*a*b^3 + (208*A*a^3*b)/5 + 8*B*a*b^3 + 2*B*a^3*b) + tan(c/2 + (d*
x)/2)^7*((15*A*a^4)/4 + 2*A*b^4 - (52*B*a^4)/5 - 20*B*b^4 + 3*A*a^2*b^2 -
72*B*a^2*b^2 - 48*A*a*b^3 - (208*A*a^3*b)/5 + 8*B*a*b^3 + 2*B*a^3*b))/(d*(
15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6
 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^
12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1117, normalized size of antiderivative = 3.45

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \text{Too large to display}$$

input
```
int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x)
```

output
```
( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a**5 - 180*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a**3*b**2 - 120*cos(
c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a*b**4 + 45*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**5 + 540*cos(c + d*x)*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3*b**2 + 360*cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**4 - 45*cos(c + d*x)*log(tan((c + d*x
)/2) - 1)*sin(c + d*x)**2*a**5 - 540*cos(c + d*x)*log(tan((c + d*x)/2) - 1
)*sin(c + d*x)**2*a**3*b**2 - 360*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*s
in(c + d*x)**2*a*b**4 + 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**5 + 1
80*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**3*b**2 + 120*cos(c + d*x)*log
(tan((c + d*x)/2) - 1)*a*b**4 + 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**6*a**5 + 180*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c +
d*x)**6*a**3*b**2 + 120*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x
)**6*a*b**4 - 45*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*
*5 - 540*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**3*b**2
- 360*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*b**4 + 45*c
os(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**5 + 540*cos(c + d
*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3*b**2 + 360*cos(c + d*x)
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**4 - 15*cos(c + d*x)*log(ta
n((c + d*x)/2) + 1)*a**5 - 180*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a...
```

3.250 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 178

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{(2a^2+b^2)(Ab-aB)x}{2b^4} - \frac{2a^3(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} - \frac{(3aAb-3a^2B-2b^2B)\sin(c+dx)}{3b^3d} + \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{B\cos^2(c+dx)\sin(c+dx)}{3bd}$$

output
```
1/2*(2*a^2+b^2)*(A*b-B*a)*x/b^4-2*a^3*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2
*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)/b^4/(a+b)^(1/2)/d-1/3*(3*A*a*b-3*B*a^
2-2*B*b^2)*sin(d*x+c)/b^3/d+1/2*(A*b-B*a)*cos(d*x+c)*sin(d*x+c)/b^2/d+1/3*
B*cos(d*x+c)^2*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{6(2a^2+b^2)(Ab-aB)(c+dx) - \frac{24a^3(-Ab+aB)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + 3b(-4aAb+4a^2B+3b^2B)\sin}{12b^4d}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(6*(2*a^2 + b^2)*(A*b - a*B)*(c + d*x) - (24*a^3*(-(A*b) + a*B)*ArcTanh[((
a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(-4*a*A
*b + 4*a^2*B + 3*b^2*B)*Sin[c + d*x] + 3*b^2*(A*b - a*B)*Sin[2*(c + d*x)]
+ b^3*B*Sin[3*(c + d*x)])/(12*b^4*d)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3469, 3042, 3528, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 3469$$
$$\frac{\int \frac{\cos(c+dx)\left(3(Ab-aB)\cos^2(c+dx)+2bB\cos(c+dx)+2aB\right)}{a+b\cos(c+dx)}dx}{3b} + \frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2bB\sin\left(c+dx+\frac{\pi}{2}\right)+2aB\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{\int \frac{-2\left(-3Ba^2+3Aba-2b^2B\right)\cos^2(c+dx)+b(3Ab+aB)\cos(c+dx)+3a(Ab-aB)}{a+b\cos(c+dx)}dx}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{-2\left(-3Ba^2+3Aba-2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(3Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(Ab-aB)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\frac{\int \frac{3\left(ab(Ab-aB)+\left(2a^2+b^2\right)\cos(c+dx)(Ab-aB)\right)}{a+b\cos(c+dx)}dx}{b}-\frac{2\left(-3a^2B+3aAb-2b^2B\right)\sin(c+dx)}{bd}}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{ab(Ab-aB)+\left(2a^2+b^2\right)\cos(c+dx)(Ab-aB)}{a+b\cos(c+dx)}dx}{b}-\frac{2\left(-3a^2B+3aAb-2b^2B\right)\sin(c+dx)}{bd}}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\int \frac{ab(Ab-aB)+\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)(Ab-aB)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2\left(-3a^2B+3aAb-2b^2B\right)\sin(c+dx)}{bd}}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\frac{3\left(\frac{x(2a^2+b^2)(Ab-aB)}{b}-\frac{2a^3(Ab-aB)\int\frac{1}{a+b\cos(c+dx)}dx}{b}\right)}{b}-\frac{2(-3a^2B+3aAb-2b^2B)\sin(c+dx)}{bd}}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(\frac{x(2a^2+b^2)(Ab-aB)}{b}-\frac{2a^3(Ab-aB)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2(-3a^2B+3aAb-2b^2B)\sin(c+dx)}{bd}}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{3\left(\frac{x(2a^2+b^2)(Ab-aB)}{b}-\frac{4a^3(Ab-aB)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{2(-3a^2B+3aAb-2b^2B)\sin(c+dx)}{bd}}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{3\left(\frac{x(2a^2+b^2)(Ab-aB)}{b}-\frac{4a^3(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{2(-3a^2B+3aAb-2b^2B)\sin(c+dx)}{bd}}{2b}+\frac{3(Ab-aB)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{B\sin(c+dx)\cos^2(c+dx)}{3bd}$$

input

```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(B*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d) + ((3*(A*b - a*B)*Cos[c + d*x]*Sin
[c + d*x])/(2*b*d) + ((3*(((2*a^2 + b^2)*(A*b - a*B)*x)/b - (4*a^3*(A*b -
a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sq
rt[a + b]*d)))/b - (2*(3*a*A*b - 3*a^2*B - 2*b^2*B)*Sin[c + d*x])/(b*d))/(
2*b))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.35

method	result
derivativedivides	$-\frac{2a^3(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(-Aa\,b^2-\frac{1}{2}A\,b^3+B\,a^2b+\frac{1}{2}Ba\,b^2+b^3B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-2Aa\,b^2+2B\,a^2b+\frac{2}{3}\right.}{\left(1+\tan\left(\frac{dx}{2}+\right.\right.}$
default	$-\frac{2a^3(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(-Aa\,b^2-\frac{1}{2}A\,b^3+B\,a^2b+\frac{1}{2}Ba\,b^2+b^3B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-2Aa\,b^2+2B\,a^2b+\frac{2}{3}\right.}{\left(1+\tan\left(\frac{dx}{2}+\right.\right.}$
risch	$\frac{xA\,a^2}{b^3}+\frac{xA}{2b}-\frac{x\,a^3B}{b^4}-\frac{aBx}{2b^2}+\frac{3i\mathrm{e}^{-i(dx+c)}B}{8db}+\frac{i\mathrm{e}^{i(dx+c)}Aa}{2d\,b^2}+\frac{i\mathrm{e}^{-i(dx+c)}a^2B}{2d\,b^3}-\frac{i\mathrm{e}^{-i(dx+c)}Aa}{2d\,b^2}-\frac{3i\mathrm{e}^{i(d}}{8}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-2*a^3*(A*b-B*a)/b^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a-b)*(a+b))^(1/2))+2/b^4*(((-A*a*b^2-1/2*A*b^3+B*a^2*b+1/2*B*a*b^2+b
^3*B)*tan(1/2*d*x+1/2*c)^5+(-2*A*a*b^2+2*B*a^2*b+2/3*b^3*B)*tan(1/2*d*x+1/
2*c)^3+(-A*a*b^2+B*a^2*b+b^3*B+1/2*A*b^3-1/2*B*a*b^2)*tan(1/2*d*x+1/2*c))/
(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*(2*A*a^2*b+A*b^3-2*B*a^3-B*a*b^2)*arctan(ta
n(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.04

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$= \left[-\frac{3\,(2\,Ba^5-2\,Aa^4b-Ba^3b^2+Aa^2b^3-Bab^4+Ab^5)dx-3\,(Ba^4-Aa^3b)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+}{}\right.}{}\right.$$

$$-\frac{3\,(2\,Ba^5-2\,Aa^4b-Ba^3b^2+Aa^2b^3-Bab^4+Ab^5)dx-6\,(Ba^4-Aa^3b)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+}{\sqrt{a^2-b^2}\sin(}\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
[-1/6*(3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d
*x - 3*(B*a^4 - A*a^3*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
 c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*B
*a^4*b - 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3 -
B*b^5)*cos(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*
x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d), -1/6*(3*(2*B*a^5 - 2*A*a^4*b -
B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d*x - 6*(B*a^4 - A*a^3*b)*sqrt(a^
2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6
*B*a^4*b - 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3
- B*b^5)*cos(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(
d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(161) = 322$.

Time = 0.18 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.02

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{\frac{3\,(2\,Ba^3-2\,Aa^2b+Bab^2-Ab^3)(dx+c)}{b^4}+\frac{12\,(Ba^4-Aa^3b)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^4}-\cdots}{\cdots}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
-1/6*(3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*(d*x + c)/b^4 + 12*(B*a^4
- A*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*b^4) - 2*(6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1
/2*c)^5 + 3*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^2*tan(1/2*d*x + 1/2*c)^5
+ 6*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*A*
a*b*tan(1/2*d*x + 1/2*c)^3 + 4*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(
1/2*d*x + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 3*B*a*b*tan(1/2*d*x + 1/
2*c) + 3*A*b^2*tan(1/2*d*x + 1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 45.84 (sec) , antiderivative size = 4568, normalized size of antiderivative = 25.66

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
((tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 2*A*a*b - B*a*b))/b^3 +
(tan(c/2 + (d*x)/2)^5*(2*B*a^2 - A*b^2 + 2*B*b^2 - 2*A*a*b + B*a*b))/b^3 +
 (4*tan(c/2 + (d*x)/2)^3*(3*B*a^2 + B*b^2 - 3*A*a*b))/(3*b^3))/(d*(3*tan(c
/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (a
tan((((2*a^2 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^
9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a
^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3
*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^
7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A
*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + ((2*
a^2 + b^2)*(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a
^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b
^12 - 2*B*a*b^12))/b^9 - (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*
a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4)))/(2*b^4) + ((2*a^2
 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*
b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*
A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6
 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*
B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 -
 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - ((2*a^2 + b^2...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.13

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{\sin(dx+c)\left(-\sin(dx+c)^2+3\right)}{3d}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
(sin(c + d*x)*( - sin(c + d*x)**2 + 3))/(3*d)
```

3.251 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 134

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = -\frac{(2aAb-2a^2B-b^2B)\,x}{2b^3} + \frac{2a^2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} + \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
-1/2*(2*A*a*b-2*B*a^2-B*b^2)*x/b^3+2*a^2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)/b^3/(a+b)^(1/2)/d+(A*b-B*a)*sin(d*
x+c)/b^2/d+1/2*B*cos(d*x+c)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{2(-2aAb+2a^2B+b^2B)(c+dx)+\frac{8a^2(-Ab+aB)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+4b(Ab-aB)\sin(c+dx)+b}{4b^3d}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(2*(-2*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) + (8*a^2*(-(A*b) + a*B)*ArcTanh[
((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*b*(A*b
- a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*b^3*d)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 3469, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3469}$$
$$\frac{\int \frac{2(Ab-aB)\cos^2(c+dx)+bB\cos(c+dx)+aB}{a+b\cos(c+dx)}dx}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

$\downarrow$ 3502

$$\frac{\frac{\int \frac{abB-\left(-2Ba^2+2Aba-b^2B\right)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}+\frac{2(Ab-aB)\sin(c+dx)}{bd}}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{abB+\left(2Ba^2-2Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{2(Ab-aB)\sin(c+dx)}{bd}}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

$\downarrow$ 3214

$$\frac{\frac{\frac{2a^2(Ab-aB)\int \frac{1}{a+b\cos(c+dx)}dx}{b}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{b}}{b}+\frac{2(Ab-aB)\sin(c+dx)}{bd}}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2a^2(Ab-aB)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{b}}{b}+\frac{2(Ab-aB)\sin(c+dx)}{bd}}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

$\downarrow$ 3138

$$\frac{\frac{\frac{4a^2(Ab-aB)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{b}}{b}+\frac{2(Ab-aB)\sin(c+dx)}{bd}}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

$\downarrow$ 218

$$\frac{\frac{\frac{4a^2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{b}}{b}+\frac{2(Ab-aB)\sin(c+dx)}{bd}}{2b}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + ((-(((2*a*A*b - 2*a^2*B - b^2*B)*x
)/b) + (4*a^2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/b + (2*(A*b - a*B)*Sin[c + d*x])/(b*d))
/(2*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{2a^2(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}-\frac{2\left(\frac{\left(-A\,b^2+Bab+\frac{1}{2}B\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-A\,b^2+Bab-\frac{1}{2}B\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{(2A}{}\right.}{b^3}}{d}$
default	$\frac{\frac{2a^2(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}-\frac{2\left(\frac{\left(-A\,b^2+Bab+\frac{1}{2}B\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-A\,b^2+Bab-\frac{1}{2}B\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{(2A}{}\right.}{b^3}}{d}$
risch	$-\frac{xAa}{b^2}+\frac{x\,a^2B}{b^3}+\frac{Bx}{2b}-\frac{i\mathrm{e}^{i(dx+c)}A}{2bd}+\frac{i\mathrm{e}^{i(dx+c)}Ba}{2b^2d}+\frac{i\mathrm{e}^{-i(dx+c)}A}{2bd}-\frac{i\mathrm{e}^{-i(dx+c)}Ba}{2b^2d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia}{}\right.}{\sqrt{-a^2}}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(2*a^2*(A*b-B*a)/b^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a-b)*(a+b))^(1/2))-2/b^3*(((-A*b^2+B*a*b+1/2*B*b^2)*tan(1/2*d*x+1/2*c
)^3+(-A*b^2+B*a*b-1/2*B*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^
2+1/2*(2*A*a*b-2*B*a^2-B*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.18

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \left[\frac{(2\,Ba^4-2\,Aa^3b-Ba^2b^2+2\,Aab^3-Bb^4)dx+(Ba^3-Aa^2b)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx}{b^2\cos}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
[1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*d*x + (B*a^3 -
 A*a^2*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b
^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^3*b - 2*A*a^
2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x +
c))/((a^2*b^3 - b^5)*d), 1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3
 - B*b^4)*d*x - 2*(B*a^3 - A*a^2*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c
) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*
b^3 + 2*A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3
- b^5)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10409 vs. 2(116) = 232.

Time = 171.83 (sec) , antiderivative size = 10409, normalized size of antiderivative = 77.68

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*cos(c))*cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (
-2*A*d*x*tan(c/2 + d*x/2)**4/(2*b*d*tan(c/2 + d*x/2)**4 + 4*b*d*tan(c/2 +
d*x/2)**2 + 2*b*d) - 4*A*d*x*tan(c/2 + d*x/2)**2/(2*b*d*tan(c/2 + d*x/2)**
4 + 4*b*d*tan(c/2 + d*x/2)**2 + 2*b*d) - 2*A*d*x/(2*b*d*tan(c/2 + d*x/2)**
4 + 4*b*d*tan(c/2 + d*x/2)**2 + 2*b*d) + 2*A*tan(c/2 + d*x/2)**5/(2*b*d*ta
n(c/2 + d*x/2)**4 + 4*b*d*tan(c/2 + d*x/2)**2 + 2*b*d) + 8*A*tan(c/2 + d*x
/2)**3/(2*b*d*tan(c/2 + d*x/2)**4 + 4*b*d*tan(c/2 + d*x/2)**2 + 2*b*d) + 6
*A*tan(c/2 + d*x/2)/(2*b*d*tan(c/2 + d*x/2)**4 + 4*b*d*tan(c/2 + d*x/2)**2
 + 2*b*d) + 3*B*d*x*tan(c/2 + d*x/2)**4/(2*b*d*tan(c/2 + d*x/2)**4 + 4*b*d
*tan(c/2 + d*x/2)**2 + 2*b*d) + 6*B*d*x*tan(c/2 + d*x/2)**2/(2*b*d*tan(c/2
 + d*x/2)**4 + 4*b*d*tan(c/2 + d*x/2)**2 + 2*b*d) + 3*B*d*x/(2*b*d*tan(c/2
 + d*x/2)**4 + 4*b*d*tan(c/2 + d*x/2)**2 + 2*b*d) - 2*B*tan(c/2 + d*x/2)**
5/(2*b*d*tan(c/2 + d*x/2)**4 + 4*b*d*tan(c/2 + d*x/2)**2 + 2*b*d) - 10*B*t
an(c/2 + d*x/2)**3/(2*b*d*tan(c/2 + d*x/2)**4 + 4*b*d*tan(c/2 + d*x/2)**2
+ 2*b*d) - 4*B*tan(c/2 + d*x/2)/(2*b*d*tan(c/2 + d*x/2)**4 + 4*b*d*tan(c/2
 + d*x/2)**2 + 2*b*d), Eq(a, b)), (2*A*d*x*tan(c/2 + d*x/2)**5/(2*b*d*tan(
c/2 + d*x/2)**5 + 4*b*d*tan(c/2 + d*x/2)**3 + 2*b*d*tan(c/2 + d*x/2)) + 4*
A*d*x*tan(c/2 + d*x/2)**3/(2*b*d*tan(c/2 + d*x/2)**5 + 4*b*d*tan(c/2 + d*x
/2)**3 + 2*b*d*tan(c/2 + d*x/2)) + 2*A*d*x*tan(c/2 + d*x/2)/(2*b*d*tan(c/2
 + d*x/2)**5 + 4*b*d*tan(c/2 + d*x/2)**3 + 2*b*d*tan(c/2 + d*x/2)) + 6*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.69

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\frac{(2\,Ba^2-2\,Aab+Bb^2)(dx+c)}{b^3} + \frac{4\,(Ba^3-Aa^2b)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^3} - \frac{2\left(2\,Ba\tan\right.}{}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/2*((2*B*a^2 - 2*A*a*b + B*b^2)*(d*x + c)/b^3 + 4*(B*a^3 - A*a^2*b)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/
2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^3) - 2
*(2*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/
2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c)
 - B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 45.35 (sec) , antiderivative size = 3761, normalized size of antiderivative = 28.07

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
((tan(c/2 + (d*x)/2)*(2*A*b - 2*B*a + B*b))/b^2 - (tan(c/2 + (d*x)/2)^3*(2
*B*a - 2*A*b + B*b))/b^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^
4 + 1)) - (atan((((((((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^
8 - 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 - (4*tan(c/2 +
 (d*x)/2)*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b
^6))/b^7)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3) - (8*tan(c/2 + (d*x)/2
)*(8*B^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A
^2*a^3*b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b
^4 - 16*B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B
*a^2*b^5 + 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4)*(B*a^2*
2i + B*b^2*1i - A*a*b*2i)*1i)/(2*b^3) - (((((8*(2*B*b^10 + 8*A*a^2*b^8 - 4
*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b
^9))/b^6 + (4*tan(c/2 + (d*x)/2)*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*(8*a*b^8
 - 16*a^2*b^7 + 8*a^3*b^6))/b^7)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3)
 + (8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b
 - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2
*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6
- 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B
*a^5*b^2))/b^4)*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*1i)/(2*b^3))/((16*(4*B^3*
a^8 - 6*B^3*a^7*b + 4*A^3*a^4*b^4 - 4*A^3*a^5*b^3 - B^3*a^3*b^5 + 2*B^3...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{\cos(dx+c)\sin(dx+c)+dx}{2d}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x) + d*x)/(2*d)
```

3.252 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 89

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{(Ab-aB)x}{b^2} - \frac{2a(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d} + \frac{B\sin(c+dx)}{bd}$$

output
```
(A*b-B*a)*x/b^2-2*a*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^
(1/2))/(a-b)^(1/2)/b^2/(a+b)^(1/2)/d+B*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{(Ab-aB)(c+dx) - \frac{2a(-Ab+aB)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + bB\sin(c+dx)}{b^2d}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
((A*b - a*B)*(c + d*x) - (2*a*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*B*Sin[c + d*x])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int \frac{(Ab-aB)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}+\frac{B\sin(c+dx)}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{(Ab-aB)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}+\frac{B\sin(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\sin(c+dx)}{bd}$$

$$\Big\downarrow \text{3214}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\int\frac{1}{a+b\cos(c+dx)}dx}{b}\right)}{b}+\frac{B\sin(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}+\frac{B\sin(c+dx)}{bd}$$

$$\Big\downarrow \text{3138}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{2a\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}+\frac{B\sin(c+dx)}{bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}+\frac{B\sin(c+dx)}{bd}$$

input

```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output

```
((A*b - a*B)*(x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(Sqrt[a - b]*b*Sqrt[a + b]*d)))/b + (B*Sin[c + d*x])/(b*d)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2(Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2(Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{xA}{b}-\frac{aBx}{b^2}-\frac{i\mathrm{e}^{i(dx+c)}B}{2db}+\frac{iB\,\mathrm{e}^{-i(dx+c)}}{2bd}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)A}{\sqrt{-a^2+b^2}\,db}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib}{b}\right.}{\sqrt{-a^2+b^2}}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*(B*b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-B*a)*arct
an(tan(1/2*d*x+1/2*c)))-2*a*(A*b-B*a)/b^2/((a-b)*(a+b))^(1/2)*arctan((a-b)
*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.62

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$=\left[-\frac{2\,(Ba^3-Aa^2b-Bab^2+Ab^3)dx-(Ba^2-Aab)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2}}{b^2\cos(dx+c)^2+2\,ab\,c}\right.}{2\,(a^2b^2-b^4)d}\right.$$
$$-\frac{(Ba^3-Aa^2b-Bab^2+Ab^3)dx-(Ba^2-Aab)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(Ba^2b-Bb^3)\,\mathrm{si}}{(a^2b^2-b^4)d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
[-1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*a*b)*sqrt(-a
^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(
-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((
a^2*b^2 - b^4)*d), -((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*
a*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
 + c))) - (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(76) = 152$.

Time = 56.42 (sec) , antiderivative size = 3225, normalized size of antiderivative = 36.24

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*cos(c)), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (A*d*x*t
an(c/2 + d*x/2)**2/(b*d*tan(c/2 + d*x/2)**2 + b*d) + A*d*x/(b*d*tan(c/2 +
d*x/2)**2 + b*d) - A*tan(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) -
 A*tan(c/2 + d*x/2)/(b*d*tan(c/2 + d*x/2)**2 + b*d) - B*d*x*tan(c/2 + d*x/
2)**2/(b*d*tan(c/2 + d*x/2)**2 + b*d) - B*d*x/(b*d*tan(c/2 + d*x/2)**2 + b
*d) + B*tan(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) + 3*B*tan(c/2
+ d*x/2)/(b*d*tan(c/2 + d*x/2)**2 + b*d), Eq(a, b)), (A*d*x*tan(c/2 + d*x/
2)**3/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A*d*x*tan(c/2 + d
*x/2)/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A*tan(c/2 + d*x/2
)**2/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A/(b*d*tan(c/2 + d
*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B*d*x*tan(c/2 + d*x/2)**3/(b*d*tan(c/2
+ d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B*d*x*tan(c/2 + d*x/2)/(b*d*tan(c/2
+ d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 3*B*tan(c/2 + d*x/2)**2/(b*d*tan(c/2
 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B/(b*d*tan(c/2 + d*x/2)**3 + b*d*ta
n(c/2 + d*x/2)), Eq(a, -b)), ((A*sin(c + d*x)/d + B*x*sin(c + d*x)**2/2 +
B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*cos(c + d*x)/(2*d))/a, Eq(b, 0)), (
x*(A + B*cos(c))*cos(c)/(a + b*cos(c)), Eq(d, 0)), (A*a*b*d*x*sqrt(-a/(a -
 b) - b/(a - b))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b)
)*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqr
t(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = -\frac{\frac{(Ba-Ab)(dx+c)}{b^2} - \frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b} + \frac{2(Ba^2-Aab)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
-((B*a - A*b)*(d*x + c)/b^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*b) + 2*(B*a^2 - A*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 42.30 (sec) , antiderivative size = 541, normalized size of antiderivative = 6.08

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^2-b^2\right)} - \frac{2\,A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^2-b^2\right)} - \frac{B\,b\,\sin\left(c+d\,x\right)}{d\,\left(a^2-b^2\right)}$$
$$+ \frac{2\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\left(a^2-b^2\right)} - \frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,\left(a^2-b^2\right)}$$
$$+ \frac{A\,a\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\sqrt{b^2-a^2}}$$
$$- \frac{A\,a\,\ln\left(\frac{b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\sqrt{b^2-a^2}}$$
$$- \frac{B\,a^2\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,\sqrt{b^2-a^2}}$$
$$+ \frac{B\,a^2\,\ln\left(\frac{b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,\sqrt{b^2-a^2}} + \frac{B\,a^2\,\sin\left(c+d\,x\right)}{b\,d\,\left(a^2-b^2\right)}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
(2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (2*A
*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (B*b*sin
(c + d*x))/(d*(a^2 - b^2)) + (2*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/(b*d*(a^2 - b^2)) - (2*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/(b^2*d*(a^2 - b^2)) + (A*a*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2
+ (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b
*d*(b^2 - a^2)^(1/2)) - (A*a*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)
/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b*d*(b^2
 - a^2)^(1/2)) - (B*a^2*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) +
 cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b^2*d*(b^2 -
a^2)^(1/2)) + (B*a^2*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + co
s(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b^2*d*(b^2 - a^2
)^(1/2)) + (B*a^2*sin(c + d*x))/(b*d*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{\sin(dx+c)}{d}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
sin(c + d*x)/d
```

3.253 $\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{Bx}{b}+\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}$$

output
```
B*x/b+2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(1/2)/b/(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B(c+dx)+\frac{2(-Ab+aB)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{bd}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
(B*(c + d*x) + (2*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/Sqrt[-a^2 + b^2])/(b*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{(Ab-aB)\int \frac{1}{a+b\cos(c+dx)}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(Ab-aB)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}+\frac{Bx}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{Bx}{b}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
(B*x)/b + (2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\dfrac{\frac{2(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
default	$\dfrac{\frac{2(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
risch	$\frac{Bx}{b}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)A}{\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)Ba}{\sqrt{-a^2+b^2}\,db}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2 \ldots}{\sqrt{-a \ldots}}\right.}{\sqrt{-a^2+b^2} \ldots}$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(A*b-B*a)/b/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a
-b)*(a+b))^(1/2))+2*B/b*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.61

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \left[\frac{2\,(Ba^2-Bb^2)dx+(Ba-Ab)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{2\,(a^2b-b^3)d}\right.$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*(B*a^2 - B*b^2)*d*x + (B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(
d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x +
c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
c) + a^2)))/((a^2*b - b^3)*d), ((B*a^2 - B*b^2)*d*x - (B*a - A*b)*sqrt(a^2
 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^
2*b - b^3)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(56) = 112$.

Time = 11.33 (sec) , antiderivative size = 524, normalized size of antiderivative = 7.82

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x(A+B\cos(c))}{\cos(c)} \\ \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd}+\frac{Bx}{b}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} \\ \frac{A}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{Bx}{b}+\frac{B}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} \\ \frac{Ax+\frac{B\sin(c+dx)}{d}}{a} \\ \frac{x(A+B\cos(c))}{a+b\cos(c)} \\ \frac{Ab\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{Ab\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{Badx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{Ba\log\left(\right.}{abd\sqrt{-}} \end{cases}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*cos(c))/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (
A*tan(c/2 + d*x/2)/(b*d) + B*x/b - B*tan(c/2 + d*x/2)/(b*d), Eq(a, b)), (A
/(b*d*tan(c/2 + d*x/2)) + B*x/b + B/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((
A*x + B*sin(c + d*x)/d)/a, Eq(b, 0)), (x*(A + B*cos(c))/(a + b*cos(c)), Eq
(d, 0)), (A*b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d
*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - A*b
*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a -
b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + B*a*d*x*sqrt(-a/(
a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(
a - b) - b/(a - b))) - B*a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d
*x/2))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a
 - b))) + B*a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*
sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - B*b*
d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**
2*d*sqrt(-a/(a - b) - b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(58) = 116.

Time = 0.20 (sec) , antiderivative size = 296, normalized size of antiderivative = 4.42

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{\frac{\left(\sqrt{a^2-b^2}B(2a-b)|a-b|-\sqrt{a^2-b^2}Ab|a-b|-\sqrt{a^2-b^2}A|a-b||b|+\sqrt{a^2-b^2}B|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{2a+\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)b^2+(a^3-2a^2b+ab^2)|b|}}{d}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-((sqrt(a^2 - b^2)*B*(2*a - b)*abs(a - b) - sqrt(a^2 - b^2)*A*b*abs(a - b)
 - sqrt(a^2 - b^2)*A*abs(a - b)*abs(b) + sqrt(a^2 - b^2)*B*abs(a - b)*abs(
b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1
/2*c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 - 2*a
*b + b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (2*B*a - A*b - B*b + A*a
bs(b) - B*abs(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*t
an(1/2*d*x + 1/2*c)/sqrt((2*a - sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))
))/(b^2 - a*abs(b)))/d
```

Mupad [B] (verification not implemented)

Time = 42.83 (sec) , antiderivative size = 344, normalized size of antiderivative = 5.13

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{a\left(B\ln\left(\frac{b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{-(a+b)(a-b)} - B\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{c}{}\right.}\right.\right.}{} + \frac{2B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{b\,d}$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x)),x)
```

output
```
(a*(B*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(-(a + b)*(a - b))^(1/2) - B*log(
(a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2))/cos(c/2 + (d*x)/2))*(b^2 - a^2)^(1/2)) - A*b*log((b*sin(c/2 + (
d*x)/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos
(c/2 + (d*x)/2))*(-(a + b)*(a - b))^(1/2) + A*b*log((a*sin(c/2 + (d*x)/2)
- b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (
d*x)/2))*(b^2 - a^2)^(1/2))/(b*d*(a^2 - b^2)) + (2*B*atan(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = x$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
x
```

3.254 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d} + \frac{A\text{arctanh}(\sin(c+dx))}{ad}$$

output
```
-2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(1
/2)/(a+b)^(1/2)/d+A*arctanh(sin(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\frac{2(Ab-aB)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + A\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{ad}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
((2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt
[-a^2 + b^2] + A*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]))/(a*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3480}$$

$$\frac{A\int \sec(c+dx)dx}{a}-\frac{(Ab-aB)\int \frac{1}{a+b\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{(Ab-aB)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2(Ab-aB)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

$$\Big\downarrow \text{218}$$

$$\frac{A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{A\text{arctanh}(\sin(c+dx))}{ad}-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqr
t[a - b]*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2(-Ab+Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$
default	$\frac{-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2(-Ab+Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$
risch	$-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)Ab}{\sqrt{-a^2+b^2}\,da}+\frac{\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)B}{\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a}}{b\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,da}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-A/a*ln(tan(1/2*d*x+1/2*c)-1)+2/a*(-A*b+B*a)/((a-b)*(a+b))^(1/2)*arct
an((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+A/a*ln(tan(1/2*d*x+1/2*c)
+1))
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$=\left[\frac{(Ba-Ab)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+(Aa^2}{2\,(a^3-ab^2)d}\right.$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
[1/2*((B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)
*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^2 - A*
b^2)*log(sin(d*x + c) + 1) - (A*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3
 - a*b^2)*d), 1/2*(2*(B*a - A*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) +
 b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^2 - A*b^2)*log(sin(d*x + c) + 1
) - (A*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$=\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ba}{\sqrt{a^2-b^2}a}}{d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
(A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
 1))/a + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(B*a - A*b)
/(sqrt(a^2 - b^2)*a))/d
```

Mupad [B] (verification not implemented)

Time = 42.94 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.50

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{2\,A\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}$$
$$+\frac{b\left(A\,\ln\left(\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\sqrt{-(a+b)\,(a-b)}-A\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d}{}\right.}{\cos\left(\frac{c}{}\right.}\right.\right.}{}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

output
```
(2*A*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (b*(A*log((a*co
s(c/2 + (d*x)/2) + b*cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*(b^2 - a^2)^(
1/2))/cos(c/2 + (d*x)/2))*(-(a + b)*(a - b))^(1/2) - A*log((a*sin(c/2 + (d
*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(
c/2 + (d*x)/2))*(b^2 - a^2)^(1/2)) - B*a*log((a*cos(c/2 + (d*x)/2) + b*cos
(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)
)*(-(a + b)*(a - b))^(1/2) + B*a*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (
d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(b^2 -
 a^2)^(1/2))/(a*d*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1))/d
```

3.255 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 99

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2b(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} - \frac{(Ab-aB)\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{A\tan(c+dx)}{ad}$$

output
```
2*b*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)
^(1/2)/(a+b)^(1/2)/d-(A*b-B*a)*arctanh(sin(d*x+c))/a^2/d+A*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{-\frac{2b(Ab-aB)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + (Ab-aB)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{a^2d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

output
```
((-2*b*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] + (A*b - a*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a*A*Tan[c + d*x])/(a^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3479, 25, 27, 3042, 3226, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int -\frac{(Ab-aB)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{A\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{\int \frac{(Ab-aB)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3226}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\left(\frac{\int \sec(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\left(\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

output
```
-(((A*b - a*B)*((-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a*Sqrt[a - b]*Sqrt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d)))/a) + (A*Tan[
c + d*x])/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{\frac{2b(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}}-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-Ab+B}{}}{d}$
default	$\frac{\frac{2b(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}}-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{A}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-Ab+B}{}}{d}$
risch	$\frac{2iA}{da\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}-i\right)Ab}{a^2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)B}{ad}-\frac{\ln\left(e^{i(dx+c)}+i\right)Ab}{a^2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)B}{ad}-\frac{b^2\ln\left(e\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(2*b*(A*b-B*a)/a^2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)
/((a-b)*(a+b))^(1/2))-A/a/(tan(1/2*d*x+1/2*c)-1)+(A*b-B*a)/a^2*ln(tan(1/2*
d*x+1/2*c)-1)-A/a/(tan(1/2*d*x+1/2*c)+1)+1/a^2*(-A*b+B*a)*ln(tan(1/2*d*x+1
/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(89) = 178$.

Time = 0.18 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.65

$$\int \frac{(A + B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \left[\frac{(Bab - Ab^2)\sqrt{-a^2+b^2}\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{}\right.$$

$$-\frac{2\,(Bab - Ab^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)\cos(dx+c) - (Ba^3 - Aa^2b - Bab^2 + Ab^3)\cos(dx}{2}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
[1/2*((B*a*b - A*b^2)*sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c
) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)
*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) +
1) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c)), -
1/2*(2*(B*a*b - A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(
a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^
3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3
)*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/
((a^4 - a^2*b^2)*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec^2{(c+dx)}}{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} - \frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a} + \frac{2\,(Bab-Ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2 \cdots}{\cdots}}{d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - (B*a - A*b)*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/a^2 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 - 1)*a) + 2*(B*a*b - A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn
(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 43.58 (sec) , antiderivative size = 675, normalized size of antiderivative = 6.82

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

output
```
(A*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/(d*(a^2 - b^2))
- (B*a*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/(d*(a^2 - b^2)
) + (A*a*tan(c + d*x))/(d*(a^2 - b^2)) - (A*b^3*atan((sin(c/2 + (d*x)/2)*1
i)/cos(c/2 + (d*x)/2))*2i)/(a^2*d*(a^2 - b^2)) + (B*b^2*atan((sin(c/2 + (d
*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/(a*d*(a^2 - b^2)) - (A*b^2*tan(c + d*x)
)/(a*d*(a^2 - b^2)) - (B*b*atan(((a^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
 + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 + (d*x)/2)*(
b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^3*b^
2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(-(a + b)*(a - b))^(1
/2)*2i)/(a*d*(a^2 - b^2)) + (A*b^2*atan(((a^5*sin(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2) + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 + (d
*x)/2)*(b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
- a^3*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2)*
(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(-(a + b)*(a
- b))^(1/2)*2i)/(a^2*d*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.18

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sin(dx+c)}{\cos(dx+c)\,d}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

output
```
sin(c + d*x)/(cos(c + d*x)*d)
```

3.256 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 143

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{2b^2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{(a^2A+2Ab^2-2abB)\,\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad}$$

output
```
-2*b^2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a
-b)^(1/2)/(a+b)^(1/2)/d+1/2*(A*a^2+2*A*b^2-2*B*a*b)*arctanh(sin(d*x+c))/a^
3/d-(A*b-B*a)*tan(d*x+c)/a^2/d+1/2*A*sec(d*x+c)*tan(d*x+c)/a/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 300 vs. $2(143) = 286$.

Time = 3.00 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.10

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\frac{8b^2(Ab-aB)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 2(a^2A+2Ab^2-2abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+ \cdots}{\cdots}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

output
```
((8*b^2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
Sqrt[-a^2 + b^2] - 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a*(-
(A*b) + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^
2*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*(-(A*b) + a*B)*Sin[(c
+ d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*a^3*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3479, 25, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int -\frac{(-Ab\cos^2(c+dx)-aA\cos(c+dx)+2(Ab-aB))\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{A\tan(c+dx)\sec(c+dx)}{2ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{(-Ab\cos^2(c+dx)-aA\cos(c+dx)+2(Ab-aB))\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{-Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-aA\sin\left(c+dx+\frac{\pi}{2}\right)+2(Ab-aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{\int -\frac{\left(Aa^2-2bBa+Ab\cos(c+dx)a+2Ab^2\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{2(Ab-aB)\tan(c+dx)}{ad}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2(Ab-aB)\tan(c+dx)}{ad}-\frac{\int \frac{\left(Aa^2-2bBa+Ab\cos(c+dx)a+2Ab^2\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2(Ab-aB)\tan(c+dx)}{ad}-\frac{\int \frac{Aa^2-2bBa+Ab\sin\left(c+dx+\frac{\pi}{2}\right)a+2Ab^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3480}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2(Ab-aB)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2A-2abB+2Ab^2\right)\int \sec(c+dx)dx}{a}-\frac{2b^2(Ab-aB)\int \frac{1}{a+b\cos(c+dx)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2(Ab-aB)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2A-2abB+2Ab^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2(Ab-aB)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2(Ab-aB)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2A-2abB+2Ab^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^2(Ab-aB)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{2a}$$

$\Big\downarrow$ 218

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2(Ab-aB)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2A-2abB+2Ab^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

$\Big\downarrow$ 4257

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2(Ab-aB)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2A-2abB+2Ab^2\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{4b^2(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

output
```
(A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - (-(((-4*b^2*(A*b - a*B)*ArcTan[(Sq
rt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) +
((a^2*A + 2*A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(a*d))/a) + (2*(A*b -
a*B)*Tan[c + d*x])/(a*d))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\frac{A}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-Aa-2Ab+2Ba}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2A-2A\,b^2+2Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^3}-\frac{2b^2(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}\right.}{\sqrt{(a-b)(a}}\right.}{a^3\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{A}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-Aa-2Ab+2Ba}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2A-2A\,b^2+2Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^3}-\frac{2b^2(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}\right.}{\sqrt{(a-b)(a}}\right.}{a^3\sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{i\left(Aa\,e^{3i(dx+c)}+2Ab\,e^{2i(dx+c)}-2Ba\,e^{2i(dx+c)}-Aa\,e^{i(dx+c)}+2Ab-2Ba\right)}{a^2d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b^3\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,d\,a^3}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/2*A/a/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-A*a-2*A*b+2*B*a)/a^2/(tan(1/2*
d*x+1/2*c)-1)+1/2/a^3*(-A*a^2-2*A*b^2+2*B*a*b)*ln(tan(1/2*d*x+1/2*c)-1)-2*
b^2*(A*b-B*a)/a^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-
b)*(a+b))^(1/2))-1/2*A/a/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-2*A*b+2*B*a)/a
^2/(tan(1/2*d*x+1/2*c)+1)+1/2*(A*a^2+2*A*b^2-2*B*a*b)/a^3*ln(tan(1/2*d*x+1
/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(129) = 258$.

Time = 2.27 (sec) , antiderivative size = 589, normalized size of antiderivative = 4.12

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \left[\frac{2\,(Bab^2-Ab^3)\sqrt{-a^2+b^2}\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{}\right.$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
[1/4*(2*(B*a*b^2 - A*b^3)*sqrt(-a^2 + b^2)*cos(d*x + c)^2*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c
) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2)) + (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x
+ c)^2*log(sin(d*x + c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3
- 2*A*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 +
2*(B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^
5 - a^3*b^2)*d*cos(d*x + c)^2), 1/4*(4*(B*a*b^2 - A*b^3)*sqrt(a^2 - b^2)*a
rctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2
 + (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4
)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 + 2*(B*a^4
- A*a^3*b - B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b
^2)*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(129) = 258$.

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\frac{(Aa^2-2\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{(Aa^2-2\,Bab+2\,Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{4\,(Bab^2-Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+\right.}{}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/2*((A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 -
(A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 4*(B*
a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(
-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt
(a^2 - b^2)*a^3) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2
*c)^3 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*ta
n(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
- 1)^2*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 45.91 (sec) , antiderivative size = 4051, normalized size of antiderivative = 28.33

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

output
```
(B*a*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b
*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a*sin
(c + d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan((sin(c
/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*
x)/2 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(
d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b^2*atan((sin(c/2 + (d*x)/2
)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2
)) + (A*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(a^3*d*(a
^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan((sin(c/2 + (d*x)/2)*1i
)/cos(c/2 + (d*x)/2))*1i)/(a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) +
 (A*b^3*sin(2*c + 2*d*x))/(2*a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2))
 - (B*b^2*sin(2*c + 2*d*x))/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2))
 - (A*a*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*
1i)/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b*atan((sin(c/2 + (d
*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(d*(a^2 - b^2)*(cos(2*
c + 2*d*x)/2 + 1/2)) - (A*b^2*sin(c + d*x))/(2*a*d*(a^2 - b^2)*(cos(2*c +
2*d*x)/2 + 1/2)) + (A*b^3*atan(((A^2*a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2) + 8*A^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A^2*b^9*sin(c/2 +
 (d*x)/2)*(b^2 - a^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2) + 8*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*A^2*a^4*b^5...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

$$\int \frac{(A + B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sin(dx+c)^2-\text{lo}}{2d\left(\sin(dx+c)^2-1\right)}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c + d*x)/2) - 1)
+ log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d*x)/2) + 1) -
sin(c + d*x))/(2*d*(sin(c + d*x)**2 - 1))
```

3.257 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2b^3(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} - \frac{(a^2+2b^2)(Ab-aB)\operatorname{arctanh}(\sin(c+dx))}{2a^4d} + \frac{(2a^2A+3Ab^2-3abB)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad}$$

output
```
2*b^3*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-
b)^(1/2)/(a+b)^(1/2)/d-1/2*(a^2+2*b^2)*(A*b-B*a)*arctanh(sin(d*x+c))/a^4/d
+1/3*(2*A*a^2+3*A*b^2-3*B*a*b)*tan(d*x+c)/a^3/d-1/2*(A*b-B*a)*sec(d*x+c)*t
an(d*x+c)/a^2/d+1/3*A*sec(d*x+c)^2*tan(d*x+c)/a/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 422 vs. $2(187) = 374$.

Time = 3.70 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.26

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\frac{24b^3(-Ab+aB)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 6(a^2+2b^2)(-Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

output
```
((24*b^3*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] - 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2 + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^3 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^3 - (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(12*a^4*d)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 3479, 25, 3042, 3534, 25, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3479$$

$$\frac{\int -\frac{(-2Ab\cos^2(c+dx)-2aA\cos(c+dx)+3(Ab-aB))\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{3a}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}$$

$$\Big\downarrow \ 25$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\int \frac{(-2Ab\cos^2(c+dx)-2aA\cos(c+dx)+3(Ab-aB))\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\int \frac{-2Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-2aA\sin\left(c+dx+\frac{\pi}{2}\right)+3(Ab-aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \ 3534$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{\int -\frac{\left(-3b(Ab-aB)\cos^2(c+dx)+a(Ab+3aB)\cos(c+dx)+2\left(2Aa^2-3bBa+3Ab^2\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}}{3a}$$

$$\Big\downarrow \ 25$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{\left(-3b(Ab-aB)\cos^2(c+dx)+a(Ab+3aB)\cos(c+dx)+2\left(2Aa^2-3bBa+3Ab^2\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a(Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(2Aa^2-3bBa+3Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{\int -\frac{3\left(\left(a^2+2b^2\right)(Ab-aB)+ab\cos(c+dx)(Ab-aB)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}}{2a}}{3a}}{}$$

$\Big\downarrow$ 27

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}-\frac{3\int\frac{\left(\left(a^2+2b^2\right)(Ab-aB)+ab\cos(c+dx)(Ab-aB)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}-\frac{3\int\frac{\left(a^2+2b^2\right)(Ab-aB)+ab\sin\left(c+dx+\frac{\pi}{2}\right)(Ab-aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3480

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{\left(a^2+2b^2\right)(Ab-aB)\int\sec(c+dx)dx}{a}-\frac{2b^3(Ab-aB)\int\frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{\left(a^2+2b^2\right)(Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^3(Ab-aB)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3138

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{\left(a^2+2b^2\right)(Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3(Ab-aB)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}}{ad}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 218$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{\left(a^2+2b^2\right)(Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 4257$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{\left(a^2+2b^2\right)(Ab-aB)\text{arctanh}(\sin(c+dx))}{ad}-\frac{4b^3(Ab-aB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

output
```
(A*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d) - ((3*(A*b - a*B)*Sec[c + d*x]*Tan
[c + d*x])/(2*a*d) - ((-3*((-4*b^3*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*(A
*b - a*B)*ArcTanh[Sin[c + d*x]])/(a*d)))/a + (2*(2*a^2*A + 3*A*b^2 - 3*a*b
*B)*Tan[c + d*x])/(a*d))/(2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{\frac{2b^3(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4\sqrt{(a-b)(a+b)}}-\frac{A}{3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-Aa-Ab+Ba}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(-A\,a^2b-2A\,b^3+a^3B+2Ba\,b^2\right)\ln}{2a^4}}{}$
default	$\frac{\frac{2b^3(Ab-Ba)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4\sqrt{(a-b)(a+b)}}-\frac{A}{3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-Aa-Ab+Ba}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(-A\,a^2b-2A\,b^3+a^3B+2Ba\,b^2\right)\ln}{2a^4}}{}$
risch	$\frac{i(3Aab\,\mathrm{e}^{5i(dx+c)}-3B\,a^2\mathrm{e}^{5i(dx+c)}+6A\,b^2\mathrm{e}^{4i(dx+c)}-6Bab\,\mathrm{e}^{4i(dx+c)}+12A\,a^2\mathrm{e}^{2i(dx+c)}+12A\,b^2\mathrm{e}^{2i(dx+c)}-12Bab\,\mathrm{e}^{2i(dx}}{3d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(2*b^3*(A*b-B*a)/a^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a-b)*(a+b))^(1/2))-1/3*A/a/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-A*a-A*b+B*a
)/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/2/a^4*(-A*a^2*b-2*A*b^3+B*a^3+2*B*a*b^2)*
ln(tan(1/2*d*x+1/2*c)+1)-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-2*B*a*b)/a^3/(ta
n(1/2*d*x+1/2*c)+1)-1/3*A/a/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(A*a+A*b-B*a)/a^2
/(tan(1/2*d*x+1/2*c)-1)^2+1/2*(A*a^2*b+2*A*b^3-B*a^3-2*B*a*b^2)/a^4*ln(tan
(1/2*d*x+1/2*c)-1)-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-2*B*a*b)/a^3/(tan(1/2*
d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.90

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
[1/12*(6*(B*a*b^3 - A*b^4)*sqrt(-a^2 + b^2)*cos(d*x + c)^3*log((2*a*b*cos(
d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x +
c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
c) + a^2)) + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*
b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^5 - A*a^4*b + B*a^3*b^2
 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
+ 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B*a^4*b + A*a^3*b^2 + 3*B*a^2*
b^3 - 3*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b
^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), -1/12*
(12*(B*a*b^3 - A*b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a
^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(B*a^5 - A*a^4*b + B*a^3*b^2 -
 A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3
*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x +
 c)^3*log(-sin(d*x + c) + 1) - 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B
*a^4*b + A*a^3*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 -
A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b
^2)*d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**4/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(170) = 340.

Time = 0.21 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.20

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{\frac{3\,(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4} - \frac{3\,(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4} + \frac{12\,(Bab^3-Ab^4)\left(\pi\right\lfloor}{}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
1/6*(3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^4 - 3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^4 + 12*(B*a*b^3 - A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/
2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - 2*(6*A*a^2*tan(1/2*d*x + 1
/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5
- 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^
2*tan(1/2*d*x + 1/2*c)^3 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(
1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x +
1/2*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A
*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 47.15 (sec) , antiderivative size = 4696, normalized size of antiderivative = 25.11

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

output
```
(atan(((((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2
*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4
+ 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^
4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2
*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5
*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + (((8*(2*B*a^13 - 4*A*a^8*b^5
 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3
+ 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (4*tan(c/2 + (d*x)/2)*(8*
a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/
a^10)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b^3 - B*a^3 +
 A*a^2*b - 2*B*a*b^2)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B
^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13
*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7
 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B
^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6
+ 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 -
 (((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2
+ 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^
9 + (4*tan(c/2 + (d*x)/2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B
*a^3 + A*a^2*b - 2*B*a*b^2))/a^10)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sin(dx+c)\left(2\sin(dx+c)^2-3\right)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

output
```
(sin(c + d*x)*(2*sin(c + d*x)**2 - 3))/(3*cos(c + d*x)*d*(sin(c + d*x)**2
- 1))
```

3.258 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 263

$$\begin{aligned}
&\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx \\
&= -\frac{(4aAb-6a^2B-b^2B)\,x}{2b^4} \\
&\quad+\frac{2a^2(2a^2Ab-3Ab^3-3a^3B+4ab^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d} \\
&\quad+\frac{(2a^2Ab-Ab^3-3a^3B+2ab^2B)\sin(c+dx)}{b^3(a^2-b^2)\,d} \\
&\quad-\frac{(2aAb-3a^2B+b^2B)\cos(c+dx)\sin(c+dx)}{2b^2(a^2-b^2)\,d} \\
&\quad+\frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))}
\end{aligned}$$

output

```
-1/2*(4*A*a*b-6*B*a^2-B*b^2)*x/b^4+2*a^2*(2*A*a^2*b-3*A*b^3-3*B*a^3+4*B*a*
b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^4/(a
+b)^(3/2)/d+(2*A*a^2*b-A*b^3-3*B*a^3+2*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)/d
-1/2*(2*A*a*b-3*B*a^2+B*b^2)*cos(d*x+c)*sin(d*x+c)/b^2/(a^2-b^2)/d+a*(A*b-
B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 2.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{2(-4aAb+6a^2B+b^2B)(c+dx) - \frac{8a^2(-2a^2Ab+3Ab^3+3a^3B-4ab^2B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + 4b(Ab-2aB}{4b^4d}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*(-4*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - (8*a^2*(-2*a^2*A*b + 3*A*b^3 +
 3*a^3*B - 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(3/2) + 4*b*(A*b - 2*a*B)*Sin[c + d*x] + (4*a^3*b*(A*b - a*
B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*B*Sin[2*(c +
 d*x)])/(4*b^4*d)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 3468, 25, 3042, 3528, 25, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(-\left(\left(-3Ba^2+2Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab-aB)\cos(c+dx)+2a(Ab-aB)\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$\downarrow$ 25

$$\frac{\int \frac{\cos(c+dx)\left(-\left(\left(-3Ba^2+2Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab-aB)\cos(c+dx)+2a(Ab-aB)\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\left(3Ba^2-2Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(Ab-aB)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3528

$$\frac{\frac{\int -\frac{-2\left(-3Ba^3+2Aba^2+2b^2Ba-Ab^3\right)\cos^2(c+dx)-b\left(-Ba^2+2Aba-b^2B\right)\cos(c+dx)+a\left(-3Ba^2+2Aba+b^2B\right)}{a+b\cos(c+dx)}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\cos}{2bd}}{b\left(a^2-b^2\right)}$$
$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 25

$$\frac{-\frac{\int \frac{-2\left(-3Ba^3+2Aba^2+2b^2Ba-Ab^3\right)\cos^2(c+dx)-b\left(-Ba^2+2Aba-b^2B\right)\cos(c+dx)+a\left(-3Ba^2+2Aba+b^2B\right)}{a+b\cos(c+dx)}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\cos}{2bd}}{b\left(a^2-b^2\right)}$$
$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{-2\left(-3Ba^3+2Aba^2+2b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(-Ba^2+2Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-3Ba^2+2Aba+b^2B\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c-}{2bd}}{b\left(a^2-b^2\right)}$$
$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3502

$$-\frac{\frac{\frac{\int \frac{ab\left(-3Ba^2+2Aba+b^2B\right)+\left(a^2-b^2\right)\left(-6Ba^2+4Aba-b^2B\right)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}-\frac{2\left(-3a^3B+2a^2Ab+2ab^2B-Ab^3\right)\sin(c+dx)}{bd}}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+}{2bd}}{b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{ab\left(-3Ba^2+2Aba+b^2B\right)+\left(a^2-b^2\right)\left(-6Ba^2+4Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2\left(-3a^3B+2a^2Ab+2ab^2B-Ab^3\right)\sin(c+dx)}{bd}}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin}{2bd}}{b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3214

$$-\frac{\frac{\frac{\frac{x\left(a^2-b^2\right)\left(-6a^2B+4aAb-b^2B\right)}{b}-\frac{2a^2\left(-3a^3B+2a^2Ab+4ab^2B-3Ab^3\right)\int\frac{1}{a+b\cos(c+dx)}dx}{b}}{b}-\frac{2\left(-3a^3B+2a^2Ab+2ab^2B-Ab^3\right)\sin(c+dx)}{bd}}{2b}-\frac{\left(-3a^2B+2a\right.}{}}{b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{x\left(a^2-b^2\right)\left(-6a^2B+4aAb-b^2B\right)}{b}-\frac{2a^2\left(-3a^3B+2a^2Ab+4ab^2B-3Ab^3\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{2\left(-3a^3B+2a^2Ab+2ab^2B-Ab^3\right)\sin(c+dx)}{bd}}{2b}-\frac{\left(-3a^2B\right.}{}}{b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3138

$$-\frac{\frac{\frac{\frac{x\left(a^2-b^2\right)\left(-6a^2B+4aAb-b^2B\right)}{b}-\frac{4a^2\left(-3a^3B+2a^2Ab+4ab^2B-3Ab^3\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{2\left(-3a^3B+2a^2Ab+2ab^2B-Ab^3\right)s}{bd}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 218

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}+\frac{-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\cos(c+dx)}{2bd}-\frac{\frac{\frac{x\left(a^2-b^2\right)\left(-6a^2B+4aAb-b^2B\right)}{b}-\frac{4a^2\left(-3a^3B+2a^2Ab+4ab^2B-3Ab^3\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}}{2b}}{b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
(a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])) + (-1/2*((2*a*A*b - 3*a^2*B + b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(
b*d) - ((((a^2 - b^2)*(4*a*A*b - 6*a^2*B - b^2*B)*x)/b - (4*a^2*(2*a^2*A*b
 - 3*A*b^3 - 3*a^3*B + 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/b - (2*(2*a^2*A*b - A*b^3 - 3*a
^3*B + 2*a*b^2*B)*Sin[c + d*x])/(b*d))/(2*b))/(b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{2a^2\left(\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(2A\,a^2b-3A\,b^3-3a^3B+4Ba\,b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^4}-\frac{2\Bigg(}{}}{d}$
default	$\frac{\frac{2a^2\left(\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(2A\,a^2b-3A\,b^3-3a^3B+4Ba\,b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^4}-\frac{2\Bigg(}{}}{d}$
risch	$-\frac{2xAa}{b^3}+\frac{3x\,a^2B}{b^4}+\frac{Bx}{2b^2}-\frac{2ia^3(-Ab+Ba)\left(a\,\mathrm{e}^{i(dx+c)}+b\right)}{b^4(a^2-b^2)d\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)}+\frac{i\mathrm{e}^{-i(dx+c)}A}{2d\,b^2}+\frac{iB\,\mathrm{e}^{-2i(dx+c)}}{8b^2d}+\frac{i\mathrm{e}^{i(}}{}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(2*a^2/b^4*(a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+(2*A*a^2*b-3*A*b^3-3*B*a^3+4*B*a*b^2)
/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a
+b))^(1/2)))-2/b^4*(((-A*b^2+2*B*a*b+1/2*B*b^2)*tan(1/2*d*x+1/2*c)^3+(-A*b
^2+2*B*a*b-1/2*B*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(
4*A*a*b-6*B*a^2-B*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 964, normalized size of antiderivative = 3.67

$$\int\frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
[1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5
- 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^
2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*d*x + (3*B*a^6 - 2*
A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b
^3 + 3*A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
 (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*si
n(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
 - (6*B*a^6*b - 4*A*a^5*b^2 - 10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2
*A*a*b^6 - (B*a^4*b^3 - 2*B*a^2*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2
 - 2*A*a^4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x
+ c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4
 - 2*a^3*b^6 + a*b^8)*d), 1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8
*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7
- 4*A*a^6*b - 11*B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a
*b^6)*d*x - 2*(3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*
b - 2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^6*b
 - 4*A*a^5*b^2 - 10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2*A*a*b^6 - (B
*a^4*b^3 - 2*B*a^2*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2 - 2*A*a^4*b^
3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.29

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{4\,(3\,Ba^5-2\,Aa^4b-4\,Ba^3b^2+3\,Aa^2b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^4-b^6)\sqrt{a^2-b^2}}-\frac{4\,(Ba^4\tan(\frac{1}{2}\,dx\cdots}{(a^2b^3-b^5)\left(a\tan(\frac{1}{2}\,dx\cdots\right.}}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
1/2*(4*(3*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 3*A*a^2*b^3)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - 4
*(B*a^4*tan(1/2*d*x + 1/2*c) - A*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b
^5)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (6*B*
a^2 - 4*A*a*b + B*b^2)*(d*x + c)/b^4 - 2*(4*B*a*tan(1/2*d*x + 1/2*c)^3 - 2
*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*tan(1/2*d
*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 38.01 (sec) , antiderivative size = 6744, normalized size of antiderivative = 25.64

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
(a^2*atan(((a^2*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^
2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a
^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3
 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*
B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a
*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 4
0*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^
8 + b^9 - a^2*b^7 - a^3*b^6) + (a^2*((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a
^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B
*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8
*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^2*tan(c/2 + (d*x)/
2)*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2
)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b
^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^
6*b^4)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B
*a*b^2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*A*b^3 + 3*B*a^3 - 2*
A*a^2*b - 4*B*a*b^2)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^2*(
-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10
 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a
^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.81

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{-4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)a^3+\cos(dx+c)^2a^2b^2dx-\cos(dx+c)^2b^4dx+\cos(dx+c)}{}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*a**3 + cos(c + d*x)**2*a**2*b**2*d*x - cos(c + d*x)**2*b**
4*d*x + cos(c + d*x)*sin(c + d*x)*a**2*b**2 - cos(c + d*x)*sin(c + d*x)*b*
*4 + sin(c + d*x)**2*a**2*b**2*d*x - sin(c + d*x)**2*b**4*d*x - 2*sin(c +
d*x)*a**3*b + 2*sin(c + d*x)*a*b**3 + 2*a**4*d*x - 2*a**2*b**2*d*x)/(2*b**
3*d*(a**2 - b**2))
```

3.259 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 155

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{(Ab-2aB)x}{b^3} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
(A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((a-b)^(1/
2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+B*sin(d*x
+c)/b^2/d-a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{(Ab-2aB)(c+dx)+\frac{2a(-a^2Ab+2Ab^3+2a^3B-3ab^2B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+bB\sin(c+dx)+\frac{a^2b(-Ab+a}{(a-b)(a+b)(a}}{b^3d}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
((A*b - 2*a*B)*(c + d*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*
B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2
) + b*B*Sin[c + d*x] + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b
)*(a + b*Cos[c + d*x])))/(b^3*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 3467, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{\int \frac{b(a^2-b^2)B\cos^2(c+dx)+(a^2-b^2)(Ab-aB)\cos(c+dx)+ab(Ab-aB)}{a+b\cos(c+dx)}dx}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)^2+(a^2-b^2)(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(Ab-aB)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\int \frac{a(Ab-aB)b^2+\left(a^2-b^2\right)(Ab-2aB)\cos(c+dx)b}{a+b\cos(c+dx)}dx}{b}+\frac{B(a^2-b^2)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a(Ab-aB)b^2+\left(a^2-b^2\right)(Ab-2aB)\sin\left(c+dx+\frac{\pi}{2}\right)b}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B(a^2-b^2)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{x(a^2-b^2)(Ab-2aB)-a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int \frac{1}{a+b\cos(c+dx)}dx}{b}+\frac{B(a^2-b^2)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{x(a^2-b^2)(Ab-2aB)-a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B(a^2-b^2)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{x(a^2-b^2)(Ab-2aB)-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}}{b}+\frac{B(a^2-b^2)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{B(a^2-b^2)\sin(c+dx)}{d}+\frac{x(a^2-b^2)(Ab-2aB)-\frac{2a(-2a^3B+a^2Ab+3ab^2B-2Ab^3)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{b}}{b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])))
 + (((a^2 - b^2)*(A*b - 2*a*B)*x - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a
*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*S
qrt[a + b]*d))/b + ((a^2 - b^2)*B*Sin[c + d*x])/d)/(b^2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{2a\left(\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(A\,a^2b-2A\,b^3-2a^3B+3Ba\,b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^3}+\frac{2Bb}{1+t}}{d}$
default	$\frac{-\frac{2a\left(\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(A\,a^2b-2A\,b^3-2a^3B+3Ba\,b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^3}+\frac{2Bb}{1+t}}{d}$
risch	$\frac{xA}{b^2}-\frac{2xBa}{b^3}-\frac{iB\,e^{i(dx+c)}}{2b^2d}+\frac{iB\,e^{-i(dx+c)}}{2b^2d}+\frac{2ia^2(-Ab+Ba)\left(a\,e^{i(dx+c)}+b\right)}{b^3(a^2-b^2)d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{a^3\ln\left(e^{i(dx+c)}-\frac{ia^2}{}\right.}{\sqrt{-a^2+b^2}\,(a}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-2*a/b^3*(a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)/(a
-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b)
)^(1/2)))+2/b^3*(B*b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-2*B*
a)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(148) = 296$.

Time = 0.13 (sec) , antiderivative size = 789, normalized size of antiderivative = 5.09

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
[-1/2*(2*(2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 -
A*b^6)*d*x*cos(d*x + c) + 2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3
 + 2*B*a^2*b^4 - A*a*b^5)*d*x - (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2
*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*s
qrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a
^3*b^3 + A*a^2*b^4 + B*a*b^5 + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(d*x +
 c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3
- 2*a^3*b^5 + a*b^7)*d), -((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*
b^4 + 2*B*a*b^5 - A*b^6)*d*x*cos(d*x + c) + (2*B*a^6 - A*a^5*b - 4*B*a^4*b
^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*d*x - (2*B*a^5 - A*a^4*b - 3*B*a
^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)
*sin(d*x + c))) - (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b
^5 + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b
^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. $2(148) = 296$.

Time = 0.25 (sec) , antiderivative size = 1116, normalized size of antiderivative = 7.20

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
((4*B*a^6*b^2 - 2*A*a^5*b^3 - 2*B*a^5*b^3 + A*a^4*b^4 - 9*B*a^4*b^4 + 5*A*
a^3*b^5 + 4*B*a^3*b^5 - 2*A*a^2*b^6 + 5*B*a^2*b^6 - 3*A*a*b^7 - 2*B*a*b^7
+ A*b^8 + 2*B*a^3*abs(-a^2*b^3 + b^5) - A*a^2*b*abs(-a^2*b^3 + b^5) - B*a^
2*b*abs(-a^2*b^3 + b^5) + A*a*b^2*abs(-a^2*b^3 + b^5) - 2*B*a*b^2*abs(-a^2
*b^3 + b^5) + A*b^3*abs(-a^2*b^3 + b^5))*(pi*floor(1/2*(d*x + c)/pi + 1/2)
 + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 + sqr
t(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) +
 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/(a^3*b^2*abs
(-a^2*b^3 + b^5) - a*b^4*abs(-a^2*b^3 + b^5) + (a^2*b^3 - b^5)^2) + ((a^2*
b - a*b^2 - b^3)*sqrt(a^2 - b^2)*A*abs(-a^2*b^3 + b^5)*abs(-a + b) - (2*a^
3 - a^2*b - 2*a*b^2)*sqrt(a^2 - b^2)*B*abs(-a^2*b^3 + b^5)*abs(-a + b) - (
2*a^5*b^3 - a^4*b^4 - 5*a^3*b^5 + 2*a^2*b^6 + 3*a*b^7 - b^8)*sqrt(a^2 - b^
2)*A*abs(-a + b) + (4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*b^5 + 5*a^2*
b^6 - 2*a*b^7)*sqrt(a^2 - b^2)*B*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi +
 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4
- sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b
^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/((a^2*b
^3 - b^5)^2*(a^2 - 2*a*b + b^2) - (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6
)*abs(-a^2*b^3 + b^5)) + 2*(2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1
/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*...
```

Mupad [B] (verification not implemented)

Time = 28.14 (sec) , antiderivative size = 3276, normalized size of antiderivative = 21.14

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
(log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*d) - ((2*tan(c/2 + (d
*x)/2)^3*(A*a^2*b - B*b^3 - 2*B*a^3 + B*a*b^2 + B*a^2*b))/(b^2*(a + b)*(a
- b)) + (2*tan(c/2 + (d*x)/2)*(B*b^3 - 2*B*a^3 + A*a^2*b + B*a*b^2 - B*a^2
*b))/(b^2*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) + 2*a
*tan(c/2 + (d*x)/2)^2)) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*2i))
/(b^3*d) - (a*atan(((a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^
2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*
A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^
4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^
2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/
(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3
*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 -
2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)
- (32*a*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3
 - A*a^2*b - 3*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2
*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7
 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3
- A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)
^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3
*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 +...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.70

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)a^2+\sin(dx+c)\,a^2b-\sin(dx+c)\,b^3-a^3dx+a\,b^2dx}{b^2d\,(a^2-b^2)}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*a**2 + sin(c + d*x)*a**2*b - sin(c + d*x)*b**3 - a**3*d*x + a
*b**2*d*x)/(b**2*d*(a**2 - b**2))
```

3.260 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 122

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{Bx}{b^2} - \frac{2(Ab^3+a^3B-2ab^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} + \frac{a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
B*x/b^2-2*(A*b^3+B*a^3-2*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d+a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2
)/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{B(c+dx) - \frac{2(Ab^3+a(a^2-2b^2)B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + \frac{ab(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{b^2d}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
(B*(c + d*x) - (2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (a*b*(A*b - a*B)*Sin[c + d*
x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(b^2*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {3042, 3447, 3042, 3500, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{b(Ab-aB)-(a^2-b^2)B\cos(c+dx)}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{b(Ab-aB)-(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}$$

$\downarrow$ 3214

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{(aB(a^2-2b^2)+Ab^3)\int\frac{1}{a+b\cos(c+dx)}dx}{b}-\frac{Bx(a^2-b^2)}{b}}{b(a^2-b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{(aB(a^2-2b^2)+Ab^3)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{Bx(a^2-b^2)}{b}}{b(a^2-b^2)}$$

$\downarrow$ 3138

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2(aB(a^2-2b^2)+Ab^3)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}-\frac{Bx(a^2-b^2)}{b}}{b(a^2-b^2)}$$

$\downarrow$ 218

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2(aB(a^2-2b^2)+Ab^3)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}-\frac{Bx(a^2-b^2)}{b}}{b(a^2-b^2)}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((-(((a^2 - b^2)*B*x)/b) + (2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTan[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/(b*(a^
2 - b^2))) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c +
d*x]))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.32

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(A\,b^3+a^3B-2Ba\,b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}+\frac{2B\arctan\left(\tan\right.}{b^2}}{d}$
default	$-\frac{\frac{2\left(-\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(A\,b^3+a^3B-2Ba\,b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}+\frac{2B\arctan\left(\tan\right.}{b^2}}{d}$
risch	$\frac{Bx}{b^2}-\frac{2ia(Ab-Ba)\left(a\,e^{i(dx+c)}+b\right)}{b^2(-a^2+b^2)d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)A}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}-\frac{a^3\ln\left(e^{i(dx+c)}-\right.}{\sqrt{-a^2+b^2}\,(}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-2/b^2*(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+(A*b^3+B*a^3-2*B*a*b^2)/(a-b)/(a+b)/((a
-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2*B
/b^2*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(114) = 228$.

Time = 0.12 (sec) , antiderivative size = 551, normalized size of antiderivative = 4.52

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$=\left[\frac{2\left(Ba^4b-2\,Ba^2b^3+Bb^5\right)dx\cos\left(dx+c\right)+2\left(Ba^5-2\,Ba^3b^2+Bab^4\right)dx+\left(Ba^4-2\,Ba^2b^2+Aab^3+\right.}{}\right.$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fri
cas")
```

output
```
[1/2*(2*(B*a^4*b - 2*B*a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + 2*(B*a^5 - 2*B*
a^3*b^2 + B*a*b^4)*d*x + (B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a
*b^3 + A*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2
*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^4*b^3 - 2*a
^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((B*a^4*b
 - 2*B*a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)
*d*x - (B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a*b^3 + A*b^4)*cos(
d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*si
n(d*x + c))) - (B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/(
(a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)
*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.63

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{2\,(Ba^3-2\,Bab^2+Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^2-b^4)\sqrt{a^2-b^2}}+\frac{(dx+c)B}{b^2}-\frac{2\,(Ba^2\tan(\frac{1}{2}\,dx+}{(a^2b-b^3)\left(a\tan(\frac{1}{2}\,dx+\right.}}{d}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="gia
c")
```

output
```
(2*(B*a^3 - 2*B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + (d*x + c)*B/b^2 - 2*(B*a^2*
tan(1/2*d*x + 1/2*c) - A*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 30.07 (sec) , antiderivative size = 3775, normalized size of antiderivative = 30.94

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
(2*B*atan(((B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7
- 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 -
 a^3*b^3) - (B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4
*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))
)*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*
a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*
B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))/b^2 - (B*((B
*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B
*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*ta
n(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5
- 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 - (32*t
an(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5
*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a
^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))/b^2)/((64*(B^3*a^5 - A*B^2*b^
5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^4*b + 2*B^3*a^2*b^3 - 3*B^3*a^3*b^2 -
3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a
^3*b^3) + (B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 -
 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 -
a^3*b^3) - (B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*
b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.68

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)a+a^2dx-b^2dx}{bd\left(a^2-b^2\right)}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*a + a**2*d*x - b**2*d*x)/(b*d*(a**2 - b**2))
```

3.261 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2(aA-bB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
2*(A*a-B*b)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)
/(a+b)^(3/2)/d-(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{2(aA-bB)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + \frac{(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

output
```
((2*(a*A - b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^
2 + b^2)^(3/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos
[c + d*x])))/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3233}$$
$$-\frac{\int -\frac{aA-bB}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{(Ab-aB)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{aA-bB}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{(Ab-aB)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}$$
$$\Big\downarrow \text{27}$$
$$\frac{(aA-bB)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{(Ab-aB)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(aA-bB)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{(Ab-aB)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}$$
$$\Big\downarrow \text{3138}$$

$$\frac{2(aA-bB)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(aA-bB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*d) - ((A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{2(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)}+\frac{2(Aa-Bb)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)}+\frac{2(Aa-Bb)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{2i(Ab-Ba)\left(a\,e^{i(dx+c)}+b\right)}{b(-a^{2}+b^{2})d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)A}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)d}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^{2}-ib^{2}+a\sqrt{-}}{b\sqrt{-a^{2}+b}}\right.}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-}$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)+2*(A*a-B*b)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.79

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \left[-\frac{(Aa^2-Bab+(Aab-Bb^2)\cos(dx+c))\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+}\right.}{2\,((a^4b-2\,a^2b^3+b^5)d\cos(dx+c)+(a^5-2}\right.$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*((A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(
a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*
b*cos(d*x + c) + a^2)) - 2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*sin(d*x + c
))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d
), ((A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(
-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (B*a^3 - A*a^2*b -
 B*a*b^2 + A*b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c)
+ (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4974 vs. $2(82) = 164$.

Time = 165.71 (sec) , antiderivative size = 4974, normalized size of antiderivative = 49.74

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*cos(c))/cos(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0))
, (A*tan(c/2 + d*x/2)**3/(6*b**2*d) + A*tan(c/2 + d*x/2)/(2*b**2*d) - B*ta
n(c/2 + d*x/2)**3/(6*b**2*d) + B*tan(c/2 + d*x/2)/(2*b**2*d), Eq(a, b)), (
-A/(2*b**2*d*tan(c/2 + d*x/2)) - A/(6*b**2*d*tan(c/2 + d*x/2)**3) + B/(2*b
**2*d*tan(c/2 + d*x/2)) - B/(6*b**2*d*tan(c/2 + d*x/2)**3), Eq(a, -b)), (x
*(A + B*cos(c))/(a + b*cos(c))**2, Eq(d, 0)), (A*a**2*log(-sqrt(-a/(a - b)
 - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a**4*d*sqrt(-a/(a -
 b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a**4*d*sqrt(-a/(a - b) - b/(a - b))
 - 2*a**3*b*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - 2*a**2*b*
*2*d*sqrt(-a/(a - b) - b/(a - b)) + 2*a*b**3*d*sqrt(-a/(a - b) - b/(a - b)
)*tan(c/2 + d*x/2)**2 - b**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/
2)**2 + b**4*d*sqrt(-a/(a - b) - b/(a - b))) + A*a**2*log(-sqrt(-a/(a - b)
 - b/(a - b)) + tan(c/2 + d*x/2))/(a**4*d*sqrt(-a/(a - b) - b/(a - b))*tan
(c/2 + d*x/2)**2 + a**4*d*sqrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sqrt(-
a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - 2*a**2*b**2*d*sqrt(-a/(a - b)
 - b/(a - b)) + 2*a*b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**
2 - b**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + b**4*d*sqrt(
-a/(a - b) - b/(a - b))) - A*a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c
/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(
c/2 + d*x/2)**2 + a**4*d*sqrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sqrt...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.59

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(Aa-Bb)}{(a^2-b^2)^{\frac{3}{2}}}-\frac{Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ab\tan\left(\frac{1}{2}dx+\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right.}\right.}{d}$$

input

```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output

```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(A*a - B*b)/(a^2
 - b^2)^(3/2) - (B*a*tan(1/2*d*x + 1/2*c) - A*b*tan(1/2*d*x + 1/2*c))/((a*
tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

Mupad [B] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)}{2\,\sqrt{a+b}\,\sqrt{a-b}}\right)\,(A\,a-B\,b)}{d\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}} - \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A\,b-B\,a)}{d\,(a+b)\,(a-b)\,\left((a-b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}$$

input

```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^2,x)
```

output

```
(2*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a - b)^(1/2)))*
(A*a - B*b))/(d*(a + b)^(3/2)*(a - b)^(3/2)) - (2*tan(c/2 + (d*x)/2)*(A*b
- B*a))/(d*(a + b)*(a - b)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)}$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2)))/(d*(a**2 - b**2))
```

3.262 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 133

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{2(2a^2Ab-Ab^3-a^3B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{A\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
-2*(2*A*a^2*b-A*b^3-B*a^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/
2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+A*arctanh(sin(d*x+c))/a^2/d+b*(A*b-B*a)*
sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.44

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\cos(c+dx)(B+A\sec(c+dx))\left(\frac{2(-2a^2Ab+Ab^3+a^3B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.}{a^2d(A+B\cos(c+}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

output
```
(Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-2*a^2*A*b + A*b^3 + a^3*B)*ArcTan
h[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - A*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (a*b*(A*b - a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c +
d*x]))))/(a^2*d*(A + B*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3042, 3479, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{(A(a^2-b^2)-a(Ab-aB)\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A(a^2-b^2)-a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{A(a^2-b^2)\int \sec(c+dx)dx}{a}-\frac{(a^3(-B)+2a^2Ab-Ab^3)\int \frac{1}{a+b\cos(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{A(a^2-b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{(a^3(-B)+2a^2Ab-Ab^3)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{A(a^2-b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2(a^3(-B)+2a^2Ab-Ab^3)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{A(a^2-b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2(a^3(-B)+2a^2Ab-Ab^3)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{A(a^2-b^2)\text{arctanh}(\sin(c+dx))}{ad}-\frac{2(a^3(-B)+2a^2Ab-Ab^3)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

output
```
((-2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqr
t[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (A*(a^2 - b^2)*ArcTanh[Sin[c +
d*x]])/(a*d))/(a*(a^2 - b^2)) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2
)*d*(a + b*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2\left(-\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(2Aa^2b-Ab^3-a^3B\right.}{(a-b)(}\right.}{a^2}}{d}$
default	$\frac{-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2\left(-\frac{a(Ab-Ba)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(2Aa^2b-Ab^3-a^3B\right.}{(a-b)(}\right.}{a^2}}{d}$
risch	$-\frac{2i(Ab-Ba)\left(a\,e^{i(dx+c)}+b\right)}{(-a^2+b^2)da\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{2b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)A}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a}{b\sqrt{-a}}\right.}{\sqrt{-a^2+b^2}\,(a+b)(}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-A/a^2*ln(tan(1/2*d*x+1/2*c)-1)+A/a^2*ln(tan(1/2*d*x+1/2*c)+1)-2/a^2*
(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b+a+b)+(2*A*a^2*b-A*b^3-B*a^3)/(a-b)/(a+b)/((a-b)*(a+b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(123) = 246$.

Time = 2.46 (sec) , antiderivative size = 684, normalized size of antiderivative = 5.14

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fri
cas")
```

output
```
[1/2*((B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^3*b - 2*A*a^2*b^2 + A*b^4)*cos(d
*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^5 - 2*A*a^3*b^2
 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c
) + 1) - (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*
cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3
+ A*a*b^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (
a^7 - 2*a^5*b^2 + a^3*b^4)*d), 1/2*(2*(B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^
3*b - 2*A*a^2*b^2 + A*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*
x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^5 - 2*A*a^3*b^2 + A*a*b
^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) -
 (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x
+ c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^
4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*
a^5*b^2 + a^3*b^4)*d)]
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.68

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \frac{\frac{2\,(Ba^3 - 2\,Aa^2b + Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a - 2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^4 - a^2b^2)\sqrt{a^2 - b^2}} + \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right)}{a^2} - \frac{A\log(|\tan}{}}{d}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="gia
c")
```

output
```
(2*(B*a^3 - 2*A*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^2 - A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 2*(B*a*b*tan(1/2*
d*x + 1/2*c) - A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x +
 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 30.39 (sec) , antiderivative size = 3763, normalized size of antiderivative = 28.29

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

output
```
- (A*atan(((A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^
2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 -
 a^4*b^2) - (32*A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*
a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))
)/a^2 - (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^
5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^
5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2 - (A*((A*
((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*
a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*A*
tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^
3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2))))/a^2 + (32*tan(c/
2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b -
5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^
3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2)/((A*((A*((32*(A*a^4*b^5 -
B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*
b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*A*tan(c/2 + (d*x)/2)
*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a
^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2))))/a^2 - (32*tan(c/2 + (d*x)/2)*(A^2*
a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*
A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)b-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2+\mathrm{lo}}{ad\left(a^2-b^2\right)}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*b - log(tan((c + d*x)/2) - 1)*a**2 + log(tan((c + d*x)/2)
- 1)*b**2 + log(tan((c + d*x)/2) + 1)*a**2 - log(tan((c + d*x)/2) + 1)*b**
2)/(a*d*(a**2 - b**2))
```

3.263 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 189

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2b(3a^2Ab-2Ab^3-2a^3B+ab^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{(2Ab-aB)\text{arctanh}(\sin(c+dx))}{a^3d} + \frac{(a^2A-2Ab^2+abB)\tan(c+dx)}{a^2(a^2-b^2)d} + \frac{b(Ab-aB)\tan(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))}$$

output
```
2*b*(3*A*a^2*b-2*A*b^3-2*B*a^3+B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d-(2*A*b-B*a)*arctanh(sin(d*x
+c))/a^3/d+(A*a^2-2*A*b^2+B*a*b)*tan(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*ta
n(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.27

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{-\frac{2b(-3a^2Ab+2Ab^3+2a^3B-ab^2B)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + 2Ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - aB}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((-2*b*(-3*a^2*A*b + 2*A*b^3 + 2*a^3*B - a*b^2*B)*ArcTanh[((a - b)*Tan[(c
+ d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 2*A*b*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] - a*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2
*A*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a*B*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*
(a + b*Cos[c + d*x])) + a*A*Tan[c + d*x])/(a^3*d)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3479, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{(Aa^2+bBa-(Ab-aB)\cos(c+dx)a-2Ab^2+b(Ab-aB)\cos^2(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Aa^2+bBa-(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^2+b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int -\frac{\left(\left(a^2-b^2\right)(2Ab-aB)-ab(Ab-aB)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}-\frac{\int \frac{\left(\left(a^2-b^2\right)(2Ab-aB)-ab(Ab-aB)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}-\frac{\int \frac{\left(a^2-b^2\right)(2Ab-aB)-ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)(2Ab-aB)\int \sec(c+dx)dx}{a}-\frac{b\left(-2a^3B+3a^2Ab+ab^2B-2Ab^3\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}-\frac{\frac{(a^2-b^2)(2Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b(-2a^3B+3a^2Ab+ab^2B-2Ab^3)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}-\frac{\frac{(a^2-b^2)(2Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b(-2a^3B+3a^2Ab+ab^2B-2Ab^3)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 218

$$\frac{\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}-\frac{\frac{(a^2-b^2)(2Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b(-2a^3B+3a^2Ab+ab^2B-2Ab^3)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a(a^2-b^2)}+\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 4257

$$\frac{b(Ab-aB)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{(a^2A+abB-2Ab^2)\tan(c+dx)}{ad}-\frac{\frac{(a^2-b^2)(2Ab-aB)\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b(-2a^3B+3a^2Ab+ab^2B-2Ab^3)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a(a^2-b^2)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + (-((
(-2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2 - b^2)*(2*
A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a*d))/a) + ((a^2*A - 2*A*b^2 + a*b*B)*T
an[c + d*x])/(a*d))/(a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{A}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(2Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{A}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-2Ab+Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}+\frac{2b\left(-\frac{\cdots}{(a^2-\cdots}\right.}{\cdots}}{d}$
default	$\frac{-\frac{A}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(2Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{A}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-2Ab+Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}+\frac{2b\left(-\frac{\cdots}{(a^2-\cdots}\right.}{\cdots}}{d}$
risch	$\frac{2i\left(-Aa\,b^2 e^{3i(dx+c)}+B\,a^2 b\,e^{3i(dx+c)}+A\,a^2 b\,e^{2i(dx+c)}-2A\,b^3 e^{2i(dx+c)}+Ba\,b^2 e^{2i(dx+c)}+2a^3 A\,e^{i(dx+c)}-3Aa\,b^2 e^{i(dx\cdots}\right.}{d\,a^2\left(e^{2i(dx+c)}+1\right)\left(a^2-b^2\right)\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-A/a^2/(tan(1/2*d*x+1/2*c)-1)+(2*A*b-B*a)/a^3*ln(tan(1/2*d*x+1/2*c)-1
)-A/a^2/(tan(1/2*d*x+1/2*c)+1)+1/a^3*(-2*A*b+B*a)*ln(tan(1/2*d*x+1/2*c)+1)
+2*b/a^3*(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+(3*A*a^2*b-2*A*b^3-2*B*a^3+B*a*b^2)/(a-b)/
(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1
/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(180) = 360$.

Time = 6.32 (sec) , antiderivative size = 1088, normalized size of antiderivative = 5.76

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
[1/2*(((2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2
*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 +
b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2
+ b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + a^2)) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4
*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*
a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(sin(d*x +
 c) + 1) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 -
 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3
+ B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(A*a^6 -
 2*A*a^4*b^2 + A*a^2*b^4 + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4
+ 2*A*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*
cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)), -1/2*(2*((2*
B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2*B*a^4*b -
 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((B*a^5*b - 2*A*a
^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (
B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos
(d*x + c))*log(sin(d*x + c) + 1) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 +
 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b -...
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(180) = 360$.

Time = 0.20 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.14

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{2\left(2Ba^3b-3Aa^2b^2-Bab^3+2Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^5-a^3b^2)\sqrt{a^2-b^2}} - \frac{2\left(Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
(2*(2*B*a^3*b - 3*A*a^2*b^2 - B*a*b^3 + 2*A*b^4)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) - 2*(A*a^3*
tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*
d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/
2*c)^3 + A*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b
^2*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d
*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*t
an(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) + (B*a - 2*A*b)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1))/a^3 - (B*a - 2*A*b)*log(abs(tan(1/2*d*x + 1/2*c)
- 1))/a^3)/d
```

Mupad [B] (verification not implemented)

Time = 30.66 (sec) , antiderivative size = 5464, normalized size of antiderivative = 28.91

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

output
```
(atan(((((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2
*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 +
 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5
*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*
a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 -
a^4*b^3 - a^5*b^2) + (((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4
 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A
*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*tan(c/2 + (
d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^
9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*(2*A*b - B*a
))/a^3)*(2*A*b - B*a)*1i)/a^3 + (((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*
a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*
a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 -
5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b
+ 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*
a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (((32*(A*a^7*b^5 - 2*A*a^6*b
^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a
^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a
^7*b^2) - (32*tan(c/2 + (d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a
^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.06

$$\int \frac{(A + B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,b^2+\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2b-\cos(}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*cos(c + d*x)*b**2 + cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*
*2*b - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b**3 - cos(c + d*x)*log(tan(
(c + d*x)/2) + 1)*a**2*b + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b**3 + s
in(c + d*x)*a**3 - sin(c + d*x)*a*b**2)/(cos(c + d*x)*a**2*d*(a**2 - b**2)
)
```

3.264 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 270

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{2b^2(4a^2Ab-3Ab^3-3a^3B+2ab^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d}\\ &\quad+\frac{(a^2A+6Ab^2-4abB)\,\mathrm{arctanh}(\sin(c+dx))}{2a^4d}\\ &\quad-\frac{(2a^2Ab-3Ab^3-a^3B+2ab^2B)\tan(c+dx)}{a^3(a^2-b^2)\,d}\\ &\quad+\frac{(a^2A-3Ab^2+2abB)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)\,d}\\ &\quad+\frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{a(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-2*b^2*(4*A*a^2*b-3*A*b^3-3*B*a^3+2*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*
x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*(A*a^2+6*A*b^2-4*B
*a*b)*arctanh(sin(d*x+c))/a^4/d-(2*A*a^2*b-3*A*b^3-B*a^3+2*B*a*b^2)*tan(d*
x+c)/a^3/(a^2-b^2)/d+1/2*(A*a^2-3*A*b^2+2*B*a*b)*sec(d*x+c)*tan(d*x+c)/a^2
/(a^2-b^2)/d+b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+
c))
```

Mathematica [A] (verified)

Time = 7.57 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.62

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= -\frac{2b^2(-4a^2Ab+3Ab^3+3a^3B-2ab^2B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^4(a^2-b^2)\sqrt{-a^2+b^2}d} + \frac{(-a^2A-6Ab^2+4abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{(a^2A+6Ab^2-4abB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{A}{4a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{A}{4a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{-2Ab\sin\left(\frac{1}{2}(c+dx)\right)+aB\sin\left(\frac{1}{2}(c+dx)\right)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{-2Ab\sin\left(\frac{1}{2}(c+dx)\right)+aB\sin\left(\frac{1}{2}(c+dx)\right)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{a^3(a-b)(a+b)d(a+b\cos(c+dx))}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(-2*b^2*(-4*a^2*A*b + 3*A*b^3 + 3*a^3*B - 2*a*b^2*B)*ArcTanh[((a - b)*Tan[
(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^4*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((
-(a^2*A) - 6*A*b^2 + 4*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2
*a^4*d) + ((a^2*A + 6*A*b^2 - 4*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]])/(2*a^4*d) + A/(4*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - A
/(4*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (-2*A*b*Sin[(c + d*x)
/2] + a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
+ (-2*A*b*Sin[(c + d*x)/2] + a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])) + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x])/(a^3*
(a - b)*(a + b)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {3042, 3479, 3042, 3534, 25, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{\left(Aa^2+2bBa-(Ab-aB)\cos(c+dx)a-3Ab^2+2b(Ab-aB)\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Aa^2+2bBa-(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-3Ab^2+2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int -\frac{\left(-b\left(Aa^2+2bBa-3Ab^2\right)\cos^2(c+dx)-a\left(Aa^2-2bBa+Ab^2\right)\cos(c+dx)+2\left(-Ba^3+2Aba^2+2b^2Ba-3Ab^3\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{\left(a^2A+2abB-3Ab^2\right)\tan(c+}{2ad}}{a\left(a^2-b^2\right)} \frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(a^2A+2abB-3Ab^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int\frac{\left(-b\left(Aa^2+2bBa-3Ab^2\right)\cos^2(c+dx)-a\left(Aa^2-2bBa+Ab^2\right)\cos(c+dx)+2\left(-Ba^3+2Aba^2+2b^2Ba-3Ab^3\right)\right)}{a+b\cos(c+dx)}}{2a}}{a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2A+2abB-3Ab^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int\frac{-b\left(Aa^2+2bBa-3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(Aa^2-2bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(-Ba^3+2Aba^2+2b^2Ba-\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{2a}}{a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3534

$$\frac{\frac{(a^2A+2abB-3Ab^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{\int-\frac{\left(\left(a^2-b^2\right)\left(Aa^2-4bBa+6Ab^2\right)+ab\left(Aa^2+2bBa-3Ab^2\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{2\left(a^3(-B)+2a^2Ab+\right.}{}}{2a}}{a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 25

$$\frac{\frac{(a^2A+2abB-3Ab^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3\right)\tan(c+dx)}{ad}-\frac{\int\frac{\left(\left(a^2-b^2\right)\left(Aa^2-4bBa+6Ab^2\right)+ab\left(Aa^2+2bBa-3Ab^2\right)\cos\right.}{a+b\cos(c+dx)}}{a}}{2a}}{a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2A+2abB-3Ab^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3\right)\tan(c+dx)}{ad}-\frac{\int\frac{\left(a^2-b^2\right)\left(Aa^2-4bBa+6Ab^2\right)+ab\left(Aa^2+2bBa-3Ab^2\right)\sin}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{a}}{2a}}{a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3480

$$\frac{\frac{\left(a^2A+2abB-3Ab^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)\left(a^2A-4abB+6Ab^2\right)\int\sec(c+dx)dx}{a}-\frac{2b^2\left(-3a^3B\cdots\right.}{a}}{2a}}{}}{a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2A+2abB-3Ab^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)\left(a^2A-4abB+6Ab^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2\left(-3\cdots\right.}{a}}{2a}}{}}{a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\left(a^2A+2abB-3Ab^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)\left(a^2A-4abB+6Ab^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^2\left(-3\cdots\right.}{}}{2a}}{}}{a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 218

$$\frac{\frac{\left(a^2A+2abB-3Ab^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)\left(a^2A-4abB+6Ab^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^2\left(-3\cdots\right.}{}}{2a}}{}}{a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 4257

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+$$

$$\frac{\frac{\left(a^2A+2abB-3Ab^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)\left(a^2A-4abB+6Ab^2\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{4b^2\left(\cdots\right.}{}}{2a}}{}}{a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])) + (((a^2*A - 3*A*b^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
- (-(((-4*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a^3*B + 2*a*b^2*B)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2
- b^2)*(a^2*A + 6*A*b^2 - 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(a*d))/a) + (2*(
2*a^2*A*b - 3*A*b^3 - a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(a*d))/(2*a))/(a*(a
^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{A}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-Aa-4Ab+2Ba}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2A-6A\,b^2+4Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^4}-\frac{2b^2\left(-\frac{a(Ab-Ba)b\,\mathrm{ta}}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\right.}\right.}{}}{}$
default	$\frac{\frac{A}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-Aa-4Ab+2Ba}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2A-6A\,b^2+4Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^4}-\frac{2b^2\left(-\frac{a(Ab-Ba)b\,\mathrm{ta}}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\right.}\right.}{}}{}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/2*A/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-A*a-4*A*b+2*B*a)/a^3/(tan(1/
2*d*x+1/2*c)-1)+1/2/a^4*(-A*a^2-6*A*b^2+4*B*a*b)*ln(tan(1/2*d*x+1/2*c)-1)-
2*b^2/a^4*(-a*(A*b-B*a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+(4*A*a^2*b-3*A*b^3-3*B*a^3+2*B*a*b^2)/(a-
b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))
^(1/2)))-1/2*A/a^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-4*A*b+2*B*a)/a^3/(ta
n(1/2*d*x+1/2*c)+1)+1/2*(A*a^2+6*A*b^2-4*B*a*b)/a^4*ln(tan(1/2*d*x+1/2*c)+
1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(256) = 512$.

Time = 11.11 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.92

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
[1/4*(2*((3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6)*cos(d*x + c)^3
+ (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c)^2)*sq
rt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*
sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a
^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3
+ (A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*
b^5 + 6*A*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^6*b - 4*B*a
^5*b^2 + 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*c
os(d*x + c)^3 + (A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*
b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*
(A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4 + 2*(B*a^6*b - 2*A*a^5*b^2 - 3*B*a^4*b^3
+ 5*A*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a
^6*b - 4*B*a^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c)
)*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2
*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2), 1/4*(4*((3*B*a^3*b^3 - 4*A*a^2*b^4
- 2*B*a*b^5 + 3*A*b^6)*cos(d*x + c)^3 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a
^2*b^4 + 3*A*a*b^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c
) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4
*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3...
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\frac{4\left(3Ba^3b^2-4Aa^2b^3-2Bab^4+3Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{a^2-b^2}}+\frac{4\left(Bab^3\tan\left(\right.\right.}{(a^5-a^3b^2)\left(a\tan\left(\right.\right.}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-1/2*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3 - 2*B*a*b^4 + 3*A*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*ta
n(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) +
4*(B*a*b^3*tan(1/2*d*x + 1/2*c) - A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*
b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - (A*a
^2 - 4*B*a*b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 + (A*a^2 -
4*B*a*b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*(A*a*tan(1/2
*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c
)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) - 4*A*b*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 31.24 (sec) , antiderivative size = 6692, normalized size of antiderivative = 24.79

$$\int \frac{(A + B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

output
```
(atan(-((((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2
*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a
^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8
 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*
B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8
 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 4
0*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (((8*
(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 1
6*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 +
 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3 - a
^10*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*B*a*b)*(8*a^13*b - 8
*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*
b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4))*(A*a^
2 + 6*A*b^2 - 4*B*a*b)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 7
2*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^
7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11
*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a
^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 -
 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B
*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{-4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)^2 b^3+4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)b^3}{}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*sin(c + d*x)**2*b**3 + 4*sqrt(a**2 - b**2)*atan((tan((c +
d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*b**3 + 2*cos(c + d*x)*s
in(c + d*x)*a**3*b - 2*cos(c + d*x)*sin(c + d*x)*a*b**3 - log(tan((c + d*x
)/2) - 1)*sin(c + d*x)**2*a**4 - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2
*a**2*b**2 + 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**4 + log(tan((c
 + d*x)/2) - 1)*a**4 + log(tan((c + d*x)/2) - 1)*a**2*b**2 - 2*log(tan((c
+ d*x)/2) - 1)*b**4 + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4 + log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 - 2*log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2*b**4 - log(tan((c + d*x)/2) + 1)*a**4 - log(tan((c +
d*x)/2) + 1)*a**2*b**2 + 2*log(tan((c + d*x)/2) + 1)*b**4 - sin(c + d*x)*a
**4 + sin(c + d*x)*a**2*b**2)/(2*a**3*d*(sin(c + d*x)**2*a**2 - sin(c + d*
x)**2*b**2 - a**2 + b**2))
```

3.265 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 398

$$\begin{aligned}
\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx &= -\frac{(6aAb-12a^2B-b^2B)\,x}{2b^5} \\
&+ \frac{a^2(6a^4Ab-15a^2Ab^3+12Ab^5-12a^5B+29a^3b^2B-20ab^4B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^5(a+b)^{5/2}d} \\
&+ \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2d} \\
&- \frac{(3a^3Ab-6aAb^3-6a^4B+10a^2b^2B-b^4B)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2d} \\
&+ \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} \\
&+ \frac{a(2a^2Ab-5Ab^3-4a^3B+7ab^2B)\cos^2(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-1/2*(6*A*a*b-12*B*a^2-B*b^2)*x/b^5+a^2*(6*A*a^4*b-15*A*a^2*b^3+12*A*b^5-1
2*B*a^5+29*B*a^3*b^2-20*B*a*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/(a-b)^(5/2)/b^5/(a+b)^(5/2)/d+1/2*(6*A*a^4*b-11*A*a^2*b^3+2*A*b^
5-12*B*a^5+21*B*a^3*b^2-6*B*a*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d-1/2*(3*A*a
^3*b-6*A*a*b^3-6*B*a^4+10*B*a^2*b^2-B*b^4)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-
b^2)^2/d+1/2*a*(A*b-B*a)*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*
x+c))^2+1/2*a*(2*A*a^2*b-5*A*b^3-4*B*a^3+7*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c
)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 6.61 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.84

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{16a^2(-6a^4Ab+15a^2Ab^3-12Ab^5+12a^5B-29a^3b^2B+20ab^4B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{-48a^7Abc+72a^5Ab^3c-24aAb^7c+96a^8Bc}{}}{}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
((16*a^2*(-6*a^4*A*b + 15*a^2*A*b^3 - 12*A*b^5 + 12*a^5*B - 29*a^3*b^2*B +
 20*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 +
 b^2)^(5/2) + (-48*a^7*A*b*c + 72*a^5*A*b^3*c - 24*a*A*b^7*c + 96*a^8*B*c
- 136*a^6*b^2*B*c - 12*a^4*b^4*B*c + 48*a^2*b^6*B*c + 4*b^8*B*c - 48*a^7*A
*b*d*x + 72*a^5*A*b^3*d*x - 24*a*A*b^7*d*x + 96*a^8*B*d*x - 136*a^6*b^2*B*
d*x - 12*a^4*b^4*B*d*x + 48*a^2*b^6*B*d*x + 4*b^8*B*d*x + 16*a*b*(a^2 - b^
2)^2*(-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[c + d*x] + 4*(-(a^2*b) +
b^3)^2*(-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[2*(c + d*x)] + 48*a^6*A
*b^2*Sin[c + d*x] - 84*a^4*A*b^4*Sin[c + d*x] + 8*a^2*A*b^6*Sin[c + d*x] +
 4*A*b^8*Sin[c + d*x] - 96*a^7*b*B*Sin[c + d*x] + 160*a^5*b^3*B*Sin[c + d*
x] - 32*a^3*b^5*B*Sin[c + d*x] - 8*a*b^7*B*Sin[c + d*x] + 36*a^5*A*b^3*Sin
[2*(c + d*x)] - 64*a^3*A*b^5*Sin[2*(c + d*x)] + 16*a*A*b^7*Sin[2*(c + d*x)
] - 72*a^6*b^2*B*Sin[2*(c + d*x)] + 130*a^4*b^4*B*Sin[2*(c + d*x)] - 48*a^
2*b^6*B*Sin[2*(c + d*x)] + 2*b^8*B*Sin[2*(c + d*x)] + 4*a^4*A*b^4*Sin[3*(c
 + d*x)] - 8*a^2*A*b^6*Sin[3*(c + d*x)] + 4*A*b^8*Sin[3*(c + d*x)] - 8*a^5
*b^3*B*Sin[3*(c + d*x)] + 16*a^3*b^5*B*Sin[3*(c + d*x)] - 8*a*b^7*B*Sin[3*
(c + d*x)] + a^4*b^4*B*Sin[4*(c + d*x)] - 2*a^2*b^6*B*Sin[4*(c + d*x)] + b
^8*B*Sin[4*(c + d*x)])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*b^5*d)
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 3468, 25, 3042, 3526, 25, 3042, 3528, 27, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int -\frac{\cos^2(c+dx)\left(-2\left(-2Ba^2+Aba+b^2B\right)\cos^2(c+dx)-2b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$\big\downarrow$ 25

$$\frac{\int \frac{\cos^2(c+dx)\left(-2\left(-2Ba^2+Aba+b^2B\right)\cos^2(c+dx)-2b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-2\left(-2Ba^2+Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3526

$$\frac{\frac{a\left(-4a^3B+2a^2Ab+7ab^2B-5Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(-2\left(-6Ba^4+3Aba^3+10b^2Ba^2-6Ab^3a-b^4B\right)\cos^2(c+dx)+b\left(Ba^3+Aba^2-4b\cdots\right)\right)}{a+b\cos(c+dx)}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{\cos(c+dx)\left(-2\left(-6Ba^4+3Aba^3+10b^2Ba^2-6Ab^3a-b^4B\right)\cos^2(c+dx)+b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\cos(c+dx)+2a\left(-4Ba^3+2Aba^2+7b^2Ba-5Ab^3\right)\right)}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}+\cdots}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-2\left(-6Ba^4+3Aba^3+10b^2Ba^2-6Ab^3a-b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a\left(-4Ba^3+2Aba^2+7b^2Ba-5A\cdots\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3528

$$\frac{\frac{\int -\frac{\frac{2\left(-\left(\left(-12Ba^5+6Aba^4+21b^2Ba^3-11Ab^3a^2-6b^4Ba+2Ab^5\right)\cos^2(c+dx)\right)-b\left(-2Ba^4+Aba^3+4b^2Ba^2-4Ab^3a+b^4B\right)\cos(c+dx)+a\left(-6Ba^4+3Aba^3+10b^2Ba\right.}{a+b\cos(c+dx)}}{2b}}{b(a^2-b^2)}}{2b\left(a^2-\right.}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{-\frac{\int\frac{-\left(\left(-12Ba^5+6Aba^4+21b^2Ba^3-11Ab^3a^2-6b^4Ba+2Ab^5\right)\cos^2(c+dx)\right)-b\left(-2Ba^4+Aba^3+4b^2Ba^2-4Ab^3a+b^4B\right)\cos(c+dx)+a\left(-6Ba^4+3Aba^3+10b^2Ba^2\right.}{a+b\cos(c+dx)}}{b}}{b(a^2-b^2)}}{2b\left(a^2-\right.}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{-\frac{\int\frac{\left(12Ba^5-6Aba^4-21b^2Ba^3+11Ab^3a^2+6b^4Ba-2Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(-2Ba^4+Aba^3+4b^2Ba^2-4Ab^3a+b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-6Ba^4+3Aba^3+10b^2B\right.}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{b}}{b(a^2-b^2)}}{2b\left(a^2-\right.}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3502

$$\frac{-\frac{\frac{\int\frac{\left(-12Ba^2+6Aba-b^2B\right)\cos(c+dx)\left(a^2-b^2\right)^2+ab\left(-6Ba^4+3Aba^3+10b^2Ba^2-6Ab^3a-b^4B\right)}{a+b\cos(c+dx)}dx}{b}-\frac{\left(-12a^5B+6a^4Ab+21a^3b^2B-11a^2Ab^3-6ab^4B+2Ab^5\right)\sin}{bd}}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{\int\frac{\left(-12Ba^2+6Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^2+ab\left(-6Ba^4+3Aba^3+10b^2Ba^2-6Ab^3a-b^4B\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(-12a^5B+6a^4Ab+21a^3b^2B-11a^2Ab^3-6ab^4B+2Ab^5\right.}{bd}}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right.}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\frac{\frac{x(a^2-b^2)^2(-12a^2B+6aAb-b^2B)}{b}-\frac{\frac{a^2(-12a^5B+6a^4Ab+29a^3b^2B-15a^2Ab^3-20ab^4B+12Ab^5)\int\frac{1}{a+b\cos(c+dx)}dx}{b}}{b}}{b}-\frac{(-12a^5B+6a^4Ab+21a^3b^2B-11a^2A\cdots}{bd}}{b(a^2-b^2)}}{2b(a\cdots}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{x(a^2-b^2)^2(-12a^2B+6aAb-b^2B)}{b}-\frac{\frac{a^2(-12a^5B+6a^4Ab+29a^3b^2B-15a^2Ab^3-20ab^4B+12Ab^5)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}}{b}-\frac{(-12a^5B+6a^4Ab+21a^3b^2B-1\cdots}{\cdots}}{b(a^2-b^2)}}{2b\cdots}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{\frac{\frac{x(a^2-b^2)^2(-12a^2B+6aAb-b^2B)}{b}-\frac{\frac{2a^2(-12a^5B+6a^4Ab+29a^3b^2B-15a^2Ab^3-20ab^4B+12Ab^5)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}}{b}-\frac{(-12a^5\cdots}{\cdots}}{b(a^2-b^2)}}{\cdots}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{a(-4a^3B+2a^2Ab+7ab^2B-5Ab^3)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{-\frac{(-6a^4B+3a^3Ab+10a^2b^2B-6aAb^3-b^4B)\sin(c+dx)\cos(c+dx)}{bd}-\frac{\frac{x(a^2-b^2)^2(-12a^2B+\cdots}{b}\cdots}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2) + ((a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B)*Cos[c + d*x
]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + (-(((3*a^3*A*b
- 6*a*A*b^3 - 6*a^4*B + 10*a^2*b^2*B - b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(
b*d)) - ((((a^2 - b^2)^2*(6*a*A*b - 12*a^2*B - b^2*B)*x)/b - (2*a^2*(6*a^4
*A*b - 15*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*Arc
Tan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b
]*d))/b - ((6*a^4*A*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*B -
 6*a*b^4*B)*Sin[c + d*x])/(b*d))/b)/(b*(a^2 - b^2)))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-A b^{2}+3 B a b+\frac{1}{2} B b^{2}\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}+\left(-A b^{2}+3 B a b-\frac{1}{2} B b^{2}\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+\frac{\left(6 A a b-12 a^{2} B-B b^{2}\right) \arctan\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}\right.}{b^{5}}$
default	$-\frac{2\left(\frac{\left(-A b^{2}+3 B a b+\frac{1}{2} B b^{2}\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}+\left(-A b^{2}+3 B a b-\frac{1}{2} B b^{2}\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{2}}+\frac{\left(6 A a b-12 a^{2} B-B b^{2}\right) \arctan\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}\right.}{b^{5}}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-2/b^5*(((-A*b^2+3*B*a*b+1/2*B*b^2)*tan(1/2*d*x+1/2*c)^3+(-A*b^2+3*B*
a*b-1/2*B*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(6*A*a*b
-12*B*a^2-B*b^2)*arctan(tan(1/2*d*x+1/2*c)))+2*a^2/b^5*((1/2*(4*A*a^2*b-A*
a*b^2-8*A*b^3-6*B*a^3+B*a^2*b+10*B*a*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3+1/2*b*a*(4*A*a^2*b+A*a*b^2-8*A*b^3-6*B*a^3-B*a^2*b+10*B*a*b
^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b+a+b)^2+1/2*(6*A*a^4*b-15*A*a^2*b^3+12*A*b^5-12*B*a^5+29*B*a^3*b
^2-20*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/
2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(378) = 756$.

Time = 0.23 (sec) , antiderivative size = 1812, normalized size of antiderivative = 4.55

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
[1/4*(2*(12*B*a^8*b^2 - 6*A*a^7*b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a
^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 + 6*A*a*b^9 - B*b^10)*d*x*cos(d*x + c)
^2 + 4*(12*B*a^9*b - 6*A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*
b^5 - 18*A*a^4*b^6 - 9*B*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*d*x*cos(d*x + c)
 + 2*(12*B*a^10 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 -
 18*A*a^5*b^5 - 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*d*x + (12*B*a^9 - 6
*A*a^8*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B*a^5*b^4 - 12*A*a^4*b^5 + (12
*B*a^7*b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 + 20*B*a^3*b^6 - 12
*A*a^2*b^7)*cos(d*x + c)^2 + 2*(12*B*a^8*b - 6*A*a^7*b^2 - 29*B*a^6*b^3 +
15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)
*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^
2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2)) - 2*(12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*b^3 +
17*A*a^6*b^4 + 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^8 - (
B*a^6*b^4 - 3*B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10)*cos(d*x + c)^3 + 2*(2*B*a^
7*b^3 - A*a^6*b^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3*A*a^2*b^8
- 2*B*a*b^9 + A*b^10)*cos(d*x + c)^2 + (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*
a^6*b^4 + 25*A*a^5*b^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*
a*b^9)*cos(d*x + c))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^
13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2712 vs. $2(378) = 756$.

Time = 0.45 (sec) , antiderivative size = 2712, normalized size of antiderivative = 6.81

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/2*((3*(2*a^5*b - a^4*b^2 - 4*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*sqrt(a^2 - b
^2)*A*abs(a^4*b^5 - 2*a^2*b^7 + b^9)*abs(-a + b) - (12*a^6 - 6*a^5*b - 23*
a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 - a*b^5 + b^6)*sqrt(a^2 - b^2)*B*abs(a^4
*b^5 - 2*a^2*b^7 + b^9)*abs(-a + b) + 3*(4*a^10*b^5 - 2*a^9*b^6 - 17*a^8*b
^7 + 8*a^7*b^8 + 28*a^6*b^9 - 12*a^5*b^10 - 21*a^4*b^11 + 8*a^3*b^12 + 6*a
^2*b^13 - 2*a*b^14)*sqrt(a^2 - b^2)*A*abs(-a + b) - (24*a^11*b^4 - 12*a^10
*b^5 - 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 - 111*a^5*b^10
+ 42*a^4*b^11 + 28*a^3*b^12 - 8*a^2*b^13 + a*b^14 - b^15)*sqrt(a^2 - b^2)*
B*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*tan(1/2*d*x +
1/2*c)/sqrt((4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 + sqrt(-16*(a^5*b^4 + a^4*b^5
 - 2*a^3*b^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2
*a^2*b^7 + a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2))/(a^5*b^4 -
a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9))))/((a^4*b^5 - 2*a^2*b^7 +
b^9)^2*(a^2 - 2*a*b + b^2) + (a^7*b^4 - 2*a^6*b^5 - a^5*b^6 + 4*a^4*b^7 -
a^3*b^8 - 2*a^2*b^9 + a*b^10)*abs(a^4*b^5 - 2*a^2*b^7 + b^9)) + (24*B*a^11
*b^4 - 12*A*a^10*b^5 - 12*B*a^10*b^5 + 6*A*a^9*b^6 - 100*B*a^9*b^6 + 51*A*
a^8*b^7 + 47*B*a^8*b^7 - 24*A*a^7*b^8 + 158*B*a^7*b^8 - 84*A*a^6*b^9 - 68*
B*a^6*b^9 + 36*A*a^5*b^10 - 111*B*a^5*b^10 + 63*A*a^4*b^11 + 42*B*a^4*b^11
 - 24*A*a^3*b^12 + 28*B*a^3*b^12 - 18*A*a^2*b^13 - 8*B*a^2*b^13 + 6*A*a*b^
14 + B*a*b^14 - B*b^15 - 12*B*a^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*A*...
```

Mupad [B] (verification not implemented)

Time = 34.68 (sec) , antiderivative size = 10598, normalized size of antiderivative = 26.63

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(3*B*b^7 - 36*B*a^7 - 2*A*b^7 + 10*A*a^2*b^5 + 16*A
*a^3*b^4 - 35*A*a^4*b^3 - 9*A*a^5*b^2 + 5*B*a^2*b^5 - 26*B*a^3*b^4 - 29*B*
a^4*b^3 + 67*B*a^5*b^2 - 4*A*a*b^6 + 18*A*a^6*b + 4*B*a*b^6 + 18*B*a^6*b))
/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) - (tan(c/2 + (d*x)/2)^3*(2*A*b^7 +
36*B*a^7 + 3*B*b^7 - 10*A*a^2*b^5 + 16*A*a^3*b^4 + 35*A*a^4*b^3 - 9*A*a^5*
b^2 + 5*B*a^2*b^5 + 26*B*a^3*b^4 - 29*B*a^4*b^3 - 67*B*a^5*b^2 - 4*A*a*b^6
 - 18*A*a^6*b - 4*B*a*b^6 + 18*B*a^6*b))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b
^4)) + (tan(c/2 + (d*x)/2)^7*(B*b^6 - 12*B*a^6 - 2*A*b^6 + 4*A*a^2*b^4 - 1
2*A*a^3*b^3 - 3*A*a^4*b^2 - 8*B*a^2*b^4 - 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*
A*a*b^5 + 6*A*a^5*b + 5*B*a*b^5 + 6*B*a^5*b))/((a*b^4 - b^5)*(a + b)^2) +
(tan(c/2 + (d*x)/2)*(2*A*b^6 - 12*B*a^6 + B*b^6 - 4*A*a^2*b^4 - 12*A*a^3*b
^3 + 3*A*a^4*b^2 - 8*B*a^2*b^4 + 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 +
 6*A*a^5*b - 5*B*a*b^5 - 6*B*a^5*b))/((a + b)*(b^6 - 2*a*b^5 + a^2*b^4)))/
(d*(2*a*b + tan(c/2 + (d*x)/2)^4*(6*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^2*(4
*a*b + 4*a^2) - tan(c/2 + (d*x)/2)^6*(4*a*b - 4*a^2) + tan(c/2 + (d*x)/2)^
8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan(((((8*tan(c/2 + (d*x)/2)*(288*
B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72
*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*
A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^
2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.57

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{-12\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos\left(dx+c\right)a^5b+16\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)}{}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output

```
( - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*cos(c + d*x)*a**5*b + 16*sqrt(a**2 - b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**3*b**3
 - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*a**6 + 16*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan
((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**4*b**2 - 3*cos(c + d*x)*sin(c + d*x
)*a**5*b**2 + 6*cos(c + d*x)*sin(c + d*x)*a**3*b**4 - 3*cos(c + d*x)*sin(c
 + d*x)*a*b**6 + 6*cos(c + d*x)*a**6*b*c + 6*cos(c + d*x)*a**6*b*d*x - 11*
cos(c + d*x)*a**4*b**3*c - 11*cos(c + d*x)*a**4*b**3*d*x + 4*cos(c + d*x)*
a**2*b**5*c + 4*cos(c + d*x)*a**2*b**5*d*x + cos(c + d*x)*b**7*c + cos(c +
 d*x)*b**7*d*x - sin(c + d*x)**3*a**4*b**3 + 2*sin(c + d*x)**3*a**2*b**5 -
 sin(c + d*x)**3*b**7 - 6*sin(c + d*x)*a**6*b + 11*sin(c + d*x)*a**4*b**3
- 6*sin(c + d*x)*a**2*b**5 + sin(c + d*x)*b**7 + 6*a**7*c + 6*a**7*d*x - 1
1*a**5*b**2*c - 11*a**5*b**2*d*x + 4*a**3*b**4*c + 4*a**3*b**4*d*x + a*b**
6*c + a*b**6*d*x)/(2*b**4*d*(cos(c + d*x)*a**4*b - 2*cos(c + d*x)*a**2*b**
3 + cos(c + d*x)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.266 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 280

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx &= \frac{(Ab-3aB)x}{b^4}\\ &-\frac{a(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2B-12ab^4B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d}\\ &-\frac{(aAb-3a^2B+2b^2B)\sin(c+dx)}{2b^3(a^2-b^2)\,d}+\frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b(a^2-b^2)\,d(a+b\cos(c+dx))^2}\\ &-\frac{a^2(a^2Ab-4Ab^3-3a^3B+6ab^2B)\sin(c+dx)}{2b^3(a^2-b^2)^2\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
(A*b-3*B*a)*x/b^4-a*(2*A*a^4*b-5*A*a^2*b^3+6*A*b^5-6*B*a^5+15*B*a^3*b^2-12
*B*a*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b
^4/(a+b)^(5/2)/d-1/2*(A*a*b-3*B*a^2+2*B*b^2)*sin(d*x+c)/b^3/(a^2-b^2)/d+1/
2*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/2
*a^2*(A*a^2*b-4*A*b^3-3*B*a^3+6*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b
*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 4.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{2(Ab-3aB)(c+dx)-\frac{2a(-2a^4Ab+5a^2Ab^3-6Ab^5+6a^5B-15a^3b^2B+12ab^4B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+2bB\sin(c}{2b^4d}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(2*(A*b - 3*a*B)*(c + d*x) - (2*a*(-2*a^4*A*b + 5*a^2*A*b^3 - 6*A*b^5 + 6*
a^5*B - 15*a^3*b^2*B + 12*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*B*Sin[c + d*x] + (a^3*b*(A*b - a*B
)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^2*b*(-3*a^2*
A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a
 + b*Cos[c + d*x])))/(2*b^4*d)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3468, 25, 3042, 3510, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int -\frac{\cos(c+dx)\left(-\left(\left(-3Ba^2+Aba+2b^2B\right)\cos^2(c+dx)\right)-2b(Ab-aB)\cos(c+dx)+2a(Ab-aB)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\cos(c+dx)\left(-\left(\left(-3Ba^2+Aba+2b^2B\right)\cos^2(c+dx)\right)-2b(Ab-aB)\cos(c+dx)+2a(Ab-aB)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\left(3Ba^2-Aba-2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3510$$

$$\frac{\frac{\int \frac{-b\left(a^2-b^2\right)\left(-3Ba^2+Aba+2b^2B\right)\cos^2(c+dx)+\left(a^2-b^2\right)\left(-3Ba^3+Aba^2+4b^2Ba-2Ab^3\right)\cos(c+dx)+ab\left(-3Ba^3+Aba^2+6b^2Ba-4Ab^3\right)}{a+b\cos(c+dx)}dx}{b^2\left(a^2-b^2\right)}-\frac{a^2\left(-3a^3B+\ldots\right.}{b^2d(\ldots}}{2b\left(a^2-b^2\right)}$$
$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{-b\left(a^2-b^2\right)\left(-3Ba^2+Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(a^2-b^2\right)\left(-3Ba^3+Aba^2+4b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+ab\left(-3Ba^3+Aba^2+6b^2Ba-4Ab^3\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\left(a^2-b^2\right)}-\frac{a^2(-\ldots}{\ldots}}{2b\left(a^2-b^2\right)}$$
$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{\frac{\int \frac{a\left(-3Ba^3+Aba^2+6b^2Ba-4Ab^3\right)b^2+2\left(a^2-b^2\right)^2(Ab-3aB)\cos(c+dx)b}{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{a^2\left(-3a^3B+a^2Ab+6ab^2B-4A\ldots\right.}{b^2d\left(a^2-b^2\right)(a+b\cos(c\ldots}}{2b\left(a^2-b^2\right)}$$
$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{a\left(-3Ba^3+Aba^2+6b^2Ba-4Ab^3\right)b^2+2\left(a^2-b^2\right)^2(Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)b}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(-3a^3B+a^2Ab+6ab^2B\cdots}{b^2d(a^2-b^2)(a+b\,\mathrm{co}\cdots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3214$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2(Ab-3aB)-a\left(-6a^5B+2a^4Ab+15a^3b^2B-5a^2Ab^3-12ab^4B+6Ab^5\right)\int\frac{1}{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(-3a^3\cdots}{\cdots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2(Ab-3aB)-a\left(-6a^5B+2a^4Ab+15a^3b^2B-5a^2Ab^3-12ab^4B+6Ab^5\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{a^2(-\cdots}{\cdots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2(Ab-3aB)-\frac{2a\left(-6a^5B+2a^4Ab+15a^3b^2B-5a^2Ab^3-12ab^4B+6Ab^5\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}}{b}-\frac{\left(a^2-b^2\right)\left(-3a^2B+aAb+2b\cdots\right.}{d}}{b^2(a^2-b^2)}\cdots}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 218$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2(Ab-3aB)-\frac{2a\left(-6a^5B+2a^4Ab+15a^3b^2B-5a^2Ab^3-12ab^4B+6Ab^5\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{\left(a^2-b^2\right)\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}\cdots}{2b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2) + (-((a^2*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d
*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))) + ((2*(a^2 - b^2)^2*(A*b -
3*a*B)*x - (2*a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*
B - 12*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[
a - b]*Sqrt[a + b]*d))/b - ((a^2 - b^2)*(a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c
+ d*x])/d)/(b^2*(a^2 - b^2)))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2(Ab-3Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{2a\left(\frac{\frac{\left(2Aa^2b-Aab^2-6Ab^3-4a^3B+Ba^2b+8Bab^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{ba}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}\right.\right.}\right.}{}$
default	$\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2(Ab-3Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{2a\left(\frac{\frac{\left(2Aa^2b-Aab^2-6Ab^3-4a^3B+Ba^2b+8Bab^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{ba}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}\right.\right.}\right.}{}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(2/b^4*(B*b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-3*B*a)*ar
ctan(tan(1/2*d*x+1/2*c)))-2/b^4*a*((1/2*(2*A*a^2*b-A*a*b^2-6*A*b^3-4*B*a^3
+B*a^2*b+8*B*a*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*b*a
*(2*A*a^2*b+A*a*b^2-6*A*b^3-4*B*a^3-B*a^2*b+8*B*a*b^2)/(a+b)/(a-b)^2*tan(1
/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2+1/2*(
2*A*a^4*b-5*A*a^2*b^3+6*A*b^5-6*B*a^5+15*B*a^3*b^2-12*B*a*b^4)/(a^4-2*a^2*
b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(268) = 536.

Time = 0.19 (sec) , antiderivative size = 1561, normalized size of antiderivative = 5.58

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
[-1/4*(4*(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^
6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*d*x*cos(d*x + c)^2 + 8*(3*B*a^8*b - A
*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2
*b^7 + A*a*b^8)*d*x*cos(d*x + c) + 4*(3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*
A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*d*x - (6*
B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^
5 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6
 - 6*A*a*b^7)*cos(d*x + c)^2 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 +
 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*
log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2
)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2
*a*b*cos(d*x + c) + a^2)) - 2*(6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*
A*a^5*b^4 + 13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(B*a^6*b^3 - 3*B*
a^4*b^5 + 3*B*a^2*b^7 - B*b^9)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3
 - 25*B*a^5*b^4 + 9*A*a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*co
s(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos
(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)
+ (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(2*(3*B*a^7*b^2 -
A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*
b^8 + A*b^9)*d*x*cos(d*x + c)^2 + 4*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(268) = 536.

Time = 0.20 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.94

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-((6*B*a^6 - 2*A*a^5*b - 15*B*a^4*b^2 + 5*A*a^3*b^3 + 12*B*a^2*b^4 - 6*A*a
*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*
a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*B*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^
5*b*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^
2*tan(1/2*d*x + 1/2*c)^3 - 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^3*b^
3*tan(1/2*d*x + 1/2*c)^3 + 8*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^6*tan(1/2*d*x + 1/2*c) - 2*A*a^5*b*tan(1/
2*d*x + 1/2*c) + 5*B*a^5*b*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^2*tan(1/2*d*x
+ 1/2*c) - 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 5*A*a^3*b^3*tan(1/2*d*x + 1/
2*c) - 8*B*a^3*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^4*tan(1/2*d*x + 1/2*c)
)/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x +
 1/2*c)^2 + a + b)^2) + (3*B*a - A*b)*(d*x + c)/b^4 - 2*B*tan(1/2*d*x + 1/
2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 30.56 (sec) , antiderivative size = 5542, normalized size of antiderivative = 19.79

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(6*B*a^5 - 2*B*b^5 + 6*A*a^2*b^3 + A*a^3*b^2 + 4*B*
a^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 - 3*B*a^4*b))/((a*b^3 - b^4
)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(6*B*a^5 + 2*B*b^5 + 6*A*a^2*b^3 - A*a^
3*b^2 - 4*B*a^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 + 3*B*a^4*b))/(
(a + b)*(b^5 - 2*a*b^4 + a^2*b^3)) + (2*tan(c/2 + (d*x)/2)^3*(6*B*a^6 - 2*
B*b^6 + 5*A*a^3*b^3 + 6*B*a^2*b^4 - 13*B*a^4*b^2 - 2*A*a^5*b))/(b*(a*b^2 -
 b^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b + 3*a^2
- b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 +
(d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) + (log(tan(c/2 + (d*x)/2) + 1i)*(A*b -
3*B*a)*1i)/(b^4*d) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*3i))/(b^4
*d) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2
*a*b^11 - 72*B^2*a^11*b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^
8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*
A^2*a^9*b^3 + 8*A^2*a^10*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a
^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8
*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b
+ 48*A*B*a^2*b^10 - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 2
88*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*
A*B*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^
5*b^8 - a^6*b^7 - a^7*b^6) + (a*((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.86

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,a^4b-6\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos}{}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output
```
(4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*cos(c + d*x)*a**4*b - 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**2*b**3 + 4*
sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2
 - b**2))*a**5 - 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d
*x)/2)*b)/sqrt(a**2 - b**2))*a**3*b**2 + cos(c + d*x)*sin(c + d*x)*a**4*b*
*2 - 2*cos(c + d*x)*sin(c + d*x)*a**2*b**4 + cos(c + d*x)*sin(c + d*x)*b**
6 - 2*cos(c + d*x)*a**5*b*c - 2*cos(c + d*x)*a**5*b*d*x + 4*cos(c + d*x)*a
**3*b**3*c + 4*cos(c + d*x)*a**3*b**3*d*x - 2*cos(c + d*x)*a*b**5*c - 2*co
s(c + d*x)*a*b**5*d*x + 2*sin(c + d*x)*a**5*b - 3*sin(c + d*x)*a**3*b**3 +
 sin(c + d*x)*a*b**5 - 2*a**6*c - 2*a**6*d*x + 4*a**4*b**2*c + 4*a**4*b**2
*d*x - 2*a**2*b**4*c - 2*a**2*b**4*d*x)/(b**3*d*(cos(c + d*x)*a**4*b - 2*c
os(c + d*x)*a**2*b**3 + cos(c + d*x)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.267 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 211

$$\begin{aligned}&\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3} \, dx \\ &= \frac{Bx}{b^3}+\frac{(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} \\ &\quad -\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a(a^2Ab-4Ab^3-3a^3B+6ab^2B)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}\end{aligned}$$

output
```
B*x/b^3+(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*a*b^4)*arctan((a-b)^(1/
2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*(
A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*a*(A*a^2*b-4*A*
b^3-3*B*a^3+6*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{2B(c+dx)+\frac{2(-a^2Ab^3-2Ab^5+2a^5B-5a^3b^2B+6ab^4B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{a^2b(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+\frac{ab(a^2}{}}{2b^3d}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(2*B*(c + d*x) + (2*(-(a^2*A*b^3) - 2*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 6*a*
b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^
(5/2) + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c
+ d*x])^2) + (a*b*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/
((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^3*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3467, 3042, 3500, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{\int \frac{2b(a^2-b^2)B\cos^2(c+dx)+(a^2-2b^2)(Ab-aB)\cos(c+dx)+2ab(Ab-aB)}{(a+b\cos(c+dx))^2}dx}{2b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2b(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)^2+(a^2-2b^2)(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2ab(Ab-aB)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3500

$$\frac{\frac{a(-3a^3B+a^2Ab+6ab^2B-4Ab^3)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int -\frac{(Ba^3+Aba^2-4b^2Ba+2Ab^3)b^2+2(a^2-b^2)^2B\cos(c+dx)b}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}}{2b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{(Ba^3+Aba^2-4b^2Ba+2Ab^3)b^2+2(a^2-b^2)^2B\cos(c+dx)b}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}+\frac{a(-3a^3B+a^2Ab+6ab^2B-4Ab^3)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{(Ba^3+Aba^2-4b^2Ba+2Ab^3)b^2+2(a^2-b^2)^2B\sin\left(c+dx+\frac{\pi}{2}\right)b}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}+\frac{a(-3a^3B+a^2Ab+6ab^2B-4Ab^3)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3214

$$\frac{\frac{(-2a^5B+5a^3b^2B+a^2Ab^3-6ab^4B+2Ab^5)\int \frac{1}{a+b\cos(c+dx)}dx+2Bx(a^2-b^2)^2}{b(a^2-b^2)}+\frac{a(-3a^3B+a^2Ab+6ab^2B-4Ab^3)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(-2a^5B+5a^3b^2B+a^2Ab^3-6ab^4B+2Ab^5\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+2Bx\left(a^2-b^2\right)^2}{b(a^2-b^2)}+\frac{a\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b^2\left(a^2-b^2\right)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{2\left(-2a^5B+5a^3b^2B+a^2Ab^3-6ab^4B+2Ab^5\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}+2Bx\left(a^2-b^2\right)^2}{b(a^2-b^2)}+\frac{a\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b^2\left(a^2-b^2\right)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{a\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}+\frac{2Bx\left(a^2-b^2\right)^2+\frac{2\left(-2a^5B+5a^3b^2B+a^2Ab^3-6ab^4B+2Ab^5\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{b(a^2-b^2)}}{2b^2\left(a^2-b^2\right)}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

input

```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output

```
-1/2*(a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2) + ((2*(a^2 - b^2)^2*B*x + (2*(a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3*b
^2*B - 6*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqr
t[a - b]*Sqrt[a + b]*d))/(b*(a^2 - b^2)) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B
 + 6*a*b^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*b^2*(
a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{2\left(-\frac{\left(A a\,b^{2}+4A\,b^{3}+2a^{3}B-B\,a^{2}b-6B a\,b^{2}\right)a b\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{b a\left(A a\,b^{2}-4A\,b^{3}-2a^{3}B-B\,a^{2}b+6B a\,b^{2}\right)\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{2(a+b)(a-b)^{2}}\right)}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{\left(A\,a^{2}b^{3}\right.}{}}{\frac{b^{3}}{d}}$
default	$\frac{\frac{2\left(-\frac{\left(A a\,b^{2}+4A\,b^{3}+2a^{3}B-B\,a^{2}b-6B a\,b^{2}\right)a b\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{b a\left(A a\,b^{2}-4A\,b^{3}-2a^{3}B-B\,a^{2}b+6B a\,b^{2}\right)\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{2(a+b)(a-b)^{2}}\right)}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{\left(A\,a^{2}b^{3}\right.}{}}{\frac{b^{3}}{d}}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(2/b^3*((-1/2*(A*a*b^2+4*A*b^3+2*B*a^3-B*a^2*b-6*B*a*b^2)*a*b/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*b*a*(A*a*b^2-4*A*b^3-2*B*a^3-B*a^2*
b+6*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^2+1/2*(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*
a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/
2*c)/((a-b)*(a+b))^(1/2)))+2*B/b^3*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(201) = 402$.

Time = 0.16 (sec) , antiderivative size = 1152, normalized size of antiderivative = 5.46

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
[1/4*(4*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x + c)^2
 + 8*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 4*
(B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x + (2*B*a^7 - 5*B*a^5*b
^2 - A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*b^4 -
A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b
^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*l
og((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)
*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*
a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b
^5 + 6*B*a^2*b^6 - 4*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5 - 3*a^
4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^
3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^
9)*d), 1/2*(2*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x
+ c)^2 + 4*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c
) + 2*(B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x - (2*B*a^7 - 5*B
*a^5*b^2 - A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*
b^4 - A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B
*a^4*b^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(201) = 402$.

Time = 0.20 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.16

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx =$$
$$-\frac{\frac{(2\,Ba^5-5\,Ba^3b^2-Aa^2b^3+6\,Bab^4-2\,Ab^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^4b^3-2\,a^2b^5+b^7)\sqrt{a^2-b^2}} - \frac{(dx+c)B}{b^3} + \frac{2\,Ba\cdots}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-((2*B*a^5 - 5*B*a^3*b^2 - A*a^2*b^3 + 6*B*a*b^4 - 2*A*b^5)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*ta
n(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^
2 - b^2)) - (d*x + c)*B/b^3 + (2*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^4*b*
tan(1/2*d*x + 1/2*c)^3 + A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^2*ta
n(1/2*d*x + 1/2*c)^3 + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 4*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^5*tan(1/2*
d*x + 1/2*c) + 3*B*a^4*b*tan(1/2*d*x + 1/2*c) - A*a^3*b^2*tan(1/2*d*x + 1/
2*c) - 5*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c)
 - 6*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 4*A*a*b^4*tan(1/2*d*x + 1/2*c))/((a^
4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 31.62 (sec) , antiderivative size = 6923, normalized size of antiderivative = 32.81

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

output

```
(2*B*atan(-((B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12
+ 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11
- 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 -
 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a
^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (B*tan(c/2 + (d*x)/2)*(8*a*b^15
- 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*
a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3
*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^
3 + (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*
b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*
a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^
3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*
a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 -
 a^6*b^5 - a^7*b^4)))/b^3 - (B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13
 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12
 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7
 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 -
3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (B*tan(c/2 + (d*
x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 4
8*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.80

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,a^3b+4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\mathrm{c}}{}$$

input

```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*cos(c + d*x)*a**3*b + 4*sqrt(a**2 - b**2)*atan((tan((c + d
*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a*b**3 - 2*
sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2
 - b**2))*a**4 + 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d
*x)/2)*b)/sqrt(a**2 - b**2))*a**2*b**2 + cos(c + d*x)*a**4*b*d*x - 2*cos(c
 + d*x)*a**2*b**3*d*x + cos(c + d*x)*b**5*d*x - sin(c + d*x)*a**4*b + sin(
c + d*x)*a**2*b**3 + a**5*d*x - 2*a**3*b**2*d*x + a*b**4*d*x)/(b**2*d*(cos
(c + d*x)*a**4*b - 2*cos(c + d*x)*a**2*b**3 + cos(c + d*x)*b**5 + a**5 - 2
*a**3*b**2 + a*b**4))
```

3.268 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 180

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = -\frac{(3aAb-a^2B-2b^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(Ab-aB)\sin(c+dx)}{2b(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)\sin(c+dx)}{2b(a^2-b^2)^2\,d(a+b\cos(c+dx))}$$

output

```
-(3*A*a*b-B*a^2-2*B*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2)
)/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*
cos(d*x+c))^2+1/2*(A*a^2*b+2*A*b^3+B*a^3-4*B*a*b^2)*sin(d*x+c)/b/(a^2-b^2)
^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$
$$=\frac{-\frac{2(-3aAb+a^2B+2b^2B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{a(Ab-aB)\sin(c+dx)}{(a-b)b(a+b)(a+b\cos(c+dx))^2}+\frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)\sin(c+dx)}{(a-b)^2b(a+b)^2(a+b\cos(c+dx))}}{2d}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
((-2*(-3*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[
-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*(A*b - a*B)*Sin[c + d*x])/((a - b)*b
*(a + b)*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B
)*Sin[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {3042, 3447, 3042, 3500, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A \sin \left(c+dx+\frac{\pi}{2}\right)+B \sin \left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b \sin \left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int \frac{2b(Ab-aB)-\left(Ba^2+Aba-2b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int \frac{2b(Ab-aB)+\left(-Ba^2-Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{-\frac{\int -\frac{b\left(-Ba^2+3Aba-2b^2B\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(a^3B+a^2Ab-4ab^2B+2Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{\int \frac{b\left(-Ba^2+3Aba-2b^2B\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(a^3B+a^2Ab-4ab^2B+2Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{b\left(a^2(-B)+3aAb-2b^2B\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(a^3B+a^2Ab-4ab^2B+2Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{b\left(a^2(-B)+3aAb-2b^2B\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\left(a^3B+a^2Ab-4ab^2B+2Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{2b\left(a^2(-B)+3aAb-2b^2B\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{\left(a^3B+a^2Ab-4ab^2B+2Ab^3\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{2b(a^2(-B)+3aAb-2b^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{(a^3B+a^2Ab-4ab^2B+2Ab^3)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) -
((2*b*(3*a*A*b - a^2*B - 2*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*d) - ((a^2*A*b + 2*A*b^3
+ a^3*B - 4*a*b^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(
2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(2a^2A+Aab+2Ab^2-a^2B-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2A-Aab+2Ab^2+a^2B-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}-\frac{\left(3Aab-a^2B-2Bb^2\cdots\right.}{\left(a^4-2a^2\cdots\right.}}{d}$
default	$-\frac{\frac{2\left(-\frac{\left(2a^2A+Aab+2Ab^2-a^2B-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2A-Aab+2Ab^2+a^2B-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}-\frac{\left(3Aab-a^2B-2Bb^2\cdots\right.}{\left(a^4-2a^2\cdots\right.}}{d}$
risch	$\frac{i\left(3Aab^4e^{3i(dx+c)}+2Ba^4be^{3i(dx+c)}-5Ba^2b^3e^{3i(dx+c)}+2Aa^4be^{2i(dx+c)}+5Aa^2b^3e^{2i(dx+c)}+2Ab^5e^{2i(dx+c)}+2Ba^5\cdots\right.}{b^2(a\cdots}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-2*(-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-4*B*a*b)/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3-1/2*(2*A*a^2-A*a*b+2*A*b^2+B*a^2-4*B*a*b)/(a+b)/(a^2-
2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b+a+b)^2-(3*A*a*b-B*a^2-2*B*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)
*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(165) = 330.

Time = 0.13 (sec) , antiderivative size = 740, normalized size of antiderivative = 4.11

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
[-1/4*((B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4
)*cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*s
qrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*A*a^5 - 3*B*a^4*b - A*a^3*b
^2 + 3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 +
4*B*a*b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3
*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((B*a^
4 - 3*A*a^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4)*cos(d*x +
c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*A*a^5 -
 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^
3*b^2 + A*a^2*b^3 + 4*B*a*b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(165) = 330$.

Time = 0.23 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.17

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{\frac{(Ba^2-3\,Aab+2\,Bb^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gia
c")
```

output
```
((B*a^2 - 3*A*a*b + 2*B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2
*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*A*a^3*tan(1/2*d*x +
1/2*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
 3*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan
(1/2*d*x + 1/2*c) + B*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2
*c) - 3*B*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan(1/2*d*x + 1/2*c) - 4*B*
a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b
^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2)
)/d
```

Mupad [B] (verification not implemented)

Time = 26.83 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.38

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,A\,a^2+2\,A\,b^2-B\,a^2+A\,a\,b-4\,B\,a\,b\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,a^2+2\,A\,b^2+B\,a^2-A\,a\,b-4\,B\,a\,b\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)} + \frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\,a\,b+b^2\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right)\,\left(B\,a^2-3\,A\,a\,b+2\,B\,b^2\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(2*A*a^2 + 2*A*b^2 - B*a^2 + A*a*b - 4*B*a*b))/((a
+ b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A*a*b -
 4*B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2
*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))
+ (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1
/2)*(a - b)^(5/2)))*(B*a^2 + 2*B*b^2 - 3*A*a*b))/(d*(a + b)^(5/2)*(a - b)^
(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.07

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos\left(dx+c\right)b^2-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)ab}{d\left(\cos\left(dx+c\right)a^4b-2\cos\left(dx+c\right)a^2b^3+\cos\left(dx+c\right)b^5+a^5-2a^3b^2+\right.}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output

```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*cos(c + d*x)*b**2 - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x
)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a*b + sin(c + d*x)*a**3 -
sin(c + d*x)*a*b**2)/(d*(cos(c + d*x)*a**4*b - 2*cos(c + d*x)*a**2*b**3 +
cos(c + d*x)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.269 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{(2a^2A+Ab^2-3abB)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(Ab-aB)\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\sin(c+dx)}{2(a^2-b^2)^2\,d(a+b\cos(c+dx))}$$

output
```
(2*A*a^2+A*b^2-3*B*a*b)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))
/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d
*x+c))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x
+c))
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{-\frac{2(2a^2A+Ab^2-3abB)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+\frac{(-3aAb+a^2B+2b^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}}{2d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

output
```
((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(
a + b)*(a + b*Cos[c + d*x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sin[c + d*x
])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3233, 25, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{2(aA-bB)-(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{2(aA-bB)+(aB-Ab)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3233$$

$$\frac{-\frac{\int -\frac{2Aa^2-3bBa+Ab^2}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{(a^2(-B)+3aAb-2b^2B)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{2Aa^2-3bBa+Ab^2}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{(a^2(-B)+3aAb-2b^2B)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(2a^2A-3abB+Ab^2)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{(a^2(-B)+3aAb-2b^2B)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(2a^2A-3abB+Ab^2)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{(a^2(-B)+3aAb-2b^2B)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3138$$

$$\frac{\frac{2(2a^2A-3abB+Ab^2)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{(a^2(-B)+3aAb-2b^2B)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2(2a^2A-3abB+Ab^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{(a^2(-B)+3aAb-2b^2B)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)}{2d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*((A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (
(2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*d) - ((3*a*A*b - a^2*B - 2*b
^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{-\frac{\left(4Aab+A\,b^{2}-2a^{2}B-Bab-2B\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{(a-b)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(4Aab-A\,b^{2}-2a^{2}B+Bab-2B\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^{2}-2ab+b^{2}\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{\left(2a^{2}A+A\,b^{2}-3Bab\right)\operatorname{arcta}}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right.}}{d}$
default	$\frac{\frac{-\frac{\left(4Aab+A\,b^{2}-2a^{2}B-Bab-2B\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{(a-b)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(4Aab-A\,b^{2}-2a^{2}B+Bab-2B\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^{2}-2ab+b^{2}\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{\left(2a^{2}A+A\,b^{2}-3Bab\right)\operatorname{arcta}}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right.}}{d}$
risch	$\frac{i\left(-2A\,a^{2}b^{2}e^{3i(dx+c)}-A\,b^{4}e^{3i(dx+c)}+3Ba\,b^{3}e^{3i(dx+c)}-6A\,a^{3}b\,e^{2i(dx+c)}-3Aa\,b^{3}e^{2i(dx+c)}+2B\,a^{4}e^{2i(dx+c)}+5B\,a^{2}b^{2}\right.}{b\left(a^{2}-b^{2}\right)^{2}d\left(b\,e^{2i(dx\right.}}$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3-1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2
*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2+(2*A*a^2+A*b^2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(150) = 300$.

Time = 0.13 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.52

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3} dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*((2*A*a^4 - 3*B*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4
)*cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*s
qrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5 - 4*A*a^4*b - B*a^3*b
^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 +
3*A*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3
*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((2*A*
a^4 - 3*B*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*cos(d*x +
c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*B*a^5 -
 4*A*a^4*b - B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^
3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3} dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(150) = 300.

Time = 0.18 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.38

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$
$$= \frac{\frac{(2\,Aa^2 - 3\,Bab + Ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a - 2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^4 - 2\,a^2b^2 + b^4)\sqrt{a^2 - b^2}} + \frac{2\,Ba^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 4\,Aa^2b\tan\left(\frac{1}{2}\,dx + \right.}{}}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
((2*A*a^2 - 3*B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2
*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*B*a^3*tan(1/2*d*x +
1/2*c)^3 - 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)
^3 + 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A
*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan
(1/2*d*x + 1/2*c) - 4*A*a^2*b*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x +
 1/2*c) - 3*A*a*b^2*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) +
A*b^3*tan(1/2*d*x + 1/2*c) + 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b
^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2)
)/d
```

Mupad [B] (verification not implemented)

Time = 26.75 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.51

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2Ba^2-Ab^2+2Bb^2-4Aab+Bab)}{(a+b)^2(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(Ab^2+2Ba^2+2Bb^2-4Aab-Bab)}{(a+b)(a^2-2ab+b^2)}}{d\left(2ab+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-2b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-2ab+b^2)+a^2+b^2\right)} + \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)(a^2-2ab+b^2)}{2\sqrt{a+b}\,(a-b)^{5/2}}\right)(2Aa^2-3Bab+Ab^2)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(2*B*a^2 - A*b^2 + 2*B*b^2 - 4*A*a*b + B*a*b))/((a
+ b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 4*A*a*b
 - B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2
*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))
+ (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1
/2)*(a - b)^(5/2)))*(2*A*a^2 + A*b^2 - 3*B*a*b))/(d*(a + b)^(5/2)*(a - b)^
(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.17

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,ab+2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)a^2 -}{d\left(\cos(dx+c)\,a^4b-2\cos(dx+c)\,a^2b^3+\cos(dx+c)\,b^5+a^5-2a^3b^2+a\right.}$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*cos(c + d*x)*a*b + 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)
*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**2 - sin(c + d*x)*a**2*b + s
in(c + d*x)*b**3)/(d*(cos(c + d*x)*a**4*b - 2*cos(c + d*x)*a**2*b**3 + cos
(c + d*x)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.270 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 214

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx &= -\frac{(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} \\ &\quad+\frac{A\text{arctanh}(\sin(c+dx))}{a^3d}+\frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} \\ &\quad+\frac{b(5a^2Ab-2Ab^3-3a^3B)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}\end{aligned}$$

output

```
-(6*A*a^4*b-5*A*a^2*b^3+2*A*b^5-2*B*a^5-B*a^3*b^2)*arctan((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+A*arctanh(sin(d*
x+c))/a^3/d+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/
2*b*(5*A*a^2*b-2*A*b^3-3*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+
c))
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.26

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{\cos(c+dx)(B+A\sec(c+dx))\left(-\frac{2(-6a^4Ab+5a^2Ab^3-2Ab^5+2a^5B+a^3b^2B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 2A\log\left(\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Cos[c + d*x]*(B + A*Sec[c + d*x])*((-2*(-6*a^4*A*b + 5*a^2*A*b^3 - 2*A*b^
5 + 2*a^5*B + a^3*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^
2]])/(-a^2 + b^2)^(5/2) - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2
*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*(A*b - a*B)*Sin[c + d
*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b*(5*a^2*A*b - 2*A*b^3
- 3*a^3*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]))))/(2*a
^3*d*(A + B*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 3479, 3042, 3534, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{\left(b(Ab-aB)\cos^2(c+dx)-2a(Ab-aB)\cos(c+dx)+2A\left(a^2-b^2\right)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2A\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(2A\left(a^2-b^2\right)^2-a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{b\left(-3a^3B+5a^2Ab-2Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2A\left(a^2-b^2\right)^2-a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b\left(-3a^3B+5a^2Ab-2Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{\frac{2A\left(a^2-b^2\right)^2\int \sec(c+dx)dx}{a}-\frac{\left(-2a^5B+6a^4Ab-a^3b^2B-5a^2Ab^3+2Ab^5\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b\left(-3a^3B+5a^2Ab-2Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2A\left(a^2-b^2\right)^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\left(-2a^5B+6a^4Ab-a^3b^2B-5a^2Ab^3+2Ab^5\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{b\left(-3a^3B+5a^2Ab-2Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{2A\left(a^2-b^2\right)^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2\left(-2a^5B+6a^4Ab-a^3b^2B-5a^2Ab^3+2Ab^5\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a(a^2-b^2)}+\frac{b(-3a^3B+5a^2Ab-2Ab^3)}{ad(a^2-b^2)(a+b\cos(c}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{2A\left(a^2-b^2\right)^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2\left(-2a^5B+6a^4Ab-a^3b^2B-5a^2Ab^3+2Ab^5\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}+\frac{b(-3a^3B+5a^2Ab-2Ab^3)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 4257

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{b(-3a^3B+5a^2Ab-2Ab^3)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{2A\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{ad}-\frac{2\left(-2a^5B+6a^4Ab-a^3b^2B-5a^2Ab^3+2Ab^5\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) +
(((-2*(6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcTan[(Sq
rt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) +
(2*A*(a^2 - b^2)^2*ArcTanh[Sin[c + d*x]])/(a*d))/(a*(a^2 - b^2)) + (b*(5*a
^2*A*b - 2*A*b^3 - 3*a^3*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.41

method	result
derivativedivides	$-\frac{-\frac{2\left(\frac{-\frac{\left(6A\,a^2b+Aa\,b^2-2A\,b^3-4a^3B-B\,a^2b\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{ba\left(6A\,a^2b-Aa\,b^2-2A\,b^3-4a^3B+B\,a^2b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a-b)^2}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\left(6A\,a^4\cdots\right.}{\cdots}\right)}{a^3}\cdots}{d}$
default	$-\frac{-\frac{2\left(\frac{-\frac{\left(6A\,a^2b+Aa\,b^2-2A\,b^3-4a^3B-B\,a^2b\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{ba\left(6A\,a^2b-Aa\,b^2-2A\,b^3-4a^3B+B\,a^2b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a-b)^2}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\left(6A\,a^4\cdots\right.}{\cdots}\right)}{a^3}\cdots}{d}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-2/a^3*((-1/2*(6*A*a^2*b+A*a*b^2-2*A*b^3-4*B*a^3-B*a^2*b)*a*b/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*a*(6*A*a^2*b-A*a*b^2-2*A*b^3-4*B
*a^3+B*a^2*b)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^2+1/2*(6*A*a^4*b-5*A*a^2*b^3+2*A*b^5-2*B*a^5-B*a
^3*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1
/2*c)/((a-b)*(a+b))^(1/2)))-A/a^3*ln(tan(1/2*d*x+1/2*c)-1)+A/a^3*ln(tan(1/
2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(199) = 398$.

Time = 11.39 (sec) , antiderivative size = 1400, normalized size of antiderivative = 6.54

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
[1/4*((2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*
a^5*b^2 - 6*A*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2
+ 2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*
x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(A*a^8 - 3*A*a^6*b^
2 + 3*A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b
^8)*cos(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos
(d*x + c))*log(sin(d*x + c) + 1) - 2*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 -
A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c)^2
 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(-si
n(d*x + c) + 1) - 2*(4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 +
 B*a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*
a^3*b^5 - 2*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3
*a^5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
 a^4*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), 1/
2*((2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*a^5
*b^2 - 6*A*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 + 2
*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(...
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(199) = 398.

Time = 0.25 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.25

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$
$$= \frac{\frac{(2\,Ba^5 - 6\,Aa^4b + Ba^3b^2 + 5\,Aa^2b^3 - 2\,Ab^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a - 2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^7 - 2\,a^5b^2 + a^3b^4)\sqrt{a^2 - b^2}} + \frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right.\right.}{a^3}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="gia
c")
```

output
```
((2*B*a^5 - 6*A*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2
 - b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - A*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (4*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^3*b^2*tan
(1/2*d*x + 1/2*c)^3 - 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*b^3*tan
(1/2*d*x + 1/2*c)^3 - B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*A*a*b^4*tan(1/2
*d*x + 1/2*c)^3 - 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^4*b*tan(1/2*d*x +
 1/2*c) - 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*B*a^3*b^2*tan(1/2*d*x + 1/2
*c) - 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - B*a^2*b^3*tan(1/2*d*x + 1/2*c) +
3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a
^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 31.37 (sec) , antiderivative size = 6913, normalized size of antiderivative = 32.30

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(2*A*b^4 - 6*A*a^2*b^2 + B*a^2*b^2 - A*a*b^3 + 4*B*
a^3*b))/((a^2*b - a^3)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(2*A*b^4 - 6*A*a^2
*b^2 - B*a^2*b^2 + A*a*b^3 + 4*B*a^3*b))/((a + b)*(a^4 - 2*a^3*b + a^2*b^2
)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^
4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (A*atan(((A*((8*tan(c/2 + (d*x)/2)*(
4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*
a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^
4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B
*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 -
a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (A*((
8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^
9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*
a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^
14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10
*b^3 - 3*a^11*b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^
7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4
- 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a
^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3)*1i)/a^3 + (A*((8*tan(c
/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*
a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.65

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

output
```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*cos(c + d*x)*a**2*b**2 + 2*sqrt(a**2 - b**2)*atan((tan((c
+ d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*b**4 - 4
*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**
2 - b**2))*a**3*b + 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c
+ d*x)/2)*b)/sqrt(a**2 - b**2))*a*b**3 - cos(c + d*x)*log(tan((c + d*x)/2)
 - 1)*a**4*b + 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**2*b**3 - cos(c
+ d*x)*log(tan((c + d*x)/2) - 1)*b**5 + cos(c + d*x)*log(tan((c + d*x)/2)
+ 1)*a**4*b - 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**2*b**3 + cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*b**5 - log(tan((c + d*x)/2) - 1)*a**5 + 2*
log(tan((c + d*x)/2) - 1)*a**3*b**2 - log(tan((c + d*x)/2) - 1)*a*b**4 + l
og(tan((c + d*x)/2) + 1)*a**5 - 2*log(tan((c + d*x)/2) + 1)*a**3*b**2 + lo
g(tan((c + d*x)/2) + 1)*a*b**4 + sin(c + d*x)*a**3*b**2 - sin(c + d*x)*a*b
**4)/(a**2*d*(cos(c + d*x)*a**4*b - 2*cos(c + d*x)*a**2*b**3 + cos(c + d*x
)*b**5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.271 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 299

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx &= \frac{b(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d}\\ &\quad-\frac{(3Ab-aB)\text{arctanh}(\sin(c+dx))}{a^4d}\\ &\quad+\frac{(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B)\tan(c+dx)}{2a^3(a^2-b^2)^2d}\\ &\quad+\frac{b(Ab-aB)\tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2}+\frac{b(6a^2Ab-3Ab^3-4a^3B+ab^2B)\tan(c+dx)}{2a^2(a^2-b^2)^2d(a+b\cos(c+dx))}\end{aligned}$$

output
```
b*(12*A*a^4*b-15*A*a^2*b^3+6*A*b^5-6*B*a^5+5*B*a^3*b^2-2*B*a*b^4)*arctan((
a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d-(
3*A*b-B*a)*arctanh(sin(d*x+c))/a^4/d+1/2*(2*A*a^4-11*A*a^2*b^2+6*A*b^4+5*B
*a^3*b-2*B*a*b^3)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*tan(d*x+c)/
a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b*(6*A*a^2*b-3*A*b^3-4*B*a^3+B*a*b^2)
*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 7.77 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{b(-12a^4Ab+15a^2Ab^3-6Ab^5+6a^5B-5a^3b^2B+2ab^4B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^4(a^2-b^2)^2\sqrt{-a^2+b^2}d}$$
$$+\frac{(3Ab-aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^4d}$$
$$+\frac{(-3Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^4d}$$
$$+\frac{A\sin\left(\frac{1}{2}(c+dx)\right)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$+\frac{A\sin\left(\frac{1}{2}(c+dx)\right)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{-Ab^3\sin(c+dx)+ab^2B\sin(c+dx)}{2a^2(a-b)(a+b)d(a+b\cos(c+dx))^2}$$
$$+\frac{-7a^2Ab^3\sin(c+dx)+4Ab^5\sin(c+dx)+5a^3b^2B\sin(c+dx)-2ab^4B\sin(c+dx)}{2a^3(a-b)^2(a+b)^2d(a+b\cos(c+dx))}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b*(-12*a^4*A*b + 15*a^2*A*b^3 - 6*A*b^5 + 6*a^5*B - 5*a^3*b^2*B + 2*a*b^4
*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^4*(a^2 - b^2)
^2*Sqrt[-a^2 + b^2]*d) + ((3*A*b - a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]])/(a^4*d) + ((-3*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)/(a^4*d) + (A*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])) + (A*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
+ (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x])/(2*a^2*(a - b)*(a + b)*d*
(a + b*Cos[c + d*x])^2) + (-7*a^2*A*b^3*Sin[c + d*x] + 4*A*b^5*Sin[c + d*x
] + 5*a^3*b^2*B*Sin[c + d*x] - 2*a*b^4*B*Sin[c + d*x])/(2*a^3*(a - b)^2*(a
 + b)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {3042, 3479, 3042, 3534, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{\left(2Aa^2+bBa-2(Ab-aB)\cos(c+dx)a-3Ab^2+2b(Ab-aB)\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2Aa^2+bBa-2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-3Ab^2+2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(2Aa^4+5bBa^3-11Ab^2a^2-2b^3Ba-\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)a+6Ab^4+b\left(-4Ba^3+6Aba^2+b^2Ba-3Ab^3\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b(-4}{}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2Aa^4+5bBa^3-11Ab^2a^2-2b^3Ba-\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+6Ab^4+b\left(-4Ba^3+6Aba^2+b^2Ba-3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b(-4a^3B}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{\int -\frac{\frac{\left(2\left(a^2-b^2\right)^2(3Ab-aB)-ab\left(-4Ba^3+6Aba^2+b^2Ba-3Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}}{a}dx+\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}}{a(a^2-b^2)}+\frac{b(-4a^3B}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}-\frac{\int\frac{\left(2\left(a^2-b^2\right)^2(3Ab-aB)-ab\left(-4Ba^3+6Aba^2+b^2Ba-3Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b(-4a^3B+}{ad}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}-\frac{\int\frac{2\left(a^2-b^2\right)^2(3Ab-aB)-ab\left(-4Ba^3+6Aba^2+b^2Ba-3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a(a^2-b^2)}+\frac{b(-4a^3B+6a^2Ab+}{ad(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3480$$

$$\frac{\frac{\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}-\frac{\frac{2\left(a^2-b^2\right)^2(3Ab-aB)\int\sec(c+dx)dx}{a}-\frac{b\left(-6a^5B+12a^4Ab+5a^3b^2B-15a^2Ab^3-2ab^4B+6Ab^5\right)\int\frac{}{a+b\cos}}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}-\frac{2\left(a^2-b^2\right)^2(3Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\left(-6a^5B+12a^4Ab+5a^3b^2B-15a^2Ab^3-2ab^4B+6Ab^5\right)\int\frac{\quad}{a+\cdots}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\big\downarrow$ 3138

$$\frac{\frac{\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}-\frac{2\left(a^2-b^2\right)^2(3Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\frac{2b\left(-6a^5B+12a^4Ab+5a^3b^2B-15a^2Ab^3-2ab^4B+6Ab^5\right)\int\frac{\quad}{(\cdots}}{ad}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}-\frac{2\left(a^2-b^2\right)^2(3Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\frac{2b\left(-6a^5B+12a^4Ab+5a^3b^2B-15a^2Ab^3-2ab^4B+6Ab^5\right)\operatorname{arct}\cdots}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\big\downarrow$ 4257

$$\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}+$$

$$\frac{\frac{b\left(-4a^3B+6a^2Ab+ab^2B-3Ab^3\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\tan(c+dx)}{ad}-\frac{\frac{2\left(a^2-b^2\right)^2(3Ab-aB)\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{2b(\cdots}{\cdots}}{a(a^2-b^2)}}{\cdots}}{2a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) +
((b*(6*a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Tan[c + d*x])/(a*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])) + (-(((-2*b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5
- 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (2*(a^2 - b^2)^2*(3*A*b - a
*B)*ArcTanh[Sin[c + d*x]])/(a*d))/a) + ((2*a^4*A - 11*a^2*A*b^2 + 6*A*b^4
+ 5*a^3*b*B - 2*a*b^3*B)*Tan[c + d*x])/(a*d))/(a*(a^2 - b^2)))/(2*a*(a^2 -
 b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{A}{a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-3Ab+Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^4}-\frac{A}{a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(3Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^4}+\frac{2b\left(-\frac{(8Aa\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{A}{a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-3Ab+Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^4}-\frac{A}{a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(3Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^4}+\frac{2b\left(-\frac{(8Aa\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-A/a^3/(tan(1/2*d*x+1/2*c)+1)+1/a^4*(-3*A*b+B*a)*ln(tan(1/2*d*x+1/2*c
)+1)-A/a^3/(tan(1/2*d*x+1/2*c)-1)+(3*A*b-B*a)/a^4*ln(tan(1/2*d*x+1/2*c)-1)
+2*b/a^4*((-1/2*(8*A*a^2*b+A*a*b^2-4*A*b^3-6*B*a^3-B*a^2*b+2*B*a*b^2)*a*b/
(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*a*(8*A*a^2*b-A*a*b^2-4*A*
b^3-6*B*a^3+B*a^2*b+2*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2+1/2*(12*A*a^4*b-15*A*a^2*b^3+
6*A*b^5-6*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(
1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(284) = 568.

Time = 24.39 (sec) , antiderivative size = 2100, normalized size of antiderivative = 7.02

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
[1/4*(((6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b^6 + 2*B*a*b^
7 - 6*A*b^8)*cos(d*x + c)^3 + 2*(6*B*a^6*b^2 - 12*A*a^5*b^3 - 5*B*a^4*b^4
+ 15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c)^2 + (6*B*a^7*b - 12
*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d
*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*((B*a^7*b^2 - 3*A*
a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8
+ 3*A*b^9)*cos(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a
^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2
 + (B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*
b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*((B
*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2
*b^7 - B*a*b^8 + 3*A*b^9)*cos(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*
a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)
*cos(d*x + c)^2 + (B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5
*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*cos(d*x + c))*log(-sin(d*x +
 c) + 1) + 2*(2*A*a^9 - 6*A*a^7*b^2 + 6*A*a^5*b^4 - 2*A*a^3*b^6 + (2*A*a^7
*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 - 7*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b
^7 - 6*A*a*b^8)*cos(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^...
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(284) = 568$.

Time = 0.24 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.92

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
((6*B*a^5*b - 12*A*a^4*b^2 - 5*B*a^3*b^3 + 15*A*a^2*b^4 + 2*B*a*b^5 - 6*A*
b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b
^2 + a^4*b^4)*sqrt(a^2 - b^2)) + (6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*A
*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*A
*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*A
*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^6
*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b^3*t
an(1/2*d*x + 1/2*c) + 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 7*A*a^2*b^4*tan(1
/2*d*x + 1/2*c) - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*A*a*b^5*tan(1/2*d*x
 + 1/2*c) - 2*B*a*b^5*tan(1/2*d*x + 1/2*c) + 4*A*b^6*tan(1/2*d*x + 1/2*c))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x +
1/2*c)^2 + a + b)^2) + (B*a - 3*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^
4 - (B*a - 3*A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*A*tan(1/2*d*x
 + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 34.52 (sec) , antiderivative size = 9312, normalized size of antiderivative = 31.14

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(6*A*b^5 - 2*A*a^5 - 12*A*a^2*b^3 + 4*A*a^3*b^2 + B
*a^2*b^3 + 6*B*a^3*b^2 - 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a^3*b - a^4
)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*A*a^5 + 6*A*b^5 - 12*A*a^2*b^3 - 4*A
*a^3*b^2 - B*a^2*b^3 + 6*B*a^3*b^2 + 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/(
(a + b)*(a^5 - 2*a^4*b + a^3*b^2)) - (2*tan(c/2 + (d*x)/2)^3*(2*A*a^6 - 6*
A*b^6 + 13*A*a^2*b^4 - 6*A*a^4*b^2 - 5*B*a^3*b^3 + 2*B*a*b^5))/(a*(a^2*b -
 a^3)*(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a*b - a^2 +
3*b^2) - tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 +
(d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) + (atan(((((8*tan(c/2 + (d*x)/2)*(72*A^
2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 28
8*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*
A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^
2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6
- 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*
A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a
^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8
*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b
^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (((8*(4*B*a^18 + 1
2*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6
 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.27

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

output
```
(6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*cos(c + d*x)*a**3*b**2 - 4*sqrt(a**2 - b**2)*atan((tan((c + d
*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a*b**4 - 6*
sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2
 - b**2))*sin(c + d*x)**2*a**2*b**3 + 4*sqrt(a**2 - b**2)*atan((tan((c + d
*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*b**5 + 6
*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**
2 - b**2))*a**2*b**3 - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan(
(c + d*x)/2)*b)/sqrt(a**2 - b**2))*b**5 + 2*cos(c + d*x)*log(tan((c + d*x)
/2) - 1)*a**5*b - 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**3*b**3 + 2*c
os(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**5 - 2*cos(c + d*x)*log(tan((c +
 d*x)/2) + 1)*a**5*b + 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**3*b**3
- 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**5 + cos(c + d*x)*sin(c + d
*x)*a**5*b - 3*cos(c + d*x)*sin(c + d*x)*a**3*b**3 + 2*cos(c + d*x)*sin(c
+ d*x)*a*b**5 - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4*b**2 + 4*
log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**4 - 2*log(tan((c + d*x)/
2) - 1)*sin(c + d*x)**2*b**6 + 2*log(tan((c + d*x)/2) - 1)*a**4*b**2 - 4*l
og(tan((c + d*x)/2) - 1)*a**2*b**4 + 2*log(tan((c + d*x)/2) - 1)*b**6 + 2*
log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4*b**2 - 4*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2*a**2*b**4 + 2*log(tan((c + d*x)/2) + 1)*sin(c +...
```

3.272 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 402

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{b^2(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} + \frac{(a^2A+12Ab^2-6abB)\operatorname{arctanh}(\sin(c+dx))}{2a^5d} - \frac{(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B)\tan(c+dx)}{2a^4(a^2-b^2)^2d} + \frac{(a^4A-10a^2Ab^2+6Ab^4+6a^3bB-3ab^3B)\sec(c+dx)\tan(c+dx)}{2a^3(a^2-b^2)^2d} + \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{b(7a^2Ab-4Ab^3-5a^3B+2ab^2B)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)^2d(a+b\cos(c+dx))}$$

output
```
-b^2*(20*A*a^4*b-29*A*a^2*b^3+12*A*b^5-12*B*a^5+15*B*a^3*b^2-6*B*a*b^4)*ar
ctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/
2)/d+1/2*(A*a^2+12*A*b^2-6*B*a*b)*arctanh(sin(d*x+c))/a^5/d-1/2*(6*A*a^4*b
-21*A*a^2*b^3+12*A*b^5-2*B*a^5+11*B*a^3*b^2-6*B*a*b^4)*tan(d*x+c)/a^4/(a^2
-b^2)^2/d+1/2*(A*a^4-10*A*a^2*b^2+6*A*b^4+6*B*a^3*b-3*B*a*b^3)*sec(d*x+c)*
tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a^2-
b^2)/d/(a+b*cos(d*x+c))^2+1/2*b*(7*A*a^2*b-4*A*b^3-5*B*a^3+2*B*a*b^2)*sec(
d*x+c)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.26

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{16b^2(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 8(a^2A+12Ab^2-6abB)\log\left(\cos\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((16*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B -
 6*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 +
b^2)^(5/2) - 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + (2*a*(4*a^7*A - 30*a^5*A*b^2 + 68*a^3*A*b^4 - 36*a*A*b^6 + 8*
a^6*b*B - 32*a^4*b^3*B + 18*a^2*b^5*B + (-16*a^6*A*b + 14*a^4*A*b^3 + 47*a
^2*A*b^5 - 36*A*b^7 + 8*a^7*B - 10*a^5*b^2*B - 25*a^3*b^4*B + 18*a*b^6*B)*
Cos[c + d*x] + 2*a*b*(-11*a^4*A*b + 32*a^2*A*b^3 - 18*A*b^5 + 4*a^5*B - 16
*a^3*b^2*B + 9*a*b^4*B)*Cos[2*(c + d*x)] - 6*a^4*A*b^3*Cos[3*(c + d*x)] +
21*a^2*A*b^5*Cos[3*(c + d*x)] - 12*A*b^7*Cos[3*(c + d*x)] + 2*a^5*b^2*B*Co
s[3*(c + d*x)] - 11*a^3*b^4*B*Cos[3*(c + d*x)] + 6*a*b^6*B*Cos[3*(c + d*x)
])*Sec[c + d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*
a^5*d)
```

Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {3042, 3479, 3042, 3534, 3042, 3534, 27, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{\left(3b(Ab-aB)\cos^2(c+dx)-2a(Ab-aB)\cos(c+dx)+2\left(Aa^2+bBa-2Ab^2\right)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(Aa^2+bBa-2Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(2b\left(-5Ba^3+7Aba^2+2b^2Ba-4Ab^3\right)\cos^2(c+dx)-a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)+2\left(Aa^4+6bBa^3-10Ab^2a^2-3b^3Ba+6Ab^4\right)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2b\left(-5Ba^3+7Aba^2+2b^2Ba-4Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(Aa^4+6bBa^3-10Ab^2a^2-3b^3Ba+6Ab^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b(-5}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3534

$$\frac{\frac{\int -\frac{\frac{2\left(-2Ba^5+6Aba^4+11b^2Ba^3-21Ab^3a^2-6b^4Ba-\left(Aa^4-4bBa^3+4Ab^2a^2+b^3Ba-2Ab^4\right)\cos(c+dx)a+12Ab^5-b\left(Aa^4+6bBa^3-10Ab^2a^2-3b^3Ba+6Ab^4\right)\cos^2(}{a+b\cos(c+dx)}}{2a}}{a(a^2-b^2)}}{2a\left(a^2-\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\int \frac{\left(-2Ba^5+6Aba^4+11b^2Ba^3-21Ab^3a^2-6b^4Ba-\left(Aa^4-4bBa^3+4Ab^2a^2+b^3Ba-2Ab^4\right)\cos(c+\right.}{a+b\cos(c+dx}}{a}}{a(a^2-b^2)}}{2a\left(a^2-\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\int \frac{-2Ba^5+6Aba^4+11b^2Ba^3-21Ab^3a^2-6b^4Ba-\left(Aa^4-4bBa^3+4Ab^2a^2+b^3Ba-2Ab^4\right)\sin\left(c+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c\right.\right.}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3534

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\int -\frac{\left(\left(Aa^2-6bBa+12Ab^2\right)\left(a^2-b^2\right)^2+ab\left(Aa^4+6bBa^3-10Ab^2a^2-3b^3Ba+6Ab^4\right)\cos(c+dx)\right)}{a+b\cos(c+dx)}}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\left(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5\right)\tan(c+dx)}{ad}-\frac{\int\frac{\left(\left(Aa^2-6bBa+12Ab^2\right)\left(\cdots\right.\right.}{\cdots}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\cdots\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\left(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5\right)\tan(c+dx)}{ad}-\frac{\int\frac{\left(Aa^2-6bBa+12Ab^2\right)\left(a\cdots\right.}{\cdots}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3480

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\left(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5\right)\tan(c+dx)}{ad}-\frac{\left(a^2-b^2\right)^2\left(a^2A-6abB+1\cdots\right.}{a}\cdots}{a(a^2-b^2)}}{2a\cdots}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\left(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5\right)\tan(c+dx)}{ad}-\frac{\left(a^2-b^2\right)^2\left(a^2A-6abB+1\cdots\right.}{\cdots}\cdots}{a(a^2-b^2)}}{\cdots}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\left(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5\right)\tan(c+dx)}{ad}-\frac{\left(a^2-b^2\right)^2\left(a^2A-6abB+1\cdots\right.}{\cdots}\cdots}{a(a^2-b^2)}}{\cdots}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\left(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5\right)\tan(c+dx)}{ad}-\frac{\left(a^2-b^2\right)^2\left(a^2A-6abB+1\right.}{}}{a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 4257

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{b(-5a^3B+7a^2Ab+2ab^2B-4Ab^3)\tan(c+dx)\sec(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\left(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\left(-2a^5B+6a^4Ab+11a^3b^2B-\right.}{}}{}}{}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^2) + ((b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)*Sec[c + d*x]*
Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + (((a^4*A - 10*a^2*A
*b^2 + 6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(a*d) -
 (-(((-2*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2
*B - 6*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqr
t[a - b]*Sqrt[a + b]*d) + ((a^2 - b^2)^2*(a^2*A + 12*A*b^2 - 6*a*b*B)*ArcT
anh[Sin[c + d*x]])/(a*d))/a) + ((6*a^4*A*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a
^5*B + 11*a^3*b^2*B - 6*a*b^4*B)*Tan[c + d*x])/(a*d))/a)/(a*(a^2 - b^2)))/
(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{A}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-Aa-6Ab+2Ba}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+12A\,b^2-6Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^5}-\frac{2b^2\left(-\frac{\left(10A\,a^2b+Aa\,b^2-6A\,b^3 \cdots\right.}{2(\cdots}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{A}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-Aa-6Ab+2Ba}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2A+12A\,b^2-6Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^5}-\frac{2b^2\left(-\frac{\left(10A\,a^2b+Aa\,b^2-6A\,b^3 \cdots\right.}{2(\cdots}\cdots\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-1/2*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-6*A*b+2*B*a)/a^4/(tan(1
/2*d*x+1/2*c)+1)+1/2*(A*a^2+12*A*b^2-6*B*a*b)/a^5*ln(tan(1/2*d*x+1/2*c)+1)
-2*b^2/a^5*((-1/2*(10*A*a^2*b+A*a*b^2-6*A*b^3-8*B*a^3-B*a^2*b+4*B*a*b^2)*a
*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*a*(10*A*a^2*b-A*a*b^2-
6*A*b^3-8*B*a^3+B*a^2*b+4*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2+1/2*(20*A*a^4*b-29*A*a^2*
b^3+12*A*b^5-12*B*a^5+15*B*a^3*b^2-6*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(
a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+1/2*A/a^
3/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-A*a-6*A*b+2*B*a)/a^4/(tan(1/2*d*x+1/2*c)-
1)+1/2/a^5*(-A*a^2-12*A*b^2+6*B*a*b)*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(382) = 764$.

Time = 37.32 (sec) , antiderivative size = 2416, normalized size of antiderivative = 6.01

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
[1/4*(((12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 + 6*B*a*
b^8 - 12*A*b^9)*cos(d*x + c)^4 + 2*(12*B*a^6*b^3 - 20*A*a^5*b^4 - 15*B*a^4
*b^5 + 29*A*a^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*cos(d*x + c)^3 + (12*B*a^7
*b^2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*a^4*b^5 + 6*B*a^3*b^6 - 12*A*a^2
*b^7)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + ((A*a^8
*b^2 - 6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*
b^7 + 35*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b -
6*B*a^8*b^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 3
5*A*a^3*b^7 + 6*B*a^2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9
*b + 9*A*a^8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b
^6 + 6*B*a^3*b^7 - 12*A*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - (
(A*a^8*b^2 - 6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*
B*a^3*b^7 + 35*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^
9*b - 6*B*a^8*b^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b
^6 + 35*A*a^3*b^7 + 6*B*a^2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6
*B*a^9*b + 9*A*a^8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A
*a^4*b^6 + 6*B*a^3*b^7 - 12*A*a^2*b^8)*cos(d*x + c)^2)*log(-sin(d*x + c) +
 1) + 2*(A*a^10 - 3*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6 + (2*B*a^8*b^2 ...
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec^3{(c+dx)}}{(a+b\cos{(c+dx)})^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(382) = 764$.

Time = 0.23 (sec) , antiderivative size = 1395, normalized size of antiderivative = 3.47

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/2*(2*(12*B*a^5*b^2 - 20*A*a^4*b^3 - 15*B*a^3*b^4 + 29*A*a^2*b^5 + 6*B*a
*b^6 - 12*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arcta
n(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a
^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) - 2*(A*a^7*tan(1/2*d*x + 1/2*c)
^7 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4
*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*
B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16
*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 +
9*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 +
 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 -
6*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*tan(1/2*d*x + 1/2*c)^7 + 3*A*a
^7*tan(1/2*d*x + 1/2*c)^5 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^6*b*tan
(1/2*d*x + 1/2*c)^5 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 5*A*a^5*b^2*tan(1
/2*d*x + 1/2*c)^5 + 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 26*A*a^4*b^3*tan
(1/2*d*x + 1/2*c)^5 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*A*a^3*b^4*t
an(1/2*d*x + 1/2*c)^5 - 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*A*a^2*b^5
*tan(1/2*d*x + 1/2*c)^5 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^6*
tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^7*tan(
1/2*d*x + 1/2*c)^5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x
+ 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*tan(1/2*d*x +...
```

Mupad [B] (verification not implemented)

Time = 33.35 (sec) , antiderivative size = 10547, normalized size of antiderivative = 26.24

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(3*A*a^7 + 36*A*b^7 + 2*B*a^7 - 67*A*a^2*b^5 - 29*A
*a^3*b^4 + 26*A*a^4*b^3 + 5*A*a^5*b^2 - 9*B*a^2*b^5 + 35*B*a^3*b^4 + 16*B*
a^4*b^3 - 10*B*a^5*b^2 + 18*A*a*b^6 - 4*A*a^6*b - 18*B*a*b^6 - 4*B*a^6*b))
/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) + (tan(c/2 + (d*x)/2)^5*(3*A*a^7 -
36*A*b^7 - 2*B*a^7 + 67*A*a^2*b^5 - 29*A*a^3*b^4 - 26*A*a^4*b^3 + 5*A*a^5*
b^2 - 9*B*a^2*b^5 - 35*B*a^3*b^4 + 16*B*a^4*b^3 + 10*B*a^5*b^2 + 18*A*a*b^
6 + 4*A*a^6*b + 18*B*a*b^6 - 4*B*a^6*b))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b
^2)) - (tan(c/2 + (d*x)/2)^7*(A*a^6 - 12*A*b^6 - 2*B*a^6 + 23*A*a^2*b^4 -
10*A*a^3*b^3 - 8*A*a^4*b^2 - 3*B*a^2*b^4 - 12*B*a^3*b^3 + 4*B*a^4*b^2 + 6*
A*a*b^5 + 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/((a^4*b - a^5)*(a + b)^2) +
(tan(c/2 + (d*x)/2)*(A*a^6 - 12*A*b^6 + 2*B*a^6 + 23*A*a^2*b^4 + 10*A*a^3*
b^3 - 8*A*a^4*b^2 + 3*B*a^2*b^4 - 12*B*a^3*b^3 - 4*B*a^4*b^2 - 6*A*a*b^5 -
 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/((a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/
(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) - tan(c/2 + (d*x)/2)^2*(4
*a*b + 4*b^2) + tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^
8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (atan(((((8*tan(c/2 + (d*x)/2)*(A^2*
a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 +
1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8
 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 -
40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 -...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1637, normalized size of antiderivative = 4.07

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

output

```
( - 16*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*cos(c + d*x)*sin(c + d*x)**2*a**2*b**4 + 12*sqrt(a**2 - b
**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos
(c + d*x)*sin(c + d*x)**2*b**6 + 16*sqrt(a**2 - b**2)*atan((tan((c + d*x)/
2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**2*b**4 - 12*
sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2
 - b**2))*cos(c + d*x)*b**6 - 16*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*
a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**3*b**3 + 12*
sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2
 - b**2))*sin(c + d*x)**2*a*b**5 + 16*sqrt(a**2 - b**2)*atan((tan((c + d*x
)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**3*b**3 - 12*sqrt(a**2 -
 b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a
*b**5 - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**6*b - 4*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4*b**3 + 11*cos(
c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**5 - 6*cos(c + d
*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**7 + cos(c + d*x)*log(tan(
(c + d*x)/2) - 1)*a**6*b + 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**4*b
**3 - 11*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**2*b**5 + 6*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*b**7 + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*a**6*b + 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c ...
```

3.273 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 409

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \frac{(Ab-4aB)x}{b^5} - \frac{a(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7-8a^7B+28a^5b^2B-35a^3b^4B+20ab^6B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}b^5(a+b)^{7/2}d} - \frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\sin(c+dx)}{6b^4(a^2-b^2)^2d} + \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)\cos^2(c+dx)\sin(c+dx)}{6b^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} - \frac{a^2(a^4Ab-2a^2Ab^3+6Ab^5-4a^5B+11a^3b^2B-12ab^4B)\sin(c+dx)}{2b^4(a^2-b^2)^3d(a+b\cos(c+dx))}$$

output
```
(A*b-4*B*a)*x/b^5-a*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-8*A*b^7-8*B*a^7+28*
B*a^5*b^2-35*B*a^3*b^4+20*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2)/d-1/6*(3*A*a^3*b-8*A*a*b^3-12*B*a^
4+23*B*a^2*b^2-6*B*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d+1/3*a*(A*b-B*a)*cos(d
*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3
-4*B*a^3+9*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x
+c))^2-1/2*a^2*(A*a^4*b-2*A*a^2*b^3+6*A*b^5-4*B*a^5+11*B*a^3*b^2-12*B*a*b^
4)*sin(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1278 vs. $2(409) = 818$.

Time = 9.36 (sec) , antiderivative size = 1278, normalized size of antiderivative = 3.12

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
-((a*(-2*a^6*A*b + 7*a^4*A*b^3 - 8*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 28*a^5*
b^2*B + 35*a^3*b^4*B - 20*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/(b^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d)) + (-24*a^9*A*b*(c +
 d*x) + 36*a^7*A*b^3*(c + d*x) + 36*a^5*A*b^5*(c + d*x) - 84*a^3*A*b^7*(c
+ d*x) + 36*a*A*b^9*(c + d*x) + 96*a^10*B*(c + d*x) - 144*a^8*b^2*B*(c + d
*x) - 144*a^6*b^4*B*(c + d*x) + 336*a^4*b^6*B*(c + d*x) - 144*a^2*b^8*B*(c
 + d*x) - 72*a^8*A*b^2*(c + d*x)*Cos[c + d*x] + 198*a^6*A*b^4*(c + d*x)*Co
s[c + d*x] - 162*a^4*A*b^6*(c + d*x)*Cos[c + d*x] + 18*a^2*A*b^8*(c + d*x)
*Cos[c + d*x] + 18*A*b^10*(c + d*x)*Cos[c + d*x] + 288*a^9*b*B*(c + d*x)*C
os[c + d*x] - 792*a^7*b^3*B*(c + d*x)*Cos[c + d*x] + 648*a^5*b^5*B*(c + d*
x)*Cos[c + d*x] - 72*a^3*b^7*B*(c + d*x)*Cos[c + d*x] - 72*a*b^9*B*(c + d*
x)*Cos[c + d*x] - 36*a^7*A*b^3*(c + d*x)*Cos[2*(c + d*x)] + 108*a^5*A*b^5*
(c + d*x)*Cos[2*(c + d*x)] - 108*a^3*A*b^7*(c + d*x)*Cos[2*(c + d*x)] + 36
*a*A*b^9*(c + d*x)*Cos[2*(c + d*x)] + 144*a^8*b^2*B*(c + d*x)*Cos[2*(c + d
*x)] - 432*a^6*b^4*B*(c + d*x)*Cos[2*(c + d*x)] + 432*a^4*b^6*B*(c + d*x)*
Cos[2*(c + d*x)] - 144*a^2*b^8*B*(c + d*x)*Cos[2*(c + d*x)] - 6*a^6*A*b^4*
(c + d*x)*Cos[3*(c + d*x)] + 18*a^4*A*b^6*(c + d*x)*Cos[3*(c + d*x)] - 18*
a^2*A*b^8*(c + d*x)*Cos[3*(c + d*x)] + 6*A*b^10*(c + d*x)*Cos[3*(c + d*x)]
 + 24*a^7*b^3*B*(c + d*x)*Cos[3*(c + d*x)] - 72*a^5*b^5*B*(c + d*x)*Cos[3*
(c + d*x)] + 72*a^3*b^7*B*(c + d*x)*Cos[3*(c + d*x)] - 24*a*b^9*B*(c + ...
```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 3468, 25, 3042, 3526, 25, 3042, 3510, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}-\frac{\int -\frac{\cos^2(c+dx)\left(-\left(\left(-4Ba^2+Aba+3b^2B\right)\cos^2(c+dx)\right)-3b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\cos^2(c+dx)\left(-\left(\left(-4Ba^2+Aba+3b^2B\right)\cos^2(c+dx)\right)-3b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\left(4Ba^2-Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \ 3526$$

$$\frac{\frac{a\left(-4a^3B+a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int -\frac{\cos(c+dx)\left(-\left(\left(-12Ba^4+3Aba^3+23b^2Ba^2-8Ab^3a-6b^4B\right)\cos^2(c+dx)\right)+2b\left(Ba^3+2Aba^2 \cdots\right)\right)}{(a+b\cos(c+dx))^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\cos(c+dx)\left(-\left(\left(-12Ba^4+3Aba^3+23b^2Ba^2-8Ab^3a-6b^4B\right)\cos^2(c+dx)\right)+2b\left(Ba^3+2Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)+2a\left(-4Ba^3+Aba^2+9b^2Ba-6Ab^3\right)\right)}{(a+b\cos(c+dx))^2}dx}{2b(a^2-b^2)} \cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\left(12Ba^4-3Aba^3-23b^2Ba^2+8Ab^3a+6b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(Ba^3+2Aba^2-6b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a\left(-4Ba^3+Aba^2+9b^2Ba-6Ab \cdots\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}}{2b(a^2-b^2)} \cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$\Big\downarrow$ 3510

$$\frac{\frac{\int \frac{-b\left(a^2-b^2\right)\left(-12Ba^4+3Aba^3+23b^2Ba^2-8Ab^3a-6b^4B\right)\cos^2(c+dx)+\left(a^2-b^2\right)\left(-12Ba^5+3Aba^4+25b^2Ba^3-4Ab^3a^2-18b^4Ba+6Ab^5\right)\cos(c+dx)+3ab\left(-4Ba\right.}{a+b\cos(c+dx)}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{-b\left(a^2-b^2\right)\left(-12Ba^4+3Aba^3+23b^2Ba^2-8Ab^3a-6b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(a^2-b^2\right)\left(-12Ba^5+3Aba^4+25b^2Ba^3-4Ab^3a^2-18b^4Ba+6Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3502

$$\frac{\frac{\frac{\int \frac{3\left(2b(Ab-4aB)\cos(c+dx)\left(a^2-b^2\right)^3+ab^2\left(-4Ba^5+Aba^4+11b^2Ba^3-2Ab^3a^2-12b^4Ba+6Ab^5\right)\right)}{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-12a^4B+3a^3Ab+23a^2b^2B-8aAb^3-6b^4B\right)\sin}{d}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}$$

$$3b\left(a^2-b^2\right.$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{2b(Ab-4aB)\cos(c+dx)\left(a^2-b^2\right)^3+ab^2\left(-4Ba^5+Aba^4+11b^2Ba^3-2Ab^3a^2-12b^4Ba+6Ab^5\right)}{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-12a^4B+3a^3Ab+23a^2b^2B-8aAb^3-6b^4B\right)\sin(}{d}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}$$

$$3b\left(a^2-b^2\right)$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\int \frac{2b(Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^3+ab^2\left(-4Ba^5+Aba^4+11b^2Ba^3-2Ab^3a^2-12b^4Ba+6Ab^5\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-12a^4B+3a^3Ab+23a^2b^2B-8aAb^3-6b^4B\right)}{d}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\,(a^2-b^2}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$\big\downarrow$ 3214

$$\frac{\frac{\frac{3\left(2x\left(a^2-b^2\right)^3(Ab-4aB)-a\left(-8a^7B+2a^6Ab+28a^5b^2B-7a^4Ab^3-35a^3b^4B+8a^2Ab^5+20ab^6B-8Ab^7\right)\int \frac{1}{a+b\cos(c+dx)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(-12a^4B+3a^3Ab+23a\right.}{}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(2x\left(a^2-b^2\right)^3(Ab-4aB)-a\left(-8a^7B+2a^6Ab+28a^5b^2B-7a^4Ab^3-35a^3b^4B+8a^2Ab^5+20ab^6B-8Ab^7\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(-12a^4B+3a^3Ab\right.}{}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$\big\downarrow$ 3138

$$\frac{\frac{\frac{3\left(2x\left(a^2-b^2\right)^3(Ab-4aB)-\frac{2a\left(-8a^7B+2a^6Ab+28a^5b^2B-7a^4Ab^3-35a^3b^4B+8a^2Ab^5+20ab^6B-8Ab^7\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{b}-\frac{\left(a\right.}{}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$\big\downarrow$ 218

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

$$\frac{\frac{a\left(-4a^3B+a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{3\left(2x\left(a^2-b^2\right)^3(Ab-4aB)-\frac{2a\left(-8a^7B+2a^6Ab+28a^5b^2B-7a^4Ab^3-35a^3b^4B+8a^2Ab^5+20a\right.}{d\sqrt{a-b}\sqrt{a+b}}\right.}{b}}{}}{}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
(a*(A*b - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^3) + ((a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^2*B)*Cos[c + d*x]^
2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((-3*a^2*(a^4
*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*a^5*B + 11*a^3*b^2*B - 12*a*b^4*B)*Sin[c
+ d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + ((3*(2*(a^2 - b^2)^3*(A
*b - 4*a*B)*x - (2*a*(2*a^6*A*b - 7*a^4*A*b^3 + 8*a^2*A*b^5 - 8*A*b^7 - 8*
a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d)))/b - ((a^2 - b^2)
*(3*a^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b^4*B)*Sin[c + d*x])
/d)/(b^2*(a^2 - b^2)))/(2*b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+2(Ab-4Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^{5}}-\frac{2a\left(\frac{\left(2A\,a^{4}b-A\,a^{3}b^{2}-6A\,a^{2}b^{3}+4Aa\,b^{4}+12A\,b^{5}-6B\,a^{5}+2B\,a^{4}b+18B\,a^{3}b}{2(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}$
default	$\frac{\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+2(Ab-4Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^{5}}-\frac{2a\left(\frac{\left(2A\,a^{4}b-A\,a^{3}b^{2}-6A\,a^{2}b^{3}+4Aa\,b^{4}+12A\,b^{5}-6B\,a^{5}+2B\,a^{4}b+18B\,a^{3}b}{2(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(2/b^5*(B*b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-4*B*a)*ar
ctan(tan(1/2*d*x+1/2*c)))-2*a/b^5*((1/2*(2*A*a^4*b-A*a^3*b^2-6*A*a^2*b^3+4
*A*a*b^4+12*A*b^5-6*B*a^5+2*B*a^4*b+18*B*a^3*b^2-5*B*a^2*b^3-20*B*a*b^4)*a
*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(3*A*a^4*b-11*
A*a^2*b^3+18*A*b^5-9*B*a^5+29*B*a^3*b^2-30*B*a*b^4)*a*b/(a^2+2*a*b+b^2)/(a
^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(2*A*a^4*b+A*a^3*b^2-6*A*a^2*b^3-4*
A*a*b^4+12*A*b^5-6*B*a^5-2*B*a^4*b+18*B*a^3*b^2+5*B*a^2*b^3-20*B*a*b^4)*a*
b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3+1/2*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-8
*A*b^7-8*B*a^7+28*B*a^5*b^2-35*B*a^3*b^4+20*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*
b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1249 vs. $2(395) = 790$.

Time = 0.30 (sec) , antiderivative size = 2567, normalized size of antiderivative = 6.28

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
[-1/12*(12*(4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^
5*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*d
*x*cos(d*x + c)^3 + 36*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*
b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^
10 - A*a*b^11)*d*x*cos(d*x + c)^2 + 36*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9
*b^3 + 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b
^8 + 4*B*a^3*b^9 - A*a^2*b^10)*d*x*cos(d*x + c) + 12*(4*B*a^12 - A*a^11*b
- 16*B*a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6
+ 4*A*a^5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*d*x - 3*(8*B*a^11 - 2*A*a^10*b -
28*B*a^9*b^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8
*A*a^4*b^7 + (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*
B*a^4*b^7 - 8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8
*B*a^9*b^2 - 2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A
*a^4*b^7 - 20*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^10*b - 2*
A*a^9*b^2 - 28*B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B
*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x
+ c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) +
 b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 a^2)) - 2*(24*B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133
*B*a^7*b^5 - 43*A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*a^3*b^9 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(395) = 790.

Time = 0.27 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.36

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-1/3*(3*(8*B*a^8 - 2*A*a^7*b - 28*B*a^6*b^2 + 7*A*a^5*b^3 + 35*B*a^4*b^4 -
 8*A*a^3*b^5 - 20*B*a^2*b^6 + 8*A*a*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(a^2 -
b^2)) - (18*B*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^8*b*tan(1/2*d*x + 1/2*c)^
5 - 42*B*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c)^
5 - 24*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c)
^5 + 117*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^5*b^4*tan(1/2*d*x + 1/2
*c)^5 - 24*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^5*tan(1/2*d*x + 1/
2*c)^5 - 105*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^3*b^6*tan(1/2*d*x +
 1/2*c)^5 + 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^2*b^7*tan(1/2*d*x
 + 1/2*c)^5 + 36*B*a^9*tan(1/2*d*x + 1/2*c)^3 - 12*A*a^8*b*tan(1/2*d*x + 1
/2*c)^3 - 152*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 56*A*a^6*b^3*tan(1/2*d*x
+ 1/2*c)^3 + 236*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^5*tan(1/2*
d*x + 1/2*c)^3 - 120*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^2*b^7*tan(1
/2*d*x + 1/2*c)^3 + 18*B*a^9*tan(1/2*d*x + 1/2*c) - 6*A*a^8*b*tan(1/2*d*x
+ 1/2*c) + 42*B*a^8*b*tan(1/2*d*x + 1/2*c) - 15*A*a^7*b^2*tan(1/2*d*x + 1/
2*c) - 24*B*a^7*b^2*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c
) - 117*B*a^6*b^3*tan(1/2*d*x + 1/2*c) + 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c)
 - 24*B*a^5*b^4*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c)...
```

Mupad [B] (verification not implemented)

Time = 37.54 (sec) , antiderivative size = 7823, normalized size of antiderivative = 19.13

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

output
```
(log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 4*B*a)*1i)/(b^5*d) - ((tan(c/2 + (d*x
)/2)^7*(12*A*a^2*b^5 - 2*B*b^7 - 8*B*a^7 + 4*A*a^3*b^4 - 6*A*a^4*b^3 - A*a
^5*b^2 + 6*B*a^2*b^5 - 26*B*a^3*b^4 - 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^
6*b + 2*B*a*b^6 + 4*B*a^6*b))/(b^4*(a + b)^3*(a - b)) - (tan(c/2 + (d*x)/2
)^3*(72*B*a^8 + 18*B*b^8 + 36*A*a^2*b^6 - 96*A*a^3*b^5 - 14*A*a^4*b^4 + 59
*A*a^5*b^3 + 3*A*a^6*b^2 - 72*B*a^2*b^6 - 60*B*a^3*b^5 + 273*B*a^4*b^4 + 4
7*B*a^5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b - 12*B*a^7*b))/(3*b^4*(a + b)^2*(
a - b)^3) - (tan(c/2 + (d*x)/2)^5*(72*B*a^8 + 18*B*b^8 - 36*A*a^2*b^6 - 96
*A*a^3*b^5 + 14*A*a^4*b^4 + 59*A*a^5*b^3 - 3*A*a^6*b^2 - 72*B*a^2*b^6 + 60
*B*a^3*b^5 + 273*B*a^4*b^4 - 47*B*a^5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b + 1
2*B*a^7*b))/(3*b^4*(a + b)^3*(a - b)^2) + (tan(c/2 + (d*x)/2)*(2*B*b^7 - 8
*B*a^7 + 12*A*a^2*b^5 - 4*A*a^3*b^4 - 6*A*a^4*b^3 + A*a^5*b^2 - 6*B*a^2*b^
5 - 26*B*a^3*b^4 + 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b + 2*B*a*b^6 - 4
*B*a^6*b))/(b^4*(a + b)*(a - b)^3))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x
)/2)^4*(6*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 - 2*b^3)
+ tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + tan(c/2 + (
d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (log(tan(c/2 + (d*x)/2) - 1i
)*(A*b*1i - B*a*4i))/(b^5*d) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b
^16 + 128*B^2*a^16 - 8*A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*a^2*b^14 + 48*
A^2*a^3*b^13 - 92*A^2*a^4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a^6*b^10 + ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1271, normalized size of antiderivative = 3.11

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4} \, dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

output
```
(24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
a**2 - b**2))*cos(c + d*x)*a**7*b - 60*sqrt(a**2 - b**2)*atan((tan((c + d*
x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**5*b**3 +
48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*cos(c + d*x)*a**3*b**5 - 12*sqrt(a**2 - b**2)*atan((tan((c +
d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**6*b*
*2 + 30*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt(a**2 - b**2))*sin(c + d*x)**2*a**4*b**4 - 24*sqrt(a**2 - b**2)*atan((t
an((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2
*a**2*b**6 + 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)
/2)*b)/sqrt(a**2 - b**2))*a**8 - 18*sqrt(a**2 - b**2)*atan((tan((c + d*x)/
2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**6*b**2 - 6*sqrt(a**2 - b*
*2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**4
*b**4 + 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b
)/sqrt(a**2 - b**2))*a**2*b**6 + 9*cos(c + d*x)*sin(c + d*x)*a**7*b**2 - 2
5*cos(c + d*x)*sin(c + d*x)*a**5*b**4 + 20*cos(c + d*x)*sin(c + d*x)*a**3*
b**6 - 4*cos(c + d*x)*sin(c + d*x)*a*b**8 - 12*cos(c + d*x)*a**8*b*c - 12*
cos(c + d*x)*a**8*b*d*x + 36*cos(c + d*x)*a**6*b**3*c + 36*cos(c + d*x)*a*
*6*b**3*d*x - 36*cos(c + d*x)*a**4*b**5*c - 36*cos(c + d*x)*a**4*b**5*d*x
+ 12*cos(c + d*x)*a**2*b**7*c + 12*cos(c + d*x)*a**2*b**7*d*x - 2*sin(c...
```

3.274 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 301

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx &= \frac{Bx}{b^4} \\ &\quad -\frac{(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} \\ &\quad +\frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^3}+\frac{a^2(5Ab^3+3a^3B-8ab^2B)\sin(c+dx)}{6b^3(a^2-b^2)^2\,d(a+b\cos(c+dx))^2} \\ &\quad -\frac{a(a^2Ab^3-16Ab^5+9a^5B-28a^3b^2B+34ab^4B)\sin(c+dx)}{6b^3(a^2-b^2)^3\,d(a+b\cos(c+dx))}\end{aligned}$$

output

```
B*x/b^4-(3*A*a^2*b^5+2*A*b^7+2*B*a^7-7*B*a^5*b^2+8*B*a^3*b^4-8*B*a*b^6)*ar
ctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/
2)/d+1/3*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c)
)^3+1/6*a^2*(5*A*b^3+3*B*a^3-8*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*
cos(d*x+c))^2-1/6*a*(A*a^2*b^3-16*A*b^5+9*B*a^5-28*B*a^3*b^2+34*B*a*b^4)*s
in(d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 717 vs. $2(301) = 602$.

Time = 6.40 (sec) , antiderivative size = 717, normalized size of antiderivative = 2.38

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{-\frac{24(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{24a^9Bc-36a^7b^2Bc-36a^5b^4Bc+84a^3b^6Bc-36a}{}}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
((-24*(3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b
^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
7/2) + (24*a^9*B*c - 36*a^7*b^2*B*c - 36*a^5*b^4*B*c + 84*a^3*b^6*B*c - 36
*a*b^8*B*c + 24*a^9*B*d*x - 36*a^7*b^2*B*d*x - 36*a^5*b^4*B*d*x + 84*a^3*b
^6*B*d*x - 36*a*b^8*B*d*x + 18*b*(a^2 - b^2)^3*(4*a^2 + b^2)*B*(c + d*x)*C
os[c + d*x] + 36*a*b^2*(a^2 - b^2)^3*B*(c + d*x)*Cos[2*(c + d*x)] + 6*a^6*
b^3*B*c*Cos[3*(c + d*x)] - 18*a^4*b^5*B*c*Cos[3*(c + d*x)] + 18*a^2*b^7*B*
c*Cos[3*(c + d*x)] - 6*b^9*B*c*Cos[3*(c + d*x)] + 6*a^6*b^3*B*d*x*Cos[3*(c
 + d*x)] - 18*a^4*b^5*B*d*x*Cos[3*(c + d*x)] + 18*a^2*b^7*B*d*x*Cos[3*(c +
 d*x)] - 6*b^9*B*d*x*Cos[3*(c + d*x)] + 18*a^5*A*b^4*Sin[c + d*x] + 39*a^3
*A*b^6*Sin[c + d*x] + 18*a*A*b^8*Sin[c + d*x] - 24*a^8*b*B*Sin[c + d*x] +
57*a^6*b^3*B*Sin[c + d*x] - 72*a^4*b^5*B*Sin[c + d*x] - 36*a^2*b^7*B*Sin[c
 + d*x] + 6*a^4*A*b^5*Sin[2*(c + d*x)] + 54*a^2*A*b^7*Sin[2*(c + d*x)] - 3
0*a^7*b^2*B*Sin[2*(c + d*x)] + 90*a^5*b^4*B*Sin[2*(c + d*x)] - 120*a^3*b^6
*B*Sin[2*(c + d*x)] + 2*a^5*A*b^4*Sin[3*(c + d*x)] - 5*a^3*A*b^6*Sin[3*(c
+ d*x)] + 18*a*A*b^8*Sin[3*(c + d*x)] - 11*a^6*b^3*B*Sin[3*(c + d*x)] + 32
*a^4*b^5*B*Sin[3*(c + d*x)] - 36*a^2*b^7*B*Sin[3*(c + d*x)])/((a^2 - b^2)^
3*(a + b*Cos[c + d*x])^3))/(24*b^4*d)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3468, 25, 3042, 3510, 25, 3042, 3500, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}-\frac{\int -\frac{\cos(c+dx)(3(a^2-b^2)B\cos^2(c+dx)-3b(Ab-aB)\cos(c+dx)+2a(Ab-aB))}{(a+b\cos(c+dx))^3}dx}{3b(a^2-b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(c+dx)(3(a^2-b^2)B\cos^2(c+dx)-3b(Ab-aB)\cos(c+dx)+2a(Ab-aB))}{(a+b\cos(c+dx))^3}dx}{3b(a^2-b^2)}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(3(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b(a^2-b^2)}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3510}$$

$$\frac{\frac{\int -\frac{-6b\left(a^2-b^2\right)^2B\cos^2(c+dx)+\left(3Ba^5-10b^2Ba^3+Ab^3a^2+12b^4Ba-6Ab^5\right)\cos(c+dx)+2ab\left(3Ba^3-8b^2Ba+5Ab^3\right)}{(a+b\cos(c+dx))^2}dx}{2b^2(a^2-b^2)}+\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+}{2b^2d(a^2-b^2)(a+b\cos(c+dx))}}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int\frac{-6b\left(a^2-b^2\right)^2B\cos^2(c+dx)+\left(3Ba^5-10b^2Ba^3+Ab^3a^2+12b^4Ba-6Ab^5\right)\cos(c+dx)+2ab\left(3Ba^3-8b^2Ba+5Ab^3\right)}{(a+b\cos(c+dx))^2}d}{2b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int\frac{-6b\left(a^2-b^2\right)^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Ba^5-10b^2Ba^3+Ab^3a^2+12b^4Ba-6Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2ab\left(3Ba^3-8b^2Ba\right.}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}}{2b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3500

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{a\left(9a^5B-28a^3b^2B+a^2Ab^3+34ab^4B-16Ab^5\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{3\left(b\left(2Ab^6-6aBb^5+3a^2Ab^4+2a^3Bb^3-a^5Bb\right)-2b\left(a\right.\right.}{a+b\cos(c+dx)}}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{3\int\frac{b\left(2Ab^6-6aBb^5+3a^2Ab^4+2a^3Bb^3-a^5Bb\right)-2b\left(a^2-b^2\right)^3B\cos(c+dx)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}+\frac{a\left(9a^5B-28a^3b^2B+a^2Ab^3+34ab^4B\right.}{d\left(a^2-b^2\right)(a+b\cos(c-\right.}}{2b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{3\int\frac{b\left(2Ab^6-6aBb^5+3a^2Ab^4+2a^3Bb^3-a^5Bb\right)-2b\left(a^2-b^2\right)^3B\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}+\frac{a\left(9a^5B-28a^3b^2B+a^2Ab^3+34a\cdots\right.}{d\left(a^2-b^2\right)(a+b\cos\cdots}}{2b^2(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3214

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{3\left(\left(2a^7B-7a^5b^2B+8a^3b^4B+3a^2Ab^5-8ab^6B+2Ab^7\right)\int\frac{1}{a+b\cos(c+dx)}dx-2Bx\left(a^2-b^2\right)^3\right)}{b\left(a^2-b^2\right)}+\frac{a\left(9a^5B-28a^3b^2B+\cdots\right.}{d\left(a^2\cdots\right.}}{2b^2(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{3\left(\left(2a^7B-7a^5b^2B+8a^3b^4B+3a^2Ab^5-8ab^6B+2Ab^7\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2Bx\left(a^2-b^2\right)^3\right)}{b\left(a^2-b^2\right)}+\frac{a\left(9a^5B-28a^3\cdots\right.}{\cdots}}{2b^2(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3138

$$\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{3\left(\frac{2\left(2a^7B-7a^5b^2B+8a^3b^4B+3a^2Ab^5-8ab^6B+2Ab^7\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}-2Bx\left(a^2-\cdots\right.\right.}{b\left(a^2-b^2\right)}\cdots}{2b^2(a^2-b^2)}\cdots}{3b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 218

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{\frac{a^2(3a^3B-8ab^2B+5Ab^3)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{a\left(9a^5B-28a^3b^2B+a^2Ab^3+34ab^4B-16Ab^5\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{3\left(\frac{2\left(2a^7B-7a^5b^2B+8a^3b^4B+3a^2Ab^5-8ab^6B+2Ab^7\right)}{d\sqrt{a-b}\sqrt{a+b}}\right.}{b\left(a^2-b\right.}}{2b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
(a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^3) + ((a^2*(5*A*b^3 + 3*a^3*B - 8*a*b^2*B)*Sin[c + d*x])/(2*b^2*
(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*(-2*(a^2 - b^2)^3*B*x + (2*(3*
a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b^6*B)*Arc
Tan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*
d)))/(b*(a^2 - b^2)) + (a*(a^2*A*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B +
 34*a*b^4*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*b^2*(a
^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.52

method	result
derivativedivides	$-\frac{2\left(-\frac{\left(2A a^2b^3+3Aa b^4+6A b^5-2B a^5+B a^4b+6B a^3b^2-4B a^2b^3-12Ba b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(A a^2b^3+9A b^5-3B a^5+11B a\cdots\right.}{3\left(a^2+2ab+b^2\right)\cdots}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\cdots\right.\right.}\cdots$
default	$-\frac{2\left(-\frac{\left(2A a^2b^3+3Aa b^4+6A b^5-2B a^5+B a^4b+6B a^3b^2-4B a^2b^3-12Ba b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(A a^2b^3+9A b^5-3B a^5+11B a\cdots\right.}{3\left(a^2+2ab+b^2\right)\cdots}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\cdots\right.\right.}\cdots$
risch	Expression too large to display

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-2/b^4*((-1/2*(2*A*a^2*b^3+3*A*a*b^4+6*A*b^5-2*B*a^5+B*a^4*b+6*B*a^3*
b^2-4*B*a^2*b^3-12*B*a*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5-2/3*(A*a^2*b^3+9*A*b^5-3*B*a^5+11*B*a^3*b^2-18*B*a*b^4)*a*b/(a^
2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*A*a^2*b^3-3*A*a*b
^4+6*A*b^5-2*B*a^5-B*a^4*b+6*B*a^3*b^2+4*B*a^2*b^3-12*B*a*b^4)*a*b/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3+1/2*(3*A*a^2*b^5+2*A*b^7+2*B*a^7-7*B*a^5*b^2+8*B*
a^3*b^4-8*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2*B/b^4*arctan(tan(1/2*d*
x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(286) = 572$.

Time = 0.23 (sec) , antiderivative size = 1857, normalized size of antiderivative = 6.17

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
[1/12*(12*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11)*d
*x*cos(d*x + c)^3 + 36*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^
8 + B*a*b^10)*d*x*cos(d*x + c)^2 + 36*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^
5 - 4*B*a^4*b^7 + B*a^2*b^9)*d*x*cos(d*x + c) + 12*(B*a^11 - 4*B*a^9*b^2 +
 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*d*x + 3*(2*B*a^10 - 7*B*a^8*b^2 +
8*B*a^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 7*B
*a^5*b^5 + 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10)*cos(d*x + c)^
3 + 3*(2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8
 + 2*A*a*b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 +
3*A*a^4*b^6 - 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*lo
g((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*
(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + a^2)) - 2*(6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*
B*a^6*b^5 - 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8 + (11*B*a^8*b^3 - 2*
A*a^7*b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36*
B*a^2*b^9 + 18*A*a*b^10)*cos(d*x + c)^2 + 3*(5*B*a^9*b^2 - 20*B*a^7*b^4 -
A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 9*A*a^2*b^9)*cos(d
*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b
^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 +
 a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(286) = 572.

Time = 0.22 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.70

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
1/3*(3*(2*B*a^7 - 7*B*a^5*b^2 + 8*B*a^3*b^4 + 3*A*a^2*b^5 - 8*B*a*b^6 + 2*
A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3
*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) + 3*(d*x + c)*B/b^4 - (6*B*a
^8*tan(1/2*d*x + 1/2*c)^5 - 15*B*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^6*b^
2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*B*a^5*b
^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b
^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*
b^5*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^
2*b^6*tan(1/2*d*x + 1/2*c)^5 - 18*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^
8*tan(1/2*d*x + 1/2*c)^3 - 56*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^5*b
^3*tan(1/2*d*x + 1/2*c)^3 + 116*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*A*a^
3*b^5*tan(1/2*d*x + 1/2*c)^3 - 72*B*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*
a*b^7*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^8*tan(1/2*d*x + 1/2*c) + 15*B*a^7*b*t
an(1/2*d*x + 1/2*c) - 6*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^5*b^3*tan(1
/2*d*x + 1/2*c) - 45*B*a^5*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^4*tan(1/2*
d*x + 1/2*c) - 6*B*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^5*tan(1/2*d*x
+ 1/2*c) + 60*B*a^3*b^5*tan(1/2*d*x + 1/2*c) - 27*A*a^2*b^6*tan(1/2*d*x +
1/2*c) + 36*B*a^2*b^6*tan(1/2*d*x + 1/2*c) - 18*A*a*b^7*tan(1/2*d*x + 1/2*
c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 ...
```

Mupad [B] (verification not implemented)

Time = 32.60 (sec) , antiderivative size = 9733, normalized size of antiderivative = 32.34

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(3*A*a^2*b^4 - 2*B*a^6 + 2*A*a^3*b^3 - 12*B*a^2*b^4
 - 4*B*a^3*b^3 + 6*B*a^4*b^2 + 6*A*a*b^5 + B*a^5*b))/((a*b^3 - b^4)*(a + b
)^3) - (tan(c/2 + (d*x)/2)*(2*B*a^6 + 3*A*a^2*b^4 - 2*A*a^3*b^3 + 12*B*a^2
*b^4 - 4*B*a^3*b^3 - 6*B*a^4*b^2 - 6*A*a*b^5 + B*a^5*b))/((a + b)*(3*a*b^5
 - b^6 - 3*a^2*b^4 + a^3*b^3)) + (4*tan(c/2 + (d*x)/2)^3*(A*a^3*b^3 - 3*B*
a^6 - 18*B*a^2*b^4 + 11*B*a^4*b^2 + 9*A*a*b^5))/(3*(a + b)^2*(b^5 - 2*a*b^
4 + a^2*b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a
^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3
*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))
) + (2*B*atan(((B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*
b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44
*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*
B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^
2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^
3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17
 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*
a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (B*((8*(4*A*b^21
 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 +
14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^1
9 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 11...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1004, normalized size of antiderivative = 3.34

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

output
```
( - 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*cos(c + d*x)*a**6*b + 20*sqrt(a**2 - b**2)*atan((tan((c +
d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**4*b**3
- 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt
(a**2 - b**2))*cos(c + d*x)*a**2*b**5 + 4*sqrt(a**2 - b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**5*b
**2 - 10*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt(a**2 - b**2))*sin(c + d*x)**2*a**3*b**4 + 12*sqrt(a**2 - b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**
2*a*b**6 - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)
*b)/sqrt(a**2 - b**2))*a**7 + 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a
 - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**5*b**2 - 2*sqrt(a**2 - b**2)*
atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**3*b**
4 - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*a*b**6 - 3*cos(c + d*x)*sin(c + d*x)*a**6*b**2 + 9*cos(c
+ d*x)*sin(c + d*x)*a**4*b**4 - 6*cos(c + d*x)*sin(c + d*x)*a**2*b**6 + 4*
cos(c + d*x)*a**7*b*d*x - 12*cos(c + d*x)*a**5*b**3*d*x + 12*cos(c + d*x)*
a**3*b**5*d*x - 4*cos(c + d*x)*a*b**7*d*x - 2*sin(c + d*x)**2*a**6*b**2*d*
x + 6*sin(c + d*x)**2*a**4*b**4*d*x - 6*sin(c + d*x)**2*a**2*b**6*d*x + 2*
sin(c + d*x)**2*b**8*d*x - 2*sin(c + d*x)*a**7*b + 7*sin(c + d*x)*a**5*...
```

3.275 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 274

$$\begin{aligned}
&\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx \\
&= \frac{(a^3A+4aAb^2-3a^2bB-2b^3B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} \\
&\quad -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)\,d(a+b\cos(c+dx))^3}+\frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)\sin(c+dx)}{6b^2(a^2-b^2)^2\,d(a+b\cos(c+dx))^2} \\
&\quad +\frac{(a^4Ab-10a^2Ab^3-6Ab^5+2a^5B-5a^3b^2B+18ab^4B)\sin(c+dx)}{6b^2(a^2-b^2)^3\,d(a+b\cos(c+dx))}
\end{aligned}$$

output
```
(A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a
^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*a^3+9*B*a*b^2)*sin
(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(A*a^4*b-10*A*a^2*b^3-6*A
*b^5+2*B*a^5-5*B*a^3*b^2+18*B*a*b^4)*sin(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*cos
(d*x+c))
```

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{-\frac{24(a^3A+4aAb^2-3a^2bB-2b^3B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{2(-25a^4Ab-14a^2Ab^3-6Ab^5+10a^5B+17a^3b^2B+18ab^4B+6a(a^4A-}{}}{24\left(a^2-b^2\right)^3d}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
((-24*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[((a - b)*Tan[(c +
d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-25*a^4*A*b - 14*a^2*A*
b^3 - 6*A*b^5 + 10*a^5*B + 17*a^3*b^2*B + 18*a*b^4*B + 6*a*(a^4*A - 9*a^2*
A*b^2 - 2*A*b^4 + a^3*b*B + 9*a*b^3*B)*Cos[c + d*x] + (a^4*A*b - 10*a^2*A*
b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 18*a*b^4*B)*Cos[2*(c + d*x)])*Sin[
c + d*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3467, 3042, 3500, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{\int \frac{3b(a^2-b^2)B\cos^2(c+dx)+(a^2-3b^2)(Ab-aB)\cos(c+dx)+3ab(Ab-aB)}{(a+b\cos(c+dx))^3}dx}{3b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\int \frac{3b(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)^2+(a^2-3b^2)(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3ab(Ab-aB)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3500

$$\frac{\frac{a(-4a^3B+a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int -\frac{2\left(Ba^3+2Aba^2-6b^2Ba+3Ab^3\right)b^2+\left(2Ba^4+Aba^3-3b^2Ba^2-6Ab^3a+6b^4B\right)\cos(c+dx)b}{(a+b\cos(c+dx))^2}dx}{2b(a^2-b^2)}}{3b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{2\left(Ba^3+2Aba^2-6b^2Ba+3Ab^3\right)b^2+\left(2Ba^4+Aba^3-3b^2Ba^2-6Ab^3a+6b^4B\right)\cos(c+dx)b}{(a+b\cos(c+dx))^2}dx}{2b(a^2-b^2)}+\frac{a(-4a^3B+a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{2\left(Ba^3+2Aba^2-6b^2Ba+3Ab^3\right)b^2+\left(2Ba^4+Aba^3-3b^2Ba^2-6Ab^3a+6b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}+\frac{a(-4a^3B+a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b^2(a^2-b^2)}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3233

$$\frac{\frac{\frac{b\left(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+18ab^4B-6Ab^5\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{3b^3\left(Aa^3-3bBa^2+4Ab^2a-2b^3B\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}+\frac{a\left(-4a^3B+a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b^2\left(a^2-b^2\right)}$$
$$\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3b^3\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\int\frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}+\frac{b\left(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+18ab^4B-6Ab^5\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2b(a^2-b^2)}+\frac{a\left(-4a^3B+a^2Ab+9ab^2B-6Ab^3}{2d(a^2-b^2)(a+b\cos(c+d}}{3b^2\left(a^2-b^2\right)}$$
$$\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3b^3\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{b\left(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+18ab^4B-6Ab^5\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2b(a^2-b^2)}+\frac{a\left(-4a^3B+a^2Ab+9ab^2B-6}{2d(a^2-b^2)(a+b\cos(}}{3b^2\left(a^2-b^2\right)}$$
$$\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{6b^3\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}+\frac{b\left(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+18ab^4B-6Ab^5\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2b(a^2-b^2)}+\frac{a(-4a^3}{}}{3b^2\left(a^2-b^2\right)}$$
$$\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{a\left(-4a^3B+a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{6b^3\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}+\frac{b\left(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+1}{d\left(a^2-b^2\right)(a+b\cos(}}{2b(a^2-b^2)}}{3b^2\left(a^2-b^2\right)}$$
$$\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
-1/3*(a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^3) + ((a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(2*(a^2
 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((6*b^3*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B
 - 2*b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a -
b]*Sqrt[a + b]*(a^2 - b^2)*d) + (b*(a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a
^5*B - 5*a^3*b^2*B + 18*a*b^4*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c
 + d*x])))/(2*b*(a^2 - b^2)))/(3*b^2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{-\frac{\left(a^{3}A+6A\,a^{2}b+2Aa\,b^{2}+2A\,b^{3}-2a^{3}B-3B\,a^{2}b-6Ba\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{4\left(7A\,a^{2}b+3A\,b^{3}-a^{3}B-9Ba\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(a^{3}A\right.}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{3}}}{d}$
default	$\frac{\frac{-\frac{\left(a^{3}A+6A\,a^{2}b+2Aa\,b^{2}+2A\,b^{3}-2a^{3}B-3B\,a^{2}b-6Ba\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{4\left(7A\,a^{2}b+3A\,b^{3}-a^{3}B-9Ba\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(a^{3}A\right.}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{3}}}{d}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(2*(-1/2*(A*a^3+6*A*a^2*b+2*A*a*b^2+2*A*b^3-2*B*a^3-3*B*a^2*b-6*B*a*b^
2)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(7*A*a^2*b+3*A
*b^3-B*a^3-9*B*a*b^2)/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
+1/2*(A*a^3-6*A*a^2*b+2*A*a*b^2-2*A*b^3+2*B*a^3-3*B*a^2*b+6*B*a*b^2)/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3+(A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a
-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(260) = 520$.

Time = 0.16 (sec) , antiderivative size = 1220, normalized size of antiderivative = 4.45

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
[-1/12*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 + (A*a^3*b^3 - 3*
B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4*b^2 - 3*B*a^3*b
^3 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b - 3*B*a^4*b^2 +
4*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c
) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2)) - 2*(4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*a^4*b^3 - 11*B*a^
3*b^4 + 2*A*a^2*b^5 + (2*B*a^7 + A*a^6*b - 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23
*B*a^3*b^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^2 + 3*(A*a^7
 + B*a^6*b - 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*
a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a
^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*
a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*
b^6 + a^3*b^8)*d), 1/6*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 +
 (A*a^3*b^3 - 3*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4
*b^2 - 3*B*a^3*b^3 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b
- 3*B*a^4*b^2 + 4*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (4*B*a^7 - 1
3*A*a^6*b + 7*B*a^5*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 + 2*A*a^2*b^5 + (...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(260) = 520.

Time = 0.23 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.51

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-1/3*(3*(A*a^3 - 3*B*a^2*b + 4*A*a*b^2 - 2*B*b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^
2 - b^2)) + (3*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*tan(1/2*d*x + 1/2*c)
^5 + 12*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b*tan(1/2*d*x + 1/2*c)^5
- 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
 + 12*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)
^5 - 6*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^4*tan(1/2*d*x + 1/2*c)^5
+ 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 - 4*B*a^5*tan(1/2*d*x + 1/2*c)^3 + 28*A*a
^4*b*tan(1/2*d*x + 1/2*c)^3 - 32*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*A*a
^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*A*b
^5*tan(1/2*d*x + 1/2*c)^3 - 3*A*a^5*tan(1/2*d*x + 1/2*c) - 6*B*a^5*tan(1/2
*d*x + 1/2*c) + 12*A*a^4*b*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b*tan(1/2*d*x +
1/2*c) + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^2*tan(1/2*d*x + 1/2
*c) + 12*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c
) + 6*A*a*b^4*tan(1/2*d*x + 1/2*c) - 18*B*a*b^4*tan(1/2*d*x + 1/2*c) + 6*A
*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 26.48 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.61

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{(a-b)}^{7/2}}\right)\,\left(A\,a^3-3\,B\,a^2\,b+4\,A\,a\,b^2-2\,B\,b^3\right)}{d\,{(a+b)}^{7/2}\,{(a-b)}^{7/2}}$$
$$-\frac{\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(-B\,a^3+7\,A\,a^2\,b-9\,B\,a\,b^2+3\,A\,b^3\right)}{3\,{(a+b)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A\,a^3+2\,A\,b^3-2\,B\,a^3+2\,A\,a\,b^2+6\,A\,a^2\,b-6\,B\,a\,b^2-3\,B\,a^2\,b\right.}{{(a+b)}^3\,(a-b)}}{d\,\left(3\,a\,b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\right.\right.}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

output
```
(atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*
(a + b)^(1/2)*(a - b)^(7/2)))*(A*a^3 - 2*B*b^3 + 4*A*a*b^2 - 3*B*a^2*b))/(
d*(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(3*A*b^3 - B*a^3
 + 7*A*a^2*b - 9*B*a*b^2))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 +
(d*x)/2)^5*(A*a^3 + 2*A*b^3 - 2*B*a^3 + 2*A*a*b^2 + 6*A*a^2*b - 6*B*a*b^2
- 3*B*a^2*b))/((a + b)^3*(a - b)) - (tan(c/2 + (d*x)/2)*(A*a^3 - 2*A*b^3 +
 2*B*a^3 + 2*A*a*b^2 - 6*A*a^2*b + 6*B*a*b^2 - 3*B*a^2*b))/((a + b)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 +
3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3
 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
+ a^3 - b^3)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.31

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

output
```
(4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*cos(c + d*x)*a**3*b + 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a*b**3 - 2*sqr
t(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 -
b**2))*sin(c + d*x)**2*a**2*b**2 - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*b**4 + 2*sq
rt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 -
 b**2))*a**4 + 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x
)/2)*b)/sqrt(a**2 - b**2))*a**2*b**2 + 4*sqrt(a**2 - b**2)*atan((tan((c +
d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*b**4 + cos(c + d*x)*sin
(c + d*x)*a**5 - 5*cos(c + d*x)*sin(c + d*x)*a**3*b**2 + 4*cos(c + d*x)*si
n(c + d*x)*a*b**4 - 3*sin(c + d*x)*a**4*b + 3*sin(c + d*x)*a**2*b**3)/(2*d
*(2*cos(c + d*x)*a**7*b - 6*cos(c + d*x)*a**5*b**3 + 6*cos(c + d*x)*a**3*b
**5 - 2*cos(c + d*x)*a*b**7 - sin(c + d*x)**2*a**6*b**2 + 3*sin(c + d*x)**
2*a**4*b**4 - 3*sin(c + d*x)**2*a**2*b**6 + sin(c + d*x)**2*b**8 + a**8 -
2*a**6*b**2 + 2*a**2*b**6 - b**8))
```

3.276 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 263

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx &= -\frac{(4a^2Ab+Ab^3-a^3B-4ab^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}\\ &\quad+\frac{a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3}+\frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B)\sin(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\cos(c+dx))^2}\\ &\quad+\frac{(2a^3Ab+13aAb^3+a^4B-10a^2b^2B-6b^4B)\sin(c+dx)}{6b\,(a^2-b^2)^3\,d(a+b\cos(c+dx))}\end{aligned}$$

output

```
-(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^
2)/d/(a+b*cos(d*x+c))^3+1/6*(2*A*a^2*b+3*A*b^3+B*a^3-6*B*a*b^2)*sin(d*x+c)
/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(2*A*a^3*b+13*A*a*b^3+B*a^4-10*B*a
^2*b^2-6*B*b^4)*sin(d*x+c)/b/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{-\frac{24(-4a^2Ab-Ab^3+a^3B+4ab^2B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{2(12a^5A+22a^3Ab^2+11aAb^4-25a^4bB-14a^2b^3B-6b^5B+6(2a^4Ab+}{}}{24\left(a^2-b^2\right)^3 d}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
((-24*(-4*a^2*A*b - A*b^3 + a^3*B + 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(12*a^5*A + 22*a^3*A*b^2
+ 11*a*A*b^4 - 25*a^4*b*B - 14*a^2*b^3*B - 6*b^5*B + 6*(2*a^4*A*b + 9*a^2*
A*b^3 - A*b^5 + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B)*Cos[c + d*x] + b*(2*a^3*A
*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Cos[2*(c + d*x)])*Sin[c
+ d*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 3447, 3042, 3500, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\int \frac{3b(Ab-aB)-\left(Ba^2+2Aba-3b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\int \frac{3b(Ab-aB)+\left(-Ba^2-2Aba+3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3233}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{-\frac{\int -\frac{2b\left(-2Ba^2+5Aba-3b^2B\right)-\left(Ba^3+2Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\int \frac{2b\left(-2Ba^2+5Aba-3b^2B\right)-\left(Ba^3+2Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\int \frac{2b\left(-2Ba^2+5Aba-3b^2B\right)+\left(-Ba^3-2Aba^2+6b^2Ba-3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3233}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{-\frac{\int -\frac{3b\left(-Ba^3+4Aba^2-4b^2Ba+Ab^3\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{3b\left(a^3(-B)+4a^2Ab-4ab^2B+Ab^3\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{3b\left(a^3(-B)+4a^2Ab-4ab^2B+Ab^3\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3138$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{6b\left(a^3(-B)+4a^2Ab-4ab^2B+Ab^3\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{6b\left(a^3(-B)+4a^2Ab-4ab^2B+Ab^3\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

output
```
(a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) -
(-1/2*((2*a^2*A*b + 3*A*b^3 + a^3*B - 6*a*b^2*B)*Sin[c + d*x])/((a^2 - b^2
)*d*(a + b*Cos[c + d*x])^2) + ((6*b*(4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
+ b]*(a^2 - b^2)*d) - ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*
b^4*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*(a^2 - b^2))
)/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-\frac{2\left(-\frac{\left(2a^3A+2A\,a^2b+6Aa\,b^2+A\,b^3-a^3B-6B\,a^2b-2Ba\,b^2-2b^3B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(3a^3A+7Aa\,b^2-7B\,a^2b-3b^3B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{3\left(a^2-2ab+b^2\right)\left(a^2+2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^3}$
default	$-\frac{2\left(-\frac{\left(2a^3A+2A\,a^2b+6Aa\,b^2+A\,b^3-a^3B-6B\,a^2b-2Ba\,b^2-2b^3B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(3a^3A+7Aa\,b^2-7B\,a^2b-3b^3B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{3\left(a^2-2ab+b^2\right)\left(a^2+2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^3}$
risch	Expression too large to display

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-2*(-1/2*(2*A*a^3+2*A*a^2*b+6*A*a*b^2+A*b^3-B*a^3-6*B*a^2*b-2*B*a*b^2
-2*B*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(3*A*a^
3+7*A*a*b^2-7*B*a^2*b-3*B*b^3)/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3-1/2*(2*A*a^3-2*A*a^2*b+6*A*a*b^2-A*b^3+B*a^3-6*B*a^2*b+2*B*a*b^2
-2*B*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3-(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2
)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d
*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(247) = 494$.

Time = 0.17 (sec) , antiderivative size = 1232, normalized size of antiderivative = 4.68

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fri
cas")
```

output
```
[-1/12*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3 + (B*a^3*b^3 - 4*A*
a^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 +
 4*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*a^
3*b^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c)
+ (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*s
in(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)
) - 2*(6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 +
2*B*a^2*b^5 + A*a*b^6 + (B*a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b
^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a
^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6
 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 -
4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a
^5*b^6 + a^3*b^8)*d), 1/6*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3
+ (B*a^3*b^3 - 4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*
b^2 - 4*A*a^3*b^3 + 4*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4
*A*a^4*b^2 + 4*B*a^3*b^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*A*a^7 - 13*B*a
^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(247) = 494$.

Time = 0.20 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.75

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gia
c")
```

output
```
-1/3*(3*(B*a^3 - 4*A*a^2*b + 4*B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2
- b^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(1/2*d*x + 1/2*c)^5
 - 6*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 +
12*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
- 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^
5 + 12*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 +
 3*A*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^
5*tan(1/2*d*x + 1/2*c)^3 - 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^
2*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 28*A*a*b^
4*tan(1/2*d*x + 1/2*c)^3 + 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1
/2*d*x + 1/2*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b*tan(1/2*d*x + 1
/2*c) - 12*B*a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c
) - 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c)
- 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*
B*a*b^4*tan(1/2*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^5*tan(
1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/
2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.71

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,A\,a^3-7\,B\,a^2\,b+7\,A\,a\,b^2-3\,B\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,a^3-A\,b^3+B\,a^3-2\,B\,b^3+6\,A\,a\,b^2-2\,A\,a^2\,b+2\,B\,a\,b^2-6\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}+}{d\,\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)\right.}$$
$$-\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(-B\,a^3+4\,A\,a^2\,b-4\,B\,a\,b^2+A\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

output
```
((4*tan(c/2 + (d*x)/2)^3*(3*A*a^3 - 3*B*b^3 + 7*A*a*b^2 - 7*B*a^2*b))/(3*(
a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)*(2*A*a^3 - A*b^3 + B*a
^3 - 2*B*b^3 + 6*A*a*b^2 - 2*A*a^2*b + 2*B*a*b^2 - 6*B*a^2*b))/((a + b)*(3
*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(c/2 + (d*x)/2)^5*(2*A*a^3 + A*b^3 -
B*a^3 - 2*B*b^3 + 6*A*a*b^2 + 2*A*a^2*b - 2*B*a*b^2 - 6*B*a^2*b))/((a + b)
^3*(a - b)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3
 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a
^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
- (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(
2*(a + b)^(1/2)*(a - b)^(7/2)))*(A*b^3 - B*a^3 + 4*A*a^2*b - 4*B*a*b^2))/(
d*(a + b)^(7/2)*(a - b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.78

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{-12\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,a^2b^2+6\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)}{2d\left(2\cos(dx+c)\right.}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

output
```
( - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*cos(c + d*x)*a**2*b**2 + 6*sqrt(a**2 - b**2)*atan((tan((c
 + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a*b*
*3 - 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*a**3*b - 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - t
an((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a*b**3 + cos(c + d*x)*sin(c + d*x)*a
**4*b + cos(c + d*x)*sin(c + d*x)*a**2*b**3 - 2*cos(c + d*x)*sin(c + d*x)*
b**5 + 2*sin(c + d*x)*a**5 - sin(c + d*x)*a**3*b**2 - sin(c + d*x)*a*b**4)
/(2*d*(2*cos(c + d*x)*a**7*b - 6*cos(c + d*x)*a**5*b**3 + 6*cos(c + d*x)*a
**3*b**5 - 2*cos(c + d*x)*a*b**7 - sin(c + d*x)**2*a**6*b**2 + 3*sin(c + d
*x)**2*a**4*b**4 - 3*sin(c + d*x)**2*a**2*b**6 + sin(c + d*x)**2*b**8 + a*
*8 - 2*a**6*b**2 + 2*a**2*b**6 - b**8))
```

3.277 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 237

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{(Ab-aB)\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\sin(c+dx)}{6(a^2-b^2)^2\,d(a+b\cos(c+dx))^2} - \frac{(11a^2Ab+4Ab^3-2a^3B-13ab^2B)\sin(c+dx)}{6(a^2-b^2)^3\,d(a+b\cos(c+dx))}$$

output
```
(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/
d/(a+b*cos(d*x+c))^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/
d/(a+b*cos(d*x+c))^2-1/6*(11*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*sin(d*x+c
)/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^4} dx$$
$$= \frac{\frac{6(2a^3A+3aAb^2-4a^2bB-b^3B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{2(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3} + \frac{(-5aAb+2a^2B+3b^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} +}{6d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

output
```
((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sin[c + d
*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b
^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + ((-11*a
^2*A*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sin[c + d*x])/((a - b)^3*(a + b)^
3*(a + b*Cos[c + d*x])))/(6*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^4} dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A + B\sin\left(c + dx + \frac{\pi}{2}\right)}{\left(a + b\sin\left(c + dx + \frac{\pi}{2}\right)\right)^4} dx$$
$$\Big\downarrow \text{ 3233}$$

$$-\frac{\int -\frac{3(aA-bB)-2(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^3}dx}{3(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^3}dx}{3(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3233$$

$$\frac{-\frac{\int -\frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)+\left(2Ba^2-5Aba+3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3233$$

$$\frac{\frac{-\frac{\int -\frac{3\left(2Aa^3-4bBa^2+3Ab^2a-b^3B\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

output
```
-1/3*((A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (
-1/2*((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Co
s[c + d*x])^2) + ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b
^2)*d) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/((a^
2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{-\frac{\left(6A\,a^{2}b+3Aa\,b^{2}+2A\,b^{3}-2a^{3}B-2B\,a^{2}b-6Ba\,b^{2}-b^{3}B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{4\left(9A\,a^{2}b+A\,b^{3}-3a^{3}B-7Ba\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}-\frac{(6A\,a}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{3}}}{d}$
default	$\frac{-\frac{\left(6A\,a^{2}b+3Aa\,b^{2}+2A\,b^{3}-2a^{3}B-2B\,a^{2}b-6Ba\,b^{2}-b^{3}B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{(a-b)\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{4\left(9A\,a^{2}b+A\,b^{3}-3a^{3}B-7Ba\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}-\frac{(6A\,a}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{3}}}{d}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^
3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(9*A*a^2*b+A*b
^3-3*B*a^3-7*B*a*b^2)/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
-1/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3+(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a
-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(222) = 444.

Time = 0.16 (sec) , antiderivative size = 1228, normalized size of antiderivative = 5.18

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 + (2*A*a^3*b^3 -
4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a^4*b^2 - 4*B*a^3
*b^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*b - 4*B*a^4*b^2
+ 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c
) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2)) - 2*(6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^
3*b^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (2*B*a^5*b^2 - 11*A*a^4*b^3 + 11
*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a
^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 +
B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a
^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*
a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*
b^6 + a^3*b^8)*d), 1/6*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 +
 (2*A*a^3*b^3 - 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a
^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*
b - 4*B*a^4*b^2 + 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*B*a^7 - 1
8*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(222) = 444.

Time = 0.21 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.92

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^4} dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^
2 - b^2)) - (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2
*c)^5 - 6*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*
c)^5 + 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2
*c)^5 - 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*x + 1/2*
c)^5 + 12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5
+ 3*B*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 36*A*
a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*A*
a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*A*b
^5*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(
1/2*d*x + 1/2*c) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A*a^3*b^2*tan(1/2*d
*x + 1/2*c) + 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x
+ 1/2*c) + 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2
*c) + 12*B*a*b^4*tan(1/2*d*x + 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B
*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 26.88 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.86

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)\,(a^3-3\,a^2\,b+3\,a\,b^2-b^3)}{2\,\sqrt{a+b}\,(a-b)^{7/2}}\right)\,(2\,A\,a^3-4\,B\,a^2\,b+3\,A\,a\,b^2-B\,b^3)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$
$$-\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(-3\,B\,a^3+9\,A\,a^2\,b-7\,B\,a\,b^2+A\,b^3)}{3\,(a+b)^2\,(a^2-2\,a\,b+b^2)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(2\,B\,a^3-2\,A\,b^3+B\,b^3-3\,A\,a\,b^2-6\,A\,a^2\,b+6\,B\,a\,b^2+2\,B\,a^2\,b}{(a+b)^3\,(a-b)}}{d\,\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\right.}$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^4,x)
```

output
```
(atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*
(a + b)^(1/2)*(a - b)^(7/2)))*(2*A*a^3 - B*b^3 + 3*A*a*b^2 - 4*B*a^2*b))/(
d*(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(A*b^3 - 3*B*a^3
 + 9*A*a^2*b - 7*B*a*b^2))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) - (tan(c/2 +
(d*x)/2)^5*(2*B*a^3 - 2*A*b^3 + B*b^3 - 3*A*a*b^2 - 6*A*a^2*b + 6*B*a*b^2
+ 2*B*a^2*b))/((a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)*(2*A*b^3 - 2*B*a^3
 + B*b^3 - 3*A*a*b^2 + 6*A*a^2*b - 6*B*a*b^2 + 2*B*a^2*b))/((a + b)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 +
3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3
 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
+ a^3 - b^3)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.65

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{8\sqrt{a^2-b^2}\, atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos\left(dx+c\right)a^3b+4\sqrt{a^2-b^2}\, atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos}{}$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

output
```
(8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*cos(c + d*x)*a**3*b + 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a*b**3 - 4*sqr
t(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 -
b**2))*sin(c + d*x)**2*a**2*b**2 - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)
/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*b**4 + 4*sq
rt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 -
 b**2))*a**4 + 6*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x
)/2)*b)/sqrt(a**2 - b**2))*a**2*b**2 + 2*sqrt(a**2 - b**2)*atan((tan((c +
d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*b**4 - 3*cos(c + d*x)*s
in(c + d*x)*a**3*b**2 + 3*cos(c + d*x)*sin(c + d*x)*a*b**4 - 4*sin(c + d*x
)*a**4*b + 5*sin(c + d*x)*a**2*b**3 - sin(c + d*x)*b**5)/(2*d*(2*cos(c + d
*x)*a**7*b - 6*cos(c + d*x)*a**5*b**3 + 6*cos(c + d*x)*a**3*b**5 - 2*cos(c
 + d*x)*a*b**7 - sin(c + d*x)**2*a**6*b**2 + 3*sin(c + d*x)**2*a**4*b**4 -
 3*sin(c + d*x)**2*a**2*b**6 + sin(c + d*x)**2*b**8 + a**8 - 2*a**6*b**2 +
 2*a**2*b**6 - b**8))
```

3.278 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 301

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx &= -\frac{(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}\\ &\quad+\frac{A\text{arctanh}(\sin(c+dx))}{a^4d}+\frac{b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^3}\\ &\quad+\frac{b(8a^2Ab-3Ab^3-5a^3B)\sin(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\cos(c+dx))^2}\\ &\quad+\frac{b(26a^4Ab-17a^2Ab^3+6Ab^5-11a^5B-4a^3b^2B)\sin(c+dx)}{6a^3\,(a^2-b^2)^3\,d(a+b\cos(c+dx))}\end{aligned}$$

output

```
-(8*A*a^6*b-8*A*a^4*b^3+7*A*a^2*b^5-2*A*b^7-2*B*a^7-3*B*a^5*b^2)*arctan((a
-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+A*
arctanh(sin(d*x+c))/a^4/d+1/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*co
s(d*x+c))^3+1/6*b*(8*A*a^2*b-3*A*b^3-5*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d
/(a+b*cos(d*x+c))^2+1/6*b*(26*A*a^4*b-17*A*a^2*b^3+6*A*b^5-11*B*a^5-4*B*a^
3*b^2)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.22

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{\cos(c+dx)(B+A\sec(c+dx))\left(\frac{24(-8a^6Ab+8a^4Ab^3-7a^2Ab^5+2Ab^7+2a^7B+3a^5b^2B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - 2\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

output
```
(Cos[c + d*x]*(B + A*Sec[c + d*x])*((24*(-8*a^6*A*b + 8*a^4*A*b^3 - 7*a^2*
A*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2]
)/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 24*A*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 24*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*a*b*(-7
2*a^6*A*b + 38*a^4*A*b^3 - 5*a^2*A*b^5 - 6*A*b^7 + 36*a^7*B + a^5*b^2*B +
8*a^3*b^4*B + 6*a*b*(-20*a^4*A*b + 15*a^2*A*b^3 - 5*A*b^5 + 9*a^5*B + a^3*
b^2*B)*Cos[c + d*x] + b^2*(-26*a^4*A*b + 17*a^2*A*b^3 - 6*A*b^5 + 11*a^5*B
 + 4*a^3*b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[
c + d*x])^3)))/(24*a^4*d*(A + B*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 3479, 3042, 3534, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$\Big\downarrow$ 3479

$$\frac{\int \frac{\left(2b(Ab-aB)\cos^2(c+dx)-3a(Ab-aB)\cos(c+dx)+3A\left(a^2-b^2\right)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3A\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3534

$$\frac{\frac{\int \frac{\left(6A\left(a^2-b^2\right)^2+b\left(-5Ba^3+8Aba^2-3Ab^3\right)\cos^2(c+dx)-2a\left(-3Ba^3+6Aba^2-2b^2Ba-Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b(-5a^3B+8a^2Ab-3Ab^3)\sin(c+}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{6A\left(a^2-b^2\right)^2+b\left(-5Ba^3+8Aba^2-3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\left(-3Ba^3+6Aba^2-2b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b(-5a^3B+8a^2Ab-3Ab^3)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3534

$$\frac{\frac{\frac{\int \frac{3\left(2A\left(a^2-b^2\right)^3-a\left(-2Ba^5+6Aba^4-3b^2Ba^3-2Ab^3a^2+Ab^5\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b\left(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab^3+6Ab^5\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+ b(-}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{\left(2A(a^2-b^2)^3-a\left(-2Ba^5+6Aba^4-3b^2Ba^3-2Ab^3a^2+Ab^5\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{b\left(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab^3+6Ab^5\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b(-\cdots}{\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\int \frac{2A(a^2-b^2)^3-a\left(-2Ba^5+6Aba^4-3b^2Ba^3-2Ab^3a^2+Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b\left(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab^3+6Ab^5\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b(-5a^3B+\cdots}{2ad(a^2-\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3480

$$\frac{\frac{3\left(\frac{2A(a^2-b^2)^3\int\sec(c+dx)dx}{a}-\frac{\left(-2a^7B+8a^6Ab-3a^5b^2B-8a^4Ab^3+7a^2Ab^5-2Ab^7\right)\int\frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a(a^2-b^2)}+\frac{b\left(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab^3+6Ab^5\right)\cdots}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a(a^2-b^2)}$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(\frac{2A(a^2-b^2)^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\left(-2a^7B+8a^6Ab-3a^5b^2B-8a^4Ab^3+7a^2Ab^5-2Ab^7\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a(a^2-b^2)}+\frac{b\left(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab\cdots\right.}{ad(a^2-b^2)(a+b\cos(c+\cdots}}{2a(a^2-b^2)}$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3138

$$\frac{\frac{3\left(\frac{2A\left(a^2-b^2\right)^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2\left(-2a^7B+8a^6Ab-3a^5b^2B-8a^4Ab^3+7a^2Ab^5-2Ab^7\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a\left(a^2-b^2\right)}+\frac{b\left(-11a^5B+26\right.}{}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(\frac{2A\left(a^2-b^2\right)^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2\left(-2a^7B+8a^6Ab-3a^5b^2B-8a^4Ab^3+7a^2Ab^5-2Ab^7\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a\left(a^2-b^2\right)}+\frac{b\left(-11a^5B+26a^4Ab-4a^3b^2B\right.}{ad\left(a^2-b^2\right)(a-}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 4257$$

$$\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

$$\frac{\frac{b(-5a^3B+8a^2Ab-3Ab^3)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{b\left(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab^3+6Ab^5\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{3\left(\frac{2A\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{ad}-\frac{2\left(-2a^7B+\right.}{}\right.}{}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

output
```
(b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
((b*(8*a^2*A*b - 3*A*b^3 - 5*a^3*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a +
b*Cos[c + d*x])^2) + ((3*((-2*(8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A
*b^7 - 2*a^7*B - 3*a^5*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (2*A*(a^2 - b^2)^3*ArcTanh[Sin[c +
 d*x]])/(a*d)))/(a*(a^2 - b^2)) + (b*(26*a^4*A*b - 17*a^2*A*b^3 + 6*A*b^5
 - 11*a^5*B - 4*a^3*b^2*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*
x])))/(2*a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.59

method	result
derivativedivides	$-\frac{2\left(-\frac{\left(12Aa^4b+4Aa^3b^2-6Aa^2b^3-Aab^4+2Ab^5-6Ba^5-3Ba^4b-2Ba^3b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(18Aa^4b-11Aa^2b^3+3Ab^5-9\right.}{3\left(a^2+2ab+b^2\right)}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\right.\right.}$
default	$-\frac{2\left(-\frac{\left(12Aa^4b+4Aa^3b^2-6Aa^2b^3-Aab^4+2Ab^5-6Ba^5-3Ba^4b-2Ba^3b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(18Aa^4b-11Aa^2b^3+3Ab^5-9\right.}{3\left(a^2+2ab+b^2\right)}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\right.\right.}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-2/a^4*((-1/2*(12*A*a^4*b+4*A*a^3*b^2-6*A*a^2*b^3-A*a*b^4+2*A*b^5-6*B
*a^5-3*B*a^4*b-2*B*a^3*b^2)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5-2/3*(18*A*a^4*b-11*A*a^2*b^3+3*A*b^5-9*B*a^5-B*a^3*b^2)*a*b/(a^
2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(12*A*a^4*b-4*A*a^3*
b^2-6*A*a^2*b^3+A*a*b^4+2*A*b^5-6*B*a^5+3*B*a^4*b-2*B*a^3*b^2)*a*b/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3+1/2*(8*A*a^6*b-8*A*a^4*b^3+7*A*a^2*b^5-2*A*b^7-2*
B*a^7-3*B*a^5*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arcta
n((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+A/a^4*ln(tan(1/2*d*x+1/2*
c)+1)-A/a^4*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(285) = 570$.

Time = 32.69 (sec) , antiderivative size = 2269, normalized size of antiderivative = 7.54

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^4} dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fri
cas")
```

output
```
[1/12*(3*(2*B*a^10 - 8*A*a^9*b + 3*B*a^8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 +
 2*A*a^3*b^7 + (2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*B*a^5*b^5 + 8*A*a^4*b^6 - 7*
A*a^2*b^8 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*
a^6*b^4 + 8*A*a^5*b^5 - 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c)^2 + 3*(2*B*a
^9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8
)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*c
os(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*(A*a^11 - 4
*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*
b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 -
4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*
a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c)
)*log(sin(d*x + c) + 1) - 6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*
b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A
*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3
*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5
- 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(18*B*
a^10*b - 36*A*a^9*b^2 - 23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a
^5*b^6 - 2*B*a^4*b^7 + 11*A*a^3*b^8 + (11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a
^6*b^5 + 43*A*a^5*b^6 - 4*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10)*cos(d*...
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^4}\,dx = \int \frac{(A + B\cos{(c + dx)})\sec{(c + dx)}}{(a + b\cos{(c + dx)})^4}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(285) = 570$.

Time = 0.28 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.78

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="gia
c")
```

output
```
1/3*(3*(2*B*a^7 - 8*A*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - 7*A*a^2*b^5 + 2*
A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*
b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) + 3*A*log(abs(tan(1/2*d*x + 1/
2*c) + 1))/a^4 - 3*A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - (18*B*a^7*b*
tan(1/2*d*x + 1/2*c)^5 - 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^6*b^
2*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5*b
^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*b
^4*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^3*
b^5*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 15*A*a*b
^7*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^7*b*ta
n(1/2*d*x + 1/2*c)^3 - 72*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 32*B*a^5*b^3*
tan(1/2*d*x + 1/2*c)^3 + 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^3*b^
5*tan(1/2*d*x + 1/2*c)^3 - 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^8*
tan(1/2*d*x + 1/2*c)^3 + 18*B*a^7*b*tan(1/2*d*x + 1/2*c) - 36*A*a^6*b^2*ta
n(1/2*d*x + 1/2*c) + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^5*b^3*tan(
1/2*d*x + 1/2*c) + 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^4*tan(1/2*
d*x + 1/2*c) + 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c) + 45*A*a^3*b^5*tan(1/2*d*x
 + 1/2*c) + 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^6*tan(1/2*d*x + 1
/2*c) - 15*A*a*b^7*tan(1/2*d*x + 1/2*c) - 6*A*b^8*tan(1/2*d*x + 1/2*c))...
```

Mupad [B] (verification not implemented)

Time = 32.36 (sec) , antiderivative size = 9727, normalized size of antiderivative = 32.32

$$\int \frac{(A + B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^4),x)
```

output
```
(A*atan(-((A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14
- 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^
2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2
*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^
12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9
 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^
6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10
*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (A*((8*(4*A*a^21 + 4*
B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 7
0*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16
*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*
a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6
*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a
^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6
+ 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*A*tan(c/2 + (d
*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 -
 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6
 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b +
a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11
*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4)*1i)...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1406, normalized size of antiderivative = 4.67

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^4}\, dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

output
```
( - 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*cos(c + d*x)*a**5*b**2 + 20*sqrt(a**2 - b**2)*atan((tan((
c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**3*b
**4 - 8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt(a**2 - b**2))*cos(c + d*x)*a*b**6 + 12*sqrt(a**2 - b**2)*atan((tan((c
+ d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**4*
b**3 - 10*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)
/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**2*b**5 + 4*sqrt(a**2 - b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**
2*b**7 - 12*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*
b)/sqrt(a**2 - b**2))*a**6*b - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*
a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**4*b**3 + 6*sqrt(a**2 - b**2)
*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**2*b*
*5 - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*b**7 - 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**7*b +
12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**5*b**3 - 12*cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*a**3*b**5 + 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*a*b**7 + 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**7*b - 12*cos(c + d*x
)*log(tan((c + d*x)/2) + 1)*a**5*b**3 + 12*cos(c + d*x)*log(tan((c + d*x)/
2) + 1)*a**3*b**5 - 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**7 + 5...
```

3.279 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 420

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx \\
&= \frac{b(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^5b^2B-7a^3b^4B+2ab^6B)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} \\
&\quad - \frac{(4Ab-aB)\text{arctanh}(\sin(c+dx))}{a^5d} \\
&\quad + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3d} \\
&\quad + \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b(9a^2Ab-4Ab^3-6a^3B+ab^2B)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2} \\
&\quad + \frac{b(12a^4Ab-11a^2Ab^3+4Ab^5-6a^5B+2a^3b^2B-ab^4B)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^3d(a+b\cos(c+dx))}
\end{aligned}$$

output
```
b*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*b^7-8*B*a^7+8*B*a^5*b^2-7*B*a^
3*b^4+2*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a
-b)^(7/2)/(a+b)^(7/2)/d-(4*A*b-B*a)*arctanh(sin(d*x+c))/a^5/d+1/6*(6*A*a^6
-65*A*a^4*b^2+68*A*a^2*b^4-24*A*b^6+26*B*a^5*b-17*B*a^3*b^3+6*B*a*b^5)*tan
(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*co
s(d*x+c))^3+1/6*b*(9*A*a^2*b-4*A*b^3-6*B*a^3+B*a*b^2)*tan(d*x+c)/a^2/(a^2-
b^2)^2/d/(a+b*cos(d*x+c))^2+1/2*b*(12*A*a^4*b-11*A*a^2*b^3+4*A*b^5-6*B*a^5
+2*B*a^3*b^2-B*a*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 4.94 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.31

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{-\frac{48b(-20a^6Ab+35a^4Ab^3-28a^2Ab^5+8Ab^7+8a^7B-8a^5b^2B+7a^3b^4B-2ab^6B)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}+48(4Ab-aB)\log}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

output
```
((-48*b*(-20*a^6*A*b + 35*a^4*A*b^3 - 28*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 8
*a^5*b^2*B + 7*a^3*b^4*B - 2*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/S
qrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + 48*(4*A*b - a*B)*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 48*(-4*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + (2*a*(24*a^9*A - 36*a^7*A*b^2 - 246*a^5*A*b^4 + 318*a^3*A*b^6
- 120*a*A*b^8 + 120*a^6*b^3*B - 90*a^4*b^5*B + 30*a^2*b^7*B + b*(72*a^8*A
- 438*a^6*A*b^2 + 305*a^4*A*b^4 + 28*a^2*A*b^6 - 72*A*b^8 + 144*a^7*b*B -
50*a^5*b^3*B - 7*a^3*b^5*B + 18*a*b^7*B)*Cos[c + d*x] + 6*a*b^2*(6*a^6*A -
 53*a^4*A*b^2 + 57*a^2*A*b^4 - 20*A*b^6 + 20*a^5*b*B - 15*a^3*b^3*B + 5*a*
b^5*B)*Cos[2*(c + d*x)] + 6*a^6*A*b^3*Cos[3*(c + d*x)] - 65*a^4*A*b^5*Cos[
3*(c + d*x)] + 68*a^2*A*b^7*Cos[3*(c + d*x)] - 24*A*b^9*Cos[3*(c + d*x)] +
 26*a^5*b^4*B*Cos[3*(c + d*x)] - 17*a^3*b^6*B*Cos[3*(c + d*x)] + 6*a*b^8*B
*Cos[3*(c + d*x)])*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(
48*a^5*d)
```

Rubi [A] (verified)

Time = 3.11 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {3042, 3479, 3042, 3534, 3042, 3534, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{(3Aa^2+bBa-3(Ab-aB)\cos(c+dx)a-4Ab^2+3b(Ab-aB)\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-4Ab^2+3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(6Aa^4+8bBa^3-23Ab^2a^2-3b^3Ba-2\left(-3Ba^3+6Aba^2-2b^2Ba-Ab^3\right)\cos(c+dx)a+12Ab^4+2b\left(-6Ba^3+9Aba^2+b^2Ba-4Ab^3\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a(a^2-b^2)}+ b}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{6Aa^4+8bBa^3-23Ab^2a^2-3b^3Ba-2\left(-3Ba^3+6Aba^2-2b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+12Ab^4+2b\left(-6Ba^3+9Aba^2+b^2Ba-4Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b(-6\ldots}{\ldots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\big\downarrow$ 3534

$$\frac{\frac{\int \frac{\left(6Aa^6+26bBa^5-65Ab^2a^4-17b^3Ba^3+68Ab^4a^2+6b^5Ba-\left(-6Ba^5+18Aba^4-8b^2Ba^3-7Ab^3a^2-b^4Ba+4Ab^5\right)\cos(c+dx)a-24Ab^6+3b\left(-6Ba^5+12Aba^4+2b^2B\ldots\right.}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{6Aa^6+26bBa^5-65Ab^2a^4-17b^3Ba^3+68Ab^4a^2+6b^5Ba-\left(-6Ba^5+18Aba^4-8b^2Ba^3-7Ab^3a^2-b^4Ba+4Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-24Ab^6+3b\left(-6Ba^5+12Aba^4+2\ldots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3\ldots}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\big\downarrow$ 3534

$$\frac{\frac{\frac{\int -\frac{3\left(2\left(a^2-b^2\right)^3(4Ab-aB)-ab\left(-6Ba^5+12Aba^4+2b^2Ba^3-11Ab^3a^2-b^4Ba+4Ab^5\right)\cos(c+dx)\right)\sec(c+dx)}{\frac{a+b\cos(c+dx)}{a}}dx+\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2A\ldots\right.}{ad}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-\ldots\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6\right)\tan(c+dx)}{ad}-\frac{3\int \frac{\left(2\left(a^2-b^2\right)^3(4Ab-aB)-ab\left(-6Ba^5+12Aba^4+2b^2Ba^3-11Ab^3a^2-b^4Ba+4A\ldots\right.\right.}{\frac{a+b\cos(c+dx)}{a}}}{\ldots}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-\ldots\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6\right)\tan(c+dx)}{ad}-\frac{3\int\frac{2\left(a^2-b^2\right)^3(4Ab-aB)-ab\left(-6Ba^5+12Aba^4+2b^2Ba^3-11Ab^3a^2-b^4Ba+4Ab^5\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3480

$$\frac{\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3(4Ab-aB)\int\sec(c+dx)dx}{a}-\frac{b\left(-8a^7B+20a^6Ab+8a^5b^2B-35a^4Ab^3\right.}{a}\right.}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3(4Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\left(-8a^7B+20a^6Ab+8a^5b^2B-35a\right.}{a}\right.}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3(4Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\left(-8a^7B+20a^6Ab+8a^5b^2B-35\right.}{}\right.}{a\left(a^2-b^2\right)}}{2a(a}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 218

$$\frac{\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3(4Ab-aB)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\left(-8a^7B+20a^6Ab+8a^5b^2B-35\right.}{a}\right.}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

$$\frac{\frac{b\left(-6a^3B+9a^2Ab+ab^2B-4Ab^3\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{3b\left(-6a^5B+12a^4Ab+2a^3b^2B-11a^2Ab^3-ab^4B+4Ab^5\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B\right.}{}}{}}{}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
((b*(9*a^2*A*b - 4*A*b^3 - 6*a^3*B + a*b^2*B)*Tan[c + d*x])/(2*a*(a^2 - b^
2)*d*(a + b*Cos[c + d*x])^2) + ((3*b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5
- 6*a^5*B + 2*a^3*b^2*B - a*b^4*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*C
os[c + d*x])) + ((-3*((-2*b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8*
A*b^7 - 8*a^7*B + 8*a^5*b^2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTan[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (2*(a^2
 - b^2)^3*(4*A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a*d)))/a + ((6*a^6*A - 65*
a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*a^5*b*B - 17*a^3*b^3*B + 6*a*b^5*
B)*Tan[c + d*x])/(a*d))/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2)))/(3*a*(a^2 - b^
2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{2b\left(-\frac{\left(20Aa^4b+5Aa^3b^2-18Aa^2b^3-2Aab^4+6Ab^5-12Ba^5-4Ba^4b+6Ba^3b^2+Ba^2b^3-2Bab^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(30Aa^4b-\cdots\right.}{\cdots}\cdots\right)}{\cdots}$
default	$\frac{2b\left(-\frac{\left(20Aa^4b+5Aa^3b^2-18Aa^2b^3-2Aab^4+6Ab^5-12Ba^5-4Ba^4b+6Ba^3b^2+Ba^2b^3-2Bab^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{2\left(30Aa^4b-\cdots\right.}{\cdots}\cdots\right)}{\cdots}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(2*b/a^5*((-1/2*(20*A*a^4*b+5*A*a^3*b^2-18*A*a^2*b^3-2*A*a*b^4+6*A*b^5
-12*B*a^5-4*B*a^4*b+6*B*a^3*b^2+B*a^2*b^3-2*B*a*b^4)*a*b/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(30*A*a^4*b-29*A*a^2*b^3+9*A*b^5-1
8*B*a^5+11*B*a^3*b^2-3*B*a*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3-1/2*(20*A*a^4*b-5*A*a^3*b^2-18*A*a^2*b^3+2*A*a*b^4+6*A*b^5-
12*B*a^5+4*B*a^4*b+6*B*a^3*b^2-B*a^2*b^3-2*B*a*b^4)*a*b/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b+a+b)^3+1/2*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*b^7-8*B*a^7+8*
B*a^5*b^2-7*B*a^3*b^4+2*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b
))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-A/a^4/(tan(
1/2*d*x+1/2*c)+1)+1/a^5*(-4*A*b+B*a)*ln(tan(1/2*d*x+1/2*c)+1)-A/a^4/(tan(1
/2*d*x+1/2*c)-1)+(4*A*b-B*a)/a^5*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1662 vs. $2(402) = 804$.

Time = 66.78 (sec) , antiderivative size = 3393, normalized size of antiderivative = 8.08

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx = \int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(402) = 804$.

Time = 0.32 (sec) , antiderivative size = 996, normalized size of antiderivative = 2.37

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
1/3*(3*(8*B*a^7*b - 20*A*a^6*b^2 - 8*B*a^5*b^3 + 35*A*a^4*b^4 + 7*B*a^3*b^
5 - 28*A*a^2*b^6 - 2*B*a*b^7 + 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b
^2)) + (36*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^6*b^3*tan(1/2*d*x + 1
/2*c)^5 - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 105*A*a^5*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^4*b^5*tan(1/2*d*x
 + 1/2*c)^5 + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 117*A*a^3*b^6*tan(1/2*
d*x + 1/2*c)^5 - 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^2*b^7*tan(1/2
*d*x + 1/2*c)^5 - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 42*A*a*b^8*tan(1/2
*d*x + 1/2*c)^5 + 6*B*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 18*A*b^9*tan(1/2*d*x
+ 1/2*c)^5 + 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^6*b^3*tan(1/2*d
*x + 1/2*c)^3 - 116*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 236*A*a^4*b^5*tan(1
/2*d*x + 1/2*c)^3 + 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 152*A*a^2*b^7*ta
n(1/2*d*x + 1/2*c)^3 - 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^9*tan(1/
2*d*x + 1/2*c)^3 + 36*B*a^7*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^6*b^3*tan(1/
2*d*x + 1/2*c) + 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) - 105*A*a^5*b^4*tan(1/2
*d*x + 1/2*c) - 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c) + 24*A*a^4*b^5*tan(1/2*d*
x + 1/2*c) - 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c) + 117*A*a^3*b^6*tan(1/2*d*x
 + 1/2*c) - 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b^7*tan(1/2*d*x...
```

Mupad [B] (verification not implemented)

Time = 41.46 (sec) , antiderivative size = 13119, normalized size of antiderivative = 31.24

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(18*A*a^8 + 72*A*b^8 - 236*A*a^2*b^6 + 47*A*a^3*b^5
 + 273*A*a^4*b^4 - 60*A*a^5*b^3 - 72*A*a^6*b^2 + 3*B*a^2*b^6 + 59*B*a^3*b^
5 - 14*B*a^4*b^4 - 96*B*a^5*b^3 + 36*B*a^6*b^2 - 12*A*a*b^7 - 18*B*a*b^7))
/(3*a^4*(a + b)^2*(a - b)^3) - (tan(c/2 + (d*x)/2)^7*(24*A*a^2*b^5 - 8*A*b
^7 - 2*A*a^7 - 11*A*a^3*b^4 - 26*A*a^4*b^3 + 6*A*a^5*b^2 - B*a^2*b^5 - 6*B
*a^3*b^4 + 4*B*a^4*b^3 + 12*B*a^5*b^2 + 4*A*a*b^6 + 2*A*a^6*b + 2*B*a*b^6)
)/(a^4*(a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)^5*(18*A*a^8 + 72*A*b^8 - 2
36*A*a^2*b^6 - 47*A*a^3*b^5 + 273*A*a^4*b^4 + 60*A*a^5*b^3 - 72*A*a^6*b^2
- 3*B*a^2*b^6 + 59*B*a^3*b^5 + 14*B*a^4*b^4 - 96*B*a^5*b^3 - 36*B*a^6*b^2
+ 12*A*a*b^7 - 18*B*a*b^7))/(3*a^4*(a + b)^3*(a - b)^2) + (tan(c/2 + (d*x)
/2)*(2*A*a^7 - 8*A*b^7 + 24*A*a^2*b^5 + 11*A*a^3*b^4 - 26*A*a^4*b^3 - 6*A*
a^5*b^2 + B*a^2*b^5 - 6*B*a^3*b^4 - 4*B*a^4*b^3 + 12*B*a^5*b^2 - 4*A*a*b^6
 + 2*A*a^6*b + 2*B*a*b^6))/(a^4*(a + b)*(a - b)^3))/(d*(3*a*b^2 + 3*a^2*b
- tan(c/2 + (d*x)/2)^4*(6*a^2*b - 6*b^3) - tan(c/2 + (d*x)/2)^2*(6*a*b^2 -
 2*a^3 + 4*b^3) - tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b
^3 - tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (atan((((((4
*A*b - B*a)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^
12 + 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 +
174*A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a
^21*b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^1...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2086, normalized size of antiderivative = 4.97

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

output

```
(24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(
a**2 - b**2))*cos(c + d*x)*sin(c + d*x)**2*a**4*b**4 - 30*sqrt(a**2 - b**2
)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c
+ d*x)*sin(c + d*x)**2*a**2*b**6 + 12*sqrt(a**2 - b**2)*atan((tan((c + d*x
)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*sin(c + d*x)*
*2*b**8 - 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)
*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**6*b**2 + 6*sqrt(a**2 - b**2)*atan((
tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a
**4*b**4 + 18*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2
)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**2*b**6 - 12*sqrt(a**2 - b**2)*atan
((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)
*b**8 + 48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b
)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**5*b**3 - 60*sqrt(a**2 - b**2)*atan
((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)
**2*a**3*b**5 + 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d
*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a*b**7 - 48*sqrt(a**2 - b**2)
*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a**5*b*
*3 + 60*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/s
qrt(a**2 - b**2))*a**3*b**5 - 24*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*
a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*a*b**7 + 6*cos(c + d*x)*log(...
```

3.280 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 547

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx =$$

$$-\frac{b^2(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-20a^7B+35a^5b^2B-28a^3b^4B+8ab^6B)\arctan\left(\frac{\sqrt{a-b}\tan(}{\sqrt{a-}}\right.}{a^6(a-b)^{7/2}(a+b)^{7/2}d}$$

$$+\frac{(a^2A+20Ab^2-8abB)\operatorname{arctanh}(\sin(c+dx))}{2a^6d}$$

$$-\frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B+24ab^6B)\tan(c+dx)}{6a^5(a^2-b^2)^3d}$$

$$+\frac{(a^6A-23a^4Ab^2+27a^2Ab^4-10Ab^6+12a^5bB-11a^3b^3B+4ab^5B)\sec(c+dx)\tan(c+dx)}{2a^4(a^2-b^2)^3d}$$

$$+\frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^3}$$

$$+\frac{b(10a^2Ab-5Ab^3-7a^3B+2ab^2B)\sec(c+dx)\tan(c+dx)}{6a^2(a^2-b^2)^2d(a+b\cos(c+dx))^2}$$

$$+\frac{b(48a^4Ab-53a^2Ab^3+20Ab^5-27a^5B+20a^3b^2B-8ab^4B)\sec(c+dx)\tan(c+dx)}{6a^3(a^2-b^2)^3d(a+b\cos(c+dx))}$$

output
```
-b^2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^5-20*A*b^7-20*B*a^7+35*B*a^5*b^2-
28*B*a^3*b^4+8*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))
/a^6/(a-b)^(7/2)/(a+b)^(7/2)/d+1/2*(A*a^2+20*A*b^2-8*B*a*b)*arctanh(sin(d*
x+c))/a^6/d-1/6*(24*A*a^6*b-146*A*a^4*b^3+167*A*a^2*b^5-60*A*b^7-6*B*a^7+6
5*B*a^5*b^2-68*B*a^3*b^4+24*B*a*b^6)*tan(d*x+c)/a^5/(a^2-b^2)^3/d+1/2*(A*a
^6-23*A*a^4*b^2+27*A*a^2*b^4-10*A*b^6+12*B*a^5*b-11*B*a^3*b^3+4*B*a*b^5)*s
ec(d*x+c)*tan(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*sec(d*x+c)*tan(d*x+
c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b*(10*A*a^2*b-5*A*b^3-7*B*a^3+2*B*
a*b^2)*sec(d*x+c)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*b*(4
8*A*a^4*b-53*A*a^2*b^3+20*A*b^5-27*B*a^5+20*B*a^3*b^2-8*B*a*b^4)*sec(d*x+c
)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Mathematica [A] (verified)

Time = 5.60 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \frac{\frac{96b^2(-40a^6Ab+84a^4Ab^3-69a^2Ab^5+20Ab^7+20a^7B-35a^5b^2B+28a^3b^4B-8ab^6B)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - 48(a^2A+20Ab}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

output
```
((96*b^2*(-40*a^6*A*b + 84*a^4*A*b^3 - 69*a^2*A*b^5 + 20*A*b^7 + 20*a^7*B
- 35*a^5*b^2*B + 28*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/
2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 48*(a^2*A + 20*A*b^2 - 8*a*b*B
)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 48*(a^2*A + 20*A*b^2 - 8*a*b*
B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*(24*a^10*A - 324*a^8*A*
b^2 + 1116*a^6*A*b^4 - 830*a^4*A*b^6 - 61*a^2*A*b^8 + 180*A*b^10 + 72*a^9*
b*B - 438*a^7*b^3*B + 305*a^5*b^5*B + 28*a^3*b^7*B - 72*a*b^9*B + 6*a*(-20
*a^8*A*b - 9*a^6*A*b^3 + 309*a^4*A*b^5 - 400*a^2*A*b^7 + 150*A*b^9 + 8*a^9
*B - 6*a^7*b^2*B - 135*a^5*b^4*B + 163*a^3*b^6*B - 60*a*b^8*B)*Cos[c + d*x
] + 12*b*(-21*a^8*A*b + 85*a^6*A*b^3 - 55*a^4*A*b^5 - 19*a^2*A*b^7 + 20*A*
b^9 + 6*a^9*B - 36*a^7*b^2*B + 20*a^5*b^4*B + 8*a^3*b^6*B - 8*a*b^8*B)*Cos
[2*(c + d*x)] - 138*a^7*A*b^3*Cos[3*(c + d*x)] + 738*a^5*A*b^5*Cos[3*(c +
d*x)] - 840*a^3*A*b^7*Cos[3*(c + d*x)] + 300*a*A*b^9*Cos[3*(c + d*x)] + 36
*a^8*b^2*B*Cos[3*(c + d*x)] - 318*a^6*b^4*B*Cos[3*(c + d*x)] + 342*a^4*b^6
*B*Cos[3*(c + d*x)] - 120*a^2*b^8*B*Cos[3*(c + d*x)] - 24*a^6*A*b^4*Cos[4*
(c + d*x)] + 146*a^4*A*b^6*Cos[4*(c + d*x)] - 167*a^2*A*b^8*Cos[4*(c + d*x
)] + 60*A*b^10*Cos[4*(c + d*x)] + 6*a^7*b^3*B*Cos[4*(c + d*x)] - 65*a^5*b^
5*B*Cos[4*(c + d*x)] + 68*a^3*b^7*B*Cos[4*(c + d*x)] - 24*a*b^9*B*Cos[4*(c
 + d*x)])*Sec[c + d*x]*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3
))/(96*a^6*d)
```

Rubi [A] (verified)

Time = 4.05 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 3479, 3042, 3534, 3042, 3534, 3042, 3534, 27, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{\left(3Aa^2+2bBa-3(Ab-aB)\cos(c+dx)a-5Ab^2+4b(Ab-aB)\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\int \frac{3Aa^2+2bBa-3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-5Ab^2+4b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3534

$$\frac{\frac{\int \frac{\left(3b\left(-7Ba^3+10Aba^2+2b^2Ba-5Ab^3\right)\cos^2(c+dx)-2a\left(-3Ba^3+6Aba^2-2b^2Ba-Ab^3\right)\cos(c+dx)+2\left(3Aa^4+9bBa^3-18Ab^2a^2-4b^3Ba+10Ab^4\right)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{3b\left(-7Ba^3+10Aba^2+2b^2Ba-5Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\left(-3Ba^3+6Aba^2-2b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(3Aa^4+9bBa^3-18Ab^2a^2-4b^3Ba+10Ab^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3534

$$\frac{\frac{\int \frac{\left(2b\left(-27Ba^5+48Aba^4+20b^2Ba^3-53Ab^3a^2-8b^4Ba+20Ab^5\right)\cos^2(c+dx)-a\left(-6Ba^5+18Aba^4-7b^2Ba^3-8Ab^3a^2-2b^4Ba+5Ab^5\right)\cos(c+dx)+6\left(Aa^6+12bBa^5\right.}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\int \frac{2b\left(-27Ba^5+48Aba^4+20b^2Ba^3-53Ab^3a^2-8b^4Ba+20Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(-6Ba^5+18Aba^4-7b^2Ba^3-8Ab^3a^2-2b^4Ba+5Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)+6\left(Aa^6+\ldots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}$$

$$\overline{\quad a\left(a^2-b^2\right)\quad}$$

$$\overline{\quad 2a(a^2-b^2)\quad}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \; 3534$$

$$\int -\frac{2\left(-6Ba^7+24Aba^6+65b^2Ba^5-146Ab^3a^4-68b^4Ba^3+167Ab^5a^2+24b^6Ba-\left(3Aa^6-18bBa^5+27Ab^2a^4+7b^3Ba^3-25Ab^4a^2-4b^5Ba+10Ab^6\right)\cos(c+dx)a-60A\ldots\right.}{a+b\cos(c+dx)}$$

$$\overline{\quad 2a \quad}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad}-\int \frac{\left(-6Ba^7+24Aba^6+65b^2Ba^5-146Ab^3a^4-68b^4Ba^3+167Ab^5a^2+24b^6\ldots\right.}{\ldots}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\int \frac{-6Ba^7+24Aba^6+65b^2Ba^5-146Ab^3a^4-68b^4Ba^3+167Ab^5a^2+24b^6B\ldots}{\ldots}}{\ldots}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^3}$$

$$\Big\downarrow \; 3534$$

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad} - \frac{\int -\frac{3\left(\left(Aa^2-8bBa+20Ab^2\right)\left(a^2-b^2\right)^3+ab\left(Aa^6+12bBa^5-23Ab^2a^4-\right.\right.}{a+b\cos(c+}}{a\left(a^2-\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 27

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad} - \frac{\frac{\left(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6\right.}{ad}}{a\left(a^2-\right.}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad} - \frac{\frac{\left(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6\right.}{ad}}{a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3480

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad} - \frac{\left(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6\right.}{ad}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad} - \frac{\left(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6\right.}{ad}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\big\downarrow$ 3138

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad} - \frac{\left(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6\right.}{ad}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\big\downarrow$ 218

$$\frac{3\left(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6\right)\tan(c+dx)\sec(c+dx)}{ad} - \frac{\left(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6\right.}{ad}$$

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\big\downarrow$ 4257

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

$$\frac{b\left(-7a^3B+10a^2Ab+2ab^2B-5Ab^3\right)\tan(c+dx)\sec(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{b\left(-27a^5B+48a^4Ab+20a^3b^2B-53a^2Ab^3-8ab^4B+20Ab^5\right)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{3\left(a^6A+\right.}{}}{}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

output
```
(b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^3) + ((b*(10*a^2*A*b - 5*A*b^3 - 7*a^3*B + 2*a*b^2*B)*Sec[c + d*x]
*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((b*(48*a^4*A*
b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B + 20*a^3*b^2*B - 8*a*b^4*B)*Sec[c +
 d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + ((3*(a^6*A -
23*a^4*A*b^2 + 27*a^2*A*b^4 - 10*A*b^6 + 12*a^5*b*B - 11*a^3*b^3*B + 4*a*b
^5*B)*Sec[c + d*x]*Tan[c + d*x])/(a*d) - ((-3*((-2*b^2*(40*a^6*A*b - 84*a^
4*A*b^3 + 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35*a^5*b^2*B - 28*a^3*b^4*B
 + 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[
a - b]*Sqrt[a + b]*d) + ((a^2 - b^2)^3*(a^2*A + 20*A*b^2 - 8*a*b*B)*ArcTan
h[Sin[c + d*x]])/(a*d)))/a + ((24*a^6*A*b - 146*a^4*A*b^3 + 167*a^2*A*b^5
- 60*A*b^7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^6*B)*Tan[c + d
*x])/(a*d))/a)/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{A}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-Aa-8Ab+2Ba}{2a^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2A-20A\,b^2+8Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^6}-\frac{2b^2\left(-\frac{\left(30A\,a^4b+6A\,a^3b^2-34A\right.}{}\right.}{}}{}$
default	$\frac{\frac{A}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-Aa-8Ab+2Ba}{2a^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2A-20A\,b^2+8Bab\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^6}-\frac{2b^2\left(-\frac{\left(30A\,a^4b+6A\,a^3b^2-34A\right.}{}\right.}{}}{}$
risch	Expression too large to display

input

```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBO
SE)
```

output

```
1/d*(1/2*A/a^4/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-A*a-8*A*b+2*B*a)/a^5/(tan(1/
2*d*x+1/2*c)-1)+1/2/a^6*(-A*a^2-20*A*b^2+8*B*a*b)*ln(tan(1/2*d*x+1/2*c)-1)
-2*b^2/a^6*((-1/2*(30*A*a^4*b+6*A*a^3*b^2-34*A*a^2*b^3-3*A*a*b^4+12*A*b^5-
20*B*a^5-5*B*a^4*b+18*B*a^3*b^2+2*B*a^2*b^3-6*B*a*b^4)*a*b/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(45*A*a^4*b-53*A*a^2*b^3+18*A*b^
5-30*B*a^5+29*B*a^3*b^2-9*B*a*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3-1/2*(30*A*a^4*b-6*A*a^3*b^2-34*A*a^2*b^3+3*A*a*b^4+12*A*
b^5-20*B*a^5+5*B*a^4*b+18*B*a^3*b^2-2*B*a^2*b^3-6*B*a*b^4)*a*b/(a+b)/(a^3-
3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b+a+b)^3+1/2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^5-20*A*b^7-20
*B*a^7+35*B*a^5*b^2-28*B*a^3*b^4+8*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-
1/2*A/a^4/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-A*a-8*A*b+2*B*a)/a^5/(tan(1/2*d*x
+1/2*c)+1)+1/2*(A*a^2+20*A*b^2-8*B*a*b)/a^6*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. 2(525) = 1050.

Time = 94.70 (sec) , antiderivative size = 3819, normalized size of antiderivative = 6.98

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec^3{(c+dx)}}{(a+b\cos{(c+dx)})^4}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1090 vs. $2(525) = 1050$.

Time = 0.29 (sec) , antiderivative size = 1090, normalized size of antiderivative = 1.99

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + b\cos(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-1/6*(6*(20*B*a^7*b^2 - 40*A*a^6*b^3 - 35*B*a^5*b^4 + 84*A*a^4*b^5 + 28*B*
a^3*b^6 - 69*A*a^2*b^7 - 8*B*a*b^8 + 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt
(a^2 - b^2)) + 2*(60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 90*A*a^6*b^4*tan(1
/2*d*x + 1/2*c)^5 - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 162*A*a^5*b^5*t
an(1/2*d*x + 1/2*c)^5 - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*b^6
*tan(1/2*d*x + 1/2*c)^5 + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 213*A*a^3
*b^7*tan(1/2*d*x + 1/2*c)^5 - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 48*A*a
^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 81*A
*a*b^9*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 36*A*b
^10*tan(1/2*d*x + 1/2*c)^5 + 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 - 180*A*
a^6*b^4*tan(1/2*d*x + 1/2*c)^3 - 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 + 39
2*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 + 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3
- 284*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^9*tan(1/2*d*x + 1/2*c)^3
 + 72*A*b^10*tan(1/2*d*x + 1/2*c)^3 + 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) -
90*A*a^6*b^4*tan(1/2*d*x + 1/2*c) + 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 1
62*A*a^5*b^5*tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c) + 48
*A*a^4*b^6*tan(1/2*d*x + 1/2*c) - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) + 213
*A*a^3*b^7*tan(1/2*d*x + 1/2*c) - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c) + 4...
```

Mupad [B] (verification not implemented)

Time = 34.42 (sec) , antiderivative size = 14398, normalized size of antiderivative = 26.32

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)*(A*a^8 + 20*A*b^8 + 2*B*a^8 - 59*A*a^2*b^6 - 27*A*a^3
*b^5 + 57*A*a^4*b^4 + 21*A*a^5*b^3 - 11*A*a^6*b^2 - 4*B*a^2*b^6 + 24*B*a^3
*b^5 + 11*B*a^4*b^4 - 26*B*a^5*b^3 - 6*B*a^6*b^2 + 10*A*a*b^7 - 7*A*a^7*b
- 8*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b)*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^
5*(9*A*a^10 + 180*A*b^10 - 611*A*a^2*b^8 + 740*A*a^4*b^6 - 324*A*a^6*b^4 +
 36*A*a^8*b^2 + 248*B*a^3*b^7 - 320*B*a^5*b^5 + 132*B*a^7*b^3 - 72*B*a*b^9
 - 18*B*a^9*b))/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)^9*(A*a^8
 + 20*A*b^8 - 2*B*a^8 - 59*A*a^2*b^6 + 27*A*a^3*b^5 + 57*A*a^4*b^4 - 21*A*
a^5*b^3 - 11*A*a^6*b^2 + 4*B*a^2*b^6 + 24*B*a^3*b^5 - 11*B*a^4*b^4 - 26*B*
a^5*b^3 + 6*B*a^6*b^2 - 10*A*a*b^7 + 7*A*a^7*b - 8*B*a*b^7 + 2*B*a^7*b))/(
a^5*(a + b)^3*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*A*a^9 - 120*A*b^9 + 6*
B*a^9 + 364*A*a^2*b^7 + 71*A*a^3*b^6 - 369*A*a^4*b^5 - 45*A*a^5*b^4 + 111*
A*a^6*b^3 + 3*A*a^7*b^2 + 12*B*a^2*b^7 - 148*B*a^3*b^6 - 29*B*a^4*b^5 + 15
9*B*a^5*b^4 + 18*B*a^6*b^3 - 30*B*a^7*b^2 - 30*A*a*b^8 - 21*A*a^8*b + 48*B
*a*b^8 - 6*B*a^8*b))/(3*a^5*(a + b)^2*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^7
*(6*A*a^9 + 120*A*b^9 - 6*B*a^9 - 364*A*a^2*b^7 + 71*A*a^3*b^6 + 369*A*a^4
*b^5 - 45*A*a^5*b^4 - 111*A*a^6*b^3 + 3*A*a^7*b^2 + 12*B*a^2*b^7 + 148*B*a
^3*b^6 - 29*B*a^4*b^5 - 159*B*a^5*b^4 + 18*B*a^6*b^3 + 30*B*a^7*b^2 - 30*A
*a*b^8 + 21*A*a^8*b - 48*B*a*b^8 - 6*B*a^8*b))/(3*a^5*(a + b)^3*(a - b)^2)
)/(d*(tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 + 10*b^3) - tan(c...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3080, normalized size of antiderivative = 5.63

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x)
```

output
```
( - 80*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sq
rt(a**2 - b**2))*cos(c + d*x)*sin(c + d*x)**2*a**5*b**4 + 116*sqrt(a**2 -
b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*co
s(c + d*x)*sin(c + d*x)**2*a**3*b**6 - 48*sqrt(a**2 - b**2)*atan((tan((c +
 d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*sin(c + d
*x)**2*a*b**8 + 80*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d
*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a**5*b**4 - 116*sqrt(a**2 - b**2
)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c
+ d*x)*a**3*b**6 + 48*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c
+ d*x)/2)*b)/sqrt(a**2 - b**2))*cos(c + d*x)*a*b**8 + 40*sqrt(a**2 - b**2)
*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c +
 d*x)**4*a**4*b**5 - 58*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((
c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**4*a**2*b**7 + 24*sqrt(a**2
 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))
*sin(c + d*x)**4*b**9 - 40*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - ta
n((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**6*b**3 - 22*sqrt(a
**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**
2))*sin(c + d*x)**2*a**4*b**5 + 92*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2
)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*sin(c + d*x)**2*a**2*b**7 - 4
8*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt...
```

3.281 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}$$

output
```
B*sin(d*x+c)/d-1/3*B*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx=B\left(\frac{\sin(c+dx)}{d}-\frac{\sin^3(c+dx)}{3d}\right)$$

input
```
Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x
]
```

output
```
B*(Sin[c + d*x]/d - Sin[c + d*x]^3/(3*d))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2011, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$-\frac{B\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
-((B*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{B\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}$	23
default	$\frac{B\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3d}$	23
parallelrisch	$\frac{B(9\sin(dx+c)+\sin(3dx+3c))}{12d}$	25
risch	$\frac{3B\sin(dx+c)}{4d}+\frac{B\sin(3dx+3c)}{12d}$	29
norman	$\frac{\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{10B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{10B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$	84

input
```
int(cos(d*x+c)^3*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
1/3/d*B*(cos(d*x+c)^2+2)*sin(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{\left(B\cos\left(dx+c\right)^2+2\,B\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/3*(B*cos(d*x + c)^2 + 2*B)*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \begin{cases} \frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d\neq 0 \\ \frac{x(Ba+Bb\cos(c))\cos^3(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((2*B*sin(c + d*x)**3/(3*d) + B*sin(c + d*x)*cos(c + d*x)**2/d, N
e(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**3/(a + b*cos(c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = -\frac{B\sin(dx+c)^3-3\,B\sin(dx+c)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
-1/3*(B*sin(d*x + c)^3 - 3*B*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 24.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\,(9\,\sin(c+d\,x)+\sin(3\,c+3\,d\,x))}{12\,d}$$

input
```
int((cos(c + d*x)^3*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
(B*(9*sin(c + d*x) + sin(3*c + 3*d*x)))/(12*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{\sin(dx+c)\,b\left(-\sin(dx+c)^2+3\right)}{3d}$$

input
```
int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
(sin(c + d*x)*b*( - sin(c + d*x)**2 + 3))/(3*d)
```

3.282 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 27

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{Bx}{2} + \frac{B\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*B*x+1/2*B*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B(2(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x
]
```

output
```
(B*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2011, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$B\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{24}$$

$$B\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

input
```
Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
B*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{Bx}{2}+\frac{B\sin(2dx+2c)}{4d}$	21
parallelrisch	$\frac{B(2dx+\sin(2dx+2c))}{4d}$	21
derivativedivides	$\frac{B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	28
default	$\frac{B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	28
norman	$\frac{\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{Bx}{2}-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2}+\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$	98

input
```
int(cos(d*x+c)^2*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
1/2*B*x+1/4*B/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{Bdx+B\cos{(dx+c)}\sin{(dx+c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/2*(B*d*x + B*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(22) = 44.

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.52

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \begin{cases} \frac{Bx\sin^2{(c+dx)}}{2}+\frac{Bx\cos^2{(c+dx)}}{2}+\frac{B\sin{(c+dx)}\cos{(c+dx)}}{2d} & \text{for } d\neq 0 \\ \frac{x(Ba+Bb\cos{(c)})\cos^2{(c)}}{a+b\cos{(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((B*x*sin(c + d*x)**2/2 + B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*
cos(c + d*x)/(2*d), Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**2/(a + b*cos(
c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{(dx+c)B+\frac{B\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
1/2*((d*x + c)*B + B*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 24.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\,x}{2}+\frac{B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int((cos(c + d*x)^2*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
(B*x)/2 + (B*tan(c/2 + (d*x)/2) - B*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d
*x)/2)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\frac{b(\cos(dx+c)\sin(dx+c)+dx)}{2d}$$

input
```
int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
(b*(cos(c + d*x)*sin(c + d*x) + d*x))/(2*d)
```

3.283 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 11

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)} \, dx = \frac{B\sin(c+dx)}{d}$$

output
```
B*sin(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)} \, dx = B\left(\frac{\cos(dx)\sin(c)}{d}+\frac{\cos(c)\sin(dx)}{d}\right)$$

input
```
Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
B*((Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2011, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cos(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{B\sin(c+dx)}{d}$$

input
```
Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(B*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (warning: unable to verify)

Time = 1.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{B\sin(dx+c)}{d}$	12
default	$\frac{B\sin(dx+c)}{d}$	12
risch	$\frac{B\sin(dx+c)}{d}$	12
parallelrisch	$\frac{B\sin(dx+c)}{d}$	12
norman	$\frac{\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	50

input
```
int(cos(d*x+c)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVERBO
SE)
```

output
```
B*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\frac{B\sin{(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
B*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \begin{cases} \frac{B\sin{(c+dx)}}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos{(c)})\cos{(c)}}{a+b\cos{(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((B*sin(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)/(a +
b*cos(c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\sin{(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
B*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 24.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\,\sin{(c+d\,x)}}{d}$$

input
```
int((cos(c + d*x)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
(B*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{\sin{(dx+c)}\,b}{d}$$

input
```
int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
(sin(c + d*x)*b)/d
```

3.284 $\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 3

$$\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx=Bx$$

output
```
B*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx=Bx$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
B*x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2011, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int 1dx$$

$$\Big\downarrow \text{ 24}$$

$$Bx$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
B*x
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

Maple [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	Bx	4
risch	Bx	4
derivativedivides	$\frac{B(dx+c)}{d}$	11
orering	$\frac{x(Ba+bB\cos(dx+c))}{a+\cos(dx+c)b}$	28
norman	$\frac{Bx+Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	35

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
B*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx = Bx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
B*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int \frac{aB + bB\cos(c + dx)}{a + b\cos(c + dx)}\,dx = Bx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
B*x
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{aB + bB\cos(c + dx)}{a + b\cos(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 3.33

$$\int \frac{aB + bB\cos(c + dx)}{a + b\cos(c + dx)}\,dx = \frac{(dx + c)B}{d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
(d*x + c)*B/d
```

Mupad [B] (verification not implemented)

Time = 24.54 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB + bB\cos(c + dx)}{a + b\cos(c + dx)}\,dx = B\,x$$

input
```
int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x)),x)
```

output
```
B*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB + bB\cos(c + dx)}{a + b\cos(c + dx)}\,dx = bx$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
b*x
```

3.285 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 12

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\operatorname{arctanh}(\sin(c+dx))}{d}$$

output
```
B*arctanh(sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\coth^{-1}(\sin(c+dx))}{d}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
(B*ArcCoth[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2011, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \sec(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{4257}$$
$$\frac{B\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
(B*ArcTanh[Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

method	result	size
derivativedivides	$\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d}$	20
default	$\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d}$	20
parallelrisch	$\frac{B\left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}$	33
norman	$\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	37
risch	$\frac{B\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{B\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$	39

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b),x,method=_RETURNVERBO
SE)
```

output
```
1/d*B*ln(sec(d*x+c)+tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\log\left(\sin\left(dx+c\right)+1\right)-B\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/2*(B*log(sin(d*x + c) + 1) - B*log(-sin(d*x + c) + 1))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(10) = 20.

Time = 1.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.25

$$\int \frac{(aB + bB\cos(c + dx))\sec(c + dx)}{a + b\cos(c + dx)}\,dx = \begin{cases} \frac{B\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for}\ d \neq 0 \\ \frac{x(Ba+Bb\cos\left(c\right))\sec\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((B*log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*b*
cos(c))*sec(c)/(a + b*cos(c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(aB + bB\cos(c + dx))\sec(c + dx)}{a + b\cos(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(12) = 24$.

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.92

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx+c) + 2\right|\right) - B\log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx+c) - 2\right|\right)}{4\,d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
1/4*(B*log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - B*log(abs(1/sin(d*x +
 c) + sin(d*x + c) - 2)))/d
```

Mupad [B] (verification not implemented)

Time = 24.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d}$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

output
```
(2*B*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

output
```
(b*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)))/d
```

3.286 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 11

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\tan(c+dx)}{d}$$

output
```
B*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\tan(c+dx)}{d}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x
]
```

output
```
(B*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2011, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \sec^2(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{4254}$$
$$-\frac{B\int 1d(-\tan(c+dx))}{d}$$
$$\Big\downarrow \text{24}$$
$$\frac{B\tan(c+dx)}{d}$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

output
```
(B*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{B\tan(dx+c)}{d}$	12
default	$\frac{B\tan(dx+c)}{d}$	12
parallelrisch	$\frac{\sin(dx+c)B}{\cos(dx+c)d}$	20
risch	$\frac{2iB}{d\left(e^{2i(dx+c)}+1\right)}$	21
norman	$\frac{-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$	65

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
B*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{a + b\cos(c + dx)}\,dx = \frac{B\sin(dx + c)}{d\cos(dx + c)}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
B*sin(d*x + c)/(d*cos(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(8) = 16.

Time = 1.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{a + b\cos(c + dx)}\,dx = \begin{cases} \frac{B\tan(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\sec^2(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((B*tan(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*sec(c)**2/(a
 + b*cos(c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(aB + bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{(aB + bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\tan(dx+c)}{d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
B*tan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 24.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \frac{(aB + bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

output
```
-(2*B*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{a + b\cos(c + dx)}\,dx = \frac{\sin(dx + c)\,b}{\cos(dx + c)\,d}$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

output
```
(sin(c + d*x)*b)/(cos(c + d*x)*d)
```

3.287 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 36

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{B\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{B\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*B*arctanh(sin(d*x+c))/d+1/2*B*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx=B\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\sec(c+dx)\tan(c+dx)}{2d}\right)$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x
]
```

output
```
B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2011, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$B\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4257}$$

$$B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

output
```
B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	37
default	$\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	37
parallelrisch	$-\frac{B\left((\cos(2dx+2c)+1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\sin(dx+c)\right)}{2d(\cos(2dx+2c)+1)}$	79
risch	$-\frac{iB\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{B\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{B\ln\left(e^{i(dx+c)}-i\right)}{2d}$	81
norman	$\frac{\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{2}}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	117

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
1/d*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{(aB + bB\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx$$
$$= \frac{B\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)} - B\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)} + 2\,B\sin{(dx+c)}}{4\,d\cos{(dx+c)}^2}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/4*(B*cos(d*x + c)^2*log(sin(d*x + c) + 1) - B*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) + 2*B*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx = B\int \sec^3{(c + dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

output
```
B*Integral(sec(c + d*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(aB + bB\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int \frac{(aB + bB\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx = \frac{B\log(|\sin(dx + c) + 1|) - B\log(|\sin(dx + c) - 1|) - \frac{2B\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
1/4*(B*log(abs(sin(d*x + c) + 1)) - B*log(abs(sin(d*x + c) - 1)) - 2*B*sin
(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 24.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.03

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} + \frac{B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

output
```
(B*tan(c/2 + (d*x)/2) + B*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 -
 2*tan(c/2 + (d*x)/2)^2 + 1)) + (B*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 + \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sin(dx+c)^2 -}{2d\left(\sin(dx+c)^2-1\right)}$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

output
```
(b*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c + d*x)/2) -
1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d*x)/2) + 1)
 - sin(c + d*x)))/(2*d*(sin(c + d*x)**2 - 1))
```

3.288 $\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{B\tan(c+dx)}{d}+\frac{B\tan^3(c+dx)}{3d}$$

output
```
B*tan(d*x+c)/d+1/3*B*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{B\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x
]
```

output
```
(B*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2011, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{B\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

output
```
-((B*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{B\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}$	25
default	$-\frac{B\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}$	25
risch	$\frac{4iB\left(3\,e^{2i(dx+c)}+1\right)}{3d\left(e^{2i(dx+c)}+1\right)^{3}}$	34
parallelrisch	$-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}+1\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}$	68
norman	$\frac{-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3d}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{3}}$	99

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-1/d*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(aB + bB\cos(c + dx))\sec^4(c + dx)}{a + b\cos(c + dx)}\,dx = \frac{(2\,B\cos(dx + c)^2 + B)\sin(dx + c)}{3\,d\cos(dx + c)^3}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/3*(2*B*cos(d*x + c)^2 + B)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [A] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{(aB + bB\cos(c + dx))\sec^4(c + dx)}{a + b\cos(c + dx)}\,dx = \begin{cases} \frac{B\left(\frac{\tan^3(c+dx)}{3} + \tan(c+dx)\right)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\sec^4(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise(((B*(tan(c + d*x)**3/3 + tan(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*
b*cos(c))*sec(c)**4/(a + b*cos(c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(aB + bB\cos(c + dx))\sec^4(c + dx)}{a + b\cos(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(aB + bB\cos(c + dx))\sec^4(c + dx)}{a + b\cos(c + dx)}\,dx = \frac{B\tan{(dx + c)}^3 + 3\,B\tan{(dx + c)}}{3\,d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
1/3*(B*tan(d*x + c)^3 + 3*B*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 24.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(aB + bB\cos(c + dx))\sec^4(c + dx)}{a + b\cos(c + dx)}\,dx = \frac{2\,B\,\sin{(c + d\,x)}\,\cos{(c + d\,x)}^2 + B\,\sin{(c + d\,x)}}{3\,d\cos{(c + d\,x)}^3}$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

output
```
(B*sin(c + d*x) + 2*B*cos(c + d*x)^2*sin(c + d*x))/(3*d*cos(c + d*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{(aB + bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sin(dx+c)\,b\left(2\sin(dx+c)^2-3\right)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

output
```
(sin(c + d*x)*b*(2*sin(c + d*x)**2 - 3))/(3*cos(c + d*x)*d*(sin(c + d*x)**
2 - 1))
```

3.289 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 114

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{(2a^2+b^2)\,Bx}{2b^3} - \frac{2a^3B\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
1/2*(2*a^2+b^2)*B*x/b^3-2*a^3*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/(a-b)^(1/2)/b^3/(a+b)^(1/2)/d-a*B*sin(d*x+c)/b^2/d+1/2*B*cos(d*x+
c)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{B\left(2(2a^2+b^2)(c+dx)+\frac{8a^3\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-4ab\sin(c+dx)+b^2\sin(2(c+dx))\right)}{4b^3d}$$

input
```
Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2
,x]
```

output
```
(B*(2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/
Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c +
d*x)]))/(4*b^3*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2011, 3042, 3272, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$\downarrow$ 3272

$$B\left(\frac{\int \frac{-2a\cos^2(c+dx)+b\cos(c+dx)+a}{a+b\cos(c+dx)}dx}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

$\downarrow$ 3042

$$B\left(\frac{\int \frac{-2a\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\sin\left(c+dx+\frac{\pi}{2}\right)+a}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

$\downarrow$ 3502

$$B\left(\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

$\downarrow$ 3042

$$B\left(\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

$\downarrow$ 3214

$$B\left(\frac{\frac{\frac{x\left(2a^2+b^2\right)}{b}-\frac{2a^3\int \frac{1}{a+b\cos(c+dx)}dx}{b}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

$\downarrow$ 3042

$$B\left(\frac{\frac{\frac{x\left(2a^2+b^2\right)}{b}-\frac{2a^3\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

$\downarrow$ 3138

$$B\left(\frac{\frac{\frac{x\left(2a^2+b^2\right)}{b}-\frac{4a^3\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

$\downarrow$ 218

$$B\left(\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}\right)$$

input
```
Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
B*((Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + ((((2*a^2 + b^2)*x)/b - (4*a^3*Ar
cTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a +
b]*d))/b - (2*a*Sin[c + d*x])/(b*d))/(2*b))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{2B\left(\frac{\frac{\left(-ab-\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-ab+\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\left(2a^2+b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{b^3}-\frac{a^3\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}\right)}{d}$
default	$\frac{2B\left(\frac{\frac{\left(-ab-\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-ab+\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\left(2a^2+b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{b^3}-\frac{a^3\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}\right)}{d}$
risch	$\frac{Bxa^2}{b^3}+\frac{Bx}{2b}+\frac{iBa\,e^{i(dx+c)}}{2b^2d}-\frac{iBa\,e^{-i(dx+c)}}{2b^2d}-\frac{a^3B\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d\,b^3}+\frac{a^3B\ln\left(e^{i(dx+c)}-\right.}{\sqrt{-}}$

input
```
int(cos(d*x+c)^3*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/d*B*(1/b^3*(((-a*b-1/2*b^2)*tan(1/2*d*x+1/2*c)^3+(-a*b+1/2*b^2)*tan(1/2*
d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2+b^2)*arctan(tan(1/2*d*x+
1/2*c)))-a^3/b^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b
)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.07

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= \left[-\frac{\sqrt{-a^2+b^2}Ba^3\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-(2\,Ba^4-}{2\,(a^2b^3-b^5)d}\right.$$

$$-\frac{2\,\sqrt{a^2-b^2}Ba^3\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(2\,Ba^4-Ba^2b^2-Bb^4)dx+(2\,Ba^3b-2\,Bab^3-(Ba^2b^2}{2\,(a^2b^3-b^5)d}$$

input
```
integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*B*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(
d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^4 - B*a^2
*b^2 - B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x +
 c))*sin(d*x + c))/((a^2*b^3 - b^5)*d), -1/2*(2*sqrt(a^2 - b^2)*B*a^3*arct
an(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^4 - B*a^
2*b^2 - B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x
+ c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.62

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba^3}{\sqrt{a^2-b^2}b^3} - \frac{(2Ba^2+Bb^2)(dx+c)}{b^3} + \frac{2\left(2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Bb\right.}{}}{2d}$$

input
```
integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^3/(sqrt(a^
2 - b^2)*b^3) - (2*B*a^2 + B*b^2)*(d*x + c)/b^3 + 2*(2*B*a*tan(1/2*d*x + 1
/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - B*b*ta
n(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 25.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.52

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{B\,\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}-\frac{B\,a\,\sin\left(c+d\,x\right)}{b^2\,d}-\frac{B\,a^3\,\mathrm{atan}\left(\frac{\left(a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2{}\mathrm{i}}{b^3\,d\,\sqrt{b^2-a^2}}$$

input
```
int((cos(c + d*x)^3*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
(B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (B*sin(2*c + 2*d*x
))/(4*b*d) + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d)
 - (B*a*sin(c + d*x))/(b^2*d) - (B*a^3*atan(((a*sin(c/2 + (d*x)/2) - b*sin
(c/2 + (d*x)/2))*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^3*d*(b
^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.88

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{-4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)a^3+\cos(dx+c)^2\,a^2b^2dx-\cos(dx+c)^2\,b^4dx+\cos(dx+c)}{}$$

input

```
int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output

```
( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*a**3 + cos(c + d*x)**2*a**2*b**2*d*x - cos(c + d*x)**2*b**
4*d*x + cos(c + d*x)*sin(c + d*x)*a**2*b**2 - cos(c + d*x)*sin(c + d*x)*b*
*4 + sin(c + d*x)**2*a**2*b**2*d*x - sin(c + d*x)**2*b**4*d*x - 2*sin(c +
d*x)*a**3*b + 2*sin(c + d*x)*a*b**3 + 2*a**4*d*x - 2*a**2*b**2*d*x)/(2*b**
2*d*(a**2 - b**2))
```

3.290 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 79

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx=-\frac{aBx}{b^2}+\frac{2a^2B\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d}+\frac{B\sin(c+dx)}{bd}$$

output
```
-a*B*x/b^2+2*a^2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b
)^(1/2)/b^2/(a+b)^(1/2)/d+B*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx=\frac{B\left(-a(c+dx)-\frac{2a^2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+b\sin(c+dx)\right)}{b^2d}$$

input
```
Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2
,x]
```

output
```
(B*(-(a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 +
 b^2]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x]))/(b^2*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2011, 3042, 3225, 25, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3225}$$

$$B\left(\frac{\int -\frac{a\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}+\frac{\sin(c+dx)}{bd}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{\sin(c+dx)}{bd}-\frac{\int \frac{a\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{\sin(c+dx)}{bd}-\frac{a\int\frac{\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}\right)$$

$\Big\downarrow$ 3042

$$B\left(\frac{\sin(c+dx)}{bd}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$\Big\downarrow$ 3214

$$B\left(\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{a\int\frac{1}{a+b\cos(c+dx)}dx}{b}\right)}{b}\right)$$

$\Big\downarrow$ 3042

$$B\left(\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{a\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}\right)$$

$\Big\downarrow$ 3138

$$B\left(\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{2a\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}\right)$$

$\Big\downarrow$ 218

$$B\left(\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}\right)$$

input
```
Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
B*(-((a*(x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(S
qrt[a - b]*b*Sqrt[a + b]*d)))/b) + Sin[c + d*x]/(b*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{2B\left(\frac{a^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}-\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}\right)}{d}$
default	$\frac{2B\left(\frac{a^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}-\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}\right)}{d}$
risch	$-\frac{aBx}{b^2}-\frac{i\mathrm{e}^{i(dx+c)}B}{2db}+\frac{iB\,\mathrm{e}^{-i(dx+c)}}{2bd}-\frac{a^2B\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d\,b^2}+\frac{a^2B\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2}{\sqrt{-}}\right.}{\sqrt{-a^2+b^2}\,d\,b}$

input
```
int(cos(d*x+c)^2*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/d*B*(a^2/b^2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*
(a+b))^(1/2))-1/b^2*(-b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+a*arct
an(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.56

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= \left[-\frac{\sqrt{-a^2+b^2}Ba^2\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\,(Ba^3-}{2\,(a^2b^2-b^4)d}\right.$$

input
```
integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*B*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(
d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(B*a^3 - B*a*b
^2)*d*x - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2
 - b^2)*B*a^2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))
 - (B*a^3 - B*a*b^2)*d*x + (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4
)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$=\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba^2}{\sqrt{a^2-b^2}b^2}-\frac{(dx+c)Ba}{b^2}+\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}}{d}$$

input
```
integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^2/(sqrt(a^2 - b
^2)*b^2) - (d*x + c)*B*a/b^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 + 1)*b))/d
```

Mupad [B] (verification not implemented)

Time = 24.81 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.44

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$=\frac{B\sin(c+dx)}{bd}-\frac{2Ba\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{b^2d}$$
$$-\frac{Ba^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b-2\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2+1\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(b^2-a^2)^{3/2}+a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}-ab\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}\right)2\mathrm{i}}{b^2d\sqrt{b^2-a^2}}$$

input
```
int((cos(c + d*x)^2*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
(B*sin(c + d*x))/(b*d) - (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
))/(b^2*d) - (B*a^2*atan((b^3*sin(c/2 + (d*x)/2)*1i - a*b^2*sin(c/2 + (d*x
)/2)*2i + a^2*b*sin(c/2 + (d*x)/2)*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/
2) + a^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a*b*cos(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2)))*2i)/(b^2*d*(b^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)a^2+\sin(dx+c)\,a^2b-\sin(dx+c)\,b^3-a^3dx+a\,b^2dx}{bd\,(a^2-b^2)}$$

input
```
int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*a**2 + sin(c + d*x)*a**2*b - sin(c + d*x)*b**3 - a**3*d*x + a
*b**2*d*x)/(b*d*(a**2 - b**2))
```

3.291 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 61

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{Bx}{b} - \frac{2aB\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}$$

output
```
B*x/b-2*a*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)
/b/(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{B\left(c+dx+\frac{2a\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{bd}$$

input
```
Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x
]
```

output
```
(B*(c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2]))/(b*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2011, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$B\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3138}$$

$$B\left(\frac{x}{b}-\frac{2a\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)$$

$$\Big\downarrow \text{218}$$

$$B\left(\frac{x}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)$$

input
```
Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
B*(x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
- b]*b*Sqrt[a + b]*d))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{2B\left(\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{a\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}\right)}{d}$	66
default	$\frac{2B\left(\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{a\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}\right)}{d}$	66
risch	$\frac{Bx}{b}-\frac{aB\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,db}+\frac{aB\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,db}$	155

input
```
int(cos(d*x+c)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNVER
BOSE)
```

output
```
2/d*B*(1/b*arctan(tan(1/2*d*x+1/2*c))-1/b*a/((a-b)*(a+b))^(1/2)*arctan((a-
b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.79

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+b^2}Ba\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\,(Ba^2-B}{2\,(a^2b-b^3)d}\right.$$
$$\left.-\frac{\sqrt{a^2-b^2}Ba\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(Ba^2-Bb^2)dx}{(a^2b-b^3)d}\right]$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*B*a*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2 - B*b^2)*
d*x)/((a^2*b - b^3)*d), -(sqrt(a^2 - b^2)*B*a*arctan(-(a*cos(d*x + c) + b)
/(sqrt(a^2 - b^2)*sin(d*x + c))) - (B*a^2 - B*b^2)*d*x)/((a^2*b - b^3)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(52) = 104$.

Time = 0.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.02

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{\frac{\left(\sqrt{a^2-b^2}B(2a-b)|a-b|+\sqrt{a^2-b^2}B|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{2a+\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)b^2+(a^3-2a^2b+ab^2)|b|}+\frac{(2Ba-Bb-B|b|)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\right.}{}}{d}$$

input
```
integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm=
"giac")
```

output
```
-((sqrt(a^2 - b^2)*B*(2*a - b)*abs(a - b) + sqrt(a^2 - b^2)*B*abs(a - b)*a
bs(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x
+ 1/2*c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 -
2*a*b + b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (2*B*a - B*b - B*abs(
b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1
/2*c)/sqrt((2*a - sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/(b^2 - a*ab
s(b)))/d
```

Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{2B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} + \frac{2B\,a\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}\right)}{b\,d\,\sqrt{b^2-a^2}}$$

input
```
int((cos(c + d*x)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
(2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*B*a*atanh((a*
sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)
^(1/2))))/(b*d*(b^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)a+a^2dx-b^2dx}{d\,(a^2-b^2)}$$

input
```
int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*a + a**2*d*x - b**2*d*x)/(d*(a**2 - b**2))
```

3.292 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 50

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\frac{2B\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

output
```
2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(
1/2)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx=-\frac{2B\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

output
```
(-2*B*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b
^2]*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2011, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3138}$$

$$\frac{2B\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2B\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2B\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	45
default	$\frac{2B\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	45
risch	$-\frac{B\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}+\frac{B\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,d}$	139

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*B/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.54

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$
$$= \left[-\frac{\sqrt{-a^2 + b^2} B \log \left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2} \right)}{2\,(a^2-b^2)d}, \frac{B\arctan\left(-\frac{}{\sqrt{}}\right.}{\sqrt{a^2}}\right.$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*B*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), B*arct
an(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*
d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(42) = 84$.

Time = 155.94 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.80

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}Bx}{\cos(c)} & \text{for}\ a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for}\ a = b \\ \frac{B}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for}\ a = -b \\ \frac{x(Ba+Bb\cos(c))}{(a+b\cos(c))^2} & \text{for}\ d = 0 \\ \frac{B\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*tan(c/2 + d
*x/2)/(b*d), Eq(a, b)), (B/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), (x*(B*a + B
*b*cos(c))/(a + b*cos(c))**2, Eq(d, 0)), (B*log(-sqrt(-a/(a - b) - b/(a -
b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a
 - b) - b/(a - b))) - B*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2
))/(a*d*sqrt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))),
True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(2a - 2b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)B}{\sqrt{a^2 - b^2}d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B/(sqrt(a^2 - b^2)*d
)
```

Mupad [B] (verification not implemented)

Time = 24.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2\,B\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a-b)}{\sqrt{a^2-b^2}}\right)}{d\,\sqrt{a^2-b^2}}$$

input
```
int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^2,x)
```

output
```
(2*B*atan((tan(c/2 + (d*x)/2)*(a - b))/(a^2 - b^2)^(1/2)))/(d*(a^2 - b^2)^
(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)b}{d\,(a^2-b^2)}$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a
**2 - b**2))*b)/(d*(a**2 - b**2))
```

3.293 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 70

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{2bB\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d} + \frac{B\operatorname{arctanh}(\sin(c+dx))}{ad}$$

output
```
-2*b*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(1/2)/(a
+b)^(1/2)/d+B*arctanh(sin(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{B\left(\frac{2b\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\right.\right.\right.}{ad}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x
]
```

output
```
(B*((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
 b^2] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]))/(a*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2011, 3042, 3226, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3226}$$

$$B\left(\frac{\int \sec(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\cos(c+dx)}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)$$

$$\Big\downarrow \text{3138}$$

$$B\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)$$

$$\Big\downarrow \text{218}$$

$$B\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{4257}$$

$$B\left(\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

output
```
B*((-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b
]*Sqrt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{2B\left(-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a}-\frac{b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a}\right)}{d}$
default	$\frac{2B\left(-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a}-\frac{b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a}\right)}{d}$
risch	$-\frac{bB\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,da}+\frac{bB\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,da}+\frac{B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}-\frac{B\ln(}{}$

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVER
BOSE)
```

output
```
2/d*B*(-1/2/a*ln(tan(1/2*d*x+1/2*c)-1)-b/a/((a-b)*(a+b))^(1/2)*arctan((a-b
)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+1/2/a*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.17

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}Bb\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right) - (Ba^2 - Bb^2}{2\,(a^3-ab^2)d} \right.$$
$$-\frac{2\,\sqrt{a^2-b^2}Bb\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - (Ba^2-Bb^2)\log(\sin(dx+c)+1) + (Ba^2-Bb^2)\log(-\sin}{2\,(a^3-ab^2)d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*B*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (B*a^2 - B*b^2)*lo
g(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^
2)*d), -1/2*(2*sqrt(a^2 - b^2)*B*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c))) - (B*a^2 - B*b^2)*log(sin(d*x + c) + 1) + (B*a^2 - B
*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

output
```
B*Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.74

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb}{\sqrt{a^2-b^2}a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}+\frac{B\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)-}{a}}{d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm=
"giac")
```

output
```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b/(sqrt(a^2 - b^
2)*a) - B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + B*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a)/d
```

Mupad [B] (verification not implemented)

Time = 24.74 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \frac{(aB + bB\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \frac{2\,B\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} + \frac{2\,B\,b\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{a\,d\,\sqrt{b^2-a^2}}$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

output
```
(2*B*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*B*b*atanh((a
*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2
)^(1/2))))/(a*d*(b^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96

$$\int \frac{(aB + bB\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \frac{b\left(-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)b - \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2 + \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2 + \right.}{ad\,(a^2-b^2)}$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

output
```
(b*( - 2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt(a**2 - b**2))*b - log(tan((c + d*x)/2) - 1)*a**2 + log(tan((c + d*x)/
2) - 1)*b**2 + log(tan((c + d*x)/2) + 1)*a**2 - log(tan((c + d*x)/2) + 1)*
b**2))/(a*d*(a**2 - b**2))
```

3.294 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 88

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\frac{2b^2B\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d}-\frac{bB\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

output
```
2*b^2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^(1/2)
/(a+b)^(1/2)/d-b*B*arctanh(sin(d*x+c))/a^2/d+B*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$=\frac{B\left(-\frac{2b^2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.\right.}{a^2d}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2
,x]
```

output
```
(B*((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^
2 + b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]) + a*Tan[c + d*x]))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2011, 3042, 3281, 25, 27, 3042, 3226, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$B\left(\frac{\int -\frac{b\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{\tan(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{\int \frac{b\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{b\int\frac{\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{b\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)$$

$$\Big\downarrow \text{3226}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int\sec(c+dx)dx}{a}-\frac{b\int\frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}\right)$$

$$\Big\downarrow \text{3138}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}\right)$$

$$\Big\downarrow \text{218}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}\right)$$

$$\Big\downarrow \text{4257}$$

$$B\left(\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}\right)$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
B*(-((b*((-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d)))/a) + Tan[c + d*x]/(
a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{2B\left(-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^2}-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^2}+\frac{b^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\right.}{\sqrt{(a-b)(a+b)}}\right.}{a^2\sqrt{(a-b)(a+b)}}\right.}{d}$
default	$\frac{2B\left(-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^2}-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^2}+\frac{b^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\right.}{\sqrt{(a-b)(a+b)}}\right.}{a^2\sqrt{(a-b)(a+b)}}\right.}{d}$
risch	$\frac{2iB}{da\left(e^{2i(dx+c)}+1\right)}-\frac{b^2B\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d\,a^2}+\frac{b^2B\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d\,a^2}+\frac{Bb\ln(}{}$

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/d*B*(-1/2/a/(tan(1/2*d*x+1/2*c)-1)+1/2*b/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1/
2/a/(tan(1/2*d*x+1/2*c)+1)-1/2*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)+b^2/a^2/((a-
b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(79) = 158$.

Time = 0.13 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.52

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+b^2}Bb^2\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-}{}\right.$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*B*b^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^
2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (B*
a^2*b - B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^2*b - B*b^3)*cos(
d*x + c)*log(-sin(d*x + c) + 1) - 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4
- a^2*b^2)*d*cos(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*B*b^2*arctan(-(a*cos(d*
x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^2*b - B*b^
3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (B*a^2*b - B*b^3)*cos(d*x + c)*log
(-sin(d*x + c) + 1) + 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d
*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^2}\,dx = B\int \frac{\sec^2(c + dx)}{a + b\cos(c + dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

output
```
B*Integral(sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^2}{\sqrt{a^2-b^2}a^2} - \frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} + \frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}{a^2}}{d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^2/(sqrt(a^2 - b
^2)*a^2) - B*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + B*b*log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 25.18 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.70

$$\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^2}\,dx = \frac{2B\left(\frac{a^3\sin(c+dx)}{2} - \frac{ab^2\sin(c+dx)}{2}\right)}{a^2 d\cos(c+dx)(a^2-b^2)} - \frac{2B\left(a^2 b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) - b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) + b^2\,\mathrm{atanh}\left(\frac{a^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}+2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(b^2-\right.\right.}{a^2 d\,(a}$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

output
```
(2*B*((a^3*sin(c + d*x))/2 - (a*b^2*sin(c + d*x))/2))/(a^2*d*cos(c + d*x)*
(a^2 - b^2)) - (2*B*(a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) -
b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + b^2*atanh((a^5*sin(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2) + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2)
- 2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2) - a^3*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b
*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2
))*(b^2 - a^2)^(1/2)))/(a^2*d*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.28

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b\left(2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\cos(dx+c)\,b^2+\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2b-\mathrm{co}\right.}{}$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

output
```
(b*(2*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/sqr
t(a**2 - b**2))*cos(c + d*x)*b**2 + cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*a**2*b - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b**3 - cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*a**2*b + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b**3
+ sin(c + d*x)*a**3 - sin(c + d*x)*a*b**2))/(cos(c + d*x)*a**2*d*(a**2 - b
**2))
```

3.295 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 123

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{2b^3B\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{(a^2+2b^2)\,B\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad}$$

output

```
-2*b^3*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(1/2
)/(a+b)^(1/2)/d+1/2*(a^2+2*b^2)*B*arctanh(sin(d*x+c))/a^3/d-b*B*tan(d*x+c)
/a^2/d+1/2*B*sec(d*x+c)*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.94

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{B\left(\frac{8b^3\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 4b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input

```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2
,x]
```

output

```
(B*((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
 + b^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
- 4*a*b*Tan[c + d*x]))/(4*a^3*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2011, 3042, 3281, 25, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$B\left(\frac{\int -\frac{\left(-b\cos^2(c+dx)-a\cos(c+dx)+2b\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{\left(-b\cos^2(c+dx)-a\cos(c+dx)+2b\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{-b\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\sin\left(c+dx+\frac{\pi}{2}\right)+2b}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}\right)$$

$$\Big\downarrow \text{3534}$$

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{\int -\frac{\left(a^2+b\cos(c+dx)a+2b^2\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{2b\tan(c+dx)}{ad}}{2a}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\int \frac{\left(a^2+b\cos(c+dx)a+2b^2\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\int \frac{a^2+b\sin\left(c+dx+\frac{\pi}{2}\right)a+2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}\right)$$

$$\Big\downarrow \text{3480}$$

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int \sec(c+dx)dx}{a}-\frac{2b^3\int \frac{1}{a+b\cos(c+dx)}dx}{a}}{a}}{2a}\right)$$

$\Big\downarrow$ 3042

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^3\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}\right)$$

$\Big\downarrow$ 3138

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{2a}\right)$$

$\Big\downarrow$ 218

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}\right)$$

$\Big\downarrow$ 4257

$$B\left(\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{4b^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}\right)$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

output
```
B*((Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - (-(((-4*b^3*ArcTan[(Sqrt[a - b]*T
an[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2 + 2*b
^2)*ArcTanh[Sin[c + d*x]])/(a*d))/a) + (2*b*Tan[c + d*x])/(a*d))/(2*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{2B\left(-\frac{b^3\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}-\frac{1}{4a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-a-2b}{4a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2+2b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4a^3}+\frac{}{4a(\text{t}}\right.}{d}$
default	$\frac{2B\left(-\frac{b^3\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}-\frac{1}{4a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-a-2b}{4a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2+2b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4a^3}+\frac{}{4a(\text{t}}\right.}{d}$
risch	$-\frac{iB\left(e^{3i(dx+c)}a+2b\,e^{2i(dx+c)}-a\,e^{i(dx+c)}+2b\right)}{d\,a^2\left(e^{2i(dx+c)}+1\right)^2}+\frac{B\ln\left(e^{i(dx+c)}+i\right)}{2ad}+\frac{B\ln\left(e^{i(dx+c)}+i\right)b^2}{a^3d}-\frac{B\ln\left(e^{i(dx+c)}-i\right)}{2ad}$

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/d*B*(-b^3/a^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)
*(a+b))^(1/2))-1/4/a/(tan(1/2*d*x+1/2*c)+1)^2-1/4*(-a-2*b)/a^2/(tan(1/2*d*
x+1/2*c)+1)+1/4*(a^2+2*b^2)/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/4/a/(tan(1/2*d*
x+1/2*c)-1)^2-1/4*(-a-2*b)/a^2/(tan(1/2*d*x+1/2*c)-1)+1/4/a^3*(-a^2-2*b^2)
*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.96

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$=\left[-\frac{2\sqrt{-a^2+b^2}Bb^3\cos(dx+c)^2\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right.}{}\right.$$
$$-\frac{4\sqrt{a^2-b^2}Bb^3\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)\cos(dx+c)^2-(Ba^4+Ba^2b^2-2Bb^4)\cos(dx+c)^2\log(\sin}{}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
[-1/4*(2*sqrt(-a^2 + b^2)*B*b^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (
2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(
d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) -
 (B*a^4 + B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a
^4 + B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4
 - B*a^2*b^2 - 2*(B*a^3*b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a
^3*b^2)*d*cos(d*x + c)^2), -1/4*(4*sqrt(a^2 - b^2)*B*b^3*arctan(-(a*cos(d*
x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (B*a^4 + B*a^
2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a^4 + B*a^2*b^2
 - 2*B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4 - B*a^2*b^2 -
 2*(B*a^3*b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(
d*x + c)^2)]
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

output
```
B*Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(110) = 220$.

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.80

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^3}{\sqrt{a^2-b^2}a^3} - \frac{(Ba^2+2Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} + \frac{(Ba^2+2B}{}}{2d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^3/(sqrt(a^
2 - b^2)*a^3) - (B*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 +
 (B*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(B*a*tan(1/2
*d*x + 1/2*c)^3 + 2*B*b*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1/2*d*x + 1/2*c)
- 2*B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 25.22 (sec) , antiderivative size = 1099, normalized size of antiderivative = 8.93

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

output
```
((B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (B*b^2*sin(c + d
*x))/2 + (B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d
*x))/2)/(a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (a*((B*sin(c + d*x)
)/2 + (B*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (B*atanh(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2))/(d*(a^2 - b^2)*(cos
(2*c + 2*d*x)/2 + 1/2)) - (B*b*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c
 + 2*d*x)/2 + 1/2)) - (B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b^3*sin(2*c + 2*d*x))
/(2*a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan(((a^9*sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3
/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) -
 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x
)/2)*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) -
a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2
 - a^3)*(a^7 - 3*a^3*b^4 + 2*a^5*b^2)))*1i)/(a^3*d*(b^2 - a^2)^(1/2)*(cos(
2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan(((a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^9*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3
*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*sin(c/2 + (d*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.90

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b\left(-4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\sin(dx+c)^2 b^3 + 4\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{\sqrt{a^2-b^2}}\right)\right.}{}$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

output
```
(b*( - 4*sqrt(a**2 - b**2)*atan((tan((c + d*x)/2)*a - tan((c + d*x)/2)*b)/
sqrt(a**2 - b**2))*sin(c + d*x)**2*b**3 + 4*sqrt(a**2 - b**2)*atan((tan((c
 + d*x)/2)*a - tan((c + d*x)/2)*b)/sqrt(a**2 - b**2))*b**3 + 2*cos(c + d*x
)*sin(c + d*x)*a**3*b - 2*cos(c + d*x)*sin(c + d*x)*a*b**3 - log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**2*a**4 - log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**2*a**2*b**2 + 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**4 + log(tan
((c + d*x)/2) - 1)*a**4 + log(tan((c + d*x)/2) - 1)*a**2*b**2 - 2*log(tan(
(c + d*x)/2) - 1)*b**4 + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4 +
log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 - 2*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2*b**4 - log(tan((c + d*x)/2) + 1)*a**4 - log(tan((c
 + d*x)/2) + 1)*a**2*b**2 + 2*log(tan((c + d*x)/2) + 1)*b**4 - sin(c + d*x
)*a**4 + sin(c + d*x)*a**2*b**2))/(2*a**3*d*(sin(c + d*x)**2*a**2 - sin(c
+ d*x)**2*b**2 - a**2 + b**2))
```

3.296 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 386

$$\begin{aligned}
&\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx \\
&= \frac{2(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+147b^4B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{2(a^2-b^2)(24a^2Ab+75Ab^3-16a^3B-36ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{315b^4d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{2(24a^2Ab+75Ab^3-16a^3B-36ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^3d} \\
&\quad - \frac{2(36aAb-24a^2B-49b^2B)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315b^3d} \\
&\quad + \frac{2(3Ab-2aB)\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{21b^2d} \\
&\quad + \frac{2B\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9bd}
\end{aligned}$$

output
```
2/315*(24*A*a^3*b+57*A*a*b^3-16*B*a^4-24*B*a^2*b^2+147*B*b^4)*(a+b*cos(d*x
+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^4/d/((a
+b*cos(d*x+c))/(a+b))^(1/2)-2/315*(a^2-b^2)*(24*A*a^2*b+75*A*b^3-16*B*a^3-
36*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2)*(b/(a+b))^(1/2))/b^4/d/(a+b*cos(d*x+c))^(1/2)+2/315*(24*A*a^2*b+75*
A*b^3-16*B*a^3-36*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^3/d-2/315*(
36*A*a*b-24*B*a^2-49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/21*(
3*A*b-2*B*a)*cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/9*B*cos(
d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.76

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$= \frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(6a^2Ab+75Ab^3-4a^3B+111ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+(24a^3Ab+57aAb^3-\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(6*a^2*A*b + 75*A*b^3 - 4*a^3*B
 + 111*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (24*a^3*A*b + 57*a
*A*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d*x)
/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) - b*(a + b*
Cos[c + d*x])*(-2*(-48*a^2*A*b + 345*A*b^3 + 32*a^3*B + 57*a*b^2*B)*Sin[c
+ d*x] - b*((36*a*A*b - 24*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*(9
*A*b + a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^4*d*Sqrt
[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3502, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2\int \frac{1}{2}\cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(3(3Ab-2aB)\cos^2(c+dx)+7bB\cos(c+dx)+4aB\right)dx}{9b}+\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\left(3(3Ab-2aB)\cos^2(c+dx)+7bB\cos(c+dx)+4aB\right)dx}{9b}+\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3(3Ab-2aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+7bB\sin\left(c+dx+\frac{\pi}{2}\right)+4aB\right)dx}{9b}+\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{2\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(-\left(\left(-24Ba^2+36Aba-49b^2B\right)\cos^2(c+dx)\right)+b(45Ab-2aB)\cos(c+dx)+6a(3Ab-2aB)\right)dx}{7b}+\frac{6(3Ab-2aB)\sin(c+dx)\cos}{7}}{9b}+\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \sqrt{a+b\cos(c+dx)}\left(-\left(\left(-24Ba^2+36Aba-49b^2B\right)\cos^2(c+dx)\right)+b(45Ab-2aB)\cos(c+dx)+6a(3Ab-2aB)\right)dx}{7b}+\frac{6(3Ab-2aB)\sin(c+dx)\cos(c-}{7bd}}{9b}$$
$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\left(24Ba^2-36Aba+49b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(45Ab-2aB)\sin\left(c+dx+\frac{\pi}{2}\right)+6a(3Ab-2aB)\right)dx}{7b}+6(3Ab-2aB)\sin(c+dx)}{9b}$$
$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3502

$$\frac{\frac{\frac{2\int -\frac{3}{2}\sqrt{a+b\cos(c+dx)}\left(b\left(-4Ba^2+6Aba-49b^2B\right)-\left(-16Ba^3+24Aba^2-36b^2Ba+75Ab^3\right)\cos(c+dx)\right)dx}{5b}-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx}{5bd}}{7b}}{9b}$$
$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{3\int \sqrt{a+b\cos(c+dx)}\left(b\left(-4Ba^2+6Aba-49b^2B\right)-\left(-16Ba^3+24Aba^2-36b^2Ba+75Ab^3\right)\cos(c+dx)\right)dx}{5b}-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))}{5bd}}{7b}}{9b}$$
$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\frac{3\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(-4Ba^2+6Aba-49b^2B\right)+\left(16Ba^3-24Aba^2+36b^2Ba-75Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(}{5bd}}{7b}}{9b}$$
$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3232

$$-\frac{3\left(\frac{2}{3}\int -\frac{b\left(-4Ba^3+6Aba^2+111b^2Ba+75Ab^3\right)+\left(-16Ba^4+24Aba^3-24b^2Ba^2+57Ab^3a+147b^4B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\sin(c-}{3d}}{5b}}{7b}$$

$$9b$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\downarrow 27$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{b\left(-4Ba^3+6Aba^2+111b^2Ba+75Ab^3\right)+\left(-16Ba^4+24Aba^3-24b^2Ba^2+57Ab^3a+147b^4B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\sin(c-}{3d}}{5b}}{7b}$$

$$9b$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\downarrow 3042$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{b\left(-4Ba^3+6Aba^2+111b^2Ba+75Ab^3\right)+\left(-16Ba^4+24Aba^3-24b^2Ba^2+57Ab^3a+147b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\text{s}}{3d}}{5b}}{7b}$$

$$9b$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\downarrow 3231$$

$$-\frac{3\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}\right)-2(-}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\downarrow 3042$$

$$-\frac{3\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right.\right.}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$$\downarrow 3134$$

$$-\frac{3\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}\cdots}}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\cdots\right)\cdots}{5b}\cdots\right)}{7b}\cdots$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\big\downarrow$ 3042

$$-\frac{3\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}\cdots}}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\cdots\right)\cdots}{5b}\cdots\right)}{7b}\cdots$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\big\downarrow$ 3132

$$-\frac{3\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\cdots\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\cdots\right)\cdots}{5b}\cdots\right)}{7b}\cdots$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\big\downarrow$ 3142

$$-\frac{3\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B\right)\sqrt{a+b\cos\cdots}}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\cdots\right)\cdots}{5b}\cdots\right)}{7\cdots}\cdots$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\big\downarrow$ 3042

$$-\frac{3\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B\right)\sqrt{a}}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.}{5b}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3140

$$-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}-\frac{3\left(\frac{1}{3}\left(\frac{2\left(a^2-b^2\right)\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right.}{bd\sqrt{a+b\cos(c+dx)}}\right.\right.}{}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}}{9bd}$$

input
```
Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*b*d) + ((6
*(3*A*b - 2*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*
b*d) + ((-2*(36*a*A*b - 24*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(5*b*d) - (3*(((-2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^
2*b^2*B + 147*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(24*a
^2*A*b + 75*A*b^3 - 16*a^3*B - 36*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))
/3 - (2*(24*a^2*A*b + 75*A*b^3 - 16*a^3*B - 36*a*b^2*B)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(3*d)))/(5*b))/(7*b))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(367) = 734$.

Time = 29.65 (sec) , antiderivative size = 1635, normalized size of antiderivative = 4.24

method	result	size
default	Expression too large to display	1635
parts	Expression too large to display	1824

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(720*A*b^5+640*B*a*b^4+2240
*B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-432*A*a*b^4-1080*A*b^5+8
*B*a^2*b^3-960*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
+(-12*A*a^2*b^3+432*A*a*b^4+840*A*b^5+8*B*a^3*b^2-8*B*a^2*b^3+728*B*a*b^4+
952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(24*A*a^3*b^2+6*A*a^2*b
^3-258*A*a*b^4-240*A*b^5-16*B*a^4*b-4*B*a^3*b^2-24*B*a^2*b^3-204*B*a*b^4-1
68*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-24*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-51*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^4*b-24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^3*b^2+57*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b^3-57*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.66

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/945*(sqrt(1/2)*(32*I*B*a^5 - 48*I*A*a^4*b + 36*I*B*a^3*b^2 - 96*I*A*a^2
*b^3 + 39*I*B*a*b^4 + 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*s
in(d*x + c) + 2*a)/b) + sqrt(1/2)*(-32*I*B*a^5 + 48*I*A*a^4*b - 36*I*B*a^3
*b^2 + 96*I*A*a^2*b^3 - 39*I*B*a*b^4 - 225*I*A*b^5)*sqrt(b)*weierstrassPIn
verse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(16*I*B*a^4*b - 24*I*
A*a^3*b^2 + 24*I*B*a^2*b^3 - 57*I*A*a*b^4 - 147*I*B*b^5)*sqrt(b)*weierstra
ssZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(-16*I*B*a^4*b + 24
*I*A*a^3*b^2 - 24*I*B*a^2*b^3 + 57*I*A*a*b^4 + 147*I*B*b^5)*sqrt(b)*weiers
trassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(35*B*b^5*cos(d*x + c)^3 +
 8*B*a^3*b^2 - 12*A*a^2*b^3 + 13*B*a*b^4 + 75*A*b^5 + 5*(B*a*b^4 + 9*A*b^5
)*cos(d*x + c)^2 - (6*B*a^2*b^3 - 9*A*a*b^4 - 49*B*b^5)*cos(d*x + c))*sqrt
(b*cos(d*x + c) + a)*sin(d*x + c))/(b^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^4\,dx\right)b + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)a$$

input
```
int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**4,x)*b + int(sqrt(cos(c + d*x)*
b + a)*cos(c + d*x)**3,x)*a
```

3.297 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 303

$$\begin{aligned}
&\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx \\
&= -\frac{2(14a^2Ab-63Ab^3-8a^3B-19ab^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad+\frac{2(a^2-b^2)(14aAb-8a^2B-25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\cos(c+dx)}} \\
&\quad-\frac{2(14aAb-8a^2B-25b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} \\
&\quad+\frac{2(7Ab-4aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d} \\
&\quad+\frac{2B\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd}
\end{aligned}$$

output
```
-2/105*(14*A*a^2*b-63*A*b^3-8*B*a^3-19*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^3/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)+2/105*(a^2-b^2)*(14*A*a*b-8*B*a^2-25*B*b^2)*((a+b*cos(d*x+c)
)/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^3/
d/(a+b*cos(d*x+c))^(1/2)-2/105*(14*A*a*b-8*B*a^2-25*B*b^2)*(a+b*cos(d*x+c)
)^(1/2)*sin(d*x+c)/b^2/d+2/35*(7*A*b-4*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x
+c)/b^2/d+2/7*B*cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{4\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(49aAb+2a^2B+25b^2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+(-14a^2Ab+63Ab^3+8a^3B+1\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(49*a*A*b + 2*a^2*B + 25*b^2*B)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B
 + 19*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((28*a*A*b - 16*a
^2*B + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + a*B)*Sin[2*(c + d*x)] + 5
*b*B*Sin[3*(c + d*x)])))/(210*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left((7Ab-4aB)\cos^2(c+dx)+5bB\cos(c+dx)+2aB\right)dx}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\cos(c+dx)}\left((7Ab-4aB)\cos^2(c+dx)+5bB\cos(c+dx)+2aB\right)dx}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left((7Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+5bB\sin\left(c+dx+\frac{\pi}{2}\right)+2aB\right)dx}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(b(21Ab-2aB)-\left(-8Ba^2+14Aba-25b^2B\right)\cos(c+dx)\right)dx}{5b}+\frac{2(7Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \sqrt{a+b\cos(c+dx)}\left(b(21Ab-2aB)-\left(-8Ba^2+14Aba-25b^2B\right)\cos(c+dx)\right)dx}{5b}+\frac{2(7Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(21Ab-2aB)+\left(8Ba^2-14Aba+25b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2(7Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3232

$$\frac{\frac{\frac{2}{3}\int \frac{b\left(2Ba^2+49Aba+25b^2B\right)-\left(-8Ba^3+14Aba^2-19b^2Ba-63Ab^3\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2(7Ab-4aB)\,\mathrm{si}}{}}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{3}\int \frac{b\left(2Ba^2+49Aba+25b^2B\right)-\left(-8Ba^3+14Aba^2-19b^2Ba-63Ab^3\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2(7Ab-4aB)\,\mathrm{si}}{}}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{1}{3}\int \frac{b\left(2Ba^2+49Aba+25b^2B\right)+\left(8Ba^3-14Aba^2+19b^2Ba+63Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2(7Ab-4aB)}{}}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3231

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(-8a^3B+14a^2Ab-19ab^2B-63Ab^3\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c-}{3d}}{5b}}{7b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^3B+14a^2Ab-19ab^2B-63Ab^3\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)-\frac{2\left(-8a^2B+14aAb-25b^2\right.}{}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^3B+14a^2Ab-19ab^2B-63Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(-8a^2\right.}{}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^3B+14a^2Ab-19ab^2B-63Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(-\right.}{}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3132

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-8a^3B+14a^2Ab-19ab^2B-63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(-8a^2B+14\right.}{}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3142

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^3B+14a^2Ab-19ab^2B-63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{1}{3}\left(\frac{(a^2-b^2)(-8a^2B+14aAb-25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx+\frac{\pi}{2})}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2(-8a^3B+14a^2Ab-19ab^2B-63Ab^3)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$\downarrow$ 3140

$$\frac{\frac{\frac{1}{3}\left(\frac{2(a^2-b^2)(-8a^2B+14aAb-25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2(-8a^3B+14a^2Ab-19ab^2B-63Ab^3)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*b*d) + ((2*(
7*A*b - 4*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (((-2*(1
4*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) + (2*(a^2 - b^2)*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
 d*x]]))/3 - (2*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(3*d))/(5*b))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(288) = 576$.

Time = 16.41 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.31

method	result	size
default	Expression too large to display	1305
parts	Expression too large to display	1494

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output

```
-2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4-144*B*a*b^3-360*B*
b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(112*A*a*b^3+168*A*b^4-4*B*a^
2*b^2+144*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*
A*a^2*b^2-56*A*a*b^3-42*A*b^4+8*B*a^3*b+2*B*a^2*b^2-86*B*a*b^3-80*B*b^4)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^3*b-14*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3-14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^3*b+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a*b^3-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-8*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.85

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/315*(sqrt(1/2)*(-16*I*B*a^4 + 28*I*A*a^3*b - 32*I*B*a^2*b^2 + 21*I*A*a*
b^3 + 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a
)/b) + sqrt(1/2)*(16*I*B*a^4 - 28*I*A*a^3*b + 32*I*B*a^2*b^2 - 21*I*A*a*b^
3 - 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/
b) + 3*sqrt(1/2)*(-8*I*B*a^3*b + 14*I*A*a^2*b^2 - 19*I*B*a*b^3 - 63*I*A*b^
4)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)
*(8*I*B*a^3*b - 14*I*A*a^2*b^2 + 19*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b)*weiers
trassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(15*B*b^4*cos(d*x + c)^2 -
 4*B*a^2*b^2 + 7*A*a*b^3 + 25*B*b^4 + 3*(B*a*b^3 + 7*A*b^4)*cos(d*x + c))*
sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d)
```

Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)a$$

input
```
int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x)*
b + a)*cos(c + d*x)**2,x)*a
```

3.298 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 231

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx &= \frac{2(5aAb-2a^2B+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad - \frac{2(a^2-b^2)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}} \\ &\quad + \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} \\ &\quad + \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}\end{aligned}$$

output

```
2/15*(5*A*a*b-2*B*a^2+9*B*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/1
5*(a^2-b^2)*(5*A*b-2*B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^2/d/(a+b*cos(d*x+c))^(1/2)+2/15*(5
*A*b-2*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d+2/5*B*(a+b*cos(d*x+c))^(
3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.77

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(5Ab+7aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+(5aAb-2a^2B+9b^2B)\left((a+b)E\left(\frac{1}{2}(c+dx\right.\right.\right.}{15b^2d\sqrt{a+}}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 7*a*B)*EllipticF[(c +
d*x)/2, (2*b)/(a + b)] + (5*a*A*b - 2*a^2*B + 9*b^2*B)*((a + b)*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) +
2*b*(a + b*Cos[c + d*x])*(5*A*b + a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x])/
(15*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sqrt{a+b\cos(c+dx)}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}(3bB+(5Ab-2aB)\cos(c+dx))dx}{5b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\cos(c+dx)}(3bB+(5Ab-2aB)\cos(c+dx))dx}{5b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3bB+(5Ab-2aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{2}{3}\int \frac{b(5Ab+7aB)+(-2Ba^2+5Aba+9b^2B)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2(5Ab-2aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\int \frac{b(5Ab+7aB)+(-2Ba^2+5Aba+9b^2B)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2(5Ab-2aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \frac{b(5Ab+7aB)+(-2Ba^2+5Aba+9b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(5Ab-2aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{1}{3}\left(\frac{(-2a^2B+5aAb+9b^2B)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{(a^2-b^2)(5Ab-2aB)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2(5Ab-2aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{3}\left(\frac{(-2a^2B+5aAb+9b^2B)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(a^2-b^2)(5Ab-2aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2(5Ab-2aB)\sin(c+dx)\sqrt{a+b\cos}}{3d}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$\downarrow$ 3134

$$\frac{\frac{1}{3}\left(\frac{(-2a^2B+5aAb+9b^2B)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(5Ab-2aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2(5Ab-2aB)\sin(}{}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{3}\left(\frac{(-2a^2B+5aAb+9b^2B)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(5Ab-2aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2(5Ab-2aB)s}{}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$\downarrow$ 3132

$$\frac{\frac{1}{3}\left(\frac{2(-2a^2B+5aAb+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(5Ab-2aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2(5Ab-2aB)\sin(c+dx}{3d}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$\downarrow$ 3142

$$\frac{\frac{1}{3}\left(\frac{2\left(-2a^2B+5aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)+\frac{2(5Ab-}{}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{3}\left(\frac{2\left(-2a^2B+5aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)+\frac{2(5}{}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \; 3140$$

$$\frac{\frac{1}{3}\left(\frac{2\left(-2a^2B+5aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}\right)+\frac{2(5A}{}}{5b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (((2*(5*a*A*b - 2*
a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(5*A*b -
2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 + (2*(5*A*b - 2*a*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3*d))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(220) = 440$.

Time = 15.57 (sec) , antiderivative size = 993, normalized size of antiderivative = 4.30

method	result	size
default	Expression too large to display	993
parts	Expression too large to display	1119

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3+16*B*a*b^2+24*B*b^3)*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-2*B*a^2*b-8*
B*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^2*b-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a*b^2+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^3-2*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.13

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/45*(sqrt(1/2)*(4*I*B*a^3 - 10*I*A*a^2*b + 3*I*B*a*b^2 + 15*I*A*b^3)*sqr
t(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(-4*
I*B*a^3 + 10*I*A*a^2*b - 3*I*B*a*b^2 - 15*I*A*b^3)*sqrt(b)*weierstrassPInv
erse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(2*I*B*a^2*b - 5*I*A*a
*b^2 - 9*I*B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)
) + 3*sqrt(1/2)*(-2*I*B*a^2*b + 5*I*A*a*b^2 + 9*I*B*b^3)*sqrt(b)*weierstra
ssZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(3*B*b^3*cos(d*x + c) + B*a*b
^2 + 5*A*b^3)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)\,(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos (dx+c)\, b+a}\, \cos (dx+c)\, dx\right) a + \left(\int \sqrt{\cos (dx+c)\, b+a}\, \cos (dx+c)^2\, dx\right) b$$

input
```
int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)*b +
 a)*cos(c + d*x)**2,x)*b
```

3.299 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2(3Ab+aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(a^2-b^2)\,B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*(3*A*b+B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
)*(b/(a+b))^(1/2))/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*B*((a+
b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))
^(1/2))/b/d/(a+b*cos(d*x+c))^(1/2)+2/3*B*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)
/d
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{2(a+b)(3Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a^2-b^2)\,B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2}{a}\right.}{3bd\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b*Cos[c + d*x])*Sin[
c + d*x])/(3*b*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{3}\int \frac{3aA+bB+(3Ab+aB)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3aA+bB+(3Ab+aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\int \frac{3aA+bB+(3Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3231

$$\frac{1}{3}\left(\frac{(aB+3Ab)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(aB+3Ab)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3134

$$\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3132

$$\frac{1}{3}\left(\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{B\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3142

$$\frac{1}{3}\left(\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3140

$$\frac{1}{3}\left(\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
((2*(3*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*B*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[a + b*Cos[c + d*x]]))/3 + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(164) = 328$.

Time = 10.84 (sec), antiderivative size = 600, normalized size of antiderivative = 3.51

method	result
default	$-\frac{2\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}b^{2}+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}$
parts	$-\frac{2A\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)(a-b)}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}\,d} -$

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(
1/2*d*x+1/2*c)^5*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a*b-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*B*cos(1/2*d
*x+1/2*c)^3*a*b-6*B*cos(1/2*d*x+1/2*c)^3*b^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^2+B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*B*cos(1/2*d*x+1/2*c)*a*b+2*B*
cos(1/2*d*x+1/2*c)*b^2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.56

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(3\sqrt{b\cos(dx+c)+a}Bb^2\sin(dx+c) - \sqrt{\frac{1}{2}}(-2i\,Ba^2+3i\,Aab+3i\,Bb^2)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4\ldots}{}\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(b*cos(d*x + c) + a)*B*b^2*sin(d*x + c) - sqrt(1/2)*(-2*I*B*a^2
 + 3*I*A*a*b + 3*I*B*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
c) + 2*a)/b) - sqrt(1/2)*(2*I*B*a^2 - 3*I*A*a*b - 3*I*B*b^2)*sqrt(b)*weier
strassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(1/2)*(-I*B*a*b -
3*I*A*b^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*s
qrt(1/2)*(I*B*a*b + 3*I*A*b^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x +
 c) + 2*a)/b)))/(b^2*d)
```

Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos(dx+c)\,b+a}dx\right)a + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)b$$

input

```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output

```
int(sqrt(cos(c + d*x)*b + a),x)*a + int(sqrt(cos(c + d*x)*b + a)*cos(c + d
*x),x)*b
```

3.300 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 178

$$\begin{aligned}\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx &= \frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &+ \frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ &+ \frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

output
```
2*B*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*A*b*((a+b*cos(d*x+c))/(a+b))^(1/
2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c
))^(1/2)+2*a*A*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c
),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 12.80 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.60

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)BE\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+A\left(b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+a\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx)\right.\right.\right.}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2
*b)/(a + b)] + A*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {3042, 3481, 3042, 3134, 3042, 3132, 3282, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3481}$$

$$A\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)dx+B\int \sqrt{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+B\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3134}$$

$$A\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$A\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3282}$$

$$A\left(b\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx+a\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$A\left(b\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3142}$$

$$A\left(a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$\downarrow$ 3042

$$A\left(a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$\downarrow$ 3140

$$A\left(a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$\downarrow$ 3286

$$A\left(\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$\downarrow$ 3042

$$A\left(\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$\Big\downarrow$ 3284

$$A\left(\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]) + A*((2*b*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 7.99 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.39

method	result
default	$-\frac{2\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\,\left(Ab\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)-Aa\,\text{El}\cdots\right.}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{d}{2}\cdots\right.}$
parts	$-\frac{2A\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)b-\text{Ellipt}\cdots\right.}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}\,d}$

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(A*b*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-A*a*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a-B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)\,dx\right)b$$
$$+ \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(cos
(c + d*x)*b + a)*sec(c + d*x),x)*a
```

3.301 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\begin{aligned}\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad+\frac{(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ &\quad+\frac{(Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ &\quad+\frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d}\end{aligned}$$

output
```
-A*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(
1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+(A*a+2*B*b)*((a+b*cos(d*x+c))/(a+b)
)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(
d*x+c))^(1/2)+(A*b+2*B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/
2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+A*(a+b*co
s(d*x+c))^(1/2)*tan(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.25 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.75

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$= \frac{\frac{8bB\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2(Ab+4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2iA\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1-}{}}}{}}{}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
((8*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c +
 d*x]] - ((2*I)*A*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Co
s[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt
[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Elli
pticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a
+ b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3478, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$\Big\downarrow$ 3478

$$\int \frac{\left(-Ab\cos^2(c+dx)+2bB\cos(c+dx)+Ab+2aB\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\Big\downarrow$ 27

$$\frac{1}{2}\int \frac{\left(-Ab\cos^2(c+dx)+2bB\cos(c+dx)+Ab+2aB\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\int \frac{-Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+2bB\sin\left(c+dx+\frac{\pi}{2}\right)+Ab+2aB}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\Big\downarrow$ 3538

$$\frac{1}{2}\left(-\frac{\int -\frac{(b(Ab+2aB)+b(aA+2bB)\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-A\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{(b(Ab+2aB)+b(aA+2bB)\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-A\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\int \frac{b(Ab+2aB)+b(aA+2bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-A\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{2}\left(\frac{\int \frac{b(Ab+2aB)+b(aA+2bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{A\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{b(Ab+2aB)+b(aA+2bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{A\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{2}\left(\frac{\int \frac{b(Ab+2aB)+b(aA+2bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{2}\left(\frac{b(aA+2bB)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx+b(2aB+Ab)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{b(aA+2bB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2aB+Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{2}\left(\frac{b(2aB+Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2A\sqrt{a+b\cos(c}{d\sqrt{}}\right.$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{b(2aB+Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2A\sqrt{a+b\,\mathrm{co}}}{}\right.$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\big\downarrow$ 3140

$$\frac{1}{2}\left(\frac{b(2aB+Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2A\sqrt{a+b\,\mathrm{co}}}{d}\right.$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\big\downarrow$ 3286

$$\frac{1}{2}\left(\frac{\frac{b(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2A\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a}{}}}\right.$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\frac{b(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2A\sqrt{}}{}\right.$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$\downarrow$ 3284

$$\frac{1}{2}\left(\frac{\frac{2b(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2A\sqrt{a+b\cos}}{d}\right.$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
((-2*A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b*(a*A + 2*b*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Co
s[c + d*x]]) + (2*b*(A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/2 +
 (A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(213) = 426$.

Time = 10.09 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.50

method	result
default	$-\frac{\sqrt{-\left(-2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}-\frac{2(Ab\cdots}{\cdots}\right.}{\cdots}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*(A*b+B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))+2*A*a*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))+1/2*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}\sec^2{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b$$
$$+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^2\,dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(sqrt(
cos(c + d*x)*b + a)*sec(c + d*x)**2,x)*a
```

3.302 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 292

$$\begin{aligned}
&\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx \\
&= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad+\frac{(3Ab+4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&\quad+\frac{(4a^2A-Ab^2+4abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} \\
&\quad+\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} \\
&\quad+\frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

output
```
-1/4*(A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2)*(b/(a+b))^(1/2))/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/4*(3*A*b+4*B*a)*(
(a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+
b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2-A*b^2+4*B*a*b)*((a+b*cos(
d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/
2))/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*(A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*tan(
d*x+c)/a/d+1/2*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.44

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$

$$= \frac{\frac{8Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2A-3Ab^2+4abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}}-\frac{2i(Ab+4aB)\sqrt{-\frac{b(-}{}}}{}}{}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
((8*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A - 3*A*b^2 + 4*a*b*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a
*Sqrt[a + b*Cos[c + d*x]]) - ((2*I)*(A*b + 4*a*B)*Sqrt[-((b*(-1 + Cos[c +
d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*
(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
 + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[
Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a^2*b*
Sqrt[-(a + b)^(-1)]) + (4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (A*b + 4*a*B)*
Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/a)/(16*d)
```

Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3478, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3478}$$

$$\frac{1}{2}\int \frac{\left(Ab\cos^2(c+dx)+2(aA+2bB)\cos(c+dx)+Ab+4aB\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\left(Ab\cos^2(c+dx)+2(aA+2bB)\cos(c+dx)+Ab+4aB\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+2(aA+2bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab+4aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(4Aa^2+4bBa+2Ab\cos(c+dx)a-Ab^2-b(Ab+4aB)\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx}}{ad}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(4Aa^2+4bBa+2Ab\cos(c+dx)a-Ab^2-b(Ab+4aB)\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx}}{ad}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\int \frac{4Aa^2+4bBa+2Ab\sin\left(c+dx+\frac{\pi}{2}\right)a-Ab^2-b(Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3538}$$

$$\frac{1}{4}\left(\frac{-\frac{\int -\frac{\left(b\left(4Aa^2+4bBa-Ab^2\right)+ab(3Ab+4aB)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left((4aB+Ab)\int\sqrt{a+b\cos(c+dx)}dx\right)}{2a}+\frac{(4aB+Ab)\,\mathrm{t}}{}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{\left(b\left(4Aa^2+4bBa-Ab^2\right)+ab(3Ab+4aB)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-(4aB+Ab)\int\sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{(4aB+Ab)\tan(c+}{}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{b\left(4Aa^2+4bBa-Ab^2\right)+ab(3Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-(4aB+Ab)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}+\frac{(4aB+Ab)\tan(c+\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3134

$$\frac{1}{4}\left(\frac{\frac{\int \frac{b\left(4Aa^2+4bBa-Ab^2\right)+ab(3Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{(4aB+Ab)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(4aB+Ab)\tan(c+\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{\int \frac{b\left(4Aa^2+4bBa-Ab^2\right)+ab(3Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{(4aB+Ab)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(4aB+Ab)\tan(\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3132

$$\frac{1}{4}\left(\frac{\frac{\int \frac{b\left(4Aa^2+4bBa-Ab^2\right)+ab(3Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(4aB+Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(4aB+Ab)\tan(c+dx)}{ad}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3481

$$\frac{1}{4}\left(\frac{\frac{b(4a^2A+4abB-Ab^2)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+ab(4aB+3Ab)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2(4aB+Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(4a\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{b(4a^2A+4abB-Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(4aB+3Ab)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(4aB+Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3142

$$\frac{1}{4}\left(\frac{\frac{b(4a^2A+4abB-Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab(4aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(4aB+Ab)\sqrt{a+b\cos}}{d\sqrt{\frac{a+}{}}}}{2a}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{b(4a^2A+4abB-Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab(4aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(4aB+Ab)\sqrt{a+b}}{d\sqrt{}}}{2a}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3140

$$\frac{1}{4}\left(\frac{\frac{b(4a^2A+4abB-Ab^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab(4aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(4aB+Ab)\sqrt{a+b\cos}}{d\sqrt{\frac{a+}{}}}}{2a}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3286

$$\frac{1}{4}\left(\frac{\frac{\frac{b\left(4a^2A+4abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab(4aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(4aB+Ab)\sqrt{a+b\cos}}{d\sqrt{\frac{a-}{}}}}{2a}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{\frac{b\left(4a^2A+4abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab(4aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(4}{}}{2a}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3284

$$\frac{1}{4}\left(\frac{\frac{\frac{2b(4a^2A+4abB-Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab(4aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(4aB+Ab)\sqrt{a+\cdots}}{d\sqrt{\cdots}}}{2a}\right.$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-2*(A*b
+ 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a*b*(3*A*b + 4*a*B)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a
+ b*Cos[c + d*x]]) + (2*b*(4*a^2*A - A*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Co
s[c + d*x]]))/b)/(2*a) + ((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d
*x])/(a*d))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(280) = 560$.

Time = 13.64 (sec) , antiderivative size = 1290, normalized size of antiderivative = 4.42

method	result	size
default	Expression too large to display	1290
parts	Expression too large to display	1601

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a
)*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+1/2*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A*a*(-1/2
*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*...
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int (A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}\sec^3{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b$$
$$+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^3\,dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(sqrt(
cos(c + d*x)*b + a)*sec(c + d*x)**3,x)*a
```

3.303 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 378

$$\begin{aligned}
&\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx \\
&= -\frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad+\frac{(16a^2A-Ab^2+18abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\cos(c+dx)}} \\
&\quad+\frac{(4a^2Ab+Ab^3+8a^3B-2ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{8a^2d\sqrt{a+b\cos(c+dx)}} \\
&\quad+\frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24a^2d} \\
&\quad+\frac{(Ab+6aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12ad} \\
&\quad+\frac{A\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}$$

output
```
-1/24*(16*A*a^2-3*A*b^2+6*B*a*b)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/a^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1
/24*(16*A*a^2-A*b^2+18*B*a*b)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/a/d/(a+b*cos(d*x+c))^(1/2)+1/8*(
4*A*a^2*b+A*b^3+8*B*a^3-2*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a^2/d/(a+b*cos(d*x+c))^(1
/2)+1/24*(16*A*a^2-3*A*b^2+6*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a^2/
d+1/12*(A*b+6*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d*x+c)/a/d+1/3*A*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.19 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.68

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$

$$= \frac{\frac{2(4aAb^2+24a^2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2Ab+9Ab^3+48a^3B-18ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+\cdots}\right.}{\sqrt{a+b\cos(c+dx)}}}{\cdots}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec^2(c+dx)(Ab\sin(c+dx)+6aB\sin(c+dx))}{12a}+\frac{\sec(c+dx)(16a^2A\sin(c+dx)-3Ab^2\sin(c+dx)+6abB\sin(c+dx\cdots}{24a^2}\right.}{d}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
((2*(4*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A*b + 9*
A*b^3 + 48*a^3*B - 18*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
Pi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*
a^2*A*b + 3*A*b^3 - 6*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((
b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*Ar
cSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*
(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
 + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqr
t[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^
(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]
) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x])
+ 2*(a + b*Cos[c + d*x])^2)))/(96*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec
[c + d*x]^2*(A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/(12*a) + (Sec[c + d*x
]*(16*a^2*A*Sin[c + d*x] - 3*A*b^2*Sin[c + d*x] + 6*a*b*B*Sin[c + d*x]))/(
24*a^2) + (A*Sec[c + d*x]^2*Tan[c + d*x])/3))/d
```

Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.03, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3478, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 3478$$

$$\frac{1}{3}\int \frac{\left(3Ab\cos^2(c+dx)+2(2aA+3bB)\cos(c+dx)+Ab+6aB\right)\sec^3(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\int \frac{\left(3Ab\cos^2(c+dx)+2(2aA+3bB)\cos(c+dx)+Ab+6aB\right)\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\int \frac{3Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+2(2aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab+6aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3534

$$\frac{1}{6}\left(\frac{\int \frac{\left(16Aa^2+6bBa+2(7Ab+6aB)\cos(c+dx)a-3Ab^2+b(Ab+6aB)\cos^2(c+dx)\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{(6aB+Ab)\tan(c+dx)\sec(c+}{2ad}\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(\frac{\int \frac{\left(16Aa^2+6bBa+2(7Ab+6aB)\cos(c+dx)a-3Ab^2+b(Ab+6aB)\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{4a}+\frac{(6aB+Ab)\tan(c+dx)\sec(c+}{2ad}\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\int \frac{16Aa^2+6bBa+2(7Ab+6aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-3Ab^2+b(Ab+6aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{(6aB+Ab)\tan(c+dx)\sec(c+dx)\surd}{2ad}\right.$$
$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3534

$$\frac{1}{6}\left(\frac{\frac{\int \frac{\left(-b\left(16Aa^2+6bBa-3Ab^2\right)\cos^2(c+dx)+2ab(Ab+6aB)\cos(c+dx)+3\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{(16a^2A+6abB-3Ab^2)\tan(c+a}{ad}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 27$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{\left(-b\left(16Aa^2+6bBa-3Ab^2\right)\cos^2(c+dx)+2ab(Ab+6aB)\cos(c+dx)+3\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{(16a^2A+6abB-3Ab^2)\tan(c+a}{ad}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{-b\left(16Aa^2+6bBa-3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab(Ab+6aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{(16a^2A+6abB-3Ab^2)\tan(c+dx)\sqrt{}}{ad}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3538$$

$$\frac{1}{6}\left(\frac{\frac{-\left(\left(16a^2A+6abB-3Ab^2\right)\int\sqrt{a+b\cos(c+dx)}dx\right)-\frac{\int -\frac{\left(3b\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)+ab\left(16Aa^2+18bBa-Ab^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{2a}+\frac{(1}{}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 25$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{\left(3b\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)+ab\left(16Aa^2+18bBa-Ab^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-(16a^2A+6abB-3Ab^2)\int\sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{(16a^2A+\cdots}{\cdots}}{4a}\cdots\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{3b\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)+ab\left(16Aa^2+18bBa-Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-(16a^2A+6abB-3Ab^2)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}+\frac{(16a^2A+6a\cdots}{\cdots}}{4a}\cdots\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{3b\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)+ab\left(16Aa^2+18bBa-Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(16a^2A+6abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(1\cdots}{\cdots}}{4a}\cdots\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{3b\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)+ab\left(16Aa^2+18bBa-Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(16a^2A+6abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\cdots}{4a}\cdots\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3132

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{3b\left(8Ba^3+4Aba^2-2b^2Ba+Ab^3\right)+ab\left(16Aa^2+18bBa-Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(16a^2A+6abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(16a^2A}{}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3481

$$\frac{1}{6}\left(\frac{\frac{\frac{ab\left(16a^2A+18abB-Ab^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3b\left(8a^3B+4a^2Ab-2ab^2B+Ab^3\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\left(16a^2A+6abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\frac{ab\left(16a^2A+18abB-Ab^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b\left(8a^3B+4a^2Ab-2ab^2B+Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(16a^2A+6abB-3Ab^2\right)\sqrt{}}{d\sqrt{a}}}{2a}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3142

$$\frac{1}{6}\left(\frac{\frac{\frac{\frac{ab\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3b\left(8a^3B+4a^2Ab-2ab^2B+Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(16a^2\right.}{}}{2a}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\frac{\frac{ab\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3b\left(8a^3B+4a^2Ab-2ab^2B+Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(\right.}{}}{2a}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3140

$$\frac{1}{6}\left(\frac{\frac{\frac{3b\left(8a^3B+4a^2Ab-2ab^2B+Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(16a^2\right.}{}}{2a}}{4a}\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3286

$$\frac{1}{6}\left(\frac{\frac{\frac{3b\left(8a^3B+4a^2Ab-2ab^2B+Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(1\cdots}{\cdots}}{2a}}{4a}\cdots\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\frac{3b\left(8a^3B+4a^2Ab-2ab^2B+Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(\cdots\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}\cdots}{2a}}{4a}\cdots\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3284

$$\frac{1}{6}\left(\frac{\frac{\left(16a^2A+6abB-3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{\frac{\frac{2ab\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{6b\left(8a^3B+4a^2Ab-2a\cdots\right.}{\cdots}}{b}}{\cdots}}{4a}\cdots\right.$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((A*b +
6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + (((-2
*(16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a*b*(16
*a^2*A - A*b^2 + 18*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*b*(4*a^2*A*b
+ A*b^3 + 8*a^3*B - 2*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
Pi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(2*a)
+ ((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(
a*d))/(4*a))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3478

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3481

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2212 vs. $2(362) = 724$.

Time = 16.37 (sec) , antiderivative size = 2213, normalized size of antiderivative = 5.85

method	result	size
default	Expression too large to display	2213
parts	Expression too large to display	2716

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a
)*(-1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^
(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))*b^2)+2*A*a*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/
12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*...
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{4}dx\right)b+\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{4}dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(sqrt(
cos(c + d*x)*b + a)*sec(c + d*x)**4,x)*a
```

3.304 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 378

$$\begin{aligned}
&\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \\
&-\frac{2(18a^3Ab-246aAb^3-8a^4B-33a^2b^2B-147b^4B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{2(a^2-b^2)(18a^2Ab-75Ab^3-8a^3B-39ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{315b^3d\sqrt{a+b\cos(c+dx)}} \\
&-\frac{2(18a^2Ab-75Ab^3-8a^3B-39ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^2d} \\
&-\frac{2(18aAb-8a^2B-49b^2B)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315b^2d} \\
&+\frac{2(9Ab-4aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63b^2d} \\
&+\frac{2B\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9bd}
\end{aligned}$$

output
```
-2/315*(18*A*a^3*b-246*A*a*b^3-8*B*a^4-33*B*a^2*b^2-147*B*b^4)*(a+b*cos(d*
x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^3/d/((
a+b*cos(d*x+c))/(a+b))^(1/2)+2/315*(a^2-b^2)*(18*A*a^2*b-75*A*b^3-8*B*a^3-
39*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2)*(b/(a+b))^(1/2))/b^3/d/(a+b*cos(d*x+c))^(1/2)-2/315*(18*A*a^2*b-75*
A*b^3-8*B*a^3-39*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d-2/315*(1
8*A*a*b-8*B*a^2-49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/63*(9*
A*b-4*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/9*B*cos(d*x+c)*(a+b*c
os(d*x+c))^(5/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(153a^2Ab+75Ab^3+2a^3B+186ab^2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
(8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(153*a^2*A*b + 75*A*b^3 + 2*a^3
*B + 186*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-18*a^3*A*b + 2
46*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d
*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a +
 b*Cos[c + d*x])*((72*a^2*A*b + 690*A*b^3 - 32*a^3*B + 804*a*b^2*B)*Sin[c
+ d*x] + b*(2*(144*a*A*b + 6*a^2*B + 133*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*
(9*A*b + 10*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^3*d
*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.04, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}\left((9Ab-4aB)\cos^2(c+dx)+7bB\cos(c+dx)+2aB\right)dx}{9b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\cos(c+dx))^{3/2}\left((9Ab-4aB)\cos^2(c+dx)+7bB\cos(c+dx)+2aB\right)dx}{9b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left((9Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+7bB\sin\left(c+dx+\frac{\pi}{2}\right)+2aB\right)dx}{9b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}\left(3b(15Ab-2aB)-\left(-8Ba^2+18Aba-49b^2B\right)\cos(c+dx)\right)dx}{7b}+\frac{2(9Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}}{9b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 27

$$\frac{\frac{\int (a+b\cos(c+dx))^{3/2}(3b(15Ab-2aB)-(-8Ba^2+18Aba-49b^2B)\cos(c+dx))dx}{7b}+\frac{2(9Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}}{9b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(3b(15Ab-2aB)+(8Ba^2-18Aba+49b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2(9Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}}{9b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3232

$$\frac{\frac{\frac{2}{5}\int \frac{3}{2}\sqrt{a+b\cos(c+dx)}\left(b\left(-2Ba^2+57Aba+49b^2B\right)-\left(-8Ba^3+18Aba^2-39b^2Ba-75Ab^3\right)\cos(c+dx)\right)dx-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\sin(c+dx)(a+b}{5d}}{7b}}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3}{5}\int \sqrt{a+b\cos(c+dx)}\left(b\left(-2Ba^2+57Aba+49b^2B\right)-\left(-8Ba^3+18Aba^2-39b^2Ba-75Ab^3\right)\cos(c+dx)\right)dx-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\sin(c+dx)(a+b\cos}{5d}}{7b}}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(-2Ba^2+57Aba+49b^2B\right)+\left(8Ba^3-18Aba^2+39b^2Ba+75Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\sin(c+dx)(}{5d}}{7b}}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3232

$$\frac{\frac{\frac{3}{5}\left(\frac{2}{3}\int \frac{b\left(2Ba^3+153Aba^2+186b^2Ba+75Ab^3\right)-\left(-8Ba^4+18Aba^3-33b^2Ba^2-246Ab^3a-147b^4B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sin(c+d}{3d}\right.}{7b}}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{5}\left(\frac{\frac{1}{3}\int \frac{b\left(2Ba^3+153Aba^2+186b^2Ba+75Ab^3\right)-\left(-8Ba^4+18Aba^3-33b^2Ba^2-246Ab^3a-147b^4B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sin(c+d}{3d}}{7b}\right.}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{5}\left(\frac{\frac{1}{3}\int \frac{b\left(2Ba^3+153Aba^2+186b^2Ba+75Ab^3\right)+\left(8Ba^4-18Aba^3+33b^2Ba^2+246Ab^3a+147b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sin(c-}{3d}}{7b}\right.}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3231$$

$$\frac{\frac{3}{5}\left(\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{2\left(-8\right.}{}}{7b}\right.}{}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{5}\left(\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{7b}\right.}{}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3134$$

$$\frac{\frac{3}{5}\left(\frac{\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+}}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.}{7b}\right.}{}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+}}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3132

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3142

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\sqrt{a+b\cos}}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\sqrt{a+}}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3140

$$\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{2\left(a^2-b^2\right)\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B\right)\sqrt{a+b\cos \cdots}}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\cdots\right.\right.}{7\cdots}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d) + ((2*(
9*A*b - 4*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((-2*(18
*a*A*b - 8*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d
) + (3*(((-2*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B - 147*b^4*
B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*A*b - 75*A*b^3
- 8*a^3*B - 39*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 - (2*(18*a^2*A*b
 - 75*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(3*d)))/5)/(7*b))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(359) = 718$.

Time = 25.27 (sec) , antiderivative size = 1635, normalized size of antiderivative = 4.33

method	result	size
default	Expression too large to display	1635
parts	Expression too large to display	1824

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output

```
-2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(720*A*b^5+1360*B*a*b^4+224
0*B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-936*A*a*b^4-1080*A*b^5-
424*B*a^2*b^3-2040*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+(324*A*a^2*b^3+936*A*a*b^4+840*A*b^5-4*B*a^3*b^2+424*B*a^2*b^3+1568*B
*a*b^4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A*a^3*b^2-1
62*A*a^2*b^3-384*A*a*b^4-240*A*b^5+8*B*a^4*b+2*B*a^3*b^2-282*B*a^2*b^3-444
*B*a*b^4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+18*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-93*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^4*b+18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b^2+246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2*b^3-246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.69

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/945*(sqrt(1/2)*(-16*I*B*a^5 + 36*I*A*a^4*b - 60*I*B*a^3*b^2 - 33*I*A*a^
2*b^3 + 264*I*B*a*b^4 + 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b
*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(16*I*B*a^5 - 36*I*A*a^4*b + 60*I*B*a^
3*b^2 + 33*I*A*a^2*b^3 - 264*I*B*a*b^4 - 225*I*A*b^5)*sqrt(b)*weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos
(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(-8*I*B*a^4*b + 18*
I*A*a^3*b^2 - 33*I*B*a^2*b^3 - 246*I*A*a*b^4 - 147*I*B*b^5)*sqrt(b)*weiers
trassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(8*I*B*a^4*b - 1
8*I*A*a^3*b^2 + 33*I*B*a^2*b^3 + 246*I*A*a*b^4 + 147*I*B*b^5)*sqrt(b)*weie
rstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstr
assPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(35*B*b^5*cos(d*x + c)^3
 - 4*B*a^3*b^2 + 9*A*a^2*b^3 + 88*B*a*b^4 + 75*A*b^5 + 5*(10*B*a*b^4 + 9*A
*b^5)*cos(d*x + c)^2 + (3*B*a^2*b^3 + 72*A*a*b^4 + 49*B*b^5)*cos(d*x + c))
*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2,
x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^4\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)a^2$$

input
```
int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**4,x)*b**2 + 2*int(sqrt(cos(c +
d*x)*b + a)*cos(c + d*x)**3,x)*a*b + int(sqrt(cos(c + d*x)*b + a)*cos(c +
d*x)**2,x)*a**2
```

3.305 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{2(21a^2Ab+63Ab^3-6a^3B+82ab^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(21aAb-6a^2B+25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(21aAb-6a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd}+\frac{2(7Ab-2aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd}+\frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

output
```
2/105*(21*A*a^2*b+63*A*b^3-6*B*a^3+82*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^2/d/((a+b*cos(d*x+c))/
(a+b))^(1/2)-2/105*(a^2-b^2)*(21*A*a*b-6*B*a^2+25*B*b^2)*((a+b*cos(d*x+c))
/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^2/d
/(a+b*cos(d*x+c))^(1/2)+2/105*(21*A*a*b-6*B*a^2+25*B*b^2)*(a+b*cos(d*x+c))
^(1/2)*sin(d*x+c)/b/d+2/35*(7*A*b-2*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)
/b/d+2/7*B*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 4.93 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{4\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(84aAb+51a^2B+25b^2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+(21a^2Ab+\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(84*a*A*b + 51*a^2*B + 25*b^2*B
)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (21*a^2*A*b + 63*A*b^3 - 6*a^3*B
 + 82*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((168*a*A*b + 12*
a^2*B + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + 8*a*B)*Sin[2*(c + d*x)]
+ 5*b*B*Sin[3*(c + d*x)])))/(210*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.645$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3447$$

$$\int (a+b\cos(c+dx))^{3/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}(5bB+(7Ab-2aB)\cos(c+dx))dx}{7b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\cos(c+dx))^{3/2}(5bB+(7Ab-2aB)\cos(c+dx))dx}{7b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(5bB+(7Ab-2aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(b(21Ab+19aB)+\left(-6Ba^2+21Aba+25b^2B\right)\cos(c+dx)\right)dx+\frac{2(7Ab-2aB)\sin(c+dx)(a\cdot}{5d}}{7b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}\left(b(21Ab+19aB)+\left(-6Ba^2+21Aba+25b^2B\right)\cos(c+dx)\right)dx+\frac{2(7Ab-2aB)\sin(c+dx)(a+b}{5d}}{7b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(21Ab+19aB)+\left(-6Ba^2+21Aba+25b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2(7Ab-2aB)\sin}{}}{7b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3232

$$\frac{1}{5}\left(\frac{\frac{2}{3}\int \frac{b(51Ba^2+84Aba+25b^2B)+(-6Ba^3+21Aba^2+82b^2Ba+63Ab^3)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2(-6a^2B+21aAb+25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{7b}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{\frac{1}{3}\int \frac{b(51Ba^2+84Aba+25b^2B)+(-6Ba^3+21Aba^2+82b^2Ba+63Ab^3)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2(-6a^2B+21aAb+25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{7b}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{1}{3}\int \frac{b(51Ba^2+84Aba+25b^2B)+(-6Ba^3+21Aba^2+82b^2Ba+63Ab^3)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(-6a^2B+21aAb+25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{7b}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3231

$$\frac{1}{5}\left(\frac{\frac{1}{3}\left(\frac{(-6a^3B+21a^2Ab+82ab^2B+63Ab^3)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{(a^2-b^2)(-6a^2B+21aAb+25b^2B)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2(-6a^2B+}{}}{7b}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{1}{3}\left(\frac{(-6a^3B+21a^2Ab+82ab^2B+63Ab^3)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(a^2-b^2)(-6a^2B+21aAb+25b^2B)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2(}{}}{7b}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3134

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(-6a^3B+21a^2Ab+82ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\right.}}}{b}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(-6a^3B+21a^2Ab+82ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+\right.}}}{b}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3132$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(-6a^3B+21a^2Ab+82ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}d}{b}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3142$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(-6a^3B+21a^2Ab+82ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a-}}}}{b\sqrt{a+b\cos(c+dx)}}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(-6a^3B+21a^2Ab+82ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a-}}}}{b\sqrt{a+b\cos(c+dx)}}\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{1}{5}\left(\frac{2(-6a^2B+21aAb+25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{1}{3}\left(\frac{2(-6a^3B+21a^2Ab+82ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\right.\right.}{7b}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((2*(7*A*b - 2*a*B
)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((2*(21*a^2*A*b + 63*A
*b^3 - 6*a^3*B + 82*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^
2)*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 + (2*
(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*
d))/5)/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(282) = 564$.

Time = 20.63 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.39

method	result	size
default	Expression too large to display	1305
parts	Expression too large to display	1492

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4-312*B*a*b^3-360*B*
b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(252*A*a*b^3+168*A*b^4+108*B*
a^2*b^2+312*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8
4*A*a^2*b^2-126*A*a*b^3-42*A*b^4-6*B*a^3*b-54*B*a^2*b^2-128*B*a*b^3-80*B*b
^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-21*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+21*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^3+21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3*b-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^2*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a*b^3-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^
4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.89

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/315*(sqrt(1/2)*(12*I*B*a^4 - 42*I*A*a^3*b - 11*I*B*a^2*b^2 + 126*I*A*a*
b^3 + 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a
)/b) + sqrt(1/2)*(-12*I*B*a^4 + 42*I*A*a^3*b + 11*I*B*a^2*b^2 - 126*I*A*a*
b^3 - 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a
)/b) + 3*sqrt(1/2)*(6*I*B*a^3*b - 21*I*A*a^2*b^2 - 82*I*B*a*b^3 - 63*I*A*b
^4)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^
2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^
2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2
)*(-6*I*B*a^3*b + 21*I*A*a^2*b^2 + 82*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b)*weie
rstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstr
assPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(15*B*b^4*cos(d*x + c)^2
 + 3*B*a^2*b^2 + 42*A*a*b^3 + 25*B*b^4 + 3*(8*B*a*b^3 + 7*A*b^4)*cos(d*x +
 c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A + B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\,(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)a^2 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)ab$$

input
```
int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x),x)*a**2 + int(sqrt(cos(c + d*x)*
b + a)*cos(c + d*x)**3,x)*b**2 + 2*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*
x)**2,x)*a*b
```

3.306 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 225

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2(20aAb+3a^2B+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(a^2-b^2)(5Ab+3aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}} + \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/15*(20*A*a*b+3*B*a^2+9*B*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15
*(a^2-b^2)*(5*A*b+3*B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b/d/(a+b*cos(d*x+c))^(1/2)+2/15*(5*A*
b+3*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*B*(a+b*cos(d*x+c))^(3/2)*
sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.90

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2\left(b(15a^2A+5Ab^2+12abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+(20aAb+3a\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*(b*(15*a^2*A + 5*A*b^2 + 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)] + (20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a +
b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]) + b*(a + b*Cos[c + d*x])*(5
*A*b + 6*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Sqrt[a + b*Cos[c
 + d*x]])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}(5aA+3bB+(5Ab+3aB)\cos(c+dx))dx+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}(5aA+3bB+(5Ab+3aB)\cos(c+dx))dx+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(5aA+3bB+(5Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2(3aB+5Ab)\sin(c+dx)\sqrt{a+b\,c}}{3d}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2(3aB+5Ab)\sin(c+dx)\sqrt{a+b\,c}}{3d}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(3aB+5Ab)\sin(c+dx)\sqrt{a}}{3d}\right.$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3231$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2B+20aAb+9b^2B)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{(a^2-b^2)(3aB+5Ab)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2(3a$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2B+20aAb+9b^2B)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(a^2-b^2)(3aB+5Ab)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3134

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2B+20aAb+9b^2B)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(3aB+5Ab)\int\frac{}{\sqrt{a+b\sin}}}{b}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2B+20aAb+9b^2B)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(3aB+5Ab)\int\frac{}{\sqrt{a+b}}}{b}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(3a^2B+20aAb+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(3aB+5Ab)\int\frac{1}{\sqrt{a+b\sin(c+a}}}{b}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3142

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(3a^2B+20aAb+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{b\sqrt{a+b\cos(c+d}}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(3a^2B+20aAb+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{b\sqrt{a+b\cos(c+}}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(3a^2B+20aAb+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx}{a+b}}}{bd\sqrt{a+b\cos(}}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((2*(20*a*A*b + 3*a
^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(5*A*b + 3
*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(214) = 428$.

Time = 16.27 (sec) , antiderivative size = 993, normalized size of antiderivative = 4.41

method	result	size
default	Expression too large to display	993
parts	Expression too large to display	1115

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3+36*B*a*b^2+24*B*b^3)*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-12*B*a^2*b-1
8*B*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^3+3*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-3
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.19

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-2/45*(sqrt(1/2)*(-6*I*B*a^3 + 5*I*A*a^2*b + 18*I*B*a*b^2 + 15*I*A*b^3)*sq
rt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(6*
I*B*a^3 - 5*I*A*a^2*b - 18*I*B*a*b^2 - 15*I*A*b^3)*sqrt(b)*weierstrassPInv
erse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(-3*I*B*a^2*b - 20*I*A
*a*b^2 - 9*I*B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/
b)) + 3*sqrt(1/2)*(3*I*B*a^2*b + 20*I*A*a*b^2 + 9*I*B*b^3)*sqrt(b)*weierst
rassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrass
PInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(3*B*b^3*cos(d*x + c) + 6*B
*a*b^2 + 5*A*b^3)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (A + B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}dx\right)a^2 + 2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)b^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a),x)*a**2 + 2*int(sqrt(cos(c + d*x)*b + a)*cos(
c + d*x),x)*a*b + int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2,x)*b**2
```

3.307 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 236

$$\begin{aligned}\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c&+dx)\,dx = \frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\
&+\frac{2(3aAb-a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}\\
&+\frac{2a^2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\\
&+\frac{2bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*(3*A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(b/(a+b))^(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(3*A*a*b-B*a^2+B
*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+2*a^2*A*((a+b*cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*co
s(d*x+c))^(1/2)+2/3*b*B*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.74 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.72

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{\frac{4(6aAb+3a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(6a^2A+3Ab^2+4abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\right.}{\sqrt{a+b\cos(c+dx)}}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
((4*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^2*A + 3
*A*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b + 4*a*B)*Sq
rt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b
)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt
[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi
[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*b*B*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(6*d)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 3469, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow 3469$$

$$\frac{2}{3}\int \frac{\left(3Aa^2+b(3Ab+4aB)\cos^2(c+dx)+\left(3Ba^2+6Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\int \frac{\left(3Aa^2+b(3Ab+4aB)\cos^2(c+dx)+\left(3Ba^2+6Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\int \frac{3Aa^2+b(3Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Ba^2+6Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3538$$

$$\frac{1}{3}\left((4aB+3Ab)\int \sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(3Aba^2+b\left(-Ba^2+3Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 25$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(3Aba^2+b\left(-Ba^2+3Aba+b^2B\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+(4aB+3Ab)\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\left(\frac{\int \frac{3Aba^2+b\left(-Ba^2+3Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+(4aB+3Ab)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3134

$$\frac{1}{3}\left(\frac{\int \frac{3Aba^2+b\left(-Ba^2+3Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(4aB+3Ab)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{\int \frac{3Aba^2+b\left(-Ba^2+3Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(4aB+3Ab)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3132

$$\frac{1}{3}\left(\frac{\int \frac{3Aba^2+b\left(-Ba^2+3Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2(4aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3481

$$\frac{1}{3}\left(\frac{b\left(a^2(-B)+3aAb+b^2B\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3a^2Ab\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2(4aB+3Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{b\left(a^2(-B)+3aAb+b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2Ab\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2(4aB+3Ab)\sqrt{}}{}\right.$$

$$\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3142

$$\frac{1}{3}\left(\frac{\frac{b\left(a^2(-B)+3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3a^2Ab\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2(4aB+3A}{}\right.$$

$$\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{\frac{b\left(a^2(-B)+3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3a^2Ab\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2(4aB+}{}\right.$$

$$\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3140

$$\frac{1}{3}\left(\frac{3a^2Ab\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(a^2(-B)+3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2(4aB+3}{}\right.$$

$$\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3286

$$\frac{1}{3}\left(\frac{\frac{3a^2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(a^2(-B)+3aAb+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2(4aB+3A\right.$$

$$\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(\frac{\frac{3a^2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(a^2(-B)+3aAb+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\right.$$

$$\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{1}{3}\left(\frac{\frac{2b(a^2(-B)+3aAb+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{6a^2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2(4aB+\right.$$

$$\frac{2bB\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
((2*(3*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b*(3*a*A*b - a^2*B
+ b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*a^2*A*b*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c
 + d*x]]))/b)/3 + (2*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(230) = 460$.

Time = 11.95 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.97

method	result
parts	$\frac{2A\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b}{a-b}}\left(\text{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)a^2-b\,\text{Ell}\right.}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\right.}$
default	$-\frac{2\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5b^2+3Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b}{a-b}}\,\text{EllipticF}\left(\cos\left(\frac{d}{}\right.\right.\right.}{}$

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
2*A*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-b*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a-b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a+b^2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d-2/3*B*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*
a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(
1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{3/2}}{\cos(c+dx)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = 2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b^2 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
2*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x),x)*a*b + int(sqrt
(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x),x)*b**2 + int(sqrt(cos(c
 + d*x)*b + a)*sec(c + d*x),x)*a**2
```

3.308 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 232

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = -\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(a^2A+2Ab^2+2abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{a(3Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{aA\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d}$$

output
```
-(A*a-2*B*b)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+(A*a^2+2*A*b^2+2*B*a*b)*(
(a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+
b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+a*(3*A*b+2*B*a)*((a+b*cos(d*x+c))/(a+b
))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*c
os(d*x+c))^(1/2)+a*A*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.72

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{\frac{8b(Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(5aAb+4a^2B+2b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x
]
```

output
```
((8*b*(A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(5*a*A*b + 4*a^2*B + 2*b
^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-(a*A) + 2*b*B)*Sqrt[-((b*(-1
 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c +
d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[
c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
)))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x
])/(4*d)
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3468, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3468}$$

$$\int \frac{\left(-b(aA-2bB)\cos^2(c+dx)+2b(Ab+2aB)\cos(c+dx)+a(3Ab+2aB)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\left(-b(aA-2bB)\cos^2(c+dx)+2b(Ab+2aB)\cos(c+dx)+a(3Ab+2aB)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{-b(aA-2bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(3Ab+2aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3538}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\left(ab(3Ab+2aB)+b\left(Aa^2+2bBa+2Ab^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left((aA-2bB)\int \sqrt{a+b\cos(c+dx)}dx\right)\right)+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\left(ab(3Ab+2aB)+b\left(Aa^2+2bBa+2Ab^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-(aA-2bB)\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{ab(3Ab+2aB)+b\left(Aa^2+2bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-(aA-2bB)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{2}\left(\frac{\int \frac{ab(3Ab+2aB)+b\left(Aa^2+2bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{ab(3Ab+2aB)+b\left(Aa^2+2bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{2}\left(\frac{\int \frac{ab(3Ab+2aB)+b\left(Aa^2+2bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(aA-2bB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{2}\left(\frac{b\left(a^2A+2abB+2Ab^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx+ab(2aB+3Ab)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2(aA-2bB)\sqrt{a+b\cos(c}{d\sqrt{\frac{a+b\,c}{}}}\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{b\left(a^2A+2abB+2Ab^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(2aB+3Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(aA-2\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{2}\left(\frac{\frac{b\left(a^2A+2abB+2Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+ab(2aB+3Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(aA\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\frac{b\left(a^2A+2abB+2Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+ab(2aB+3Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{2}\left(\frac{ab(2aB+3Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(a^2A+2abB+2Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(a\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{1}{2}\left(\frac{\frac{ab(2aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(a^2A+2abB+2Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(aA-2\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\frac{ab(2aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(a^2A+2abB+2Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{2}\left(\frac{\frac{2b(a^2A+2abB+2Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab(2aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2(aA\right.$$

$$\frac{aA\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
((-2*(a*A - 2*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b*(a^2*A + 2*A*b^2 +
 2*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*b*(3*A*b + 2*a*B)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[a + b*Cos[c + d*x]]))/b)/2 + (a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])
/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(232) = 464$.

Time = 12.23 (sec) , antiderivative size = 991, normalized size of antiderivative = 4.27

method	result	size
parts	Expression too large to display	991
default	Expression too large to display	1167

input

```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output

```
-A*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b+(-2*a^2-2*a*b)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2+2*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+b*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a-3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a
*b)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2+2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-3*a*b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*
sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d+2*B*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2,
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = 2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b^2 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^2\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
2*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**2,x)*a*b + int(s
qrt(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**2,x)*b**2 + int(sqrt
(cos(c + d*x)*b + a)*sec(c + d*x)**2,x)*a**2
```

3.309 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 295

$$\begin{aligned}
\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx =
&-\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{(7aAb+4a^2B+8b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{(4a^2A+3Ab^2+12abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} \\
&+\frac{aA\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

output
```
-1/4*(5*A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/4*(7*A*a*b+4*B*a^
2+8*B*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+3*A*b^2+12*B*
a*b)*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2
)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+1/4*(5*A*b+4*B*a)*(a+b*cos(d*x
+c))^(1/2)*tan(d*x+c)/d+1/2*a*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d*x+
c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.43

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{\frac{8b(aA+4bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2A+Ab^2+20abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x
]
```

output
```
((8*b*(a*A + 4*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + A*b^2 + 20*a*
b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(5*A*b + 4*a*B)*Sqrt[-((b*(-1 +
 Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*
x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
+ d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))
)/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (5*A*b +
 4*a*B)*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Rubi [A] (verified)

Time = 2.62 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3468, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{2}\int \frac{\left(b(aA+4bB)\cos^2(c+dx)+2\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+a(5Ab+4aB)\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\left(b(aA+4bB)\cos^2(c+dx)+2\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+a(5Ab+4aB)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{b(aA+4bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(Aa^2+4bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5Ab+4aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(-ab(5Ab+4aB)\cos^2(c+dx)+2ab(aA+4bB)\cos(c+dx)+a\left(4Aa^2+12bBa+3Ab^2\right)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{(4aB+5Ab)\tan(c+dx)\sqrt{}}{d}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(\frac{\int \frac{\left(-ab(5Ab+4aB)\cos^2(c+dx)+2ab(aA+4bB)\cos(c+dx)+a\left(4Aa^2+12bBa+3Ab^2\right)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{(4aB+5Ab)\tan(c+dx)\sqrt{}}{d}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\int \frac{-ab(5Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab(aA+4bB)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(4Aa^2+12bBa+3Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{(4aB+5Ab)\tan(c+dx)\sqrt{a+}}{d}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3538

$$\frac{1}{4}\left(\frac{-\frac{\int -\frac{\left(ab\left(4Aa^2+12bBa+3Ab^2\right)+ab\left(4Ba^2+7Aba+8b^2B\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a(4aB+5Ab)\int\sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{(4a}{}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 25

$$\frac{1}{4}\left(\frac{\frac{\int \frac{\left(ab\left(4Aa^2+12bBa+3Ab^2\right)+ab\left(4Ba^2+7Aba+8b^2B\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a(4aB+5Ab)\int\sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{(4aB+}{}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{\int \frac{ab\left(4Aa^2+12bBa+3Ab^2\right)+ab\left(4Ba^2+7Aba+8b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-a(4aB+5Ab)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}+\frac{(4aB+\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{ab\left(4Aa^2+12bBa+3Ab^2\right)+ab\left(4Ba^2+7Aba+8b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a(4aB+5Ab)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(4aB+5\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{ab\left(4Aa^2+12bBa+3Ab^2\right)+ab\left(4Ba^2+7Aba+8b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a(4aB+5Ab)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(4aB\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{ab\left(4Aa^2+12bBa+3Ab^2\right)+ab\left(4Ba^2+7Aba+8b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a(4aB+5Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{(4aB+5Ab\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{4}\left(\frac{\frac{ab(4a^2B+7aAb+8b^2B)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+ab(4a^2A+12abB+3Ab^2)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a(4aB+5Ab)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{ab(4a^2B+7aAb+8b^2B)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(4a^2A+12abB+3Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a(4aB+5Ab)\sqrt{a+b\cos}}{d\sqrt{\frac{a-}{}}}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3142

$$\frac{1}{4}\left(\frac{\frac{ab(4a^2A+12abB+3Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab(4a^2B+7aAb+8b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a(4aB+}{}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{ab(4a^2A+12abB+3Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab(4a^2B+7aAb+8b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a(4a}{}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3140

$$\frac{1}{4}\left(\frac{\frac{ab\left(4a^2A+12abB+3Ab^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(4a^2B+7aAb+8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a(4aB-}{}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3286

$$\frac{1}{4}\left(\frac{\frac{\frac{ab\left(4a^2A+12abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(4a^2B+7aAb+8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a(4aB-}{}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\frac{\frac{ab\left(4a^2A+12abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(4a^2B+7aAb+8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{}{a}\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3284

$$\frac{1}{4}\left(\frac{\frac{\frac{2ab\left(4a^2B+7aAb+8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(4a^2A+12abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a(4a}{}}{2a}\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-2*a*(
5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a*b*(7*a*A*b + 4*a^2*B +
 8*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*b*(4*a^2*A + 3*A*b^2 + 12*a*
b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(2*a) + ((5*A*b + 4*a*B)*Sqrt[a
+ b*Cos[c + d*x]]*Tan[c + d*x])/d)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(283) = 566$.

Time = 14.72 (sec) , antiderivative size = 1403, normalized size of antiderivative = 4.76

method	result	size
default	Expression too large to display	1403
parts	Expression too large to display	1722

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a*(2*A*b+B*a)*(-cos(1/2*d*x+1/2*c)/a*(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2*b/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a^2*A*(-1/2*cos(1/2*d*x+1/2*c)/a*(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*...
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3,
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)ab+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b^2+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^3\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
2*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**3,x)*a*b + int(s
qrt(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**3,x)*b**2 + int(sqrt
(cos(c + d*x)*b + a)*sec(c + d*x)**3,x)*a**2
```

3.310 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 375

$$
\begin{aligned}
\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \\
&-\frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{(16a^2A+17Ab^2+42abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{(12a^2Ab-Ab^3+8a^3B+6ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{8ad\sqrt{a+b\cos(c+dx)}} \\
&+\frac{(16a^2A+3Ab^2+30abB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24ad} \\
&+\frac{(7Ab+6aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d} \\
&+\frac{aA\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}
$$

output
```
-1/24*(16*A*a^2+3*A*b^2+30*B*a*b)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/
24*(16*A*a^2+17*A*b^2+42*B*a*b)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJaco
biAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+1/8*(
12*A*a^2*b-A*b^3+8*B*a^3+6*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Ellipti
cPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a/d/(a+b*cos(d*x+c))^(1/
2)+1/24*(16*A*a^2+3*A*b^2+30*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+
1/12*(7*A*b+6*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.19 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.69

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{\frac{2(28aAb^2+24a^2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(56a^2Ab-9Ab^3+48a^3B+6ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\right.}{\sqrt{a+b\cos(c+dx)}}}{} + \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{12}\sec^2(c+dx)(7Ab\sin(c+dx)+6aB\sin(c+dx))+\frac{\sec(c+dx)(16a^2A\sin(c+dx)+3Ab^2\sin(}{24a}\right.}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x
]
```

output
```
((2*(28*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(56*a^2*A*b -
9*A*b^3 + 48*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipti
cPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16
*a^2*A*b - 3*A*b^3 - 30*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-
((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*
ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
(a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*S
qrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b
)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*
x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]
) + 2*(a + b*Cos[c + d*x])^2)))/(96*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec
[c + d*x]^2*(7*A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*
(16*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 30*a*b*B*Sin[c + d*x]))/(2
4*a) + (a*A*Sec[c + d*x]^2*Tan[c + d*x])/3))/d
```

Rubi [A] (verified)

Time = 3.37 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3468, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{3}\int \frac{\left(3b(aA+2bB)\cos^2(c+dx)+2\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(7Ab+6aB)\right)\sec^3(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\int \frac{\left(3b(aA+2bB)\cos^2(c+dx)+2\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(7Ab+6aB)\right)\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\int \frac{3b(aA+2bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(7Ab+6aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{6}\left(\frac{\int \frac{\left(ab(7Ab+6aB)\cos^2(c+dx)+2a\left(6Ba^2+13Aba+12b^2B\right)\cos(c+dx)+a\left(16Aa^2+30bBa+3Ab^2\right)\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{(6aB+7Ab)\tan(\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\frac{\int \frac{\left(ab(7Ab+6aB)\cos^2(c+dx)+2a\left(6Ba^2+13Aba+12b^2B\right)\cos(c+dx)+a\left(16Aa^2+30bBa+3Ab^2\right)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{4a}+\frac{(6aB+7Ab)\tan(\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\int \frac{ab(7Ab+6aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(6Ba^2+13Aba+12b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(16Aa^2+30bBa+3Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{(6aB+7Ab)\tan(c+\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3534

$$\frac{1}{6}\left(\frac{\frac{\int \frac{\left(2b(7Ab+6aB)\cos(c+dx)a^2-b\left(16Aa^2+30bBa+3Ab^2\right)\cos^2(c+dx)a+3\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}}{4a}+\frac{\left(16a^2A+30abB+3Ab^2\right)\text{ta}}{a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{6}\left(\frac{\frac{\int \frac{\left(2b(7Ab+6aB)\cos(c+dx)a^2-b\left(16Aa^2+30bBa+3Ab^2\right)\cos^2(c+dx)a+3\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}}{4a}+\frac{\left(16a^2A+30abB+3Ab^2\right)\text{ta}}{a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\int \frac{2b(7Ab+6aB)\sin\left(c+dx+\frac{\pi}{2}\right)a^2-b\left(16Aa^2+30bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+3\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}}{4a}+\frac{\left(16a^2A+30abB+3Ab^2\right)\tan(c}{d}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3538

$$\frac{1}{6}\left(\frac{\frac{-a(16a^2A+30abB+3Ab^2)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int-\frac{\left(b\left(16Aa^2+42bBa+17Ab^2\right)\cos(c+dx)a^2+3b\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{2a}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int\frac{\left(b\left(16Aa^2+42bBa+17Ab^2\right)\cos(c+dx)a^2+3b\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a(16a^2A+30abB+3Ab^2)\int\sqrt{a+b\cos(c+dx)}dx}{2a}}{4a}+\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int\frac{b\left(16Aa^2+42bBa+17Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-a(16a^2A+30abB+3Ab^2)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}}{4a}+\frac{(16a}{}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \ 3134$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int\frac{b\left(16Aa^2+42bBa+17Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(16a^2A+30abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(16Aa^2+42bBa+17Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(16a^2A+30abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(16Aa^2+42bBa+17Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(8Ba^3+12Aba^2+6b^2Ba-Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(16a^2A+30abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3481$$

$$\frac{1}{6}\left(\frac{\frac{\frac{a^2b\left(16a^2A+42abB+17Ab^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3ab\left(8a^3B+12a^2Ab+6ab^2B-Ab^3\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a\left(16a^2A+30abB+3Ab^2\right)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\frac{\frac{a^2b\left(16a^2A+42abB+17Ab^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3ab\left(8a^3B+12a^2Ab+6ab^2B-Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-2a\left(16a^2A+30abB+\right.}{2a}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3142

$$\frac{1}{6}\left(\frac{\frac{\frac{\frac{a^2b\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3ab\left(8a^3B+12a^2Ab+6ab^2B-Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\ldots}{2a}}{4a}\ldots\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\frac{\frac{a^2b\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3ab\left(8a^3B+12a^2Ab+6ab^2B-Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\ldots}{b}\ldots}{2a}}{4a}\ldots\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3140

$$\frac{1}{6}\left(\frac{\frac{\frac{3ab\left(8a^3B+12a^2Ab+6ab^2B-Ab^3\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2b\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\ldots}{2a}}{4a}\ldots\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\Big\downarrow$ 3286

$$\frac{1}{6}\left(\frac{\frac{\frac{3ab\left(8a^3B+12a^2Ab+6ab^2B-Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2a^2b\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{\frac{3ab\left(8a^3B+12a^2Ab+6ab^2B-Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2a^2b\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Elliptic}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{6}\left(\frac{\frac{\left(16a^2A+30abB+3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{\frac{2a^2b\left(16a^2A+42abB+17Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{6ab\left(8a^3B+12a\right.}{}}{b}}{4a}\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((7*A*
b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((
-2*a*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a^
2*b*(16*a^2*A + 17*A*b^2 + 42*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*a*b
*(12*a^2*A*b - A*b^3 + 8*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]))/b)/(2*a) + ((16*a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*T
an[c + d*x])/d)/(4*a))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2326 vs. $2(359) = 718$.

Time = 18.17 (sec) , antiderivative size = 2327, normalized size of antiderivative = 6.21

method	result	size
default	Expression too large to display	2327
parts	Expression too large to display	2727

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*b^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a*(2*A*b+B*a)*(-1/2*cos(1/2*d*x+1/
2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*
d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b...
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4,
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = 2\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{4}dx\right)ab+\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{4}dx\right)b^{2}+\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{4}dx\right)a^{2}$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
2*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**4,x)*a*b + int(s
qrt(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**4,x)*b**2 + int(sqrt
(cos(c + d*x)*b + a)*sec(c + d*x)**4,x)*a**2
```

3.311 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 462

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = -\frac{2(110a^4Ab-3069a^2Ab^3-1617Ab^5-40a^5B-255a^3b^2B-3705ab^4B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{3465b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{2(a^2-b^2)(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-675b^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right.}{3465b^3d\sqrt{a+b\cos(c+dx)}}$$
$$-\frac{2(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-675b^4B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3465b^2d}$$
$$-\frac{2(110a^2Ab-539Ab^3-40a^3B-335ab^2B)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3465b^2d}$$
$$-\frac{2(22aAb-8a^2B-81b^2B)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{693b^2d}$$
$$+\frac{2(11Ab-4aB)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{99b^2d}$$
$$+\frac{2B\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{11bd}$$

output
```
-2/3465*(110*A*a^4*b-3069*A*a^2*b^3-1617*A*b^5-40*B*a^5-255*B*a^3*b^2-3705
*B*a*b^4)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(
a+b))^(1/2))/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3465*(a^2-b^2)*(110*A*
a^3*b-1254*A*a*b^3-40*B*a^4-285*B*a^2*b^2-675*B*b^4)*((a+b*cos(d*x+c))/(a+
b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^3/d/(a+
b*cos(d*x+c))^(1/2)-2/3465*(110*A*a^3*b-1254*A*a*b^3-40*B*a^4-285*B*a^2*b^
2-675*B*b^4)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d-2/3465*(110*A*a^2*b-5
39*A*b^3-40*B*a^3-335*B*a*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d-2/6
93*(22*A*a*b-8*B*a^2-81*B*b^2)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/9
9*(11*A*b-4*B*a)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/d+2/11*B*cos(d*x+c)
*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{16\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(1705a^3Ab+2871aAb^3+10a^4B+3315a^2b^2B+675b^4B)\,\text{Elliptic}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
(16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(1705*a^3*A*b + 2871*a*A*b^3 +
 10*a^4*B + 3315*a^2*b^2*B + 675*b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2
*B + 3705*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((880*a^3*A*b
 + 32868*a*A*b^3 - 320*a^4*B + 18660*a^2*b^2*B + 13050*b^4*B)*Sin[c + d*x]
 + b*(4*(1650*a^2*A*b + 1463*A*b^3 + 30*a^3*B + 3095*a*b^2*B)*Sin[2*(c + d
*x)] + 5*b*((836*a*A*b + 452*a^2*B + 513*b^2*B)*Sin[3*(c + d*x)] + 7*b*((2
2*A*b + 46*a*B)*Sin[4*(c + d*x)] + 9*b*B*Sin[5*(c + d*x)]))))))/(27720*b^3*
d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2\int \frac{1}{2}(a+b\cos(c+dx))^{5/2}\left((11Ab-4aB)\cos^2(c+dx)+9bB\cos(c+dx)+2aB\right)dx}{11b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\cos(c+dx))^{5/2}\left((11Ab-4aB)\cos^2(c+dx)+9bB\cos(c+dx)+2aB\right)dx}{11b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left((11Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+9bB\sin\left(c+dx+\frac{\pi}{2}\right)+2aB\right)dx}{11b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}(a+b\cos(c+dx))^{5/2}\left(b(77Ab-10aB)-\left(-8Ba^2+22Aba-81b^2B\right)\cos(c+dx)\right)dx}{9b}+\frac{2(11Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}}{11b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 27

$$\frac{\frac{\int (a+b\cos(c+dx))^{5/2}(b(77Ab-10aB)-(-8Ba^2+22Aba-81b^2B)\cos(c+dx))dx}{9b}+\frac{2(11Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}}{11b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$\frac{\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b(77Ab-10aB)+(8Ba^2-22Aba+81b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2(11Ab-4aB)\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}}{11b}+\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3232

$$\frac{\frac{\frac{2}{7}\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}\left(3b(-10Ba^2+143Aba+135b^2B)-(-40Ba^3+110Aba^2-335b^2Ba-539Ab^3)\cos(c+dx)\right)dx-\frac{2\left(-8a^2B+22aAb-81b^2B\right)\text{si}}{7\text{d}}}{9b}}{11b}$$
$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{7}\int (a+b\cos(c+dx))^{3/2}\left(3b(-10Ba^2+143Aba+135b^2B)-(-40Ba^3+110Aba^2-335b^2Ba-539Ab^3)\cos(c+dx)\right)dx-\frac{2\left(-8a^2B+22aAb-81b^2B\right)\sin(}{7d}}{9b}}{11b}$$
$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{1}{7}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(3b(-10Ba^2+143Aba+135b^2B)+(40Ba^3-110Aba^2+335b^2Ba+539Ab^3)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(-8a^2B+22aAb-81b^2\right.}{}}{9b}}{11b}$$
$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3232

$$\frac{\frac{1}{7}\left(\frac{2}{5}\int \frac{3}{2}\sqrt{a+b\cos(c+dx)}\left(b(-10Ba^3+605Aba^2+1010b^2Ba+539Ab^3)-(-40Ba^4+110Aba^3-285b^2Ba^2-1254Ab^3a-675b^4B)\cos(c+dx)\right)dx-\right.}{9b}$$
$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int\sqrt{a+b\cos(c+dx)}\left(b\left(-10Ba^3+605Aba^2+1010b^2Ba+539Ab^3\right)-\left(-40Ba^4+110Aba^3-285b^2Ba^2-1254Ab^3a-675b^4B\right)\cos(c+dx)\right)dx-\frac{2(}{}\right.}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(-10Ba^3+605Aba^2+1010b^2Ba+539Ab^3\right)+\left(40Ba^4-110Aba^3+285b^2Ba^2+1254Ab^3a+675b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)d\right.}{9b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3232

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int\frac{b\left(10Ba^4+1705Aba^3+3315b^2Ba^2+2871Ab^3a+675b^4B\right)-\left(-40Ba^5+110Aba^4-255b^2Ba^3-3069Ab^3a^2-3705b^4Ba-1617Ab^5\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-40\right.}{}\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int\frac{b\left(10Ba^4+1705Aba^3+3315b^2Ba^2+2871Ab^3a+675b^4B\right)-\left(-40Ba^5+110Aba^4-255b^2Ba^3-3069Ab^3a^2-3705b^4Ba-1617Ab^5\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(-40\right.}{}\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int\frac{b\left(10Ba^4+1705Aba^3+3315b^2Ba^2+2871Ab^3a+675b^4B\right)+\left(40Ba^5-110Aba^4+255b^2Ba^3+3069Ab^3a^2+3705b^4Ba+1617Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(-\right.}{}\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3231

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(-40a^5B+110a^4Ab-255a^3b^2B-3069a^2Ab^3-3705ab^4B-1\right.}{b}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-40a^5B+110a^4Ab-255a^3b^2B-3069a^2Ab^3-3705ab^4\right.}{b}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3134

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-40a^5B+110a^4Ab-255a^3b^2B-3069a^2Ab^3-3705ab^4\right.}{b\sqrt{\frac{a+b\cos}{}}}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-40a^5B+110a^4Ab-255a^3b^2B-3069a^2Ab^3-3705ab^4\right.}{b\sqrt{\frac{a+b}{}}}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3132

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-40a^5B+110a^4Ab-255a^3b^2B-3069a^2Ab^3-3705ab^4\right.}{bd\sqrt{\frac{a+b\cos(}{a+}}}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\big\downarrow$ 3142

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-40a^5B+110a^4Ab-255a^3b^2B-306\right.}{}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-40a^5B+110a^4Ab-255a^3b^2B\right.}{}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\big\downarrow$ 3140

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a^2-b^2)\left(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-40a^5B+110a^4Ab-255a^3b^2B-30\right.}{}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d) + ((2*
(11*A*b - 4*a*B)*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((-2*(
22*a*A*b - 8*a^2*B - 81*b^2*B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*d) + ((-2*(110*a^2*A*b - 539*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (3*(((-2*(110*a^4*A*b - 3069*a^2*A*b^
3 - 1617*A*b^5 - 40*a^5*B - 255*a^3*b^2*B - 3705*a*b^4*B)*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]) + (2*(a^2 - b^2)*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 2
85*a^2*b^2*B - 675*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 - (2*(110*a^3*
A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)))/5)/7)/(9*b))/(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(439) = 878$.

Time = 39.42 (sec) , antiderivative size = 1983, normalized size of antiderivative = 4.29

method	result	size
default	Expression too large to display	1983
parts	Expression too large to display	2137

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/3465*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-40*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6+675*b^6*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+40*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-1617*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^6+(-12320*A*b^6-35840*B*a*b^5-50400*B*b
^6)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(22880*A*a*b^5+24640*A*b^6+21
920*B*a^2*b^4+71680*B*a*b^5+56880*B*b^6)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+
1/2*c)+(-14960*A*a^2*b^4-34320*A*a*b^5-22792*A*b^6-4640*B*a^3*b^3-32880*B*
a^2*b^4-66160*B*a*b^5-34920*B*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
+(3520*A*a^3*b^3+14960*A*a^2*b^4+26488*A*a*b^5+10472*A*b^6-20*B*a^4*b^2+46
40*B*a^3*b^3+25120*B*a^2*b^4+30320*B*a*b^5+13860*B*b^6)*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+(-110*A*a^4*b^2-1760*A*a^3*b^3-7326*A*a^2*b^4-7524*A
*a*b^5-1848*A*b^6+40*B*a^5*b+10*B*a^4*b^2-3210*B*a^3*b^3-7080*B*a^2*b^4-66
90*B*a*b^5-2790*B*b^6)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+110*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b+110*A*(sin(1/2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.57

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/10395*(sqrt(1/2)*(-80*I*B*a^6 + 220*I*A*a^5*b - 480*I*B*a^4*b^2 - 1023*
I*A*a^3*b^3 + 2535*I*B*a^2*b^4 + 5379*I*A*a*b^5 + 2025*I*B*b^6)*sqrt(b)*we
ierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(80*I*B*a^6
 - 220*I*A*a^5*b + 480*I*B*a^4*b^2 + 1023*I*A*a^3*b^3 - 2535*I*B*a^2*b^4 -
 5379*I*A*a*b^5 - 2025*I*B*b^6)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(
d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(-40*I*B*a^5*b + 110*I*A*a^4*b^2 - 255*I*
B*a^3*b^3 - 3069*I*A*a^2*b^4 - 3705*I*B*a*b^5 - 1617*I*A*b^6)*sqrt(b)*weie
rstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstr
assPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(40*I*B*a^5*b
- 110*I*A*a^4*b^2 + 255*I*B*a^3*b^3 + 3069*I*A*a^2*b^4 + 3705*I*B*a*b^5 +
1617*I*A*b^6)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) -
3*(315*B*b^6*cos(d*x + c)^4 - 20*B*a^4*b^2 + 55*A*a^3*b^3 + 1025*B*a^2*b^4
 + 1793*A*a*b^5 + 675*B*b^6 + 35*(23*B*a*b^5 + 11*A*b^6)*cos(d*x + c)^3 +
5*(113*B*a^2*b^4 + 209*A*a*b^5 + 81*B*b^6)*cos(d*x + c)^2 + (15*B*a^3*b^3
+ 825*A*a^2*b^4 + 1145*B*a*b^5 + 539*A*b^6)*cos(d*x + c))*sqrt(b*cos(d*...
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2,
x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A&+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^5\,dx\right)b^3 \\ &+3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^4\,dx\right)a\,b^2 \\ &+3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)a^2b \\ &+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)a^3\end{aligned}$$

input
```
int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**5,x)*b**3 + 3*int(sqrt(cos(c +
d*x)*b + a)*cos(c + d*x)**4,x)*a*b**2 + 3*int(sqrt(cos(c + d*x)*b + a)*cos
(c + d*x)**3,x)*a**2*b + int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2,x)*a
**3
```

3.312 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 372

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B+147b^4B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$-\frac{2(a^2-b^2)(45a^2Ab+75Ab^3-10a^3B+114ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{315b^2d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{2(45a^2Ab+75Ab^3-10a^3B+114ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd}$$
$$+\frac{2(45aAb-10a^2B+49b^2B)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315bd}$$
$$+\frac{2(9Ab-2aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd}+\frac{2B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

output
```
2/315*(45*A*a^3*b+435*A*a*b^3-10*B*a^4+279*B*a^2*b^2+147*B*b^4)*(a+b*cos(d
*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^2/d/(
(a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*(a^2-b^2)*(45*A*a^2*b+75*A*b^3-10*B*a^
3+114*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2)*(b/(a+b))^(1/2))/b^2/d/(a+b*cos(d*x+c))^(1/2)+2/315*(45*A*a^2*b+
75*A*b^3-10*B*a^3+114*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d+2/315
*(45*A*a*b-10*B*a^2+49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/63*(
9*A*b-2*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/9*B*(a+b*cos(d*x+c))^
(7/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 5.97 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(405a^2Ab+75Ab^3+155a^3B+261ab^2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(405*a^2*A*b + 75*A*b^3 + 155*a
^3*B + 261*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (45*a^3*A*b +
435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(
a + b*Cos[c + d*x])*(2*(540*a^2*A*b + 345*A*b^3 + 20*a^3*B + 747*a*b^2*B)*
Sin[c + d*x] + b*((540*a*A*b + 300*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5
*b*(2*(9*A*b + 19*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260
*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.03, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.742$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a+b\cos(c+dx))^{5/2}\left(A\cos(c+dx)+B\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}(a+b\cos(c+dx))^{5/2}(7bB+(9Ab-2aB)\cos(c+dx))dx}{9b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\cos(c+dx))^{5/2}(7bB+(9Ab-2aB)\cos(c+dx))dx}{9b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(7bB+(9Ab-2aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3232

$$\frac{\frac{2}{7}\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}\left(3b(15Ab+13aB)+\left(-10Ba^2+45Aba+49b^2B\right)\cos(c+dx)\right)dx+\frac{2(9Ab-2aB)\sin(c+}{}}{9b}$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\int (a+b\cos(c+dx))^{3/2}\left(3b(15Ab+13aB)+\left(-10Ba^2+45Aba+49b^2B\right)\cos(c+dx)\right)dx+\frac{2(9Ab-2aB)\sin(c+dx}{7}}{9b}$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{7}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(3b(15Ab+13aB)+\left(-10Ba^2+45Aba+49b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2(9Ab-}{}}{9b}$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3232

$$\frac{\frac{1}{7}\left(\frac{2}{5}\int \frac{3}{2}\sqrt{a+b\cos(c+dx)}\left(b\left(55Ba^2+120Aba+49b^2B\right)+\left(-10Ba^3+45Aba^2+114b^2Ba+75Ab^3\right)\cos(c+dx\right.\right.}{9b}$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \sqrt{a+b\cos(c+dx)}\left(b\left(55Ba^2+120Aba+49b^2B\right)+\left(-10Ba^3+45Aba^2+114b^2Ba+75Ab^3\right)\cos(c+dx)\right.\right.}{9b}$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(55Ba^2+120Aba+49b^2B\right)+\left(-10Ba^3+45Aba^2+114b^2Ba+75Ab^3\right)\sin\left(c\right.\right.\right.}{9b}$$
$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3232

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int \frac{b(155Ba^3+405Aba^2+261b^2Ba+75Ab^3)+(-10Ba^4+45Aba^3+279b^2Ba^2+435Ab^3a+147b^4B)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2(-10a^3B+45a^2A\cdots}{\cdots}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{b(155Ba^3+405Aba^2+261b^2Ba+75Ab^3)+(-10Ba^4+45Aba^3+279b^2Ba^2+435Ab^3a+147b^4B)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2(-10a^3B+45a^2A\cdots}{\cdots}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{b(155Ba^3+405Aba^2+261b^2Ba+75Ab^3)+(-10Ba^4+45Aba^3+279b^2Ba^2+435Ab^3a+147b^4B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(-10a^3B+45\cdots}{\cdots}\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3231$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{(a^2-b^2)(-10a^3B+45a^2Ab+114ab^2B+75Ab^3)\int\frac{\cdots}{\sqrt{a\cdots}}}{b}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(a^2-b^2)(-10a^3B+45a^2Ab+114ab^2B+75Ab^3)\cdots}{b}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-10a^3B+45a^2Ab+114\cdots\right.}{\cdots}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

↓ 3042

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-10a^3B+45a^2Ab+\cdots\right.}{\cdots}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

↓ 3132

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2\left(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-10a^3B+45a^2Ab+114ab^2\cdots\right.}{b\cdots}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

↓ 3142

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2\left(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-10a^3B+45a^2Ab+114ab^2\cdots\right.}{b\sqrt{\cdots}}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

↓ 3042

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2\left(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-10a^3B+45a^2Ab+114ab^2\cdots\right.}{b\cdots}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow 3140$$

$$\frac{\frac{1}{7}\left(\frac{2(-10a^2B+45aAb+49b^2B)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}+\frac{3}{5}\left(\frac{2(-10a^3B+45a^2Ab+114ab^2B+75Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\right.\right.}{}$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((2*(9*A*b - 2*a*B
)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + ((2*(45*a*A*b - 10*a^2*
B + 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (3*(((2*(45
*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sqrt[a + b*
Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*A*b + 75*A*b^3 - 10*a^3*B + 1
14*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 + (2*(45*a^2*A*b + 75*A*b^3
- 10*a^3*B + 114*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5
)/7)/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(353) = 706$.

Time = 27.94 (sec) , antiderivative size = 1635, normalized size of antiderivative = 4.40

method	result	size
default	Expression too large to display	1635
parts	Expression too large to display	1824

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(720*A*b^5+2080*B*a*b^4+224
0*B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1440*A*a*b^4-1080*A*b^5
-1360*B*a^2*b^3-3120*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+(1080*A*a^2*b^3+1440*A*a*b^4+840*A*b^5+320*B*a^3*b^2+1360*B*a^2*b^3
+2408*B*a*b^4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-270*A*a
^3*b^2-540*A*a^2*b^3-510*A*a*b^4-240*A*b^5-10*B*a^4*b-160*B*a^3*b^2-666*B*
a^2*b^3-684*B*a*b^4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-45*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-30*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+45*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+435*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-435*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.72

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/945*(sqrt(1/2)*(20*I*B*a^5 - 90*I*A*a^4*b - 93*I*B*a^3*b^2 + 345*I*A*a^
2*b^3 + 489*I*B*a*b^4 + 225*I*A*b^5)*sqrt(b)*weierstrassPInverse(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b
*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(-20*I*B*a^5 + 90*I*A*a^4*b + 93*I*B*a
^3*b^2 - 345*I*A*a^2*b^3 - 489*I*B*a*b^4 - 225*I*A*b^5)*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(10*I*B*a^4*b - 4
5*I*A*a^3*b^2 - 279*I*B*a^2*b^3 - 435*I*A*a*b^4 - 147*I*B*b^5)*sqrt(b)*wei
erstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierst
rassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(-10*I*B*a^4*
b + 45*I*A*a^3*b^2 + 279*I*B*a^2*b^3 + 435*I*A*a*b^4 + 147*I*B*b^5)*sqrt(b
)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, we
ierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(35*B*b^5*cos(d*x
+ c)^3 + 5*B*a^3*b^2 + 135*A*a^2*b^3 + 163*B*a*b^4 + 75*A*b^5 + 5*(19*B*a*
b^4 + 9*A*b^5)*cos(d*x + c)^2 + (75*B*a^2*b^3 + 135*A*a*b^4 + 49*B*b^5)*co
s(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\,(A+B\,\cos(c+dx))\,(a+b\,\cos(c+dx))^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)a^3 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^4\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)a\,b^2 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)a^2b$$

input
```
int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x),x)*a**3 + int(sqrt(cos(c + d*x)*
b + a)*cos(c + d*x)**4,x)*b**3 + 3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*
x)**3,x)*a*b**2 + 3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2,x)*a**2*b
```

3.313 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 288

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(a^2-b^2)(56aAb+15a^2B+25b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105bd\sqrt{a+b\cos(c+dx)}} + \frac{2(56aAb+15a^2B+25b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2(7Ab+5aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
2/105*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)-2/105*(a^2-b^2)*(56*A*a*b+15*B*a^2+25*B*b^2)*((a+b*cos(d*x+c
))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b/d
/(a+b*cos(d*x+c))^(1/2)+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*(a+b*cos(d*x+c)
)^(1/2)*sin(d*x+c)/d+2/35*(7*A*b+5*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/
d+2/7*B*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2b(105a^3A+119aAb^2+135a^2bB+25b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*(105*a^3*A + 119*a*A*b^2 + 135*a^2*b*B + 25*b^3*B)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*(161*a^2*A*b +
63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a
+ b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b
)/(a + b)]) + b*(a + b*Cos[c + d*x])*(154*a*A*b + 90*a^2*B + 65*b^2*B + 6*
b*(7*A*b + 15*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c + d*x)])*Sin[c + d*x])
/(105*b*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}(7aA+5bB+(7Ab+5aB)\cos(c+dx))dx+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int (a+b\cos(c+dx))^{3/2}(7aA+5bB+(7Ab+5aB)\cos(c+dx))dx+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(7aA+5bB+(7Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56Aba+25b^2B\right)\cos(c+dx)\right)dx+\frac{2(5aB+\cdots}{\cdots}\right.$$
$$+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56Aba+25b^2B\right)\cos(c+dx)\right)dx+\frac{2(5aB+\cdots}{\cdots}\right.$$
$$+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56Aba+25b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx\cdots\right.$$
$$+\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int\frac{105Aa^3+135bBa^2+119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2+145b^2Ba+63Ab^3\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{105Aa^3+135bBa^2+119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2+145b^2Ba+63Ab^3\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{105Aa^3+135bBa^2+119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2+145b^2Ba+63Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3231

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(15a^3B+161a^2Ab+145ab^2B+63Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(15a^2B+56aAb+25b\right.}{b}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(15a^3B+161a^2Ab+145ab^2B+63Ab^3\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(15a^2B+56aAb\right.}{}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3134

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(15\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3132

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(15a^2E\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3142

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(15a^2E\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(15a^2E\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\downarrow$ 3140

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2(15a^2B+56aAb+25b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{1}{3}\left(\frac{2(15a^3B+161a^2Ab+145ab^2B+63A}{bd\sqrt{\frac{a+}{}}}\right.\right.\right.$$

$$\frac{2B\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + ((2*(7*A*b + 5*a*B)*
(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((2*(161*a^2*A*b + 63*A*
b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b
^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 + (
2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(3*d))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(273) = 546$.

Time = 20.30 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.53

method	result	size
default	Expression too large to display	1305
parts	Expression too large to display	1491

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4-480*B*a*b^3-360*B*
b^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(392*A*a*b^3+168*A*b^4+360*B*
a^2*b^2+480*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
54*A*a^2*b^2-196*A*a*b^3-42*A*b^4-90*B*a^3*b-180*B*a^2*b^2-170*B*a*b^3-80*
B*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+56*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a^3*b-161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^3-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^4-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-10
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.95

$$\int (a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-2/315*(sqrt(1/2)*(-30*I*B*a^4 - 7*I*A*a^3*b + 115*I*B*a^2*b^2 + 231*I*A*a
*b^3 + 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*
a)/b) + sqrt(1/2)*(30*I*B*a^4 + 7*I*A*a^3*b - 115*I*B*a^2*b^2 - 231*I*A*a*
b^3 - 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a
)/b) + 3*sqrt(1/2)*(-15*I*B*a^3*b - 161*I*A*a^2*b^2 - 145*I*B*a*b^3 - 63*I
*A*b^4)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*
a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*
a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt
(1/2)*(15*I*B*a^3*b + 161*I*A*a^2*b^2 + 145*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b
)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, we
ierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(15*B*b^4*cos(d*x
+ c)^2 + 45*B*a^2*b^2 + 77*A*a*b^3 + 25*B*b^4 + 3*(15*B*a*b^3 + 7*A*b^4)*c
os(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A + B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}dx\right)a^3 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)a^2b + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)a\,b^2$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a),x)*a**3 + 3*int(sqrt(cos(c + d*x)*b + a)*cos(
c + d*x),x)*a**2*b + int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3,x)*b**3
+ 3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2,x)*a*b**2
```

3.314 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 292

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c&+dx)\,dx = \frac{2(35aAb+23a^2B+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\&+\frac{2(10a^2Ab+5Ab^3-8a^3B+8ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}\\&+\frac{2a^3A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\\&+\frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d}\\&+\frac{2bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/15*(35*A*a*b+23*B*a^2+9*B*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/15*
(10*A*a^2*b+5*A*b^3-8*B*a^3+8*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2
)+2*a^3*A*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2
^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+2/15*b*(5*A*b+8*B*a)*(a+b
*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/
d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.38 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.55

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{\frac{4(45a^2Ab+5Ab^3+15a^3B+17ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(30a^3A+35aAb^2+23a^2bB+9b^3B)\sqrt{\frac{a+b\cos(c+}{a+b}}}{\sqrt{a+b\cos(c+dx)}}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
((4*(45*a^2*A*b + 5*A*b^3 + 15*a^3*B + 17*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]
] + (2*(30*a^3*A + 35*a*A*b^2 + 23*a^2*b*B + 9*b^3*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
 + d*x]] + ((2*I)*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[-((b*(-1 + Cos[c +
d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*
a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*
Sqrt[-(a + b)^(-1)]) + 4*b*Sqrt[a + b*Cos[c + d*x]]*(5*A*b + 11*a*B + 3*b*
B*Cos[c + d*x])*Sin[c + d*x])/(30*d)
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.04, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.677$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(5Aa^2+b(5Ab+8aB)\cos^2(c+dx)+\left(3Bb^2+5a(2Ab+aB)\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}\left(5Aa^2+b(5Ab+8aB)\cos^2(c+dx)+\left(3Bb^2+5a(2Ab+aB)\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(5Aa^2+b(5Ab+8aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Bb^2+5a(2Ab+aB)\right)\sin\left(c+dx+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)}$$
$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{\left(15Aa^3+b\left(23Ba^2+35Aba+9b^2B\right)\cos^2(c+dx)+\left(15Ba^3+45Aba^2+17b^2Ba+5Ab^3\right)\cos(c+dx)\right)\cdots}{2\sqrt{a+b\cos(c+dx)}}\cdots\right.$$
$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15Aa^3+b\left(23Ba^2+35Aba+9b^2B\right)\cos^2(c+dx)+\left(15Ba^3+45Aba^2+17b^2Ba+5Ab^3\right)\cos(c+dx)\right)\cdots}{\sqrt{a+b\cos(c+dx)}}\cdots\right.$$
$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15Aa^3+b\left(23Ba^2+35Aba+9b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(15Ba^3+45Aba^2+17b^2Ba+5Ab^3\right)\sin\left(c+\cdots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\cdots\right.$$
$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3538

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2B+35aAb+9b^2B\right)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(15Aba^3+b\left(-8Ba^3+10Aba^2+8b^2Ba+5Ab^3\right)\cos(c+dx)\right)\sec\cdots}{\sqrt{a+b\cos(c+dx)}}}{b}\cdots\right.\right.$$
$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 25

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2B+35aAb+9b^2B\right)\int\sqrt{a+b\cos(c+dx)}dx+\frac{\int \frac{\left(15Aba^3+b\left(-8Ba^3+10Aba^2+8b^2Ba+5Ab^3\right)\cos(c+dx)\right)\sec(\cdots}{\sqrt{a+b\cos(c+dx)}}}{b}\cdots\right.\right.$$
$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2B+35aAb+9b^2B\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{15Aba^3+b\left(-8Ba^3+10Aba^2+8b^2Ba+5Ab^3\right)\sin\left(c+dx\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{b}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3134

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2B+35aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\int\frac{15Aba^3+b\left(-8Ba^3+10Aba^2+8b^2Ba+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c\right.}}}{b}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2B+35aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\int\frac{15Aba^3+b\left(-8Ba^3+10Aba^2+8b^2B\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\right.}}}{b}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int\frac{15Aba^3+b\left(-8Ba^3+10Aba^2+8b^2Ba+5Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(23a^2B+35aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3481

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15a^3Ab\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+b\left(-8a^3B+10a^2Ab+8ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2\left(23a^2B+35a\right.}{}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15a^3Ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(-8a^3B+10a^2Ab+8ab^2B+5Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\big\downarrow$ 3142

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15a^3Ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\left(-8a^3B+10a^2Ab+8ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15a^3Ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\left(-8a^3B+10a^2Ab+8ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\big\downarrow$ 3140

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15a^3Ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(-8a^3B+10a^2Ab+8ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$\big\downarrow$ 3286

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\frac{15a^3Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(-8a^3B+10a^2Ab+8ab^2B+5Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\frac{15a^3Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(-8a^3B+10a^2Ab+8ab^2B+5Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23a^2B+35aAb+9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{30a^3Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx)\right.}{d\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((2*(35*a*A*b + 2
3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b*(10*a^2*A*b + 5*A*
b^3 - 8*a^3*B + 8*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (30*a^3*A*b*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/
(d*Sqrt[a + b*Cos[c + d*x]]))/b)/3 + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(3*d))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. $2(282) = 564$.

Time = 18.59 (sec) , antiderivative size = 1067, normalized size of antiderivative = 3.65

method	result	size
default	Expression too large to display	1067
parts	Expression too large to display	1192

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3+56*B*a*b^2+24*B*b^3)*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-22*B*a^2*b-2
8*B*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b^2-15*a^3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^3+8*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
b^2+23*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

Reduce [F]

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx &= 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)\,dx\right)a^2b \\ &+ \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)b^3 \\ &+ 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a\,b^2 \\ &+ \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)a^3\end{aligned}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x),x)*a**2*b + int(s
qrt(cos(c + d*x)*b + a)*cos(c + d*x)**3*sec(c + d*x),x)*b**3 + 3*int(sqrt(
cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x),x)*a*b**2 + int(sqrt(cos(
c + d*x)*b + a)*sec(c + d*x),x)*a**3
```

3.315 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 296

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = & -\frac{(3a^2A-6Ab^2-14abB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ & +\frac{(3a^3A+12aAb^2+4a^2bB+2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} \\ & +\frac{a^2(5Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ & -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} \\ & +\frac{aA(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{d}\end{aligned}$$

output
```
-1/3*(3*A*a^2-6*A*b^2-14*B*a*b)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/3*(3
*A*a^3+12*A*a*b^2+4*B*a^2*b+2*B*b^3)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Invers
eJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+
a^2*(5*A*b+2*B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)-1/3*b*(3*A*a-2*B*
b)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+a*A*(a+b*cos(d*x+c))^(3/2)*tan(d*x+
c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.49

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{\frac{8b(9aAb+9a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(27a^2Ab+6Ab^3+12a^3B+14ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}}{\sqrt{a+b\cos(c+dx)}}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x
]
```

output
```
((8*b*(9*a*A*b + 9*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(27*a^2*A*
b + 6*A*b^3 + 12*a^3*B + 14*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)
*(-3*a^2*A + 6*A*b^2 + 14*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*
Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
 + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]
) + 4*Sqrt[a + b*Cos[c + d*x]]*(3*a^2*A + 2*b^2*B*Cos[c + d*x])*Tan[c + d*
x])/(12*d)
```

Rubi [A] (verified)

Time = 2.64 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3468, 27, 3042, 3528, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3468}$$

$$\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(-b(3aA-2bB)\cos^2(c+dx)+2b(Ab+2aB)\cos(c+dx)+a(5Ab+2aB)\right)\sec(c+dx)dx+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \sqrt{a+b\cos(c+dx)}\left(-b(3aA-2bB)\cos^2(c+dx)+2b(Ab+2aB)\cos(c+dx)+a(5Ab+2aB)\right)\sec(c+dx)dx+\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-b(3aA-2bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5Ab+2a}{\sin\left(c+dx+\frac{\pi}{2}\right)}$$
$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{1}{2}\left(\frac{2}{3}\int \frac{\left(3(5Ab+2aB)a^2-b\left(3Aa^2-14bBa-6Ab^2\right)\cos^2(c+dx)+2b\left(9Ba^2+9Aba+b^2B\right)\cos(c+dx)\right)\sec(}{2\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{\left(3(5Ab+2aB)a^2-b\left(3Aa^2-14bBa-6Ab^2\right)\cos^2(c+dx)+2b\left(9Ba^2+9Aba+b^2B\right)\cos(c+dx)\right)\sec(}{\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{3(5Ab+2aB)a^2-b\left(3Aa^2-14bBa-6Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(9Ba^2+9Aba+b^2B\right)\sin\left(c+dx+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3538}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\left(\left(3a^2A-14abB-6Ab^2\right)\int\sqrt{a+b\cos(c+dx)}dx\right)-\frac{\int -\frac{\left(3b(5Ab+2aB)a^2+b\left(3Aa^3+4bBa^2+12Ab^2a+2b^3B\right)\cos\right.}{\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$
$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\int\frac{\left(3b(5Ab+2aB)a^2+b\left(3Aa^3+4bBa^2+12Ab^2a+2b^3B\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left(3a^2A-14abB-6Ab^2\right)\int\sqrt{a+b\cos}\right.\right.$$
$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\int \frac{3b(5Ab+2aB)a^2+b(3Aa^3+4bBa^2+12Ab^2a+2b^3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b} - \left(3a^2A-14abB-6Ab^2\right)\int \sqrt{a+b\sin\left(c+\right.}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3134

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\int \frac{3b(5Ab+2aB)a^2+b(3Aa^3+4bBa^2+12Ab^2a+2b^3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b} - \frac{\left(3a^2A-14abB-6Ab^2\right)\sqrt{a+b\cos(c+dx)}}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\int \frac{3b(5Ab+2aB)a^2+b(3Aa^3+4bBa^2+12Ab^2a+2b^3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b} - \frac{\left(3a^2A-14abB-6Ab^2\right)\sqrt{a+b\cos(c+dx)}}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3132

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\int \frac{3b(5Ab+2aB)a^2+b(3Aa^3+4bBa^2+12Ab^2a+2b^3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b} - \frac{2\left(3a^2A-14abB-6Ab^2\right)\sqrt{a+b\cos(c+dx}}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3481

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3a^2b(2aB+5Ab)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx + b\left(3a^3A+4a^2bB+12aAb^2+2b^3B\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b} - \frac{2\left(3a^2A\right.}{}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3a^2b(2aB+5Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(3a^3A+4a^2bB+12aAb^2+2b^3B\right)\int \frac{1}{\sqrt{a+b\sin(c+dx}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3142

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3a^2b(2aB+5Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\left(3a^3A+4a^2bB+12aAb^2+2b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c}{a+}}}}{\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3a^2b(2aB+5Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\left(3a^3A+4a^2bB+12aAb^2+2b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c\right.}{}}}}{\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3140

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3a^2b(2aB+5Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(3a^3A+4a^2bB+12aAb^2+2b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$\downarrow$ 3286

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\frac{3a^2b(2aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(3a^3A+4a^2bB+12aAb^2+2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2}{a+}\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\frac{3a^2b(2aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(3a^3A+4a^2bB+12aAb^2+2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{Ellipt}}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\frac{6a^2b(2aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b(3a^3A+4a^2bB+12aAb^2+2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+da\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)(a+b\cos(c+dx))^{3/2}}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(((-2*(3*a^2*A - 6*A*b^2 - 14*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b*
(3*a^3*A + 12*a*A*b^2 + 4*a^2*b*B + 2*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
+ (6*a^2*b*(5*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/3 - (2*b*(
3*a*A - 2*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/2 + (a*A*(a +
 b*Cos[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1490 vs. $2(288) = 576$.

Time = 25.99 (sec) , antiderivative size = 1491, normalized size of antiderivative = 5.04

method	result	size
parts	Expression too large to display	1491
default	Expression too large to display	1563

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-A*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b+(-2*a^3-2*a^2*b)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-2*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+2*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*b^3+5*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))*a^2*b-EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-4*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2)*sin(1/2*d*x+1/2*c)^2-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3-5*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2,
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a^2b+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b^3+3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a\,b^2+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^2\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**2,x)*a**2*b + in
t(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3*sec(c + d*x)**2,x)*b**3 + 3*int
(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**2,x)*a*b**2 + int(
sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2,x)*a**3
```

3.316 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 315

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=&-\frac{(9aAb+4a^2B-8b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\&+\frac{(11a^2Ab+8Ab^3+4a^3B+16ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}\\&+\frac{a(4a^2A+15Ab^2+20abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}\\&+\frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}\\&+\frac{aA(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
-1/4*(9*A*a*b+4*B*a^2-8*B*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/4*(11
*A*a^2*b+8*A*b^3+4*B*a^3+16*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Invers
eJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+
1/4*a*(4*A*a^2+15*A*b^2+20*B*a*b)*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticP
i(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+1
/4*a*(7*A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a*A*(a+b*cos(d*
x+c))^(3/2)*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.18 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.87

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{\frac{2(4a^2Ab+16Ab^3+48ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2(8a^3A+21aAb^2+36a^2bB+8b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{Ellipti}}{\sqrt{a+b\cos(c+dx)}}}{} + \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{4}\sec(c+dx)\,(9aAb\sin(c+dx)+4a^2B\sin(c+dx))+\frac{1}{2}a^2A\sec(c+dx)\tan(c+dx)\right.}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x
]
```

output
```
((2*(4*a^2*A*b + 16*A*b^3 + 48*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a
^3*A + 21*a*A*b^2 + 36*a^2*b*B + 8*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - (
(2*I)*(-9*a*A*b^2 - 4*a^2*b*B + 8*b^3*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)
]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*Elli
pticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)
^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt
[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Co
s[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[
c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(16*d) + (Sqrt[a + b*Cos[c + d*x]]
*((Sec[c + d*x]*(9*a*A*b*Sin[c + d*x] + 4*a^2*B*Sin[c + d*x]))/4 + (a^2*A*
Sec[c + d*x]*Tan[c + d*x])/2))/d
```

Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.03, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{2}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(-b(aA-4bB)\cos^2(c+dx)+2\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+a(7Ab+4aB)\right)\text{s}$$
$$dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\int \sqrt{a+b\cos(c+dx)}\left(-b(aA-4bB)\cos^2(c+dx)+2\left(Aa^2+4bBa+2Ab^2\right)\cos(c+dx)+a(7Ab+4aB)\right)\sec(c+dx)dx+\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-b(aA-4bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(Aa^2+4bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(7\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3526

$$\frac{1}{4}\left(\int \frac{\left(-b\left(4Ba^2+9Aba-8b^2B\right)\cos^2(c+dx)+2b\left(Aa^2+12bBa+4Ab^2\right)\cos(c+dx)+a\left(4Aa^2+20bBa+15\right.\right.}{2\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\left(-b\left(4Ba^2+9Aba-8b^2B\right)\cos^2(c+dx)+2b\left(Aa^2+12bBa+4Ab^2\right)\cos(c+dx)+a\left(4Aa^2+20bBa+\right.\right.}{\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{-b\left(4Ba^2+9Aba-8b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(Aa^2+12bBa+4Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(4Aa^2+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3538

$$\frac{1}{4}\left(\frac{1}{2}\left(-\left(\left(4a^2B+9aAb-8b^2B\right)\int\sqrt{a+b\cos(c+dx)}dx\right)-\frac{\int-\frac{\left(ab\left(4Aa^2+20bBa+15Ab^2\right)+b\left(4Ba^3+11Aba^2+16b^2Ba\right.\right.}{\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 25

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{\left(ab\left(4Aa^2+20bBa+15Ab^2\right)+b\left(4Ba^3+11Aba^2+16b^2Ba+8Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left(4a^2B+9aAb-8b^2B\right)\int\sqrt{\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{ab\left(4Aa^2+20bBa+15Ab^2\right)+b\left(4Ba^3+11Aba^2+16b^2Ba+8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\left(4a^2B+9aAb-8b^2B\right)\int\sqrt{a+b\sin\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3134

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{ab\left(4Aa^2+20bBa+15Ab^2\right)+b\left(4Ba^3+11Aba^2+16b^2Ba+8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(4a^2B+9aAb-8b^2B\right)\sqrt{a+b\cos(}}{\sqrt{\frac{a+b\cos(c}{a+b}}}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{ab\left(4Aa^2+20bBa+15Ab^2\right)+b\left(4Ba^3+11Aba^2+16b^2Ba+8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(4a^2B+9aAb-8b^2B\right)\sqrt{a+b\cos(}}{\sqrt{\frac{a+b\cos}{a+}}}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3132

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{ab\left(4Aa^2+20bBa+15Ab^2\right)+b\left(4Ba^3+11Aba^2+16b^2Ba+8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(4a^2B+9aAb-8b^2B\right)\sqrt{a+b\cos}}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 3481

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{ab\left(4a^2A+20abB+15Ab^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+b\left(4a^3B+11a^2Ab+16ab^2B+8Ab^3\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}d}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{ab\left(4a^2A+20abB+15Ab^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(4a^3B+11a^2Ab+16ab^2B+8Ab^3\right)\int -}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 3142

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{ab\left(4a^2A+20abB+15Ab^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\left(4a^3B+11a^2Ab+16ab^2B+8Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int}{\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{ab(4a^2A+20abB+15Ab^2)\int\frac{1}{\sin(c+dx+\frac{\pi}{2})\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx+\frac{b(4a^3B+11a^2Ab+16ab^2B+8Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int}{\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{ab(4a^2A+20abB+15Ab^2)\int\frac{1}{\sin(c+dx+\frac{\pi}{2})\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx+\frac{2b(4a^3B+11a^2Ab+16ab^2B+8Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\frac{ab(4a^2A+20abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(4a^3B+11a^2Ab+16ab^2B+8Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\frac{ab(4a^2A+20abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(4a^3B+11a^2Ab+16ab^2B+8Ab^3)\sqrt{\frac{a+b\cos(c}{a+b}}}{d\sqrt{a+b\cos(c+dx}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\frac{2ab(4a^2A+20abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b(4a^3B+11a^2Ab+16ab^2B+8Ab^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{Ellipt}}{d\sqrt{a+b\cos(c+dx)}}}{b}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^{3/2}}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-2*(
9*a*A*b + 4*a^2*B - 8*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b*(11*a^2*
A*b + 8*A*b^3 + 4*a^3*B + 16*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*
b*(4*a^2*A + 15*A*b^2 + 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/2 +
 (a*(7*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(303) = 606$.

Time = 70.81 (sec) , antiderivative size = 1742, normalized size of antiderivative = 5.53

method	result	size
default	Expression too large to display	1742
parts	Expression too large to display	2096

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^2*(3*A*b+B*a)*(-cos(1/2*d*x+1/2*c)/a
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2*b/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*a^3*A*(-1/2*cos(1/2*d*x+1/2*c)/a*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*...
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3,
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a^2b + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a\,b^2 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^3\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**3,x)*a**2*b + in
t(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3*sec(c + d*x)**3,x)*b**3 + 3*int
(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**3,x)*a*b**2 + int(
sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**3,x)*a**3
```

3.317 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 376

$$\begin{aligned}
&\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \\
&-\frac{(16a^2A+33Ab^2+54abB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{(16a^3A+59aAb^2+66a^2bB+48b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{(20a^2Ab+5Ab^3+8a^3B+30ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{8d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{(16a^2A+33Ab^2+54abB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d} \\
&+\frac{a(3Ab+2aB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{4d} \\
&+\frac{aA(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}$$

output
```
-1/24*(16*A*a^2+33*A*b^2+54*B*a*b)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/2
4*(16*A*a^3+59*A*a*b^2+66*B*a^2*b+48*B*b^3)*((a+b*cos(d*x+c))/(a+b))^(1/2)
*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))
^(1/2)+1/8*(20*A*a^2*b+5*A*b^3+8*B*a^3+30*B*a*b^2)*((a+b*cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*co
s(d*x+c))^(1/2)+1/24*(16*A*a^2+33*A*b^2+54*B*a*b)*(a+b*cos(d*x+c))^(1/2)*t
an(d*x+c)/d+1/4*a*(3*A*b+2*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d*x+
c)/d+1/3*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.35 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.70

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{\frac{2(52aAb^2+24a^2bB+96b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(104a^2Ab-3Ab^3+48a^3B+126ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{E}}{\sqrt{a+b\cos(c+dx)}}}{} + \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{12}\sec^2(c+dx)\left(13aAb\sin(c+dx)+6a^2B\sin(c+dx)\right)+\frac{1}{24}\sec(c+dx)\left(16a^2A\sin(\right.\right.}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x
]
```

output
```
((2*(52*a*A*b^2 + 24*a^2*b*B + 96*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(10
4*a^2*A*b - 3*A*b^3 + 48*a^3*B + 126*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
 - ((2*I)*(-16*a^2*A*b - 33*A*b^3 - 54*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/
(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/
(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a
 + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a +
 b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(96*d) + (Sqrt[a + b*Cos[c
+ d*x]]*((Sec[c + d*x]^2*(13*a*A*b*Sin[c + d*x] + 6*a^2*B*Sin[c + d*x]))/1
2 + (Sec[c + d*x]*(16*a^2*A*Sin[c + d*x] + 33*A*b^2*Sin[c + d*x] + 54*a*b*
B*Sin[c + d*x]))/24 + (a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/3))/d
```

Rubi [A] (verified)

Time = 3.38 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.05, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{3}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(b(aA+6bB)\cos^2(c+dx)+2\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(3Ab+2aB)\right) \text{s}$$
$$dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\int \sqrt{a+b\cos(c+dx)}\left(b(aA+6bB)\cos^2(c+dx)+2\left(2Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(3Ab+2aB)\right)\sec$$
$$dx)dx+\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(aA+6bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3526$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{\left(b\left(6Ba^2+13Aba+24b^2B\right)\cos^2(c+dx)+2\left(6Ba^3+19Aba^2+36b^2Ba+12Ab^3\right)\cos(c+dx)+a(16A\right.}{2\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{\left(b\left(6Ba^2+13Aba+24b^2B\right)\cos^2(c+dx)+2\left(6Ba^3+19Aba^2+36b^2Ba+12Ab^3\right)\cos(c+dx)+a(16A\right.}{\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{b\left(6Ba^2+13Aba+24b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(6Ba^3+19Aba^2+36b^2Ba+12Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int \frac{\left(-ab\left(16Aa^2+54bBa+33Ab^2\right)\cos^2(c+dx)+2ab\left(6Ba^2+13Aba+24b^2B\right)\cos(c+dx)+3a\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}}{a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int \frac{\left(-ab\left(16Aa^2+54bBa+33Ab^2\right)\cos^2(c+dx)+2ab\left(6Ba^2+13Aba+24b^2B\right)\cos(c+dx)+3a\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int \frac{-ab\left(16Aa^2+54bBa+33Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab\left(6Ba^2+13Aba+24b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3538

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{-a\left(16a^2A+54abB+33Ab^2\right)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(3ab\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)+ab\left(16Aa^3+66bBa^2+59\right.\right.}{\sqrt{a+b\cos(c+dx)}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\left(3ab\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)+ab\left(16Aa^3+66bBa^2+59Ab^2a+48b^3B\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a\left(16a^2A+54abB+33Ab^2\right.}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{3ab\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)+ab\left(16Aa^3+66bBa^2+59Ab^2a+48b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-a\left(16a^2A+54abB+33Ab^2\right)\int\sqrt{\cdots}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3134

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{3ab\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)+ab\left(16Aa^3+66bBa^2+59Ab^2a+48b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(16a^2A+54abB+33Ab^2\right)\sqrt{a+b\cos(c+dx}\cdots}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\cdots}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{3ab\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)+ab\left(16Aa^3+66bBa^2+59Ab^2a+48b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(16a^2A+54abB+33Ab^2\right)\sqrt{a+b\cos(c+dx}\cdots}{\sqrt{\frac{a+b\cos(c+dx}{a+b}\cdots}}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3132

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{3ab\left(8Ba^3+20Aba^2+30b^2Ba+5Ab^3\right)+ab\left(16Aa^3+66bBa^2+59Ab^2a+48b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(16a^2A+54abB+33Ab^2\right)\sqrt{a+b\cos(c+d}\cdots}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\cdots}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3481

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{ab(16a^3A+66a^2bB+59aAb^2+48b^3B)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3ab(8a^3B+20a^2Ab+30ab^2B+5Ab^3)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a(16a^2A+5}{}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{ab(16a^3A+66a^2bB+59aAb^2+48b^3B)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3ab(8a^3B+20a^2Ab+30ab^2B+5Ab^3)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3142

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{3ab(8a^3B+20a^2Ab+30ab^2B+5Ab^3)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+b}}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{3ab(8a^3B+20a^2Ab+30ab^2B+5Ab^3)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+b}}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3140

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{3ab\left(8a^3B+20a^2Ab+30ab^2B+5Ab^3\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Elliptic}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3286

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{\frac{3ab\left(8a^3B+20a^2Ab+30ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\frac{\frac{3ab\left(8a^3B+20a^2Ab+30ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\,c}{}}}{d\sqrt{a+b\cos(c+}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3284

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(16a^2A+54abB+33Ab^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{\frac{2ab\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Elli}}{d\sqrt{a+b\cos(c+dx)}}}{}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^{3/2}}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((3*a
*(3*A*b + 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
 + (((-2*a*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 ((2*a*b*(16*a^3*A + 59*a*A*b^2 + 66*a^2*b*B + 48*b^3*B)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos
[c + d*x]]) + (6*a*b*(20*a^2*A*b + 5*A*b^3 + 8*a^3*B + 30*a*b^2*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*
Sqrt[a + b*Cos[c + d*x]]))/b)/(2*a) + ((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sq
rt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d)/4)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2437 vs. $2(360) = 720$.

Time = 192.19 (sec) , antiderivative size = 2438, normalized size of antiderivative = 6.48

method	result	size
default	Expression too large to display	2438
parts	Expression too large to display	2878

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output

```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^2*(3*A*b+B*a)*(-1/2*cos(1/2*d*x+1/2*
c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2...
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\text{Timed out}$$

input

```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="fricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4,
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^4}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)a^2b+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\sec(dx+c)^4\,dx\right)b^3+3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)a\,b^2+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^4\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**4,x)*a**2*b + in
t(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3*sec(c + d*x)**4,x)*b**3 + 3*int
(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**4,x)*a*b**2 + int(
sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**4,x)*a**3
```

3.318 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 465

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = -\frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{192ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{(356a^2Ab+133Ab^3+128a^3B+472ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{192d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{(48a^4A+120a^2Ab^2-5Ab^4+160a^3bB+40ab^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{192ad}$$
$$+\frac{(36a^2A+59Ab^2+104abB)\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{96d}$$
$$+\frac{a(11Ab+8aB)\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{24d}$$
$$+\frac{aA(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
-1/192*(284*A*a^2*b+15*A*b^3+128*B*a^3+264*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/a/d/((a+b*cos(d*x+c
))/(a+b))^(1/2)+1/192*(356*A*a^2*b+133*A*b^3+128*B*a^3+472*B*a*b^2)*((a+b*
cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(
1/2))/d/(a+b*cos(d*x+c))^(1/2)+1/64*(48*A*a^4+120*A*a^2*b^2-5*A*b^4+160*B*
a^3*b+40*B*a*b^3)*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a/d/(a+b*cos(d*x+c))^(1/2)+1/192*(284*A*a^
2*b+15*A*b^3+128*B*a^3+264*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+
1/96*(36*A*a^2+59*A*b^2+104*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d
*x+c)/d+1/24*a*(11*A*b+8*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^2*tan(d*x+
c)/d+1/4*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.51 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.57

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{\frac{2(144a^3Ab+236aAb^3+416a^2b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(288a^4A+436a^2Ab^2-45Ab^4+832a^3bB-24ab^3B}{\sqrt{a+b\cos(}}}{} + \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{24}\sec^3(c+dx)\,(17aAb\sin(c+dx)+8a^2B\sin(c+dx))+\frac{1}{96}\sec^2(c+dx)\,(36a^2A\sin\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x
]
```

output
```
((2*(144*a^3*A*b + 236*a*A*b^3 + 416*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] +
 (2*(288*a^4*A + 436*a^2*A*b^2 - 45*A*b^4 + 832*a^3*b*B - 24*a*b^3*B)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/
Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-284*a^2*A*b^2 - 15*A*b^4 - 128*a^3*b*B
 - 264*a*b^3*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d
*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
- b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
 d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 -
Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[
c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[
c + d*x])^2)))/(768*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^3*(17*
a*A*b*Sin[c + d*x] + 8*a^2*B*Sin[c + d*x]))/24 + (Sec[c + d*x]^2*(36*a^2*A
*Sin[c + d*x] + 59*A*b^2*Sin[c + d*x] + 104*a*b*B*Sin[c + d*x]))/96 + (Sec
[c + d*x]*(284*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 128*a^3*B*Si
n[c + d*x] + 264*a*b^2*B*Sin[c + d*x]))/(192*a) + (a^2*A*Sec[c + d*x]^3*Ta
n[c + d*x])/4))/d
```

Rubi [A] (verified)

Time = 4.23 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.04, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{1}{4}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(b(3aA+8bB)\cos^2(c+dx)+2\left(3Aa^2+8bBa+4Ab^2\right)\cos(c+dx)+a(11Ab+8aB)\right)$$
$$dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 27

$$\frac{1}{8}\int \sqrt{a+b\cos(c+dx)}\left(b(3aA+8bB)\cos^2(c+dx)+2\left(3Aa^2+8bBa+4Ab^2\right)\cos(c+dx)+a(11Ab+8aB)\right) \text{s}$$
$$dx)dx+\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3042

$$\frac{1}{8}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(3aA+8bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(3Aa^2+8bBa+4Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3526

$$\frac{1}{8}\left(\frac{1}{3}\int \frac{\left(3b\left(8Ba^2+17Aba+16b^2B\right)\cos^2(c+dx)+2\left(16Ba^3+49Aba^2+72b^2Ba+24Ab^3\right)\cos(c+dx)+a(36\right.}{2\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\int \frac{\left(3b\left(8Ba^2+17Aba+16b^2B\right)\cos^2(c+dx)+2\left(16Ba^3+49Aba^2+72b^2Ba+24Ab^3\right)\cos(c+dx)+a(36\right.}{\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\int \frac{3b\left(8Ba^2+17Aba+16b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(16Ba^3+49Aba^2+72b^2Ba+24Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3534

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int \frac{\left(ab\left(36Aa^2+104bBa+59Ab^2\right)\cos^2(c+dx)+2a\left(36Aa^3+152bBa^2+161Ab^2a+96b^3B\right)\cos(c+dx)+a\left(128Ba^3+284Aba^2+264b^2Ba+15\right.\right.}{2\sqrt{a+b\cos(c+dx)}}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int \frac{\left(ab\left(36Aa^2+104bBa+59Ab^2\right)\cos^2(c+dx)+2a\left(36Aa^3+152bBa^2+161Ab^2a+96b^3B\right)\cos(c+dx)+a\left(128Ba^3+284Aba^2+264b^2Ba+15\right.\right.}{\sqrt{a+b\cos(c+dx)}}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int \frac{ab\left(36Aa^2+104bBa+59Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(36Aa^3+152bBa^2+161Ab^2a+96b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(128Ba^3+284Aba^2+264b^2\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\int \frac{\left(2b\left(36Aa^2+104bBa+59Ab^2\right)\cos(c+dx)a^2-b\left(128Ba^3+284Aba^2+264b^2Ba+15Ab^3\right)\cos^2(c+dx)a+3\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba-5Ab^4\right.\right.}{2\sqrt{a+b\cos(c+dx)}}}{a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\int \frac{\left(2b\left(36Aa^2+104bBa+59Ab^2\right)\cos(c+dx)a^2-b\left(128Ba^3+284Aba^2+264b^2Ba+15Ab^3\right)\cos^2(c+dx)a+3\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba-5Ab^4\right.\right.}{\sqrt{a+b\cos(c+dx)}}}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\int \frac{2b\left(36Aa^2+104bBa+59Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2-b\left(128Ba^3+284Aba^2+264b^2Ba+15Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+3\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3538}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{-a\left(128a^3B+284a^2Ab+264ab^2B+15Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(b\left(128Ba^3+356Aba^2+472b^2Ba+133Ab^3\right)\cos(c+dx)a^2+3b\left(48Aa^4+16\right.\right.}{\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{\left(b\left(128Ba^3+356Aba^2+472b^2Ba+133Ab^3\right)\cos(c+dx)a^2+3b\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba-5Ab^4\right)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a\left(128a^3B+284a^2A\right.}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(128Ba^3+356Aba^2+472b^2Ba+133Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba-5Ab^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-a\left(128a^3B+284a^2Ab+264a\right.}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(128Ba^3+356Aba^2+472b^2Ba+133Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba-5Ab^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(128a^3B+284a^2Ab+264ab^2\right.}{}}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(128Ba^3+356Aba^2+472b^2Ba+133Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba-5Ab^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(128a^3B+284a^2Ab+264ab^2\right.}{}}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3132

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(128Ba^3+356Aba^2+472b^2Ba+133Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(48Aa^4+160bBa^3+120Ab^2a^2+40b^3Ba-5Ab^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(128a^3B+284a^2Ab+264ab^2\right.}{a}}{2a}}{4a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3481

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{a^2b\left(128a^3B+356a^2Ab+472ab^2B+133Ab^3\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3ab\left(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a\left(12\right.}{}}{2a}}{4}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{a^2b\left(128a^3B+356a^2Ab+472ab^2B+133Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3ab\left(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\big\downarrow$ 3142

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{a^2b\left(128a^3B+356a^2Ab+472ab^2B+133Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3ab\left(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4\right)\int\frac{}{\sin\left(c\right.}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{a^2b\left(128a^3B+356a^2Ab+472ab^2B+133Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3ab\left(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4\right)\int\frac{}{\sin\ldots}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{3ab\left(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2b\left(128a^3B+356a^2Ab+472ab^2B+133Ab^3\right)\sqrt{\frac{a+b\cos\ldots}{a+\ldots}}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{ 3286}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{3ab\left(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2a^2b\left(128a^3B+356a^2Ab+472ab^2B+133Ab^3\right)\sqrt{\frac{a+b\ldots}{\ldots}}}{d\sqrt{a+b\cos(c+dx\ldots}}}{b}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\frac{\frac{3ab\left(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2a^2b\left(128a^3B+356a^2Ab+472ab^2\right.}{}}{2a}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(36a^2A+104abB+59Ab^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}+\frac{\left(128a^3B+284a^2Ab+264ab^2B+15Ab^3\right.}{d}\right.\right.$$

$$\frac{aA\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((a*(
11*A*b + 8*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d
) + (((36*a^2*A + 59*A*b^2 + 104*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]*Tan[c + d*x])/(2*d) + (((-2*a*(284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 26
4*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a^2*b*(356*a^2*A*b + 133*A*b
^3 + 128*a^3*B + 472*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*a*b*(48*a^
4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*b*B + 40*a*b^3*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
b*Cos[c + d*x]]))/b)/(2*a) + ((284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*
b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d)/(4*a))/6)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3547 vs. $2(445) = 890$.

Time = 537.63 (sec) , antiderivative size = 3548, normalized size of antiderivative = 7.63

method	result	size
default	Expression too large to display	3548
parts	Expression too large to display	4064

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(3*A
*b+B*a)*(-1/3*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+
1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^
2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1
/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2...
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5,
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = 3\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^5dx\right)a^2b + \left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^5dx\right)b^3 + 3\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^5dx\right)a\,b^2 + \left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^5dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output
```
3*int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)**5,x)*a**2*b + in
t(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3*sec(c + d*x)**5,x)*b**3 + 3*int
(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2*sec(c + d*x)**5,x)*a*b**2 + int(
sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**5,x)*a**3
```

3.319 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 320

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx &= \frac{2(56a^2Ab+63Ab^3-48a^3B-44ab^2B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &- \frac{2(56a^3Ab+49aAb^3-48a^4B-32a^2b^2B-25b^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105b^4d\sqrt{a+b\cos(c+dx)}} \\ &- \frac{2(28aAb-24a^2B-25b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^3d} \\ &+ \frac{2(7Ab-6aB)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35b^2d} \\ &+ \frac{2B\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7bd}\end{aligned}$$

output
```
2/105*(56*A*a^2*b+63*A*b^3-48*B*a^3-44*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^4/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)-2/105*(56*A*a^3*b+49*A*a*b^3-48*B*a^4-32*B*a^2*b^2-25*B*b^4)
*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(
a+b))^(1/2))/b^4/d/(a+b*cos(d*x+c))^(1/2)-2/105*(28*A*a*b-24*B*a^2-25*B*b^
2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^3/d+2/35*(7*A*b-6*B*a)*cos(d*x+c)*(
a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d+2/7*B*cos(d*x+c)^2*(a+b*cos(d*x+c))
^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.72

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{4\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(14aAb-12a^2B+25b^2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)-(-56a^2Ab-63Ab^3+48a^3B\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x
]
```

output
```
(4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(14*a*A*b - 12*a^2*B + 25*b^2*B
)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-56*a^2*A*b - 63*A*b^3 + 48*a^3
*B + 44*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(-56*a*A*b +
48*a^2*B + 65*b^2*B + 6*b*(7*A*b - 6*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c
 + d*x)])*Sin[c + d*x])/(210*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2\int \frac{\cos(c+dx)\left((7Ab-6aB)\cos^2(c+dx)+5bB\cos(c+dx)+4aB\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(c+dx)\left((7Ab-6aB)\cos^2(c+dx)+5bB\cos(c+dx)+4aB\right)}{\sqrt{a+b\cos(c+dx)}}dx}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left((7Ab-6aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+5bB\sin\left(c+dx+\frac{\pi}{2}\right)+4aB\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{2\int \frac{-\left(\left(-24Ba^2+28Aba-25b^2B\right)\cos^2(c+dx)\right)+b(21Ab+2aB)\cos(c+dx)+2a(7Ab-6aB)}{2\sqrt{a+b\cos(c+dx)}}dx}{5b}+\frac{2(7Ab-6aB)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-\left(\left(-24Ba^2+28Aba-25b^2B\right)\cos^2(c+dx)\right)+b(21Ab+2aB)\cos(c+dx)+2a(7Ab-6aB)}{\sqrt{a+b\cos(c+dx)}}dx}{5b}+\frac{2(7Ab-6aB)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\left(24Ba^2-28Aba+25b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(21Ab+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(7Ab-6aB)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{2(7Ab-6aB)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{\frac{2\int \frac{b\left(-12Ba^2+14Aba+25b^2B\right)+\left(-48Ba^3+56Aba^2-44b^2Ba+63Ab^3\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(-24a^2B+28aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2(7Ab-6aB)}{}}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{b\left(-12Ba^2+14Aba+25b^2B\right)+\left(-48Ba^3+56Aba^2-44b^2Ba+63Ab^3\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(-24a^2B+28aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2(7Ab-6aB)}{}}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{b\left(-12Ba^2+14Aba+25b^2B\right)+\left(-48Ba^3+56Aba^2-44b^2Ba+63Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(-24a^2B+28aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2(7Ab-6\cdots}{\cdots}}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$\Big\downarrow$ 3231

$$\frac{\frac{\frac{\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b}-\frac{2\left(-24a^2B+28aAb-25b^2\cdots\right)}{\cdots}}{5b}\cdots}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{2\left(-24a^2B+28a\cdots\right)}{\cdots}}{5b}\cdots}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$\Big\downarrow$ 3134

$$\frac{\frac{\frac{\frac{\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\cdots}{5b}\cdots}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\cdots}{b}}{3b}\cdots}{5b}\cdots}{7b}+\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$\Big\downarrow$ 3132

$$\frac{\frac{\frac{\frac{2\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{2(-2}{}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{\frac{2\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c}{a+}}}}{b\sqrt{a+b\cos(c+dx)}}}{3b}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\frac{2\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(}{}}}}{b\sqrt{a+b\cos(c+dx)}}}{3b}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{\frac{\frac{2\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(-48a^4B+56a^3Ab-32a^2b^2B+49aAb^3-25b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+\right.}{bd\sqrt{a+b\cos(c+dx)}}}{3b}}{5b}}{7b}$$

$$\frac{2B\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}}{7bd}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*B*Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*b*d) + ((2*(
7*A*b - 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
 + (((2*(56*a^2*A*b + 63*A*b^3 - 48*a^3*B - 44*a*b^2*B)*Sqrt[a + b*Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]) - (2*(56*a^3*A*b + 49*a*A*b^3 - 48*a^4*B - 32*a^2*b^2*B - 25*
b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(3*b) - (2*(28*a*A*b - 24*a^2*B - 2
5*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/(5*b))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(305) = 610$.

Time = 18.13 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.08

method	result	size
default	Expression too large to display	1305
parts	Expression too large to display	1494

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^4+(-168*A*b^4+24*B*a*b^3-360*B*b
^4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-28*A*a*b^3+168*A*b^4+24*B*a^
2*b^2-24*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(56*A*
a^2*b^2+14*A*a*b^3-42*A*b^4-48*B*a^3*b-12*B*a^2*b^2-44*B*a*b^3-80*B*b^4)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^3*b-49*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3+56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^3*b-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a*b^3-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+48*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+32*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.76

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/315*(sqrt(1/2)*(96*I*B*a^4 - 112*I*A*a^3*b + 52*I*B*a^2*b^2 - 84*I*A*a*
b^3 + 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a
)/b) + sqrt(1/2)*(-96*I*B*a^4 + 112*I*A*a^3*b - 52*I*B*a^2*b^2 + 84*I*A*a*
b^3 - 75*I*B*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a
)/b) + 3*sqrt(1/2)*(48*I*B*a^3*b - 56*I*A*a^2*b^2 + 44*I*B*a*b^3 - 63*I*A*
b^4)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/
2)*(-48*I*B*a^3*b + 56*I*A*a^2*b^2 - 44*I*B*a*b^3 + 63*I*A*b^4)*sqrt(b)*we
ierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weiers
trassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3
*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(15*B*b^4*cos(d*x + c)
^2 + 24*B*a^2*b^2 - 28*A*a*b^3 + 25*B*b^4 - 3*(6*B*a*b^3 - 7*A*b^4)*cos(d*
x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3,x)
```

3.320 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 246

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx
&= -\frac{2(10aAb-8a^2B-9b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad+\frac{2(10a^2Ab+5Ab^3-8a^3B-7ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}} \\
&\quad+\frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} \\
&\quad+\frac{2B\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd}
\end{aligned}$$

output
```
-2/15*(10*A*a*b-8*B*a^2-9*B*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2
/15*(10*A*a^2*b+5*A*b^3-8*B*a^3-7*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*
InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^3/d/(a+b*cos(d*x+
c))^(1/2)+2/15*(5*A*b-4*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d+2/5*B
*cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2(5Ab+2aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+(-10aAb+8a^2B+9b^2B)\left((a+b)E\left(\frac{1}{2}(c+\right.\right.\right.}{15b^3d\sqrt{a}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x
]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 2*a*B)*EllipticF[(c +
d*x)/2, (2*b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*((a + b)*Elliptic
E[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))
+ 2*b*(a + b*Cos[c + d*x])*(5*A*b - 4*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*
x])/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2\int \frac{(5Ab-4aB)\cos^2(c+dx)+3bB\cos(c+dx)+2aB}{2\sqrt{a+b\cos(c+dx)}}dx}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(5Ab-4aB)\cos^2(c+dx)+3bB\cos(c+dx)+2aB}{\sqrt{a+b\cos(c+dx)}}dx}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(5Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+3bB\sin\left(c+dx+\frac{\pi}{2}\right)+2aB}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{b(5Ab+2aB)-\left(-8Ba^2+10Aba-9b^2B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2(5Ab-4aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{b(5Ab+2aB)-\left(-8Ba^2+10Aba-9b^2B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2(5Ab-4aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{b(5Ab+2aB)+\left(8Ba^2-10Aba+9b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2(5Ab-4aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{\frac{\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(-8a^2B+10aAb-9b^2B\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}}{3b}+\frac{2(5Ab-4aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^2B+10aAb-9b^2B\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}+\frac{2(5Ab-4aB)\sin(c+dx)\sqrt{a+b\cos(c+}{3bd}}{5b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \ 3134$$

$$\frac{\frac{\frac{\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2(5Ab-4aB)\sin(c+d}{3bd}}{5b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+2(5Ab-4aB)\sin(}{5b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \ 3132$$

$$\frac{\frac{\frac{\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2(5Ab-4aB)\sin(c+dx)\sqrt{a}{3bd}}{5b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \ 3142$$

$$\frac{\frac{\frac{\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+2(5Ab-4aB}{5b}$$

$$\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2(5Ab-\cdots}{\cdots}}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$\Big\downarrow$ 3140

$$\frac{\frac{\frac{2\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2(5Ab-4a\cdots}{\cdots}}{5b}+\frac{2B\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*B*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d) + (((-2*(
10*a*A*b - 8*a^2*B - 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(10*a^2*
A*b + 5*A*b^3 - 8*a^3*B - 7*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(3*b)
 + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/(5*b
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(235) = 470$.

Time = 14.19 (sec) , antiderivative size = 993, normalized size of antiderivative = 4.04

method	result	size
default	Expression too large to display	993
parts	Expression too large to display	1120

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+(20*A*b^3-4*B*a*b^2+24*B*b^3)*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3+8*B*a^2*b+2*B
*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^2*b+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a*b^2-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^3-7*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^2+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-8*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.00

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/45*(sqrt(1/2)*(-16*I*B*a^3 + 20*I*A*a^2*b - 12*I*B*a*b^2 + 15*I*A*b^3)*
sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^
2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(
16*I*B*a^3 - 20*I*A*a^2*b + 12*I*B*a*b^2 - 15*I*A*b^3)*sqrt(b)*weierstrass
PInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(-8*I*B*a^2*b + 10
*I*A*a*b^2 - 9*I*B*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*a)/b)) + 3*sqrt(1/2)*(8*I*B*a^2*b - 10*I*A*a*b^2 + 9*I*B*b^3)*sqrt(b)*wei
erstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierst
rassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(3*B*b^3*cos(d*x + c) -
 4*B*a*b^2 + 5*A*b^3)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*cos(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2,x)
```

3.321 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 183

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx &= \frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad - \frac{2(3aAb-2a^2B-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}} \\ &\quad + \frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd}\end{aligned}$$

output
```
2/3*(3*A*b-2*B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(b/(a+b))^(1/2))/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(3*A*a*b-2*B
*a^2-B*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2)*(b/(a+b))^(1/2))/b^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*B*(a+b*cos(d*x+c)
)^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{-2(a+b)(-3Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(-3aAb+2a^2B+b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Ellip}}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(-2*(a + b)*(-3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] + 2*(-3*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b
*Cos[c + d*x])*Sin[c + d*x])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3447}$$
$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{bB+(3Ab-2aB)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bB+(3Ab-2aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{bB+(3Ab-2aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{(3Ab-2aB)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{(-2a^2B+3aAb-b^2B)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3Ab-2aB)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(-2a^2B+3aAb-b^2B)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(-2a^2B+3aAb-b^2B)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(-2a^2B+3aAb-b^2B)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3132

$$\frac{\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(-2a^2B+3aAb-b^2B)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3142

$$\frac{\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(-2a^2B+3aAb-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(-2a^2B+3aAb-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3140

$$\frac{\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(-2a^2B+3aAb-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{3b}+\frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
((2*(3*A*b - 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a*A*b - 2*a^2*B
 - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(3*b) + (2*B*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(176) = 352$.

Time = 9.66 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.67

method	result
default	$\frac{2\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}b^{2}+3Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}$
parts	$\frac{2A\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-\mathrm{Elliptic}\right.}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(
1/2*d*x+1/2*c)^5*b^2+3*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*cos(1/2*d*x+1/2*c)^3*a*b+6
*B*cos(1/2*d*x+1/2*c)^3*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-2*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a*b+2*B*cos(1/2*d*x+1/2*c)*a*b-2*B*cos(1/2*d*x
+1/2*c)*b^2)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.39

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\left(3\sqrt{b\cos(dx+c)+a}Bb^2\sin(dx+c)-\sqrt{\frac{1}{2}}(4i\,Ba^2-6i\,Aab+3i\,Bb^2)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2}{3\,b}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
2/9*(3*sqrt(b*cos(d*x + c) + a)*B*b^2*sin(d*x + c) - sqrt(1/2)*(4*I*B*a^2
- 6*I*A*a*b + 3*I*B*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b
^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c
) + 2*a)/b) - sqrt(1/2)*(-4*I*B*a^2 + 6*I*A*a*b - 3*I*B*b^2)*sqrt(b)*weier
strassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(1/2)*(2*I*B*a*b -
 3*I*A*b^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*
sqrt(1/2)*(-2*I*B*a*b + 3*I*A*b^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d
*x + c) + 2*a)/b)))/(b^3*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*cos(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 24.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd} + \frac{2A\left(\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)(a+b)-a\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{bd\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)(2a^2+b^2)-2a\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)(a+b)\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
(2*B*sin(c + d*x)*(a + b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*(ellipticE(c/
2 + (d*x)/2, (2*b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*x)/2, (2*b)/(a
+ b)))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos(c + d*x))^(1/
2)) + (2*B*((a + b*cos(c + d*x))/(a + b))^(1/2)*(ellipticF(c/2 + (d*x)/2,
(2*b)/(a + b))*(2*a^2 + b^2) - 2*a*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))
*(a + b)))/(3*b^2*d*(a + b*cos(c + d*x))^(1/2))
```

Reduce [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*cos(c + d*x),x)
```

3.322 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

output
```
2*B*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*(A*b-B*a)*((a+b*cos(d*x+c))/(a
+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b/d/(a+b
*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)BE\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(Ab-aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2
*b)/(a + b)] + (A*b - a*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sq
rt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3231}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{B\int \sqrt{a+b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

input
```
Int[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c
 + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 5.83 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\,\left(Ab\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)-B\,\mathrm{Ell}\right.}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\sin\left(\frac{dx}{2}\right.}$
parts	$\frac{2A\sqrt{\frac{a+\cos(dx+c)b}{a+b}}\,\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{a+\cos(dx+c)b}}+\frac{2B\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\cos\left(\frac{dx}{2}\right.}{}}}{\sqrt{-2b\sin\left(\frac{dx}{2}+\right.}}$
risch	Expression too large to display

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(A*b*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
-B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.85

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{2\left(-3i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab\right.\right.\right.}{27\,b^3}}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(-3*I*sqrt(1/2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*
a)/b)) + 3*I*sqrt(1/2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
2*a)/b)) + sqrt(1/2)*(-2*I*B*a + 3*I*A*b)*sqrt(b)*weierstrassPInverse(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(2*I*B*a - 3*I*A*b)*sqrt(b)*weier
strassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/(b^2*d)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \int \frac{A + B\cos{(c + dx)}}{\sqrt{a + b\cos{(c + dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \int \frac{B\cos{(dx + c)} + A}{\sqrt{b\cos{(dx + c)} + a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \int \frac{B\cos(dx + c) + A}{\sqrt{b\cos(dx + c) + a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 25.01 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \frac{2\,A\,\mathrm{F}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{d\,\sqrt{a + b\,\cos\left(c + d\,x\right)}} + \frac{2\,B\,\left(\mathrm{E}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\left(a + b\right) - a\,\mathrm{F}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{b\,d\,\sqrt{a + b\,\cos\left(c + d\,x\right)}}$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
(2*A*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*cos(c + d*x))/(a + b)
)^(1/2))/(d*(a + b*cos(c + d*x))^(1/2)) + (2*B*(ellipticE(c/2 + (d*x)/2, (
2*b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b)))*((a + b
*cos(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos(c + d*x))^(1/2))
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \int \sqrt{\cos(dx + c)\,b + a}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a),x)
```

3.323 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*B*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(
b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)+2*A*((a+b*cos(d*x+c))/(a+b))^(1/2
)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+
c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+A\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(B*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)] + A*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c +
 d*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3481}$$

$$A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right) \sqrt{a+b\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+B\int \frac{1}{\sqrt{a+b\sin \left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3142}$$

$$A\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right) \sqrt{a+b\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right) \sqrt{a+b\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$A\int \frac{1}{\sin \left(c+dx+\frac{\pi}{2}\right) \sqrt{a+b\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.64

method	result
default	$\frac{2\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\,\left(A\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)-B\,\mathrm{Elli}\right.}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}\,d}$
parts	$\frac{2A\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a-b}{a-b}}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}\,d}-\frac{2B\sqrt{}}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(A*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2)))/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sec(c + d*x),x)
```

3.324 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= -\frac{A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ &\quad-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} \\ &\quad+\frac{A\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad}\end{aligned}$$

output
```
-A*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(
1/2))/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+A*((a+b*cos(d*x+c))/(a+b))^(1/2)*
InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^
(1/2)-(A*b-2*B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a/d/(a+b*cos(d*x+c))^(1/2)+A*(a+b*cos(d*x+
c))^(1/2)*tan(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.74 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.48

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\frac{2(-3Ab+4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2iA\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\text{arcsinh}\right.\right.}{}}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x
]
```

output
```
((2*(-3*A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*A*Sqrt[-((b*(-1
 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c +
d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[
c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
)))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])
/(4*a*d)
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3479, 27, 3042, 3539, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int -\frac{(Ab\cos^2(c+dx)+Ab-2aB)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int \frac{(Ab\cos^2(c+dx)+Ab-2aB)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int \frac{Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+Ab-2aB}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}$$

$$\Big\downarrow \text{3539}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{A\int \sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{(b(Ab-2aB)-aAb\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{(b(Ab-2aB)-aAb\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+A\int \sqrt{a+b\cos(c+dx)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{b(Ab-2aB)-aAb\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+A\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}$$

$$\Big\downarrow \text{3134}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{b(Ab-2aB)-aAb\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{A\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{b(Ab-2aB)-aAb\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{A\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \text{3132}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{b(Ab-2aB)-aAb\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \text{3481}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b(Ab-2aB)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-aAb\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b(Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-aAb\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \text{3142}$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b(Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b(Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \; 3140$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b(Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \; 3286$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{b(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{b(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \; 3284$$

$$\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{2b(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2A\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
-1/2*((2*A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((-2*a*A*b*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*b*(A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/a + (A*S
qrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3539
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp
[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Simp[1/(b*d)   Int[Simp[a
*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(216) = 432$.

Time = 7.00 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.96

method	result
default	$-\frac{\sqrt{-\left(-2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{a\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\ldots}{\ldots}}}{2\sqrt{-2b\ldots}}\ldots\right.\right.}{\ldots}$
parts	$-\frac{A\sqrt{-\left(-2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{a\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos(\ldots}{\ldots}}}{\sqrt{-2b\sin\ldots}}\ldots\right.}{\ldots}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-cos(
1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(
1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+1/2*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))-2*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^2dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2,x)
```

3.325 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 299

$$\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx
&= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{(4a^2A+3Ab^2-4abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} \\
&\quad + \frac{A\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

output
```
1/4*(3*A*b-4*B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(b/(a+b))^(1/2))/a^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-1/4*(A*b-4*B*a)*
((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a
+b))^(1/2))/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+3*A*b^2-4*B*a*b)*((a+b
*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b)
)^(1/2))/a^2/d/(a+b*cos(d*x+c))^(1/2)-1/4*(3*A*b-4*B*a)*(a+b*cos(d*x+c))^(
1/2)*tan(d*x+c)/a^2/d+1/2*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)*tan(d*x+c)/a
/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.23 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.86

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\frac{8aAb\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2A+9Ab^2-12abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i(3Ab^2-4abB)\sqrt{}}{}}{}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)(-3Ab\sin(c+dx)+4aB\sin(c+dx))}{4a^2}+\frac{A\sec(c+dx)\tan(c+dx)}{2a}\right)}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x
]
```

output
```
((8*a*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + 9*A*b^2 - 12*a*b*B)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])
/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(3*A*b^2 - 4*a*b*B)*Sqrt[(b - b*Cos[c +
 d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*
a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[
a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c +
 d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 -
 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4
*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(16*a^2*d) + (Sqrt[a
 + b*Cos[c + d*x]]*((Sec[c + d*x]*(-3*A*b*Sin[c + d*x] + 4*a*B*Sin[c + d*x
]))/(4*a^2) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a)))/d
```

Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int -\frac{\left(-Ab\cos^2(c+dx)-2aA\cos(c+dx)+3Ab-4aB\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\int\frac{(-Ab\cos^2(c+dx)-2aA\cos(c+dx)+3Ab-4aB)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{4a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\int\frac{-Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-2aA\sin\left(c+dx+\frac{\pi}{2}\right)+3Ab-4aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}$$

$\Big\downarrow$ 3534

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{\int-\frac{\left(4Aa^2-4bBa+2Ab\cos(c+dx)a+3Ab^2+b(3Ab-4aB)\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}}{4a}$$

$\Big\downarrow$ 27

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int\frac{\left(4Aa^2-4bBa+2Ab\cos(c+dx)a+3Ab^2+b(3Ab-4aB)\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int\frac{4Aa^2-4bBa+2Ab\sin\left(c+dx+\frac{\pi}{2}\right)a+3Ab^2+b(3Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}}{4a}$$

$\Big\downarrow$ 3538

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{(3Ab-4aB)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int-\frac{\left(b\left(4Aa^2-4bBa+3Ab^2\right)-ab(Ab-4aB)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{2a}}{4a}$$

$\Big\downarrow$ 25

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{\left(b\left(4Aa^2-4bBa+3Ab^2\right)-ab(Ab-4aB)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+(3Ab-4aB)\int\sqrt{a+b\cos(c+dx)}dx}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4Aa^2-4bBa+3Ab^2\right)-ab(Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+(3Ab-4aB)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}}{4a}$$

$\Big\downarrow$ 3134

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4Aa^2-4bBa+3Ab^2\right)-ab(Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4Aa^2-4bBa+3Ab^2\right)-ab(Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

$\Big\downarrow$ 3132

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4Aa^2-4bBa+3Ab^2\right)-ab(Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2(3Ab-4aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

$\Big\downarrow$ 3481

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-$$
$$\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2A-4abB+3Ab^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab(Ab-4aB)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2(3Ab-4aB)\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a+b\cos}{a+}}}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-$$
$$\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2A-4abB+3Ab^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab(Ab-4aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+2(3}{2a}}{4a}$$

$\downarrow$ 3142

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-$$
$$\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2A-4abB+3Ab^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos}{}}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-$$
$$\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2A-4abB+3Ab^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+b}+\frac{b\sin}{}}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}$$

$\downarrow$ 3140

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-$$
$$\frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2A-4abB+3Ab^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}$$

$\downarrow$ 3286

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad} - \frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad} - \frac{\frac{\frac{b\left(4a^2A-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}} - \frac{2ab(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}(}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad} - \frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad} - \frac{\frac{\frac{b\left(4a^2A-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}} - \frac{2ab(Ab-4aB)\sqrt{\frac{a+b\cos}{}}}{d\sqrt{}}}{b}}{2a}}{4a}$$

$\Big\downarrow$ 3284

$$\frac{A\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad} - \frac{\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad} - \frac{\frac{\frac{2b\left(4a^2A-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} - \frac{2ab(Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Ellipti}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - (-1/2*((2
*(3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((-2*a*b*(A*b - 4*a*B)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*
Sqrt[a + b*Cos[c + d*x]]) + (2*b*(4*a^2*A + 3*A*b^2 - 4*a*b*B)*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt
[a + b*Cos[c + d*x]]))/b)/a + ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(a*d))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(287) = 574$.

Time = 7.89 (sec) , antiderivative size = 1182, normalized size of antiderivative = 3.95

method	result	size
default	Expression too large to display	1182
parts	Expression too large to display	1244

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/2*
cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*b^2)+2*B*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec^3{(c+dx)}}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^3dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**3,x)
```

3.326 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 387

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx &= \\
&-\frac{2(40a^3Ab-25aAb^3-48a^4B+24a^2b^2B+9b^4B)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15b^4(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{2(40a^2Ab+5Ab^3-48a^3B-12ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^4d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{2(20a^2Ab-5Ab^3-24a^3B+9ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^3(a^2-b^2)\,d} \\
&-\frac{2(5aAb-6a^2B+b^2B)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5b^2(a^2-b^2)\,d}
\end{aligned}$$

output
```
-2/15*(40*A*a^3*b-25*A*a*b^3-48*B*a^4+24*B*a^2*b^2+9*B*b^4)*(a+b*cos(d*x+c
))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^4/(a^2-b^
2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/15*(40*A*a^2*b+5*A*b^3-48*B*a^3-12*B
*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/
2)*(b/(a+b))^(1/2))/b^4/d/(a+b*cos(d*x+c))^(1/2)+2*a*(A*b-B*a)*cos(d*x+c)^
2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/15*(20*A*a^2*b-5*A*b^3
-24*B*a^3+9*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^3/(a^2-b^2)/d-2/5
*(5*A*a*b-6*B*a^2+B*b^2)*cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/
(a^2-b^2)/d
```

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.79

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{2b^2(-10a^2Ab-5Ab^3+12a^3B+3ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{(a-b)(a+b)} + \frac{2(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2)
,x]
```

output
```
((2*b^2*(-10*a^2*A*b - 5*A*b^3 + 12*a^3*B + 3*a*b^2*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/((a - b)*(a + b)) +
 (2*(-40*a^3*A*b + 25*a*A*b^3 + 48*a^4*B - 24*a^2*b^2*B - 9*b^4*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
 - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/((a - b)*(a + b)) + (30*a^3*b
*(-(A*b) + a*B)*Sin[c + d*x])/(-a^2 + b^2) + 2*b*(5*A*b - 9*a*B)*(a + b*Co
s[c + d*x])*Sin[c + d*x] + 3*b^2*B*(a + b*Cos[c + d*x])*Sin[2*(c + d*x)])/
(15*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3468, 27, 3042, 3528, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{\cos(c+dx)\left(-\left(\left(-6Ba^2+5Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab-aB)\cos(c+dx)+4a(Ab-aB)\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(c+dx)\left(-\left(\left(-6Ba^2+5Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab-aB)\cos(c+dx)+4a(Ab-aB)\right)}{\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\left(6Ba^2-5Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+4a(Ab-aB)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{2\int -\frac{-\left(\left(-24Ba^3+20Aba^2+9b^2Ba-5Ab^3\right)\cos^2(c+dx)\right)-b\left(-2Ba^2+5Aba-3b^2B\right)\cos(c+dx)+2a\left(-6Ba^2+5Aba+b^2B\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{5b}-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin(\ldots}{\ldots}}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{-\left(\left(-24Ba^3+20Aba^2+9b^2Ba-5Ab^3\right)\cos^2(c+dx)\right)-b\left(-2Ba^2+5Aba-3b^2B\right)\cos(c+dx)+2a\left(-6Ba^2+5Aba+b^2B\right)}{\sqrt{a+b\cos(c+dx)}}dx}{5b}-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin(c\ldots}{\ldots}}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{\left(24Ba^3-20Aba^2-9b^2Ba+5Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(-2Ba^2+5Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a\left(-6Ba^2+5Aba+b^2B\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin\ldots}{\ldots}}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3502$$

$$\frac{-\frac{\frac{2\int \frac{b\left(-12Ba^3+10Aba^2-3b^2Ba+5Ab^3\right)+\left(-48Ba^4+40Aba^3+24b^2Ba^2-25Ab^3a+9b^4B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(-24a^3B+20a^2Ab+9ab^2B-5Ab^3\right)\sin(c+dx)\sqrt{a+\ldots}}{3bd}}{5b}\ldots}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{\int \frac{b\left(-12Ba^3+10Aba^2-3b^2Ba+5Ab^3\right)+\left(-48Ba^4+40Aba^3+24b^2Ba^2-25Ab^3a+9b^4B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(-24a^3B+20a^2Ab+9ab^2B-5Ab^3\right)\sin(c+dx)\sqrt{a+b\,\mathrm{c}\ldots}}{3bd}}{5b}\ldots}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{b\left(-12Ba^3+10Aba^2-3b^2Ba+5Ab^3\right)+\left(-48Ba^4+40Aba^3+24b^2Ba^2-25Ab^3a+9b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(-24a^3B+20a^2Ab+9ab^2B-5Ab^3\right)\sin(c+dx)\sqrt{\cdots}}{3bd}}{5b}}{b\left(a^2-b^2\right)}$$
$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3231

$$-\frac{\frac{\frac{\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-48a^3B+40a^2Ab-12ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b}-\frac{2\left(-24a^3B+20\cdots\right.}{\cdots}}{5b}}{b\left(a^2-b^2\right)}$$
$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-48a^3B+40a^2Ab-12ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{2\left(-24\cdots\right.}{\cdots}}{5b}}{b\left(a^2-b^2\right)}$$
$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3134

$$-\frac{\frac{\frac{\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-48a^3B+40a^2Ab-12ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\cdots\right.}}}{b}}{3b}\cdots}{5b}}{b\left(a^2-b^2\cdots\right.}$$
$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-48a^3B+40a^2Ab-12ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c\cdots\right.}}}{b}}{3b}\cdots}{5b}}{b\left(a^2-\cdots\right.}$$
$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3132

$$-\frac{\frac{\frac{2\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-48a^3B+40a^2Ab-12ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}d}{b}}{3b}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3142

$$-\frac{\frac{\frac{2\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-48a^3B+40a^2Ab-12ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+}}}}{b\sqrt{a+b\cos(c+dx)}}}{3b}}{5b}}{b\left(a^2-\right.}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{2\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-48a^3B+40a^2Ab-12ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+}}}}{b\sqrt{a+b\cos(c+dx)}}}{3b}}{5b}}{b\left(a^2\right.}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3140

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+$$

$$\frac{-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}-\frac{\frac{2\left(-48a^4B+40a^3Ab+24a^2b^2B-25aAb^3+9b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{}}{b\left(a^2\right.}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]]) + ((-2*(5*a*A*b - 6*a^2*B + b^2*B)*Cos[c + d*x]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(5*b*d) - (((2*(40*a^3*A*b - 25*a*A*b^3 - 48*a
^4*B + 24*a^2*b^2*B + 9*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 -
b^2)*(40*a^2*A*b + 5*A*b^3 - 48*a^3*B - 12*a*b^2*B)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c
+ d*x]]))/(3*b) - (2*(20*a^2*A*b - 5*A*b^3 - 24*a^3*B + 9*a*b^2*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/(5*b))/(b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. $2(374) = 748$.

Time = 16.44 (sec) , antiderivative size = 1312, normalized size of antiderivative = 3.39

method	result	size
default	Expression too large to display	1312
parts	Expression too large to display	2030

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*a^2*b
+A*a*b^2+A*b^3-B*a^3-B*a^2*b-B*a*b^2-B*b^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))+2/b^4*(A*a*b+2*A*b^2-B*a^2-2*B*a*b-3*B*b^2)*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+8
/b^2*(A*b-B*a-3*B*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6/b*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(
1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+16/b*B*(-1/10/b*cos(1/2*d
*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1
/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.30

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
2/45*(sqrt(1/2)*(96*I*B*a^6 - 80*I*A*a^5*b - 84*I*B*a^4*b^2 + 80*I*A*a^3*b
^3 - 27*I*B*a^2*b^4 + 15*I*A*a*b^5 + (96*I*B*a^5*b - 80*I*A*a^4*b^2 - 84*I
*B*a^3*b^3 + 80*I*A*a^2*b^4 - 27*I*B*a*b^5 + 15*I*A*b^6)*cos(d*x + c))*sqr
t(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(-96
*I*B*a^6 + 80*I*A*a^5*b + 84*I*B*a^4*b^2 - 80*I*A*a^3*b^3 + 27*I*B*a^2*b^4
 - 15*I*A*a*b^5 + (-96*I*B*a^5*b + 80*I*A*a^4*b^2 + 84*I*B*a^3*b^3 - 80*I*
A*a^2*b^4 + 27*I*B*a*b^5 - 15*I*A*b^6)*cos(d*x + c))*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(48*I*B*a^5*b - 40*I
*A*a^4*b^2 - 24*I*B*a^3*b^3 + 25*I*A*a^2*b^4 - 9*I*B*a*b^5 + (48*I*B*a^4*b
^2 - 40*I*A*a^3*b^3 - 24*I*B*a^2*b^4 + 25*I*A*a*b^5 - 9*I*B*b^6)*cos(d*x +
 c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/
2)*(-48*I*B*a^5*b + 40*I*A*a^4*b^2 + 24*I*B*a^3*b^3 - 25*I*A*a^2*b^4 + 9*I
*B*a*b^5 + (-48*I*B*a^4*b^2 + 40*I*A*a^3*b^3 + 24*I*B*a^2*b^4 - 25*I*A*a*b
^5 + 9*I*B*b^6)*cos(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2),
x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3}{\cos(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3)/(cos(c + d*x)*b + a),x)
```

3.327 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 262

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(6a^2Ab-3Ab^3-8a^3B+5ab^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{3b^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$-\frac{2(6aAb-8a^2B-b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}$$
$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

output
```
2/3*(6*A*a^2*b-3*A*b^3-8*B*a^3+5*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^3/(a^2-b^2)/d/((a+b*cos(d*x
+c))/(a+b))^(1/2)-2/3*(6*A*a*b-8*B*a^2-B*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/
2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^3/d/(a+b*cos(d
*x+c))^(1/2)-2*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(
1/2)+2/3*B*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.72

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((6a^2Ab-3Ab^3-8a^3B+5ab^2B)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+(a-b)(-6aAb+\right.}{a-b}}{3b}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2)
,x]
```

output
```
(2*((Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((6*a^2*A*b - 3*A*b^3 - 8*a^3*B +
5*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (a - b)*(-6*a*A*b + 8*a
^2*B + b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b) + b*((a*(3*a
*A*b - 4*a^2*B + b^2*B))/(-a^2 + b^2) + b*B*Cos[c + d*x])*Sin[c + d*x]))/(
3*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3467, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{2\int \frac{b(a^2-b^2)B\cos^2(c+dx)+(2a^2-b^2)(Ab-aB)\cos(c+dx)+ab(Ab-aB)}{2\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)} - \frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 27

$$\frac{\int \frac{b(a^2-b^2)B\cos^2(c+dx)+(2a^2-b^2)(Ab-aB)\cos(c+dx)+ab(Ab-aB)}{\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)} - \frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\int \frac{b(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)^2+(2a^2-b^2)(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(Ab-aB)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2(a^2-b^2)} - \frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3502

$$\frac{\frac{2\int \frac{\left(-2Ba^2+3Aba-b^2B\right)b^2+\left(-8Ba^3+6Aba^2+5b^2Ba-3Ab^3\right)\cos(c+dx)b}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2B(a^2-b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{b^2(a^2-b^2)} - \frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(-2Ba^2+3Aba-b^2B\right)b^2+\left(-8Ba^3+6Aba^2+5b^2Ba-3Ab^3\right)\cos(c+dx)b}{\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2B(a^2-b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{b^2(a^2-b^2)} - \frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(-2Ba^2+3Aba-b^2B\right)b^2+\left(-8Ba^3+6Aba^2+5b^2Ba-3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2B(a^2-b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{b^2(a^2-b^2)} - \frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3231

$$\frac{\frac{\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx-\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2B\left(a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos}}{3d}}{b^2\left(a^2-b^2\right)}$$
$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2B\left(a^2-b^2\right)\sin(c+dx)\sqrt{}}{3d}}{b^2\left(a^2-b^2\right)}$$
$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2B\left(a^2-b^2\right)\sin}{}}{b^2\left(a^2-b^2\right)}$$
$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2B\left(a^2-b^2\right.}{}}{b^2\left(a^2-b^2\right)}$$
$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3132

$$\frac{\frac{\frac{2\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2B\left(a^2-b^2\right)\sin(c+}{3}}{b^2\left(a^2-b^2\right)}$$
$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3142

$$\frac{\frac{\frac{2\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{3b}+\frac{2B(a^2}{}}{b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{3b}+\frac{2B}{}}{b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3140

$$\frac{\frac{2B\left(a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{\frac{2\left(-8a^3B+6a^2Ab+5ab^2B-3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\left(-8a^2B+6aAb-b^2B\right)\sqrt{a+b}}{d\sqrt{a+b\cos}}}{3b}}{b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*
x]]) + (((2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Sqrt[a + b*Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) - (2*(a^2 - b^2)*(6*a*A*b - 8*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[
c + d*x]]))/(3*b) + (2*(a^2 - b^2)*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(3*d))/(b^2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1274 vs. $2(253) = 506$.

Time = 15.99 (sec) , antiderivative size = 1275, normalized size of antiderivative = 4.87

method	result	size
parts	Expression too large to display	1275
default	Expression too large to display	1336

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2*A*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b-2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^3-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)/b^2/(a-b)/(a+b)
/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d-2/3*B*(4*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b^2-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4*b^4-8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3*b-2*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b^2+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2*a*b^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^4+8*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-7*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 767, normalized size of antiderivative = 2.93

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
2/9*(sqrt(1/2)*(-16*I*B*a^5 + 12*I*A*a^4*b + 16*I*B*a^3*b^2 - 15*I*A*a^2*b
^3 + 3*I*B*a*b^4 + (-16*I*B*a^4*b + 12*I*A*a^3*b^2 + 16*I*B*a^2*b^3 - 15*I
*A*a*b^4 + 3*I*B*b^5)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*
sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(16*I*B*a^5 - 12*I*A*a^4*b - 16*I*B*a^3
*b^2 + 15*I*A*a^2*b^3 - 3*I*B*a*b^4 + (16*I*B*a^4*b - 12*I*A*a^3*b^2 - 16*
I*B*a^2*b^3 + 15*I*A*a*b^4 - 3*I*B*b^5)*cos(d*x + c))*sqrt(b)*weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos
(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(-8*I*B*a^4*b + 6*I
*A*a^3*b^2 + 5*I*B*a^2*b^3 - 3*I*A*a*b^4 + (-8*I*B*a^3*b^2 + 6*I*A*a^2*b^3
 + 5*I*B*a*b^4 - 3*I*A*b^5)*cos(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*
b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(8*I*B*a^4*b - 6*I*A*a^3*b^2 - 5*I
*B*a^2*b^3 + 3*I*A*a*b^4 + (8*I*B*a^3*b^2 - 6*I*A*a^2*b^3 - 5*I*B*a*b^4 +
3*I*A*b^5)*cos(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
2*a)/b)) + 3*(4*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + (B*a^2*b^3 - B*b^5)*co
s(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/((a^2*b^5 - b^7)*d*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2),
x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2}{\cos(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2)/(cos(c + d*x)*b + a),x)
```

3.328 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 204

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(aAb-2a^2B+b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\cos(c+dx)}} + \frac{2a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*(A*a*b-2*B*a^2+B*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))/b^2/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
+2*(A*b-2*B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2)*(b/(a+b))^(1/2))/b^2/d/(a+b*cos(d*x+c))^(1/2)+2*a*(A*b-B*a)*sin(
d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(-\left((a+b)\left(-aAb+2a^2B-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)+\left(a^2-b^2\right)(-Ab+2aB)\sqrt{\frac{a+b\cos(c+dx)}{a}}\right.}{(a-b)b^2(a+b)d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x
]
```

output
```
(-2*(-((a + b)*(-(a*A*b) + 2*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*(-(A*b) + 2*a*B)
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
+ a*b*(-(A*b) + a*B)*Sin[c + d*x]))/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[
c + d*x]])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\int \frac{b(Ab-aB)+\left(-2Ba^2+Aba+b^2B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{b(Ab-aB)+\left(-2Ba^2+Aba+b^2B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{b(Ab-aB)+\left(-2Ba^2+Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3231}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\left(-2a^2B+aAb+b^2B\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-2aB)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\left(-2a^2B+aAb+b^2B\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-2aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3134}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\left(-2a^2B+aAb+b^2B\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-2aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(-2a^2B+aAb+b^2B)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(Ab-2aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\,(a^2-b^2)}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(-2a^2B+aAb+b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(Ab-2aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\,(a^2-b^2)}$$

$$\Big\downarrow \text{3142}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(-2a^2B+aAb+b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b\,(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(-2a^2B+aAb+b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b\,(a^2-b^2)}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(-2a^2B+aAb+b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{b\,(a^2-b^2)}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
-(((2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 -
 b^2)*(A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(b*(a^2 - b^2))) + (2*
a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(203) = 406$.

Time = 12.37 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.54

method	result
default	$-\frac{\sqrt{-\left(-2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2\sqrt{-\frac{2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a-b}+\frac{a+b}{a-b}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(Ab\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)-2B\right.}{b^2\sqrt{-2b}}\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*b*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a-B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2*a*(A*b-B*a)/b^
2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*
d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.25

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
2/3*(sqrt(1/2)*(4*I*B*a^4 - 2*I*A*a^3*b - 5*I*B*a^2*b^2 + 3*I*A*a*b^3 + (4
*I*B*a^3*b - 2*I*A*a^2*b^2 - 5*I*B*a*b^3 + 3*I*A*b^4)*cos(d*x + c))*sqrt(b
)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3
, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(-4*I*B
*a^4 + 2*I*A*a^3*b + 5*I*B*a^2*b^2 - 3*I*A*a*b^3 + (-4*I*B*a^3*b + 2*I*A*a
^2*b^2 + 5*I*B*a*b^3 - 3*I*A*b^4)*cos(d*x + c))*sqrt(b)*weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
 c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(2*I*B*a^3*b - I*A*a^2*b^
2 - I*B*a*b^3 + (2*I*B*a^2*b^2 - I*A*a*b^3 - I*B*b^4)*cos(d*x + c))*sqrt(b
)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, we
ierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(-2*I*B*
a^3*b + I*A*a^2*b^2 + I*B*a*b^3 + (-2*I*B*a^2*b^2 + I*A*a*b^3 + I*B*b^4)*c
os(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 3
*(B*a^2*b^2 - A*a*b^3)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/((a^2*b^4 -
b^6)*d*cos(d*x + c) + (a^3*b^3 - a*b^5)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\cos(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/(cos(c + d*x)*b + a),x)
```

3.329 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 185

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b\,(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}} - \frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*(A*b-B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b
/(a+b))^(1/2))/b/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*B*((a+b*cos(
d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2)
)/b/d/(a+b*cos(d*x+c))^(1/2)-2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d
*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-\left((a+b)(-Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+(a^2-b^2)\,B\sqrt{\cdots}\right.}{(a-b)b(a+b)d\sqrt{a+\cdots}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(-((a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[
(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-(A*b) + a*B)*Sin[c + d*x
]))/((a - b)*b*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3233}$$

$$-\frac{2\int -\frac{aA-bB+(Ab-aB)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{aA-bB+(Ab-aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{aA-bB+(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3231

$$\frac{\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{(Ab-aB)\int \sqrt{a+b\cos(c+dx)}dx}{b}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(Ab-aB)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3134

$$\frac{\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(Ab-aB)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(Ab-aB)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3132

$$\frac{\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3142

$$\frac{\frac{B(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{B(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{2B(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*B*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[a + b*Cos[c + d*x]]))/(a^2 - b^2) - (2*(A*b - a*B)*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(184) = 368$.

Time = 6.70 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.34

method	result
default	$-\frac{\sqrt{-\left(-2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b}{a-b}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)}{b\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{2(Ab-}{}\right.}{}$
parts	$-\frac{2A\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\frac{2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a-b}+\frac{a+b}{a-b}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}\right.}{(a-b)(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}\,d}$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*(A*b-B*a)/b/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2
*c)^2*b+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.18

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(1/2)*(-2*I*B*a^3 - I*A*a^2*b + 3*I*B*a*b^2 + (-2*I*B*a^2*b - I*A
*a*b^2 + 3*I*B*b^3)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*si
n(d*x + c) + 2*a)/b) + sqrt(1/2)*(2*I*B*a^3 + I*A*a^2*b - 3*I*B*a*b^2 + (2
*I*B*a^2*b + I*A*a*b^2 - 3*I*B*b^3)*cos(d*x + c))*sqrt(b)*weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(-I*B*a^2*b + I*A*a*b^2
 + (-I*B*a*b^2 + I*A*b^3)*cos(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*
sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(I*B*a^2*b - I*A*a*b^2 + (I*B*a*b^2
- I*A*b^3)*cos(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
2*a)/b)) + 3*(B*a*b^2 - A*b^3)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/((a^
2*b^3 - b^5)*d*cos(d*x + c) + (a^3*b^2 - a*b^4)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\, dx = \int \frac{\sqrt{\cos(dx + c)\, b + a}}{\cos(dx + c)\, b + a} dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)/(cos(c + d*x)*b + a),x)
```

3.330 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 190

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*(A*b-B*a)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))/a/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*A*((a+b*cos
(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1
/2))/a/d/(a+b*cos(d*x+c))^(1/2)+2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+
b*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.83 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.42

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)(B+A\sec(c+dx))\left(-\frac{4a(-Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}\right.}{\sqrt{a+b\cos(c+dx)}}\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x
]
```

output
```
(Cos[c + d*x]*(B + A*Sec[c + d*x])*(-(((4*a*(-(A*b) + a*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[
c + d*x]] + (2*(2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] -
((2*I)*(A*b - a*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + C
os[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*
Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((-a + b)*(
a + b))) + (4*b*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c +
d*x]])))/(2*a*d*(A + B*Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3479, 27, 3042, 3538, 25, 27, 3042, 3134, 3042, 3132, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{\left(-b(Ab-aB)\cos^2(c+dx)-a(Ab-aB)\cos(c+dx)+A\left(a^2-b^2\right)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(-b(Ab-aB)\cos^2(c+dx)-a(Ab-aB)\cos(c+dx)+A\left(a^2-b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+A\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3538}$$

$$\frac{-\frac{\int -\frac{Ab\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left((Ab-aB)\int \sqrt{a+b\cos(c+dx)}dx\right)}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{Ab\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-(Ab-aB)\int \sqrt{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{A(a^2-b^2)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-(Ab-aB)\int \sqrt{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A(a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(Ab-aB)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{A(a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{(Ab-aB)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A(a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{(Ab-aB)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{A(a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3286}$$

$$\frac{\frac{A(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{A(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2A(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
((-2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*(a^2 - b^2)*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sq
rt[a + b*Cos[c + d*x]]))/(a*(a^2 - b^2)) + (2*b*(A*b - a*B)*Sin[c + d*x])/
(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(190) = 380$.

Time = 8.32 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.28

method	result
default	$-\frac{\sqrt{-\left(-2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b}{a-b}}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)}{a\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}+\frac{2(\cdots}{\cdots}\cdots\right.}{\cdots}$
parts	$\frac{2A\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\frac{2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a-b}+\frac{a+b}{a-b}}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)a^2-\sqrt{\frac{1}{2}-\frac{\cos(d\cdots}{2}}\cdots\right.}{\cdots}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(A*b-B*a)/a/sin(1/2*d*x+1/2*c)^2/(2*
sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\cos(c+dx)}{\cos(c+dx)\,(a+b\cos(c+dx))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)}{\cos(dx+c)\,b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x))/(cos(c + d*x)*b + a),x)
```

3.331 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 303

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{(a^2A-3Ab^2+2abB)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{(3Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}} + \frac{b(a^2A-3Ab^2+2abB)\sin(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}$$

output

```
-(A*a^2-3*A*b^2+2*B*a*b)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))/a^2/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
+A*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b
/(a+b))^(1/2))/a/d/(a+b*cos(d*x+c))^(1/2)-(3*A*b-2*B*a)*((a+b*cos(d*x+c))/
(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a^2/
d/(a+b*cos(d*x+c))^(1/2)+b*(A*a^2-3*A*b^2+2*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2
)/d/(a+b*cos(d*x+c))^(1/2)+A*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.59

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{-\frac{8ab(-Ab+aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-7a^2Ab+9Ab^3+4a^3B-6ab^2B\right)\cdot}{\sqrt{a}}}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2)
,x]
```

output
```
(((-8*a*b*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-7*a^2*A*b + 9*A*b
^3 + 4*a^3*B - 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(a^2*A - 3
*A*b^2 + 2*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 +
Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*Elli
pticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((a - b)*(a
+ b)) + (4*(a*A*(a^2 - b^2) + b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Cos[c + d*x])*
Tan[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.11, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3479, 27, 3042, 3535, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{3479}$$
$$\frac{\int -\frac{\left(-Ab\cos^2(c+dx)+3Ab-2aB\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{a}+\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}$$
$$\Big\downarrow \text{27}$$
$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(-Ab\cos^2(c+dx)+3Ab-2aB\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{2a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{-Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+3Ab-2aB}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}$$
$$\Big\downarrow \text{3535}$$
$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\int \frac{\left(b\left(Aa^2+2bBa-3Ab^2\right)\cos^2(c+dx)-2ab(Ab-aB)\cos(c+dx)+\left(a^2-b^2\right)(3Ab-2aB)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$
$$\Big\downarrow \text{27}$$
$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int \frac{\left(b\left(Aa^2+2bBa-3Ab^2\right)\cos^2(c+dx)-2ab(Ab-aB)\cos(c+dx)+\left(a^2-b^2\right)(3Ab-2aB)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int \frac{b\left(Aa^2+2bBa-3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+\left(a^2-b^2\right)(3Ab-2aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$\downarrow$ 3538

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(a^2A+2abB-3Ab^2)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int-\frac{\left(b\left(a^2-b^2\right)(3Ab-2aB)-aAb\left(a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$\downarrow$ 25

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(a^2A+2abB-3Ab^2)\int\sqrt{a+b\cos(c+dx)}dx+\frac{\int\frac{\left(b\left(a^2-b^2\right)(3Ab-2aB)-aAb\left(a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(a^2A+2abB-3Ab^2)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{b\left(a^2-b^2\right)(3Ab-2aB)-aAb\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$\downarrow$ 3134

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{b\left(a^2-b^2\right)(3Ab-2aB)-aAb\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{b\left(a^2-b^2\right)(3Ab-2aB)-aAb\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$\downarrow$ 3132

$$\frac{\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{b\left(a^2-b^2\right)(3Ab-2aB)-aAb\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2A+2abB-3Ab^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$\Big\downarrow$ 3481

$$\frac{\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(a^2-b^2\right)(3Ab-2aB)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-aAb\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2A+2ab}{ad(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(a^2-b^2\right)(3Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-aAb\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3142

$$\frac{\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(a^2-b^2\right)(3Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{aAb\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a+b\cos(c+d}{a+b}}}}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(a^2-b^2\right)(3Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{aAb\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos}}{d\sqrt{\frac{a+b\cos(}{a+}}}}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3140

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b\left(a^2-b^2\right)(3Ab-2aB)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2aAb\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3286

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\frac{b\left(a^2-b^2\right)(3Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2aAb\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos}}{d\sqrt{\frac{a+b\cos(}{a+}}}}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\frac{b\left(a^2-b^2\right)(3Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2aAb\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+2\left(a^2A+2ab\right.}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3284

$$\frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{2\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\frac{2b\left(a^2-b^2\right)(3Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2aAb\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+}{a+b}}}{d\sqrt{a+b\,c}}}{b}}{a(a^2-b^2)}}{2a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*(((2*(a^2*A - 3*A*b^2 + 2*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((-2*
a*A*b*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(a^2 - b^2)*(3*A*b -
 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(a*(a^2 - b^2)) - (2*b*(a^2*A
 - 3*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*
x]]))/a + (A*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(301) = 602$.

Time = 13.00 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	912
parts	Expression too large to display	1276

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(-co
s(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/2*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*(A*b-B*a)/a^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*b*(A*b-B*a)/a^2/sin(1/2*d*x+1/2*c)
^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}}{\cos\left(dx+c\right)b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2)/(cos(c + d*x)*b + a),x)
```

3.332 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 398

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{4a^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$-\frac{(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{(4a^2A+15Ab^2-12abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4a^3d\sqrt{a+b\cos(c+dx)}}$$
$$-\frac{b(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\sin(c+dx)}{4a^3(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$
$$-\frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}}+\frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}$$

output
```
1/4*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/a^3/(a^2-b^2)/d/((a+b*cos(d
*x+c))/(a+b))^(1/2)-1/4*(5*A*b-4*B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/a^2/d/(a+b*cos(d*x+c))^(
1/2)+1/4*(4*A*a^2+15*A*b^2-12*B*a*b)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a^3/d/(a+b*cos(d*x+c))^
(1/2)-1/4*b*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^
2)/d/(a+b*cos(d*x+c))^(1/2)-1/4*(5*A*b-4*B*a)*tan(d*x+c)/a^2/d/(a+b*cos(d*
x+c))^(1/2)+1/2*A*sec(d*x+c)*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.42 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{\frac{2\left(4a^3Ab-20aAb^3+16a^2b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^4A+29a^2Ab^2-45Ab^4-28a^3bB+36ab^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{Elli}}{\sqrt{a+b\cos(c+dx)}}}{}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)(-7Ab\sin(c+dx)+4aB\sin(c+dx))}{4a^3}-\frac{2\left(-Ab^4\sin(c+dx)+ab^3B\sin(c+dx)\right)}{a^3(a^2-b^2)(a+b\cos(c+dx))}+\frac{A\sec(c+dx)\tan(c+dx)}{2a^2}\right.}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2)
,x]
```

output
```
-1/16*((2*(4*a^3*A*b - 20*a*A*b^3 + 16*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
 + (2*(8*a^4*A + 29*a^2*A*b^2 - 45*A*b^4 - 28*a^3*b*B + 36*a*b^3*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sq
rt[a + b*Cos[c + d*x]] - ((2*I)*(7*a^2*A*b^2 - 15*A*b^4 - 4*a^3*b*B + 12*a
*b^3*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a
- b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c +
d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x]
)^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x]
)^2)))/(a^3*(-a + b)*(a + b)*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]
*(-7*A*b*Sin[c + d*x] + 4*a*B*Sin[c + d*x]))/(4*a^3) - (2*(-(A*b^4*Sin[c +
 d*x]) + a*b^3*B*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (
A*Sec[c + d*x]*Tan[c + d*x])/(2*a^2)))/d
```

Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.07, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int -\frac{(-3Ab\cos^2(c+dx)-2aA\cos(c+dx)+5Ab-4aB)\sec^2(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{2a}+\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(-3Ab\cos^2(c+dx)-2aA\cos(c+dx)+5Ab-4aB\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{-3Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-2aA\sin\left(c+dx+\frac{\pi}{2}\right)+5Ab-4aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4a}$$

$$\Big\downarrow \; 3534$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int -\frac{\left(4Aa^2-12bBa+6Ab\cos(c+dx)a+15Ab^2-b(5Ab-4aB)\cos^2(c+dx)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{a}+\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(4Aa^2-12bBa+6Ab\cos(c+dx)a+15Ab^2-b(5Ab-4aB)\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{4Aa^2-12bBa+6Ab\sin\left(c+dx+\frac{\pi}{2}\right)a+15Ab^2-b(5Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 3534$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\int \frac{\left(b\left(-4Ba^3+7Aba^2+12b^2Ba-15Ab^3\right)\cos^2(c+dx)+2ab\left(Aa^2+4bBa-5Ab^2\right)\cos(c+dx)+\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)\right)\sec(}{2\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{\int \frac{\left(b\left(-4Ba^3+7Aba^2+12b^2Ba-15Ab^3\right)\cos^2(c+dx)+2ab\left(Aa^2+4bBa-5Ab^2\right)\cos(c+dx)+\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)\right)\sec(c}{\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{\int \frac{b\left(-4Ba^3+7Aba^2+12b^2Ba-15Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab\left(Aa^2+4bBa-5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3538

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{\left(-4a^3B+7a^2Ab+12ab^2B-15Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx - \frac{\int -\frac{\left(b\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)-ab\left(a^2-b^2\right)(5Ab-4aB)\cos(c+\right.}{\sqrt{a+b\cos(c+dx)}}}{b}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 25

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{\frac{\int \frac{\left(b\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)-ab\left(a^2-b^2\right)(5Ab-4aB)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\left(-4a^3B+7a^2Ab+12ab^2B-15Ab^3\right)\int\sqrt{}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{\frac{\frac{\int \frac{b\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)-ab\left(a^2-b^2\right)(5Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\left(-4a^3B+7a^2Ab+12ab^2B-15Ab^3\right)\int\sqrt{a+b\sin(}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3134

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{b\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)-ab\left(a^2-b^2\right)(5Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(-4a^3B+7a^2Ab+12ab^2B-15Ab^3\right)\sqrt{a+b\cos(c-}}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{b\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)-ab\left(a^2-b^2\right)(5Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(-4a^3B+7a^2Ab+12ab^2B-15Ab^3\right)\sqrt{a+b\cos(c-}}{\sqrt{\frac{a+b\cos(c+d}{a+b}}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \; 3132$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{b\left(a^2-b^2\right)\left(4Aa^2-12bBa+15Ab^2\right)-ab\left(a^2-b^2\right)(5Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(-4a^3B+7a^2Ab+12ab^2B-15Ab^3\right)\sqrt{a+b\cos(c}}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \; 3481$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b\left(a^2-b^2\right)\left(4a^2A-12abB+15Ab^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab\left(a^2-b^2\right)(5Ab-4aB)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2\left(-4a^3B+7a^2Ab+12ab\right.}{}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b(a^2-b^2)(4a^2A-12abB+15Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab(a^2-b^2)(5Ab-4aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+}{a(a^2-b^2)}}{2a}}{4a}$$

$\Big\downarrow$ 3142

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b(a^2-b^2)(4a^2A-12abB+15Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab(a^2-b^2)(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+b}+}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{a(a^2-b^2)}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b(a^2-b^2)(4a^2A-12abB+15Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab(a^2-b^2)(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+b}+}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{a(a^2-b^2)}}{4a}$$

$\Big\downarrow$ 3140

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b(a^2-b^2)(4a^2A-12abB+15Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab(a^2-b^2)(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\Big(}{d\sqrt{a+b\cos(c+dx)}}}{b}}{a(a^2-b^2)}}{4a}$$

$\Big\downarrow$ 3286

$$\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b(a^2-b^2)(4a^2A-12abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab(a^2-b^2)(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Ellipti}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{a(a^2-b^2)}}{4a}$$

$\downarrow$ 3042

$$\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b(a^2-b^2)(4a^2A-12abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab(a^2-b^2)(5Ab-4aB)\sqrt{a+}}{d\sqrt{a}}}{b}}{a(a^2-b^2)}}{}$$

$\downarrow$ 3284

$$\frac{\frac{(5Ab-4aB)\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2b(a^2-b^2)(4a^2A-12abB+15Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2ab(a^2-b^2)(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{Ell}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{a(a^2-b^2)}}{4a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]]) - (-1/2*(((
2*(7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) + ((-2*a*b*(a^2 - b^2)*(5*A*b - 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (
2*b*(a^2 - b^2)*(4*a^2*A + 15*A*b^2 - 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
d*x]]))/b)/(a*(a^2 - b^2)) - (2*b*(7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b
^2*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/a + ((5*A*
b - 4*a*B)*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c + d*x]]))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(382) = 764$.

Time = 16.99 (sec) , antiderivative size = 1568, normalized size of antiderivative = 3.94

method	result	size
default	Expression too large to display	1568
parts	Expression too large to display	2446

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(-1/
2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1
/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))*b^2)-2*(A*b-B*a)/a^2*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**(3/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)^3\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^3}{\cos\left(dx+c\right)b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**3)/(cos(c + d*x)*b + a),x)
```

3.333 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 550

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(80a^5Ab-140a^3Ab^3+40aAb^5-128a^6B+212a^4b^2B-55a^2b^4B-9b^6B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{15b^5(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{2(80a^4Ab-80a^2Ab^3-5Ab^5-128a^5B+116a^3b^2B+17ab^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^5(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}}$$
$$+\frac{2a(5a^2Ab-9Ab^3-8a^3B+12ab^2B)\cos^2(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{2(40a^4Ab-65a^2Ab^3+5Ab^5-64a^5B+98a^3b^2B-14ab^4B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^4(a^2-b^2)^2d}$$
$$-\frac{2(30a^3Ab-50aAb^3-48a^4B+71a^2b^2B-3b^4B)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^3(a^2-b^2)^2d}$$

output
```
-2/15*(80*A*a^5*b-140*A*a^3*b^3+40*A*a*b^5-128*B*a^6+212*B*a^4*b^2-55*B*a^
2*b^4-9*B*b^6)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
*(b/(a+b))^(1/2))/b^5/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/15*(8
0*A*a^4*b-80*A*a^2*b^3-5*A*b^5-128*B*a^5+116*B*a^3*b^2+17*B*a*b^4)*((a+b*c
os(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1
/2))/b^5/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*a*(A*b-B*a)*cos(d*x+c)^3*s
in(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*a*(5*A*a^2*b-9*A*b^3-8*
B*a^3+12*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c
))^(1/2)+2/15*(40*A*a^4*b-65*A*a^2*b^3+5*A*b^5-64*B*a^5+98*B*a^3*b^2-14*B*
a*b^4)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^4/(a^2-b^2)^2/d-2/15*(30*A*a^3*
b-50*A*a*b^3-48*B*a^4+71*B*a^2*b^2-3*B*b^4)*cos(d*x+c)*(a+b*cos(d*x+c))^(1
/2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d
```

Mathematica [A] (verified)

Time = 5.49 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.68

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(20a^4Ab-35a^2Ab^3-5Ab^5-32a^5B+44a^3b^2B+8ab^4B\right)\text{Ellipti}\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2)
,x]
```

output
```
((-2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(20*a^4*A*b - 35*a^2*A*b^3
- 5*A*b^5 - 32*a^5*B + 44*a^3*b^2*B + 8*a*b^4*B)*EllipticF[(c + d*x)/2, (2
*b)/(a + b)] - (-80*a^5*A*b + 140*a^3*A*b^3 - 40*a*A*b^5 + 128*a^6*B - 212
*a^4*b^2*B + 55*a^2*b^4*B + 9*b^6*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)
/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b))
 + b*((10*a^4*(-(A*b) + a*B)*Sin[c + d*x])/(a^2 - b^2) - (10*a^3*(-8*a^2*A
*b + 12*A*b^3 + 11*a^3*B - 15*a*b^2*B)*(a + b*Cos[c + d*x])*Sin[c + d*x])/
(a^2 - b^2)^2 + 2*(5*A*b - 14*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x] + 3
*b*B*(a + b*Cos[c + d*x])^2*Sin[2*(c + d*x)]))/(15*b^5*d*(a + b*Cos[c + d*
x])^(3/2))
```

Rubi [A] (verified)

Time = 3.10 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3528, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{\cos^2(c+dx)\left(-\left(\left(-8Ba^2+5Aba+3b^2B\right)\cos^2(c+dx)\right)-3b(Ab-aB)\cos(c+dx)+6a(Ab-aB)\right)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^2(c+dx)\left(-\left(\left(-8Ba^2+5Aba+3b^2B\right)\cos^2(c+dx)\right)-3b(Ab-aB)\cos(c+dx)+6a(Ab-aB)\right)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\left(8Ba^2-5Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+6a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3526}$$

$$\frac{\frac{2a\left(-8a^3B+5a^2Ab+12ab^2B-9Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}} - \frac{2\int -\frac{\cos(c+dx)\left(-\left(\left(-48Ba^4+30Aba^3+71b^2Ba^2-50Ab^3a-3b^4B\right)\cos^2(c+dx)\right)+b\left(2Ba^3\right.}{2\sqrt{a+b\cos(c+}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\cos(c+dx)\left(-\left(\left(-48Ba^4+30Aba^3+71b^2Ba^2-50Ab^3a-3b^4B\right)\cos^2(c+dx)\right)+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)+4a\left(-8Ba^3+5Aba^2+12b^2Ba-9Ab^3\right)\right)}{\sqrt{a+b\cos(c+dx)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\left(48Ba^4-30Aba^3-71b^2Ba^2+50Ab^3a+3b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+4a\left(-8Ba^3+5Aba^2+12b^2Ba-\right.\right.}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{\frac{2\int -\frac{-3\left(-64Ba^5+40Aba^4+98b^2Ba^3-65Ab^3a^2-14b^4Ba+5Ab^5\right)\cos^2(c+dx)-b\left(-16Ba^4+10Aba^3+27b^2Ba^2-30Ab^3a+9b^4B\right)\cos(c+dx)+2a\left(-48Ba^4+30Aba\right.}{2\sqrt{a+b\cos(c+dx)}}}{5b}}{b(a^2-b^2)}}{}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{\int \frac{-3\left(-64Ba^5+40Aba^4+98b^2Ba^3-65Ab^3a^2-14b^4Ba+5Ab^5\right)\cos^2(c+dx)-b\left(-16Ba^4+10Aba^3+27b^2Ba^2-30Ab^3a+9b^4B\right)\cos(c+dx)+2a\left(-48Ba^4+30Aba^3\right.}{\sqrt{a+b\cos(c+dx)}}}{5b}}{b(a^2-b^2)}}{}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{-3\left(-64Ba^5+40Aba^4+98b^2Ba^3-65Ab^3a^2-14b^4Ba+5Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(-16Ba^4+10Aba^3+27b^2Ba^2-30Ab^3a+9b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a\left(-48Ba^4\right.\cdots}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{5b}\cdots}{b(a^2-b^2)}\cdots$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{\frac{\frac{2\int \frac{3\left(b\left(-32Ba^5+20Aba^4+44b^2Ba^3-35Ab^3a^2+8b^4Ba-5Ab^5\right)+\left(-128Ba^6+80Aba^5+212b^2Ba^4-140Ab^3a^3-55b^4Ba^2+40Ab^5a-9b^6B\right)\cos(c+dx)\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(\cdots\right.}{}}{5b}\cdots}{\cdots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\int \frac{b\left(-32Ba^5+20Aba^4+44b^2Ba^3-35Ab^3a^2+8b^4Ba-5Ab^5\right)+\left(-128Ba^6+80Aba^5+212b^2Ba^4-140Ab^3a^3-55b^4Ba^2+40Ab^5a-9b^6B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\left(-64a^5\cdots\right.}{}}{5b}\cdots}{b(\cdots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\int \frac{b\left(-32Ba^5+20Aba^4+44b^2Ba^3-35Ab^3a^2+8b^4Ba-5Ab^5\right)+\left(-128Ba^6+80Aba^5+212b^2Ba^4-140Ab^3a^3-55b^4Ba^2+40Ab^5a-9b^6B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(\cdots\right.}{}}{5b}\cdots}{\cdots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3231}$$

$$-\frac{\frac{\frac{\frac{\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-128a^5B+80a^4Ab+116a^3b^2B-80a^2Ab^3+17ab^4B\cdots\right.}{b}}{b}\cdots}{5b}\cdots}{\cdots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\frac{\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(-128a^5B+80a^4Ab+116a^3b^2B-80a^2Ab^3+17a\cdots}{b}}{b}}{5b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3134

$$-\frac{\frac{\frac{\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-128a^5B+80a^4Ab+116a^3b^2B\cdots}{\cdots}}{b}}{5b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\frac{\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-128a^5B+80a^4Ab+116a^3\cdots}{\cdots}}{b}}{5b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3132

$$-\frac{\frac{\frac{2\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-128a^5B+80a^4Ab+116a^3b^2B-80a\cdots}{\cdots}}{b}}{5b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3142

$$-\frac{\frac{\frac{2\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-128a^5B+80a^4Ab+116a^3b^2B-80a\cdots\right.}{\cdots}}{b}}{5b\cdots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{2\left(-128a^6B+80a^5Ab+212a^4b^2B-140a^3Ab^3-55a^2b^4B+40aAb^5-9b^6B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-128a^5B+80a^4Ab+116a^3b^2B-80a\cdots\right.}{\cdots}}{b}}{\cdots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3140

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{\frac{2a\left(-8a^3B+5a^2Ab+12ab^2B-9Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{-\frac{2\left(-48a^4B+30a^3Ab+71a^2b^2B-50aAb^3-3b^4B\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}-\cdots}{\cdots}}{\cdots}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*a*(A*b - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])^(3/2)) + ((2*a*(5*a^2*A*b - 9*A*b^3 - 8*a^3*B + 12*a*b^2*B)*Co
s[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((
-2*(30*a^3*A*b - 50*a*A*b^3 - 48*a^4*B + 71*a^2*b^2*B - 3*b^4*B)*Cos[c + d
*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d) - (((2*(80*a^5*A*b - 14
0*a^3*A*b^3 + 40*a*A*b^5 - 128*a^6*B + 212*a^4*b^2*B - 55*a^2*b^4*B - 9*b^
6*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(80*a^4*A*b - 80*a^2*
A*b^3 - 5*A*b^5 - 128*a^5*B + 116*a^3*b^2*B + 17*a*b^4*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*
Cos[c + d*x]]))/b - (2*(40*a^4*A*b - 65*a^2*A*b^3 + 5*A*b^5 - 64*a^5*B + 9
8*a^3*b^2*B - 14*a*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d))/(5
*b))/(b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1749 vs. $2(531) = 1062$.

Time = 20.35 (sec) , antiderivative size = 1750, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	1750
parts	Expression too large to display	2985

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(3*A*a^2
*b+2*A*a*b^2+A*b^3-4*B*a^3-3*B*a^2*b-2*B*a*b^2-B*b^3)/b^5*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))+2*a^4*(A*b-B*a)/b^5*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2
*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos
(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1
/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^5*(2*A*a*b+2*A*b^2-3*B*a^2-4*B
*a*b-3*B*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^
2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+8/b^3*(A*b-2*B*a-3*B*b)*(-1/6/b*cos(1/2*d
*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1504, normalized size of antiderivative = 2.73

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/45*(sqrt(1/2)*(-256*I*B*a^9 + 160*I*A*a^8*b + 520*I*B*a^7*b^2 - 340*I*A
*a^6*b^3 - 242*I*B*a^5*b^4 + 185*I*A*a^4*b^5 - 42*I*B*a^3*b^6 + 15*I*A*a^2
*b^7 + (-256*I*B*a^7*b^2 + 160*I*A*a^6*b^3 + 520*I*B*a^5*b^4 - 340*I*A*a^4
*b^5 - 242*I*B*a^3*b^6 + 185*I*A*a^2*b^7 - 42*I*B*a*b^8 + 15*I*A*b^9)*cos(
d*x + c)^2 + 2*(-256*I*B*a^8*b + 160*I*A*a^7*b^2 + 520*I*B*a^6*b^3 - 340*I
*A*a^5*b^4 - 242*I*B*a^4*b^5 + 185*I*A*a^3*b^6 - 42*I*B*a^2*b^7 + 15*I*A*a
*b^8)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*a)/b) + sqrt(1/2)*(256*I*B*a^9 - 160*I*A*a^8*b - 520*I*B*a^7*b^2 + 340*I*
A*a^6*b^3 + 242*I*B*a^5*b^4 - 185*I*A*a^4*b^5 + 42*I*B*a^3*b^6 - 15*I*A*a^
2*b^7 + (256*I*B*a^7*b^2 - 160*I*A*a^6*b^3 - 520*I*B*a^5*b^4 + 340*I*A*a^4
*b^5 + 242*I*B*a^3*b^6 - 185*I*A*a^2*b^7 + 42*I*B*a*b^8 - 15*I*A*b^9)*cos(
d*x + c)^2 + 2*(256*I*B*a^8*b - 160*I*A*a^7*b^2 - 520*I*B*a^6*b^3 + 340*I*
A*a^5*b^4 + 242*I*B*a^4*b^5 - 185*I*A*a^3*b^6 + 42*I*B*a^2*b^7 - 15*I*A*a*
b^8)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*
a)/b) + 3*sqrt(1/2)*(-128*I*B*a^8*b + 80*I*A*a^7*b^2 + 212*I*B*a^6*b^3 - 1
40*I*A*a^5*b^4 - 55*I*B*a^4*b^5 + 40*I*A*a^3*b^6 - 9*I*B*a^2*b^7 + (-128*I
*B*a^6*b^3 + 80*I*A*a^5*b^4 + 212*I*B*a^4*b^5 - 140*I*A*a^3*b^6 - 55*I*B*a
^2*b^7 + 40*I*A*a*b^8 - 9*I*B*b^9)*cos(d*x + c)^2 + 2*(-128*I*B*a^7*b^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2),
x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^4\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^4}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**4)/(cos(c + d*x)**2*b**2 + 2*c
os(c + d*x)*a*b + a**2),x)
```

3.334 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 413

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B)\sqrt{a+}}{3b^4(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$-\frac{2(8a^3Ab-9aAb^3-16a^4B+16a^2b^2B+b^4B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^4(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}$$

$$-\frac{2a^2(3a^2Ab-7Ab^3-6a^3B+10ab^2B)\sin(c+dx)}{3b^3(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$-\frac{2(aAb-2a^2B+b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^3(a^2-b^2)\,d}$$

output
```
2/3*(8*A*a^4*b-15*A*a^2*b^3+3*A*b^5-16*B*a^5+28*B*a^3*b^2-8*B*a*b^4)*(a+b*
cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^
4/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(8*A*a^3*b-9*A*a*b^3-16
*B*a^4+16*B*a^2*b^2+B*b^4)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^4/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)+2/3*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
^(3/2)-2/3*a^2*(3*A*a^2*b-7*A*b^3-6*B*a^3+10*B*a*b^2)*sin(d*x+c)/b^3/(a^2-
b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-2/3*(A*a*b-2*B*a^2+B*b^2)*(a+b*cos(d*x+c))
^(1/2)*sin(d*x+c)/b^3/(a^2-b^2)/d
```

Mathematica [A] (verified)

Time = 4.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.81

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(2a^3Ab-6aAb^3-4a^4B+7a^2b^2B+b^4B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{}{}\right.\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2)
,x]
```

output
```
(2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(2*a^3*A*b - 6*a*A*b^3 - 4*
a^4*B + 7*a^2*b^2*B + b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-8*a
^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*((a
 + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])))/((a - b)^2*(a + b)) + (b*(-8*a^5*A*b + 16*a^3*A*b^3 + 16*a^
6*B - 25*a^4*b^2*B + b^6*B + 2*a*b*(-5*a^3*A*b + 9*a*A*b^3 + 10*a^4*B - 16
*a^2*b^2*B + 2*b^4*B)*Cos[c + d*x] + (-(a^2*b) + b^3)^2*B*Cos[2*(c + d*x)]
)*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 3468, 27, 3042, 3510, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{\cos(c+dx)\left(-3\left(-2Ba^2+Aba+b^2B\right)\cos^2(c+dx)-3b(Ab-aB)\cos(c+dx)+4a(Ab-aB)\right)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(c+dx)\left(-3\left(-2Ba^2+Aba+b^2B\right)\cos^2(c+dx)-3b(Ab-aB)\cos(c+dx)+4a(Ab-aB)\right)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-3\left(-2Ba^2+Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+4a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3510}$$

$$\frac{\frac{2\int \frac{-3b(a^2-b^2)(-2Ba^2+Aba+b^2B)\cos^2(c+dx)+(-12Ba^5+6Aba^4+22b^2Ba^3-13Ab^3a^2-6b^4Ba+3Ab^5)\cos(c+dx)+ab(-6Ba^3+3Aba^2+10b^2Ba-7Ab^3)}{2\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{-3b(a^2-b^2)(-2Ba^2+Aba+b^2B)\cos^2(c+dx)+(-12Ba^5+6Aba^4+22b^2Ba^3-13Ab^3a^2-6b^4Ba+3Ab^5)\cos(c+dx)+ab(-6Ba^3+3Aba^2+10b^2Ba-7Ab^3)}{\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)} -}{3b(a^2-b^2)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-3b(a^2-b^2)(-2Ba^2+Aba+b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+(-12Ba^5+6Aba^4+22b^2Ba^3-13Ab^3a^2-6b^4Ba+3Ab^5)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(-6Ba^3+3Aba^2+10b^2Ba-7Ab}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{b^2(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3502

$$\frac{\frac{\frac{2\int \frac{3((-4Ba^4+2Aba^3+7b^2Ba^2-6Ab^3a+b^4B)b^2+(-16Ba^5+8Aba^4+28b^2Ba^3-15Ab^3a^2-8b^4Ba+3Ab^5)\cos(c+dx)b)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b} - \frac{2(a^2-b^2)(-2a^2B+aAb+b^2B)\sin}{d}}{b^2(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{(-4Ba^4+2Aba^3+7b^2Ba^2-6Ab^3a+b^4B)b^2+(-16Ba^5+8Aba^4+28b^2Ba^3-15Ab^3a^2-8b^4Ba+3Ab^5)\cos(c+dx)b}{\sqrt{a+b\cos(c+dx)}}dx}{b} - \frac{2(a^2-b^2)(-2a^2B+aAb+b^2B)\sin(c+d}{d}}{b^2(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{\left(-4Ba^4+2Aba^3+7b^2Ba^2-6Ab^3a+b^4B\right)b^2+\left(-16Ba^5+8Aba^4+28b^2Ba^3-15Ab^3a^2-8b^4Ba+3Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(-2a^2B+aAb+b^2B\right)\sin(}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3231

$$\frac{\frac{\frac{\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\int\sqrt{a+b\cos(c+dx)}dx-\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2(a}{}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{\frac{\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\int\frac{}{\sqrt{a+}}}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\int}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3132

$$\frac{\frac{\frac{2\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\int\frac{}{\sqrt{a+b\sin(}}}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3142

$$\frac{\frac{\frac{2\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\sqrt{\frac{a+b\cos(c}{a+b}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-\right.}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\sqrt{\frac{a+b\cos(c}{a+b}}}{\sqrt{a+b\cos(c+dx)}}}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-\right.}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3140

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{\frac{\frac{2\left(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\left(-16a^4B+8a^3Ab+16a^2b^2B-9aAb^3+b^4B\right)\sqrt{\frac{a+b\cos(}{a+b}}}{d\sqrt{a+b\cos(c+dx)}}}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-\right.}$$

input

```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])^(3/2)) + ((-2*a^2*(3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*a*b^2*B)
*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (((2*(8*a^4*
A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*B)*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*C
os[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(8*a^3*A*b - 9*a*A*b^3 - 16*a^4*B
+ 16*a^2*b^2*B + b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b - (2*(a^2 - b^2)*(
a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/d)/(b^2*(a
^2 - b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. $2(398) = 796$.

Time = 20.77 (sec) , antiderivative size = 1412, normalized size of antiderivative = 3.42

method	result	size
default	Expression too large to display	1412
parts	Expression too large to display	2208

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2+9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a*b+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b+2*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*b^2-17*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-8*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b)-2*a^2/b^4*(3
*A*b-4*B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1316, normalized size of antiderivative = 3.19

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/9*(sqrt(1/2)*(32*I*B*a^8 - 16*I*A*a^7*b - 68*I*B*a^6*b^2 + 36*I*A*a^5*b
^3 + 37*I*B*a^4*b^4 - 24*I*A*a^3*b^5 + 3*I*B*a^2*b^6 + (32*I*B*a^6*b^2 - 1
6*I*A*a^5*b^3 - 68*I*B*a^4*b^4 + 36*I*A*a^3*b^5 + 37*I*B*a^2*b^6 - 24*I*A*
a*b^7 + 3*I*B*b^8)*cos(d*x + c)^2 + 2*(32*I*B*a^7*b - 16*I*A*a^6*b^2 - 68*
I*B*a^5*b^3 + 36*I*A*a^4*b^4 + 37*I*B*a^3*b^5 - 24*I*A*a^2*b^6 + 3*I*B*a*b
^7)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a
)/b) + sqrt(1/2)*(-32*I*B*a^8 + 16*I*A*a^7*b + 68*I*B*a^6*b^2 - 36*I*A*a^5
*b^3 - 37*I*B*a^4*b^4 + 24*I*A*a^3*b^5 - 3*I*B*a^2*b^6 + (-32*I*B*a^6*b^2
+ 16*I*A*a^5*b^3 + 68*I*B*a^4*b^4 - 36*I*A*a^3*b^5 - 37*I*B*a^2*b^6 + 24*I
*A*a*b^7 - 3*I*B*b^8)*cos(d*x + c)^2 + 2*(-32*I*B*a^7*b + 16*I*A*a^6*b^2 +
 68*I*B*a^5*b^3 - 36*I*A*a^4*b^4 - 37*I*B*a^3*b^5 + 24*I*A*a^2*b^6 - 3*I*B
*a*b^7)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
 2*a)/b) + 3*sqrt(1/2)*(16*I*B*a^7*b - 8*I*A*a^6*b^2 - 28*I*B*a^5*b^3 + 15
*I*A*a^4*b^4 + 8*I*B*a^3*b^5 - 3*I*A*a^2*b^6 + (16*I*B*a^5*b^3 - 8*I*A*a^4
*b^4 - 28*I*B*a^3*b^5 + 15*I*A*a^2*b^6 + 8*I*B*a*b^7 - 3*I*A*b^8)*cos(d*x
+ c)^2 + 2*(16*I*B*a^6*b^2 - 8*I*A*a^5*b^3 - 28*I*B*a^4*b^4 + 15*I*A*a^3*b
^5 + 8*I*B*a^2*b^6 - 3*I*A*a*b^7)*cos(d*x + c))*sqrt(b)*weierstrassZeta(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2),
x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3)/(cos(c + d*x)**2*b**2 + 2*c
os(c + d*x)*a*b + a**2),x)
```

3.335 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 331

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(2a^2Ab-3Ab^3-8a^3B+9ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^3(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} - \frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B+9ab^2B)\sin(c+dx)}{3b^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/3*(2*A*a^3*b-6*A*a*b^3-8*B*a^4+15*B*a^2*b^2-3*B*b^4)*(a+b*cos(d*x+c))^(
1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^3/(a^2-b^2)^2
/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(2*A*a^2*b-3*A*b^3-8*B*a^3+9*B*a*b^2
)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/
(a+b))^(1/2))/b^3/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2/3*a^2*(A*b-B*a)*sin
(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*a*(2*A*a^2*b-6*A*b^3-5*
B*a^3+9*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+(-\right.}{}\right.}{}$$

input

```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2)
,x]
```

output

```
(2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(a^2*A*b + 3*A*b^3 + 2*a^3*
B - 6*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-2*a^3*A*b + 6*a*A
*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (
2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a +
 b)) - (a*b*(a*(-(a^2*A*b) + 5*A*b^3 + 4*a^3*B - 8*a*b^2*B) + b*(-2*a^2*A*
b + 6*A*b^3 + 5*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)
^2))/(3*b^3*d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3467, 27, 3042, 3500, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{2\int \frac{3b(a^2-b^2)B\cos^2(c+dx)+(2a^2-3b^2)(Ab-aB)\cos(c+dx)+3ab(Ab-aB)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b^2(a^2-b^2)}-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3b(a^2-b^2)B\cos^2(c+dx)+(2a^2-3b^2)(Ab-aB)\cos(c+dx)+3ab(Ab-aB)}{(a+b\cos(c+dx))^{3/2}}dx}{3b^2(a^2-b^2)}-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3b(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)^2+(2a^2-3b^2)(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3ab(Ab-aB)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^2(a^2-b^2)}-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3500$$

$$\frac{\frac{2a(-5a^3B+2a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{b^2\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)-b\left(-8Ba^4+2Aba^3+15b^2Ba^2-6Ab^3a-3b^4B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)}}{3b^2(a^2-b^2)}-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{b^2\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)-b\left(-8Ba^4+2Aba^3+15b^2Ba^2-6Ab^3a-3b^4B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)}+\frac{2a(-5a^3B+2a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b^2(a^2-b^2)}-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{b^2\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)-b\left(-8Ba^4+2Aba^3+15b^2Ba^2-6Ab^3a-3b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}+\frac{2a(-5a^3B+2a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b^2(a^2-b^2)}-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3231

$$\frac{\frac{(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx-\left(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B\right)\int\sqrt{a+b\cos(c+dx)}dx}{b(a^2-b^2)}+\frac{2a(-5a^3B}{}}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}+\frac{2a}{}}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3134

$$\frac{\frac{(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\left(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}d}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{b(a^2-b^2)}}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\left(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\right.}{a+b}}}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{b(a^2-b^2)}}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3132

$$\frac{\frac{(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{b(a^2-b^2)}+}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3142$$

$$\frac{\frac{\frac{(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{b(a^2-b^2)}}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{b(a^2-b^2)}}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$\frac{\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2(a^2-b^2)\left(-8a^3B+2a^2Ab+9ab^2B-3Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^4B+2a^3\right.}{}}{b(a^2-b^2)}}{3b^2\left(a^2-b^2\right)}$$

$$\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input

```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

output

```
(-2*a^2*(A*b - a*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)) + (((-2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B
)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(2*a^2*A*b - 3*A*b^3 - 8*a
^3*B + 9*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/(b*(a^2 - b^2)) + (2*a*(2*
a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]))/(3*b^2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(320) = 640$.

Time = 17.21 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	954
parts	Expression too large to display	1763

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(A*b*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3*B*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a-B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+2*a^2*(A*b-B*a)/
b^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(
1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*
x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2
-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)))+2*a/b^3*(2*A*b-3*B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a
-b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1159, normalized size of antiderivative = 3.50

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/9*(sqrt(1/2)*(-16*I*B*a^7 + 4*I*A*a^6*b + 36*I*B*a^5*b^2 - 9*I*A*a^4*b^
3 - 24*I*B*a^3*b^4 + 9*I*A*a^2*b^5 + (-16*I*B*a^5*b^2 + 4*I*A*a^4*b^3 + 36
*I*B*a^3*b^4 - 9*I*A*a^2*b^5 - 24*I*B*a*b^6 + 9*I*A*b^7)*cos(d*x + c)^2 +
2*(-16*I*B*a^6*b + 4*I*A*a^5*b^2 + 36*I*B*a^4*b^3 - 9*I*A*a^3*b^4 - 24*I*B
*a^2*b^5 + 9*I*A*a*b^6)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*
b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(16*I*B*a^7 - 4*I*A*a^6*b - 36*I*B*a^
5*b^2 + 9*I*A*a^4*b^3 + 24*I*B*a^3*b^4 - 9*I*A*a^2*b^5 + (16*I*B*a^5*b^2 -
 4*I*A*a^4*b^3 - 36*I*B*a^3*b^4 + 9*I*A*a^2*b^5 + 24*I*B*a*b^6 - 9*I*A*b^7
)*cos(d*x + c)^2 + 2*(16*I*B*a^6*b - 4*I*A*a^5*b^2 - 36*I*B*a^4*b^3 + 9*I*
A*a^3*b^4 + 24*I*B*a^2*b^5 - 9*I*A*a*b^6)*cos(d*x + c))*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(-8*I*B*a^6*b + 2
*I*A*a^5*b^2 + 15*I*B*a^4*b^3 - 6*I*A*a^3*b^4 - 3*I*B*a^2*b^5 + (-8*I*B*a^
4*b^3 + 2*I*A*a^3*b^4 + 15*I*B*a^2*b^5 - 6*I*A*a*b^6 - 3*I*B*b^7)*cos(d*x
+ c)^2 + 2*(-8*I*B*a^5*b^2 + 2*I*A*a^4*b^3 + 15*I*B*a^3*b^4 - 6*I*A*a^2*b^
5 - 3*I*B*a*b^6)*cos(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
 c) + 2*a)/b)) + 3*sqrt(1/2)*(8*I*B*a^6*b - 2*I*A*a^5*b^2 - 15*I*B*a^4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2),
x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2)/(cos(c + d*x)**2*b**2 + 2*c
os(c + d*x)*a*b + a**2),x)
```

3.336 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 307

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(aAb+2a^2B-3b^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^2(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{2a(Ab-aB)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\sin(c+dx)}{3b(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}$$

output

```
-2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/b^2/(a^2-b^2)^2/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)+2/3*(A*a*b+2*B*a^2-3*B*b^2)*((a+b*cos(d*x+c))/(a+b))^(1
/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/b^2/(a^2-b^2)/d
/(a+b*cos(d*x+c))^(1/2)+2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(
d*x+c))^(3/2)+2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*sin(d*x+c)/b/(a^2-b^
2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.73

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a-b)(aAb\right.}{(a-b)^2}\right.}{}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x
]
```

output
```
(2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2*A*b + 3*A*b^3 + 2*a^3*B
- 6*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(a*A*b + 2*a^
2*B - 3*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (b*(a*
(2*a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B) + b*(a^2*A*b + 3*A*b^3 + 2*a^3*B
 - 6*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*
Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A\sin\left(c+dx+\frac{\pi}{2}\right)+B\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3500$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int \frac{3b(Ab-aB)-\left(2Ba^2+Aba-3b^2B\right)\cos(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{3b(Ab-aB)-\left(2Ba^2+Aba-3b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{3b(Ab-aB)+\left(-2Ba^2-Aba+3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3233$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{b\left(-Ba^2+4Aba-3b^2B\right)+\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{b\left(-Ba^2+4Aba-3b^2B\right)+\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{b\left(-Ba^2+4Aba-3b^2B\right)+\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3231

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a^2-b^2}-\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sin(}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3134

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2(2a^3B+a^2Ab-6ab^2}{d(a^2-b^2)\sqrt{a+}}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2(2a^3B+a^2Ab-}{d(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3132

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2(2a^3B+a^2Ab-6ab^2B+3}{d(a^2-b^2)\sqrt{a+b\,co}}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3142

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2(2a^3B+a}{d(a}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2(2a^3I}{}}{3b\left(a^2-b^2\right)}$$

$\downarrow$ 3140

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\left(2a^2B+aAb-3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2(2a^3B+}{d(}}{3b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/
2)) - (((2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[a + b*Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]) - (2*(a^2 - b^2)*(a*A*b + 2*a^2*B - 3*b^2*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos
[c + d*x]]))/(a^2 - b^2) - (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Si
n[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(296) = 592$.

Time = 13.33 (sec) , antiderivative size = 864, normalized size of antiderivative = 2.81

method	result	size
default	Expression too large to display	864
parts	Expression too large to display	1598

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2/b^2*(A*b-2*B*a)/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))-2*a*(A*b-B*a)/b^2*(1/6/b/(a-b)/(a
+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2*d*x+1/2*c)^2*b/
(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 1044, normalized size of antiderivative = 3.40

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
-2/9*(sqrt(1/2)*(4*I*B*a^6 + 2*I*A*a^5*b - 9*I*B*a^4*b^2 - 6*I*A*a^3*b^3 +
 9*I*B*a^2*b^4 + (4*I*B*a^4*b^2 + 2*I*A*a^3*b^3 - 9*I*B*a^2*b^4 - 6*I*A*a*
b^5 + 9*I*B*b^6)*cos(d*x + c)^2 + 2*(4*I*B*a^5*b + 2*I*A*a^4*b^2 - 9*I*B*a
^3*b^3 - 6*I*A*a^2*b^4 + 9*I*B*a*b^5)*cos(d*x + c))*sqrt(b)*weierstrassPIn
verse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(-4*I*B*a^6 - 2*I*A*a^5
*b + 9*I*B*a^4*b^2 + 6*I*A*a^3*b^3 - 9*I*B*a^2*b^4 + (-4*I*B*a^4*b^2 - 2*I
*A*a^3*b^3 + 9*I*B*a^2*b^4 + 6*I*A*a*b^5 - 9*I*B*b^6)*cos(d*x + c)^2 + 2*(
-4*I*B*a^5*b - 2*I*A*a^4*b^2 + 9*I*B*a^3*b^3 + 6*I*A*a^2*b^4 - 9*I*B*a*b^5
)*cos(d*x + c))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/
b) + 3*sqrt(1/2)*(2*I*B*a^5*b + I*A*a^4*b^2 - 6*I*B*a^3*b^3 + 3*I*A*a^2*b^
4 + (2*I*B*a^3*b^3 + I*A*a^2*b^4 - 6*I*B*a*b^5 + 3*I*A*b^6)*cos(d*x + c)^2
 + 2*(2*I*B*a^4*b^2 + I*A*a^3*b^3 - 6*I*B*a^2*b^4 + 3*I*A*a*b^5)*cos(d*x +
 c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/
2)*(-2*I*B*a^5*b - I*A*a^4*b^2 + 6*I*B*a^3*b^3 - 3*I*A*a^2*b^4 + (-2*I*B*a
^3*b^3 - I*A*a^2*b^4 + 6*I*B*a*b^5 - 3*I*A*b^6)*cos(d*x + c)^2 + 2*(-2*I*B
*a^4*b^2 - I*A*a^3*b^3 + 6*I*B*a^2*b^4 - 3*I*A*a*b^5)*cos(d*x + c))*sqr...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(
c + d*x)*a*b + a**2),x)
```

3.337 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 275

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(4aAb-a^2B-3b^2B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2(4aAb-a^2B-3b^2B)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*(4*A*a*b-B*a^2-3*B*b^2)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))/b/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1
/2)-2/3*(A*b-B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2)*(b/(a+b))^(1/2))/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2/3*(A*
b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-2/3*(4*A*a*b-B*a^2-3*
B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.70

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left((-4aAb+a^2B+3b^2B)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a-b)(-Ab+aB)\operatorname{EllipticF}\left(\frac{1}{2}(c+\cdots\right.\right.}{(a-b)^2b}\cdots\right.}{3d(a+b\cos(c\cdots}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*
EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(
c + d*x)/2, (2*b)/(a + b)]))/((a - b)^2*b)) + ((-5*a^2*A*b + A*b^3 + 2*a^3
*B + 2*a*b^2*B + b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x])*Sin[c + d*x]
)/(a^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3233$$

$$-\frac{2\int -\frac{3(aA-bB)-(Ab-aB)\cos(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(aA-bB)-(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3(aA-bB)+(aB-Ab)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\frac{2\int -\frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(a^2(-B)+4aAb-3b^2B)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(a^2(-B)+4aAb-3b^2B)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(a^2(-B)+4aAb-3b^2B)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-aB)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a^2-b^2}-\frac{2(a^2(-B)+4aAb-3b^2B)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3134

$$\frac{\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3132

$$\frac{\frac{\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3142

$$\frac{\frac{\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2(a^2(-B)+4aAb-3b^2B)\,s}{d(a^2-b^2)\sqrt{a+b\cos(c}}}{3\left(a^2-b^2\right)}$$

$$\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2(a^2(-B)+4aAb-3b^2}{d(a^2-b^2)\sqrt{a+b\,c}}}{3\left(a^2-b^2\right)}$$

$$\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3140

$$\frac{\frac{\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2(a^2(-B)+4aAb-3b^2B)}{d(a^2-b^2)\sqrt{a+b\cos(}}}{3\left(a^2-b^2\right)}$$

$$\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input

```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

output

```
(-2*(A*b - a*B)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2))
 + (((2*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a
^2 - b^2)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*
x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(a^2 - b^2) - (2*(4*
a*A*b - a^2*B - 3*b^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(264) = 528$.

Time = 10.92 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.74

method	result
default	$-\frac{\sqrt{-\left(-2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\frac{2B\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+\sqrt{\frac{1}{2}-\frac{\cos}{}}\right.}{}\right.}{}$
parts	Expression too large to display

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B/b/sin
(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(A*b-B*a)/b*
(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2*
d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/
2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b
^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))
/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.36

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(sqrt(1/2)*(2*I*B*a^5 + I*A*a^4*b - 6*I*B*a^3*b^2 + 3*I*A*a^2*b^3 + (
2*I*B*a^3*b^2 + I*A*a^2*b^3 - 6*I*B*a*b^4 + 3*I*A*b^5)*cos(d*x + c)^2 + 2*
(2*I*B*a^4*b + I*A*a^3*b^2 - 6*I*B*a^2*b^3 + 3*I*A*a*b^4)*cos(d*x + c))*sq
rt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(1/2)*(-2
*I*B*a^5 - I*A*a^4*b + 6*I*B*a^3*b^2 - 3*I*A*a^2*b^3 + (-2*I*B*a^3*b^2 - I
*A*a^2*b^3 + 6*I*B*a*b^4 - 3*I*A*b^5)*cos(d*x + c)^2 + 2*(-2*I*B*a^4*b - I
*A*a^3*b^2 + 6*I*B*a^2*b^3 - 3*I*A*a*b^4)*cos(d*x + c))*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(1/2)*(I*B*a^4*b - 4*I*
A*a^3*b^2 + 3*I*B*a^2*b^3 + (I*B*a^2*b^3 - 4*I*A*a*b^4 + 3*I*B*b^5)*cos(d*
x + c)^2 + 2*(I*B*a^3*b^2 - 4*I*A*a^2*b^3 + 3*I*B*a*b^4)*cos(d*x + c))*sqr
t(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(1/2)*(-I*B
*a^4*b + 4*I*A*a^3*b^2 - 3*I*B*a^2*b^3 + (-I*B*a^2*b^3 + 4*I*A*a*b^4 - 3*I
*B*b^5)*cos(d*x + c)^2 + 2*(-I*B*a^3*b^2 + 4*I*A*a^2*b^3 - 3*I*B*a*b^4)*co
s(d*x + c))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}}\, dx = \int \frac{\sqrt{\cos(dx + c)\, b + a}}{\cos(dx + c)^2\, b^2 + 2\cos(dx + c)\, ab + a^2} dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)/(cos(c + d*x)**2*b**2 + 2*cos(c + d*x)*a*b +
a**2),x)
```

3.338 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 349

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx =&-\frac{2(7a^2Ab-3Ab^3-4a^3B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\&+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3a(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}\\&+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}}\\&+\frac{2b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}}+\frac{2b(7a^2Ab-3Ab^3-4a^3B)\sin(c+dx)}{3a^2(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

output
```
-2/3*(7*A*a^2*b-3*A*b^3-4*B*a^3)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/a^2/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a
+b))^(1/2)+2/3*(A*b-B*a)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+
2*A*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)
*(b/(a+b))^(1/2))/a^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*b*(A*b-B*a)*sin(d*x+c)/
a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*b*(7*A*a^2*b-3*A*b^3-4*B*a^3)*sin
(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.15 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.13

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)(B+A\sec(c+dx))\left(\frac{2(-12a^3Ab+4aAb^3+6a^4B+2a^2b^2B)\sqrt{\frac{a\cdots}{}}}{\sqrt{a+b\cos(\cdots}}\cdots\right.}{\cdots} + \frac{\cos(c+dx)\sqrt{a+b\cos(c+dx)}(B+A\sec(c+dx))\left(-\frac{2(-Ab^2\sin(c+dx)+abB\sin(c+dx))}{3a(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{2(-7a^2Ab^2\sin(c+dx)+3A\cdots}{3a^2(a^2-b^2)^{\cdots}}\cdots\right.}{d(A+B\cos(c+dx))}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x
]
```

output
```
(Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-12*a^3*A*b + 4*a*A*b^3 + 6*a^4*B
+ 2*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4*A - 19*a^2*A*b^2 + 9*A
*b^4 + 4*a^3*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-7*a^2*A*b^2 + 3*
A*b^4 + 4*a^3*b*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c
+ d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*Ellipti
cF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[
c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1
 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*C
os[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*C
os[c + d*x])^2))))/(6*a^2*(a - b)^2*(a + b)^2*d*(A + B*Cos[c + d*x])) + (C
os[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(B + A*Sec[c + d*x])*((-2*(-(A*b^2*Si
n[c + d*x]) + a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2
) - (2*(-7*a^2*A*b^2*Sin[c + d*x] + 3*A*b^4*Sin[c + d*x] + 4*a^3*b*B*Sin[c
 + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/(d*(A + B*Cos[c + d
*x]))
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.677$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{\left(b(Ab-aB)\cos^2(c+dx)-3a(Ab-aB)\cos(c+dx)+3A\left(a^2-b^2\right)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\left(b(Ab-aB)\cos^2(c+dx)-3a(Ab-aB)\cos(c+dx)+3A\left(a^2-b^2\right)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3A\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3534

$$\frac{\frac{2\int \frac{\left(3A\left(a^2-b^2\right)^2-b\left(-4Ba^3+7Aba^2-3Ab^3\right)\cos^2(c+dx)-a\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b\left(-4a^3B+7a^2Ab-3Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(3A\left(a^2-b^2\right)^2-b\left(-4Ba^3+7Aba^2-3Ab^3\right)\cos^2(c+dx)-a\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b\left(-4a^3B+7a^2Ab-3Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3A\left(a^2-b^2\right)^2-b\left(-4Ba^3+7Aba^2-3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b\left(-4a^3B+7a^2Ab-3Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3538

$$\frac{\frac{-\frac{\int -\frac{\left(3Ab\left(a^2-b^2\right)^2+ab(Ab-aB)\cos(c+dx)\left(a^2-b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left(\left(-4a^3B+7a^2Ab-3Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx\right)}{a(a^2-b^2)}+\frac{2b(-4a^3B+7a^2Ab-3Ab^3}{ad(a^2-b^2)\sqrt{a+b\cos}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int\frac{\left(3Ab\left(a^2-b^2\right)^2+ab(Ab-aB)\cos(c+dx)\left(a^2-b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left(-4a^3B+7a^2Ab-3Ab^3\right)\int\sqrt{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{2b\left(-4a^3B+7a^2Ab-3Ab^3\right)\sin(}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\int\frac{3Ab\left(a^2-b^2\right)^2+ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\left(-4a^3B+7a^2Ab-3Ab^3\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{2b\left(-4a^3B+7a^2Ab-3Ab^3\right)\sin(c+d}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3134

$$\frac{\frac{\frac{\int\frac{3Ab\left(a^2-b^2\right)^2+ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b\left(-4a^3B+7a^2Ab-3Ab^3\right.}{ad(a^2-b^2)\sqrt{a+b\cos}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{3Ab\left(a^2-b^2\right)^2+ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b(-4a^3B+7a^2Ab-3\cdots}{ad(a^2-b^2)\sqrt{a+b\cdots}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\frac{\frac{\int \frac{3Ab\left(a^2-b^2\right)^2+ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b(-4a^3B+7a^2Ab-3Ab^3)\sin\cdots}{ad(a^2-b^2)\sqrt{a+b\cos(c+d\cdots}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3Ab\left(a^2-b^2\right)^2\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b(-4a^3\cdots}{ad(\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3Ab\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\cdots\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{\frac{ab\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3Ab\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+b\cos\cdots}}{d\sqrt{\frac{a+b\cos(c\cdots}{a+b}}}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{ab\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3Ab\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+\cdots}}{d\sqrt{\frac{a+b\cos\cdots}{\cdots}}}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3140

$$\frac{\frac{\frac{3Ab\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+b\cos\cdots}}{d\sqrt{\frac{a+b\cos(c\cdots}{a+b}}}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3286

$$\frac{\frac{\frac{\frac{3Ab\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(-4a^3B+7a^2Ab-3Ab^3\right)\sqrt{a+\cdots}}{d\sqrt{\frac{a+b\cos(\cdots}{\cdots}}}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{3Ab\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(-4a^3B+7\cdots\right.}{\cdots}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{\frac{2b\left(-4a^3B+7a^2Ab-3Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{\frac{2ab\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{6Ab\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\cdots\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}\cdots}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/
2)) + (((-2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 ((2*a*b*(a^2 - b^2)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*A*b*(a^
2 - b^2)^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(a*(a^2 - b^2)) + (2*b*(7*
a^2*A*b - 3*A*b^3 - 4*a^3*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos
[c + d*x]]))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(339) = 678$.

Time = 12.94 (sec) , antiderivative size = 858, normalized size of antiderivative = 2.46

method	result	size
default	Expression too large to display	858
parts	Expression too large to display	1340

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A/a^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*(A*b-B*a)/a*(1/6/b/(a-b)/(a+b)*cos
(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*sin(1/2*d*x+1/2*c)^2*b/(a-b)^2
/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*c
os(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2*A*b/a^2/sin(1/2*d*x+
1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^{2}b^{2}+2\cos\left(dx+c\right)ab+a^{2}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(
c + d*x)*a*b + a**2),x)
```

3.339 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 437

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \\
&-\frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{(3a^2A-5Ab^2+2abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3a^2(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} \\
&-\frac{(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a^3d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{b(3a^2A-5Ab^2+2abB)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
&+\frac{b(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\sin(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}
\end{aligned}$$

output
```
-1/3*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+14*B*a^3*b-6*B*a*b^3)*(a+b*cos(d*x+c))
^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))/a^3/(a^2-b^2)
^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/3*(3*A*a^2-5*A*b^2+2*B*a*b)*((a+b*co
s(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/
2))/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-(5*A*b-2*B*a)*((a+b*cos(d*x+c))
/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a^3
/d/(a+b*cos(d*x+c))^(1/2)+1/3*b*(3*A*a^2-5*A*b^2+2*B*a*b)*sin(d*x+c)/a^2/(
a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+1/3*b*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+14*
B*a^3*b-6*B*a*b^3)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+A*t
an(d*x+c)/a/d/(a+b*cos(d*x+c))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.83 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.72

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{2(36a^3Ab^2-20aAb^4-24a^4bB+8a^2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \cdots}{\cdots} + \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{2(-Ab^3\sin(c+dx)+ab^2B\sin(c+dx))}{3a^2(a^2-b^2)(a+b\cos(c+dx))^2} + \frac{2(-10a^2Ab^3\sin(c+dx)+6Ab^5\sin(c+dx)+7a^3b^2B\sin(c+dx)-3ab^4B\sin(\cdots}{3a^3(a^2-b^2)^2(a+b\cos(c+dx))}\cdots\right)}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2)
,x]
```

output
```
((2*(36*a^3*A*b^2 - 20*a*A*b^4 - 24*a^4*b*B + 8*a^2*b^3*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[
c + d*x]] + (2*(-33*a^4*A*b + 86*a^2*A*b^3 - 45*A*b^5 + 12*a^5*B - 38*a^3*
b^2*B + 18*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-3*a^4*A*b + 26
*a^2*A*b^3 - 15*A*b^5 - 14*a^3*b^2*B + 6*a*b^4*B)*Sqrt[(b - b*Cos[c + d*x]
)/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a
- b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
 + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt
[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x]
)/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*
(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a
 + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(12*a^3*(-a + b)^2*(a + b
)^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Si
n[c + d*x]))/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-10*a^2*A*b^
3*Sin[c + d*x] + 6*A*b^5*Sin[c + d*x] + 7*a^3*b^2*B*Sin[c + d*x] - 3*a*b^4
*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (A*Tan[c +
d*x])/a^3))/d
```

Rubi [A] (verified)

Time = 3.78 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.08, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3479, 27, 3042, 3535, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int -\frac{\left(-3Ab\cos^2(c+dx)+5Ab-2aB\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{5/2}}dx}{a}+\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{\left(-3Ab\cos^2(c+dx)+5Ab-2aB\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{-3Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+5Ab-2aB}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}$$

$$\Big\downarrow \; 3535$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{2\int \frac{\left(-b\left(3Aa^2+2bBa-5Ab^2\right)\cos^2(c+dx)-6ab(Ab-aB)\cos(c+dx)+3\left(a^2-b^2\right)(5Ab-2aB)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2A+2abB-5Ab^2)\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \; 27$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{\left(-b\left(3Aa^2+2bBa-5Ab^2\right)\cos^2(c+dx)-6ab(Ab-aB)\cos(c+dx)+3\left(a^2-b^2\right)(5Ab-2aB)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2A+2abB-5Ab^2)\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{-b\left(3Aa^2+2bBa-5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-6ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)(5Ab-2aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2A+2abB-5Ab^2)\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \; 3534$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{2\int \frac{\left(3(5Ab-2aB)\left(a^2-b^2\right)^2+b\left(3Aa^4+14bBa^3-26Ab^2a^2-6b^3Ba+15Ab^4\right)\cos^2(c+dx)-2ab\left(-6Ba^3+9Aba^2+2b^2Ba-5Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4 \cdots\right.}{\cdots}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \ 27$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{\left(3(5Ab-2aB)\left(a^2-b^2\right)^2+b\left(3Aa^4+14bBa^3-26Ab^2a^2-6b^3Ba+15Ab^4\right)\cos^2(c+dx)-2ab\left(-6Ba^3+9Aba^2+2b^2Ba-5Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4\right.}{}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{3(5Ab-2aB)\left(a^2-b^2\right)^2+b\left(3Aa^4+14bBa^3-26Ab^2a^2-6b^3Ba+15Ab^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(-6Ba^3+9Aba^2+2b^2Ba-5Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4A+14\right.}{}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \ 3538$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4\right)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(3b\left(a^2-b^2\right)^2(5Ab-2aB)-ab\left(a^2-b^2\right)\left(3Aa^2+2bBa-5Ab^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a\left(a^2-b^2\right)}-}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \ 25$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int \frac{\left(3b\left(a^2-b^2\right)^2(5Ab-2aB)-ab\left(a^2-b^2\right)\left(3Aa^2+2bBa-5Ab^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\left(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4\right)\int\sqrt{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}-2}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int \frac{3b\left(a^2-b^2\right)^2(5Ab-2aB)-ab\left(a^2-b^2\right)\left(3Aa^2+2bBa-5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\left(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a\right.}{}}{3a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3134

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int\frac{3b(a^2-b^2)^2(5Ab-2aB)-ab(a^2-b^2)(3Aa^2+2bBa-5Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c\dots}{a+\dots}}\dots}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}\dots}{3a(a^2-b^2)}\dots}{2a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int\frac{3b(a^2-b^2)^2(5Ab-2aB)-ab(a^2-b^2)(3Aa^2+2bBa-5Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c\dots\right)}{\dots}}\dots}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}\dots}{3a(a^2-b^2)}\dots}{2a}$$

$\Big\downarrow$ 3132

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int\frac{3b(a^2-b^2)^2(5Ab-2aB)-ab(a^2-b^2)(3Aa^2+2bBa-5Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+\dots}\right)\dots}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}\dots}{3a(a^2-b^2)}\dots}{2a}$$

$\Big\downarrow$ 3481

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{3b(a^2-b^2)^2(5Ab-2aB)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab(a^2-b^2)(3a^2A+2abB-5Ab^2)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\sqrt{a+b\cos\dots}}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}\dots}{3a(a^2-b^2)}\dots}{2a}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{3b\left(a^2-b^2\right)^2(5Ab-2aB)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\left(a^2-b^2\right)\left(3a^2A+2abB-5Ab^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(3a^4A+14a^3bB-26a^2Ab^2-6\right.}{}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \; 3142$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{3b\left(a^2-b^2\right)^2(5Ab-2aB)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\left(a^2-b^2\right)\left(3a^2A+2abB-5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(3a^4A+14a\right.}{}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{3b\left(a^2-b^2\right)^2(5Ab-2aB)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\left(a^2-b^2\right)\left(3a^2A+2abB-5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(3a^4A\right.}{}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}$$

$$\Big\downarrow \; 3140$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{3b\left(a^2-b^2\right)^2(5Ab-2aB)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab\left(a^2-b^2\right)\left(3a^2A+2abB-5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(3a^4A+14\right.}{}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}$$

$$\Big\downarrow \; 3286$$

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{3b(a^2-b^2)^2(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab(a^2-b^2)(3a^2A+2abB-5Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2(3a^4A-}{}}{a(a^2-b^2)}}{3a(a^2-b^2)}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{3b(a^2-b^2)^2(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab(a^2-b^2)(3a^2A+2abB-5Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}}{a(a^2-b^2)}}{3a(a^2-b^2}$$

$\downarrow$ 3284

$$\frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{6b(a^2-b^2)^2(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2ab(a^2-b^2)(3a^2A+2abB-5Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2(3a^4}{}}{a(a^2-b^2)}}{3a(a^2-b^2)}$$

input

```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
-1/2*((-2*b*(3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d
*(a + b*Cos[c + d*x])^(3/2)) + (((2*(3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 1
4*a^3*b*B - 6*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((-2*a*b*(a^2 - b^2)
*(3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*b*(a^2 -
b^2)^2*(5*A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(a*(a^2 - b^2)
) - (2*b*(3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sin[
c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b^2)))/a
 + (A*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. $2(425) = 850$.

Time = 17.55 (sec) , antiderivative size = 1346, normalized size of antiderivative = 3.08

method	result	size
default	Expression too large to display	1346
parts	Expression too large to display	2175

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(-
cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*
cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(
1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))+1/2*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*(2*A*b-B*a)/a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*b*(2*A*b-B*a)/a^3/sin(1/2*d*x+
1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2*b-a-b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**(5/2),
 x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2),
x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)^2\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^2}{\cos\left(dx+c\right)^2 b^2+2\cos\left(dx+c\right)ab+a^2}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2)/(cos(c + d*x)**2*b**2 + 2*c
os(c + d*x)*a*b + a**2),x)
```

3.340 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 532

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)}{12a^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+a}{a+b}}}$$

$$-\frac{(27a^2Ab-35Ab^3-12a^3B+20ab^2B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{12a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{(4a^2A+35Ab^2-20abB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4a^4d\sqrt{a+b\cos(c+dx)}}$$

$$-\frac{b(27a^2Ab-35Ab^3-12a^3B+20ab^2B)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}$$

$$-\frac{b(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)\sin(c+dx)}{12a^4\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$-\frac{(7Ab-4aB)\tan(c+dx)}{4a^2d(a+b\cos(c+dx))^{3/2}}+\frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}$$

output
```
1/12*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^5+104*B*a^3*b^2-60*B*a*b^4
)*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))/a^4/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-1/12*(27*A*a^2*b-35*
A*b^3-12*B*a^3+20*B*a*b^2)*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2)*(b/(a+b))^(1/2))/a^3/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)+1/4*(4*A*a^2+35*A*b^2-20*B*a*b)*((a+b*cos(d*x+c))/(a+b))^(1/2)*Ellipti
cPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a^4/d/(a+b*cos(d*x+c))^(
1/2)-1/12*b*(27*A*a^2*b-35*A*b^3-12*B*a^3+20*B*a*b^2)*sin(d*x+c)/a^3/(a^2-
b^2)/d/(a+b*cos(d*x+c))^(3/2)-1/12*b*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-1
2*B*a^5+104*B*a^3*b^2-60*B*a*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d/(a+b*cos(d*
x+c))^(1/2)-1/4*(7*A*b-4*B*a)*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(3/2)+1/2*
A*sec(d*x+c)*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.54

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{2\left(12a^5Ab-216a^3Ab^3+140aAb^5+144a^4b^2B-80a^2b^4B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+\cdots\right.}{\sqrt{a+b\cos(c+dx)}}}{\cdots} + \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)(-11Ab\sin(c+dx)+4aB\sin(c+dx))}{4a^4} - \frac{2\left(-Ab^4\sin(c+dx)+ab^3B\sin(c+dx)\right)}{3a^3(a^2-b^2)(a+b\cos(c+dx))^2} - \frac{2\left(-13a^2Ab^4\sin(c+dx\cdots\right.}{\cdots}\right.}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2)
,x]
```

output

```
((2*(12*a^5*A*b - 216*a^3*A*b^3 + 140*a*A*b^5 + 144*a^4*b^2*B - 80*a^2*b^4
*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/Sqrt[a + b*Cos[c + d*x]] + (2*(24*a^6*A + 195*a^4*A*b^2 - 566*a^2*A*b^
4 + 315*A*b^6 - 132*a^5*b*B + 344*a^3*b^3*B - 180*a*b^5*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*
Cos[c + d*x]] - ((2*I)*(33*a^4*A*b^2 - 170*a^2*A*b^4 + 105*A*b^6 - 12*a^5*
b*B + 104*a^3*b^3*B - 60*a*b^5*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[
-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a +
b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d
*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x
]) + 2*(a + b*Cos[c + d*x])^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[a
+ b*Cos[c + d*x]]*((Sec[c + d*x]*(-11*A*b*Sin[c + d*x] + 4*a*B*Sin[c + d*x
]))/(4*a^4) - (2*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x]))/(3*a^3*(a
^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-13*a^2*A*b^4*Sin[c + d*x] + 9*A*b
^6*Sin[c + d*x] + 10*a^3*b^3*B*Sin[c + d*x] - 6*a*b^5*B*Sin[c + d*x]))/(3*
a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*Tan[c + d*x])...
```

Rubi [A] (verified)

Time = 4.66 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.07, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int -\frac{\left(-5Ab\cos^2(c+dx)-2aA\cos(c+dx)+7Ab-4aB\right)\sec^2(c+dx)}{2(a+b\cos(c+dx))^{5/2}}dx}{2a}+\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{\left(-5Ab\cos^2(c+dx)-2aA\cos(c+dx)+7Ab-4aB\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{-5Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-2aA\sin\left(c+dx+\frac{\pi}{2}\right)+7Ab-4aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{4a}$$

$$\Big\downarrow \ 3534$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int -\frac{\left(4Aa^2-20bBa+10Ab\cos(c+dx)a+35Ab^2-3b(7Ab-4aB)\cos^2(c+dx)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{5/2}}dx}{a}+\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}}{4a}$$

$$\Big\downarrow \ 27$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{\left(4Aa^2-20bBa+10Ab\cos(c+dx)a+35Ab^2-3b(7Ab-4aB)\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{4Aa^2-20bBa+10Ab\sin\left(c+dx+\frac{\pi}{2}\right)a+35Ab^2-3b(7Ab-4aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3534$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{2\int \frac{\left(-b\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\cos^2(c+dx)+6ab\left(3Aa^2+4bBa-7Ab^2\right)\cos(c+dx)+3\left(a^2-b^2\right)\left(4Aa^2-20bBa+35Ab^2\right.}{2(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(-b\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\cos^2(c+dx)+6ab\left(3Aa^2+4bBa-7Ab^2\right)\cos(c+dx)+3\left(a^2-b^2\right)\left(4Aa^2-20bBa+35Ab^2\right)}{(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}}{2a}}{4a}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int\frac{-b\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+6ab\left(3Aa^2+4bBa-7Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)\left(4Aa^2-20bBa+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}}{3a\left(a^2-b^2\right)}}{2a}}{4a}}{}$$

$$\Big\downarrow \text{3534}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{2\int\frac{\left(3\left(4Aa^2-20bBa+35Ab^2\right)\left(a^2-b^2\right)^2+b\left(-12Ba^5+33Aba^4+104b^2Ba^3-170Ab^3a^2-60b^4Ba+105Ab^5\right)\cos^2(c+dx)+2ab\left(3Aa^4\right.\right.}{2\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}}{}}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(3\left(4Aa^2-20bBa+35Ab^2\right)\left(a^2-b^2\right)^2+b\left(-12Ba^5+33Aba^4+104b^2Ba^3-170Ab^3a^2-60b^4Ba+105Ab^5\right)\cos^2(c+dx)+2ab\left(3Aa^4+\right.\right.}{\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}}{3}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int\frac{3\left(4Aa^2-20bBa+35Ab^2\right)\left(a^2-b^2\right)^2+b\left(-12Ba^5+33Aba^4+104b^2Ba^3-170Ab^3a^2-60b^4Ba+105Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab\left(3Aa\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{a\left(a^2-b^2\right)}}{3a}}{}$$

$$\Big\downarrow \text{3538}$$

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\left(-12a^5B+33a^4Ab+104a^3b^2B-170a^2Ab^3-60ab^4B+105Ab^5\right)\int\sqrt{a+b\cos(c+dx)}dx-\int-\frac{\left(3b\left(a^2-b^2\right)^2\left(4Aa^2-20bBa+35Ab^2\right)-\cdots\right.}{\cdots}}{a\left(a^2-b^2\right)}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 25

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int\frac{\left(3b\left(a^2-b^2\right)^2\left(4Aa^2-20bBa+35Ab^2\right)-ab\left(a^2-b^2\right)\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\left(-12a^5B\cdots\right.}{a\left(a^2-b^2\right)}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 3042

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int\frac{3b\left(a^2-b^2\right)^2\left(4Aa^2-20bBa+35Ab^2\right)-ab\left(a^2-b^2\right)\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\left(-12a^5B+33a^4Ab\cdots\right.}{a\left(a^2-b^2\right)}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 3134

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int\frac{3b\left(a^2-b^2\right)^2\left(4Aa^2-20bBa+35Ab^2\right)-ab\left(a^2-b^2\right)\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(-12a^5B+33a^4Ab\cdots\right.}{\cdots}}{a\left(a^2-b^2\right)}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-$$

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{3b\left(a^2-b^2\right)^2\left(4Aa^2-20bBa+35Ab^2\right)-ab\left(a^2-b^2\right)\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(-12a^5B+33a^4Ab\right.}{}}{}}{a\left(a^2-b^2\right)}}{}$$

$\downarrow$ 3132

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-$$

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{3b\left(a^2-b^2\right)^2\left(4Aa^2-20bBa+35Ab^2\right)-ab\left(a^2-b^2\right)\left(-12Ba^3+27Aba^2+20b^2Ba-35Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(-12a^5B+33a^4A\right.}{}}{}}{a\left(a^2-b^2\right)}}{}$$

$\downarrow$ 3481

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-$$

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{3b\left(a^2-b^2\right)^2\left(4a^2A-20abB+35Ab^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab\left(a^2-b^2\right)\left(-12a^3B+27a^2Ab+20ab^2B-35Ab^3\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}}{b}}{a\left(a^2-b^2\right)}}{}$$

$\downarrow$ 3042

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-$$

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{3b\left(a^2-b^2\right)^2\left(4a^2A-20abB+35Ab^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\left(a^2-b^2\right)\left(-12a^3B+27a^2Ab+20ab^2B-35Ab^3\right)}{b}}{a\left(a^2-\right.}}{}$$

$\downarrow$ 3142

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{3b(a^2-b^2)^2(4a^2A-20abB+35Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab(a^2-b^2)(-12a^3B+27a^2Ab+20ab^2B-35Ab^3)}{\sqrt{a+b\cos(c}}}{b}}{}}{}$$

3042

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{3b(a^2-b^2)^2(4a^2A-20abB+35Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab(a^2-b^2)(-12a^3B+27a^2Ab+20ab^2B-35Ab^3)}{\sqrt{a+b\cos}}}{b}}{}}{}$$

3140

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{3b(a^2-b^2)^2(4a^2A-20abB+35Ab^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab(a^2-b^2)(-12a^3B+27a^2Ab+20ab^2B-35Ab^3}{d\sqrt{a+b\cos}}}{b}}{}}{}$$

3286

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{3b(a^2-b^2)^2(4a^2A-20abB+35Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab(a^2-b^2)(-12a^3B+27a^2Ab+20ab^2B-35}{d\sqrt{a+b}}}{b}}{}}{}$$

3042

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-$$

$$\frac{\frac{3b\left(a^2-b^2\right)^2\left(4a^2A-20abB+35Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2ab\left(a^2-b^2\right)\left(-12a^3B+27a\right.}{}$$

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-$$

$\downarrow$ 3284

$$\frac{A\tan(c+dx)\sec(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}}-$$

$$\frac{\frac{6b\left(a^2-b^2\right)^2\left(4a^2A-20abB+35Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2ab\left(a^2-b^2\right)\left(-12a^3B+27a^2Ab+20ab^2B-3\right.}{d\sqrt{a+}}$$

$$\frac{(7Ab-4aB)\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*(a + b*Cos[c + d*x])^(3/2)) - (-1/2*(
(-2*b*(27*a^2*A*b - 35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sin[c + d*x])/(3*a*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((2*(33*a^4*A*b - 170*a^2*A*b^
3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/
(a + b)]) + ((-2*a*b*(a^2 - b^2)*(27*a^2*A*b - 35*A*b^3 - 12*a^3*B + 20*a*
b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*b*(a^2 - b^2)^2*(4*a^2*A + 35*A*b
^2 - 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(a*(a^2 - b^2)) - (2*b
*(33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a
*b^4*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/(3*a*(a^
2 - b^2)))/a + ((7*A*b - 4*a*B)*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3
/2)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2004 vs. $2(512) = 1024$.

Time = 21.60 (sec) , antiderivative size = 2005, normalized size of antiderivative = 3.77

method	result	size
default	Expression too large to display	2005
parts	Expression too large to display	3298

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(-
1/2*cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+
a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/
(a-b))^(1/2))*b^2)-2*(2*A*b-B*a)/a^3*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+
1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec^3{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{3}}{\cos\left(dx+c\right)^{2}b^{2}+2\cos\left(dx+c\right)ab+a^{2}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**3)/(cos(c + d*x)**2*b**2 + 2*c
os(c + d*x)*a*b + a**2),x)
```

3.341 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 58

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*B*((a+b*cos(d*x+c))/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)*(
b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2011, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 2011$$

$$B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$B\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3142$$

$$\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \ 3140$$

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

method	result
default	$\frac{2B\sqrt{\frac{a+\cos(dx+c)b}{a+b}}\,\operatorname{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{a+\cos(dx+c)b}}$
parts	$-\frac{2Ba\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}{(a-b)(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}\,d}$

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*B/d/(a+cos(d*x+c)*b)^(1/2)*((a+cos(d*x+c)*b)/(a+b))^(1/2)*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.57

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)-i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weier}\right.}{bd}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas
")
```

output
```
-2*(I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a
)/b) - I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
2*a)/b))/(b*d)
```

Sympy [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
B*Integral(1/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{(a + b\,\cos(c + d\,x))^{3/2}}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\cos(dx + c)\,b + a}}{\cos(dx + c)\,b + a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)/(cos(c + d*x)*b + a),x)*b
```

3.342 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 59

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*B*((a+b*cos(d*x+c))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)
*(b/(a+b))^(1/2))/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/
2),x]
```

output
```
(2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2011, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3286}$$

$$\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(60) = 120$.

Time = 6.50 (sec), antiderivative size = 167, normalized size of antiderivative = 2.83

method	result
default	$\frac{2\sqrt{\left(2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a-b}{a-b}}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)}{\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}\,d}$
parts	$2B\Bigg(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a-b}+\frac{a+b}{a-b}}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)a^2-\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}$

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b
+a+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
B*Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx+c) + Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2)
, x)
```

Giac [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx+c) + Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^{3/2}}\, dx = \int \frac{B\,a + B\,b\,\cos\left(c + d\,x\right)}{\cos\left(c + d\,x\right)\,\left(a + b\,\cos\left(c + d\,x\right)\right)^{3/2}}\, dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^{3/2}}\, dx = \left(\int \frac{\sqrt{\cos\left(dx + c\right)b + a}\,\sec\left(dx + c\right)}{\cos\left(dx + c\right)b + a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x))/(cos(c + d*x)*b + a),x)*b
```

3.343 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 108

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2bB\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*B*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2*b*B*sin(d*x+c)/(a^2-b^
2)/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\frac{B\left(2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)\right)}{(a-b)(a+b)d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(B*(2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2
*b)/(a + b)] - 2*b*Sin[c + d*x]))/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2011, 3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$B\left(-\frac{2\int -\frac{1}{2}\sqrt{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{\int \sqrt{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3134}$$

$$B\left(\frac{\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3132}$$

$$B\left(\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
B*((2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^
2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(107) = 214$.

Time = 8.92 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.02

method	result
default	$-\frac{2B\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{(a-b)(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}\,d}$
parts	Expression too large to display

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*B*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2)))/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.45

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(3\sqrt{b\cos(dx+c)+a}Bb^2\sin(dx+c)-\sqrt{\frac{1}{2}}(-i\,Bab\cos(dx+c)-i\,Ba^2)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4\right.}{}\right.}{}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas
")
```

output
```
-2/3*(3*sqrt(b*cos(d*x + c) + a)*B*b^2*sin(d*x + c) - sqrt(1/2)*(-I*B*a*b*
cos(d*x + c) - I*B*a^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c)
 + 2*a)/b) - sqrt(1/2)*(I*B*a*b*cos(d*x + c) + I*B*a^2)*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(1/2)*(I*B*b^2*cos(d*x
+ c) + I*B*a*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b))
- 3*sqrt(1/2)*(-I*B*b^2*cos(d*x + c) - I*B*a*b)*sqrt(b)*weierstrassZeta(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
- 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^2 - b^4)*d*cos(d*x + c) + (a^3*b
- a*b^3)*d)
```

Sympy [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = B\int \frac{1}{a\sqrt{a+b\cos(c+dx)}+b\sqrt{a+b\cos(c+dx)}\cos(c+dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
B*Integral(1/(a*sqrt(a + b*cos(c + d*x)) + b*sqrt(a + b*cos(c + d*x))*cos(
c + d*x)), x)
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}}\, dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima
")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}}\, dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}}\, dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{(a + b\,\cos(c + d\,x))^{5/2}}\, dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)/(cos(c + d*x)**2*b**2 + 2*cos(c + d*x)*a*b +
a**2),x)*b
```

3.344 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 179

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2b^2B\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*b*B*(a+b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b
))^(1/2))/a/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*B*((a+b*cos(d*x+c
))/(a+b))^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))/a
/d/(a+b*cos(d*x+c))^(1/2)+2*b^2*B*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c)
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.25 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.25

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{B\left(-\frac{-\frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(2a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{Ellip}}{\sqrt{a+b\cos(c+}}}{}\right.}{}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/
2),x]
```

output
```
(B*(-(((-4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a +
b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*
(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[
a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a +
b)*(a + b))) + (4*b^2*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])
))/(2*a*d)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {2011, 3042, 3281, 27, 3042, 3538, 25, 27, 3042, 3134, 3042, 3132, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$B\left(\frac{2\int \frac{\left(a^2-b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{\int \frac{\left(a^2-b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3538}$$

$$B\left(\frac{-\frac{\int -\frac{b\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-b\int\sqrt{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{\frac{\int \frac{b\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-b\int\sqrt{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{(a^2-b^2)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-b\int\sqrt{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$B\left(\frac{(a^2-b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 3134$$

$$B\left(\frac{(a^2-b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$B\left(\frac{(a^2-b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 3132$$

$$B\left(\frac{(a^2-b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 3286$$

$$B\left(\frac{\frac{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\frac{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\,(a^2-b^2)}+\frac{2b^2\sin(c+}{ad\,(a^2-b^2)\sqrt{a+b}}\right.$$

$$\Big\downarrow \text{3284}$$

$$B\left(\frac{2b^2\sin(c+dx)}{ad\,(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\,(a^2-b^2)}\right.$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
B*(((-2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/
(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*
Cos[c + d*x]]))/(a*(a^2 - b^2)) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sq
rt[a + b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(179) = 358$.

Time = 11.98 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.11

method	result
default	$\frac{2B\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\frac{2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\ \text{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)a^{2}-\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{}$
parts	Expression too large to display

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*B*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))*b^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a-b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
)/a/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = B\int \frac{\sec(c+dx)}{a\sqrt{a+b\cos(c+dx)} + b\sqrt{a+b\cos(c+dx)}\cos(c+dx)}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
B*Integral(sec(c + d*x)/(a*sqrt(a + b*cos(c + d*x)) + b*sqrt(a + b*cos(c +
 d*x))*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2)
, x)
```

Giac [F]

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sec(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(
c + d*x)*a*b + a**2),x)*b
```

3.345 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 170

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10(Ab+aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{10(Ab+aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2(9aA+7bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{2(Ab+aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2bB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}$$

output
```
2/15*(9*A*a+7*B*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*(A*b+B*a)
*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+10/21*(A*b+B*a)*cos(d*x+c)^(1/2)
*sin(d*x+c)/d+2/45*(9*A*a+7*B*b)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*(A*b+B*
a)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*b*B*cos(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{84(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300(Ab+aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7(36aA+43}{630}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(84*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2] + 300*(A*b + a*B)*EllipticF[
(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a*A + 43*b*B)*Cos[c + d*x] + 5
*(78*A*b + 78*a*B + 18*(A*b + a*B)*Cos[2*(c + d*x)] + 7*b*B*Cos[3*(c + d*x
)]))*Sin[c + d*x])/(630*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos^{\frac{5}{2}}(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left((aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\downarrow \text{3502}$$

$$\frac{2}{9}\int \frac{1}{2}\cos^{\frac{5}{2}}(c+dx)(9aA+7bB+9(Ab+aB)\cos(c+dx))dx+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\downarrow \text{27}$$

$$\frac{1}{9}\int \cos^{\frac{5}{2}}(c+dx)(9aA+7bB+9(Ab+aB)\cos(c+dx))dx+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(9aA+7bB+9(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\downarrow \text{3227}$$

$$\frac{1}{9}\left((9aA+7bB)\int \cos^{\frac{5}{2}}(c+dx)dx+9(aB+Ab)\int \cos^{\frac{7}{2}}(c+dx)dx\right)+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{9}\left((9aA+7bB)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+9(aB+Ab)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx\right)+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\downarrow \text{3115}$$

$$\frac{1}{9}\left((9aA+7bB)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9(aB+Ab)\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{9}\left((9aA+7bB)\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9(aB+Ab)\left(\frac{5}{7}\int \sin\left(c+dx+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{9}\left(9(aB+Ab)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+(9a\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(9(aB+Ab)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(9(aB+Ab)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{9}\left(9(aB+Ab)\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + ((9*a*A + 7*b*B)*((6*Ellip
ticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)) +
 9*(A*b + a*B)*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(153) = 306$.

Time = 25.84 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.65

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}b+(720Ab+720Ba+2240Bb)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}\right.\right.}{}$
parts	$-\frac{2(Ab+Ba)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^9-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^7+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3+5\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos}}$

input
```
int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*b)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b+(720*A*b+720*B*a+2240*B*b)*sin(1/
2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*a-1080*A*b-1080*B*a-2072*B*b)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A*a+840*A*b+840*B*a+952*B*b)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A*a-240*A*b-240*B*a-168*B*b)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18
9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a+75*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.24

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(35\,Bb\cos\left(dx+c\right)^{3}+45\,(Ba+Ab)\cos\left(dx+c\right)^{2}+75\,Ba+75\,Ab+7\,(9\,Aa+7\,Bb)\cos\left(dx+c\right)\right)\sqrt{c}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/315*(2*(35*B*b*cos(d*x + c)^3 + 45*(B*a + A*b)*cos(d*x + c)^2 + 75*B*a +
 75*A*b + 7*(9*A*a + 7*B*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
- 75*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) - 75*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A*a - 7*I*B*b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*
sqrt(2)*(9*I*A*a + 7*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= -\frac{2\,A\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,A\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,B\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,B\,b\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

output
```
- (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b*cos(c + d*x)^(9/2)*sin(c
 + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/
4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b*cos(c + d*x)^(11
/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(
c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4 dx\right) b^2 + 2\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3 dx\right) ab$$
$$+ \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2 dx\right) a^2$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*b**2 + 2*int(sqrt(cos(c + d*x))*
cos(c + d*x)**3,x)*a*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a**2
```

3.346 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 140

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx &= \frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7aA+5bB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\&\quad+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\&\quad+\frac{2(Ab+aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2bB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
6/5*(A*b+B*a)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(7*A*a+5*B*b)*I
nverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/21*(7*A*a+5*B*b)*cos(d*x+c)^(1/2
)*sin(d*x+c)/d+2/5*(A*b+B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*B*cos(d*x
+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$=\frac{126(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7aA+5bB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(70aA+65bE}{105d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(126*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + 10*(7*a*A + 5*b*B)*EllipticF[
(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*a*A + 65*b*B + 42*(A*b + a*B)*Cos
[c + d*x] + 15*b*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \cos^{\frac{3}{2}}(c+dx)\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left((aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 3502$$

$$\frac{2}{7}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(7aA+5bB+7(Ab+aB)\cos(c+dx))dx+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \cos^{\frac{3}{2}}(c+dx)(7aA+5bB+7(Ab+aB)\cos(c+dx))dx+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(7aA+5bB+7(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 3227$$

$$\frac{1}{7}\left((7aA+5bB)\int \cos^{\frac{3}{2}}(c+dx)dx+7(aB+Ab)\int \cos^{\frac{5}{2}}(c+dx)dx\right)+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left((7aA+5bB)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+7(aB+Ab)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{7}\left(7(aB+Ab)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+(7aA+5bB)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(7(aB+Ab)\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+(7aA+5bB)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(c+d}}\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{7}\left((7aA+5bB)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7(aB+Ab)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right.\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \ 3120$$

$$\frac{1}{7}\left(7(aB+Ab)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+(7aA+5bB)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right.\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + ((7*a*A + 5*b*B)*((2*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)) +
 7*(A*b + a*B)*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(5*d)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(127) = 254$.

Time = 15.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.95

method	result
default	$-\frac{2\sqrt{(2\cos(\frac{dx}{2}+\frac{c}{2})^2-1)\sin(\frac{dx}{2}+\frac{c}{2})^2}\,(240B\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})^8 b+(-168Ab-168Ba-360Bb)\sin(\frac{dx}{2}+\frac{c}{2})^6\cos(\frac{dx}{2}+\frac{c}{2}}{}$
parts	$-\frac{2(Ab+Ba)\sqrt{(2\cos(\frac{dx}{2}+\frac{c}{2})^2-1)\sin(\frac{dx}{2}+\frac{c}{2})^2}\,(-8\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})^6+8\sin(\frac{dx}{2}+\frac{c}{2})^4\cos(\frac{dx}{2}+\frac{c}{2})-2\sin(\frac{dx}{2}+\frac{c}{2})^2 c}{5\sqrt{-2\sin(\frac{dx}{2}+\frac{c}{2})^4+\sin(\frac{dx}{2}+\frac{c}{2})^2}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2\cos(\frac{d}{2}}}$

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*b+25*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.37

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(15\,Bb\cos\left(dx+c\right)^2+35\,Aa+25\,Bb+21\,(Ba+Ab)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\,\sqrt{2}(}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/105*(2*(15*B*b*cos(d*x + c)^2 + 35*A*a + 25*B*b + 21*(B*a + A*b)*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(7*I*A*a + 5*I*B*b)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*A
*a - 5*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) -
63*sqrt(2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) - 63*sqrt(2)*(I*B*a + I*A*b)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/
d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx &= \frac{2\,A\,a\left(\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d} \\ &\quad - \frac{2\,A\,b\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\,\frac{11}{4};\,\cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}} \\ &\quad - \frac{2\,B\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\,\frac{11}{4};\,\cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}} \\ &\quad - \frac{2\,B\,b\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\,\frac{13}{4};\,\cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

output
```
(2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3
*d) - (2*A*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, c
os(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*s
in(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2)) - (2*B*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4],
 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^2 + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)ab$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a**2 + int(sqrt(cos(c + d*x))*cos(c
 + d*x)**3,x)*b**2 + 2*int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a*b
```

3.347 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 108

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/5*(5*A*a+3*B*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*In
verseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*(A*b+B*a)*cos(d*x+c)^(1/2)*sin(
d*x+c)/d+2/5*b*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(3(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(Ab+aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5Ab+5aB+\right.}{15d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*(3*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(A*b + a*B)*EllipticF[
(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*
Sin[c + d*x]))/(15*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sqrt{\cos(c+dx)}\left((aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left((aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\cos(c+dx)}(5aA+3bB+5(Ab+aB)\cos(c+dx))dx+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\cos(c+dx)}(5aA+3bB+5(Ab+aB)\cos(c+dx))dx+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(5aA+3bB+5(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(5(aB+Ab)\int \cos^{\frac{3}{2}}(c+dx)dx+(5aA+3bB)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left((5aA+3bB)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5(aB+Ab)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{5}\left((5aA+3bB)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5(aB+Ab)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left((5aA+3bB)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5(aB+Ab)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5(aB+Ab)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + ((2*(5*a*A + 3*b*B)*Ellipt
icE[(c + d*x)/2, 2])/d + 5*(A*b + a*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d
) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(99) = 198$.

Time = 10.51 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.44

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}b+(20Ab+20Ba+24Bb)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(\cdots\right.}{\cdots}$
parts	$-\frac{2(Ab+Ba)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\cdots}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a+5*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(3\,Bb\cos\left(dx+c\right)+5\,Ba+5\,Ab\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\,\sqrt{2}(i\,Ba+i\,Ab)\mathrm{weierstrassPInverse}(\cdots}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/15*(2*(3*B*b*cos(d*x + c) + 5*B*a + 5*A*b)*sqrt(cos(d*x + c))*sin(d*x +
c) - 5*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - 5*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A*a - 3*I*B*b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*s
qrt(2)*(5*I*A*a + 3*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.19

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \frac{2\,A\,b\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}$$
$$+\frac{2\,B\,a\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$
$$-\frac{2\,B\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

output
```
(2*A*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3
*d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2
)))/(3*d) + (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d - (2*B*b*cos(c + d*x)^(7
/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c
+ d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos\left(dx+c\right)}dx\right)a^2+2\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)ab$$
$$+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)b^2$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)),x)*a**2 + 2*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*
a*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b**2
```

3.348 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3aA+bB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*(A*b+B*a)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a+B*b)*Invers
eJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*b*B*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\left(3(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(3aA+bB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+bB\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d}$$

input
```
Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x
]
```

output
```
(2*(3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (3*a*A + b*B)*EllipticF[(c +
 d*x)/2, 2] + b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2}{3}\int \frac{3aA+bB+3(Ab+aB)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3aA+bB+3(Ab+aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\int \frac{3aA+bB+3(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3227

$$\frac{1}{3}\left((3aA+bB)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3(aB+Ab)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left((3aA+bB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3(aB+Ab)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3119

$$\frac{1}{3}\left((3aA+bB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$\big\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2(3aA+bB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

input
```
Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
((6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[
(c + d*x)/2, 2])/d)/3 + (2*b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(72) = 144$.

Time = 7.00 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.35

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+3Aa\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$
parts	$\frac{2(Ab+Ba)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}+\frac{2Aa\ \mathrm{Inve}}{}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b+3*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*b-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+B*b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$=\frac{2\,Bb\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}(-3i\,Aa-i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/3*(2*B*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-3*I*A*a - I*B*b)*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A*a
 + I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sq
rt(2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 24.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,B\,b\left(\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d} + \frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

output
```
(2*B*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3
*d) + (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b*ellipticE(c/2 + (d*x)
/2, 2))/d + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^2 + 2\left(\int \sqrt{\cos(dx+c)}dx\right)ab$$
$$+ \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b^2$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**2 + 2*int(sqrt(cos(c + d*x)),x)*
a*b + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b**2
```

3.349 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 71

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(Ab+aB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*(A*a-B*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*(A*b+B*a)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2*a*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left((-aA+bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(Ab+aB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{aA\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

input
```
Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x
]
```

output
```
(2*((-(a*A) + b*B)*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*EllipticF[(c +
d*x)/2, 2] + (a*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$2\int \frac{Ab+aB-(aA-bB)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Ab+aB-(aA-bB)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB+(bB-aA)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$(aB+Ab)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-(aA-bB)\int \sqrt{\cos(c+dx)}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$(aB+Ab)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(aA-bB)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$(aB+Ab)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(aB+Ab)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c
 + d*x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(72) = 144$.

Time = 6.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.46

method	result
default	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2Ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(}$
parts	$\frac{2(Ab+Ba)\,\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d}-\frac{2Aa\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a-A*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.61

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\,Aa\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}(-i\,Ba-i\,Ab)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
(2*A*a*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-I*B*a - I*A*b)*cos(d*x
+ c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(
I*B*a + I*A*b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + sqrt(2)*(-I*A*a + I*B*b)*cos(d*x + c)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A
*a - I*B*b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 24.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,A\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

output
```
(2*A*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*B*b*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeo
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2
)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=2\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)ab+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^2+\left(\int \sqrt{\cos(dx+c)}dx\right)b^2$$

input

```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output

```
2*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a*b + int(sqrt(cos(c + d*x))/cos(
c + d*x)**2,x)*a**2 + int(sqrt(cos(c + d*x)),x)*b**2
```

3.350 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 103

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA+3bB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\\&\quad+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
-2*(A*b+B*a)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a+3*B*b)*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*a*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)+
2*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2\left(-3(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(aA+3bB)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\right.}{3d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x
]
```

output
```
(2*(-3*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (a*A + 3
*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin[c + d*x] +
3*a*B*Sin[c + d*x] + a*A*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3447}$$

$$\int \frac{(aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{2}{3}\int \frac{3(Ab+aB)+(aA+3bB)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{3(Ab+aB)+(aA+3bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3(Ab+aB)+(aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(3(aB+Ab)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+(aA+3bB)\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3(aB+Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+(aA+3bB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{3}\left((aA+3bB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3(aB+Ab)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left((aA+3bB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3(aB+Ab)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left((aA+3bB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3(aB+Ab)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2(aA+3bB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3(aB+Ab)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((2*(a*A + 3*b*B)*Elliptic
F[(c + d*x)/2, 2])/d + 3*(A*b + a*B)*((-2*EllipticE[(c + d*x)/2, 2])/d + (
2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(98) = 196$.

Time = 6.78 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.89

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}+\frac{2(Ab+Ba)\sqrt{-2\sin\cdots}}{\cdots}\right.}{\cdots}$
parts	$-\frac{2(Ab+Ba)\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{dx}{2}\cdots\right.}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
(A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.07

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-i\,Aa-3i\,Bb)\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(i\,Aa+3}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
1/3*(sqrt(2)*(-I*A*a - 3*I*B*b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a + 3*I*B*b)*cos(d*x + c)^2*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B
*a + I*A*b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x +
c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))) + 2*(A*a + 3*(B*a + A*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,B\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,A\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

output
```
(2*B*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1
/2)) + (2*A*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeo
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2
)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b^2+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a^2 + 2\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)ab$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b**2 + int(sqrt(cos(c + d*x))/cos(c
 + d*x)**3,x)*a**2 + 2*int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a*b
```

3.351 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 140

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= -\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\\&\quad+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aA+5bB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
-2/5*(3*A*a+5*B*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*I
nverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/5*a*A*sin(d*x+c)/d/cos(d*x+c)^(5
/2)+2/3*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*A*a+5*B*b)*sin(d*x+
c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-6(3aA+5bB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(Ab+aB)\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10}{15d\cos^{\frac{3}{2}}(c}$$

input
```
Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x
]
```

output
```
(-6*(3*a*A + 5*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(A*b
 + a*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x]
 + 10*a*B*Sin[c + d*x] + 9*a*A*Sin[2*(c + d*x)] + 15*b*B*Sin[2*(c + d*x)]
+ 6*a*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3447, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \text{3447}$$
$$\int \frac{(aB+Ab)\cos(c+dx)+aA+bB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aB+Ab)\sin\left(c+dx+\frac{\pi}{2}\right)+aA+bB\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{2}{5}\int \frac{5(Ab+aB)+(3aA+5bB)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5(Ab+aB)+(3aA+5bB)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{5(Ab+aB)+(3aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(5(aB+Ab)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+(3aA+5bB)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5(aB+Ab)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+(3aA+5bB)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+(3aA+5bB)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx}\right.\right.$$
$$\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+(3aA+5bB)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(\right.}\right.\right.$$
$$\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+(3aA+5bB)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+\right.}{d}\right.\right.$$
$$\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+(3aA+5bB)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+\right.}{d}\right.\right.$$
$$\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (5*(A*b + a*B)*((2*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))) + (
3*a*A + 5*b*B)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqr
t[Cos[c + d*x]])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(127) = 254$.

Time = 8.74 (sec) , antiderivative size = 636, normalized size of antiderivative = 4.54

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2(Ab+Ba)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos(}}{3\sqrt{-2\sin(}}\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-
1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*A*a/(8*sin(1/2*d*x+1/2*c)^6-
12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*B*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.68

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{5\sqrt{2}(i\,Ba+i\,Ab)\cos{(dx+c)}^3\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-i\,B}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
-1/15*(5*sqrt(2)*(I*B*a + I*A*b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^
3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3
*I*A*a + 5*I*B*b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a - 5*I*B*b)*
cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*(3*(3*A*a + 5*B*b)*cos(d*x + c)^2 + 3*A*a + 5*(
B*a + A*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{2\,A\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{2\,B\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

output
```
(2*A*a*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos
(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*b*sin(c + d*x)*hypergeom([-
3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^
(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/
(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b*sin(c + d*x)*hype
rgeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*
x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^4}dx\right)a^2+2\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^3}dx\right)ab$$
$$+\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)b^2$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a**2 + 2*int(sqrt(cos(c + d*x))/
cos(c + d*x)**3,x)*a*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*b**2
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 264

$$\begin{aligned}
&\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx \\
&= \frac{2(9a^2A+7Ab^2+14abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} \\
&\quad+\frac{10(9b^2B+11a(2Ab+aB))\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d} \\
&\quad+\frac{10(9b^2B+11a(2Ab+aB))\sqrt{\cos(c+dx)}\sin(c+dx)}{231d} \\
&\quad+\frac{2(9a^2A+7Ab^2+14abB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\
&\quad+\frac{2(9b^2B+11a(2Ab+aB))\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{77d} \\
&\quad+\frac{2b(11Ab+13aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} \\
&\quad+\frac{2bB\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{11d}
\end{aligned}$$

output
```
2/15*(9*A*a^2+7*A*b^2+14*B*a*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10
/231*(9*b^2*B+11*a*(2*A*b+B*a))*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+1
0/231*(9*b^2*B+11*a*(2*A*b+B*a))*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/45*(9*A*a
^2+7*A*b^2+14*B*a*b)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/77*(9*b^2*B+11*a*(2*A
*b+B*a))*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/99*b*(11*A*b+13*B*a)*cos(d*x+c)^(
7/2)*sin(d*x+c)/d+2/11*b*B*cos(d*x+c)^(7/2)*(a+b*cos(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{3696(9a^2A+7Ab^2+14abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+1200(22aAb+11a^2B+9b^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x
]
```

output
```
(3696*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2] + 1200*(22*
a*A*b + 11*a^2*B + 9*b^2*B)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x
]]*(154*(36*a^2*A + 43*A*b^2 + 86*a*b*B)*Cos[c + d*x] + 180*(22*a*A*b + 11
*a^2*B + 16*b^2*B)*Cos[2*(c + d*x)] + 770*b*(A*b + 2*a*B)*Cos[3*(c + d*x)]
 + 15*(1144*a*A*b + 572*a^2*B + 531*b^2*B + 21*b^2*B*Cos[4*(c + d*x)]))*Si
n[c + d*x])/(27720*d)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.88, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2}{11}\int \frac{1}{2}\cos^{\frac{5}{2}}(c+dx)\left(b(11Ab+13aB)\cos^2(c+dx)+\left(9Bb^2+11a(2Ab+aB)\right)\cos(c+dx)+a(11aA+7bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \cos^{\frac{5}{2}}(c+dx)\left(b(11Ab+13aB)\cos^2(c+dx)+\left(9Bb^2+11a(2Ab+aB)\right)\cos(c+dx)+a(11aA+7bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b(11Ab+13aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(9Bb^2+11a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)+\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{11}\left(\frac{2}{9}\int \frac{1}{2}\cos^{\frac{5}{2}}(c+dx)\left(11\left(9Aa^2+14bBa+7Ab^2\right)+9\left(9Bb^2+11a(2Ab+aB)\right)\cos(c+dx)\right)dx+\frac{2b(13aB+\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \cos^{\frac{5}{2}}(c+dx)\left(11\left(9Aa^2+14bBa+7Ab^2\right)+9\left(9Bb^2+11a(2Ab+aB)\right)\cos(c+dx)\right)dx+\frac{2b(13aB+1\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(11\left(9Aa^2+14bBa+7Ab^2\right)+9\left(9Bb^2+11a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3227

$$\frac{1}{11}\left(\frac{1}{9}\left(11\left(9a^2A+14abB+7Ab^2\right)\int\cos^{\frac{5}{2}}(c+dx)dx+9\left(11a(aB+2Ab)+9b^2B\right)\int\cos^{\frac{7}{2}}(c+dx)dx\right)+\frac{2b(13\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(11\left(9a^2A+14abB+7Ab^2\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+9\left(11a(aB+2Ab)+9b^2B\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{7}\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3115

$$\frac{1}{11}\left(\frac{1}{9}\left(11\left(9a^2A+14abB+7Ab^2\right)\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9\left(11a(aB+2Ab)+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(11\left(9a^2A+14abB+7Ab^2\right)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9\left(11a(aB+2\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3115

$$\frac{1}{11}\left(\frac{1}{9}\left(11\left(9a^2A+14abB+7Ab^2\right)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9\left(11a(aB+2\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(11\left(9a^2A+14abB+7Ab^2\right)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9(11a(aB+\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3119

$$\frac{1}{11}\left(\frac{1}{9}\left(9\left(11a(aB+2Ab)+9b^2B\right)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

$\downarrow$ 3120

$$\frac{1}{11}\left(\frac{1}{9}\left(11\left(9a^2A+14abB+7Ab^2\right)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9(11a(aB+2Ab)+9b\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))}{11d}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(11*d) + ((2*
b*(11*A*b + 13*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (11*(9*a^2*A
+ 7*A*b^2 + 14*a*b*B)*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)) + 9*(9*b^2*B + 11*a*(2*A*b + a*B))*((2*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*EllipticF[(c + d*x)/2, 2])/(3*d
) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(243) = 486$.

Time = 57.68 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.52

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(20160B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}b^{2}+\left(-12320A\,b^{2}-24640Bab-50400B\,b^{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*b^2+(-12320*A*b^2-24640*B*a*b-5040
0*B*b^2)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(15840*A*a*b+24640*A*b^2
+7920*B*a^2+49280*B*a*b+56880*B*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*
c)+(-5544*A*a^2-23760*A*a*b-22792*A*b^2-11880*B*a^2-45584*B*a*b-34920*B*b^
2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(5544*A*a^2+18480*A*a*b+10472*A
*b^2+9240*B*a^2+20944*B*a*b+13860*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+(-1386*A*a^2-5280*A*a*b-1848*A*b^2-2640*B*a^2-3696*B*a*b-2790*B*b^2
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1650*A*a*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*b^2+825*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675*B*b^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-3234*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.13

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(315\,Bb^2\cos\left(dx+c\right)^4+385\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^3+825\,Ba^2+1650\,Aab+675\,Bb^2+45\,(11\,Ba\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
1/3465*(2*(315*B*b^2*cos(d*x + c)^4 + 385*(2*B*a*b + A*b^2)*cos(d*x + c)^3
 + 825*B*a^2 + 1650*A*a*b + 675*B*b^2 + 45*(11*B*a^2 + 22*A*a*b + 9*B*b^2)
*cos(d*x + c)^2 + 77*(9*A*a^2 + 14*B*a*b + 7*A*b^2)*cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c) - 75*sqrt(2)*(11*I*B*a^2 + 22*I*A*a*b + 9*I*B*b^2)
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 75*sqrt(2)*(-
11*I*B*a^2 - 22*I*A*a*b - 9*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 231*sqrt(2)*(-9*I*A*a^2 - 14*I*B*a*b - 7*I*A*b^2)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 231*sqrt(2)*(9*I*A*a^2 + 14*I*B*a*b + 7*I*A*b^2)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2),
x)
```

Mupad [B] (verification not implemented)

Time = 24.77 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx \\
&= -\frac{2\,A\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&\quad-\frac{2\,B\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&\quad-\frac{2\,A\,b^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&\quad-\frac{2\,B\,b^2\cos\left(c+d\,x\right)^{13/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{4};\ \frac{17}{4};\ \cos\left(c+d\,x\right)^2\right)}{13\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&\quad-\frac{4\,A\,a\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&\quad-\frac{4\,B\,a\,b\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}
\end{aligned}$$

input
```
int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

output
```
- (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos
(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(9/2)*s
in(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x
)^2)^(1/2)) - (2*A*b^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11
/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^2*cos(c
 + d*x)^(13/2)*sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))/
(13*d*(sin(c + d*x)^2)^(1/2)) - (4*A*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*h
ypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) -
 (4*B*a*b*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, co
s(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\left(\int \sqrt{\cos (dx+c)}\,\cos (dx+c)^5\,dx\right)b^3+3\left(\int \sqrt{\cos (dx+c)}\,\cos (dx+c)^4\,dx\right)a\,b^2$$
$$+3\left(\int \sqrt{\cos (dx+c)}\,\cos (dx+c)^3\,dx\right)a^2b+\left(\int \sqrt{\cos (dx+c)}\,\cos (dx+c)^2\,dx\right)a^3$$

input
```
int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**5,x)*b**3 + 3*int(sqrt(cos(c + d*x))*
cos(c + d*x)**4,x)*a*b**2 + 3*int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*a*
*2*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a**3
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 223

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx
&= \frac{2(7b^2B+9a(2Ab+aB))\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} \\
&+ \frac{2(7a^2A+5Ab^2+10abB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\
&+ \frac{2(7a^2A+5Ab^2+10abB)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\
&+ \frac{2(7b^2B+9a(2Ab+aB))\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\
&+ \frac{2b(9Ab+11aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} \\
&+ \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{9d}
\end{aligned}$$

output
```
2/15*(7*b^2*B+9*a*(2*A*b+B*a))*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/2
1*(7*A*a^2+5*A*b^2+10*B*a*b)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/21
*(7*A*a^2+5*A*b^2+10*B*a*b)*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/45*(7*b^2*B+9*
a*(2*A*b+B*a))*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/63*b*(9*A*b+11*B*a)*cos(d*x
+c)^(5/2)*sin(d*x+c)/d+2/9*b*B*cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*sin(d*x+c
)/d
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{84(18aAb+9a^2B+7b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+60(7a^2A+5Ab^2+10abB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{c}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x
]
```

output
```
(84*(18*a*A*b + 9*a^2*B + 7*b^2*B)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^2*A
 + 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(
72*a*A*b + 36*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(84*a^2*A + 78*A*b^2 + 15
6*a*b*B + 18*b*(A*b + 2*a*B)*Cos[2*(c + d*x)] + 7*b^2*B*Cos[3*(c + d*x)]))
*Sin[c + d*x])/(630*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3469$$

$$\frac{2}{9}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\left(b(9Ab+11aB)\cos^2(c+dx)+\left(7Bb^2+9a(2Ab+aB)\right)\cos(c+dx)+a(9aA+5bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(b(9Ab+11aB)\cos^2(c+dx)+\left(7Bb^2+9a(2Ab+aB)\right)\cos(c+dx)+a(9aA+5bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b(9Ab+11aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Bb^2+9a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(9\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\left(9\left(7Aa^2+10bBa+5Ab^2\right)+7\left(7Bb^2+9a(2Ab+aB)\right)\cos(c+dx)\right)dx+\frac{2b(11aB+9A\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{1}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(9\left(7Aa^2+10bBa+5Ab^2\right)+7\left(7Bb^2+9a(2Ab+aB)\right)\cos(c+dx)\right)dx+\frac{2b(11aB+9Ab)\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9\left(7Aa^2+10bBa+5Ab^2\right)+7\left(7Bb^2+9a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2b}{}\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9\left(7a^2A+10abB+5Ab^2\right)\int \cos^{\frac{3}{2}}(c+dx)dx+7\left(9a(aB+2Ab)+7b^2B\right)\int \cos^{\frac{5}{2}}(c+dx)dx\right)+\frac{2b(11aB}{}\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9\left(7a^2A+10abB+5Ab^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+7\left(9a(aB+2Ab)+7b^2B\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}d\right.\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9\left(7a^2A+10abB+5Ab^2\right)\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7\left(9a(aB+2Ab)+7\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9\left(7a^2A+10abB+5Ab^2\right)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7\left(9a(aB+2A\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9\left(7a^2A+10abB+5Ab^2\right)\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7\left(9a(aB+2A\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$\big\downarrow$ 3120

$$\frac{1}{9}\left(\frac{1}{7}\left(9\left(7a^2A+10abB+5Ab^2\right)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7(9a(aB+2Ab)\right.\right.$$

$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d) + ((2*b
*(9*A*b + 11*a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (9*(7*a^2*A + 5
*A*b^2 + 10*a*b*B)*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(3*d)) + 7*(7*b^2*B + 9*a*(2*A*b + a*B))*((6*EllipticE
[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(206) = 412$.

Time = 20.77 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.74

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}b^{2}+\left(720A\,b^{2}+1440Bab+2240B\,b^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cdots\right.}{\cdots}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^2+(720*A*b^2+1440*B*a*b+2240*B*b^
2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1008*A*a*b-1080*A*b^2-504*B*a
^2-2160*B*a*b-2072*B*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a
^2+1008*A*a*b+840*A*b^2+504*B*a^2+1680*B*a*b+952*B*b^2)*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+(-210*A*a^2-252*A*a*b-240*A*b^2-126*B*a^2-480*B*a*b-
168*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^2*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+75*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b+150*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.22

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(35\,Bb^2\cos\left(dx+c\right)^3+105\,Aa^2+150\,Bab+75\,Ab^2+45\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+7\left(9\,Ba^2+18\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
1/315*(2*(35*B*b^2*cos(d*x + c)^3 + 105*A*a^2 + 150*B*a*b + 75*A*b^2 + 45*
(2*B*a*b + A*b^2)*cos(d*x + c)^2 + 7*(9*B*a^2 + 18*A*a*b + 7*B*b^2)*cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(7*I*A*a^2 + 10*I*B*a
*b + 5*I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
- 15*sqrt(2)*(-7*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*B*a^2 - 18*I*A*a*b -
 7*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*B*a^2 + 18*I*A*a*b + 7*I*B*b^2)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 25.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.18

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$=\frac{2\,A\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}$$
$$-\frac{2\,B\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$
$$-\frac{2\,A\,b^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$
$$-\frac{2\,B\,b^2\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$
$$-\frac{4\,A\,a\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$
$$-\frac{4\,B\,a\,b\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

output
```
(2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/
(3*d) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b^2*cos(c + d*x)^(
9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c
 + d*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1
/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (4*A*a*b
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^
2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a*b*cos(c + d*x)^(9/2)*sin(c + d*x
)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)
)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^3 + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a\,b^2 + 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a^2b$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a**3 + int(sqrt(cos(c + d*x))*cos(c
 + d*x)**4,x)*b**3 + 3*int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*a*b**2 +
3*int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a**2*b
```

3.354 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 182

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx\\ =\frac{2(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}\\ +\frac{2(5b^2B+7a(2Ab+aB))\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ +\frac{2(5b^2B+7a(2Ab+aB))\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\ +\frac{2b(7Ab+9aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d}\\ +\frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21
*(5*b^2*B+7*a*(2*A*b+B*a))*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/21*(
5*b^2*B+7*a*(2*A*b+B*a))*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/35*b*(7*A*b+9*B*a
)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*
sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{42(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(14aAb+7a^2B+5b^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos}}{10}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x
]
```

output
```
(42*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + 10*(14*a*A*b
 + 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*b
*(A*b + 2*a*B)*Cos[c + d*x] + 5*(28*a*A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*
Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\cos(c+dx)}\left(b(7Ab+9aB)\cos^2(c+dx)+\left(5Bb^2+7a(2Ab+aB)\right)\cos(c+dx)+a(7aA+3bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \sqrt{\cos(c+dx)}\left(b(7Ab+9aB)\cos^2(c+dx)+\left(5Bb^2+7a(2Ab+aB)\right)\cos(c+dx)+a(7aA+3bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(7Ab+9aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Bb^2+7a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(7a\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{\cos(c+dx)}\left(7\left(5Aa^2+6bBa+3Ab^2\right)+5\left(5Bb^2+7a(2Ab+aB)\right)\cos(c+dx)\right)dx+\frac{2b(9aB+7Ab)}{}\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\cos(c+dx)}\left(7\left(5Aa^2+6bBa+3Ab^2\right)+5\left(5Bb^2+7a(2Ab+aB)\right)\cos(c+dx)\right)dx+\frac{2b(9aB+7Ab)\,\mathrm{si}}{}\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(7\left(5Aa^2+6bBa+3Ab^2\right)+5\left(5Bb^2+7a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2b(9}{}\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \; 3227$$

$$\frac{1}{7}\left(\frac{1}{5}\left(7\left(5a^2A+6abB+3Ab^2\right)\int \sqrt{\cos(c+dx)}dx+5\left(7a(aB+2Ab)+5b^2B\right)\int \cos^{\frac{3}{2}}(c+dx)dx\right)+\frac{2b(9aB+}{}\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(7\left(5a^2A+6abB+3Ab^2\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5\left(7a(aB+2Ab)+5b^2B\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(7\left(5a^2A+6abB+3Ab^2\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5\left(7a(aB+2Ab)+5b^2B\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(7\left(5a^2A+6abB+3Ab^2\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5\left(7a(aB+2Ab)+5b^2B\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5\left(7a(aB+2Ab)+5b^2B\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{14\left(5a^2A+6ab\right.}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{14\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5\left(7a(aB+2Ab)+5b^2B\right)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\,\text{s}}{}\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d) + ((2*b
*(7*A*b + 9*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + ((14*(5*a^2*A +
3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + 5*(5*b^2*B + 7*a*(2*A*b
+ a*B))*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d)))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(169) = 338$.

Time = 11.41 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.01

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\Big(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8 b^2+\left(-168A\,b^2-336Bab-360B\,b^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos(}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^2+(-168*A*b^2-336*B*a*b-360*B*b^2)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*
B*a*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*
b^2-70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70
*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*b^2+35*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*b^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.34

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(15\,Bb^2\cos\left(dx+c\right)^2+35\,Ba^2+70\,Aab+25\,Bb^2+21\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
1/105*(2*(15*B*b^2*cos(d*x + c)^2 + 35*B*a^2 + 70*A*a*b + 25*B*b^2 + 21*(2
*B*a*b + A*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*
(7*I*B*a^2 + 14*I*A*a*b + 5*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*B*a^2 - 14*I*A*a*b - 5*I*B*b^2)*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-5*I*
A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(5*I*A*a^2 + 6*I*B*a*b
 + 3*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 25.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.26

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2\,B\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d} + \frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,a\,b\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d} - \frac{2\,A\,b^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} - \frac{2\,B\,b^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} - \frac{4\,B\,a\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

output
```
(2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/
(3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*b*((2*cos(c + d*x
)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*b^2
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^
2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*x)^(9/2)*sin(c + d*x
)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)
) - (4*B*a*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, c
os(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos(dx+c)}dx\right)a^3+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^2b$$
$$+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b^3$$
$$+3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a\,b^2$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)),x)*a**3 + 3*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*
a**2*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b**3 + 3*int(sqrt(cos(c
 + d*x))*cos(c + d*x)**2,x)*a*b**2
```

3.355 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 140

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2(3b^2B+5a(2Ab+aB))\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2(3a^2A+Ab^2+2abB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2b(5Ab+7aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2bB\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)}{5d}$$

output

```
2/5*(3*b^2*B+5*a*(2*A*b+B*a))*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*
(3*A*a^2+A*b^2+2*B*a*b)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/15*b*(5
*A*b+7*B*a)*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*b*B*cos(d*x+c)^(1/2)*(a+b*co
s(d*x+c))*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\left(3(10aAb+5a^2B+3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(3a^2A+Ab^2+2abB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b\sqrt{\cos}\right.}{15d}$$

input
```
Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]]
,x]
```

output
```
(2*(3*(10*a*A*b + 5*a^2*B + 3*b^2*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*
A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(5*A
*b + 10*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3469, 27, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3469$$

$$\frac{2}{5}\int \frac{b(5Ab+7aB)\cos^2(c+dx)+\left(3Bb^2+5a(2Ab+aB)\right)\cos(c+dx)+a(5aA+bB)}{2\sqrt{\cos(c+dx)}}dx+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{b(5Ab+7aB)\cos^2(c+dx)+\left(3Bb^2+5a(2Ab+aB)\right)\cos(c+dx)+a(5aA+bB)}{\sqrt{\cos(c+dx)}}dx+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{b(5Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Bb^2+5a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5aA+bB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{5\left(3Aa^2+2bBa+Ab^2\right)+3\left(3Bb^2+5a(2Ab+aB)\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2b(7aB+5Ab)\sin(c+dx)\sqrt{\cos(c}}{3d}\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{5\left(3Aa^2+2bBa+Ab^2\right)+3\left(3Bb^2+5a(2Ab+aB)\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2b(7aB+5Ab)\sin(c+dx)\sqrt{\cos(c}}{3d}\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{5\left(3Aa^2+2bBa+Ab^2\right)+3\left(3Bb^2+5a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b(7aB+5Ab)\sin(c+dx)\sqrt{}}{3d}\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(3a^2A+2abB+Ab^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3\left(5a(aB+2Ab)+3b^2B\right)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2b(7aB\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(3a^2A+2abB+Ab^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3\left(5a(aB+2Ab)+3b^2B\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$\Big\downarrow$ 3119

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(3a^2A+2abB+Ab^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6\left(5a(aB+2Ab)+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b(7a\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$\Big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{10\left(3a^2A+2abB+Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6\left(5a(aB+2Ab)+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b(7a\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

input
```
Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) + (((6*
(3*b^2*B + 5*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/d + (10*(3*a^2*A
+ A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/d)/3 + (2*b*(5*A*b + 7*a*B)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(131) = 262$.

Time = 11.00 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.48

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}b^{2}+\left(20A\,b^{2}+40Bab+24B\,b^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}{}$
parts	$-\frac{2\left(A\,b^{2}+2Bab\right)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\left(3\,Bb^2\cos\left(dx+c\right)+10\,Bab+5\,Ab^2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\,\sqrt{2}(3i\,Aa^2+2i\,Bab+i\,Ab^2)\text{weier}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
1/15*(2*(3*B*b^2*cos(d*x + c) + 10*B*a*b + 5*A*b^2)*sqrt(cos(d*x + c))*sin
(d*x + c) - 5*sqrt(2)*(3*I*A*a^2 + 2*I*B*a*b + I*A*b^2)*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-3*I*A*a^2 - 2*I*B*a*
b - I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3
*sqrt(2)*(-5*I*B*a^2 - 10*I*A*a*b - 3*I*B*b^2)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(5*I*B*
a^2 + 10*I*A*a*b + 3*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 25.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{A\,b^2\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d} + \frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,B\,a\,b\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d} + \frac{4\,A\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} - \frac{2\,B\,b^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

output
```
(A*b^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2
, 2))/3))/d + (2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticE
(c/2 + (d*x)/2, 2))/d + (2*B*a*b*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 +
(2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (4*A*a*b*ellipticE(c/2 + (d*x)/2,
2))/d - (2*B*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^3 + 3\left(\int \sqrt{\cos(dx+c)}dx\right)a^2b$$
$$+ 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a\,b^2 + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b^3$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**3 + 3*int(sqrt(cos(c + d*x)),x)*
a**2*b + 3*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a*b**2 + int(sqrt(cos(c
+ d*x))*cos(c + d*x)**2,x)*b**3
```

3.356 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 121

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2(a^2A-Ab^2-2abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(6aAb+3a^2B+b^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
-2*(A*a^2-A*b^2-2*B*a*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(6*A*
a*b+3*B*a^2+B*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2*a^2*A*sin(d*
x+c)/d/cos(d*x+c)^(1/2)+2/3*b^2*B*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{2\Big((-3a^2A+3Ab^2+6abB)\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+(6aAb+3a^2B+b^2B)\,\text{EllipticF}\left(\tfrac{1}{2}(c+dx),2\right)+\frac{(3a^2A+b^2}{}}{3d}$$

input
```
Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2)
,x]
```

output
```
(2*((-3*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b +
3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + ((3*a^2*A + b^2*B*Cos[c + d*x
])*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3467, 27, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 3467}$$
$$\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-2\int -\frac{b^2B\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(2Ab+aB)}{2\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{b^2B\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(2Ab+aB)}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-Aa^2+2bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(2Ab+aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{2}{3}\int \frac{3Ba^2+6Aba+b^2B-3\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3Ba^2+6Aba+b^2B-3\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3Ba^2+6Aba+b^2B-3\left(Aa^2-2bBa-Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(\left(3a^2B+6aAb+b^2B\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3\left(a^2A-2abB-Ab^2\right)\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2B+6aAb+b^2B\right)\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx-3\left(a^2A-2abB-Ab^2\right)\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left(\left(3a^2B+6aAb+b^2B\right)\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2\left(3a^2B+6aAb+b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

input

```
Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output

```
((-6*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b
+ 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/d)/3 + (2*a^2*A*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(116) = 232$.

Time = 7.98 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.35

method	result
default	$\frac{-\frac{8B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b^{2}}{3}+4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a^{2}-4Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
parts	$\frac{2\left(A\,b^{2}+2Bab\right)\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}+\frac{2(2A}{}$

input

```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output

```
2/3*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2+6*A*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*a^2-6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+2*B*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2*b^2-3*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*b^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.98

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(-3i\,Ba^2-6i\,Aab-i\,Bb^2)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(-3*I*B*a^2 - 6*I*A*a*b - I*B*b^2)*cos(d*x + c)*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*B*a^2 + 6*I*A*
a*b + I*B*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 3*sqrt(2)*(I*A*a^2 - 2*I*B*a*b - I*A*b^2)*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) - 3*sqrt(2)*(-I*A*a^2 + 2*I*B*a*b + I*A*b^2)*cos(d*x + c)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(
B*b^2*cos(d*x + c) + 3*A*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 25.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{B\,b^2\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

output
```
(B*b^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2
, 2))/3))/d + (2*A*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticF
(c/2 + (d*x)/2, 2))/d + (4*A*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (4*B*a*b
*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = 3\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)a^2b+\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)a^3+3\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)a\,b^2+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)b^3$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
3*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**2*b + int(sqrt(cos(c + d*x))/c
os(c + d*x)**2,x)*a**3 + 3*int(sqrt(cos(c + d*x)),x)*a*b**2 + int(sqrt(cos
(c + d*x))*cos(c + d*x),x)*b**3
```

3.357 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 126

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{2(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} \\ &\quad+\frac{2(a^2A+3Ab^2+6abB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\ &\quad+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(2Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

output

```
-2*(2*A*a*b+B*a^2-B*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a^
2+3*A*b^2+6*B*a*b)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*a^2*A*sin(
d*x+c)/d/cos(d*x+c)^(3/2)+2*a*(2*A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(-3(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^2A+3Ab^2+6abB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{a(aA+3(2A\cdots}{\cdots}\right)}{3d}$$

input
```
Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2)
,x]
```

output
```
(2*(-3*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*
b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c
 + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3467, 27, 3042, 3500, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 3467}$$

$$\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int -\frac{3b^2B\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(2Ab+aB)}{2\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3b^2B\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(2Ab+aB)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(2Ab+aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\left(2\int \frac{Aa^2+6bBa+3Ab^2+3\left(b^2B-a(2Ab+aB)\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{6a(aB+2Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \frac{Aa^2+6bBa+3Ab^2+3\left(b^2B-a(2Ab+aB)\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{6a(aB+2Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{Aa^2+6bBa+3Ab^2+3\left(b^2B-a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a(aB+2Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(\left(a^2A+6abB+3Ab^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3\left(b^2B-a(aB+2Ab)\right)\int \sqrt{\cos(c+dx)}dx+\frac{6a(aB+2Ab)\sin}{d\sqrt{\cos(c+}}\right.$$
$$\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\left(a^2A+6abB+3Ab^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3\left(b^2B-a(aB+2Ab)\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a(aB}{d}\right.$$
$$\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{3}\left(\left(a^2A+6abB+3Ab^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6\left(b^2B-a(aB+2Ab)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a(aB+2Ab)}{d\sqrt{\cos(c}}\right.$$
$$\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2\left(a^2A+6abB+3Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6\left(b^2B-a(aB+2Ab)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a(aB+2Ab)}{d\sqrt{\cos(c}}\right.$$
$$\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a^2*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((6*(b^2*B - a*(2*A*b +
a*B))*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Ellipt
icF[(c + d*x)/2, 2])/d + (6*a*(2*A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(121) = 242$.

Time = 8.03 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.56

method	result
parts	$\frac{2(A b^2+2Bab)\,\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d} - \frac{2(2Aab+a^2B)\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\cdots}}$
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2Ab^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}+\frac{2a(2Ab+Ba)\sqrt{-2\cdots}}{\cdots}\cdots\right.}{\cdots}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
2*(A*b^2+2*B*a*b)/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))-2*(2*A*a*b+B*a^
2)*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d+2*B*b^2*(
(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2/3*a^2*A*(-2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.02

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-i\,Aa^2-6i\,Bab-3i\,Ab^2)\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(-I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*cos(d*x + c)^2*weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^2 + 6*I*B*
a*b + 3*I*A*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a^2 + 2*I*A*a*b - I*B*b^2)*cos(d*x + c)^2
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 3*sqrt(2)*(-I*B*a^2 - 2*I*A*a*b + I*B*b^2)*cos(d*x + c)^2*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) + 2*(A*a^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2),
x)
```

Mupad [B] (verification not implemented)

Time = 26.36 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,A\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,B\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,B\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}} + \frac{2\,B\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}} + \frac{4\,A\,a\,b\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(5/2),x)
```

output
```
(2*A*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*b^2*ellipticE(c/2 + (d*x)/2
, 2))/d + (4*B*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*
hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(
c + d*x)^2)^(1/2)) + (2*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos
(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (4*A*a*b*sin
(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/
2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = 3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a\,b^2+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a^3 + 3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^2b+\left(\int \sqrt{\cos(dx+c)}dx\right)b^3$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
3*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a*b**2 + int(sqrt(cos(c + d*x))/c
os(c + d*x)**3,x)*a**3 + 3*int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a**2*
b + int(sqrt(cos(c + d*x)),x)*b**3
```

3.358 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 172

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2(3a^2A+5Ab^2+10abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2(2aAb+a^2B+3b^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(3a^2A+5Ab^2+10abB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output

```
-2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/
3*(2*A*a*b+B*a^2+3*B*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/5*a^2
*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/3*a*(2*A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)
^(3/2)+2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-6(3a^2A+5Ab^2+10abB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(2aAb+a^2B+3b^2B)\cos^{\frac{3}{2}}(c+dx)\,\text{Ellipt}}{}$$

input
```
Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2)
,x]
```

output
```
(-6*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/
2, 2] + 10*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d
*x)/2, 2] + 20*a*A*b*Sin[c + d*x] + 10*a^2*B*Sin[c + d*x] + 9*a^2*A*Sin[2*
(c + d*x)] + 15*A*b^2*Sin[2*(c + d*x)] + 30*a*b*B*Sin[2*(c + d*x)] + 6*a^2
*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3467, 27, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5b^2B\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+5a(2Ab+aB)}{2\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{5b^2B\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+5a(2Ab+aB)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{5b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+10bBa+5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+5a(2Ab+aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3\left(3Aa^2+10bBa+5Ab^2\right)+5\left(Ba^2+2Aba+3b^2B\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{10a(aB+2Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(3Aa^2+10bBa+5Ab^2\right)+5\left(Ba^2+2Aba+3b^2B\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{10a(aB+2Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(3Aa^2+10bBa+5Ab^2\right)+5\left(Ba^2+2Aba+3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{10a(aB+2Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^2A+10abB+5Ab^2\right)\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+5\left(a^2B+2aAb+3b^2B\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{10a(aB\right.$$
$$\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^2A+10abB+5Ab^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+5\left(a^2B+2aAb+3b^2B\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)\right.$$
$$\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3116

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^2B+2aAb+3b^2B\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3\left(3a^2A+10abB+5Ab^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\mathrm{co}\right.\right.\right.$$
$$\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^2B+2aAb+3b^2B\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3\left(3a^2A+10abB+5Ab^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\mathrm{si}\right.\right.\right.$$
$$\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^2B+2aAb+3b^2B\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3\left(3a^2A+10abB+5Ab^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}\right.}{}\right.\right.\right.$$
$$\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{10(a^2B+2aAb+3b^2B)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3(3a^2A+10abB+5Ab^2)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}\right.}{}\right.\right.\right.$$

$$\frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a^2*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((10*a*(2*A*b + a*B)*Sin
[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((10*(2*a*A*b + a^2*B + 3*b^2*B)*Ell
ipticF[(c + d*x)/2, 2])/d + 3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*((-2*Elliptic
E[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(159) = 318$.

Time = 9.19 (sec) , antiderivative size = 723, normalized size of antiderivative = 4.20

method	result	size
default	Expression too large to display	723
parts	Expression too large to display	799

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*a*(2*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*a^2*A/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(
1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(A*b+2*B*a)/sin(1/2*d*x+1/2*c)^2/(2*si
n(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{5\sqrt{2}(i\,Ba^2+2i\,Aab+3i\,Bb^2)\cos{(dx+c)}^3\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-1/15*(5*sqrt(2)*(I*B*a^2 + 2*I*A*a*b + 3*I*B*b^2)*cos(d*x + c)^3*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a^2 -
2*I*A*a*b - 3*I*B*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a^2 + 10*I*B*a*b + 5*I*A*b^2)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*cos(
d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(3*A*a^2 + 3*(3*A*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x
 + c)^2 + 5*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2),
x)
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{6\,A\,a^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\;-\frac{1}{4};\;\cos(c+d\,x)^2\right)+30\,A\,b^2\cos(c+d\,x)^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\mathrm{c}\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(\mathrm{c}}}$$
$$+\frac{2\,B\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$
$$+\frac{4\,B\,a\,b\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(7/2),x)
```

output
```
(6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*A*
b^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2
) + 20*A*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c +
 d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*B*b^2*
ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*sin(c + d*x)*hypergeom([-3/4, 1/
2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
+ (4*B*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b^3 + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a^3 + 3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a^2b + 3\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a\,b^2$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b**3 + int(sqrt(cos(c + d*x))/cos(c
 + d*x)**4,x)*a**3 + 3*int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a**2*b +
3*int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a*b**2
```

3.359 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 305

$$\begin{aligned}
&\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx \\
&= \frac{2(27a^2Ab+7Ab^3+9a^3B+21ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} \\
&+ \frac{2(77a^3A+165aAb^2+165a^2bB+45b^3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d} \\
&+ \frac{2(77a^3A+165aAb^2+165a^2bB+45b^3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{231d} \\
&+ \frac{2(27a^2Ab+7Ab^3+9a^3B+21ab^2B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\
&+ \frac{2b(33aAb+26a^2B+9b^2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{77d} \\
&+ \frac{2b^2(11Ab+15aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} \\
&+ \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{11d}
\end{aligned}$$

output
```
2/15*(27*A*a^2*b+7*A*b^3+9*B*a^3+21*B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))/d+2/231*(77*A*a^3+165*A*a*b^2+165*B*a^2*b+45*B*b^3)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))/d+2/231*(77*A*a^3+165*A*a*b^2+165*B*a^2*b+45*B*b
^3)*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/45*(27*A*a^2*b+7*A*b^3+9*B*a^3+21*B*a*
b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/77*b*(33*A*a*b+26*B*a^2+9*B*b^2)*cos(
d*x+c)^(5/2)*sin(d*x+c)/d+2/99*b^2*(11*A*b+15*B*a)*cos(d*x+c)^(7/2)*sin(d*
x+c)/d+2/11*b*B*cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.70 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{3696(27a^2Ab+7Ab^3+9a^3B+21ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+240(77a^3A+165aAb^2+165a^2bB+45b^3B)}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x
]
```

output
```
(3696*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2,
 2] + 240*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c +
 d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(154*(108*a^2*A*b + 43*A*b^3 + 36*a^3*B
 + 129*a*b^2*B)*Cos[c + d*x] + 180*b*(33*a*A*b + 33*a^2*B + 16*b^2*B)*Cos[
2*(c + d*x)] + 770*b^2*(A*b + 3*a*B)*Cos[3*(c + d*x)] + 15*(616*a^3*A + 17
16*a*A*b^2 + 1716*a^2*b*B + 531*b^3*B + 21*b^3*B*Cos[4*(c + d*x)]))*Sin[c
+ d*x])/(27720*d)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3469, 27, 3042, 3512, 27, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2}{11}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(b(11Ab+15aB)\cos^2(c+dx)+\left(9Bb^2+11a(2Ab+aB)\right)\cos(c+dx)+a(11aA+5bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\left(b(11Ab+15aB)\cos^2(c+dx)+\left(9Bb^2+11a(2Ab+aB)\right)\cos(c+dx)+a(11aA+5bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b(11Ab+15aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(9Bb^2+11a(2Ab+\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$$\Big\downarrow \text{3512}$$

$$\frac{1}{11}\left(\frac{2}{9}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\left(9(11aA+5bB)a^2+9b\left(26Ba^2+33Aba+9b^2B\right)\cos^2(c+dx)+11\left(9Ba^3+27Aba^2+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(9(11aA+5bB)a^2+9b\left(26Ba^2+33Aba+9b^2B\right)\cos^2(c+dx)+11\left(9Ba^3+27Aba^2+21\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9(11aA+5bB)a^2+9b\left(26Ba^2+33Aba+9b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+11\left(9Ba^3\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\big\downarrow$ 3502

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2}{7}\int\frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\left(9\left(77Aa^3+165bBa^2+165Ab^2a+45b^3B\right)+77\left(9Ba^3+27Aba^2+21b^2Ba+7Ab^3\right)\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{1}{7}\int\cos^{\frac{3}{2}}(c+dx)\left(9\left(77Aa^3+165bBa^2+165Ab^2a+45b^3B\right)+77\left(9Ba^3+27Aba^2+21b^2Ba+7Ab^3\right)\text{c}\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{1}{7}\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9\left(77Aa^3+165bBa^2+165Ab^2a+45b^3B\right)+77\left(9Ba^3+27Aba^2+21b^2Ba+\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\big\downarrow$ 3227

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{1}{7}\left(9\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)\int\cos^{\frac{3}{2}}(c+dx)dx+77\left(9a^3B+27a^2Ab+21ab^2B+7Ab^3\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{1}{7}\left(9\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+77\left(9a^3B+27a^2Ab+21ab^2B\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\big\downarrow$ 3115

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{1}{7}\left(77\left(9a^3B+27a^2Ab+21ab^2B+7Ab^3\right)\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9(77\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{1}{7}\left(77\left(9a^3B+27a^2Ab+21ab^2B+7Ab^3\right)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\downarrow$ 3119

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{1}{7}\left(9\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx}}{3d}\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

$\downarrow$ 3120

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{18b\left(26a^2B+33aAb+9b^2B\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{1}{7}\left(77\left(9a^3B+27a^2Ab+21ab^2B+7Ab^3\right)\left(\frac{6I}{}\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}{11d}$$

input

```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output

```
(2*b*B*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d) + ((
2*b^2*(11*A*b + 15*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + ((18*b*(3
3*a*A*b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (9*
(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*((2*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 77*(27*a^2*A*b
 + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*((6*EllipticE[(c + d*x)/2, 2])/(5*d) +
(2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/7)/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(284) = 568$.

Time = 29.32 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.70

method	result	size
default	Expression too large to display	825
parts	Expression too large to display	1063

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*b^3+(-12320*A*b^3-36960*B*a*b^2-50
400*B*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(23760*A*a*b^2+24640*A
*b^3+23760*B*a^2*b+73920*B*a*b^2+56880*B*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2
*d*x+1/2*c)+(-16632*A*a^2*b-35640*A*a*b^2-22792*A*b^3-5544*B*a^3-35640*B*a
^2*b-68376*B*a*b^2-34920*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(4
620*A*a^3+16632*A*a^2*b+27720*A*a*b^2+10472*A*b^3+5544*B*a^3+27720*B*a^2*b
+31416*B*a*b^2+13860*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310
*A*a^3-4158*A*a^2*b-7920*A*a*b^2-1848*A*b^3-1386*B*a^3-7920*B*a^2*b-5544*B
*a*b^2-2790*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1155*a^3*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+2475*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6237*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a^2*b-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+2475*B*a^2*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+675*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.17

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(315\,Bb^3\cos\left(dx+c\right)^4+1155\,Aa^3+2475\,Ba^2b+2475\,Aab^2+675\,Bb^3+385\left(3\,Bab^2+Ab^3\right)\cos\left(dx+\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
1/3465*(2*(315*B*b^3*cos(d*x + c)^4 + 1155*A*a^3 + 2475*B*a^2*b + 2475*A*a
*b^2 + 675*B*b^3 + 385*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + 135*(11*B*a^2*
b + 11*A*a*b^2 + 3*B*b^3)*cos(d*x + c)^2 + 77*(9*B*a^3 + 27*A*a^2*b + 21*B
*a*b^2 + 7*A*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(
2)*(77*I*A*a^3 + 165*I*B*a^2*b + 165*I*A*a*b^2 + 45*I*B*b^3)*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*sqrt(2)*(-77*I*A*a^3 - 1
65*I*B*a^2*b - 165*I*A*a*b^2 - 45*I*B*b^3)*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) - 231*sqrt(2)*(-9*I*B*a^3 - 27*I*A*a^2*b - 21*I
*B*a*b^2 - 7*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) - 231*sqrt(2)*(9*I*B*a^3 + 27*I*A*a^2*b + 21
*I*B*a*b^2 + 7*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{A\,a^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d} - \frac{2\,B\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,A\,b^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,b^3\cos\left(c+d\,x\right)^{13/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{4};\ \frac{17}{4};\ \cos\left(c+d\,x\right)^2\right)}{13\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{6\,A\,a^2\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,A\,a\,b^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,B\,a^2\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{6\,B\,a\,b^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

output
```
(A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2
, 2))/3))/d - (2*B*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4
], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b^3*cos(c +
d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11
*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(13/2)*sin(c + d*x)*hyp
ergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*d*(sin(c + d*x)^2)^(1/2)) -
 (6*A*a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, co
s(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a*b^2*cos(c + d*x)^(9/2
)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c +
d*x)^2)^(1/2)) - (2*B*a^2*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2
, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a*b^2*c
os(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^
2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\begin{aligned}
&\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx \\
&= \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^4 + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^5\,dx\right)b^4 \\
&\quad + 4\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)a\,b^3 \\
&\quad + 6\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a^2b^2 \\
&\quad + 4\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a^3b
\end{aligned}$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a**4 + int(sqrt(cos(c + d*x))*cos(c
 + d*x)**5,x)*b**4 + 4*int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*a*b**3 +
6*int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*a**2*b**2 + 4*int(sqrt(cos(c +
 d*x))*cos(c + d*x)**2,x)*a**3*b
```

3.360 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 255

$$\begin{aligned}
&\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx \\
&= \frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} \\
&\quad+\frac{2(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\
&\quad+\frac{2(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\
&\quad+\frac{2b(27aAb+22a^2B+7b^2B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\
&\quad+\frac{2b^2(9Ab+13aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} \\
&\quad+\frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{9d}
\end{aligned}$$

output
```
2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))/d+2/21*(21*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))/d+2/21*(21*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*cos(d
*x+c)^(1/2)*sin(d*x+c)/d+2/45*b*(27*A*a*b+22*B*a^2+7*B*b^2)*cos(d*x+c)^(3/
2)*sin(d*x+c)/d+2/63*b^2*(9*A*b+13*B*a)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*
b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.70 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{84(15a^3A+27aAb^2+27a^2bB+7b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+60(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\,\text{Ellipti}}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x
]
```

output
```
(84*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2,
2] + 60*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c + d*x)/
2, 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c +
 d*x] + 5*(252*a^2*A*b + 78*A*b^3 + 84*a^3*B + 234*a*b^2*B + 18*b^2*(A*b +
 3*a*B)*Cos[2*(c + d*x)] + 7*b^3*B*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d
)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3469, 27, 3042, 3512, 27, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{2}{9}\int \frac{1}{2}\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(b(9Ab+13aB)\cos^2(c+dx)+\left(7Bb^2+9a(2Ab+aB)\right)\cos(c+dx)+3a(3aA+bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))\left(b(9Ab+13aB)\cos^2(c+dx)+\left(7Bb^2+9a(2Ab+aB)\right)\cos(c+dx)+3a(3aA+bB)\right)dx+\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b(9Ab+13aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Bb^2+9a(2Ab+aB\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3512}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{1}{2}\sqrt{\cos(c+dx)}\left(21(3aA+bB)a^2+7b\left(22Ba^2+27Aba+7b^2B\right)\cos^2(c+dx)+9\left(7Ba^3+21Aba^2+15b\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \sqrt{\cos(c+dx)}\left(21(3aA+bB)a^2+7b\left(22Ba^2+27Aba+7b^2B\right)\cos^2(c+dx)+9\left(7Ba^3+21Aba^2+15b^2\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(21(3aA+bB)a^2+7b\left(22Ba^2+27Aba+7b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+9\left(7Ba^3+21\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3502

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{2}{5}\int\frac{3}{2}\sqrt{\cos(c+dx)}\left(7\left(15Aa^3+27bBa^2+27Ab^2a+7b^3B\right)+15\left(7Ba^3+21Aba^2+15b^2Ba+5Ab^3\right)\cos(\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}\left(7\left(15Aa^3+27bBa^2+27Ab^2a+7b^3B\right)+15\left(7Ba^3+21Aba^2+15b^2Ba+5Ab^3\right)\cos(c\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(7\left(15Aa^3+27bBa^2+27Ab^2a+7b^3B\right)+15\left(7Ba^3+21Aba^2+15b^2Ba+5Ab^3\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3227

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}\left(15\left(7a^3B+21a^2Ab+15ab^2B+5Ab^3\right)\int\cos^{\frac{3}{2}}(c+dx)dx+7\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)\int\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}\left(7\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+15\left(7a^3B+21a^2Ab+15ab^2B+5A\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3115

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}\left(7(15a^3A+27a^2bB+27aAb^2+7b^3B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+15(7a^3B+21a^2Ab+15ab^2B+5A\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}\left(7(15a^3A+27a^2bB+27aAb^2+7b^3B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+15(7a^3B+21a^2Ab+15ab^2B+5A\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3119

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}\left(15(7a^3B+21a^2Ab+15ab^2B+5Ab^3)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

$\downarrow$ 3120

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{14b(22a^2B+27aAb+7b^2B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{14(15a^3A+27a^2bB+27aAb^2+7b^3B)E(\right.}{d}\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}{9d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

output
```
(2*b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(9*d) + ((2
*b^2*(9*A*b + 13*a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + ((14*b*(27*
a*A*b + 22*a^2*B + 7*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (3*((
14*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2
])/d + 15*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*((2*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))))/5)/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(238) = 476$.

Time = 21.04 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.92

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}b^3+(720A\,b^3+2160Ba\,b^2+2240b^3B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(720*A*b^3+2160*B*a*b^2+2240*B*
b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-151
2*B*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
+(1260*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+
952*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b
^2-240*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.26

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(35\,Bb^3\cos\left(dx+c\right)^3+105\,Ba^3+315\,Aa^2b+225\,Bab^2+75\,Ab^3+45\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^2+\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="fricas")
```

output
```
1/315*(2*(35*B*b^3*cos(d*x + c)^3 + 105*B*a^3 + 315*A*a^2*b + 225*B*a*b^2
+ 75*A*b^3 + 45*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^2 + 7*(27*B*a^2*b + 27*A*
a*b^2 + 7*B*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2
)*(7*I*B*a^3 + 21*I*A*a^2*b + 15*I*B*a*b^2 + 5*I*A*b^3)*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*sqrt(2)*(-7*I*B*a^3 - 21*I*A*
a^2*b - 15*I*B*a*b^2 - 5*I*A*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) - 21*sqrt(2)*(-15*I*A*a^3 - 27*I*B*a^2*b - 27*I*A*a*b^2
- 7*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - 21*sqrt(2)*(15*I*A*a^3 + 27*I*B*a^2*b + 27*I*A*a*b^
2 + 7*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 24.77 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.29

$$\begin{aligned}
&\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx \\
&= \frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d} \\
&\quad+\frac{B\,a^3\,\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d} \\
&\quad-\frac{2\,A\,b^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} \\
&\quad-\frac{2\,B\,b^3\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} \\
&\quad-\frac{6\,A\,a\,b^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} \\
&\quad-\frac{6\,B\,a^2\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}} \\
&\quad-\frac{2\,B\,a\,b^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}
\end{aligned}$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

output
```
(2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + A*a^2*b*ellipticF(c/2 + (d*x)/2, 2
) + A*a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (B*a^3*((2*cos(c + d*x)^
(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*b^3*c
os(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2)
)/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(11/2)*sin(c + d*x)
*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2
)) - (6*A*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4
, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a^2*b*cos(c + d*x)^
(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(
c + d*x)^2)^(1/2)) - (2*B*a*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom(
[1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos(dx+c)}dx\right)a^4 + 4\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^3b$$
$$+ \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)b^4$$
$$+ 4\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a\,b^3$$
$$+ 6\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a^2b^2$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)),x)*a**4 + 4*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*
a**3*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*b**4 + 4*int(sqrt(cos(c
 + d*x))*cos(c + d*x)**3,x)*a*b**3 + 6*int(sqrt(cos(c + d*x))*cos(c + d*x)
**2,x)*a**2*b**2
```

3.361 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 205

$$\begin{aligned}
\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx
&= \frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\
&\quad+\frac{2(21a^3A+21aAb^2+21a^2bB+5b^3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\
&\quad+\frac{2b(21aAb+18a^2B+5b^2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\
&\quad+\frac{2b^2(7Ab+11aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} \\
&\quad+\frac{2bB\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\sin(c+dx)}{7d}
\end{aligned}$$

output
```
2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))/d+2/21*(21*A*a^3+21*A*a*b^2+21*B*a^2*b+5*B*b^3)*InverseJacobiAM(1/2
*d*x+1/2*c,2^(1/2))/d+2/21*b*(21*A*a*b+18*B*a^2+5*B*b^2)*cos(d*x+c)^(1/2)*
sin(d*x+c)/d+2/35*b^2*(7*A*b+11*B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*B
*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{42(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(21a^3A+21aAb^2+21a^2bB+5b^3B)\,\text{Elliptic}$$

input
```
Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]]
,x]
```

output
```
(42*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2]
 + 10*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*x)/2
, 2] + b*Sqrt[Cos[c + d*x]]*(42*b*(A*b + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b
 + 42*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3469, 27, 3042, 3512, 27, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3469$$

$$\frac{2}{7}\int \frac{(a+b\cos(c+dx))\left(b(7Ab+11aB)\cos^2(c+dx)+\left(5Bb^2+7a(2Ab+aB)\right)\cos(c+dx)+a(7aA+bB)\right)}{2\sqrt{\cos(c+dx)}}dx$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\downarrow$ 27

$$\frac{1}{7}\int \frac{(a+b\cos(c+dx))\left(b(7Ab+11aB)\cos^2(c+dx)+\left(5Bb^2+7a(2Ab+aB)\right)\cos(c+dx)+a(7aA+bB)\right)}{\sqrt{\cos(c+dx)}}dx$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b(7Ab+11aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Bb^2+7a(2Ab+aB)\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\downarrow$ 3512

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{5(7aA+bB)a^2+5b\left(18Ba^2+21Aba+5b^2B\right)\cos^2(c+dx)+7\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\cos(c\right.}{2\sqrt{\cos(c+dx)}}$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5(7aA+bB)a^2+5b\left(18Ba^2+21Aba+5b^2B\right)\cos^2(c+dx)+7\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\cos(c\right.}{\sqrt{\cos(c+dx)}}$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5(7aA+bB)a^2+5b\left(18Ba^2+21Aba+5b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+7\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\downarrow$ 3502

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{5\left(21Aa^3+21bBa^2+21Ab^2a+5b^3B\right)+21\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{10b\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{5\left(21Aa^3+21bBa^2+21Ab^2a+5b^3B\right)+21\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{10b\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{5\left(21Aa^3+21bBa^2+21Ab^2a+5b^3B\right)+21\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3227

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(5\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+21\left(5a^3B+15a^2Ab+9ab^2B+3Ab^3\right)\int\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(5\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21\left(5a^3B+15a^2Ab+9ab^2B+3Ab\right.\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3119

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(5\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{42\left(5a^3B+15a^2Ab+9ab^2B+3A\right.}{d}\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{10b(18a^2B+21aAb+5b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{1}{3}\left(\frac{10(21a^3A+21a^2bB+21aAb^2+5b^3B)\,\text{Elli}}{d}\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}{7d}$$

input
```
Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) + ((2
*b^2*(7*A*b + 11*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (((42*(15*a
^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (10
*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*x)/2, 2])
/d)/3 + (10*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(3*d))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(192) = 384$.

Time = 16.23 (sec) , antiderivative size = 664, normalized size of antiderivative = 3.24

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8 b^3+\left(-168A\,b^3-504Ba\,b^2-360b^3B\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos(\cdots\right.}{\cdots}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-168*A*b^3-504*B*a*b^2-360*B*b^3)
*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*
b+504*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a
*b^2-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)+105*a^3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*a*b^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^3+105*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\left(15\,Bb^3\cos\left(dx+c\right)^2+105\,Ba^2b+105\,Aab^2+25\,Bb^3+21\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right.}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
1/105*(2*(15*B*b^3*cos(d*x + c)^2 + 105*B*a^2*b + 105*A*a*b^2 + 25*B*b^3 +
 21*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*
sqrt(2)*(21*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 5*I*B*b^3)*weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-21*I*A*a^3 -
21*I*B*a^2*b - 21*I*A*a*b^2 - 5*I*B*b^3)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-5*I*B*a^3 - 15*I*A*a^2*b - 9*I*B*a
*b^2 - 3*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(5*I*B*a^3 + 15*I*A*a^2*b + 9*I*B*a
*b^2 + 3*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 25.42 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}$$
$$+\frac{2\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$
$$+\frac{3\,A\,a\,b^2\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}$$
$$-\frac{2\,A\,b^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$
$$-\frac{2\,B\,b^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$
$$-\frac{6\,B\,a\,b^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(1/2),x)
```

output
```
(2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + B*a^2*b*ellipticF(c/2 + (d*x)/2, 2
) + B*a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (2*A*a^3*ellipticF(c/2 +
 (d*x)/2, 2))/d + (6*A*a^2*b*ellipticE(c/2 + (d*x)/2, 2))/d + (3*A*a*b^2*(
(2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3)
)/d - (2*A*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(9/
2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c +
 d*x)^2)^(1/2)) - (6*B*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/
2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^4 + 4\left(\int \sqrt{\cos(dx+c)}dx\right)a^3b$$
$$+ 6\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^2b^2 + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b^4$$
$$+ 4\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a\,b^3$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**4 + 4*int(sqrt(cos(c + d*x)),x)*
a**3*b + 6*int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a**2*b**2 + int(sqrt(cos
(c + d*x))*cos(c + d*x)**3,x)*b**4 + 4*int(sqrt(cos(c + d*x))*cos(c + d*x)
**2,x)*a*b**3
```

3.362 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 202

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\\ &+\frac{2(9a^2Ab+Ab^3+3a^3B+3ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\\ &-\frac{2b(6a^2A-Ab^2-3abB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}\\ &-\frac{2b^2(5aA-bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

output

```
-2/5*(5*A*a^3-15*A*a*b^2-15*B*a^2*b-3*B*b^3)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))/d+2/3*(9*A*a^2*b+A*b^3+3*B*a^3+3*B*a*b^2)*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/d-2/3*b*(6*A*a^2-A*b^2-3*B*a*b)*cos(d*x+c)^(1/2)*sin(d*x+c
)/d-2/5*b^2*(5*A*a-B*b)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*A*(a+b*cos(d*x+c
))^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{(-30a^3A+90aAb^2+90a^2bB+18b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(9a^2Ab+Ab^3+3a^3B+3ab^2B)\,\text{EllipticF}}{15d}$$

input
```
Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2)
,x]
```

output
```
((-30*a^3*A + 90*a*A*b^2 + 90*a^2*b*B + 18*b^3*B)*EllipticE[(c + d*x)/2, 2
] + 10*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2]
 + ((10*b^2*(A*b + 3*a*B)*Cos[c + d*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2
*(c + d*x)]))*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(15*d)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3468, 27, 3042, 3512, 27, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$2\int \frac{(a+b\cos(c+dx))\left(-b(5aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(5Ab+aB)\right)}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 27

$$\int \frac{(a+b\cos(c+dx))\left(-b(5aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(5Ab+aB)\right)}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-b(5aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-Aa^2+2bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5A\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3512

$$\frac{2}{5}\int \frac{5(5Ab+aB)a^2-5b\left(6Aa^2-3bBa-Ab^2\right)\cos^2(c+dx)-\left(5Aa^3-15bBa^2-15Ab^2a-3b^3B\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}c$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\int \frac{5(5Ab+aB)a^2-5b\left(6Aa^2-3bBa-Ab^2\right)\cos^2(c+dx)-\left(5Aa^3-15bBa^2-15Ab^2a-3b^3B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}c$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{5(5Ab+aB)a^2-5b\left(6Aa^2-3bBa-Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-5Aa^3+15bBa^2+15Ab^2a+3b^3B\right)\sin\left(c\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{5\left(3Ba^3+9Aba^2+3b^2Ba+Ab^3\right)-3\left(5Aa^3-15bBa^2-15Ab^2a-3b^3B\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx-\frac{10b\left(6a^2A-\right.}{}\right.$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{5\left(3Ba^3+9Aba^2+3b^2Ba+Ab^3\right)-3\left(5Aa^3-15bBa^2-15Ab^2a-3b^3B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx-\frac{10b\left(6a^2A-\right.}{}\right.$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{5\left(3Ba^3+9Aba^2+3b^2Ba+Ab^3\right)-3\left(5Aa^3-15bBa^2-15Ab^2a-3b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{10b(6a}{}\right.$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx-3\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\int\sqrt{\cos(c\right.\right.$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\int\sqrt{\right.\right.$$
$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)E\left(\frac{1}{2}(\right.}{d}\right.\right.$$

$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{10\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)E\left(\frac{1}{2}(\right.}{d}\right.\right.$$

$$\frac{2b^2(5aA-bB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*b^2*(5*a*A - b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*A*(a +
 b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (((-6*(5*a^3*A -
 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*EllipticE[(c + d*x)/2, 2])/d + (10*(9*
a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2])/d)/3 - (
10*b*(6*a^2*A - A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(191) = 382$.

Time = 12.42 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.17

method	result
default	$-\frac{2\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}b^{3}+20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b^{3}+60B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a\,b^{2}+24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/15*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+20*A*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3+60*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4*a*b^2+24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-30*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3-10*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2*b^3+45*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b
^2-30*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-6*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*b^3+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*a*b^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-9*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.50

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{5\sqrt{2}(3i\,Ba^3+9i\,Aa^2b+3i\,Bab^2+i\,Ab^3)\cos{(dx+c)}\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-1/15*(5*sqrt(2)*(3*I*B*a^3 + 9*I*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*cos(d*x
 + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2
)*(-3*I*B*a^3 - 9*I*A*a^2*b - 3*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*A*a^3 -
 15*I*B*a^2*b - 15*I*A*a*b^2 - 3*I*B*b^3)*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)
*(-5*I*A*a^3 + 15*I*B*a^2*b + 15*I*A*a*b^2 + 3*I*B*b^3)*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - 2*(3*B*b^3*cos(d*x + c)^2 + 15*A*a^3 + 5*(3*B*a*b^2 + A*b^3)*cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 25.07 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{A\,b^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$
$$+\frac{6\,A\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,B\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$
$$+\frac{3\,B\,a\,b^2\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}$$
$$+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,B\,b^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(3/2),x)
```

output
```
(A*b^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2
, 2))/3))/d + (2*B*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*A*a*b^2*ellipti
cE(c/2 + (d*x)/2, 2))/d + (6*A*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (6*B
*a^2*b*ellipticE(c/2 + (d*x)/2, 2))/d + (3*B*a*b^2*((2*cos(c + d*x)^(1/2)*
sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (2*A*a^3*sin(c +
 d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(
sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeo
m([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= 4\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^3b + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^4 + 6\left(\int \sqrt{\cos(dx+c)}dx\right)a^2b^2 + 4\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a\,b^3 + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b^4$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
4*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**3*b + int(sqrt(cos(c + d*x))/c
os(c + d*x)**2,x)*a**4 + 6*int(sqrt(cos(c + d*x)),x)*a**2*b**2 + 4*int(sqr
t(cos(c + d*x))*cos(c + d*x),x)*a*b**3 + int(sqrt(cos(c + d*x))*cos(c + d*
x)**2,x)*b**4
```

3.363 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 192

$$\begin{aligned}
&\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= -\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} \\
&\quad+\frac{2(a^3A+9aAb^2+9a^2bB+b^3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\
&\quad+\frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{2b^2(aA-bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\
&\quad+\frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

output
```
-2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
/d+2/3*(A*a^3+9*A*a*b^2+9*B*a^2*b+B*b^3)*InverseJacobiAM(1/2*d*x+1/2*c,2^(
1/2))/d+2/3*a^2*(7*A*b+3*B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b^2*(A*a-B
*b)*cos(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a*A*(a+b*cos(d*x+c))^2*sin(d*x+c)/d/
cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-6(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^3A+9aAb^2+9a^2bB+b^3B)\sqrt{c}}{3}$$

input
```
Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2)
,x]
```

output
```
(-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 18*a^2*A*b*Sin[c + d*x] + 6*a^3*B*Sin[c +
d*x] + b^3*B*Sin[2*(c + d*x)] + 2*a^3*A*Tan[c + d*x])/(3*d*Sqrt[Cos[c + d*
x]])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3468, 27, 3042, 3510, 27, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{3}\int \frac{(a+b\cos(c+dx))\left(-3b(aA-bB)\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(7Ab+3aB)\right)}{2\cos^{\frac{3}{2}}(c+dx)}dx+$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(a+b\cos(c+dx))\left(-3b(aA-bB)\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(7Ab+3aB)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx+$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-3b(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(7A\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3510}$$

$$\frac{1}{3}\left(\frac{2a^2(3aB+7Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-2\int -\frac{-3b^2(aA-bB)\cos^2(c+dx)-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cos(c-}{2\sqrt{\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \frac{-3b^2(aA-bB)\cos^2(c+dx)-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cos(c+dx)+a\left(Aa^2+9bBa+10Ab^2\right)}{\sqrt{\cos(c+dx)}}dx\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{-3b^2(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(Aa^2+9bBa+\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{3\left(Aa^3+9bBa^2+9Ab^2a+b^3B-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)}}dx+\frac{2a^2(3aB+7Ab)\,\mathrm{s}}{d\sqrt{\cos(c+}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \frac{Aa^3+9bBa^2+9Ab^2a+b^3B-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2(3aB+7Ab)\sin(c+}{d\sqrt{\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{Aa^3+9bBa^2+9Ab^2a+b^3B-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(3aB+7Ab)\,\mathrm{si}}{d\sqrt{\cos(c+}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \sqrt{\cos(c+dx)}dx+\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \sqrt{\sin\left(c+dx\right.}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left(\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(3aB+7Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{6\left(a^3B+3a^2Ab-\right.}{}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{1}{3}\left(\frac{2a^2(3aB+7Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6\left(a^3B+3a^2Ab-\right.}{}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((-
6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (
2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*EllipticF[(c + d*x)/2, 2])/d + (
2*a^2*(7*A*b + 3*a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (2*b^2*(a*A -
 b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(181) = 362$.

Time = 10.10 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.02

method	result	size
parts	Expression too large to display	771
default	Expression too large to display	1210

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
2*(A*b^3+3*B*a*b^2)*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d+2*(3*A*a*b^2
+3*B*a^2*b)/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))-2*(3*A*a^2*b+B*a^3)*(
-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2/3*b^3*B*((2
*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2/3*a^3*A*(-2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.59

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-iAa^3 - 9iBa^2b - 9iAab^2 - iBb^3)\cos(dx+c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(d$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(-I*A*a^3 - 9*I*B*a^2*b - 9*I*A*a*b^2 - I*B*b^3)*cos(d*x + c)
^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*
A*a^3 + 9*I*B*a^2*b + 9*I*A*a*b^2 + I*B*b^3)*cos(d*x + c)^2*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a^3 + 3*I*A*a
^2*b - 3*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a^3
 - 3*I*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*cos(d*x + c)^2*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(B*b^3*
cos(d*x + c)^2 + A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2),
x)
```

Mupad [B] (verification not implemented)

Time = 25.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(A\operatorname{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)b^3+3\,A\,a\operatorname{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)b^2\right)}{d}$$
$$+\frac{B\,b^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\operatorname{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,B\,a\,b^2\operatorname{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}$$
$$+\frac{6\,B\,a^2\,b\operatorname{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{2\,A\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\sqrt{\sin(c+d\,x)^2}}$$
$$+\frac{2\,B\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\sqrt{\cos(c+d\,x)}\sqrt{\sin(c+d\,x)^2}}$$
$$+\frac{6\,A\,a^2\,b\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\sqrt{\cos(c+d\,x)}\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(5/2),x)
```

output
```
(2*(A*b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*A*a*b^2*ellipticF(c/2 + (d*x)/2,
 2)))/d + (B*b^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2
 + (d*x)/2, 2))/3))/d + (6*B*a*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (6*B*a
^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1
/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/
(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (6*A*a^2*b*sin(c + d*x)*hy
pergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c +
d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= 6\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^2b^2 + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a^4$$
$$+ 4\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^3b + 4\left(\int \sqrt{\cos(dx+c)}dx\right)a\,b^3$$
$$+ \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b^4$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
6*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a**2*b**2 + int(sqrt(cos(c + d*x)
)/cos(c + d*x)**3,x)*a**4 + 4*int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a*
*3*b + 4*int(sqrt(cos(c + d*x)),x)*a*b**3 + int(sqrt(cos(c + d*x))*cos(c +
 d*x),x)*b**4
```

3.364 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 204

$$\begin{aligned}
&\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= -\frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\
&\quad+\frac{2(3a^2Ab+3Ab^3+a^3B+9ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\
&\quad+\frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3a^2A+14Ab^2+15abB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

output
```
-2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))/d+2/3*(3*A*a^2*b+3*A*b^3+B*a^3+9*B*a*b^2)*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/d+2/15*a^2*(9*A*b+5*B*a)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5
*a*(3*A*a^2+14*A*b^2+15*B*a*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/5*a*A*(a+b*
cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 4.68 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-6(3a^3A+15aAb^2+15a^2bB-5b^3B)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+10(3a^2Ab+3Ab^3+a^3B+9ab^2B}{}$$

input
```
Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2)
,x]
```

output
```
(-6*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Cos[c + d*x]^(3/2)*Ellip
ticE[(c + d*x)/2, 2] + 10*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*Cos[c
+ d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*(3*A*b + a*B)*Sin[c + d*x]
 + 9*a*(a^2*A + 5*A*b^2 + 5*a*b*B)*Sin[2*(c + d*x)] + 6*a^3*A*Tan[c + d*x]
)/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3468, 27, 3042, 3510, 27, 3042, 3500, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{5}\int \frac{(a+b\cos(c+dx))\left(-b(aA-5bB)\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(9Ab+5aB)\right)}{2\cos^{\frac{5}{2}}(c+dx)}dx$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(a+b\cos(c+dx))\left(-b(aA-5bB)\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(9Ab+5aB)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-b(aA-5bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+10bBa+5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3510}$$

$$\frac{1}{5}\left(\frac{2a^2(5aB+9Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int -\frac{-3b^2(aA-5bB)\cos^2(c+dx)+5\left(Ba^3+3Aba^2+9b^2Ba+3Ab^3\right)\cos(}{2\cos^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{-3b^2(aA-5bB)\cos^2(c+dx)+5\left(Ba^3+3Aba^2+9b^2Ba+3Ab^3\right)\cos(c+dx)+3a\left(3Aa^2+15bBa+1\right.}{\cos^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{-3b^2(aA-5bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+5\left(Ba^3+3Aba^2+9b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\left(3Aa^2+1\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(2\int \frac{5\left(Ba^3+3Aba^2+9b^2Ba+3Ab^3\right)-3\left(3Aa^3+15bBa^2+15Ab^2a-5b^3B\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{6a\left(3a^2A\right.}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{5\left(Ba^3+3Aba^2+9b^2Ba+3Ab^3\right)-3\left(3Aa^3+15bBa^2+15Ab^2a-5b^3B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{6a\left(3a^2A+\right.}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{5\left(Ba^3+3Aba^2+9b^2Ba+3Ab^3\right)-3\left(3Aa^3+15bBa^2+15Ab^2a-5b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a(3a}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\int \sqrt{\cos(c}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\int \sqrt{s}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\right.\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3120$$

$$\frac{1}{5}\left(\frac{2a^2(5aB+9Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{6a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{10\left(a^3B+3a^2Ab+9ab^2B+3\right.}{d}\right.\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((2
*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((-6*(3*a^3*
A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2])/d + (10*
(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*EllipticF[(c + d*x)/2, 2])/d + (
6*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/
3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(191) = 382$.

Time = 11.11 (sec) , antiderivative size = 950, normalized size of antiderivative = 4.66

method	result	size
parts	Expression too large to display	950
default	Expression too large to display	970

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
2*(A*b^3+3*B*a*b^2)/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))-2*(3*A*a*b^2+
3*B*a^2*b)*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2
/3*(3*A*a^2*b+B*a^3)*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*((2*cos
(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1
/2*c)/d-2/5*a^3*A*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1
)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{5\sqrt{2}(i\,Ba^3+3i\,Aa^2b+9i\,Bab^2+3i\,Ab^3)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-1/15*(5*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b + 9*I*B*a*b^2 + 3*I*A*b^3)*cos(d*x
 + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt
(2)*(-I*B*a^3 - 3*I*A*a^2*b - 9*I*B*a*b^2 - 3*I*A*b^3)*cos(d*x + c)^3*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a
^3 + 15*I*B*a^2*b + 15*I*A*a*b^2 - 5*I*B*b^3)*cos(d*x + c)^3*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*s
qrt(2)*(-3*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b^2 + 5*I*B*b^3)*cos(d*x + c)
^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - 2*(3*A*a^3 + 9*(A*a^3 + 5*B*a^2*b + 5*A*a*b^2)*cos(d*x + c)^2
 + 5*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2),
x)
```

Mupad [B] (verification not implemented)

Time = 26.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(B\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\,b^3+3\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\,b^2\right)}{d}+\frac{2\,A\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}$$
$$+\frac{2\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{2\,B\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{6\,A\,a\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{2\,A\,a^2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$+\frac{6\,B\,a^2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(7/2),x)
```

output
```
(2*(B*b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*B*a*b^2*ellipticF(c/2 + (d*x)/2,
 2)))/d + (2*A*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*sin(c + d*x)*
hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin
(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, co
s(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*A*a*b^
2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x
)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a^2*b*sin(c + d*x)*hypergeom([-3/4,
 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
 + (6*B*a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= 4\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right) a\, b^3 + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right) a^4$$
$$+ 4\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right) a^3 b$$
$$+ 6\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right) a^2 b^2 + \left(\int \sqrt{\cos(dx+c)}dx\right) b^4$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
4*int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a*b**3 + int(sqrt(cos(c + d*x))/c
os(c + d*x)**4,x)*a**4 + 4*int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*a**3*
b + 6*int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a**2*b**2 + int(sqrt(cos(c
 + d*x)),x)*b**4
```

3.365 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 182

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= -\frac{2(5aAb-5a^2B-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^3d} \\ &\quad+\frac{2(3a^2+b^2)\,(Ab-aB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^4d} \\ &\quad-\frac{2a^3(Ab-aB)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^4(a+b)d} \\ &\quad+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd}\end{aligned}$$

output
```
-2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d
+2/3*(3*a^2+b^2)*(A*b-B*a)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^4/d-2*
a^3*(A*b-B*a)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^4/(a+b)/d
+2/3*(A*b-B*a)*cos(d*x+c)^(1/2)*sin(d*x+c)/b^2/d+2/5*B*cos(d*x+c)^(3/2)*si
n(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 13.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.43

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\frac{2b^2(-5aAb+5a^2B+9b^2B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+2b^2(5Ab+4aB)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b}\right.}{a+b}\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x
]
```

output
```
((2*b^2*(-5*a*A*b + 5*a^2*B + 9*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b) + 2*b^2*(5*A*b + 4*a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*
a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*b^2*Sqrt[Cos[c +
 d*x]]*(5*A*b - 5*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x] + (6*(-5*a*A*b +
5*a^2*B + 3*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Ellipt
icPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c
+ d*x]^2]))/(30*b^4*d)
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3538, 27, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$\Big\downarrow$ 3469

$$\frac{2\int \frac{\sqrt{\cos(c+dx)}\left(5(Ab-aB)\cos^2(c+dx)+3bB\cos(c+dx)+3aB\right)}{2(a+b\cos(c+dx))}dx}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{\cos(c+dx)}\left(5(Ab-aB)\cos^2(c+dx)+3bB\cos(c+dx)+3aB\right)}{a+b\cos(c+dx)}dx}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(5(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+3bB\sin\left(c+dx+\frac{\pi}{2}\right)+3aB\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\Big\downarrow$ 3528

$$\frac{\frac{2\int \frac{-3\left(-5Ba^2+5Aba-3b^2B\right)\cos^2(c+dx)+b(5Ab+4aB)\cos(c+dx)+5a(Ab-aB)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{10(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-3\left(-5Ba^2+5Aba-3b^2B\right)\cos^2(c+dx)+b(5Ab+4aB)\cos(c+dx)+5a(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{10(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{-3\left(-5Ba^2+5Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(5Ab+4aB)\sin\left(c+dx+\frac{\pi}{2}\right)+5a(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}+\frac{10(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\Big\downarrow$ 3538

$$\frac{\frac{-\frac{3(-5a^2B+5aAb-3b^2B)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int-\frac{5(ab(Ab-aB)+(3a^2+b^2)\cos(c+dx)(Ab-aB))}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{3b}+\frac{10(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\downarrow$ 27

$$\frac{\frac{\frac{5\int\frac{ab(Ab-aB)+(3a^2+b^2)\cos(c+dx)(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3(-5a^2B+5aAb-3b^2B)\int\sqrt{\cos(c+dx)}dx}{b}}{3b}+\frac{10(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{5\int\frac{ab(Ab-aB)+(3a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{3(-5a^2B+5aAb-3b^2B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}+\frac{10(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{5\int\frac{ab(Ab-aB)+(3a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6(-5a^2B+5aAb-3b^2B)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{10(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\downarrow$ 3481

$$\frac{\frac{\frac{5\left(\frac{(3a^2+b^2)(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{3a^3(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}\right)}{b}-\frac{6(-5a^2B+5aAb-3b^2B)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{10(Ab-aB)\sin(c\text{·}}{3\text{·}}}{5b}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{5\left(\frac{\left(3a^2+b^2\right)(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a^3(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{6\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}+\frac{10(A}{}}{5b}$$

$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{5\left(\frac{2\left(3a^2+b^2\right)(Ab-aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{3a^3(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{6\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}+\frac{10(}{}}{5b}$$

$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

$\downarrow$ 3284

$$\frac{\frac{\frac{5\left(\frac{2\left(3a^2+b^2\right)(Ab-aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{6a^3(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}\right)}{b}-\frac{6\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}+\frac{10(Ab-aB)\sin(c}{3}}{5b}$$

$$\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5bd}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(2*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*b*d) + (((-6*(5*a*A*b - 5*a^2*B -
 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b*d) + (5*((2*(3*a^2 + b^2)*(A*b - a
*B)*EllipticF[(c + d*x)/2, 2])/(b*d) - (6*a^3*(A*b - a*B)*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d)))/b)/(3*b) + (10*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(175) = 350$.

Time = 9.27 (sec) , antiderivative size = 1074, normalized size of antiderivative = 5.90

method	result	size
default	Expression too large to display	1074

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*B*a*b^
3+24*B*b^4)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*a*b^3-20*A*b^4-2
0*B*a^2*b^2+44*B*a*b^3-24*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(
-10*A*a*b^3+10*A*b^4+10*B*a^2*b^2-16*B*a*b^3+6*B*b^4)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2*b^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-5*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*b^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-15*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^3*b-15*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^4+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-5*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*a^2*b^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(...
```

Fricas [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(b*cos(d
*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}dx$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)
```

3.366 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 137

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} - \frac{2(3aAb-3a^2B-b^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d} + \frac{2a^2(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^3(a+b)d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
2*(A*b-B*a)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d-2/3*(3*A*a*b-3*B*a
^2-B*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^3/d+2*a^2*(A*b-B*a)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^3/(a+b)/d+2/3*B*cos(d*x+c)
^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.87 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\frac{(3Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + B\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right) + 2B\sqrt{\cos(c+}}{}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x
]
```

output
```
(((3*A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + B*(2*
EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a + b)) + 2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (3*(A*b - a*B)*(-2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[S
qrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/(3*b*d)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3469, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3469$$

$$\frac{2\int \frac{3(Ab-aB)\cos^2(c+dx)+bB\cos(c+dx)+aB}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(Ab-aB)\cos^2(c+dx)+bB\cos(c+dx)+aB}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3538}$$

$$\frac{\frac{3(Ab-aB)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{abB-\left(-3Ba^2+3Aba-b^2B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{abB-\left(-3Ba^2+3Aba-b^2B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+\frac{3(Ab-aB)\int\sqrt{\cos(c+dx)}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{abB+\left(3Ba^2-3Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{3(Ab-aB)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\int \frac{abB+\left(3Ba^2-3Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{\frac{3a^2(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{\left(-3a^2B+3aAb-b^2B\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3a^2(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\left(-3a^2B+3aAb-b^2B\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{3a^2(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(-3a^2B+3aAb-b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3284

$$\frac{\frac{\frac{6a^2(Ab-aB)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}-\frac{2\left(-3a^2B+3aAb-b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

input

```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output

```
((6*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((-2*(3*a*A*b - 3*a^2*B
 - b^2*B)*EllipticF[(c + d*x)/2, 2])/(b*d) + (6*a^2*(A*b - a*B)*EllipticPi
[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))/b)/(3*b) + (2*B*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(3*b*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(136) = 272$.

Time = 8.01 (sec) , antiderivative size = 822, normalized size of antiderivative = 6.00

method	result	size
default	Expression too large to display	822

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2+4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4*b^3+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^3-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b+2*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*a*b^2-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
b^3-3*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
B*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*B*(si...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x),x)
```

3.367 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 89

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(Ab-aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d} - \frac{2a(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2(a+b)d}$$

output
```
2*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d+2*(A*b-B*a)*InverseJacobiAM(
1/2*d*x+1/2*c,2^(1/2))/b^2/d-2*a*(A*b-B*a)*EllipticPi(sin(1/2*d*x+1/2*c),2
*b/(a+b),2^(1/2))/b^2/(a+b)/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{Ab\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right) - \frac{2B\left(bE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)\,\text{EllipticF}\left(\arcsi\right.\right.}{}}{b^2d}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x
]
```

output
```
(A*b*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(a + b)) - (2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (
a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), Ar
cSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3042, 3481, 3042, 3119, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3481}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx}{b}+\frac{B\int \sqrt{\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$$\Big\downarrow \; 3282$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}\right)}{b}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$$\Big\downarrow \; 3120$$

$$\frac{(Ab-aB)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$$\Big\downarrow \; 3284$$

$$\frac{(Ab-aB)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}\right)}{b}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + ((A*b - a*B)*((2*EllipticF[(c + d*
x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a +
b)*d)))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(94) = 188$.

Time = 5.83 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.31

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab-A\,\mathrm{EllipticF}\left(c\right.\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-A*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+B
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \sqrt{\cos\left(dx+c\right)}dx$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)),x)
```

3.368 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 61

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd} + \frac{2(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b(a+b)d}$$

output
```
2*B*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/d+2*(A*b-B*a)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2\left((a+b)B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\right)}{b(a+b)d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x
]
```

output
```
(2*((a + b)*B*EllipticF[(c + d*x)/2, 2] + (A*b - a*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2]))/(b*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3481}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+\frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}+\frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

output
```
(2*B*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(b*(a + b)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(64) = 128$.

Time = 3.62 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.56

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(A\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)b+B\,\mathrm{Ellipt}\right.}{(a-b)b\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))*b+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))*a)/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))}\,dx = \int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)
```

3.369 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 86

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a+b)d} + \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

output
```
-2*A*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-2*(A*b-B*a)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*A*sin(d*x+c)/a/d/cos(d*x+c)^
(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 206 vs. $2(86) = 172$.

Time = 2.91 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.40

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{\frac{2(-3Ab+2aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}-\frac{2aA\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{b}+\frac{4A\sin(c+dx)}{\sqrt{\cos(c+dx)}}-\frac{2A\left(-2ab\right.}{}}{2}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x
]
```

output
```
((2*(-3*A*b + 2*a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) -
(2*a*A*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(a + b)))/b + (4*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (2*A*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sq
rt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3479, 27, 3042, 3538, 25, 27, 3042, 3119, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

↓ 3479

$$\frac{2\int -\frac{Ab\cos^2(c+dx)+aA\cos(c+dx)+Ab-aB}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

↓ 27

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{Ab\cos^2(c+dx)+aA\cos(c+dx)+Ab-aB}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}$$

↓ 3042

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+aA\sin\left(c+dx+\frac{\pi}{2}\right)+Ab-aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

↓ 3538

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{A\int \sqrt{\cos(c+dx)}dx-\frac{\int -\frac{b(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}$$

↓ 25

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{b(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+A\int \sqrt{\cos(c+dx)}dx}{a}$$

↓ 27

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+A\int \sqrt{\cos(c+dx)}dx}{a}$$

↓ 3042

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+A\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

↓ 3119

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

↓ 3284

$$\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{2(Ab-aB)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

output
```
-(((2*A*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b - a*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/((a + b)*d))/a) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[
c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(88) = 176$.

Time = 4.32 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.49

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{2A\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin(}\right.}{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos(}}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*A/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+4*(A*b-B*a)/a/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\, dx = \int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^2} dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)
```

3.370 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 150

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{2b(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a+b)d} + \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

output
```
2*(A*b-B*a)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*A*InverseJacob
iAM(1/2*d*x+1/2*c,2^(1/2))/a/d+2*b*(A*b-B*a)*EllipticPi(sin(1/2*d*x+1/2*c)
,2*b/(a+b),2^(1/2))/a^2/(a+b)/d+2/3*A*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*(A
*b-B*a)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.69 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.73

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$
$$= \frac{\frac{2a(2a^2A+9Ab^2-9abB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{a(8aAb-6a^2B)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{b}+\frac{4a^2A\,\text{si}}{\cos^{\frac{3}{2}}(}}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x
]
```

output
```
((2*a*(2*a^2*A + 9*A*b^2 - 9*a*b*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(a + b) + (a*(8*a*A*b - 6*a^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (4*a^2*A*Sin[c +
d*x])/Cos[c + d*x]^(3/2) + (12*a*(-(A*b) + a*B)*Sin[c + d*x])/Sqrt[Cos[c +
 d*x]] + (6*(A*b - a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1]
+ 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*E
llipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[S
in[c + d*x]^2]))/(6*a^3*d)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$\Big\downarrow$ 3479

$$\frac{2\int -\frac{-Ab\cos^2(c+dx)-aA\cos(c+dx)+3(Ab-aB)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}+\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-Ab\cos^2(c+dx)-aA\cos(c+dx)+3(Ab-aB)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-aA\sin\left(c+dx+\frac{\pi}{2}\right)+3(Ab-aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$\Big\downarrow$ 3534

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2\int -\frac{Aa^2-3bBa+(4Ab-3aB)\cos(c+dx)a+3Ab^2+3b(Ab-aB)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}}{3a}$$

$\Big\downarrow$ 27

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{Aa^2-3bBa+(4Ab-3aB)\cos(c+dx)a+3Ab^2+3b(Ab-aB)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{Aa^2-3bBa+(4Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)a+3Ab^2+3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{3a}$$

$\Big\downarrow$ 3538

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3(Ab-aB)\int\sqrt{\cos(c+dx)}dx-\frac{\int -\frac{aA\cos(c+dx)b^2+\left(Aa^2-3bBa+3Ab^2\right)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{3a}$$

$\downarrow$ 25

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{aA\cos(c+dx)b^2+\left(Aa^2-3bBa+3Ab^2\right)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+3(Ab-aB)\int\sqrt{\cos(c+dx)}dx}{a}}{3a}$$

$\downarrow$ 3042

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{aA\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(Aa^2-3bBa+3Ab^2\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+3(Ab-aB)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{3a}$$

$\downarrow$ 3119

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{aA\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(Aa^2-3bBa+3Ab^2\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$\downarrow$ 3481

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^2(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+aAb\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$\downarrow$ 3042

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^2(Ab-aB)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+aAb\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\frac{3b^2(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx + \frac{2aAb\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b} + \frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{\frac{6(Ab-aB)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\frac{\frac{6b^2(Ab-aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)} + \frac{2aAb\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b} + \frac{6(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

output
```
(2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (-(((6*(A*b - a*B)*Ellipti
cE[(c + d*x)/2, 2])/d + ((2*a*A*b*EllipticF[(c + d*x)/2, 2])/d + (6*b^2*(A
*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/a) +
(6*(A*b - a*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(147) = 294$.

Time = 5.76 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.94

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{2A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+}}\right.}{a}\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)/a^2/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
-4*b^2*(A*b-B*a)/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^3}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)
```

3.371 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 303

$$\begin{aligned}
&\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx \\
&= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a^2-b^2)\,d} \\
&\quad - \frac{(9a^3Ab-12aAb^3-15a^4B+16a^2b^2B+2b^4B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^4(a^2-b^2)\,d} \\
&\quad + \frac{a^2(3a^2Ab-5Ab^3-5a^3B+7ab^2B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^4(a+b)^2d} \\
&\quad - \frac{(3aAb-5a^2B+2b^2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)\,d} \\
&\quad + \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))}
\end{aligned}$$

output
```
(3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/b^3/(a^2-b^2)/d-1/3*(9*A*a^3*b-12*A*a*b^3-15*B*a^4+16*B*a^2*b^2+2*B*b^4)
*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^4/(a^2-b^2)/d+a^2*(3*A*a^2*b-5*A
*b^3-5*B*a^3+7*B*a*b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(
a-b)/b^4/(a+b)^2/d-1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*cos(d*x+c)^(1/2)*sin(d*x+
c)/b^2/(a^2-b^2)/d+a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(
a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.01 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.05

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{4\sqrt{\cos(c+dx)}\left(2B+\frac{3a^2(-Ab+aB)}{(a^2-b^2)(a+b\cos(c+dx))}\right)\sin(c+dx)-\frac{\frac{2\left(-3a^2Ab+6Ab^3+5a^3B-8ab^2B\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8\left(-3\right.}{}}{}}{}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2
,x]
```

output
```
(4*Sqrt[Cos[c + d*x]]*(2*B + (3*a^2*(-(A*b) + a*B))/((a^2 - b^2)*(a + b*Co
s[c + d*x])))*Sin[c + d*x] - ((2*(-3*a^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2
*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-3*a*A*b + 2*
a^2*B + b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2]))/(a + b) + (6*(-3*a^2*A*b + 2*A*b^3 + 5*a^3*B - 4*a
*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*El
lipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^
2]))/((a - b)*(a + b)))/(12*b^2*d)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3468, 27, 3042, 3528, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+3Aba+2b^2B\right)\cos^2(c+dx)\right)-2b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{2(a+b\cos(c+dx))}dx}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+3Aba+2b^2B\right)\cos^2(c+dx)\right)-2b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{a+b\cos(c+dx)}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5Ba^2-3Aba-2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(Ab-aB)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3528}$$

$$\frac{\frac{2\int -\frac{-3\left(-5Ba^3+3Aba^2+4b^2Ba-2Ab^3\right)\cos^2(c+dx)-2b\left(-2Ba^2+3Aba-b^2B\right)\cos(c+dx)+a\left(-5Ba^2+3Aba+2b^2B\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}-\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+}{3bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{-3\left(-5Ba^3+3Aba^2+4b^2Ba-2Ab^3\right)\cos^2(c+dx)-2b\left(-2Ba^2+3Aba-b^2B\right)\cos(c+dx)+a\left(-5Ba^2+3Aba+2b^2B\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}-\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+}{3bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{-3\left(-5Ba^3+3Aba^2+4b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2b\left(-2Ba^2+3Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-5Ba^2+3Aba+2b^2B\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}-\frac{2\left(-5a^2B+3aAb+2b^2B\right)}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3538

$$-\frac{\frac{-\frac{3\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{ab\left(-5Ba^2+3Aba+2b^2B\right)+\left(-15Ba^4+9Aba^3+16b^2Ba^2-12Ab^3a+2b^4B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{3b}-\frac{2(-5}{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{ab\left(-5Ba^2+3Aba+2b^2B\right)+\left(-15Ba^4+9Aba^3+16b^2Ba^2-12Ab^3a+2b^4B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\int\sqrt{\cos(c+dx)}dx}{b}}{3b}-\frac{2(-5a^2}{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{ab\left(-5Ba^2+3Aba+2b^2B\right)+\left(-15Ba^4+9Aba^3+16b^2Ba^2-12Ab^3a+2b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{3\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}-\ldots}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3119

$$-\frac{\frac{\frac{\int \frac{ab\left(-5Ba^2+3Aba+2b^2B\right)+\left(-15Ba^4+9Aba^3+16b^2Ba^2-12Ab^3a+2b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}-\frac{2(-5\ldots}{\ldots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3481

$$-\frac{\frac{\frac{\left(-15a^4B+9a^3Ab+16a^2b^2B-12aAb^3+2b^4B\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{6\left(-5a^3B+3a^2A\ldots\right.}{\ldots}}{3b}\ldots}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\frac{\left(-15a^4B+9a^3Ab+16a^2b^2B-12aAb^3+2b^4B\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6(\ldots}{\ldots}}{3b}\ldots}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3120

$$-\frac{\frac{\frac{2\left(-15a^4B+9a^3Ab+16a^2b^2B-12aAb^3+2b^4B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6\ldots}{\ldots}}{3b}\ldots}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3284

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+$$

$$\frac{-\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}-\frac{\frac{\frac{2\left(-15a^4B+9a^3Ab+16a^2b^2B-12aAb^3+2b^4B\right)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{6a^2\left(-5a^3B+3a^2Ab+7ab^2 \cdots\right.}{\cdots}}{b}\cdots}{3b}}{2b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
(a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])) + (-1/3*((-6*(3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Elli
pticE[(c + d*x)/2, 2])/(b*d) + ((2*(9*a^3*A*b - 12*a*A*b^3 - 15*a^4*B + 16
*a^2*b^2*B + 2*b^4*B)*EllipticF[(c + d*x)/2, 2])/(b*d) - (6*a^2*(3*a^2*A*b
 - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(b*(a + b)*d))/b)/b - (2*(3*a*A*b - 5*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*b*d))/(2*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(300) = 600$.

Time = 20.85 (sec) , antiderivative size = 1066, normalized size of antiderivative = 3.52

method	result	size
default	Expression too large to display	1066

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^4*b^2+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-9*a^2*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)-2*a^3*(A*b-B*a)/b^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a
/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2}{\cos\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)*b + a),x)
```

3.372 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 224

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= -\frac{(aAb-3a^2B+2b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2(a^2-b^2)\,d} \\ &+ \frac{(a^2Ab-2Ab^3-3a^3B+4ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^3(a^2-b^2)\,d} \\ &- \frac{a(a^2Ab-3Ab^3-3a^3B+5ab^2B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^3(a+b)^2d} \\ &+ \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-(A*a*b-3*B*a^2+2*B*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^
2)/d+(A*a^2*b-2*A*b^3-3*B*a^3+4*B*a*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(
1/2))/b^3/(a^2-b^2)/d-a*(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^2)*EllipticPi(sin
(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^3/(a+b)^2/d+a*(A*b-B*a)*cos(d*x
+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 3.27 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.25

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{-\frac{4a(-Ab+aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))} + \frac{\frac{2\left(aAb+a^2B-2b^2B\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{8(-Ab+aB)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-a\,\text{EllipticPi}\left(\cdots\right.\right.}{a+b}}{\cdots}}{\cdots}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2
,x]
```

output
```
((-4*a*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b
*Cos[c + d*x])) + ((2*(a*A*b + a^2*B - 2*b^2*B)*EllipticPi[(2*b)/(a + b),
(c + d*x)/2, 2])/(a + b) + (8*(-(A*b) + a*B)*((a + b)*EllipticF[(c + d*x)/
2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(-(a*A*
b) + 3*a^2*B - 2*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1]
+ 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*E
llipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sq
rt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3468, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{-\left(\left(-3Ba^2+Aba+2b^2B\right)\cos^2(c+dx)\right)-2b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\left(\left(-3Ba^2+Aba+2b^2B\right)\cos^2(c+dx)\right)-2b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(3Ba^2-Aba-2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3538}$$

$$\frac{-\frac{\left(-3a^2B+aAb+2b^2B\right)\int \sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{ab(Ab-aB)+\left(-3Ba^3+Aba^2+4b^2Ba-2Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{ab(Ab-aB)+\left(-3Ba^3+Aba^2+4b^2Ba-2Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{\left(-3a^2B+aAb+2b^2B\right)\int \sqrt{\cos(c+dx)}dx}{b}}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{ab(Ab-aB)+\left(-3Ba^3+Aba^2+4b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\left(-3a^2B+aAb+2b^2B\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\int \frac{ab(Ab-aB)+\left(-3Ba^3+Aba^2+4b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2(-3a^2B+aAb+2b^2B)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3481$$

$$\frac{\frac{\frac{\left(-3a^3B+a^2Ab+4ab^2B-2Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{2(-3a^2B+aAb+2b^2B)E\left(\frac{1}{2}(c+\ldots\right.}{bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(-3a^3B+a^2Ab+4ab^2B-2Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2(-3a^2B+aAb+2b\ldots}{b\ldots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{2\left(-3a^3B+a^2Ab+4ab^2B-2Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2(-3a^2B+aAb+\ldots}{\ldots}}{2b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3284$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}+$$

$$\frac{\frac{\frac{2\left(-3a^3B+a^2Ab+4ab^2B-2Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{2(-3a^2B+aAb+2b^2B)E\left(\frac{1}{2}(\ldots\right.}{bd}}{2b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
((-2*(a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*(a
^2*A*b - 2*A*b^3 - 3*a^3*B + 4*a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(b*d) -
 (2*a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*EllipticPi[(2*b)/(a + b),
(c + d*x)/2, 2])/(b*(a + b)*d))/b)/(2*b*(a^2 - b^2)) + (a*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(227) = 454$.

Time = 8.07 (sec) , antiderivative size = 849, normalized size of antiderivative = 3.79

method	result	size
default	Expression too large to display	849

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*a^2*(A*b-B*a)/b^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)
^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b
^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2)))+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x)*b + a),x)
```

3.373 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 198

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(aAb+a^2B-2b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2\left(a^2-b^2\right)d} \\ &\quad - \frac{\left(a^2Ab+Ab^3+a^3B-3ab^2B\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^2(a+b)^2d} \\ &\quad - \frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

output
```
(A*b-B*a)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d+(A*a*b+B*a^2
-2*B*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^2/(a^2-b^2)/d-(A*a^2*b+
A*b^3+B*a^3-3*B*a*b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a
-b)/b^2/(a+b)^2/d-(A*b-B*a)*cos(d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+b*c
os(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.92 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\frac{4(-Ab+aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))} - \frac{\frac{2(-Ab+aB)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{(4aA-4bB)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right.}{b}}{}}{}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2
,x]
```

output
```
((4*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Co
s[c + d*x])) - ((2*(-(A*b) + a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(a + b) + ((4*a*A - 4*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(A*b - a*B)*(-2*a*b*El
lipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a
+ b)))/(4*d)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3478, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3478}$$

$$-\frac{\int \frac{-\left((Ab-aB)\cos^2(c+dx)\right)-2(aA-bB)\cos(c+dx)+Ab-aB}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a^2-b^2}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-\left((Ab-aB)\cos^2(c+dx)\right)-2(aA-bB)\cos(c+dx)+Ab-aB}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(aB-Ab)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab-aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3538}$$

$$-\frac{-\frac{\int -\frac{b(Ab-aB)-\left(Ba^2+Aba-2b^2B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{(Ab-aB)\int \sqrt{\cos(c+dx)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{b(Ab-aB)-\left(Ba^2+Aba-2b^2B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{(Ab-aB)\int \sqrt{\cos(c+dx)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{b(Ab-aB)+\left(-Ba^2-Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{(Ab-aB)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{\int \frac{b(Ab-aB)+\left(-Ba^2-Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 3481

$$-\frac{\frac{\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{\left(a^2B+aAb-2b^2B\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\left(a^2B+aAb-2b^2B\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(a^2B+aAb-2b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 3284

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}-\frac{\frac{\frac{2\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}-\frac{2\left(a^2B+aAb-2b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

output
```
-1/2*((-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((-2*(a*A*b + a^2
*B - 2*b^2*B)*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(a^2*A*b + A*b^3 + a^3
*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))/
b)/(a^2 - b^2) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2
)*d*(a + b*Cos[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(201) = 402$.

Time = 6.28 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.08

method	result	size
default	Expression too large to display	808

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
4/b*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-b^2/a
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^
(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)b+a}dx$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*b + a),x)
```

3.374 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 200

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a(a^2-b^2)d} - \frac{(Ab-aB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b(a^2-b^2)d} + \frac{(3a^2Ab-Ab^3-a^3B-ab^2B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a-b)b(a+b)^2d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))}$$

output
```
-(A*b-B*a)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d-(A*b-B*a)*I
nverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/(a^2-b^2)/d+(3*A*a^2*b-A*b^3-B*a^3
-B*a*b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a-b)/b/(a+b)
^2/d+b*(A*b-B*a)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c)
)
```

Mathematica [A] (warning: unable to verify)

Time = 3.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.37

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^2}\,dx$$

$$= \frac{\frac{4b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))} + \frac{\frac{2\left(4a^2A-3Ab^2-abB\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{4a(-Ab+aB)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\ldots\right)}{a+b}\ldots\right.}{b}\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2)
,x]
```

output
```
((4*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos
[c + d*x])) + ((2*(4*a^2*A - 3*A*b^2 - a*b*B)*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(a + b) + (4*a*(-(A*b) + a*B)*(2*EllipticF[(c + d*x)/2, 2]
- (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(-(A*b)
 + a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*El
lipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]
))/((a - b)*(a + b)))/(4*a*d)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3479, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{2Aa^2-bBa-2(Ab-aB)\cos(c+dx)a-Ab^2-b(Ab-aB)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Aa^2-bBa-2(Ab-aB)\cos(c+dx)a-Ab^2-b(Ab-aB)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2Aa^2-bBa-2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-Ab^2-b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3538}$$

$$\frac{-\frac{\int -\frac{b\left(2Aa^2-bBa-Ab^2\right)-ab(Ab-aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\left((Ab-aB)\int \sqrt{\cos(c+dx)}dx\right)}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\left(2Aa^2-bBa-Ab^2\right)-ab(Ab-aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-(Ab-aB)\int \sqrt{\cos(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{b\left(2Aa^2-bBa-Ab^2\right)-ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-(Ab-aB)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3119

$$\frac{\frac{\int \frac{b\left(2Aa^2-bBa-Ab^2\right)-ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3481

$$\frac{\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-a(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3042

$$\frac{\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-a(Ab-aB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3120

$$\frac{\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2a(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3284

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\,(a^2-b^2)\,(a+b\cos(c+dx))}+\frac{\frac{\frac{2\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{2a(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\,(a^2-b^2)}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

output
```
((-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/d + ((-2*a*(A*b - a*B)*Ellipti
cF[(c + d*x)/2, 2])/d + (2*(3*a^2*A*b - A*b^3 - a^3*B - a*b^2*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/(2*a*(a^2 - b^2)) + (b*(
A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(203) = 406$.

Time = 5.89 (sec) , antiderivative size = 721, normalized size of antiderivative = 3.60

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\frac{4B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}+\frac{2(Ab-Ba)}{}\right.}{}$

input

```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2
*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2
*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+
2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{A+B\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,(a+b\cos(c+d\,x))^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2 b+\cos(dx+c)\,a}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*b + cos(c + d*x)*a),x)
```

3.375 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 256

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx \\
&= -\frac{(2a^2A-3Ab^2+abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2(a^2-b^2)\,d}+\frac{(Ab-aB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a(a^2-b^2)\,d} \\
&\quad-\frac{(5a^2Ab-3Ab^3-3a^3B+ab^2B)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a-b)(a+b)^2d} \\
&\quad+\frac{(2a^2A-3Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}}+\frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}
\end{aligned}$$

output

```
-(2*A*a^2-3*A*b^2+B*a*b)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^
2)/d+(A*b-B*a)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a/(a^2-b^2)/d-(5*A*a
^2*b-3*A*b^3-3*B*a^3+B*a*b^2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1
/2))/a^2/(a-b)/(a+b)^2/d+(2*A*a^2-3*A*b^2+B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/
d/cos(d*x+c)^(1/2)+b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(
a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.19 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.23

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{-\frac{\frac{2(-10a^2Ab+9Ab^3+4a^3B-3ab^2B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}-\frac{8a(a^2A-2Ab^2+abB)\left((a+b)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\right)}{b(a+b)}-2(\cdots}{(\cdots}}{\cdots}$$

input

```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2)
,x]
```

output

```
(-(((2*(-10*a^2*A*b + 9*A*b^3 + 4*a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2])/(a + b) - (8*a*(a^2*A - 2*A*b^2 + a*b*B)*((a + b)*El
lipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b
*(a + b)) - (2*(2*a^2*A - 3*A*b^2 + a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[C
os[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1]
 + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[
c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))) + 4*Sqrt[Cos[c +
 d*x]]*((b^2*(A*b - a*B)*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]))
 + 2*A*Tan[c + d*x]))/(4*a^2*d)
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{2Aa^2+bBa-2(Ab-aB)\cos(c+dx)a-3Ab^2+b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Aa^2+bBa-2(Ab-aB)\cos(c+dx)a-3Ab^2+b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2Aa^2+bBa-2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-3Ab^2+b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{2\int -\frac{-2Ba^3+4Aba^2+b^2Ba+2\left(Aa^2+bBa-2Ab^2\right)\cos(c+dx)a-3Ab^3+b\left(2Aa^2+bBa-3Ab^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{2\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{-2Ba^3+4Aba^2+b^2Ba+2\left(Aa^2+bBa-2Ab^2\right)\cos(c+dx)a-3Ab^3+b\left(2Aa^2+bBa-3Ab^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int\frac{-2Ba^3+4Aba^2+b^2Ba+2\left(Aa^2+bBa-2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-3Ab^3+b\left(2Aa^2+bBa-3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3538

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(2a^2A+abB-3Ab^2)\int\sqrt{\cos(c+dx)}dx-\int-\frac{b\left(-2Ba^3+4Aba^2+b^2Ba-3Ab^3\right)-ab^2(Ab-aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 25

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(2a^2A+abB-3Ab^2)\int\sqrt{\cos(c+dx)}dx+\int\frac{b\left(-2Ba^3+4Aba^2+b^2Ba-3Ab^3\right)-ab^2(Ab-aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(2a^2A+abB-3Ab^2)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{b\left(-2Ba^3+4Aba^2+b^2Ba-3Ab^3\right)-ab^2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3119

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int\frac{b\left(-2Ba^3+4Aba^2+b^2Ba-3Ab^3\right)-ab^2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-ab(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}+\frac{2\left(2a^2A+abB-3A\cdots\right.}{\cdots}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-ab(Ab-aB)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(2a\cdots\right.}{\cdots}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2ab(Ab-aB)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}+\frac{2\left(\cdots\right.}{\cdots}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}+$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{2\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)|2\right)}{d}+\frac{\frac{2b\left(-3a^3B+5a^2Ab+ab^2B-3Ab^3\right)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{2ab(Ab-aB)\,\mathrm{I}\cdots}{\cdots}}{b}}{a}}{2a\left(a^2-b^2\right)}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

output
```
(b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Co
s[c + d*x])) + (-(((2*(2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2
])/d + ((-2*a*b*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/d + (2*b*(5*a^2*A*b
 - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/((a + b)*d))/b)/a) + (2*(2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a*d*Sq
rt[Cos[c + d*x]]))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(257) = 514$.

Time = 6.50 (sec) , antiderivative size = 856, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	856

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2/sin(1/
2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))-2*(A*b-B*a)/a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a
-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/
(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2)))+4*A*b^2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3} b+\cos\left(dx+c\right)^{2} a}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*b + cos(c + d*x)**2*a),x)
```

3.376 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 345

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx \\
&= \frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3\,(a^2-b^2)\,d} \\
&+ \frac{(2a^2A-5Ab^2+3abB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2\,(a^2-b^2)\,d} \\
&+ \frac{b(7a^2Ab-5Ab^3-5a^3B+3ab^2B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^3(a-b)(a+b)^2d} \\
&+ \frac{(2a^2A-5Ab^2+3abB)\sin(c+dx)}{3a^2\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)} \\
&- \frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sin(c+dx)}{a^3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} \\
&+ \frac{b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}
\end{aligned}$$

output
```
(4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/a^3/(a^2-b^2)/d+1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/a^2/(a^2-b^2)/d+b*(7*A*a^2*b-5*A*b^3-5*B*a^3+3*B*a*b^2)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+1/3*(2*A*a
^2-5*A*b^2+3*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)-(4*A*a^2*b
-5*A*b^3-2*B*a^3+3*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)+b*
(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.35 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\frac{2(4a^4A+44a^2Ab^2-45Ab^4-30a^3bB+27ab^3B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{(28a^3Ab-40aAb^3-12a^4B+24a^2b^2B)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{b}}{}$$

$$+\frac{\sqrt{\cos(c+dx)}\left(\frac{2\sec(c+dx)(-2Ab\sin(c+dx)+aB\sin(c+dx))}{a^3}+\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{a^3(a^2-b^2)(a+b\cos(c+dx))}+\frac{2A\sec(c+dx)\tan(c+dx)}{3a^2}\right)}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2)
,x]
```

output
```
((2*(4*a^4*A + 44*a^2*A*b^2 - 45*A*b^4 - 30*a^3*b*B + 27*a*b^3*B)*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + ((28*a^3*A*b - 40*a*A*b^3 - 1
2*a^4*B + 24*a^2*b^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(12*a^2*A*b^2 - 15*A*b^4 - 6
*a^3*b*B + 9*a*b^3*B)*Cos[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (
-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c +
d*x])/(a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(12*a^3*(a
 - b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*
x] + a*B*Sin[c + d*x]))/a^3 + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x])/
(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(
3*a^2)))/d
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{2Aa^2+3bBa-2(Ab-aB)\cos(c+dx)a-5Ab^2+3b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Aa^2+3bBa-2(Ab-aB)\cos(c+dx)a-5Ab^2+3b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2Aa^2+3bBa-2(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-5Ab^2+3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{2\int -\frac{-b\left(2Aa^2+3bBa-5Ab^2\right)\cos^2(c+dx)-2a\left(Aa^2-3bBa+2Ab^2\right)\cos(c+dx)+3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}+\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 27

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(2Aa^2+3bBa-5Ab^2\right)\cos^2(c+dx)-2a\left(Aa^2-3bBa+2Ab^2\right)\cos(c+dx)+3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(2Aa^2+3bBa-5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\left(Aa^2-3bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{3a}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3534

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2\int -\frac{2Aa^4-12bBa^3+16Ab^2a^2+9b^3Ba+2\left(-3Ba^3+7Aba^2+6b^2Ba-10Ab^3\right)\cos(c+dx)a-15Ab^4+3b\left(-2Ba^3+4Aba^2+\right.}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 27

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{2Aa^4-12bBa^3+16Ab^2a^2+9b^3Ba+2\left(-3Ba^3+7Aba^2+6b^2Ba-10Ab^3\right.}{\sqrt{\cos(c+dx)}(}}{}}{3a}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int\frac{2Aa^4-12bBa^3+16Ab^2a^2+9b^3Ba+2\left(-3Ba^3+7Aba^2+6b^2Ba-10Ab^3\cdots\right.}{\sqrt{\sin\left(c+dx+\cdots\right.}}}{3a}}{\cdots}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\downarrow$ 3538

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\int\sqrt{\cos(c+dx)}dx-\frac{\int-\frac{a\left(2Aa^2+3bB\cdots\right.}{\cdots}}{\cdots}}{3a}}{\cdots}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\downarrow$ 25

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\int\sqrt{\cos(c+dx)}dx+\frac{\int\frac{a\left(2Aa^2+3bBa\cdots\right.}{\cdots}}{\cdots}}{3a}}{\cdots}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{a\left(2Aa^2+3\cdots\right.}{\cdots}}{\cdots}}{3a}}{\cdots}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\downarrow$ 3119

$$\frac{\frac{2\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(2Aa^2+3bBa-5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(2Aa^4-12bBa^3+16Ab^2a^2\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{b}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3481

$$\frac{\frac{2\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{ab\left(2a^2A+3abB-5Ab^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b^2\left(-5a^3B+7a^2Ab+3ab^2\right.}{b}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{ab\left(2a^2A+3abB-5Ab^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2\left(-5a^3B+7a^2Ab+3\right.}{b}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b^2\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx\right.\right.}}{b}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3284

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+$$

$$\frac{\frac{2\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{6\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\left(2a^2A+3abB-\right.}{}}{3a}}{2a\left(a^2-b^2\right)}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

output
```
(b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Co
s[c + d*x])) + ((2*(2*a^2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a*d*Cos[
c + d*x]^(3/2)) - (-(((6*(4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Ellip
ticE[(c + d*x)/2, 2])/d + ((2*a*b*(2*a^2*A - 5*A*b^2 + 3*a*b*B)*EllipticF[
(c + d*x)/2, 2])/d + (6*b^2*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/a) + (6*(4*a^2*A*
b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]))
/(3*a))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(340) = 680$.

Time = 8.00 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.91

method	result	size
default	Expression too large to display	1005

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(2*A*b-B*a)/a^3/sin(1/2*d*x+1/2*c)
^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))-4*b^2*(2*A*b-B*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(A*b-B*a)*b/
a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b
/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))^2} \, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))^2} \, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))^2} \, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))^2} \, dx = \int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}}} \, dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4} b+\cos\left(dx+c\right)^{3} a}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*b + cos(c + d*x)**3*a),x)
```

3.377 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 367

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx &= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3(a^2-b^2)^2d}\\
&\quad+\frac{(3a^4Ab-5a^2Ab^3+8Ab^5-15a^5B+33a^3b^2B-24ab^4B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^4(a^2-b^2)^2d}\\
&\quad-\frac{a(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^4(a+b)^3d}\\
&\quad+\frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2}\\
&\quad+\frac{a(a^2Ab-7Ab^3-5a^3B+11ab^2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d+1/4*(3*A*a^4*b-5*A*a^2*b^3+8*A*b^5-
15*B*a^5+33*B*a^3*b^2-24*B*a*b^4)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b
^4/(a^2-b^2)^2/d-1/4*a*(3*A*a^4*b-6*A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b
^2-35*B*a*b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^
4/(a+b)^3/d+1/2*a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b
*cos(d*x+c))^2+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*cos(d*x+c)^(1/2)
*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 6.11 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.06

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{-\frac{2a\sqrt{\cos(c+dx)}\left(a\left(-a^2Ab+7Ab^3+5a^3B-11ab^2B\right)+b\left(-3a^2Ab+9Ab^3+7a^3B-13ab^2B\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{\left(-a^3Ab-5aAb^3+5a^4B-7a\right.}{}}{}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3
,x]
```

output
```
((-2*a*Sqrt[Cos[c + d*x]]*(a*(-(a^2*A*b) + 7*A*b^3 + 5*a^3*B - 11*a*b^2*B)
 + b*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*Cos[c + d*x])*Sin[c + d
*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((-(a^3*A*b) - 5*a*A*b^3 +
5*a^4*B - 7*a^2*b^2*B + 8*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a + b) + (8*(a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*((a + b)*EllipticF[
(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) +
((-3*a^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*(-2*a*b*Elli
pticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c
+ d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a +
 b)^2))/(8*b^2*d)
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int -\frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+Aba+4b^2B\right)\cos^2(c+dx)\right)-4b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{2(a+b\cos(c+dx))^2}dx}{2b(a^2-b^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+Aba+4b^2B\right)\cos^2(c+dx)\right)-4b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{(a+b\cos(c+dx))^2}dx}{4b(a^2-b^2)}+\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5Ba^2-Aba-4b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b(a^2-b^2)}+\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3526}$$

$$\frac{\frac{a(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int -\frac{-\left(\left(\left(-15Ba^4+3Aba^3+29b^2Ba^2-9Ab^3a-8b^4B\right)\cos^2(c+dx)\right)+4b\left(Ba^3+Aba^2-4b^2Ba+\right.\right.}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-\left(\left(\left(-15Ba^4+3Aba^3+29b^2Ba^2-9Ab^3a-8b^4B\right)\cos^2(c+dx)\right)+4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\cos(c+dx)+a\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2b(a^2-b^2)}+\frac{a(-5a^3}{}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(15Ba^4-3Aba^3-29b^2Ba^2+9Ab^3a+8b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}+\frac{a(-5a}{}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3538

$$\frac{\frac{-\frac{\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{ab\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)+\left(-15Ba^5+3Aba^4+33b^2Ba^3-5Ab^3a^2-24b^4Ba+8Ab\right.}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{ab\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)+\left(-15Ba^5+3Aba^4+33b^2Ba^3-5Ab^3a^2-24b^4Ba+8Ab^5\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\int\sqrt{}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{ab\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)+\left(-15Ba^5+3Aba^4+33b^2Ba^3-5Ab^3a^2-24b^4Ba+8Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{\int \frac{ab\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)+\left(-15Ba^5+3Aba^4+33b^2Ba^3-5Ab^3a^2-24b^4Ba+8Ab^5\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right.}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3481

$$\frac{\frac{\frac{\left(-15a^5B+3a^4Ab+33a^3b^2B-5a^2Ab^3-24ab^4B+8Ab^5\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+}}{b}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(-15a^5B+3a^4Ab+33a^3b^2B-5a^2Ab^3-24ab^4B+8Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\right.}}{b}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-\right.}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{2\left(-15a^5B+3a^4Ab+33a^3b^2B-5a^2Ab^3-24ab^4B+8Ab^5\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int\frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\right.}}{b}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-\right.}$$

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3284

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\,(a^2-b^2)\,(a+b\cos(c+dx))^2}+\frac{\frac{a(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{2\left(-15a^5B+3a^4Ab+33a^3b^2B-5a^2Ab^3-24ab^4B+8Ab^5\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\left(-15a\right.}{}}{b}}{}}{4b\,(a^2-b^2)}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2) + (((-2*(3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B
- 8*b^4*B)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*(3*a^4*A*b - 5*a^2*A*b^3
 + 8*A*b^5 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4*B)*EllipticF[(c + d*x)/2,
2])/(b*d) - (2*a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b
^2*B - 35*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d
))/b)/(2*b*(a^2 - b^2)) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(4*
b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(358) = 716$.

Time = 83.55 (sec) , antiderivative size = 1977, normalized size of antiderivative = 5.39

method	result	size
default	Expression too large to display	1977

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*a^2/b^4*(3*A*b-4*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/
2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b
+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2)))+12/b^3*a*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos(c+d\,x)^{5/2}\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)**2*b**2 + 2*cos(c +
 d*x)*a*b + a**2),x)
```

3.378 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3} dx$

Optimal result

Integrand size = 33, antiderivative size = 344

$$
\begin{aligned}
&\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2(a^2-b^2)^2d} \\
&\quad+\frac{(a^3Ab-7aAb^3+3a^4B-5a^2b^2B+8b^4B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^3(a^2-b^2)^2d} \\
&\quad-\frac{(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^3(a+b)^3d} \\
&\quad+\frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b(a^2-b^2)\,d(a+b\cos(c+dx))^2} \\
&\quad+\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b(a^2-b^2)^2\,d(a+b\cos(c+dx))}
\end{aligned}
$$

output
```
-1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))/b^2/(a^2-b^2)^2/d+1/4*(A*a^3*b-7*A*a*b^3+3*B*a^4-5*B*a^2*b^2+8*B*b^4)
*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^3/(a^2-b^2)^2/d-1/4*(A*a^4*b-10*
A*a^2*b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*B*a*b^4)*EllipticPi(sin(1/2*d*x+1
/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d+1/2*a*(A*b-B*a)*cos(d*x+c)^
(1/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/4*(A*a^2*b+5*A*b^3+3*B
*a^3-9*B*a*b^2)*cos(d*x+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c
))
```

Mathematica [A] (warning: unable to verify)

Time = 4.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.05

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{2\sqrt{\cos(c+dx)}\left(a(3a^2Ab+3Ab^3+a^3B-7ab^2B)+b(a^2Ab+5Ab^3+3a^3B-9ab^2B)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} - \frac{\frac{\left(-5a^2Ab-Ab^3+a^3B+5ab^2B\right)\text{EllipticPi}}{a+b}}{}}{}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3
,x]
```

output
```
((2*Sqrt[Cos[c + d*x]]*(a*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B) + b*(a
^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2
- b^2)^2*(a + b*Cos[c + d*x])^2) - (((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2
*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(-3*a*A*b + a^
2*B + 2*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2]))/(a + b) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*
B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ellipti
cF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), Arc
Sin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/
((a - b)^2*(a + b)^2))/(8*b*d)
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3468, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int -\frac{\left(3Ba^2+Aba-4b^2B\right)\cos^2(c+dx)-4b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(3Ba^2+Aba-4b^2B\right)\cos^2(c+dx)-4b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{4b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(3Ba^2+Aba-4b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{-a\left(3Ba^3+Aba^2-9b^2Ba+5Ab^3\right)\cos^2(c+dx)-4ab\left(-Ba^2+3Aba-2b^2B\right)\cos(c+dx)+a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{(3a^3B+a^2Ab-9ab^2B+5Ab^3)}{d(a^2-b^2)(a+b\cos}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{-a\left(3Ba^3+Aba^2-9b^2Ba+5Ab^3\right)\cos^2(c+dx)-4ab\left(-Ba^2+3Aba-2b^2B\right)\cos(c+dx)+a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{(3a^3B+a^2Ab-9ab^2B+5Ab^3)}{d(a^2-b^2)(a+b\cos}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-a\left(3Ba^3+Aba^2-9b^2Ba+5Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4ab\left(-Ba^2+3Aba-2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{(3a^3B+a^2Ab-9ab^2B}{d(a^2-b^2}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3538

$$\frac{\frac{-\frac{a\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{ab\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)+a\left(3Ba^4+Aba^3-5b^2Ba^2-7Ab^3a+8b^4B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2a(a^2-b^2)}+(3a^3B+}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{ab\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)+a\left(3Ba^4+Aba^3-5b^2Ba^2-7Ab^3a+8b^4B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{a\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\int\sqrt{\cos(c+dx)}dx}{b}}{2a(a^2-b^2)}+(3a^3B+a^2A}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{ab\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)+a\left(3Ba^4+Aba^3-5b^2Ba^2-7Ab^3a+8b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2a(a^2-b^2)}+\frac{(3a}{}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{\int \frac{ab\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)+a\left(3Ba^4+Aba^3-5b^2Ba^2-7Ab^3a+8b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2a\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2a(a^2-b^2)}+\frac{(3a^3B+}{}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3481

$$\frac{\frac{\frac{\frac{a\left(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\left(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{2a\left(3a^3B\right.}{}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{a\left(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{b}}{b}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{\frac{2a\left(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\left(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{b}}{b}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\Big\downarrow$ 3284

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+$$
$$\frac{\frac{\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{2a\left(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B\right)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\left(3a^5B+a^4Ab-6a^3b^2B-1\right.}{}}{b}}{2}}{4b\left(a^2-b^2\right)}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2) + (((-2*a*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Ellip
ticE[(c + d*x)/2, 2])/(b*d) + ((2*a*(a^3*A*b - 7*a*A*b^3 + 3*a^4*B - 5*a^2
*b^2*B + 8*b^4*B)*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*(a^4*A*b - 10*a^
2*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*b)/(
a + b), (c + d*x)/2, 2])/(b*(a + b)*d))/b)/(2*a*(a^2 - b^2)) + ((a^2*A*b +
 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b
^2)*d*(a + b*Cos[c + d*x])))/(4*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1936 vs. $2(335) = 670$.

Time = 8.41 (sec) , antiderivative size = 1937, normalized size of antiderivative = 5.63

method	result	size
default	Expression too large to display	1937

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
4/b^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-B*a)/b^3*(-1
/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+
b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos{(c+d\,x)}^{3/2}\,(A+B\,\cos{(c+d\,x)})}{(a+b\,\cos{(c+d\,x)})^3}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}\,\cos{(dx+c)}}{\cos{(dx+c)}^2\,b^2+2\cos{(dx+c)}\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(c + d*
x)*a*b + a**2),x)
```

3.379 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 337

$$\begin{aligned}
&\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx \\
&= \frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\,(a^2-b^2)^2\,d} \\
&\quad+\frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2\,(a^2-b^2)^2\,d} \\
&\quad-\frac{(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a(a-b)^2b^2(a+b)^3d} \\
&\quad-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} \\
&\quad-\frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\,\sqrt{\cos(c+dx)}\sin(c+dx)}{4a\,(a^2-b^2)^2\,d(a+b\cos(c+dx))}
\end{aligned}$$

output
```
1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/a/b/(a^2-b^2)^2/d+1/4*(3*A*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2))/b^2/(a^2-b^2)^2/d-1/4*(3*A*a^4*b+10*A*a^2*b^3-A*b
^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2
^(1/2))/a/(a-b)^2/b^2/(a+b)^3/d-1/2*(A*b-B*a)*cos(d*x+c)^(1/2)*sin(d*x+c)/
(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*cos(d
*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.96 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{4\sqrt{\cos(c+dx)}\left(a\left(-7a^2Ab+Ab^3+3a^3B+3ab^2B\right)+b\left(-5a^2Ab-Ab^3+a^3B+5ab^2B\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{\frac{2\left(-9a^2Ab+3Ab^3+5a^3B+ab^2B\right)\,\mathrm{Ellip}\ldots}{a+b}\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3
,x]
```

output
```
((4*Sqrt[Cos[c + d*x]]*(a*(-7*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B) + b*(
-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2
- b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a
*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*a*(2*a^2*
A + A*b^2 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) - (2*(-5*a^2*A*b - A*b^3 + a^3*B
 + 5*a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
 b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[
-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c +
 d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3478, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3478}$$

$$-\frac{\int \frac{(Ab-aB)\cos^2(c+dx)-4(aA-bB)\cos(c+dx)+Ab-aB}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(Ab-aB)\cos^2(c+dx)-4(aA-bB)\cos(c+dx)+Ab-aB}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{4\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab-aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$-\frac{\frac{\int \frac{-3Ba^3+7Aba^2-3b^2Ba-4\left(2Aa^2-3bBa+Ab^2\right)\cos(c+dx)a-Ab^3-\left(-Ba^3+5Aba^2-5b^2Ba+Ab^3\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{(a^3(-B)+5a^2Ab-5ab^2B+Ab^3)\,\text{si}}{ad(a^2-b^2)(a+b\cos(}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{-3Ba^3+7Aba^2-3b^2Ba-4\left(2Aa^2-3bBa+Ab^2\right)\cos(c+dx)a-Ab^3-\left(-Ba^3+5Aba^2-5b^2Ba+Ab^3\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\mathrm{si}}{ad(a^2-b^2)(a+b\cos(}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{-3Ba^3+7Aba^2-3b^2Ba-4\left(2Aa^2-3bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-Ab^3+\left(Ba^3-5Aba^2+5b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{\left(a^3(-B)+5a^2Ab-5ab^2B+\right.}{ad(a^2-b^2)(a-}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3538

$$-\frac{\frac{-\frac{\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{b\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)-a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2a(a^2-b^2)}+\frac{\left(a^3(-B)+5a\right.}{}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{b\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)-a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\int\sqrt{\cos(c+dx)}dx}{b}}{2a(a^2-b^2)}+\frac{\left(a^3(-B)+5a^2A\right.}{ad}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{b\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)-a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2a(a^2-b^2)}+\frac{\left(a^3(-B\right.}{}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3119

$$-\frac{\frac{\frac{\int \frac{b\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)-a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}+\frac{(a^3(-B)+5a}{}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3481

$$-\frac{\frac{\frac{\left(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{a\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}}{b}-\frac{2\left(a^3(-B)+5a^2Ab\right.}{}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\left(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{2\left(a\right.}{}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3120

$$-\frac{\frac{\frac{\left(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2a\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}}{b}-\frac{2\left(\right.}{}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3284

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-$$

$$\frac{\frac{(a^3(-B)+5a^2Ab-5ab^2B+Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{2\left(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}-\frac{2a\left(a^3B+3\right.}{}}{b}}{2a(}}{4\left(a^2-b^2\right)}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*C
os[c + d*x])^2) - (((-2*(5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[
(c + d*x)/2, 2])/(b*d) + ((-2*a*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*
EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(3*a^4*A*b + 10*a^2*A*b^3 - A*b^5 +
a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(b*(a + b)*d))/b)/(2*a*(a^2 - b^2)) + ((5*a^2*A*b + A*b^3 - a^3*B - 5*a
*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])))/(4*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1849 vs. $2(328) = 656$.

Time = 8.18 (sec) , antiderivative size = 1850, normalized size of antiderivative = 5.49

method	result	size
default	Expression too large to display	1850

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/b/(-2*a*b
+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d
*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-
2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a*(A*b-B*a)/b^2*(-1/2*b^2/a/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*co...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2 b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(c + d*x)*a*b + a**2),
x)
```

3.380 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 345

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2(a^2-b^2)^2d} \\
&\quad -\frac{(7a^2Ab-Ab^3-3a^3B-3ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4ab(a^2-b^2)^2d} \\
&\quad +\frac{(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^2(a-b)^2b(a+b)^3d} \\
&\quad +\frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a(a^2-b^2)\,d(a+b\cos(c+dx))^2} \\
&\quad +\frac{b(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a^2(a^2-b^2)^2\,d(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))/a^2/(a^2-b^2)^2/d-1/4*(7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*InverseJaco
biAM(1/2*d*x+1/2*c,2^(1/2))/a/b/(a^2-b^2)^2/d+1/4*(15*A*a^4*b-6*A*a^2*b^3+
3*A*b^5-3*B*a^5-10*B*a^3*b^2+B*a*b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a
+b),2^(1/2))/a^2/(a-b)^2/b/(a+b)^3/d+1/2*b*(A*b-B*a)*cos(d*x+c)^(1/2)*sin(
d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/4*b*(9*A*a^2*b-3*A*b^3-5*B*a^3-B
*a*b^2)*cos(d*x+c)^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.39 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.11

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{-\frac{2b\sqrt{\cos(c+dx)}\left(a\left(-11a^2Ab+5Ab^3+7a^3B-ab^2B\right)+b\left(-9a^2Ab+3Ab^3+5a^3B+ab^2B\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{\left(16a^4A-19a^2Ab^2+9Ab^4-9a^3\right.}{}}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3)
,x]
```

output
```
((-2*b*Sqrt[Cos[c + d*x]]*(a*(-11*a^2*A*b + 5*A*b^3 + 7*a^3*B - a*b^2*B) +
 b*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/
((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((16*a^4*A - 19*a^2*A*b^2 + 9*A*
b^4 - 9*a^3*b*B + 3*a*b^3*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
 + b) + (8*a*(-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*((a + b)*EllipticF[(
c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b))
 + ((-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]],
 -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*
Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*a^2*d)
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{4Aa^2-bBa-4(Ab-aB)\cos(c+dx)a-3Ab^2+b(Ab-aB)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4Aa^2-bBa-4(Ab-aB)\cos(c+dx)a-3Ab^2+b(Ab-aB)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4Aa^2-bBa-4(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-3Ab^2+b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)a+3Ab^4-b\left(-5Ba^3+9Aba^2-b^2Ba-3Ab^3\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{b(-5a^3B+9a^2Ab-}{ad(a}}{4a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)a+3Ab^4-b\left(-5Ba^3+9Aba^2-b^2Ba-3Ab^3\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{b(-5a^3B+9a^2Ab-}{ad(a}}{4a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3Ab^4-b\left(-5Ba^3+9Aba^2-b^2Ba-3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b(-5a^3B+}{}}{4a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3538

$$\frac{\frac{-\left(\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\int\sqrt{\cos(c+dx)}dx\right)-\frac{\int -\frac{b\left(8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba+3Ab^4\right)-ab\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2a(a^2-b^2)}+}{4a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{b\left(8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba+3Ab^4\right)-ab\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\int\sqrt{\cos(c+dx)}dx}{2a(a^2-b^2)}+b(-5}{4a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{b\left(8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba+3Ab^4\right)-ab\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a(a^2-b^2)}\cdots}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{\int \frac{b\left(8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba+3Ab^4\right)-ab\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}+\frac{b(-5a\cdots}{\cdots}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3481

$$\frac{\frac{\frac{\left(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-a\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{2\left(-5a^3B+9a^2Ab\cdots\right)}{\cdots}}{2a(a^2-b^2)}\cdots}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-a\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(\cdots\right)}{\cdots}}{2a(a^2-b^2)}\cdots}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{\left(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2a\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2\cdots}{\cdots}}{2a(a^2-b^2)}\cdots}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3284

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{b\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{2\left(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{2a\left(-3a\cdots\right.}{\cdots}}{b}}{2\cdots}}{4a\left(a^2-b^2\right)}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

output
```
(b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2) + (((-2*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Ellipti
cE[(c + d*x)/2, 2])/d + ((-2*a*(7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*E
llipticF[(c + d*x)/2, 2])/d + (2*(15*a^4*A*b - 6*a^2*A*b^3 + 3*A*b^5 - 3*a
^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
((a + b)*d))/b)/(2*a*(a^2 - b^2)) + (b*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*
b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*
x])))/(4*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. $2(336) = 672$.

Time = 7.03 (sec) , antiderivative size = 1744, normalized size of antiderivative = 5.06

method	result	size
default	Expression too large to display	1744

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-b^2/a/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(A*b-B*a)/b*(-
1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2
/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3} b^{2}+2\cos\left(dx+c\right)^{2} ab+\cos\left(dx+c\right) a^{2}}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*b**2 + 2*cos(c + d*x)**2*a*b + cos
(c + d*x)*a**2),x)
```

3.381 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 420

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a^2-b^2)^2 d} \\
&\quad+\frac{(11a^2Ab-5Ab^3-7a^3B+ab^2B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2(a^2-b^2)^2 d} \\
&\quad-\frac{(35a^4Ab-38a^2Ab^3+15Ab^5-15a^5B+6a^3b^2B-3ab^4B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^3(a-b)^2(a+b)^3 d} \\
&\quad+\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\sin(c+dx)}{4a^3(a^2-b^2)^2 d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{b(Ab-aB)\sin(c+dx)}{2a(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} \\
&\quad+\frac{b(11a^2Ab-5Ab^3-7a^3B+ab^2B)\sin(c+dx)}{4a^2(a^2-b^2)^2 d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a
*b^2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*(35*A*a
^4*b-38*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d+1/4*(8*A*a^4-29*A*
a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x
+c)^(1/2)+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*c
os(d*x+c))^2+1/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^
2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.95 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{-\frac{\frac{\left(56a^4Ab-95a^2Ab^3+45Ab^5-16a^5B+19a^3b^2B-9ab^4B\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8a\left(2a^4A-10a^2Ab^2+5Ab^4+4a^3bB-ab^3B\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx\right.\right.}{b(a+b)}}{}}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3)
,x]
```

output
```
(-((((56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B - 9*a
*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^4*A
 - 10*a^2*A*b^2 + 5*A*b^4 + 4*a^3*b*B - a*b^3*B)*((a + b)*EllipticF[(c + d
*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((
8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*(-2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos
[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d
*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2
)) + (Sqrt[Cos[c + d*x]]*(2*a*b*(16*a^4*A - 47*a^2*A*b^2 + 25*A*b^4 + 11*a
^3*b*B - 5*a*b^3*B)*Sin[c + d*x] + b^2*(8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4
+ 9*a^3*b*B - 3*a*b^3*B)*Sin[2*(c + d*x)] + 16*A*(a^3 - a*b^2)^2*Tan[c + d
*x]))/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(8*a^3*d)
```

Rubi [A] (verified)

Time = 3.08 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{4Aa^2+bBa-4(Ab-aB)\cos(c+dx)a-5Ab^2+3b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4Aa^2+bBa-4(Ab-aB)\cos(c+dx)a-5Ab^2+3b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4Aa^2+bBa-4(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-5Ab^2+3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)a+15Ab^4+b\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{b(-7a^3B+11a}{ad(a^2-b^2)\sqrt{}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)a+15Ab^4+b\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{b(-7a^3B+11a}{ad(a^2-b^2)\sqrt{}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+15Ab^4+b\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b(-7a}{ad(a}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3534

$$\frac{\frac{\frac{2\int -\frac{-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2-3b^4Ba+4\left(2Aa^4+4bBa^3-10Ab^2a^2-b^3Ba+5Ab^4\right)\cos(c+dx)a+15Ab^5+b\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2-3b^4Ba+4\left(2Aa^4+4bBa^3-10Ab^2a^2-b^3Ba+5Ab^4\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2-3b^4Ba+4\left(2Aa^4+4bBa^3-10Ab^2a^2-b^3Ba+5Ab^4\right)\sin\left(c+dx+\cdots\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx-\cdots\right.\right.}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\cdots$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3538$$

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\int\sqrt{\cos(c+dx)}dx-\frac{\int -\frac{b\left(-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3\cdots\right.}{\cdots}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\cdots$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\int\sqrt{\cos(c+dx)}dx+\frac{\int \frac{b\left(-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2\cdots\right.}{\cdots}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\cdots$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\left(-8Ba^5+24Aba^4+5b^2Ba^3-33A\cdots\right.}{\cdots}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\cdots$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3119$$

$$\frac{\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{b\left(-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2-3b^4Ba+15Ab^5\right)-ab^2\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\sin\left(c\cdots\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{b}\cdots}{a}\cdots}{2a(a^2-b^2)}\cdots}{4a\left(a^2-b^2\right)}\cdots$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3481$$

$$\frac{\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-ab\left(-7a^3\cdots\right.}{b}\cdots}{2a(a^2-b^2)}\cdots}{4a\left(a^2\cdots\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\cdots}{b}\cdots}{2a(a^2-b^2)}\cdots}{4\cdots}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\cdots}{b}\cdots}{2a(a^2-b^2)}\cdots$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{b\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}+\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)E\left(\cdots\right.}{d}\cdots}{\cdots}}{4a\left(a\cdots\right.}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

output
```
(b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*
Cos[c + d*x])^2) + ((b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sin[c +
d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])) + (-(((2*(
8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE[(c +
d*x)/2, 2])/d + ((-2*a*b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Ellipt
icF[(c + d*x)/2, 2])/d + (2*b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a
^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/((a + b)*d))/b)/a) + (2*(8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B -
3*a*b^3*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]))/(2*a*(a^2 - b^2)))/(4*a
*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1974 vs. $2(407) = 814$.

Time = 8.94 (sec) , antiderivative size = 1975, normalized size of antiderivative = 4.70

method	result	size
default	Expression too large to display	1975

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3/sin(1/
2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))-2*(A*b-B*a)/a*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)
^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)
-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^
2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$
$$=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4} b^{2}+2\cos\left(dx+c\right)^{3} ab+\cos\left(dx+c\right)^{2} a^{2}}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*b**2 + 2*cos(c + d*x)**3*a*b + cos
(c + d*x)**2*a**2),x)
```

3.382 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 523

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx \\
&= \frac{(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d} \\
&+ \frac{(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12a^3\left(a^2-b^2\right)^2d} \\
&+ \frac{b(63a^4Ab-86a^2Ab^3+35Ab^5-35a^5B+38a^3b^2B-15ab^4B)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^4(a-b)^2(a+b)^3d} \\
&+ \frac{(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} \\
&- \frac{(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B)\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} \\
&+ \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} \\
&+ \frac{b(13a^2Ab-7Ab^3-9a^3B+3ab^2B)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}
\end{aligned}$$

output
```
1/4*(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*A*a^4-61*A*a^
2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^3)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
))/a^3/(a^2-b^2)^2/d+1/4*b*(63*A*a^4*b-86*A*a^2*b^3+35*A*b^5-35*B*a^5+38*B
*a^3*b^2-15*B*a*b^4)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^4/
(a-b)^2/(a+b)^3/d+1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^
3)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)-1/4*(24*A*a^4*b-65*A*a^2*
b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d
/cos(d*x+c)^(1/2)+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2
)/(a+b*cos(d*x+c))^2+1/4*b*(13*A*a^2*b-7*A*b^3-9*B*a^3+3*B*a*b^2)*sin(d*x+
c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.68 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{2(16a^6A+328a^4Ab^2-641a^2Ab^4+315Ab^6-168a^5bB+285a^3b^3B-135ab^5B)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{(160a^5Ab-512a^3Ab^3+280aAb^5-}{}}{} + \frac{\sqrt{\cos(c+dx)}\left(\frac{2\sec(c+dx)(-3Ab\sin(c+dx)+aB\sin(c+dx))}{a^4}+\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{2a^3(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{17a^2Ab^4\sin(c+dx)-11Ab^6\sin(}{4a^4(a^2}\right.}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3)
,x]
```

output
```
((2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 315*A*b^6 - 168*a^5*b*B +
285*a^3*b^3*B - 135*a*b^5*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
 + b) + ((160*a^5*A*b - 512*a^3*A*b^3 + 280*a*A*b^5 - 48*a^6*B + 240*a^4*b
^2*B - 120*a^2*b^4*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(72*a^4*A*b^2 - 195*a^2*A*b^4
+ 105*A*b^6 - 24*a^5*b*B + 87*a^3*b^3*B - 45*a*b^5*B)*Cos[2*(c + d*x)]*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sq
rt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1
+ 2*Cos[c + d*x]^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]
*((2*Sec[c + d*x]*(-3*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/a^4 + (A*b^4*S
in[c + d*x] - a*b^3*B*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x]
)^2) + (17*a^2*A*b^4*Sin[c + d*x] - 11*A*b^6*Sin[c + d*x] - 13*a^3*b^3*B*S
in[c + d*x] + 7*a*b^5*B*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(a + b*Cos[c +
d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3))/d
```

Rubi [A] (verified)

Time = 4.08 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.98, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{\int \frac{4Aa^2+3bBa-4(Ab-aB)\cos(c+dx)a-7Ab^2+5b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4Aa^2+3bBa-4(Ab-aB)\cos(c+dx)a-7Ab^2+5b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{4Aa^2+3bBa-4(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-7Ab^2+5b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\int \frac{8Aa^4+33bBa^3-61Ab^2a^2-15b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)a+35Ab^4+3b\left(-9Ba^3+13Aba^2+3b^2Ba-7Ab^3\right)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{b(-9a^3B+\cdots}{ad(a^2-\cdots}}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{8Aa^4+33bBa^3-61Ab^2a^2-15b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\cos(c+dx)a+35Ab^4+3b\left(-9Ba^3+13Aba^2+3b^2Ba-7Ab^3\right)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{b(-9a^3B+\cdots}{ad(a^2-\cdots}}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{8Aa^4+33bBa^3-61Ab^2a^2-15b^3Ba-4\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+35Ab^4+3b\left(-9Ba^3+13Aba^2+3b^2Ba-7Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b(\cdots}{\cdots}}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{2\int -\frac{-b\left(8Aa^4+33bBa^3-61Ab^2a^2-15b^3Ba+35Ab^4\right)\cos^2(c+dx)-4a\left(2Aa^4-12bBa^3+14Ab^2a^2+3b^3Ba-7Ab^4\right)\cos(c+dx)+3\left(-8Ba^5+24Aba^4+29b^2Ba^3-65A\right.}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}}{3a}$$

$$2a(a^2-b^2)$$

$$4a\left(a^2-b^2\right)$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(8Aa^4+33bBa^3-61Ab^2a^2-15b^3Ba+35Ab^4\right)\cos^2(c+dx)-4a\left(2Aa^4-12bBa^3+14Ab^2a^2+3b^3Ba\right.}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+}}{3a}$$

$$2a(a^2-b^2)$$

$$4a\left(a^2-b^2\right)$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(8Aa^4+33bBa^3-61Ab^2a^2-15b^3Ba+35Ab^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(2Aa^4-12bBa^3+14Ab^2a^2+3b\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\,\mathrm{s}\right.}}{3a}$$

$$2a(a^2-b^2)$$

$$4a\left(a^2-b^2\right.$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3534

$$\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{8Aa^6-72bBa^5+128Ab^2a^4+99b^3Ba^3-223Ab^4a^2-45b^5Ba+4\left(-6Ba^5+20Aba^4+30b^2Ba^3-64A\right.}{2\sqrt{}}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int\frac{8Aa^6-72bBa^5+128Ab^2a^4\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int\frac{8Aa^6-72bBa^5+128Ab^2a^4\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3538$$

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3\left(-8a^5B+24a^4Ab+29a^3b^2\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3\left(-8a^5B+24a^4Ab+29a^3b^2\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3\left(-8a^5B+24a^4Ab+29a^3b^2\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3119

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(8Aa^4+33bBa^3-61Ab^2\right.}{}}{}}{}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3481

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{ab\left(8a^4A+33a^3bB-61a^2Ab^2\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{ab\left(8a^4A+33a^3bB-61a^2Ab^2\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3120

$$\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b^2\left(-35a^5B+63a^4Ab+38a^3\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{b\left(-9a^3B+13a^2Ab+3ab^2B-7Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{\frac{2\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{6\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15\cdots\right.}{ad\sqrt{\cos(c+dx)}}\cdots}{\cdots}}{\cdots}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

output
```
(b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*
Cos[c + d*x])^2) + ((b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c
+ d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])) + ((2*(8
*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*Sin[c + d*x])/
(3*a*d*Cos[c + d*x]^(3/2)) - (-(((6*(24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5
- 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B)*EllipticE[(c + d*x)/2, 2])/d + ((2*
a*b*(8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*Elliptic
F[(c + d*x)/2, 2])/d + (6*b^2*(63*a^4*A*b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a
^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/((a + b)*d))/b)/a) + (6*(24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B
 + 29*a^3*b^2*B - 15*a*b^4*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]))/(3*a
))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2131 vs. $2(506) = 1012$.

Time = 10.34 (sec) , antiderivative size = 2132, normalized size of antiderivative = 4.08

method	result	size
default	Expression too large to display	2132

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(3*A*b-B*a)/a^4/sin(1/2*d*x+1/2*c)
^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))+2*b*(2*A*b-B*a)/a^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/
2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b
^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5 b^2+2\cos(dx+c)^4 ab+\cos(dx+c)^3 a^2}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**5*b**2 + 2*cos(c + d*x)**4*a*b + cos
(c + d*x)**3*a**2),x)
```

3.383 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 44

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
6/5*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*B*cos(d*x+c)^(3/2)*sin(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
]),x]
```

output
```
(B*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5
*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2011, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$B\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$B\left(\frac{6E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

input
```
Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
B*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(40) = 80$.

Time = 5.23 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.61

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, B\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{2\,B\cos{(dx+c)}^{\frac{3}{2}}\sin{(dx+c)} + 3i\,\sqrt{2}B\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+$$

input
```
integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
1/5*(2*B*cos(d*x + c)^(3/2)*sin(d*x + c) + 3*I*sqrt(2)*B*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqr
t(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{\cos(c+d\,x)^{5/2}\,(B\,a+B\,b\,\cos(c+d\,x))}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b$$

input
```
int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b
```

3.384 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 44

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*B*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*B*cos(d*x+c)^(1/2)*sin(
d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
]),x]
```

output
```
(2*B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2011, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cos^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$B\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$B\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
B*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(39) = 78$.

Time = 3.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.09

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,B\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{d}{}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(cos(d*x+c)^(3/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2\,B\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-i\,\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}B}{3\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
1/3*(2*B*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*B*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{\cos(c+d\,x)^{3/2}\,(B\,a+B\,b\,\cos(c+d\,x))}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)b$$

input
```
int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(c + d*x),x)*b
```

3.385 $\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 17

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

output
```
2*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
]),x]
```

output
```
(2*B*EllipticE[(c + d*x)/2, 2])/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2011, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$\Big\downarrow \text{ 2011}$$
$$B\int \sqrt{\cos(c+dx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$B\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3119}$$
$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

output
```
(2*B*EllipticE[(c + d*x)/2, 2])/d
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(19) = 38$.

Time = 2.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 7.88

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\, d}$
risch	$-\frac{i\sqrt{2}\, B\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{d} - \frac{i\left(-\frac{2\left(\mathrm{e}^{2i(dx+c)}+1\right)}{\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{i(dx+c)}}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2i\, \mathrm{Ellipti}\right.}{\sqrt{\mathrm{e}^{3i(dx+}}}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.47

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$= \frac{i\sqrt{2}B\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - i\sqrt{2}B\mathrm{weierstr}}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
(I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$
$$=\int \frac{\sqrt{\cos(c+d\,x)}\,(B\,a+B\,b\,\cos(c+d\,x))}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx=\left(\int \sqrt{\cos(dx+c)}dx\right)b$$

input
```
int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)),x)*b
```

3.386 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 17

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

output
```
2*B*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]
)),x]
```

output
```
(2*B*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2011, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3120}$$
$$\frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

output
```
(2*B*EllipticF[(c + d*x)/2, 2])/d
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2B\ \text{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d}$	19

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
2*B*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.12

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{-i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
(-I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx=\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx=\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b
```

3.387 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 40

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*B*sin(d*x+c)/d/cos(d*x+c)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = B\left(-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)),x]
```

output
```
B*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2011, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

output
```
B*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(40) = 80$.

Time = 1.49 (sec) , antiderivative size = 183, normalized size of antiderivative = 4.58

method	result
default	$-\frac{2B\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+ \cdots}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
-2*B*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.40

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$
$$= \frac{-i\sqrt{2}B\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+}{}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
(-I*sqrt(2)*B*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*B*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*
B*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2
)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{\cos(c + d\,x)^{3/2}\,(a + b\,\cos(c + d\,x))}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^2}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*b
```

3.388 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 44

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx=\frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/3*B*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d+2/3*B*sin(d*x+c)/d/cos(d*x+
c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx=\frac{2B\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x]
)),x]
```

output
```
(2*B*(EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Cos[c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2011, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$B\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3120}$$

$$B\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

output
```
B*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(39) = 78$.

Time = 1.74 (sec) , antiderivative size = 214, normalized size of antiderivative = 4.86

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}$

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*B*((2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \frac{-i\sqrt{2}B\cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + i\sqrt{2}B\cos(dx + c)^2 \text{we}}{3d\cos(dx + c)^2}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
1/3*(-I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*B*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2
)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{\cos(c + d\,x)^{5/2}\,(a + b\,\cos(c + d\,x))}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))}\, dx = \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^3} dx\right) b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x)*b
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 116

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2(3a^2+b^2)\,B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d} - \frac{2a^3B\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^3(a+b)d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
-2*a*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d+2/3*(3*a^2+b^2)*B*Inver
seJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^3/d-2*a^3*B*EllipticPi(sin(1/2*d*x+1/2
*c),2*b/(a+b),2^(1/2))/b^3/(a+b)/d+2/3*B*cos(d*x+c)^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 2.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.37

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{B\left(4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{6a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+4\sqrt{\cos(c+dx)}\sin(c+dx)-\frac{6\left(-2abE\left(\arcsin\left(\sqrt{c\ldots}\right.\right.\right.}{}\right.}{6bd}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
])^2,x]
```

output
```
(B*(4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE
[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x
]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2])))/(6*b*d)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {2011, 3042, 3272, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3272$$

$$B\left(\frac{2\int \frac{-3a\cos^2(c+dx)+b\cos(c+dx)+a}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 27$$

$$B\left(\frac{\int \frac{-3a\cos^2(c+dx)+b\cos(c+dx)+a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3042$$

$$B\left(\frac{\int \frac{-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3538$$

$$B\left(\frac{-\frac{\int -\frac{ab+\left(3a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3a\int \sqrt{\cos(c+dx)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 25$$

$$B\left(\frac{\frac{\int \frac{ab+\left(3a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3a\int \sqrt{\cos(c+dx)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3042$$

$$B\left(\frac{\frac{\int \frac{ab+\left(3a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{3a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3119$$

$$B\left(\frac{\frac{\int \frac{ab+(3a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$\downarrow$ 3481

$$B\left(\frac{\frac{\frac{(3a^2+b^2)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{3a^3\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$\downarrow$ 3042

$$B\left(\frac{\frac{\frac{(3a^2+b^2)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$\downarrow$ 3120

$$B\left(\frac{\frac{\frac{2(3a^2+b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{3a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

$\downarrow$ 3284

$$B\left(\frac{\frac{\frac{2(3a^2+b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{6a^3\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}\right)$$

input
```
Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x
]
```

output
```
B*(((-6*a*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*(3*a^2 + b^2)*EllipticF[(
c + d*x)/2, 2])/(b*d) - (6*a^3*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(b*(a + b)*d))/b)/(3*b) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(115) = 230$.

Time = 7.32 (sec) , antiderivative size = 553, normalized size of antiderivative = 4.77

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,B\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a\,b^{2}-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b^{3}-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.\right.}{}$

input

```
int(cos(d*x+c)^(5/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RET
URNVERBOSE)
```

output

```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4*b^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2+2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2*b^3+3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*
b^2-3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)
^2, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^{5/2}(Ba+Bb\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x
)
```

output
```
int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\cos(dx+c)\,b+a}dx\right)b$$

input
```
int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c + d*x)*b + a),x)*b
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 78

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2aB\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d} + \frac{2a^2B\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2(a+b)d}$$

output
```
2*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d-2*a*B*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))/b^2/d+2*a^2*B*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(
1/2))/b^2/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{2B\left(bE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cos(c+dx)}\right),-1\right)+a\,\text{Elliptic}}{b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
])^2,x]
```

output
```
(-2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2011, 3042, 3283, 3042, 3119, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3283}$$

$$B\left(\frac{\int\sqrt{\cos(c+dx)}dx}{b}-\frac{a\int\frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3119}$$

$$B\left(\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3282}$$

$$B\left(\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3120}$$

$$B\left(\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3284}$$

$$B\left(\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}\right)}{b}\right)$$

input
```
Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x
]
```

output
```
B*((2*EllipticE[(c + d*x)/2, 2])/(b*d) - (a*((2*EllipticF[(c + d*x)/2, 2])
/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d)))/b
)
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3283
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[b/d   Int[Sqrt[a + b*Sin[e + f*x]], x], x
] - Simp[(b*c - a*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(83) = 166$.

Time = 4.92 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.92

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2-\text{EllipticF}\left(\cos\left(\frac{d}{2}\right.\right.\right.}{b^2(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\,\mathrm{s}}}$

input
```
int(cos(d*x+c)^(3/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RET
URNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*
EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
^2, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^{3/2}\,(B\,a+B\,b\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x
)
```

output
```
int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)\,b+a}dx\right)b$$

input
```
int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
int((sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*x)*b + a),x)*b
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 55

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd} - \frac{2aB\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b(a+b)d}$$

output
```
2*B*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/d-2*a*B*EllipticPi(sin(1/2*d*
x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{B\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{bd}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
])^2,x]
```

output
```
(B*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b)))/(b*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2011, 3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3282}$$

$$B\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3120}$$

$$B\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3284}$$

$$B\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}\right)$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x
]
```

output
```
B*((2*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(b*(a + b)*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(58) = 116$.

Time = 3.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.44

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-\text{EllipticF}\left(\cos\left(\right.\right.\right.}{(a-b)b\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}$

input
```
int(cos(d*x+c)^(1/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^2,x,method=_RET
URNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)
^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algo
rithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$
$$=\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x
)
```

output
```
int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx=\left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)b+a}dx\right)b$$

input
```
int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)*b + a),x)*b
```

3.392 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 30

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx=\frac{2B\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

output
```
2*B*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx=\frac{2B\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]
)^2),x]
```

output
```
(2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2011, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} \, dx$$

$$\Big\downarrow \; 2011$$

$$B\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \; 3042$$

$$B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 3284$$

$$\frac{2B\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x
]
```

output
```
(2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(32) = 64$.

Time = 1.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 5.03

method	result	size
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\, \sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\, d}$	151

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RET
URNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^2} dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^2} dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^2} dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^2\sqrt{\cos(dx + c)}} dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x +
c))), x)
```

Giac [F]

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x +
c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x
)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2 b+\cos(dx+c)\,a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**2*b + cos(c + d*x)*a),x)*b
```

3.393 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 80

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2bB\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a+b)d} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

output
```
-2*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-2*b*B*EllipticPi(sin(1/2*d*
x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*B*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 196 vs. 2(80) = 160.

Time = 3.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.45

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2} dx =$$

$$-\frac{B\left(\frac{6b \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + \frac{2a\left(2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{2a \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b}\right)}{b} - \frac{4\sin(c+dx)}{\sqrt{\cos(c+dx)}} + \frac{2\left(-2abE\left(\operatorname{arcsi}\right.\right.}{}\right.}{2}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^2),x]
```

output
```
-1/2*(B*((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2
*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(a + b)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*Elliptic
E[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2])))/(a*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {2011, 3042, 3281, 27, 3042, 3538, 25, 27, 3042, 3119, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2} dx$$

$$\Big\downarrow \text{ 2011}$$

$$B \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))} dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$B\left(\frac{2\int -\frac{b\cos^2(c+dx)+a\cos(c+dx)+b}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\cos^2(c+dx)+a\cos(c+dx)+b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)$$

$$\Big\downarrow \text{3538}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \sqrt{\cos(c+dx)}dx-\frac{\int -\frac{b^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{b^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+\int \sqrt{\cos(c+dx)}dx}{a}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\int \sqrt{\cos(c+dx)}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)$$

$$\Big\downarrow \text{3119}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}\right)$$

$$\Big\downarrow \text{3284}$$

$$B\left(\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{2b\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}\right)$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x
]
```

output
```
B*(-(((2*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2])/((a + b)*d))/a) + (2*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(82) = 164$.

Time = 4.12 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.44

method	result
default	$-\frac{2B\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RET
URNVERBOSE)
```

output
```
-2*B*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/a/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(a-b)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^2\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3
/2)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^2\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2}\,dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{\cos(c + d\,x)^{3/2}\,(a + b\,\cos(c + d\,x))^2}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x
)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^2}\,dx = \left(\int \frac{\sqrt{\cos(dx + c)}}{\cos(dx + c)^3 b + \cos(dx + c)^2 a}dx\right) b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**3*b + cos(c + d*x)**2*a),x)*b
```

3.394 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 133

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{2b^2B\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a+b)d} + \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

output
```
2*b*B*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*B*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))/a/d+2*b^2*B*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2
^(1/2))/a^2/(a+b)/d+2/3*B*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*b*B*sin(d*x+c)
/a^2/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.68 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.59

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{B\left(\frac{2(2a^2+9b^2)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + 8a\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right) + \frac{4(a-3b\,\text{co}}{\text{c}}\right.}{}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x]
)^2),x]
```

output
```
(B*((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)
+ 8*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]
^(3/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b
)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(
b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^
2])))/(6*a^2*d)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2011, 3042, 3281, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \; 2011$$
$$B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$B\left(\frac{2\int -\frac{-b\cos^2(c+dx)-a\cos(c+dx)+3b}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}+\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\cos^2(c+dx)-a\cos(c+dx)+3b}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\sin\left(c+dx+\frac{\pi}{2}\right)+3b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}\right)$$

$$\Big\downarrow \text{3534}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2\int -\frac{a^2+4b\cos(c+dx)a+3b^2+3b^2\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}}{3a}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{a^2+4b\cos(c+dx)a+3b^2+3b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{3a}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{a^2+4b\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{3a}\right)$$

$$\Big\downarrow \text{3538}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\int\sqrt{\cos(c+dx)}dx-\frac{\int-\frac{a\cos(c+dx)b^2+(a^2+3b^2)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{3a}\right)$$

$\downarrow$ 25

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{a\cos(c+dx)b^2+(a^2+3b^2)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+3b\int\sqrt{\cos(c+dx)}dx}{a}}{3a}\right)$$

$\downarrow$ 3042

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+(a^2+3b^2)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+3b\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{3a}\right)$$

$\downarrow$ 3119

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+(a^2+3b^2)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}}{3a}\right)$$

$\downarrow$ 3481

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+ab\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}}{3a}\right)$$

$\downarrow$ 3042

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+ab\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}\right)$$

$$\Big\downarrow \text{3120}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}\right)$$

$$\Big\downarrow \text{3284}$$

$$B\left(\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\frac{6b^3\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}\right)$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x
]
```

output
```
B*((2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (-(((6*b*EllipticE[(c + d
*x)/2, 2])/d + ((2*a*b*EllipticF[(c + d*x)/2, 2])/d + (6*b^3*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/a) + (6*b*Sin[c + d*x])/(a*d
*Sqrt[Cos[c + d*x]]))/(3*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(130) = 260$.

Time = 5.54 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.20

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,B\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\cdots}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\cdots}}}{a}\cdots\right.}{\cdots}$

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RET
URNVERBOSE)
```

output
```
-2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(1/a*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*b^3/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^
(1/2))-b/a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5
/2)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algo
rithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{B\,a + B\,b\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x
)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4} b+\cos\left(dx+c\right)^{3} a}dx\right) b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(cos(c + d*x))/(cos(c + d*x)**4*b + cos(c + d*x)**3*a),x)*b
```

3.395 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 560

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(6aAb-3a^2B+16b^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{24ab^2d}$$

$$+\frac{\sqrt{a+b}(a+2b)(6Ab-3aB+8bB)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{24b^2d}$$

$$+\frac{\sqrt{a+b}(2a^2Ab-8Ab^3-a^3B-4ab^2B)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{}}{8b^3d}$$

$$+\frac{(6aAb-3a^2B+16b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24b^2d\sqrt{\cos(c+dx)}}$$

$$+\frac{(2Ab-aB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$

$$+\frac{B\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd}$$

output
```
-1/24*(a-b)*(a+b)^(1/2)*(6*A*a*b-3*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((a
+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a
*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d+1/24*(
a+b)^(1/2)*(a+2*b)*(6*A*b-3*B*a+8*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/8*(a+b)^(1/2)*(2*A
*a^2*b-8*A*b^3-B*a^3-4*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+1/24*(6*A*a*b-3*B*
a^2+16*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)+1/4
*(2*A*b-B*a)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d+1/3*B*
cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.88 (sec) , antiderivative size = 1224, normalized size of antiderivative = 2.19

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])
,x]
```

output
```
-1/48*((-4*a*(-18*a*A*b + a^2*B - 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
) - 4*a*(-24*A*b^2 - 28*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a*A*b + 3*a^2*B - 16*b^2*B)*((I*Cos[
(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (
2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)...
```

Rubi [A] (verified)

Time = 2.73 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{\int \frac{\sqrt{a+b\cos(c+dx)}(3(2Ab-aB)\cos^2(c+dx)+4bB\cos(c+dx)+aB)}{2\sqrt{\cos(c+dx)}}dx}{3b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{a+b\cos(c+dx)}(3(2Ab-aB)\cos^2(c+dx)+4bB\cos(c+dx)+aB)}{\sqrt{\cos(c+dx)}}dx}{6b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3(2Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+4bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 3528$$

$$\frac{\frac{1}{2}\int \frac{(-3Ba^2+6Aba+16b^2B)\cos^2(c+dx)+2b(6Ab+7aB)\cos(c+dx)+a(6Ab+aB)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3(2Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}}{6b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}\int \frac{(-3Ba^2+6Aba+16b^2B)\cos^2(c+dx)+2b(6Ab+7aB)\cos(c+dx)+a(6Ab+aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3(2Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}}{6b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{4}\int \frac{(-3Ba^2+6Aba+16b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(6Ab+aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3(2Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c}}{2d}}{6b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 3540$$

$$\frac{1}{4}\left(\frac{\int -\frac{3\left(-Ba^3+2Aba^2-4b^2Ba-8Ab^3\right)\cos^2(c+dx)-2ab(6Ab+aB)\cos(c+dx)+a\left(-3Ba^2+6Aba+16b^2B\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a-}}{bd\sqrt{\cos(c+dx)}}\right.$$
$$\overline{\qquad\qquad 6b}$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 25

$$\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{3\left(-Ba^3+2Aba^2-4b^2Ba-8Ab^3\right)\cos^2(c+dx)-2ab(6Ab+aB)\cos(c+dx)+a\left(-3Ba^2+6Aba\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.$$
$$\overline{\qquad\qquad 6b}$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{3\left(-Ba^3+2Aba^2-4b^2Ba-8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab(6Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-3Ba\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}\right.$$
$$\overline{\qquad\qquad 6b}$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3532

$$\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{a\left(-3Ba^2+6Aba+16b^2B\right)-2ab(6Ab+aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3\left(a^3(-B)+2a^2Ab-4ab^2B-\right.}{2b}\right.$$
$$\overline{\qquad\qquad 6b}$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-3Ba^2+6Aba+16b^2B\right)-2ab(6Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3\left(a^3(-B)+2a^2Ab-4ab^2\right)}{2b}\right)}{6b}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3288

$$\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-3Ba^2+6Aba+16b^2B\right)-2ab(6Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^3(-B)+2a^2Ab-4\right)}{}}{}\right)$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3477

$$\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-3a^2B+6aAb+16b^2B\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a+2b)(-3aB+6Ab+8)}{}\right)$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-3a^2B+6aAb+16b^2B\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a+2b)(-3a)}{}\right)$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3295

$$\frac{\frac{1}{4}\left(\frac{(-3a^2B+6aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(-3a^2B+6aAb+16b^2B)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^3(-B\right.}{}}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3473

$$\frac{\frac{1}{4}\left(\frac{(-3a^2B+6aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{2(a-b)\sqrt{a+b}\left(-3a^2B+6aAb+16b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{arc}\right.}{ad}\right.}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3bd}$$

input

```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output

```
(B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d) + (
(3*(2*A*b - a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(2*d) + (-1/2*((2*(a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*(a + 2*b)*(6*A*b -
 3*a*B + 8*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*Sqrt[a + b]*(2
*a^2*A*b - 8*A*b^3 - a^3*B - 4*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b,
 ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(b*d))/b + ((6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/4)/(6*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1699 vs. $2(503) = 1006$.

Time = 25.00 (sec) , antiderivative size = 1700, normalized size of antiderivative = 3.04

method	result	size
default	Expression too large to display	1700
parts	Expression too large to display	1728

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*((12*cos(d*x+c)^2+24*cos(d*x+c)+12)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticPi(cot(d*
x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-48*cos(d*x+c)^2-96*cos(d*x+c)-4
8)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(
-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticPi(cot(d*x+c)-csc(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))+(-24*cos(d*x+c)^2-48*cos(d*x+c)-24)*B*((a+cos
(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x
+c)^2-12*cos(d*x+c)-6)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*A*((a+cos(d*x+c)*b)/(co
s(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+
3)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(3*co
s(d*x+c)^2+6*cos(d*x+c)+3)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))+(-16*cos(d*x+c)^2-32*cos(d*x+c)-16)*B*((a+cos(d*x+...
```

Fricas [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \int \cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a$$
$$+ \left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}dx\right)b$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sq
rt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b
```

3.396 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 473

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(4Ab+aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{4abd}$$

$$+\frac{\sqrt{a+b}(4Ab+(a+2b)B)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-}{}}}{4bd}$$

$$-\frac{\sqrt{a+b}(4aAb-a^2B+4b^2B)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{4b^2d}$$

$$+\frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}}$$

$$+\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
-1/4*(a-b)*(a+b)^(1/2)*(4*A*b+B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/4*(a+b)^(1/2)*(4*A*b+(
a+2*b)*B)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/b/d-1/4*(a+b)^(1/2)*(4*A*a*b-B*a^2+4*B*b^2)*cot(d*x+c)*
EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(
a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/b^2/d+1/4*(4*A*b+B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d/cos(d*x
+c)^(1/2)+1/2*B*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.97 (sec) , antiderivative size = 1175, normalized size of antiderivative = 2.48

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])
,x]
```

output
```
(B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*
a*(4*A*b + 3*a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a*A + 4*b*B)*((
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2
*(4*A*b + a*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*A
rcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x]
)/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[
c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[...
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3482, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3482}$$

$$\frac{1}{4}\int \frac{(4Ab+aB)\cos^2(c+dx)+2(2aA+bB)\cos(c+dx)+aB}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \frac{(4Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2(2aA+bB)\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3540

$$\frac{1}{4}\left(\frac{\int -\frac{-((-Ba^2+4Aba+4b^2B)\cos^2(c+dx))-2abB\cos(c+dx)+a(4Ab+aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\right)+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 25

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{-((-Ba^2+4Aba+4b^2B)\cos^2(c+dx))-2abB\cos(c+dx)+a(4Ab+aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{(Ba^2-4Aba-4b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2abB\sin\left(c+dx+\frac{\pi}{2}\right)+a(4Ab+aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3532

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab+aB)-2abB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\left(a^2(-B)+4aAb+4b^2B\right)\int\frac{}{\sqrt{}}}{2b}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab+aB)-2abB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(a^2(-B)+4aAb+4b^2B\right.}{2b}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3288

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab+aB)-2abB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)}{}}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3477

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(aB+4Ab)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(B(a+2b)+4A}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{-a(B(a+2b)+4Ab)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a(a\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\big\downarrow$ 3295

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(aB+4Ab)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a^2(-}{}}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\big\downarrow$ 3473

$$\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{2\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{El}}{bd}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2
*((2*(a - b)*Sqrt[a + b]*(4*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d) - (2*Sqrt[a + b]*(4*A*b + (a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/d + (2*Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/b + ((4*A*b + a*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1186 vs. $2(422) = 844$.

Time = 12.80 (sec) , antiderivative size = 1187, normalized size of antiderivative = 2.51

method	result	size
default	Expression too large to display	1187
parts	Expression too large to display	1217

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*((-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(2+2*cos(d*x+c)^2+4*cos(d*x+c))*B*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/
2)*a^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-8*cos(d
*x+c)^2-16*cos(d*x+c)-8)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c
)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*Elliptic
E(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c
)-4)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/
(a+b))^(1/2)*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-c
os(d*x+c)^2-2*cos(d*x+c)-1)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*
x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*B*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticE(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8
)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+
b))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-2*co
s(d*x+c)^2-4*cos(d*x+c)-2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(...
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)),
 x)
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
, x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b$$
$$+\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx\right)a$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b + int(sq
rt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a
```

3.397 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 385

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

$$+\frac{\sqrt{a+b}(2A+B)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$-\frac{\sqrt{a+b}(2Ab+aB)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+s}{}}}{bd}$$

$$+\frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-(a-b)*(a+b)^(1/2)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+(a+b)^(1/2)*(2*A+B)*cot(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-(a+b)^(1/
2)*(2*A*b+B*a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+B*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 14.76 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\left(2(a+b)B\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-4(Ab+a(-A+B))\sqrt{\right.}{}}{}$$

input
```
Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x
]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(a + b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(
A*b + a*(-A + B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 8*A*b*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] + 4*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + b*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c
 + d*x))/2] + 2*a*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]
 - b*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(2*d*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3482, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3482}$$

$$\frac{1}{2}\int -\frac{-\left((2Ab+aB)\cos^2(c+dx)\right)-2aA\cos(c+dx)+aB}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int \frac{-\left((2Ab+aB)\cos^2(c+dx)\right)-2aA\cos(c+dx)+aB}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int \frac{(-2Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2aA\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3532}$$

$$\frac{1}{2}\left((aB+2Ab)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx-\int \frac{aB-2aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left((aB+2Ab)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{aB-2aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\big\downarrow$ 3288

$$\frac{1}{2}\left(-\int \frac{aB-2aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(se}{}}}{}\right.$$
$$\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\big\downarrow$ 3477

$$\frac{1}{2}\left(a(2A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-aB\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\frac{2\sqrt{a+b}(a}{}\right.$$
$$\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(a(2A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-aB\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx\right.}}\right.$$
$$\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\big\downarrow$ 3295

$$\frac{1}{2}\left(-aB\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(s}{}}}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}-\right.$$

$$\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
((-2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2
*Sqrt[a + b]*(2*A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*Sqrt[a
+ b]*(2*A*b + a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))
/2 + (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 15.95 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.73

method	result
parts	$-\frac{2A\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\left(\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)a-\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)b+2b\,\operatorname{Ellipt}\right.}{d\sqrt{a+\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$
default	$-\frac{\left(\left(4\cos(dx+c)^2+8\cos(dx+c)+4\right)A\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,b\,\operatorname{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)+\left(2+\right.\right.}{}$

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*A/d*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(EllipticF(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))*b+2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(
1/2)))/(a+cos(d*x+c)*b)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)
^(1/2)*(cos(d*x+c)+1)+B/d*((-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*Ellipt
icPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d
*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-c
os(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+
c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))+(2+2*cos(d*x+c)^2+4*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)^2*b+cos(d*x+c)*s
in(d*x+c)*a)*(a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c)^2+a*cos
(d*x+c)+cos(d*x+c)*b+a)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c))
, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx\right)b$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a + int(
sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*b
```

3.398 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 351

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$
$$+ \frac{2\sqrt{a+b}(Ab-a(A-B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-}{}}}{ad}$$
$$- \frac{2\sqrt{a+b}B\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

output
```
2*A*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2*(a+b)^(1/2)*(A*b-a*(A-B))*cot(d*x+c)*
EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b
))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/
d-2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 14.30 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-2A(a+b)\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\mid\frac{-a+b}{a+b}\right)+2(b(A-B)+a(A}{}$$

input
```
Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/
2),x]
```

output
```
(-2*A*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
 2*(b*(A - B) + a*(A + B))*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] + 4*b*B*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + (2*A*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3470, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3470}$$

$$\int \frac{aA+(Ab+aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+bB\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA+(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+bB\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3288}$$

$$\int \frac{aA+(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{3477}$$

$$(Ab-a(A-B))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+aA\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$(Ab-a(A-B))\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+aA\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{3295}$$

$$aA\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$\Big\downarrow$ 3473

$$\frac{2\sqrt{a+b}(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$
$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-$$
$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

input
```
Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*S
qrt[a + b]*(A*b - a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) -
(2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3470
```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[B*(d
/b^2)   Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*
c + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*
x]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(318) = 636$.

Time = 21.26 (sec), antiderivative size = 654, normalized size of antiderivative = 1.86

method	result
parts	$-\frac{2B\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\left(\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)a-\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)b+2b\,\text{Ellipt}\right.}{d\sqrt{a+\cos(dx+c)b}\sqrt{\cos(dx+c)}}$
default	$\frac{2\sqrt{a+\cos(dx+c)b}\left(-\frac{A\left(\csc(dx+c)^2(1-\cos(dx+c))^2-1\right)\left(\left(\csc(dx+c)^3(1-\cos(dx+c))^3-\cot(dx+c)+\csc(dx+c)\right)a+\left(-\csc(dx+c)^3(1-\cos(dx+c\right.\right.}{}\right.}{}$

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*B/d*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(EllipticF(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))*b+2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(
1/2)))/(a+cos(d*x+c)*b)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)
^(1/2)*(cos(d*x+c)+1)+2*A/d*((cos(d*x+c)^2+2*cos(d*x+c)+1)*((a+cos(d*x+c)*
b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+
1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x
+c)^2-2*cos(d*x+c)-1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/
(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*EllipticF(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))+b*cos(d*x+c)*sin(d*x+c)+a*sin(d*x+c))*(a+cos(
d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b
+a)
```

Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*b + int(
(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a
```

3.399 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 284

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}(Ab+3aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d}$$
$$+ \frac{2(a-b)\sqrt{a+b}(A-3B)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad}$$
$$+ \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output

```
2/3*(a-b)*(a+b)^(1/2)*(A*b+3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2/3*(a-b)*(a+b)^(1/2)*(A-
3*B)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d+2/3*A*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/
2)
```

Mathematica [A] (verified)

Time = 14.60 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)^{5/2}\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{3/2}\sqrt{1+\cos(c+dx)}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\Big(-2(a+b)(Ab+$$
$$+\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(Ab\sin(c+dx)+3aB\sin(c+dx))}{3a}+\frac{2}{3}A\sec(c+dx)\tan(c+dx)\right)}{d}$$

input
```
Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/
2),x]
```

output
```
(4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt
[1 + Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(A*b
+ 3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)] + 2*a*(a + b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Cos[c + d*x]^(5/2)*S
qrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((
2*Sec[c + d*x]*(A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a) + (2*A*Sec[c
 + d*x]*Tan[c + d*x])/3))/d
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3478, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3478}$$

$$\frac{2}{3}\int \frac{Ab+3aB+(aA+3bB)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{Ab+3aB+(aA+3bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{Ab+3aB+(aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3477}$$

$$\frac{1}{3}\left((3aB+Ab)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)(A-3B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right.$$
$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left((a-b)(A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+(3aB+Ab)\int \frac{\sin\left(c+dx+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a-}}\right.$$
$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3295}$$

$$\frac{1}{3}\left((3aB+Ab)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}(A-3B)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}\right.$$
$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3473

$$\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}+\right.$$
$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
((2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(A - 3*B)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a*d))/3 + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c +
 d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(252) = 504$.

Time = 30.07 (sec) , antiderivative size = 1022, normalized size of antiderivative = 3.60

method	result	size
parts	Expression too large to display	1022
default	Expression too large to display	1040

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2*B/d*((cos(d*x+c)^2+2*cos(d*x+c)+1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*((a+cos(d*x+c)*b)/(
cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*EllipticE(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)
^2-2*cos(d*x+c)-1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))+b*cos(d*x+c)*sin(d*x+c)+a*sin(d*x+c))*(a+cos(d*x+c)*b)^(1/2)/cos(d*
x+c)^(1/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)+2/3*A/d*((cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b
*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)^3+2*cos
(d*x+c)^2+cos(d*x+c))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/
(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a
+b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*EllipticF(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-c
os(d*x+c))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/...
```

Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(5/2)
, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2)
, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}}dx\right)a + \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)b$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*a + i
nt((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*b
```

3.400 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 350

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(9a^2A-2Ab^2+5abB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{15a^3d}$$
$$-\frac{2(a-b)\sqrt{a+b}(9aA+2Ab-5aB)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{15a^2d}$$
$$+\frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2-2*A*b^2+5*B*a*b)*cot(d*x+c)*EllipticE((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/15*(a-b)
*(a+b)^(1/2)*(9*A*a+2*A*b-5*B*a)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2/5*A*(a+b*cos(d*x+c))^(1/
2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*(A*b+5*B*a)*(a+b*cos(d*x+c))^(1/2)*s
in(d*x+c)/a/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.85 (sec) , antiderivative size = 1315, normalized size of antiderivative = 3.76

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/
2),x]
```

output
```
-1/15*((-4*a*(2*a^2*A*b - 2*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(9*a^3*A - 2*a*A*b^2 + 5*a^2*b*B)*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b - 2*A
*b^3 + 5*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[
I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d
*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*S
ec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C...
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3478, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3478}$$

$$\frac{2}{5}\int \frac{2Ab\cos^2(c+dx)+(3aA+5bB)\cos(c+dx)+Ab+5aB}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{2Ab\cos^2(c+dx)+(3aA+5bB)\cos(c+dx)+Ab+5aB}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{2Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+(3aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab+5aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{5}\left(\frac{2\int \frac{9Aa^2+5bBa+(7Ab+5aB)\cos(c+dx)a-2Ab^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(\frac{\int \frac{9Aa^2+5bBa+(7Ab+5aB)\cos(c+dx)a-2Ab^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\int \frac{9Aa^2+5bBa+(7Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3477

$$\frac{1}{5}\left(\frac{(9a^2A+5abB-2Ab^2)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(9aA-5aB+2Ab)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}d}{3a}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{(9a^2A+5abB-2Ab^2)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(9aA-5aB+2Ab)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+}}}{3a}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3295

$$\frac{1}{5}\left(\frac{(9a^2A+5abB-2Ab^2)\int \frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx-\frac{2(a-b)\sqrt{a+b}(9aA-5aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{3a}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3473

$$\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}(9a^2A+5abB-2Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a-b)\sqrt{a+b}(9a}{}}{3a}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((
2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(3*a) + (2*(A*b + 5*a*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1504 vs. $2(312) = 624$.

Time = 38.19 (sec) , antiderivative size = 1505, normalized size of antiderivative = 4.30

method	result	size
default	Expression too large to display	1505
parts	Expression too large to display	1512

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(9*cos(d*x+c)^4+18*cos(d*x+c)^3+9*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(9*cos(d*x+c)^4+18*cos(d*x+c)^3+
9*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos
(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(-2*cos(d*x+c)^4-4*cos(d*x+c)^3-2*cos(d*x+c)^2)+A*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^3*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)^4-4*cos(d
*x+c)^3-2*cos(d*x+c)^2)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)
*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*(5*cos(d*x+c)^4+10*cos(d*x+c)^3+5*cos(d*x+c)^2)+B*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*
a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(5*cos(d*x+c)^
4+10*cos(d*x+c)^3+5*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+
cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*EllipticF(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*(-9*cos(d*x+c)^4-18*cos(d*x+c)^3-9*cos(d*x+c)^2)
+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-...
```

Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2)
, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}}dx\right)a + \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}}dx\right)b$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*a + i
nt((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*b
```

3.401 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 433

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2(a-b)\sqrt{a+b}(19a^2Ab+8Ab^3+63a^3B-14ab^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{105a^4d}$$

$$+ \frac{2(a-b)\sqrt{a+b}(8Ab^2+a^2(25A-63B)+2ab(3A-7B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{105a^3d}$$

$$+ \frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2(25a^2A-4Ab^2+7abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105a^2d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/105*(a-b)*(a+b)^(1/2)*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*cot(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a
-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
a^4/d+2/105*(a-b)*(a+b)^(1/2)*(8*A*b^2+a^2*(25*A-63*B)+2*a*b*(3*A-7*B))*co
t(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(
a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a^3/d+2/7*A*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/
35*(A*b+7*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)+2/10
5*(25*A*a^2-4*A*b^2+7*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a^2/d/cos(d
*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.99 (sec) , antiderivative size = 1408, normalized size of antiderivative = 3.25

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/
2),x]
```

output
```
((-4*a*(25*a^4*A - 17*a^2*A*b^2 - 8*A*b^4 - 14*a^3*b*B + 14*a*b^3*B)*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*A*b - 8*a*A*b^3 - 63*a^4*B + 14*
a^2*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) + 2*(-19*a^2*A*b^2 - 8*A*b^4 - 63*a^3*b*B + 14*a*b^3*B)*((I*Cos[
(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (
2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos...
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3478, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3478}$$

$$\frac{2}{7}\int \frac{4Ab\cos^2(c+dx)+(5aA+7bB)\cos(c+dx)+Ab+7aB}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\int \frac{4Ab\cos^2(c+dx)+(5aA+7bB)\cos(c+dx)+Ab+7aB}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\int \frac{4Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+(5aA+7bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab+7aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3534

$$\frac{1}{7}\left(\frac{2\int \frac{25Aa^2+7bBa+(23Ab+21aB)\cos(c+dx)a-4Ab^2+2b(Ab+7aB)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(7aB+Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{\int \frac{25Aa^2+7bBa+(23Ab+21aB)\cos(c+dx)a-4Ab^2+2b(Ab+7aB)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(7aB+Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\int \frac{25Aa^2+7bBa+(23Ab+21aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-4Ab^2+2b(Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2(7aB+Ab)\sin(c+dx)\sqrt{a+b\cos}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{7}\left(\frac{\frac{2\int \frac{63Ba^3+19Aba^2-14b^2Ba+\left(25Aa^2+49bBa+2Ab^2\right)\cos(c+dx)a+8Ab^3}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(7aB}{}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{63Ba^3+19Aba^2-14b^2Ba+\left(25Aa^2+49bBa+2Ab^2\right)\cos(c+dx)a+8Ab^3}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(7aB+}{}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{63Ba^3+19Aba^2-14b^2Ba+\left(25Aa^2+49bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+8Ab^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(7a}{}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

$$\frac{1}{7}\left(\frac{\frac{(a-b)\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\left(63a^3B+19a^2Ab-14ab^2B+8Ab^3\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a}}{5a}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{(a-b)\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(63a^3B+19a^2Ab-14ab^2B+8Ab^3\right)\int\frac{\sin\left(c+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{3a}}{5a}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{7}\left(\frac{\frac{\left(63a^3B+19a^2Ab-14ab^2B+8Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}}{3a}}{5a}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3473

$$\frac{1}{7}\left(\frac{\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-b}}}{ad}}{}\right.$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2
*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^
(5/2)) + (((2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*
b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a +
b]*(8*A*b^2 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7*B))*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(a*d))/(3*a) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))/(5*a))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2082 vs. $2(389) = 778$.

Time = 51.95 (sec) , antiderivative size = 2083, normalized size of antiderivative = 4.81

method	result	size
default	Expression too large to display	2083
parts	Expression too large to display	2105

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/105/d*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c
)+1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*(-19*cos(d*x+c)^5-38*cos(d*x+c)^4-19*cos(d*x+c)^3)+A*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*Ell
ipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-19*cos(d*x+c)^5-38*co
s(d*x+c)^4-19*cos(d*x+c)^3)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*
x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*(-8*cos(d*x+c)^5-16*cos(d*x+c)^4-8*cos(d*x+c)^3)+A*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(
1/2)*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-8*cos(d*x
+c)^5-16*cos(d*x+c)^4-8*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*EllipticE(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*(-63*cos(d*x+c)^5-126*cos(d*x+c)^4-63*cos(d*
x+c)^3)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(-63*cos(d*x+c)^5-126*cos(d*x+c)^4-63*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(14*cos(d*x+c)^5+28*cos(
d*x+c)^4+14*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+
c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c)...
```

Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2)
, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{5}}dx\right)a + \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}}dx\right)b$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*a + i
nt((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*b
```

3.402 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 670

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{a}}{192ab^2d}$$

$$-\frac{\sqrt{a+b}(9a^3B-6a^2b(4A+B)-8b^3(16A+9B)-4ab^2(28A+39B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{192b^2d}$$

$$+\frac{\sqrt{a+b}(8a^3Ab-96aAb^3-3a^4B-24a^2b^2B-48b^4B)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{64b^3d}$$

$$+\frac{(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{192b^2d\sqrt{\cos(c+dx)}}$$

$$+\frac{(8aAb-3a^2B+12b^2B)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd}$$

$$+\frac{(8Ab-3aB)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24bd}$$

$$+\frac{B\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd}$$

output
```
-1/192*(a-b)*(a+b)^(1/2)*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B*a*b^2)*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)
/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a/b^2/d-1/192*(a+b)^(1/2)*(9*B*a^3-6*a^2*b*(4*A+B)-8*b^3*(16*A+9*B)-4*a
*b^2*(28*A+39*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/64*(a+b)^(1/2)*(8*A*a^3*b-96*A*a*b^3-3*
B*a^4-24*B*a^2*b^2-48*B*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+1/192*(24*A*a^2*b+128
*A*b^3-9*B*a^3+156*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d/cos(d*
x+c)^(1/2)+1/32*(8*A*a*b-3*B*a^2+12*B*b^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c
))^(1/2)*sin(d*x+c)/b/d+1/24*(8*A*b-3*B*a)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c
))^(3/2)*sin(d*x+c)/b/d+1/4*B*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.68 (sec) , antiderivative size = 1284, normalized size of antiderivative = 1.92

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
-1/384*((-4*a*(-136*a^2*A*b - 128*A*b^3 + 3*a^3*B - 228*a*b^2*B)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - 4*a*(-416*a*A*b^2 - 228*a^2*b*B - 144*b^3*B)*((Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-24*a^2*A*b - 128*A*b^3 + 9*a^3*B - 156*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt
[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c ...
```

Rubi [A] (verified)

Time = 3.42 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{\int \frac{(a+b\cos(c+dx))^{3/2}((8Ab-3aB)\cos^2(c+dx)+6bB\cos(c+dx)+aB)}{2\sqrt{\cos(c+dx)}}dx}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\cos(c+dx))^{3/2}((8Ab-3aB)\cos^2(c+dx)+6bB\cos(c+dx)+aB)}{\sqrt{\cos(c+dx)}}dx}{8b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left((8Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+6bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{8b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{ 3528}$$

$$\frac{\frac{1}{3}\int \frac{\sqrt{a+b\cos(c+dx)}(3(-3Ba^2+8Aba+12b^2B)\cos^2(c+dx)+2b(16Ab+15aB)\cos(c+dx)+a(8Ab+3aB))}{2\sqrt{\cos(c+dx)}}dx+\frac{(8Ab-3aB)\sin(c+dx)\sqrt{\cos(c}{3d}}{8b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{6}\int \frac{\sqrt{a+b\cos(c+dx)}(3(-3Ba^2+8Aba+12b^2B)\cos^2(c+dx)+2b(16Ab+15aB)\cos(c+dx)+a(8Ab+3aB))}{\sqrt{\cos(c+dx)}}dx+\frac{(8Ab-3aB)\sin(c+dx)\sqrt{\cos(c}{3d}}{8b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{6}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3(-3Ba^2+8Aba+12b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(16Ab+15aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(8Ab+3aB)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(8Ab-3aB)\sin(c-}{}}{8b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{ 3528}$$

$$\frac{\frac{1}{6}\left(\frac{1}{2}\int \frac{\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(57Ba^2+104Aba+36b^2B\right)\cos(c+dx)+a\left(3Ba^2+56Aba+36b^2B\right)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3(-3a^2\ldots}{}\right.}{8b}$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int \frac{\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(57Ba^2+104Aba+36b^2B\right)\cos(c+dx)+a\left(3Ba^2+56Aba+36b^2B\right)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3(-3a^2\ldots}{}\right.}{8b}$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int \frac{\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(57Ba^2+104Aba+36b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(3Ba^2+56Aba+36b^2B\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\ldots\right.}{8b}$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 3540

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int -\frac{3\left(-3Ba^4+8Aba^3-24b^2Ba^2-96Ab^3a-48b^4B\right)\cos^2(c+dx)-2ab\left(3Ba^2+56Aba+36b^2B\right)\cos(c+dx)+a\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\ldots\right.\right.$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{3\left(-3Ba^4+8Aba^3-24b^2Ba^2-96Ab^3a-48b^4B\right)\cos^2(c+dx)-2ab\left(3B\ldots\right.}{\cos^{\frac{3}{2}}(c+dx)\ldots}}{}\right.\right.$$
$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-9a^3B+24a^2Ab+156ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{3\left(-3Ba^4+8Aba^3-24b^2Ba^2-96Ab^3a-48b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab}{\sin\left(c+dx+\frac{\pi}{2}\right)}}{}\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 3532

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-9a^3B+24a^2Ab+156ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)-2ab\left(3Ba^2+56Aba+36b^2B\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{}\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-9a^3B+24a^2Ab+156ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)-2ab\left(3Ba^2+56Aba+36b^2B\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 3288

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-9a^3B+24a^2Ab+156ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)-2ab\left(3Ba^2+56Aba+36b^2B\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$\downarrow$ 3477

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}}}{}\right.\right.}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(9a^3B-6a^2b(4A+B)-4ab^2(28A+39B)-8b^3(16A+9B)\right)\int\frac{}{\sqrt{\text{si}}}}{}\right.\right.}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{3295}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b}}}{}\right.\right.}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\frac{1}{6}\left(\frac{3\left(-3a^2B+8aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+d}}{bd\sqrt{\cos(c+dx)}}\right.\right.}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{4bd}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d) + (
((8*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(3*d) + ((3*(8*a*A*b - 3*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*Sqrt[a + b]*(24*a^
2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(a*d) + (2*Sqrt[a + b]*(9*a^3*B - 6*a^2*b*(4*A + B) - 8*b^3*(16*A +
9*B) - 4*a*b^2*(28*A + 39*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*
Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 - 3*a^4*B - 24*a^2*b^2*B - 48*b^4*B)*C
ot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/b + ((24*a^2*A*b +
128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(b*d*Sqrt[Cos[c + d*x]]))/4)/6)/(8*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2335 vs. $2(607) = 1214$.

Time = 32.50 (sec) , antiderivative size = 2336, normalized size of antiderivative = 3.49

method	result	size
default	Expression too large to display	2336
parts	Expression too large to display	2361

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*((48*cos(d*x+c)^2+96*cos(d*x+c)+48)*A*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*EllipticPi(cot(d
*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-576*cos(d*x+c)^2-1152*cos(d*x+
c)-576)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))+(-144*cos(d*x+c)^2-288*cos(d*x+c)-144)*B*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*EllipticPi(co
t(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-24*cos(d*x+c)^2-48*cos(d*x+
c)-24)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
+(-24*cos(d*x+c)^2-48*cos(d*x+c)-24)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
(a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-128*cos(d*x+c)^2-256*cos(d*x+c)-128)*A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^
(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(9*cos(d
*x+c)^2+18*cos(d*x+c)+9)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c
)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(
a-b)/(a+b))^(1/2))+(-156*cos(d*x+c)^2-312*cos(d*x+c)-156)*B*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-156*cos(d*x+c)^...
```

Fricas [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A + B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A + B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^2 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b^2 + 2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)ab$$

input
```
int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a**2 + int
(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b**2 + 2*i
nt(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a*b
```

3.403 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 566

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(30aAb+3a^2B+16b^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{24abd}$$

$$+\frac{\sqrt{a+b}(30aAb+12Ab^2+3a^2B+14abB+16b^2B)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right.}{24bd}$$

$$-\frac{\sqrt{a+b}(6a^2Ab+8Ab^3-a^3B+12ab^2B)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{}}{8b^2d}$$

$$+\frac{(30aAb+3a^2B+16b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}}$$

$$+\frac{(6Ab+7aB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d}$$

$$+\frac{bB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
-1/24*(a-b)*(a+b)^(1/2)*(30*A*a*b+3*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/24*(a
+b)^(1/2)*(30*A*a*b+12*A*b^2+3*B*a^2+14*B*a*b+16*B*b^2)*cot(d*x+c)*Ellipti
cF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2
))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/8*(
a+b)^(1/2)*(6*A*a^2*b+8*A*b^3-B*a^3+12*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/
2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/
24*(30*A*a*b+3*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d/cos(d
*x+c)^(1/2)+1/12*(6*A*b+7*B*a)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*sin
(d*x+c)/d+1/3*b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.45 (sec) , antiderivative size = 1227, normalized size of antiderivative = 2.17

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
((-4*a*(42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
4*a*(48*a^2*A + 24*A*b^2 + 52*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(30*a*A*b + 3*a^2*B + 16*b^2*B)*((
I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c ...
```

Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{3}\int \frac{\sqrt{\cos(c+dx)}\left(b(6Ab+7aB)\cos^2(c+dx)+2\left(3Ba^2+6Aba+2b^2B\right)\cos(c+dx)+3a(2aA+bB)\right)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{\sqrt{\cos(c+dx)}\left(b(6Ab+7aB)\cos^2(c+dx)+2\left(3Ba^2+6Aba+2b^2B\right)\cos(c+dx)+3a(2aA+bB)\right)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(3Ba^2+6Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(2aA\right.}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{1}{6}\left(\frac{\int \frac{b\left(3Ba^2+30Aba+16b^2B\right)\cos^2(c+dx)+2b\left(12Aa^2+13bBa+6Ab^2\right)\cos(c+dx)+ab(6Ab+7aB)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(7aB+6Ab)\sin(c+dx)\sqrt{\text{co}}}{2}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{\int \frac{b\left(3Ba^2+30Aba+16b^2B\right)\cos^2(c+dx)+2b\left(12Aa^2+13bBa+6Ab^2\right)\cos(c+dx)+ab(6Ab+7aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{(7aB+6Ab)\sin(c+dx)\sqrt{\text{co}}}{2}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\int \frac{b(3Ba^2+30Aba+16b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(12Aa^2+13bBa+6Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(6Ab+7aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{(7aB+6Ab)\sin(c+d\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3540}$$

$$\frac{1}{6}\left(\frac{\frac{\int -\frac{-2a(6Ab+7aB)\cos(c+dx)b^2-3\left(-Ba^3+6Aba^2+12b^2Ba+8Ab^3\right)\cos^2(c+dx)b+a\left(3Ba^2+30Aba+16b^2B\right)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx}{d\sqrt{\cos(c+dx)}}}{4b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-2a(6Ab+7aB)\cos(c+dx)b^2-3\left(-Ba^3+6Aba^2+12b^2Ba+8Ab^3\right)\cos^2(c+dx)b+a\left(3Ba^2+3\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}}{4b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-2a(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)b^2-3\left(-Ba^3+6Aba^2+12b^2Ba+8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+a\left(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}}{4b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3532}$$

$$\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3Ba^2+30Aba+16b^2B\right)-2ab^2(6Ab+7aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-3b(a^3(-B)+6a^2Ab+12ab^2}{2b}}{4b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3Ba^2+30Aba+16b^2B\right)-2ab^2(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3b(a^3(-B)+6a^2Ab+1}{2b}}{4b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3288

$$\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3Ba^2+30Aba+16b^2B\right)-2ab^2(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6\sqrt{a+b}\left(a^3(-B)+6a^2Ab\right.}{}}{}}{4b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3477

$$\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(3a^2B+30aAb+16b^2B\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\left(3a^2B+30aAb+14ab\right.}{}}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab\left(3a^2B+30aAb+14abB+12Ab^2+16b^2B\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{}}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3295

$$\frac{1}{6}\left(\frac{\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(3a^2B+30aAb+16b^2B\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\left(3a^2B-\right.}{}}{}}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$\downarrow$ 3473

$$\frac{1}{6}\left(\frac{\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b\sqrt{a+b}\left(3a^2B+30aAb+14abB+12Ab^2+16b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+d}{a-b}}}{d}}{}}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(b*B*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (((
6*A*b + 7*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
2*d) + (-1/2*((2*(a - b)*b*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(30*a*A*b + 12*
A*b^2 + 3*a^2*B + 14*a*b*B + 16*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/d + (6*Sqrt[a + b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + ((30*a*A*b + 3*a^2*B + 16*b^
2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/(4*b))
/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. $2(509) = 1018$.

Time = 26.54 (sec) , antiderivative size = 1803, normalized size of antiderivative = 3.19

method	result	size
default	Expression too large to display	1803
parts	Expression too large to display	1830

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*((-36*cos(d*x+c)^2-72*cos(d*x+c)-36)*A*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticPi(cot(d
*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-48*cos(d*x+c)^2-96*cos(d*x+c)-
48)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+
(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+c
os(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*EllipticPi(cot(d*x+c)-csc(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))+(-72*cos(d*x+c)^2-144*cos(d*x+c)-72)*B*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*
a*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-30*cos(d
*x+c)^2-60*cos(d*x+c)-30)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+
c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c)-30)*A*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-3*cos(d*x+c)^2-6*cos(d
*x+c)-3)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+
cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))+(-16*cos(d*x+c)^2-32*cos(d*x+c)-16)*B*((a+c...
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = 2\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)ab + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b^2 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx\right)a^2$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
2*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a*b + in
t(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b**2 + in
t(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a**2
```

3.404 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 472

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(4Ab+5aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4ad}$$

$$+\frac{\sqrt{a+b}(8aA+4Ab+5aB+2bB)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d}$$

$$-\frac{\sqrt{a+b}(12aAb+3a^2B+4b^2B)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{4bd}$$

$$+\frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$+\frac{bB\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
-1/4*(a-b)*(a+b)^(1/2)*(4*A*b+5*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/4*(a+b)^(1/2)*(8*A*a+4
*A*b+5*B*a+2*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(a+b)^(1/2)*(12*A*a*b+3*B*a^2+4*B*b^2)*co
t(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a
+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/b/d+1/4*(4*A*b+5*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/
cos(d*x+c)^(1/2)+1/2*b*B*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c
)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.64 (sec) , antiderivative size = 1198, normalized size of antiderivative = 2.54

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d
*x]],x]
```

output
```
(b*B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-
4*a*(8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(1
6*a*A*b + 8*a^2*B + 4*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b^2 + 5*a*b*B)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[...
```

Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3469, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{2}\int \frac{b(4Ab+5aB)\cos^2(c+dx)+2\left(2Ba^2+4Aba+b^2B\right)\cos(c+dx)+a(4aA+bB)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 27

$$\frac{1}{4}\int \frac{b(4Ab+5aB)\cos^2(c+dx)+2\left(2Ba^2+4Aba+b^2B\right)\cos(c+dx)+a(4aA+bB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \frac{b(4Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2Ba^2+4Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(4aA+bB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3540

$$\frac{1}{4}\left(\frac{\int -\frac{-b\left(3Ba^2+12Aba+4b^2B\right)\cos^2(c+dx)-2ab(4aA+bB)\cos(c+dx)+ab(4Ab+5aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos}}{d\sqrt{\cos(c+dx)}}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 25

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-b\left(3Ba^2+12Aba+4b^2B\right)\cos^2(c+dx)-2ab(4aA+bB)\cos(c+dx)+ab(4Ab+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-b\left(3Ba^2+12Aba+4b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab(4aA+bB)\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3532}$$

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab(4Ab+5aB)-2ab(4aA+bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-b\big(3a^2B+12aAb+4b\right.$$
$$\frac{2b}{}$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab(4Ab+5aB)-2ab(4aA+bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b\big(3a^2B+12aAb+\right.$$
$$\frac{2b}{}$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3288}$$

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab(4Ab+5aB)-2ab(4aA+bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\big(3a^2B+12aAb+}{}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3477}$$

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(5aB+4Ab)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(8aA+5aB\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab(8aA+5aB+4Ab+2bB)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\right.}}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\big\downarrow$ 3295

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(5aB+4Ab)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(3}{}}{}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$\big\downarrow$ 3473

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\frac{2\sqrt{a+b}(3a^2B+12aAb+4b^2B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{El}}{d}}{}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x
]
```

output
```
(b*B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1
/2*((2*(a - b)*b*Sqrt[a + b]*(4*A*b + 5*a*B)*Cot[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a*d) - (2*b*Sqrt[a + b]*(8*a*A + 4*A*b + 5*a*B + 2*b*B)*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/d + (2*Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2
*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + ((4*A*b + 5*a*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. $2(421) = 842$.

Time = 21.75 (sec) , antiderivative size = 1389, normalized size of antiderivative = 2.94

method	result	size
default	Expression too large to display	1389
parts	Expression too large to display	1427

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*((-24*cos(d*x+c)^2-48*cos(d*x+c)-24)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticPi(cot(d*x+
c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*B
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))
^(1/2)*a^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-8*c
os(d*x+c)^2-16*cos(d*x+c)-8)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d
*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d
*x+c)-4)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
+(-5*cos(d*x+c)^2-10*cos(d*x+c)-5)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a
+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))+(-5*cos(d*x+c)^2-10*cos(d*x+c)-5)*B*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*E
llipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-8*cos(d*x+c)^2-16*c
os(d*x+c)-8)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*
x+c)+1)/(a+b))^(1/2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))+(16*cos(d*x+c)^2+32*cos(d*x+c)+16)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^{\frac{3}{2}}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/sqrt(cos(c + d*x
)), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c
)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)a^2 + \left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)b^2 + 2\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}dx\right)ab$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a**2 + i
nt(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b**2 + 2*in
t(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a*b
```

3.405 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 449

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a-b)\sqrt{a+b}(2aA-bB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\sqrt{}}\right.\right.}{ad}$$

$$-\frac{\sqrt{a+b}(2a(A-B)-b(4A+B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d}$$

$$-\frac{\sqrt{a+b}(2Ab+3aB)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec}{a}}}{d}$$

$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{(2aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
(a-b)*(a+b)^(1/2)*(2*A*a-B*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-(a+b)^(1/2)*(2*a*(A-B)-b*(4*A+B
))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(
a-b))^(1/2)/d-(a+b)^(1/2)*(2*A*b+3*B*a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d+2*a*A*(a+b*cos
(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-(2*A*a-B*b)*(a+b*cos(d*x+c))^
(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 1196, normalized size of antiderivative = 2.66

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
3/2),x]
```

output
```
(2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4
*a*(-2*a*A*b - 2*a^2*B - b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(2*a
^2*A - 2*A*b^2 - 4*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])) - 2*(2*a*A*b - b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]
], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + ...
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3468, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$2\int \frac{-b(2aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(2Ab+aB)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 27

$$\int \frac{-b(2aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(2Ab+aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\int \frac{-b(2aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-Aa^2+2bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(2Ab+aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3540

$$\frac{\int \frac{b^2(2Ab+3aB)\cos^2(c+dx)+2ab(2Ab+aB)\cos(c+dx)+ab(2aA-bB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{b^2(2Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab(2Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(2aA-bB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3532

$$\frac{b^2(3aB+2Ab)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+\int \frac{ab(2aA-bB)+2ab(2Ab+aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b^2(3aB+2Ab)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{ab(2aA-bB)+2ab(2Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b} - \frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3288

$$\frac{\int \frac{ab(2aA-bB)+2ab(2Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}(3aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\right.\right.}{d}}{2b} \\ \frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3477

$$\frac{ab(2aA-bB)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(2aA-2aB-4Ab-bB)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{2b\sqrt{a+b}(}{}}{2b} \\ \frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{-ab(2aA-2aB-4Ab-bB)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(2aA-bB)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin(c+dx+}}}{2b} \\ \frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3295

$$\frac{ab(2aA-bB)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}(2aA-2aB-4Ab-bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{d}}{} \\ \frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3473

$$-\frac{2b\sqrt{a+b}(2aA-2aB-4Ab-bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}+\frac{2b(a-b)\sqrt{a+b}(\ldots}{}$$

$$\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x
]
```

output
```
((2*(a - b)*b*Sqrt[a + b]*(2*a*A - b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*d) - (2*b*Sqrt[a + b]*(2*a*A - 4*A*b - 2*a*B - b*B)*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/d - (2*b*Sqrt[a + b]*(2*A*b + 3*a*B)*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(2*b) + (2*a*A*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*a*A - b*B)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(408) = 816$.

Time = 27.58 (sec) , antiderivative size = 1239, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	1239
parts	Expression too large to display	1281

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/d*((-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticPi(cot(d*x+c)-cs
c(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*B*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2
)*a*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(2+2*cos(d
*x+c)^2+4*cos(d*x+c))*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b
)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))+(2+2*cos(d*x+c)^2+4*cos(d*x+c))*A*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticE(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*B*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/
2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^
2-2*cos(d*x+c)-1)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(c
os(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*EllipticF(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2
)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(2+2*cos(d*x+c
)^2+4*cos(d*x+c))*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3
/2), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=2\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)ab+\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)a^{2}+\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}dx\right)b^{2}$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
2*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a*b +
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a**2
+ int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*b**2
```

3.406 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 419

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(4Ab+3aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{3ad}$$
$$+\frac{2\sqrt{a+b}(3Ab^2+a^2(A-3B)-a(4Ab-6bB))\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{}}{3ad}$$
$$-\frac{2b\sqrt{a+b}B\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/3*(a-b)*(a+b)^(1/2)*(4*A*b+3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/3*(a+b)^(1/2)*(3*A*b^2+
a^2*(A-3*B)-a*(4*A*b-6*B*b))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*b*(a+b)^(1/2)*B*cot(d*x+c)*Ell
ipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/d+2/3*a*A*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 1236, normalized size of antiderivative = 2.95

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
5/2),x]
```

output
```
((-4*a*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-4
*a*A*b - 3*a^2*B + 3*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])) + 2*(-4*A*b^2 - 3*a*b*B)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[...
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3042, 3468, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{3}\int \frac{3b^2B\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(4Ab+3aB)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3b^2B\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(4Ab+3aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(4Ab+3aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3532}$$

$$\frac{1}{3}\left(\int \frac{a(4Ab+3aB)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3b^2B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{a(4Ab+3aB)+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3288

$$\frac{1}{3}\left(\int \frac{a(4Ab+3aB)+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6bB\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(s}{}}}{}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3477

$$\frac{1}{3}\left(\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+a(3aB+4Ab)\int \frac{\text{co}}{\cos^{\frac{3}{2}}(c+}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a(3aB+4Ab)\int\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{3}\left(a(3aB+4Ab)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+\right.}{}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\right.}{ad}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x
]
```

output
```
((2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*d) + (2*Sqrt[a + b]*(3*A*b^2 + a^2*(A - 3*B) - a*(4*A*b - 6*b*B))*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (6*b*Sqrt[a + b]*B*Cot[c + d*x]
*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/3 + (2*a*A*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(376) = 752$.

Time = 31.03 (sec) , antiderivative size = 1363, normalized size of antiderivative = 3.25

method	result	size
parts	Expression too large to display	1363
default	Expression too large to display	1375

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*A/d*(((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(
4*cos(d*x+c)^3+8*cos(d*x+c)^2+4*cos(d*x+c))+((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*EllipticE(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(4*cos(d*x+c)^3+8*cos(d*x+c)^2+4*cos(d*x
+c))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a
+b))^(1/2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-cos
(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*(-4*cos(d*x+c)^3-8*cos(d*x+c)^2-4*cos(d*x+c))+(
(a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-3*cos(d*x
+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+(cos(d*x+c)+1)*sin(d*x+c)*a^2+sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)+5)*a*b+4*b^2*cos(d*x+c)^2*sin(d*x+c))*(a+cos(d*x+
c)*b)^(1/2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)-
2*B/d*((-csc(d*x+c)^3*(1-cos(d*x+c))^3-cot(d*x+c)+csc(d*x+c))*a*b+(csc(d*x
+c)^3*(1-cos(d*x+c))^3-cot(d*x+c)+csc(d*x+c))*a^2-2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticF(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(5
/2), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)b^2+\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^3}dx\right)a^2+2\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^2}dx\right)ab$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*b**2 + i
nt((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*a**2 +
 2*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a*
b
```

3.407 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 353

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(9a^2A+3Ab^2+20abB)\cot(c+dx)E\Big(\ldots}{\ldots}$$

$$-\frac{2(a-b)\sqrt{a+b}(9aA-3Ab-5aB+15bB)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1\ldots}{\ldots}}}{15ad}$$

$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2+3*A*b^2+20*B*a*b)*cot(d*x+c)*EllipticE((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2/15*(a-b
)*(a+b)^(1/2)*(9*A*a-3*A*b-5*B*a+15*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/5*a*A*(a+b*cos(d*x
+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*(6*A*b+5*B*a)*(a+b*cos(d*x+c
))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.02 (sec) , antiderivative size = 1314, normalized size of antiderivative = 3.72

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
7/2),x]
```

output
```
-1/15*((-4*a*(-3*a^2*A*b + 3*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - 4*a*(9*a^3*A + 3*a*A*b^2 + 20*a^2*b*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 3
*A*b^3 + 20*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c
+ d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x]
)*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + ...
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3468, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3468}$$

$$\frac{2}{5}\int \frac{b(2aA+5bB)\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(6Ab+5aB)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{b(2aA+5bB)\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(6Ab+5aB)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{b(2aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+10bBa+5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(6Ab+5aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{1}{5}\left(\frac{2\int \frac{a(9Aa^2+20bBa+3Ab^2)+a(5Ba^2+12Aba+15b^2B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+6Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{\int \frac{a(9Aa^2+20bBa+3Ab^2)+a(5Ba^2+12Aba+15b^2B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+6Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\int \frac{a(9Aa^2+20bBa+3Ab^2)+a(5Ba^2+12Aba+15b^2B)\sin(c+dx+\frac{\pi}{2})}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx}{3a}+\frac{2(5aB+6Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3477

$$\frac{1}{5}\left(\frac{a(9a^2A+20abB+3Ab^2)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(9aA-5aB-3Ab+15bB)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a}}}{3a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{a(9a^2A+20abB+3Ab^2)\int \frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx-a(a-b)(9aA-5aB-3Ab+15bB)\int \frac{}{\sqrt{\sin(}}}{3a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{5}\left(\frac{a\left(9a^2A+20abB+3Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(9aA-5aB-3Ab+15bB)\cot(c+dx)\sqrt{\frac{a(1-s}{}}}{}}{3a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}(9}{}}{3a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x
]
```

output
```
(2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
((2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B +
15*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(3*a) + (2*(6*A*b + 5*a*B)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(315) = 630$.

Time = 38.10 (sec) , antiderivative size = 1621, normalized size of antiderivative = 4.59

method	result	size
default	Expression too large to display	1621
parts	Expression too large to display	1626

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(9*cos(d*x+c)^4+18*cos(d*x+c)^3+9*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(9*cos(d*x+c)^4+18*cos(d*x+c)^3+
9*cos(d*x+c)^2)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(3*cos(d*x+c)^4+6*cos(d*x+c)^3+3*cos(d*x+c)^2)+A*((a+cos(d*x+c)*
b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*Ellip
ticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(3*cos(d*x+c)^4+6*cos(d*x
+c)^3+3*cos(d*x+c)^2)+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))*(20*cos(d*x+c)^4+40*cos(d*x+c)^3+20*cos(d*x+c)^2)+B*((a+co
s(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(20*cos(d*x+c)
^4+40*cos(d*x+c)^3+20*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((
a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*EllipticF(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*(-9*cos(d*x+c)^4-18*cos(d*x+c)^3-9*cos(d*x+c)^
2)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}{\cos(c+d\,x)^{7/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a^2 + 2\left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)ab + \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)b^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*a**2
+ 2*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*a
*b + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*
b**2
```

3.408 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 433

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(82a^2Ab-6Ab^3+63a^3B+21ab^2B)\,\mathrm{co}\ldots}{\ldots}$$

$$-\frac{2(a-b)\sqrt{a+b}(6Ab^2-a^2(25A-63B)+3ab(19A-7B))\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\ldots\right.\right.}{105a^2d}$$

$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(25a^2A+3Ab^2+42abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105ad\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/105*(a-b)*(a+b)^(1/2)*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*cot(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a
-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
a^3/d-2/105*(a-b)*(a+b)^(1/2)*(6*A*b^2-a^2*(25*A-63*B)+3*a*b*(19*A-7*B))*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-
(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a^2/d+2/7*a*A*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)
+2/35*(8*A*b+7*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2
/105*(25*A*a^2+3*A*b^2+42*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a/d/cos
(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.10 (sec) , antiderivative size = 1407, normalized size of antiderivative = 3.25

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
9/2),x]
```

output
```
((-4*a*(25*a^4*A - 31*a^2*A*b^2 + 6*A*b^4 + 21*a^3*b*B - 21*a*b^3*B)*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*A*b + 6*a*A*b^3 - 63*a^4*B - 21*
a^2*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) + 2*(-82*a^2*A*b^2 + 6*A*b^4 - 63*a^3*b*B - 21*a*b^3*B)*((I*Cos[
(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (
2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos...
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3468, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{7}\int \frac{b(4aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos(c+dx)+a(8Ab+7aB)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{7}\int \frac{b(4aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos(c+dx)+a(8Ab+7aB)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\int \frac{b(4aA+7bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Aa^2+14bBa+7Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(8Ab+7aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3534

$$\frac{1}{7}\left(\frac{2\int \frac{2ab(8Ab+7aB)\cos^2(c+dx)+a\left(21Ba^2+44Aba+35b^2B\right)\cos(c+dx)+a\left(25Aa^2+42bBa+3Ab^2\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(7aB+8Ab)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{7}\left(\frac{\int \frac{2ab(8Ab+7aB)\cos^2(c+dx)+a\left(21Ba^2+44Aba+35b^2B\right)\cos(c+dx)+a\left(25Aa^2+42bBa+3Ab^2\right)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(7aB+8Ab)\sin(c+dx)\sqrt{}}{5d\cos^{\frac{5}{2}}(c+\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\int \frac{2ab(8Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\left(21Ba^2+44Aba+35b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(25Aa^2+42bBa+3Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2(7aB+8Ab)\sin(c+\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3534

$$\frac{1}{7}\left(\frac{\frac{2\int \frac{\left(25Aa^2+84bBa+51Ab^2\right)\cos(c+dx)a^2+\left(63Ba^3+82Aba^2+21b^2Ba-6Ab^3\right)a}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25Aa^2+84bBa+51Ab^2\right)\cos(c+dx)a^2+\left(63Ba^3+82Aba^2+21b^2Ba-6Ab^3\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(7\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25Aa^2+84bBa+51Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(63Ba^3+82Aba^2+21b^2Ba-6Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3477

$$\frac{1}{7}\left(\frac{\frac{a\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(-\left(a^2(25A-63B)\right)+a(57Ab-21bB)+6Ab^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+}}}{3a}}{5a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{a\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(-\left(a^2(25A-63B)\right)+a(57Ab-21bB)+6Ab^2\right)\int\frac{}{\sqrt{\sin\left(c\right.}}}{3a}}{5a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{7}\left(\frac{\frac{a\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(-\left(a^2(25A-63B)\right)+a(57Ab-21bB)+6Ab^2\right)\cot(c+dx)}{}}{3a}}{5a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3473

$$\frac{1}{7}\left(\frac{\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{\frac{2(a-b)\sqrt{a+b}\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx}{a-b}}}{ad}}{}}{}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x
]
```

output
```
(2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (
(2*(8*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x
]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*
a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a +
b]*(6*A*b^2 - a^2*(25*A - 63*B) + a*(57*A*b - 21*b*B))*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/d)/(3*a) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/(5*a))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2084 vs. $2(389) = 778$.

Time = 54.74 (sec) , antiderivative size = 2085, normalized size of antiderivative = 4.82

method	result	size
default	Expression too large to display	2085
parts	Expression too large to display	2096

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/105/d*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c
)+1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*(-82*cos(d*x+c)^5-164*cos(d*x+c)^4-82*cos(d*x+c)^3)+A*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*El
lipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-82*cos(d*x+c)^5-164*
cos(d*x+c)^4-82*cos(d*x+c)^3)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(
d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c
),(-(a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^5+12*cos(d*x+c)^4+6*cos(d*x+c)^3)+A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^
(1/2)*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(6*cos(d*x
+c)^5+12*cos(d*x+c)^4+6*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*EllipticE(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*(-63*cos(d*x+c)^5-126*cos(d*x+c)^4-63*cos(d*
x+c)^3)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(-63*cos(d*x+c)^5-126*cos(d*x+c)^4-63*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-21*cos(d*x+c)^5-42*cos
(d*x+c)^4-21*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x
+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{5}}dx\right)a^{2}+2\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}}dx\right)ab+\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}}dx\right)b^{2}$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*a**2
+ 2*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*a
*b + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*
b**2
```

3.409 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 522

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB}{}$$

$$+\frac{2(a-b)\sqrt{a+b}(8Ab^3-a^3(147A-75B)+3a^2b(13A-57B)+6ab^2(A-3B))\cot(c+dx)\,\mathrm{EllipticF}\left(\mathrm{ar}\right.}{315a^3d}$$

$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{2(10Ab+9aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$+\frac{2(49a^2A+3Ab^2+72abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315ad\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315a^2d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/315*(a-b)*(a+b)^(1/2)*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B*a^3*b-18*B*a
*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^4/d+2/315*(a-b)*(a+b)^(1/2)*(8*A*b^3-a^3*(147*A-75*B)+3*a
^2*b*(13*A-57*B)+6*a*b^2*(A-3*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+2/9*a*A*(a+b*cos(d*x+c))^
(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+2/63*(10*A*b+9*B*a)*(a+b*cos(d*x+c))^(
1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/315*(49*A*a^2+3*A*b^2+72*B*a*b)*(a+b*
cos(d*x+c))^(1/2)*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)+2/315*(88*A*a^2*b-4*A*b^
3+75*B*a^3+9*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a^2/d/cos(d*x+c)^(
3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.44 (sec) , antiderivative size = 1515, normalized size of antiderivative = 2.90

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
11/2),x]
```

output
```
-1/315*((-4*a*(-39*a^4*A*b + 31*a^2*A*b^3 + 8*A*b^5 - 75*a^5*B + 93*a^3*b^
2*B - 18*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 33*a^
3*A*b^2 + 8*a*A*b^4 + 246*a^4*b*B - 18*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 33*a^2
*A*b^3 + 8*A*b^5 + 246*a^3*b^2*B - 18*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]
], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a...
```

Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {3042, 3468, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{9}\int \frac{3b(2aA+3bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos(c+dx)+a(10Ab+9aB)}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\int \frac{3b(2aA+3bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos(c+dx)+a(10Ab+9aB)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\int \frac{3b(2aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Aa^2+18bBa+9Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(10Ab+9aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3534

$$\frac{1}{9}\left(\frac{2\int \frac{4ab(10Ab+9aB)\cos^2(c+dx)+a\left(45Ba^2+92Aba+63b^2B\right)\cos(c+dx)+a\left(49Aa^2+72bBa+3Ab^2\right)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{2(9aB+10Ab)\sin(c+d}{7d\cos^{\frac{7}{2}}(c}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{\int \frac{4ab(10Ab+9aB)\cos^2(c+dx)+a\left(45Ba^2+92Aba+63b^2B\right)\cos(c+dx)+a\left(49Aa^2+72bBa+3Ab^2\right)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{2(9aB+10Ab)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\int \frac{4ab(10Ab+9aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\left(45Ba^2+92Aba+63b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(49Aa^2+72bBa+3Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{2(9aB+10Ab)\sin(c}{7d\,\mathrm{co}}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3534

$$\frac{1}{9}\left(\frac{\frac{2\int \frac{\left(147Aa^2+396bBa+209Ab^2\right)\cos(c+dx)a^2+2b\left(49Aa^2+72bBa+3Ab^2\right)\cos^2(c+dx)a+3\left(75Ba^3+88Aba^2+9b^2Ba-4Ab^3\right)a}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(49a^2A+72abB+3}{5}}{7a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147Aa^2+396bBa+209Ab^2\right)\cos(c+dx)a^2+2b\left(49Aa^2+72bBa+3Ab^2\right)\cos^2(c+dx)a+3\left(75Ba^3+88Aba^2+9b^2Ba-4Ab^3\right)a}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(49a^2A+72abB+3A}{5d}}{7a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147Aa^2+396bBa+209Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(49Aa^2+72bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+3\left(75Ba^3+88Aba^2+9b^2Ba-4Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2(49a^2A+72a}{}}{7a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3534

$$\frac{1}{9}\left(\frac{\frac{\frac{2\int \frac{3\left(\left(75Ba^3+186Aba^2+153b^2Ba+2Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+246bBa^3+33Ab^2a^2-18b^3Ba+8Ab^4\right)a\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(75a^3B+88a^2Ab+9ab^2B-4Ab^3\right)\sin(c+\cdots}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}}{7a}\cdots\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{\left(75Ba^3+186Aba^2+153b^2Ba+2Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+246bBa^3+33Ab^2a^2-18b^3Ba+8Ab^4\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2\left(75a^3B+88a^2Ab+9ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{\cdots}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}}{7a}\cdots\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{\left(75Ba^3+186Aba^2+153b^2Ba+2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(147Aa^4+246bBa^3+33Ab^2a^2-18b^3Ba+8Ab^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2\left(75a^3B+88a^2Ab+9ab^2B-4Ab^3\right)\sin(c+\cdots}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}}{7a}\cdots\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3477

$$\frac{1}{9}\left(\frac{\frac{a(a-b)\left(-3a^3(49A-25B)+a^2(39Ab-171bB)+6ab^2(A-3B)+8Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+a\left(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+8Ab^4\right)\int}{a}}{5a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{a(a-b)\left(-3a^3(49A-25B)+a^2(39Ab-171bB)+6ab^2(A-3B)+8Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+\right.}{a}}{5a}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{9}\left(\frac{\frac{a\left(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+8Ab^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(-3a^3(49A-25B)+a^2(39Ab-171bB)+6ab^2(A\right.}{}}{a}}{}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3473

$$\frac{1}{9}\left(\frac{\frac{2(49a^2A+72abB+3Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{\frac{2(75a^3B+88a^2Ab+9ab^2B-4Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}(-3a^3(49A-}{}}{}}{}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),
x]
```

output
```
(2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (
(2*(10*A*b + 9*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*
x]^(7/2)) + ((2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(147*a^4*
A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(a*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 - 3*a^3*(49*A -
25*B) + 6*a*b^2*(A - 3*B) + a^2*(39*A*b - 171*b*B))*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/d)/a + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)))/(5*a))/(7*a))/
9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(472) = 944$.

Time = 77.43 (sec) , antiderivative size = 2683, normalized size of antiderivative = 5.14

method	result	size
default	Expression too large to display	2683
parts	Expression too large to display	2699

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/315/d*(sin(d*x+c)*cos(d*x+c)^3*(246*cos(d*x+c)^2+81*cos(d*x+c)+81)*B*a^3
*b^2+sin(d*x+c)*cos(d*x+c)^4*(9*cos(d*x+c)-9)*B*a^2*b^3+sin(d*x+c)*cos(d*x
+c)*(147*cos(d*x+c)^4+137*cos(d*x+c)^3+137*cos(d*x+c)^2+85*cos(d*x+c)+85)*
A*a^4*b+sin(d*x+c)*cos(d*x+c)^2*(88*cos(d*x+c)^3+121*cos(d*x+c)^2+53*cos(d
*x+c)+53)*A*a^3*b^2+sin(d*x+c)*cos(d*x+c)^3*(33*cos(d*x+c)^2-cos(d*x+c)-1)
*A*a^2*b^3+sin(d*x+c)*cos(d*x+c)^4*(-4*cos(d*x+c)+4)*A*a*b^4+sin(d*x+c)*co
s(d*x+c)^2*(75*cos(d*x+c)^3+321*cos(d*x+c)^2+117*cos(d*x+c)+117)*B*a^4*b+A
*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*a^5*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(147*cos(
d*x+c)^6+294*cos(d*x+c)^5+147*cos(d*x+c)^4)+A*((a+cos(d*x+c)*b)/(cos(d*x+c
)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^5*EllipticE(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^6+16*cos(d*x+c)^5+8*cos(
d*x+c)^4)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a^5*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(-147*cos(d*x+c)^6-294*cos(d*x+c)^5-147*cos(d*x+c)^4)+B*((a+cos(d*x+c)*b
)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*Ellipt
icF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-75*cos(d*x+c)^6-150*cos(
d*x+c)^5-75*cos(d*x+c)^4)+8*A*b^5*cos(d*x+c)^5*sin(d*x+c)+(147*cos(d*x+c)^
4+49*cos(d*x+c)^3+49*cos(d*x+c)^2+35*cos(d*x+c)+35)*sin(d*x+c)*A*a^5-18*B*
a*b^4*cos(d*x+c)^5*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)*(75*cos(d*x+c)^3+75...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11
/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11
/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),
x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),
 x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{6}}dx\right)a^{2}+2\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{5}}dx\right)ab+\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}}dx\right)b^{2}$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**6,x)*a**2
+ 2*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*a
*b + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*
b**2
```

3.410 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 779

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{1920ab^2d}$$

$$-\frac{\sqrt{a+b}(45a^4B-30a^3b(5A+B)-16b^4(45A+64B)-8ab^3(355A+193B)-4a^2b^2(295A+423B))\cot}{1920b^2d}$$

$$+\frac{\sqrt{a+b}(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5B-40a^3b^2B-240ab^4B)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\right.}{128b^3d}$$

$$+\frac{(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{1920b^2d\sqrt{\cos(c+dx)}}$$

$$+\frac{(50a^2Ab+120Ab^3-15a^3B+172ab^2B)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{320bd}$$

$$+\frac{(50aAb-15a^2B+64b^2B)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{240bd}$$

$$+\frac{(10Ab-3aB)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd}$$

$$+\frac{B\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd}$$

output
```
-1/1920*(a-b)*(a+b)^(1/2)*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^
2+1024*B*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(
d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/b^2/d-1/1920*(a+b)^(1/2)*(45*B*a^4-30*a^3*b*(5*A+
B)-16*b^4*(45*A+64*B)-8*a*b^3*(355*A+193*B)-4*a^2*b^2*(295*A+423*B))*cot(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/b^2/d+1/128*(a+b)^(1/2)*(10*A*a^4*b-240*A*a^2*b^3-96*A*b^5-3*B*a^5-40*
B*a^3*b^2-240*B*a*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+1/1920*(150*A*a^3*b+2840*A*
a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*B*b^4)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c
)/b^2/d/cos(d*x+c)^(1/2)+1/320*(50*A*a^2*b+120*A*b^3-15*B*a^3+172*B*a*b^2)
*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d+1/240*(50*A*a*b-15
*B*a^2+64*B*b^2)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+1/
40*(10*A*b-3*B*a)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+1
/5*B*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 1353, normalized size of antiderivative = 1.74

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
-1/3840*((-4*a*(-1330*a^3*A*b - 3560*a*A*b^3 + 15*a^4*B - 3236*a^2*b^2*B -
 1024*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6440*a^2*A*b^2 - 1440
*A*b^4 - 2292*a^3*b*B - 4624*a*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-150*a^3*A*b - 2840*a*A*b^3 + 45
*a^4*B - 1692*a^2*b^2*B - 1024*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)
/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a
 + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot...
```

Rubi [A] (verified)

Time = 4.30 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.01, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.657$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{\int \frac{(a+b\cos(c+dx))^{5/2}((10Ab-3aB)\cos^2(c+dx)+8bB\cos(c+dx)+aB)}{2\sqrt{\cos(c+dx)}}dx}{5b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 27

$$\frac{\int \frac{(a+b\cos(c+dx))^{5/2}((10Ab-3aB)\cos^2(c+dx)+8bB\cos(c+dx)+aB)}{\sqrt{\cos(c+dx)}}dx}{10b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left((10Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+8bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{10b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3528

$$\frac{\frac{1}{4}\int \frac{(a+b\cos(c+dx))^{3/2}((-15Ba^2+50Aba+64b^2B)\cos^2(c+dx)+6b(10Ab+9aB)\cos(c+dx)+5a(2Ab+aB))}{2\sqrt{\cos(c+dx)}}dx+\frac{(10Ab-3aB)\sin(c+dx)\sqrt{\mathrm{c}\cdots}}{4\cdots}}{10b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 27

$$\frac{\frac{1}{8}\int \frac{(a+b\cos(c+dx))^{3/2}((-15Ba^2+50Aba+64b^2B)\cos^2(c+dx)+6b(10Ab+9aB)\cos(c+dx)+5a(2Ab+aB))}{\sqrt{\cos(c+dx)}}dx+\frac{(10Ab-3aB)\sin(c+dx)\sqrt{\mathrm{c}\cdots}}{4\cdots}}{10b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{8}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left((-15Ba^2+50Aba+64b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+6b(10Ab+9aB)\sin\left(c+dx+\frac{\pi}{2}\right)+5a(2Ab+aB)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(10Ab-3aB)\,\mathrm{si}\cdots}{\cdots}}{10b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3528

$$\frac{1}{8}\left(\frac{1}{3}\int\frac{\sqrt{a+b\cos(c+dx)}\left(3\left(-15Ba^3+50Aba^2+172b^2Ba+120Ab^3\right)\cos^2(c+dx)+2b\left(147Ba^2+310Aba+128b^2B\right)\cos(c+dx)+a\left(15Ba^2+110Aba\right.\right.}{2\sqrt{\cos(c+dx)}}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\int\frac{\sqrt{a+b\cos(c+dx)}\left(3\left(-15Ba^3+50Aba^2+172b^2Ba+120Ab^3\right)\cos^2(c+dx)+2b\left(147Ba^2+310Aba+128b^2B\right)\cos(c+dx)+a\left(15Ba^2+110Aba\right.\right.}{\sqrt{\cos(c+dx)}}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\int\frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3\left(-15Ba^3+50Aba^2+172b^2Ba+120Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(147Ba^2+310Aba+128b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(15B\right.\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3528

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3a+1024b^4B\right)\cos^2(c+dx)+2b\left(573Ba^3+1610Aba^2+1156b^2Ba+360Ab^3\right)\cos(c+dx)+a\left(1\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\int\frac{\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3a+1024b^4B\right)\cos^2(c+dx)+2b\left(573Ba^3+1610Aba^2+1156b^2Ba+360Ab^3\right)\cos(c+dx)+a\left(1\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\int \frac{\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3a+1024b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(573Ba^3+1610Aba^2+1156b^2Ba+360Ab^3\right)\sin\left(c+dx+\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3540

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int -\frac{15\left(-3Ba^5+10Aba^4-40b^2Ba^3-240Ab^3a^2-240b^4Ba-96Ab^5\right)\cos^2(c+dx)-2ab\left(15Ba^3+590Aba^2+772b^2Ba+360Ab^3\right)\cos(c+dx)+a\left(-45Ba\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 25

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{15\left(-3Ba^5+10Aba^4-40b^2Ba^3-240Ab^3a^2-\right.}{}}{}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{15\left(-3Ba^5+10Aba^4-40b^2Ba^3-240Ab^3a^2-\right.}{}}{}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\downarrow$ 3532

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{a\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3\right.}{\mathrm{co}}}{}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3\right.}{\sin\left(c\right.}}{}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3288

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3\right.}{\sin\left(c\right.}}{}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3477

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840a\right.}{}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(45a^4B-30a^3b(5A+B)-4a^2b^2(295A+4\right.}{}\right.\right.\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3295

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840a\right.}{}\right.\right.\right.}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\frac{1}{8}\left(\frac{\left(-15a^2B+50aAb+64b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}+\frac{1}{6}\left(\frac{3\left(-15a^3B+50a^2Ab+172ab^2B+120Ab^3\right)\sin(c+dx)\sqrt{\cos(c+\right.}{2d}\right.\right.}{}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{7/2}}{5bd}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d) + (
((10*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*
x])/(4*d) + (((50*a*A*b - 15*a^2*B + 64*b^2*B)*Sqrt[Cos[c + d*x]]*(a + b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + ((3*(50*a^2*A*b + 120*A*b^3 - 15*
a^3*B + 172*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(2*d) + (-1/2*((2*(a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 4
5*a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d) + (2*Sqrt[a + b]*(45*a^4*B - 30*a^3*b*(5*A + B) - 16*b^4*(45*A + 6
4*B) - 8*a*b^3*(355*A + 193*B) - 4*a^2*b^2*(295*A + 423*B))*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/d - (30*Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 9
6*A*b^5 - 3*a^5*B - 40*a^3*b^2*B - 240*a*b^4*B)*Cot[c + d*x]*EllipticPi[(a
 + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(b*d))/b + ((150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B
+ 1692*a^2*b^2*B + 1024*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d
*Sqrt[Cos[c + d*x]]))/4)/6)/8)/(10*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2991 vs. 2(710) = 1420.

Time = 38.85 (sec) , antiderivative size = 2992, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	2992
parts	Expression too large to display	3010

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/1920/d*((-720*cos(d*x+c)^2-1440*cos(d*x+c)-720)*A*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^4*EllipticF(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c
)-30)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)
/(a+b))^(1/2)*a^4*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+
(2292*cos(d*x+c)^2+4584*cos(d*x+c)+2292)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b^2*EllipticF(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-1544*cos(d*x+c)^2-3088*cos(d*x+c)-1
544)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/
(a+b))^(1/2)*a^2*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
+(4624*cos(d*x+c)^2+9248*cos(d*x+c)+4624)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^4*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(300*cos(d*x+c)^2+600*cos(d*x+c)+300)*
A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b)
)^(1/2)*a^4*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-
7200*cos(d*x+c)^2-14400*cos(d*x+c)-7200)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^3*EllipticPi(cot(d*
x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-1200*cos(d*x+c)^2-2400*cos(d*x+
c)-1200)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*a^3*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a...
```

Fricas [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int \cos(c+dx)^{3/2}\,(A+B\,\cos(c+dx))\,(a+b\,\cos(c+dx))^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^3 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a\,b^2 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a^2 b$$

input

```
int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output

```
int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a**3 + int
(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*b**3 + 3*i
nt(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*a*b**2 +
 3*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a**2
*b
```

3.411 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 664

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{192abd}$$

$$+\frac{\sqrt{a+b}(15a^3B+8b^3(16A+9B)+2a^2b(132A+59B)+4ab^2(52A+71B))\cot(c+dx)\,\text{EllipticF}\left(\text{arcsin}\right.}{192bd}$$

$$-\frac{\sqrt{a+b}(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c-}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{64b^2d}$$

$$+\frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{192bd\sqrt{\cos(c+dx)}}$$

$$+\frac{(24aAb+5a^2B+12b^2B)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32d}$$

$$+\frac{(8Ab+11aB)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24d}$$

$$+\frac{bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d}$$

output
```
-1/192*(a-b)*(a+b)^(1/2)*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*cot(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a/b/d+1/192*(a+b)^(1/2)*(15*B*a^3+8*b^3*(16*A+9*B)+2*a^2*b*(132*A+59*
B)+4*a*b^2*(52*A+71*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/64*(a+b)^(1/2)*(40*A*a^3*b+160*A*a*
b^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/192*(264*A*a
^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d
/cos(d*x+c)^(1/2)+1/32*(24*A*a*b+5*B*a^2+12*B*b^2)*cos(d*x+c)^(1/2)*(a+b*c
os(d*x+c))^(1/2)*sin(d*x+c)/d+1/24*(8*A*b+11*B*a)*cos(d*x+c)^(1/2)*(a+b*co
s(d*x+c))^(3/2)*sin(d*x+c)/d+1/4*b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/
2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.77 (sec) , antiderivative size = 1287, normalized size of antiderivative = 1.94

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x
]),x]
```

output
```
((-4*a*(472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - 4*a*(384*a^3*A + 608*a*A*b^2 + 644*a^2*b*B + 144*b^3*B)
*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
+ 2*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
 + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C...
```

Rubi [A] (verified)

Time = 3.56 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{4}\int \frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(b(8Ab+11aB)\cos^2(c+dx)+2\left(4Ba^2+8Aba+3b^2B\right)\cos(c+dx)+a\right.$$
$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(b(8Ab+11aB)\cos^2(c+dx)+2\left(4Ba^2+8Aba+3b^2B\right)\cos(c+dx)+a(8\right.$$
$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(8Ab+11aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(4Ba^2+8Aba+3b^2B\right.\right.$$
$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3528

$$\frac{1}{8}\left(\frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(3b\left(5Ba^2+24Aba+12b^2B\right)\cos^2(c+dx)+2b\left(24Aa^2+31bBa+16Ab^2\right)\cos(c+dx)+ab(8Ab+11aB)\right)}{2\sqrt{\cos(c+dx)}}dx}{3b}+\frac{(11aB+8\right.$$
$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(3b\left(5Ba^2+24Aba+12b^2B\right)\cos^2(c+dx)+2b\left(24Aa^2+31bBa+16Ab^2\right)\cos(c+dx)+ab(8Ab+11aB)\right)}{\sqrt{\cos(c+dx)}}dx}{6b}+\frac{(11aB+8\right.$$
$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3b\left(5Ba^2+24Aba+12b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(24Aa^2+31bBa+16Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(8Ab+11aB)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6b}+\frac{(\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{1}{8}\left(\frac{\frac{1}{2}\int \frac{b\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(96Aa^3+161bBa^2+152Ab^2a+36b^3B\right)\cos(c+dx)+ab\left(59Ba^2+104Aba+36b^2B\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{6b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{\frac{1}{4}\int \frac{b\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(96Aa^3+161bBa^2+152Ab^2a+36b^3B\right)\cos(c+dx)+ab\left(59Ba^2+104Aba+36b^2B\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{6b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{\frac{1}{4}\int \frac{b\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(96Aa^3+161bBa^2+152Ab^2a+36b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+ab\left(59Ba^2+104Aba\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{6b}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3540}$$

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{\int -\frac{-2a\left(59Ba^2+104Aba+36b^2B\right)\cos(c+dx)b^2-3\left(-5Ba^4+40Aba^3+120b^2Ba^2+160Ab^3a+48b^4B\right)\cos^2(c+dx)b+a\left(15Ba^3+264Aba^2+284b^2Ba+128\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-2a\left(59Ba^2+104Aba+36b^2B\right)\cos(c+dx)b^2-3\left(-5Ba^4+40Aba^3+12\right.}{\cos^{\frac{3}{2}}(c}}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-2a\left(59Ba^2+104Aba+36b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2-3\left(-5Ba^4+40Aba^3\right.}{\sin\left(c+\right.}}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \; 3532$$

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)-2ab^2\left(59Ba^2+104Aba+3\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)-2ab^2\left(59Ba^2+104Aba+3\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3288

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)-2ab^2\left(59Ba^2+104Aba+3\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3477

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos}}}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab(15a^3B+2a^2b(132A+59B)+4ab^2(52A+71B)+8b^3(16A+9}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3295

$$\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\int\frac{\sin\left(c+dx+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a}}}{}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3473

$$\frac{1}{8}\left(\frac{\frac{3b(5a^2B+24aAb+12b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+}{d\sqrt{\cos(c+dx)}}\right.}{}\right.$$

$$\frac{bB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}{4d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + (
((8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(3*d) + ((3*b*(24*a*A*b + 5*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b*Sqrt[a + b]*
(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(15*a^3*B + 8*b^3*(16*A + 9*B) + 2*
a^2*b*(132*A + 59*B) + 4*a*b^2*(52*A + 71*B))*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/d + (6*Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a^2*
b^2*B + 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b +
((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/4)/(6*b))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2438 vs. $2(601) = 1202$.

Time = 31.96 (sec) , antiderivative size = 2439, normalized size of antiderivative = 3.67

method	result	size
default	Expression too large to display	2439
parts	Expression too large to display	2459

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*((-284*cos(d*x+c)^2-568*cos(d*x+c)-284)*B*((a+cos(d*x+c)*b)/(cos(d
*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-284*cos(d*x+c)^2-568*cos(d*x
+c)-284)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))+(384*cos(d*x+c)^2+768*cos(d*x+c)+384)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b*EllipticF(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-208*cos(d*x+c)^2-416*cos(d*x+c)-208)*
A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(608
*cos(d*x+c)^2+1216*cos(d*x+c)+608)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*EllipticF(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))+(-118*cos(d*x+c)^2-236*cos(d*x+c)-118)*B*((a+
cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*a^3*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(644*cos(d*x
+c)^2+1288*cos(d*x+c)+644)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))+(-128*cos(d*x+c)^2-256*cos(d*x+c)-128)*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^
3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-72*cos(d*x+c)...
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A + B\cos(c+dx))\,dx = 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a^2b + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a\,b^2 + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx\right)a^3$$

input
```
int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
3*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a**2*b +
 int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**3,x)*b**3 +
 3*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a*b*
*2 + int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a**3
```

3.412 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 564

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{(a-b)\sqrt{a+b}(54aAb+33a^2B+16b^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{24ad}$$
$$+\frac{\sqrt{a+b}(4b^2(3A+4B)+a^2(48A+33B)+a(54Ab+26bB))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{24d}$$
$$-\frac{\sqrt{a+b}(30a^2Ab+8Ab^3+5a^3B+20ab^2B)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{8bd}$$
$$+\frac{(54aAb+33a^2B+16b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}$$
$$+\frac{b(2Ab+3aB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d}$$
$$+\frac{bB\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d}$$

output
```
-1/24*(a-b)*(a+b)^(1/2)*(54*A*a*b+33*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/24*(a+
b)^(1/2)*(4*b^2*(3*A+4*B)+a^2*(48*A+33*B)+a*(54*A*b+26*B*b))*cot(d*x+c)*El
lipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))
^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/
8*(a+b)^(1/2)*(30*A*a^2*b+8*A*b^3+5*B*a^3+20*B*a*b^2)*cot(d*x+c)*EllipticP
i((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b
))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/
d+1/24*(54*A*a*b+33*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/co
s(d*x+c)^(1/2)+1/4*b*(2*A*b+3*B*a)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)
*sin(d*x+c)/d+1/3*b*B*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.30 (sec) , antiderivative size = 1251, normalized size of antiderivative = 2.22

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d
*x]],x]
```

output
```
((-4*a*(48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - 4*a*(144*a^2*A*b + 24*A*b^3 + 48*a^3*B + 76*a*b^2*B)*((Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(54
*a*A*b^2 + 33*a^2*b*B + 16*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a
 - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)...
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {3042, 3469, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{1}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(3b(2Ab+3aB)\cos^2(c+dx)+2\left(3Ba^2+6Aba+2b^2B\right)\cos(c+dx)+a(6aA+bB)\right)}{2\sqrt{\cos(c+dx)}}dx$$
$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{\sqrt{a+b\cos(c+dx)}\left(3b(2Ab+3aB)\cos^2(c+dx)+2\left(3Ba^2+6Aba+2b^2B\right)\cos(c+dx)+a(6aA+bB)\right)}{\sqrt{\cos(c+dx)}}dx$$
$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3b(2Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(3Ba^2+6Aba+2b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}$$
$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{b\left(33Ba^2+54Aba+16b^2B\right)\cos^2(c+dx)+2\left(12Ba^3+36Aba^2+19b^2Ba+6Ab^3\right)\cos(c+dx)+a(24A}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{b\left(33Ba^2+54Aba+16b^2B\right)\cos^2(c+dx)+2\left(12Ba^3+36Aba^2+19b^2Ba+6Ab^3\right)\cos(c+dx)+a(24A}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{b\left(33Ba^2+54Aba+16b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(12Ba^3+36Aba^2+19b^2Ba+6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3540

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int -\frac{-3b\left(5Ba^3+30Aba^2+20b^2Ba+8Ab^3\right)\cos^2(c+dx)-2ab\left(24Aa^2+13bBa+6Ab^2\right)\cos(c+dx)+ab\left(33Ba^2+54Aba+16b^2B\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(3\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-3b\left(5Ba^3+30Aba^2+20b^2Ba+8Ab^3\right)\cos^2(c+dx)-2}{\cos^{\frac{3}{2}}(c+}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-3b\left(5Ba^3+30Aba^2+20b^2Ba+8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin(c+dx}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3532

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab\left(33Ba^2+54Aba+16b^2B\right)-2ab\left(24Aa^2+13bBa+6A\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab\left(33Ba^2+54Aba+16b^2B\right)-2ab\left(24Aa^2+13bBa+6A\cdots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\cdots\right.}}}{\cdots}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3288

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab\left(33Ba^2+54Aba+16b^2B\right)-2ab\left(24Aa^2+13bBa+6A\cdots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\cdots\right.}}}{\cdots}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3477

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(33a^2B+54aAb+16b^2B\right)\int \frac{\cos(c+\cdots}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cdots}}}{\cdots}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab\left(a^2(48A+33B)+a(54Ab+26bB)+\cdots\right.}{\cdots}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3295

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(33a^2B+54aAb+16b^2B)\int\frac{\sin(}{\sin(c+dx+\frac{\pi}{2})}}{}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b\sqrt{a+b}(a^2(48A+33B)+a(54Ab+26bB)+4b^2(3A+4}{}}{}\right.\right.$$

$$\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

input

```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x
]
```

output

```
(b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (
(3*b*(2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(2*d) + (-1/2*((2*(a - b)*b*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(4*b^2*(
3*A + 4*B) + a^2*(48*A + 33*B) + a*(54*A*b + 26*b*B))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/d + (6*Sqrt[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a
*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + ((54*a*A*b
 + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos
[c + d*x]]))/4)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2003 vs. $2(507) = 1014$.

Time = 30.73 (sec) , antiderivative size = 2004, normalized size of antiderivative = 3.55

method	result	size
default	Expression too large to display	2004
parts	Expression too large to display	2039

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*((-180*cos(d*x+c)^2-360*cos(d*x+c)-180)*A*((a+cos(d*x+c)*b)/(cos(d*
x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticPi(co
t(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-48*cos(d*x+c)^2-96*cos(d*x+
c)-48)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))+(-30*cos(d*x+c)^2-60*cos(d*x+c)-30)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/
(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-120*cos(d*x+c)^2-240*cos(d*x+c)-120)*
B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-
54*cos(d*x+c)^2-108*cos(d*x+c)-54)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))+(-54*cos(d*x+c)^2-108*cos(d*x+c)-54)*A*((a+co
s(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-33*cos(d*x+c
)^2-66*cos(d*x+c)-33)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))+(-33*cos(d*x+c)^2-66*cos(d*x+c)-33)*B*((a+cos(d*x+c)*b)/(cos
(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-16*cos(d*x+c)^2-32*cos(d*...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c
)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)a^3 + 3\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a\,b^2 + \left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)b^3 + 3\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}dx\right)a^2b$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a**3 + 3
*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*a*b**2 +
int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b**3 +
3*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a**2*b
```

3.413 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 547

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a-b)\sqrt{a+b}(8a^2A-4Ab^2-9abB)\cot(c+dx)E\Big(\text{arc}}{}$$

$$-\frac{\sqrt{a+b}(8a^2(A-B)-2b^2(2A+B)-3ab(8A+3B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\right.}{4d}$$

$$-\frac{\sqrt{a+b}(20aAb+15a^2B+4b^2B)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c-}{a+b}}}{4d}$$

$$-\frac{(8a^2A-4Ab^2-9abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}$$

$$-\frac{b(4aA-bB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d}$$

$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
1/4*(a-b)*(a+b)^(1/2)*(8*A*a^2-4*A*b^2-9*B*a*b)*cot(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(a+b)^(1/
2)*(8*a^2*(A-B)-2*b^2*(2*A+B)-3*a*b*(8*A+3*B))*cot(d*x+c)*EllipticF((a+b*c
os(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(a+b)^(1/2)*
(20*A*a*b+15*B*a^2+4*B*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(8*A*a^2-4*A*b^2-9*B*a
*b)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/2*b*(4*A*a-B*b)
*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2*a*A*(a+b*cos(d*x+c
))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.07 (sec) , antiderivative size = 1241, normalized size of antiderivative = 2.27

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
3/2),x]
```

output
```
((4*a*(-16*a^2*A*b - 4*A*b^3 - 8*a^3*B - 11*a*b^2*B)*Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF
[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) + 4*a*(8*a^3*A - 24*a*A*b^2 - 24*a^2*b*B - 4*b^3*B)*((Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(8*a^2*A
*b - 4*A*b^3 - 9*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*El
lipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*S
ec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
 d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(...
```

Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3468, 27, 3042, 3528, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$2\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(4aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(4Ab+aB)\right)}{2\sqrt{\cos(c+dx)}}dx+ \frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 27

$$\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(4aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(4Ab+aB)\right)}{\sqrt{\cos(c+dx)}}dx+ \frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-b(4aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-Aa^2+2bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(4Ab\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}} \frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3528

$$\frac{1}{2}\int \frac{-b\left(8Aa^2-9bBa-4Ab^2\right)\cos^2(c+dx)-2\left(2Aa^3-6bBa^2-6Ab^2a-b^3B\right)\cos(c+dx)+a\left(4Ba^2+12Aba\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} \frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+ \frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 27

$$\frac{1}{4}\int \frac{-b\left(8Aa^2-9bBa-4Ab^2\right)\cos^2(c+dx)-2\left(2Aa^3-6bBa^2-6Ab^2a-b^3B\right)\cos(c+dx)+a\left(4Ba^2+12Aba\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} \frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+ \frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{-b\left(8Aa^2-9bBa-4Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2\left(2Aa^3-6bBa^2-6Ab^2a-b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(4Ba\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3540

$$\frac{1}{4}\left(\frac{\int\frac{b^2\left(15Ba^2+20Aba+4b^2B\right)\cos^2(c+dx)+2ab\left(4Ba^2+12Aba+b^2B\right)\cos(c+dx)+ab\left(8Aa^2-9bBa-4Ab^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(8a^2A-9abB-4Ab\right.}{d}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\int\frac{b^2\left(15Ba^2+20Aba+4b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab\left(4Ba^2+12Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+ab\left(8Aa^2-9bBa-4Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(8a^2A-9abB\right.}{}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3532

$$\frac{1}{4}\left(\frac{\int\frac{ab\left(8Aa^2-9bBa-4Ab^2\right)+2ab\left(4Ba^2+12Aba+b^2B\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+b^2\left(15a^2B+20aAb+4b^2B\right)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(8}{}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{b^2\left(15a^2B+20aAb+4b^2B\right)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{ab\left(8Aa^2-9bBa-4Ab^2\right)+2ab\left(4Ba^2+12Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3288$$

$$\frac{1}{4}\left(\frac{\int\frac{ab\left(8Aa^2-9bBa-4Ab^2\right)+2ab\left(4Ba^2+12Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\left(15a^2B+20aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{}}{2b}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3477$$

$$\frac{1}{4}\left(\frac{ab\left(8a^2A-9abB-4Ab^2\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\int}{}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{-ab\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(8a^2A-9abB-\right.}{}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3295

$$\frac{1}{4}\left(\frac{ab\left(8a^2A-9abB-4Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\cot(c+}{}}{}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3473

$$\frac{1}{4}\left(\frac{-\frac{2b\sqrt{a+b}\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a}{a}\right.}{d}}{}\right.$$

$$\frac{b(4aA-bB)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x
]
```

output
```
-1/2*(b*(4*a*A - b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/d + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) + (((2*(a - b)*b*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Cot[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(8*a^2*(A - B) - 2*b
^2*(2*A + B) - 3*a*b*(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d -
 (2*b*Sqrt[a + b]*(20*a*A*b + 15*a^2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/d)/(2*b) - ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(492) = 984$.

Time = 31.28 (sec) , antiderivative size = 1848, normalized size of antiderivative = 3.38

method	result	size
default	Expression too large to display	1848
parts	Expression too large to display	1880

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*((-40*cos(d*x+c)^2-80*cos(d*x+c)-40)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticPi(cot(d*
x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c)-3
0)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a
+b))^(1/2)*a^2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a
+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^3*EllipticPi(cot(d*x+c)-csc(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8)*A*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^
3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(8*cos(d*x+c)^2+16
*cos(d*x+c)+8)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(
d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticE(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/
2)*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-9*cos(d*x+c
)^2-18*cos(d*x+c)-9)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)
/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9)*B*(cos(d*x+c)/(cos(d*x...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = 3\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx\right)a^{2}b + \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)a^{3} + \left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)b^{3} + 3\left(\int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}dx\right)a\,b^{2}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a**2*b
 + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)*a*
*3 + int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)*b**3
+ 3*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*a*b**2
```

3.414 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 536

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{(a-b)\sqrt{a+b}(14aAb+6a^2B-3b^2B)\cot(c+dx)E\Big(\mathrm{a}\\
&- \frac{\sqrt{a+b}(2ab(7A-9B)-2a^2(A-3B)-3b^2(6A+B))\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\right.}{3d}\\
&- \frac{b\sqrt{a+b}(2Ab+5aB)\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\mathrm{se}}{}}}{d}\\
&+ \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\
&- \frac{(14aAb+6a^2B-3b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}\\
&+ \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
1/3*(a-b)*(a+b)^(1/2)*(14*A*a*b+6*B*a^2-3*B*b^2)*cot(d*x+c)*EllipticE((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/3*(a+b)^(1
/2)*(2*a*b*(7*A-9*B)-2*a^2*(A-3*B)-3*b^2*(6*A+B))*cot(d*x+c)*EllipticF((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-b*(a+b)^(1/2)
*(2*A*b+5*B*a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d+2*a*(2*A*b+B*a)*(a+b*cos(d*x+c))^(1/2)*
sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/3*(14*A*a*b+6*B*a^2-3*B*b^2)*(a+b*cos(d*x+
c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a*A*(a+b*cos(d*x+c))^(3/2)*sin
(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.01 (sec) , antiderivative size = 1269, normalized size of antiderivative = 2.37

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
5/2),x]
```

output
```
((-4*a*(2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]) - 4*a*(-14*a^2*A*b + 6*A*b^3 - 6*a^3*B + 18*a*b^2*B)*((Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-14*a*A
*b^2 - 6*a^2*b*B + 3*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]
*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
 + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt...
```

Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(2aA-3bB)\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(2Ab+aB)\right)}{2\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(2aA-3bB)\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+3a(2Ab+aB)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-b(2aA-3bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3526

$$\frac{1}{3}\left(2\int \frac{-b\left(6Ba^2+14Aba-3b^2B\right)\cos^2(c+dx)-\left(3Ba^3+7Aba^2-9b^2Ba-3Ab^3\right)\cos(c+dx)+a\left(Aa^2+9bI\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\int \frac{-b\left(6Ba^2+14Aba-3b^2B\right)\cos^2(c+dx)-\left(3Ba^3+7Aba^2-9b^2Ba-3Ab^3\right)\cos(c+dx)+a\left(Aa^2+9bBa\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{-b\left(6Ba^2+14Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-3Ba^3-7Aba^2+9b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3540$$

$$\frac{1}{3}\left(\frac{\int \frac{3(2Ab+5aB)\cos^2(c+dx)b^3+a\left(6Ba^2+14Aba-3b^2B\right)b+2a\left(Aa^2+9bBa+9Ab^2\right)\cos(c+dx)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(}{d\sqrt{\cos(c}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{\int \frac{3(2Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+a\left(6Ba^2+14Aba-3b^2B\right)b+2a\left(Aa^2+9bBa+9Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(6a^2B+14aAb-3b^2B\right.}{d\sqrt{}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3532$$

$$\frac{1}{3}\left(\frac{\int \frac{ab\left(6Ba^2+14Aba-3b^2B\right)+2ab\left(Aa^2+9bBa+9Ab^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3b^3(5aB+2Ab)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(6a^2B+14aA\right.}{}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{\int \frac{ab\left(6Ba^2+14Aba-3b^2B\right)+2ab\left(Aa^2+9bBa+9Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3(5aB+2Ab)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(6a^2B+\right.}{}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3288$$

$$\frac{1}{3}\left(\frac{\int \frac{ab(6Ba^2+14Aba-3b^2B)+2ab(Aa^2+9bBa+9Ab^2)\sin(c+dx+\frac{\pi}{2})}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx-\frac{6b^2\sqrt{a+b}(5aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)}{a-b}}}{d}}{2b}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3477$$

$$\frac{1}{3}\left(\frac{ab\left(6a^2B+14aAb-3b^2B\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B\right.}{}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{-ab\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B)\right)\int \frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx+ab\left(6a^2B+14aA\right.}{}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3295$$

$$\frac{1}{3}\left(\frac{ab\left(6a^2B+14aAb-3b^2B\right)\int \frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx-\frac{2b\sqrt{a+b}\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B)\right)\text{co}}{}}{}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3473$$

$$\frac{1}{3}\left(-\frac{2b\sqrt{a+b}(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),\right.}{d}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x
]
```

output
```
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) +
 (((2*(a - b)*b*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Cot[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(2*a*b*(7*A - 9*B) - 2*a^2*
(A - 3*B) - 3*b^2*(6*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*b
^2*Sqrt[a + b]*(2*A*b + 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/d)/(2*b) + (6*a*(2*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]]) - ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(483) = 966$.

Time = 35.03 (sec) , antiderivative size = 1831, normalized size of antiderivative = 3.42

method	result	size
parts	Expression too large to display	1831
default	Expression too large to display	1883

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
B/d*((-10*cos(d*x+c)^2-20*cos(d*x+c)-10)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(2+2*cos(d*x+c)^2+4*cos(d*x+c))*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)
*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(2+2*cos(d*x+c)
^2+4*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(
d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^3*E
llipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*co
s(d*x+c)-2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c
)+1)/(a+b))^(1/2)*a^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+
cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*E
llipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)
^2*b^3+2*a^2*b*cos(d*x+c)*sin(d*x+c)+a*b^2*cos(d*x+c)*sin(d*x+c)+2*sin(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = 3\left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a\,b^2 + \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a^3 + 3\left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a^2b + \left(\int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx\right)b^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a*b**2
 + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*a*
*3 + 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x
)*a**2*b + int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)*b**3
```

3.415 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 493

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(9a^2A+23Ab^2+35abB)\cot(c+dx)E(\cdots}{\cdots}$$
$$+\frac{2\sqrt{a+b}(15Ab^3-ab^2(23A-45B)+a^2b(17A-35B)-a^3(9A-5B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\cdots}}{\sqrt{a\cdots}}\right.\right.}{15ad}$$
$$-\frac{2b^2\sqrt{a+b}B\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2+23*A*b^2+35*B*a*b)*cot(d*x+c)*EllipticE((a
+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a
*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/15*(a+b)
^(1/2)*(15*A*b^3-a*b^2*(23*A-45*B)+a^2*b*(17*A-35*B)-a^3*(9*A-5*B))*cot(d*
x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)
/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a/d-2*b^2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/15*a*(8*A*b+5*B*a)*(a+b*c
os(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*a*A*(a+b*cos(d*x+c))^(3
/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.22 (sec) , antiderivative size = 1319, normalized size of antiderivative = 2.68

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
7/2),x]
```

output
```
((4*a*(-8*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]) + 4*a*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B - 15*b^3*B)*((Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(9*a^2*A
*b + 23*A*b^3 + 35*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]
*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
 + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt...
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{5}\int \frac{\sqrt{a+b\cos(c+dx)}\left(5b^2B\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(8Ab+5aB)\right)}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\int \frac{\sqrt{a+b\cos(c+dx)}\left(5b^2B\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(8Ab+5aB)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(5b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+10bBa+5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(8Ab+5a\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3526

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15B\cos^2(c+dx)b^3+a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15B\cos^2(c+dx)b^3+a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15B\sin\left(c+dx+\frac{\pi}{2}\right)^2 b^3+a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\sin\left(c+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3532

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+15b^3B\int \frac{\sqrt{\cos}}{\sqrt{a+b}}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+15b^3B\int -\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3288

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30b^2B\sqrt{a+}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3477

$$\frac{1}{5}\left(\frac{1}{3}\left(a\left(9a^2A+35abB+23Ab^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\left(-\left(a^3(9A-5B)\right)+a^2b(17A-35B\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(a\left(9a^2A+35abB+23Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(-\left(a^3(9A-5B)\right)+a^2b\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3295

$$\frac{1}{5}\left(\frac{1}{3}\left(a\left(9a^2A+35abB+23Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(-\left(a^3(9A-5B\right.\right.}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3473

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(9a^2A+35abB+23Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{ad}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x
]
```

output
```
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 (((2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Cot[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(a*d) + (2*Sqrt[a + b]*(15*A*b^3 - a*b^2*(23*A - 45*B
) + a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(a*d) - (30*b^2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/d)/3 + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(3*d*Cos[c + d*x]^(3/2)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1957 vs. $2(444) = 888$.

Time = 42.41 (sec) , antiderivative size = 1958, normalized size of antiderivative = 3.97

method	result	size
default	Expression too large to display	1958
parts	Expression too large to display	1969

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/15/d*(B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*(30*cos(d*x+c)^4+60*cos(d*x+c)^3+30*cos(d*x+c)^2)+A*((a+cos(d*x+c)*b)
/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-9*cos(d*x+c)^4-18*cos(d*x
+c)^3-9*cos(d*x+c)^2)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))*(-9*cos(d*x+c)^4-18*cos(d*x+c)^3-9*cos(d*x+c)^2)+A*((a+cos
(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-23*cos(d*x+c)
^4-46*cos(d*x+c)^3-23*cos(d*x+c)^2)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+
b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*EllipticE(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*(-23*cos(d*x+c)^4-46*cos(d*x+c)^3-23*cos(d*x+c
)^2)+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(
-35*cos(d*x+c)^4-70*cos(d*x+c)^3-35*cos(d*x+c)^2)+B*((a+cos(d*x+c)*b)/(cos
(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-35*cos(d*x+c)^4-70*cos(d*x+c
)^3-35*cos(d*x+c)^2)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7
/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{7/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b^3 + \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a^3 + 3\left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a^2b + 3\left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a\,b^2$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*b**3 + i
nt((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*a**3 +
 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)*a*
*2*b + 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2
,x)*a*b**2
```

3.416 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 434

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(145a^2Ab+15Ab^3+63a^3B+161ab^2B)}{}$$

$$+\frac{2(a-b)\sqrt{a+b}(a^2(25A-63B)+15b^2(A-7B)-8ab(15A-7B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\sqrt{}}\right.\right.}{105ad}$$

$$+\frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(25a^2A+45Ab^2+77abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
2/105*(a-b)*(a+b)^(1/2)*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)
/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^2/d+2/105*(a-b)*(a+b)^(1/2)*(a^2*(25*A-63*B)+15*b^2*(A-7*B)-8*a*b*(15
*A-7*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a/d+2/35*a*(10*A*b+7*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x
+c)/d/cos(d*x+c)^(5/2)+2/105*(25*A*a^2+45*A*b^2+77*B*a*b)*(a+b*cos(d*x+c))
^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/7*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*
x+c)/d/cos(d*x+c)^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.34 (sec) , antiderivative size = 1409, normalized size of antiderivative = 3.25

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
9/2),x]
```

output
```
((-4*a*(25*a^4*A - 10*a^2*A*b^2 - 15*A*b^4 + 56*a^3*b*B - 56*a*b^3*B)*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-145*a^3*A*b - 15*a*A*b^3 - 63*a^4*B -
161*a^2*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi
[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])) + 2*(-145*a^2*A*b^2 - 15*A*b^4 - 63*a^3*b*B - 161*a*b^3*B)*(
(I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b
)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +...
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{7}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(2aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos(c+dx)+a(10Ab+7aB)\right)}{2\cos^{\frac{7}{2}}(c+dx)}a$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(2aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos(c+dx)+a(10Ab+7aB)\right)}{\cos^{\frac{7}{2}}(c+dx)}a$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(2aA+7bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Aa^2+14bBa+7Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3526

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{b\left(14Ba^2+30Aba+35b^2B\right)\cos^2(c+dx)+\left(21Ba^3+65Aba^2+105b^2Ba+35Ab^3\right)\cos(c+dx)+a(25A}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{b\left(14Ba^2+30Aba+35b^2B\right)\cos^2(c+dx)+\left(21Ba^3+65Aba^2+105b^2Ba+35Ab^3\right)\cos(c+dx)+a(25A}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{b\left(14Ba^2+30Aba+35b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(21Ba^3+65Aba^2+105b^2Ba+35Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3534

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\int \frac{a\left(63Ba^3+145Aba^2+161b^2Ba+15Ab^3\right)+a\left(25Aa^3+119bBa^2+135Ab^2a+105b^3B\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+77abB+45\right.}{3}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{\int \frac{a\left(63Ba^3+145Aba^2+161b^2Ba+15Ab^3\right)+a\left(25Aa^3+119bBa^2+135Ab^2a+105b^3B\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+77abB+45A\right.}{3d}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{\int \frac{a\left(63Ba^3+145Aba^2+161b^2Ba+15Ab^3\right)+a\left(25Aa^3+119bBa^2+135Ab^2a+105b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2A+77abB+\right.}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3477

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{a(a-b)\left(a^2(25A-63B)-8ab(15A-7B)+15b^2(A-7B)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+a\left(63a^3B+\right.}{3a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{a(a-b)\left(a^2(25A-63B)-8ab(15A-7B)+15b^2(A-7B)\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(63a\right.}{3a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3295}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{a\left(63a^3B+145a^2Ab+161ab^2B+15Ab^3\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)-8\right.}{}}{3a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3473}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)-8ab(15A-7B)+15b^2(A\right.}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x
]
```

output
```
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) +
 ((2*a*(10*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*
B + 161*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*(a - b)*S
qrt[a + b]*(a^2*(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*a*b*(15*A - 7*B))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(3*a) + (2*(25*a^2*A + 45*A*b^2 + 77*
a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/5)
/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2198 vs. $2(390) = 780$.

Time = 55.12 (sec) , antiderivative size = 2199, normalized size of antiderivative = 5.07

method	result	size
default	Expression too large to display	2199
parts	Expression too large to display	2216

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105/d*(B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*(161*cos(d*x+c)^5+322*cos(d*x+c)^4+161*cos(d*x+c)^3)+A*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*Elli
pticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-145*cos(d*x+c)^5-290*c
os(d*x+c)^4-145*cos(d*x+c)^3)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(
d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*(-135*cos(d*x+c)^5-270*cos(d*x+c)^4-135*cos(d*x+
c)^3)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)
/(a+b))^(1/2)*a*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(-15*cos(d*x+c)^5-30*cos(d*x+c)^4-15*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*EllipticF
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-119*cos(d*x+c)^5-238*cos(d*
x+c)^4-119*cos(d*x+c)^3)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c
)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*(-161*cos(d*x+c)^5-322*cos(d*x+c)^4-161*cos(d*x+c)^3)
+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*a*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-105
*cos(d*x+c)^5-210*cos(d*x+c)^4-105*cos(d*x+c)^3)+A*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*Elliptic...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9
/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{5}}dx\right)a^{3} + 3\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}}dx\right)a^{2}b + 3\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3}}dx\right)a\,b^{2} + \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{2}}dx\right)b^{3}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*a**3
+ 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)*a
**2*b + 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**
3,x)*a*b**2 + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*
x)**2,x)*b**3
```

3.417 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 522

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(147a^4A+279a^2Ab^2-10Ab^4+435a^3b\ldots}{\ldots}$$

$$-\frac{2(a-b)\sqrt{a+b}(10Ab^3-6a^2b(19A-60B)+3a^3(49A-25B)+15ab^2(11A-3B))\cot(c+dx)\,\mathrm{Elliptic}\ldots}{315a^2d\ldots}$$

$$+\frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)}$$

$$+\frac{2(49a^2A+75Ab^2+135abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315ad\cos^{\frac{3}{2}}(c+dx)}$$

$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}$$

output
```
2/315*(a-b)*(a+b)^(1/2)*(147*A*a^4+279*A*a^2*b^2-10*A*b^4+435*B*a^3*b+45*B
*a*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+
c))/(a-b))^(1/2)/a^3/d-2/315*(a-b)*(a+b)^(1/2)*(10*A*b^3-6*a^2*b*(19*A-60*
B)+3*a^3*(49*A-25*B)+15*a*b^2*(11*A-3*B))*cot(d*x+c)*EllipticF((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d
*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2/21*a*(4*A*b+3*B
*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/315*(49*A*a^2+7
5*A*b^2+135*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/
315*(163*A*a^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(
d*x+c)/a/d/cos(d*x+c)^(3/2)+2/9*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/co
s(d*x+c)^(9/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.65 (sec) , antiderivative size = 1517, normalized size of antiderivative = 2.91

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
11/2),x]
```

output
```
-1/315*((-4*a*(-114*a^4*A*b + 124*a^2*A*b^3 - 10*A*b^5 - 75*a^5*B + 30*a^3
*b^2*B + 45*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 27
9*a^3*A*b^2 - 10*a*A*b^4 + 435*a^4*b*B + 45*a^2*b^3*B)*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 2
79*a^2*A*b^3 - 10*A*b^5 + 435*a^3*b^2*B + 45*a*b^4*B)*((I*Cos[(c + d*x)/2]
*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*S...
```

Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{9}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(4aA+9bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos(c+dx)+3a(4Ab+3aB)\right)}{2\cos^{\frac{9}{2}}(c+dx)}a$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{9}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(4aA+9bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos(c+dx)+3a(4Ab+3aB)\right)}{\cos^{\frac{9}{2}}(c+dx)}a$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{9}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(4aA+9bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Aa^2+18bBa+9Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 3526

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{b\left(36Ba^2+76Aba+63b^2B\right)\cos^2(c+dx)+\left(45Ba^3+137Aba^2+189b^2Ba+63Ab^3\right)\cos(c+dx)+a(49\right.}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{b\left(36Ba^2+76Aba+63b^2B\right)\cos^2(c+dx)+\left(45Ba^3+137Aba^2+189b^2Ba+63Ab^3\right)\cos(c+dx)+a(49\right.}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{b\left(36Ba^2+76Aba+63b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(45Ba^3+137Aba^2+189b^2Ba+63Ab^3\right)\sin\left(c+dx+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3534

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{2\int\frac{2ab\left(49Aa^2+135bBa+75Ab^2\right)\cos^2(c+dx)+a\left(147Aa^3+585bBa^2+605Ab^2a+315b^3B\right)\cos(c+dx)+3a\left(75Ba^3+163Aba^2+135b^2Ba\right.}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\int\frac{2ab\left(49Aa^2+135bBa+75Ab^2\right)\cos^2(c+dx)+a\left(147Aa^3+585bBa^2+605Ab^2a+315b^3B\right)\cos(c+dx)+3a\left(75Ba^3+163Aba^2+135b^2Ba+5\right.}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\int\frac{2ab\left(49Aa^2+135bBa+75Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\left(147Aa^3+585bBa^2+605Ab^2a+315b^3B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\left(75Ba^3+163Aba^2+135\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3534

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{2\int\frac{3\left(\left(75Ba^3+261Aba^2+405b^2Ba+155Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+435bBa^3+279Ab^2a^2+45b^3Ba-10Ab^4\right)a\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(75a^3B+163a^2Ab+135a\right.}{}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(75Ba^3+261Aba^2+405b^2Ba+155Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+435bBa^3+279Ab^2a^2+45b^3Ba-10Ab^4\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2\left(75a^3B+163a^2Ab+135ab^2B\right.}{d\cos}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(75Ba^3+261Aba^2+405b^2Ba+155Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(147Aa^4+435bBa^3+279Ab^2a^2+45b^3Ba-10Ab^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2(75a^3B+163a^2Ab+135al}{}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3477

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{a\left(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(3a^3(49A-25B)-6a^2b(19A-60B)+15ab^2\right.}{a}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{a\left(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(3a^3(49A-25B)-6a^2b(19A-60B\right.}{a}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{a(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^3(49A-25B)-6a^2b(19A-60\right.}{}}{a}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3473

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{2(49a^2A+135abB+75Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{\frac{2(75a^3B+163a^2Ab+135ab^2B+5Ab^3)\sin(c+dx)\sqrt{a}}{d\cos^{\frac{3}{2}}(c+dx)}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),
x]
```

output
```
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) +
 ((6*a*(4*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
 d*x]^(7/2)) + ((2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqrt[a + b*Cos[c + d*
x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(147
*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 - 6*a^2*
b*(19*A - 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b^2*(11*A - 3*B))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/d)/a + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 13
5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)))/
(5*a))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(472) = 944$.

Time = 80.11 (sec) , antiderivative size = 2683, normalized size of antiderivative = 5.14

method	result	size
default	Expression too large to display	2683
parts	Expression too large to display	2699

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/315/d*(B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*a^2*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*(-45*cos(d*x+c)^6-90*cos(d*x+c)^5-45*cos(d*x+c)^4)+A*((a+cos(d*x+c)*b
)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b^2*El
lipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(279*cos(d*x+c)^6+558*
cos(d*x+c)^5+279*cos(d*x+c)^4)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^3*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*(279*cos(d*x+c)^6+558*cos(d*x+c)^5+279*cos(d*x+
c)^4)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*a*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(-10*cos(d*x+c)^6-20*cos(d*x+c)^5-10*cos(d*x+c)^4)+B*((a+cos(d*x+c)*b)/(co
s(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*b*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(435*cos(d*x+c)^6+870*cos(d*x
+c)^5+435*cos(d*x+c)^4)+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*(435*cos(d*x+c)^6+870*cos(d*x+c)^5+435*cos(d*x+c)^4)+B
*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*a^2*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(45*c
os(d*x+c)^6+90*cos(d*x+c)^5+45*cos(d*x+c)^4)+B*((a+cos(d*x+c)*b)/(cos(d*x+
c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^4*EllipticE(co...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11
/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11
/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{11/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),
x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),
 x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^6}dx\right)a^3 + 3\left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a^2b + 3\left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a\,b^2 + \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**6,x)*a**3
+ 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**5,x)*a
**2*b + 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**
4,x)*a*b**2 + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*
x)**3,x)*b**3
```

3.418 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 622

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(3705a^4Ab+255a^2Ab^3+40Ab^5+1617}{}$$

$$+\frac{2(a-b)\sqrt{a+b}(40Ab^4+3a^4(225A-539B)-6a^3b(505A-209B)+15a^2b^2(19A-121B)+10ab^3(3A}{3465a^3d}$$

$$+\frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)}$$

$$+\frac{2(81a^2A+113Ab^2+209abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)}$$

$$+\frac{2(1145a^2Ab+15Ab^3+539a^3B+825ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3465ad\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3465a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)}$$

output
```
2/3465*(a-b)*(a+b)^(1/2)*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+3
069*B*a^3*b^2-110*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/3465*(a-b)*(a+b)^(1/2)*(40*A*b
^4+3*a^4*(225*A-539*B)-6*a^3*b*(505*A-209*B)+15*a^2*b^2*(19*A-121*B)+10*a*
b^3*(3*A-11*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a^3/d+2/99*a*(14*A*b+11*B*a)*(a+b*cos(d*x+c))^(1/
2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+2/693*(81*A*a^2+113*A*b^2+209*B*a*b)*(a+b
*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/3465*(1145*A*a^2*b+15*A
*b^3+539*B*a^3+825*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a/d/cos(d*x+
c)^(5/2)+2/3465*(675*A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*a^3*b+55*B*a*b^3
)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)+2/11*a*A*(a+b*c
os(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(11/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.02 (sec) , antiderivative size = 1640, normalized size of antiderivative = 2.64

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(
13/2),x]
```

output
```
((-4*a*(675*a^6*A - 390*a^4*A*b^2 - 245*a^2*A*b^4 - 40*A*b^6 + 1254*a^5*b*
B - 1364*a^3*b^3*B + 110*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-
3705*a^5*A*b - 255*a^3*A*b^3 - 40*a*A*b^5 - 1617*a^6*B - 3069*a^4*b^2*B +
110*a^2*b^4*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
 b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi
[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])) + 2*(-3705*a^4*A*b^2 - 255*a^2*A*b^4 - 40*A*b^6 - 1617*a^5*b
*B - 3069*a^3*b^3*B + 110*a*b^5*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
 d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a...
```

Rubi [A] (verified)

Time = 3.37 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{13/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{11}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(6aA+11bB)\cos^2(c+dx)+\left(9Aa^2+22bBa+11Ab^2\right)\cos(c+dx)+a(14Ab+11a}{2\cos^{\frac{11}{2}}(c+dx)}$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(6aA+11bB)\cos^2(c+dx)+\left(9Aa^2+22bBa+11Ab^2\right)\cos(c+dx)+a(14Ab+11a}{\cos^{\frac{11}{2}}(c+dx)}$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(6aA+11bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(9Aa^2+22bBa+11Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 3526

$$\frac{1}{11}\left(\frac{2}{9}\int \frac{3b\left(22Ba^2+46Aba+33b^2B\right)\cos^2(c+dx)+\left(77Ba^3+233Aba^2+297b^2Ba+99Ab^3\right)\cos(c+dx)+a(}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{3b\left(22Ba^2+46Aba+33b^2B\right)\cos^2(c+dx)+\left(77Ba^3+233Aba^2+297b^2Ba+99Ab^3\right)\cos(c+dx)+a(}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{3b(22Ba^2+46Aba+33b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(77Ba^3+233Aba^2+297b^2Ba+99Ab^3\right)\sin\left(c+dx\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\int \frac{4ab(81Aa^2+209bBa+113Ab^2)\cos^2(c+dx)+a(405Aa^3+1507bBa^2+1531Ab^2a+693b^3B)\cos(c+dx)+a(539Ba^3+1145Aba^2+825}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{7a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\int \frac{4ab(81Aa^2+209bBa+113Ab^2)\cos^2(c+dx)+a(405Aa^3+1507bBa^2+1531Ab^2a+693b^3B)\cos(c+dx)+a(539Ba^3+1145Aba^2+825b^2}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{7a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\int \frac{4ab(81Aa^2+209bBa+113Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a(405Aa^3+1507bBa^2+1531Ab^2a+693b^3B)\sin\left(c+dx+\frac{\pi}{2}\right)+a(539Ba^3+1145Aba}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{7a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{2\int \frac{\left(1617Ba^3+5055Aba^2+6655b^2Ba+2305Ab^3\right)\cos(c+dx)a^2+2b\left(539Ba^3+1145Aba^2+825b^2Ba+15Ab^3\right)\cos^2(c+dx)a+3\left(675Aa^4+1793bBa^3+102\right.}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}}{7a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(1617Ba^3+5055Aba^2+6655b^2Ba+2305Ab^3\right)\cos(c+dx)a^2+2b\left(539Ba^3+1145Aba^2+825b^2Ba+15Ab^3\right)\cos^2(c+dx)a+3\left(675Aa^4+1793bBa^3+1025\right.}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}}{7a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(1617Ba^3+5055Aba^2+6655b^2Ba+2305Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(539Ba^3+1145Aba^2+825b^2Ba+15Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+3\left(675Aa^4+1793bB\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{5a}}{7a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \ 3534$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\frac{2\int \frac{3\left(\left(675Aa^4+2871bBa^3+3315Ab^2a^2+1705b^3Ba+10Ab^4\right)\cos(c+dx)a^2+\left(1617Ba^5+3705Aba^4+3069b^2Ba^3+255Ab^3a^2-110b^4Ba+40Ab^5\right)a\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}}{5a}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(675Aa^4+2871bBa^3+3315Ab^2a^2+1705b^3Ba+10Ab^4\right)\cos(c+dx)a^2+\left(1617Ba^5+3705Aba^4+3069b^2Ba^3+255Ab^3a^2-110b^4Ba+40Ab^5\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2(6}{}}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(675Aa^4+2871bBa^3+3315Ab^2a^2+1705b^3Ba+10Ab^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(1617Ba^5+3705Aba^4+3069b^2Ba^3+255Ab^3a^2-110b^4Ba+40Ab^5\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+}{5a}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{a(a-b)\left(3a^4(225A-539B)-6a^3b(505A-209B)+15a^2b^2(19A-121B)+10ab^3(3A-11B)+40Ab^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+a\left(1617a^5B+370\right.}{a}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{a(a-b)\left(3a^4(225A-539B)-6a^3b(505A-209B)+15a^2b^2(19A-121B)+10ab^3(3A-11B)+40Ab^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(1617a\right.}{a}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{a\left(1617a^5B+3705a^4Ab+3069a^3b^2B+255a^2Ab^3-110ab^4B+40Ab^5\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(3a^4(225A-539B)-\right.}{}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3473

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{\frac{2\left(539a^3B+1145a^2Ab+825ab^2B+15Ab^3\right)\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)}}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),
x]
```

output
```
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
 + ((2*a*(14*A*b + 11*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos
[c + d*x]^(9/2)) + ((2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*(1145*a^2*A*b + 15*A
*b^3 + 539*a^3*B + 825*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*
d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A
*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(a*d) + (2*(a - b)*Sqrt[a + b]*(40*A*b^4 + 3*a^4*(
225*A - 539*B) - 6*a^3*b*(505*A - 209*B) + 15*a^2*b^2*(19*A - 121*B) + 10*
a*b^3*(3*A - 11*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/a + (2*(675*a^
4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)))/(5*a))/(7*a))/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3300 vs. $2(566) = 1132$.

Time = 102.98 (sec) , antiderivative size = 3301, normalized size of antiderivative = 5.31

method	result	size
default	Expression too large to display	3301
parts	Expression too large to display	3311

input
```
int((a+cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/3465/d*(B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a^3*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*(-1705*cos(d*x+c)^7-3410*cos(d*x+c)^6-1705*cos(d*x+c)^5)+B*((a+cos(d
*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2
*b^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(110*cos(d*x+c)
^7+220*cos(d*x+c)^6+110*cos(d*x+c)^5)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(
a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*b*EllipticE(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))*(3705*cos(d*x+c)^7+7410*cos(d*x+c)^6+3705*
cos(d*x+c)^5)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*a^4*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(3705*cos(d*x+c)^7+7410*cos(d*x+c)^6+3705*cos(d*x+c)^5)+A*((a+co
s(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
a^3*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(255*cos(d*x
+c)^7+510*cos(d*x+c)^6+255*cos(d*x+c)^5)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^4*EllipticE(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(255*cos(d*x+c)^7+510*cos(d*x+c)^6+25
5*cos(d*x+c)^5)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*a*b^5*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(40*cos(d*x+c)^7+80*cos(d*x+c)^6+40*cos(d*x+c)^5)+B*((a+cos(d*x+
c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(1
3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13
/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13
/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{13/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),
x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),
 x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{7}}dx\right)a^{3}$$
$$+3\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{6}}dx\right)a^{2}b$$
$$+3\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{5}}dx\right)a\,b^{2}$$
$$+\left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}}dx\right)b^{3}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**7,x)*a**3
+ 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**6,x)*a
**2*b + 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**
5,x)*a*b**2 + int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*
x)**4,x)*b**3
```

3.419 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 418

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(a^2+3b^2)\,B\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{ad\ldots}$$
$$-\frac{(a-3b)\sqrt{a+b}(2a^2-ab+3b^2)\,B\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\ldots}{ad}$$
$$-\frac{b\sqrt{a+b}\left(5a+\frac{3b^2}{a}\right)B\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\ldots}{\ldots}}}{d}$$
$$+\frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2*(a-b)*(a+b)^(1/2)*(a^2+3*b^2)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-(a-3*b)*(a+b)^(1/2)*(2*a^2-
a*b+3*b^2)*B*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a/d-b*(a+b)^(1/2)*(5*a+3*b^2/a)*B*cot(d*x+c)*Ellipti
cPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a
-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
d+b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.30 (sec) , antiderivative size = 1236, normalized size of antiderivative = 2.96

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Co
s[c + d*x]^(5/2),x]
```

output
```
-1/2*(B*((-4*a*(-5*a^3*b - 3*a*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*
(2*a^4 + a^2*b^2 - 3*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])) + 2*(2*a^3*b + 6*a*b^3)*((I*Cos[(c + d*x)/2]*Sqrt
[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +...
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {3042, 3468, 27, 3042, 3470, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(\frac{3bB}{2a}+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2}{3}\int \frac{3\sqrt{a+b\cos(c+dx)}\left(2\left(a^2+3b^2\right)B+b\left(\frac{3b^2}{a}+5a\right)\cos(c+dx)B\right)}{4\cos^{\frac{3}{2}}(c+dx)}dx+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{a+b\cos(c+dx)}\left(2\left(a^2+3b^2\right)B+b\left(\frac{3b^2}{a}+5a\right)\cos(c+dx)B\right)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(2\left(a^2+3b^2\right)B+b\left(\frac{3b^2}{a}+5a\right)\sin\left(c+dx+\frac{\pi}{2}\right)B\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3470}$$

$$\frac{1}{2}\left(\int \frac{2a\left(a^2+3b^2\right)B+\left(2b\left(a^2+3b^2\right)B+ab\left(\frac{3b^2}{a}+5a\right)B\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+b^2B\left(\frac{3b^2}{a}+5a\right)\int \frac{\sqrt{\cos(c+}}{\sqrt{a+b\cos(c}}\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{2a\left(a^2+3b^2\right)B+\left(2b\left(a^2+3b^2\right)B+ab\left(\frac{3b^2}{a}+5a\right)B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2B\left(\frac{3b^2}{a}+5a\right)\int \frac{\sqrt{\sin}}{\sqrt{a+b\,s}}\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3288

$$\frac{1}{2}\left(\int \frac{2a\left(a^2+3b^2\right)B+\left(2b\left(a^2+3b^2\right)B+ab\left(\frac{3b^2}{a}+5a\right)B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2bB\sqrt{a+b}\left(\frac{3b^2}{a}+5a\right)\cot(c}{}\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3477

$$\frac{1}{2}\left(2aB\left(a^2+3b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-B(a-3b)\left(2a^2-ab+3b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b}}\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-B(a-3b)\left(2a^2-ab+3b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2aB\left(a^2+3b^2\right)\int \frac{}{\sin(c+}\right.$$
$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3295

$$\frac{1}{2}\left(2aB\left(a^2+3b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B(a-3b)\sqrt{a+b}\left(2a^2-ab+3b^2\right)\cot(c\right.$$

$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3473

$$\frac{1}{2}\left(-\frac{2B(a-3b)\sqrt{a+b}\left(2a^2-ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{ad}\right.$$

$$\frac{bB\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Cos[c +
d*x]^(5/2),x]
```

output
```
((4*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*d) - (2*(a - 3*b)*Sqrt[a + b]*(2*a^2 - a*b + 3*b^2)*B*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(5*a + (3*b^2)/a)*B*Cot[c
+ d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/2 + (b*B*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3470
```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[B*(d
/b^2)   Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*
c + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*
x]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(381) = 762$.

Time = 35.36 (sec) , antiderivative size = 1387, normalized size of antiderivative = 3.32

method	result	size
default	Expression too large to display	1387
parts	Expression too large to display	1834

input
```
int((a+cos(d*x+c)*b)^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,met
hod=_RETURNVERBOSE)
```

output
```
B/a/d*(((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a^2*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*(-10*cos(d*x+c)^3-20*cos(d*x+c)^2-10*cos(d*x+c))+((a+cos(d*x+c)*b)/(c
os(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*EllipticPi
(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-6*cos(d*x+c)^3-12*cos(d*
x+c)^2-6*cos(d*x+c))+((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+
b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))+((a+cos(d*x+c)*b)/
(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b*Ellipt
icE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+
c)^2+2*cos(d*x+c))+((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+c)^2+6*cos(d*x+c))+((a+cos(d*x+c)*
b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3*Ell
ipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(
d*x+c)^2+6*cos(d*x+c))+((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+((a+cos(d*x+c)*
b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b*Ell
ipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-7*cos(d*x+c)^3-14*...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(2\,B\cos\left(dx+c\right)+\frac{3\,Bb}{a}\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{2\,\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2)
,x, algorithm="fricas")
```

output
```
integral(1/2*(2*B*a*b^2*cos(d*x + c)^3 + 3*B*a^2*b + (4*B*a^2*b + 3*B*b^3)
*cos(d*x + c)^2 + 2*(B*a^3 + 3*B*a*b^2)*cos(d*x + c))*sqrt(b*cos(d*x + c)
+ a)/(a*cos(d*x + c)^(5/2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)**(5/
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(2\,B\cos\left(dx+c\right)+\frac{3\,Bb}{a}\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{2\,\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2)
,x, algorithm="maxima")
```

output
```
1/2*integrate((2*B*cos(d*x + c) + 3*B*b/a)*(b*cos(d*x + c) + a)^(5/2)/cos(
d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(2\,B\cos\left(dx+c\right)+\frac{3\,Bb}{a}\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{2\,\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2)
,x, algorithm="giac")
```

output
```
integrate(1/2*(2*B*cos(d*x + c) + 3*B*b/a)*(b*cos(d*x + c) + a)^(5/2)/cos(
d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(B\,\cos\left(c+d\,x\right)+\frac{3\,B\,b}{2\,a}\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((B*cos(c + d*x) + (3*B*b)/(2*a))*(a + b*cos(c + d*x))^(5/2))/cos(c +
d*x)^(5/2),x)
```

output
```
int(((B*cos(c + d*x) + (3*B*b)/(2*a))*(a + b*cos(c + d*x))^(5/2))/cos(c +
d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{b\left(4\left(\int \frac{\sqrt{\cos(dx+c)b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a^2b+3\left(\int \frac{\sqrt{\cos(dx+c)}}{c}\right.\right.}{}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
(b*(4*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)*a*
*2*b + 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
*b**3 + 3*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**
3,x)*a**2*b + 2*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c +
d*x)**2,x)*a**3 + 6*int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(
c + d*x)**2,x)*a*b**2 + 2*int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),
x)*a*b**2))/(2*a)
```

3.420 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 479

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(4Ab-3aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+d}{a-b}}}{4ab^2d}$$

$$+\frac{\sqrt{a+b}(4Ab-3aB+2bB)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(}}{4b^2d}$$

$$+\frac{\sqrt{a+b}(4aAb-3a^2B-4b^2B)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c-}{a+b}}}{4b^3d}$$

$$+\frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}$$

$$+\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd}$$

output
```
-1/4*(a-b)*(a+b)^(1/2)*(4*A*b-3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d+1/4*(a+b)^(1/2)*(4*A
*b-3*B*a+2*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/4*(a+b)^(1/2)*(4*A*a*b-3*B*a^2-4*B*b^2)*c
ot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+
c))/(a-b))^(1/2)/b^3/d+1/4*(4*A*b-3*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)
/b^2/d/cos(d*x+c)^(1/2)+1/2*B*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*sin(
d*x+c)/b/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.54 (sec) , antiderivative size = 1175, normalized size of antiderivative = 2.45

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x
]],x]
```

output
```
(B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) + ((-
4*a*(4*A*b - a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a*b*B*((Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b - 3
*a*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Si
n[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])
/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])...
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3469, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{\int \frac{(4Ab-3aB)\cos^2(c+dx)+2bB\cos(c+dx)+aB}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4Ab-3aB)\cos^2(c+dx)+2bB\cos(c+dx)+aB}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(4Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$$\Big\downarrow \text{3540}$$

$$\frac{\frac{\int -\frac{\left(-3Ba^2+4Aba-4b^2B\right)\cos^2(c+dx)-2abB\cos(c+dx)+a(4Ab-3aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{\left(-3Ba^2+4Aba-4b^2B\right)\cos^2(c+dx)-2abB\cos(c+dx)+a(4Ab-3aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{\left(-3Ba^2+4Aba-4b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2abB\sin\left(c+dx+\frac{\pi}{2}\right)+a(4Ab-3aB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$$\Big\downarrow \text{3532}$$

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{(-3a^2B+4aAb-4b^2B)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+\int\frac{a(4Ab-3aB)-2abB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{(-3a^2B+4aAb-4b^2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{a(4Ab-3aB)-2abB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$\Big\downarrow$ 3288

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab-3aB)-2abB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(}}{}}{2b}}{4b}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$\Big\downarrow$ 3477

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(4Ab-3aB)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(-3aB+4Ab+2bB)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-}{}}{4b}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{-a(-3aB+4Ab+2bB)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a(4Ab-3aB)\int\frac{\sin\left(c+dx+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+}}}{}}{4b}$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$\Big\downarrow$ 3295

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(4Ab-3aB)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}}{}}{}+$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

$\Big\downarrow$ 3473

$$\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{-\frac{2\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{bd}}{}}{}+$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) + (-1
/2*((2*(a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(a*d) - (2*Sqrt[a + b]*(4*A*b - 3*a*B + 2*b*B)*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/d - (2*Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Cot[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/b + ((4*A*b - 3*a*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1086 vs. $2(428) = 856$.

Time = 25.19 (sec) , antiderivative size = 1087, normalized size of antiderivative = 2.27

method	result	size
default	Expression too large to display	1087
parts	Expression too large to display	1125

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*((8*cos(d*x+c)^2+16*cos(d*x+c)+8)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticPi(cot(d*x+c)-
csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*B*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1
/2)*a^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-8*cos(
d*x+c)^2-16*cos(d*x+c)-8)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+
c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+
c)-4)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)
/(a+b))^(1/2)*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(3
*cos(d*x+c)^2+6*cos(d*x+c)+3)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(
d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*Elliptic
E(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c
)-2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/
(a+b))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(4*
cos(d*x+c)^2+8*cos(d*x+c)+4)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+co...
```

Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*cos
(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x
)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x),x)
```

3.421 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 427

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd}$$

$$+\frac{\sqrt{a+b}B\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

$$-\frac{\sqrt{a+b}(2Ab-aB)\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\mathrm{s}}{}}}{b^2d}$$

$$+\frac{aB\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
-(a-b)*(a+b)^(1/2)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+(a+b)^(1/2)*B*cot(d*x+c)*EllipticF((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-(a+b)^(1/2)
*(2*A*b-B*a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(
d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+a*B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(
a+b*cos(d*x+c))^(1/2)+B*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/
2)
```

Mathematica [A] (verified)

Time = 13.85 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(2(a+b)B\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-4Ab\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\right.}{}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x
]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(a + b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*A
*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 8*A*b*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)] - 4*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*B*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2
*a*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*B*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(2*b*d*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3482, 3042, 3530, 3042, 3288, 3472, 25, 27, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3482}$$

$$\frac{1}{2}\int \frac{(2Ab-aB)\cos^2(c+dx)+2aA\cos(c+dx)+aB}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{(2Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2aA\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3530}$$

$$\frac{1}{2}\left(\frac{\int \frac{B\cos(c+dx)a^2+bBa}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{b}+\frac{(2Ab-aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{B\sin\left(c+dx+\frac{\pi}{2}\right)a^2+bBa}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{(2Ab-aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3288

$$\frac{1}{2}\left(\frac{\int \frac{B\sin\left(c+dx+\frac{\pi}{2}\right)a^2+bBa}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Ellip}}{b^2d}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3472

$$\frac{1}{2}\left(\frac{\frac{\int -\frac{a\left(a^2-b^2\right)B}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(}{}}}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{a\left(a^2-b^2\right)B}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\text{se}}{}}}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3280

$$\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)}{b}-\frac{2\sqrt{a+b}(2A}{}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{b}-\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3295

$$\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx}{a-b}}}{}\right.}{b}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}\right.}{b}\right.$$

$$\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]) + ((-2*Sq
rt[a + b]*(2*A*b - a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
b^2*d) + (-(a*B*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))) +
 (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/b)/
2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 3530
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b
*Sin[e + f*x]], x], x] + Simp[1/b   Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((
a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d,
e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 16.75 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\left((4\cos(dx+c)+4)A\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\, b\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)+(-2\cos(dx+c)-2)B\sqrt{\frac{a+\cos(dx}{(\cos(dx+c)+}}\right.}{}$
parts	$-\frac{2A\left(-\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+2\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\sqrt{}}{d\sqrt{\cos(dx+c)}\,\sqrt{a+\cos(dx+c)b}}$

input

```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/d*((4*cos(d*x+c)+4)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b*E
llipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)-2)
*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)+1)*B*((a+cos(d*x+c)*b)/(cos
(d*x+c)+1)/(a+b))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))+(cos(d*x+c)+1)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b*Ell
ipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)-2)*A*((a
+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b*EllipticF(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)*
sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*sin(d*x+c))*cos(d*x+c)^(1
/2)/(a+cos(d*x+c)*b)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)/b
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input

```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x
)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}dx$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)),x)
```

3.422 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad} - \frac{2\sqrt{a+b}B\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
2*A*(a+b)^(1/2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a/d-2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticPi((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 15.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.01

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{4(a+b)\cos^{\frac{3}{2}}(c+dx)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\left(b(A+B)\operatorname{EllipticF}\left(\right.\right.}{abd\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+}{}\right.}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]),x]
```

output
```
(-4*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*(b*(A + B)
*EllipticF[ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]],
(2*a)/(a - b)] - (a + b)*B*EllipticPi[-(a/b), ArcSin[Sqrt[-((a + b*Cos[c +
 d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a - b)]))/(a*b*d*Sqrt[a + b*Cos[
c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a))^(3/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3485, 3042, 3288, 3295}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3485}$$

$$A\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$\Big\downarrow$ 3042

$$A\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 3288

$$A\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{bd}$$

$\Big\downarrow$ 3295

$$\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a +
b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3485
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[B/d   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Sim
p[(B*c - A*d)/d   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 17.48 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.76

method	result
default	$-\frac{2\sqrt{\cos(dx+c)}\,\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\,\left(A\,\mathrm{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+2B\,\mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\right.\right.}{d\sqrt{a+\cos(dx+c)b}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$
parts	$-\frac{2A(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}}{d\sqrt{\cos(dx+c)}\,\sqrt{a+\cos(dx+c)b}}-\frac{2B\left(-\mathrm{EllipticF}\left(\cot(dx\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/d*cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*(A*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+
2*B*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))-B*EllipticF(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)))/(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)
```

Fricas [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + b\cos(c + dx)}}\,dx = \int \frac{B\cos(dx + c) + A}{\sqrt{b\cos(dx + c) + a}\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + b\cos(c + dx)}}\,dx = \int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}\sqrt{\cos(c + dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))
), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A+B\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\sqrt{a+b\cos(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x),x)
```

3.423 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 230

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2d} - \frac{2\sqrt{a+b}(A-B)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

output
```
2*A*(a-b)*(a+b)^(1/2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*(a+b)^(1/2)*(A-B)*cot(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/
2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 15.72 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\left(A(a+b\cos(c+dx))\sin(c+dx)-\frac{2\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\left(2A(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(a\right.\right.}{}\right.}{ad}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]
]),x]
```

output
```
(2*(A*(a + b*Cos[c + d*x])*Sin[c + d*x] - (2*Sqrt[2]*(Cos[(c + d*x)/2]^2)^
(3/2)*(2*A*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(A + B)*Cos[(c + d*x)/2]^2*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(
3/2)))/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3477}$$

$$A\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3295}$$

$$A\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(A-B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

$$\Big\downarrow \text{3473}$$

$$\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(A-B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2
*Sqrt[a + b]*(A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(208) = 416$.

Time = 23.94 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.09

method	result
default	$\frac{2\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)A\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a\,\mathrm{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+\left(\cos(dx+c\right.\right.}{}$
parts	$-\frac{2B(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}}{d\sqrt{\cos(dx+c)}\,\sqrt{a+\cos(dx+c)b}}+\frac{2A\left(\left(\cos(dx+c)^2+2\cos\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/d*((cos(d*x+c)^2+2*cos(d*x+c)+1)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*A*((a+cos(d*x+c)*b)
/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1
)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*
x+c)^2-2*cos(d*x+c)-1)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))+A*b*cos(d*x+c)*sin(d*x+c)+A*a*sin(d*x+c))*(a+cos(d*x+c)*b)^(1
/2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)/a
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{A+B\cos{(c+dx)}}{\sqrt{a+b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2
)), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos{(dx+c)}+A}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2
)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{2}}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**2,x)
```

3.424 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 290

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3d}$$
$$+\frac{2\sqrt{a+b}(2Ab+a(A-3B))\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d}$$
$$+\frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-2/3*(a-b)*(a+b)^(1/2)*(2*A*b-3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+2/3*(a+b)^(1/2)*(2*A*b
+a*(A-3*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a^2/d+2/3*A*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a/d/cos
(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 17.49 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.43

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(-2(a+b)(-2Ab+3aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right.}{} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(-2Ab\sin(c+dx)+3aB\sin(c+dx))}{3a^2}+\frac{2A\sec(c+dx)\tan(c+dx)}{3a}\right)}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]
]),x]
```

output
```
(8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[C
os[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*A*b
 + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d
*x])^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*
a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a)))/d
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3479, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int -\frac{2Ab-3aB-aA\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2Ab-3aB-aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2Ab-3aB-aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}$$

$$\Big\downarrow \text{3477}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(2Ab-3aB)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a(A-3B)+2Ab)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(2Ab-3aB)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a(A-3B)+2Ab)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}$$

$$\Big\downarrow \text{3295}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(2Ab-3aB)\int\frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx - \frac{2\sqrt{a+b}(a(A-3B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Elliptic}}{ad}}{3a}$$

$$\Big\downarrow \text{3473}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d} - \frac{2\sqrt{a+b}(a(A-3B)+2Ab)\cot(c-}{}}{3a}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
-1/3*((2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a^2*d) - (2*Sqrt[a + b]*(2*A*b + a*(A - 3*B))*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(a*d))/a + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
3*a*d*Cos[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(258) = 516$.

Time = 34.38 (sec) , antiderivative size = 934, normalized size of antiderivative = 3.22

method	result	size
default	Expression too large to display	934
parts	Expression too large to display	935

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/3/d*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))+A*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticE(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(
d*x+c))+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+B*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticE(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*co
s(d*x+c))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c
)+1)/(a+b))^(1/2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticF(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d
*x+c))+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(
3*cos(d*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c))+(-cos(d*x+c)-1)*sin(d*x+c)*a^2
*A+sin(d*x+c)*cos(d*x+c)*(1-cos(d*x+c))*A*a*b+2*A*sin(d*x+c)*cos(d*x+c)^2*
b^2-3*B*a*b*cos(d*x+c)^2*sin(d*x+c)-3*B*sin(d*x+c)*cos(d*x+c)*a^2)*(a+c...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/2
)), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2
)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{5/2}\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{3}}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**3,x)
```

3.425 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 363

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2(a-b)\sqrt{a+b}(9a^2A+8Ab^2-10abB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{15a^4d}$$

$$-\frac{2\sqrt{a+b}(8Ab^2+a^2(9A-5B)-2ab(A+5B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{15a^3d}$$

$$+\frac{2A\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

output

```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2+8*A*b^2-10*B*a*b)*cot(d*x+c)*EllipticE((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d-2/15*(a+b
)^(1/2)*(8*A*b^2+a^2*(9*A-5*B)-2*a*b*(A+5*B))*cot(d*x+c)*EllipticF((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+2/5*A*(a+b*co
s(d*x+c))^(1/2)*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)-2/15*(4*A*b-5*B*a)*(a+b*co
s(d*x+c))^(1/2)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.97 (sec) , antiderivative size = 1319, normalized size of antiderivative = 3.63

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]
]),x]
```

output
```
-1/15*((-4*a*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - 4*a*(9*a^3*A + 8*a*A*b^2 - 10*a^2*b*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 8
*A*b^3 - 10*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c
+ d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x]
)*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + ...
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int -\frac{-2Ab\cos^2(c+dx)-3aA\cos(c+dx)+4Ab-5aB}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\int \frac{-2Ab\cos^2(c+dx)-3aA\cos(c+dx)+4Ab-5aB}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\int \frac{-2Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-3aA\sin\left(c+dx+\frac{\pi}{2}\right)+4Ab-5aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2\int -\frac{9Aa^2-10bBa+(2Ab+5aB)\cos(c+dx)a+8Ab^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{9Aa^2-10bBa+(2Ab+5aB)\cos(c+dx)a+8Ab^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{9Aa^2-10bBa+(2Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)a+8Ab^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{5a}$$

$\Big\downarrow$ 3477

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(9a^2A-10abB+8Ab^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a^2(9A-5B)-2ab(A+5B)+8Ab^2)\int\frac{}{\sqrt{\cos(}}}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(9a^2A-10abB+8Ab^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a^2(9A-5B)-2ab(A+5B)+8Ab^2}{3a}}{5a}$$

$\Big\downarrow$ 3295

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(9a^2A-10abB+8Ab^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2(9A-5B)-2ab(A+5B)+8\right.}{}}{3a}}{5a}$$

$\Big\downarrow$ 3473

$$\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)} -$$

$$\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(a-b)\sqrt{a+b}\left(9a^2A-10abB+8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{a^2d}}{5a}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (
-1/3*((2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*(8*A*b^2 + a^2*(9*A - 5*B)
 - 2*a*b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a + (2*(4*
A*b - 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3
/2)))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(325) = 650$.

Time = 46.76 (sec) , antiderivative size = 1507, normalized size of antiderivative = 4.15

method	result	size
default	Expression too large to display	1507
parts	Expression too large to display	1523

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(9*cos(d*x+c)^4+18*cos(d*x+c)^3+9*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(9*cos(d*x+c)^4+18*cos(d*x+c)^3+
9*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos
(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(8*cos(d*x+c)^4+16*cos(d*x+c)^3+8*cos(d*x+c)^2)+A*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^3*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^4+16*cos(d
*x+c)^3+8*cos(d*x+c)^2)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)
*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*(-10*cos(d*x+c)^4-20*cos(d*x+c)^3-10*cos(d*x+c)^2)+B*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/
2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-10*cos(d*
x+c)^4-20*cos(d*x+c)^3-10*cos(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*EllipticF(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-9*cos(d*x+c)^4-18*cos(d*x+c)^3-9*cos(d*x
+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b*cos(d*x + c)^5 + a*cos(d*x + c)^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/2
)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/cos(c + d*x)**4,x)
```

3.426 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 500

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{(2aAb-3a^2B+b^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ab^2\sqrt{a+b}d}$$
$$-\frac{(2Ab-(3a+b)B)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d}$$
$$-\frac{\sqrt{a+b}(2Ab-3aB)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec}{a-}}}{b^3d}$$
$$+\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$
$$-\frac{(2aAb-3a^2B+b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}}$$

output
```
(2*A*a*b-3*B*a^2+B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/(a+b)^(1/2)/d-(2*A*b-(3*a+b)*B)*cot
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a
+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/b^2/(a+b)^(1/2)/d-(a+b)^(1/2)*(2*A*b-3*B*a)*cot(d*x+c)*EllipticPi((a
+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(
1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+
2*a*(A*b-B*a)*cos(d*x+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(
1/2)-(2*A*a*b-3*B*a^2+B*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/(a^2-b^
2)/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.43 (sec) , antiderivative size = 1234, normalized size of antiderivative = 2.47

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(
3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(-(a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x]))/(b*(-a
^2 + b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*B - b^2*B)*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - 4*a*(-2*A*b^2 + 2*a*b*B)*((Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-2*a*A*b + 3*a^2*B - b
^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Si
n[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])
/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[...
```

Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3468, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{-\left(\left(-3Ba^2+2Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\left(\left(-3Ba^2+2Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(3Ba^2-2Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3540}$$

$$\frac{\frac{\int \frac{\left(a^2-b^2\right)(2Ab-3aB)\cos^2(c+dx)+2ab(Ab-aB)\cos(c+dx)+a\left(-3Ba^2+2Aba+b^2B\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\left(a^2-b^2\right)(2Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-3Ba^2+2Aba+b^2B\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3532}$$

$$\frac{\frac{\int \frac{a\left(-3Ba^2+2Aba+b^2B\right)+2ab(Ab-aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\left(a^2-b^2\right)(2Ab-3aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+d}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\left(a^2-b^2\right)(2Ab-3aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{a\left(-3Ba^2+2Aba+b^2B\right)+2ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3288

$$\frac{\frac{\int \frac{a\left(-3Ba^2+2Aba+b^2B\right)+2ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-b^2\right)(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\right.\right.}{bd}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3477

$$\frac{\frac{a\left(-3a^2B+2aAb+b^2B\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(2Ab-B(3a+b))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{2\sqrt{a+b}\left(a^2-b^2\right)(2Ab-3aB)\cot(c}{}}{2b}}{b\left(a^2-\right.}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{a\left(-3a^2B+2aAb+b^2B\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)(2Ab-B(3a+b))\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-b\right.}{}}{2b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3295}$$

$$\frac{a\left(-3a^2B+2aAb+b^2B\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-b^2\right)(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\right.}{bd}}{}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\frac{2(a-b)\sqrt{a+b}\left(-3a^2B+2aAb+b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2\sqrt{a+b}\left(a^2-b^2\right)(2Ab-3aB)\cot(c+}{}}{}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x
]
```

output
```
(2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a
+ b*Cos[c + d*x]]) + (((2*(a - b)*Sqrt[a + b]*(2*a*A*b - 3*a^2*B + b^2*B)*
Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*(2*A*b
 - (3*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*Sqrt[a + b]*(a
^2 - b^2)*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(b*d))/(2*b) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/(b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1627 vs. 2(459) = 918.

Time = 25.31 (sec) , antiderivative size = 1628, normalized size of antiderivative = 3.26

method	result	size
default	Expression too large to display	1628
parts	Expression too large to display	1669

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/d*((-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(
a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticPi(cot(d*x+c)-
csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*A*((a+
cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*b^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(6*cos(d*x
+c)^2+12*cos(d*x+c)+6)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*
b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*B*((a+cos(d*x+c)*b)/(
cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*Ellipti
cPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(2+2*cos(d*x+c)^2+4*cos
(d*x+c))*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))+(2+2*cos(d*x+c)^2+4*cos(d*x+c))*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*B*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-3*cos(d*x+c)^2-6*c
os(d*x+c)-3)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*
x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)...
```

Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x
)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*
x)*b + a),x)
```

3.427 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 416

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab\sqrt{a+b}d}$$
$$+\frac{2(Ab-aB)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab\sqrt{a+b}d}$$
$$-\frac{2\sqrt{a+b}B\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d}$$
$$+\frac{2a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a/b/(a+b)^(1/2)/d+2*(A*b-B*a)*cot(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/(a+b)^(1/
2)/d-2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2*a*(A*b-B*a)*sin(d*x+c)/b/(a
^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.02 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(
3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x]))/((a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]) - (-4*a*(a*A - b*B)*((Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF
[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(A*b - a*B)*((I*Cos[
(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (
2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr...
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3471, 3042, 3273, 3042, 3274, 3042, 3288, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3471}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}$$

$\downarrow$ 3042

$$\frac{(Ab-aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$\downarrow$ 3273

$$\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2-b^2}\right)}{b}+\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$\downarrow$ 3042

$$\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2-b^2}\right)}{b}+\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$\downarrow$ 3274

$$\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}\right)}{b}+\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$\downarrow$ 3042

$$\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\right.}}}{a^2-b^2}\right.}{b}$$

$$\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$\downarrow$ 3288

$$\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\right.}}}{a^2-b^2}\right.}{b}$$

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$\big\downarrow$ 3295

$$\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{}}{a^2-b^2}\right.}{b}$$

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$\big\downarrow$ 3473

$$\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{ad}}{}\right.}{b}$$

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

input

```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x
]
```

output
```
(-2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d)
 + ((A*b - a*B)*(-(((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(a*d))/(a^2 - b^2)) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3273
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Simp[d^2/(a^2 - b^2)   Int[Sqrt
[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3288

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3471

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> S
imp[B/b   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + S
imp[(A*b - a*B)/b   Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2)
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. $2(379) = 758$.

Time = 17.02 (sec) , antiderivative size = 1173, normalized size of antiderivative = 2.82

method	result	size
default	Expression too large to display	1173
parts	Expression too large to display	1198

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/d/(a-b)/(a+b)*(a+cos(d*x+c)*b)^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)*(A*cos(d*x+c)^(1/2)*(csc(d*x+c)^2*(1-co
s(d*x+c))^2+1)*((csc(d*x+c)^3*(1-cos(d*x+c))^3+cot(d*x+c)-csc(d*x+c))*a+(-
csc(d*x+c)^3*(1-cos(d*x+c))^3-cot(d*x+c)+csc(d*x+c))*b-2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticF(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a
+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c
)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)+B*cos(d*x+c)^(3/2)*(csc(
d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((-csc(d*x+c)^3*(1-cos(d*x+c))^3-cot(d*x+c)
+csc(d*x+c))*a*b+(csc(d*x+c)^3*(1-cos(d*x+c))^3+cot(d*x+c)-csc(d*x+c))*a^2
-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)
*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*(cos(d*x+c)/(cos(d...
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/
2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,(A+B\,\cos(c+dx))}{(a+b\,\cos(c+dx))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x
)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)\,b+a}dx$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)*b + a),x)
```

3.428 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 284

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(Ab-aB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\ldots}}{a^2\sqrt{a+b}d} + \frac{2(A+B)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a^2/(a+b)^(1/2)/d+2*(A+B)*cot(d*x+c)*EllipticF((a+b*c
os(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a+b)^(1/2)/d-2*
(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 1223, normalized size of antiderivative = 4.31

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3
/2)),x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(a*(a
^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*A - A*b^2)*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - 4*a*(-(a*A*b) + a^2*B)*((Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-(A*b^2) + a*b*B)*((I*Co
s[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/
2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) +
 (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C...
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3472}$$

$$\frac{\int \frac{Ab-aB+(aA-bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Ab-aB+(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{(Ab-aB)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)(A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-b)(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+(Ab-aB)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3295}$$

$$\frac{(Ab-aB)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}(A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{arcs}\right.}{ad}}{a^2-b^2}$$

$$\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\frac{2(a-b)\sqrt{a+b}(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}(A+B)\cot(c+dx)\sqrt{}}{}}{a^2-b^2}$$

$$\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

input

```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x
]
```

output

```
((2*(a - b)*Sqrt[a + b]*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a^2*d) + (2*(a - b)*Sqrt[a + b]*(A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*d))/(a^2 - b^2) - (2*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(258) = 516$.

Time = 18.05 (sec) , antiderivative size = 957, normalized size of antiderivative = 3.37

method	result	size
default	Expression too large to display	957
parts	Expression too large to display	990

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/d/(a-b)/(a+b)*(a+cos(d*x+c)*b)^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)*(A*((csc(d*x+c)^3*(1-cos(d*x+c))^3+cot(
d*x+c)-csc(d*x+c))*a*b+(-csc(d*x+c)^3*(1-cos(d*x+c))^3-cot(d*x+c)+csc(d*x+
c))*b^2-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b)
)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*Elliptic
E(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2)/a/cos(d*x+c)^(1/2)+B*co
s(d*x+c)^(1/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^3*(1-cos(d*x
+c))^3+cot(d*x+c)-csc(d*x+c))*a+(-csc(d*x+c)^3*(1-cos(d*x+c))^3-cot(d*x+c)
+csc(d*x+c))*b-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(
d*x+c)+1)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*a-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/
(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/
2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*Elli...
```

Fricas [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)^{\frac{3}{2}}\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}\sqrt{\cos(c + dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*
x))), x)
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)^{\frac{3}{2}}\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
c))), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\,b+\cos(dx+c)\,a}\,dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*b + cos
(c + d*x)*a),x)
```

3.429 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 305

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(a^2A-2Ab^2+abB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{a^3\sqrt{a+b}d}$$

$$-\frac{2(2Ab+a(A-B))\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2\sqrt{a+b}d}$$

$$+\frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2*(A*a^2-2*A*b^2+B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a+b)^(1/2)/d-2*(2*A*b+a*(A-B))*cot(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a^2/(a+b)^(1/2)/d+2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1
/2)/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.06 (sec) , antiderivative size = 1281, normalized size of antiderivative = 4.20

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3
/2)),x]
```

output
```
((-4*a*(2*a^2*A*b - 2*A*b^3 - a^3*B + a*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(a^3*A - 2*a*A*b^2 + a^2*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(a^2*A*b - 2*A*b^3 + a*b^2*B)*
((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a +
b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3042, 3479, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{Aa^2+bBa-(Ab-aB)\cos(c+dx)a-2Ab^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Aa^2+bBa-(Ab-aB)\cos(c+dx)a-2Ab^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Aa^2+bBa-(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\left(a^2A+abB-2Ab^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a(A-B)+2Ab)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2A+abB-2Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a(A-B)+2Ab)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx\right.}}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3295

$$\frac{\left(a^2A+abB-2Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a(A-B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{ad}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$\Big\downarrow$ 3473

$$\frac{\frac{2(a-b)\sqrt{a+b}\left(a^2A+abB-2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a-b)\sqrt{a+b}(a(A-B)-}{}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x
]
```

output
```
((2*(a - b)*Sqrt[a + b]*(a^2*A - 2*A*b^2 + a*b*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(2*A*b + a*(A - B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a*(a^2 - b^2)) + (2*b*(A*b - a*B)*Sin
[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(279) = 558$.

Time = 30.06 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.24

method	result	size
default	Expression too large to display	1293
parts	Expression too large to display	1348

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/d/(a-b)/(a+b)*(a+cos(d*x+c)*b)^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)*(-A*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(
(2*csc(d*x+c)^3*(1-cos(d*x+c))^3+2*cot(d*x+c)-2*csc(d*x+c))*b^3+(csc(d*x+c
)^3*(1-cos(d*x+c))^3-cot(d*x+c)+csc(d*x+c))*a^3+(-csc(d*x+c)^3*(1-cos(d*x+
c))^3-cot(d*x+c)+csc(d*x+c))*b*a^2-2*csc(d*x+c)^3*a*b^2*(1-cos(d*x+c))^3-2
*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)+2*Ellipti
cF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)+4*EllipticF(cot
(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)+2*EllipticE(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
(a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)+2*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos
(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)-4*EllipticE(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c
)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)-4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(co
s(d*x+c)+1)/(a+b))^(1/2))/a^2/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/cos(d*x+c)
^(3/2)+B*((csc(d*x+c)^3*(1-cos(d*x+c))^3+cot(d*x+c)-csc(d*x+c))*a*b+(-c...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\cos{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{3}{2}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*cos(c + d*x)**(
3/2)), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos{(dx+c)}+A}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos{(dx+c)}+A}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right) b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{3} b+\cos\left(dx+c\right)^{2} a}d x$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*b + cos
(c + d*x)**2*a),x)
```

3.430 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 393

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2(5a^2Ab-8Ab^3-3a^3B+6ab^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\cdots}{3a^4\sqrt{a+b}d}$$

$$+\frac{2(a+2b)(4Ab+a(A-3B))\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\cdots}{3a^3\sqrt{a+b}d}$$

$$+\frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{2(a^2A-4Ab^2+3abB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a+b)^(1/2)/d+2/3*
(a+2*b)*(4*A*b+a*(A-3*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a+b)^(1/2)/d+2*b*(A*b-B*a)*sin(d*x
+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^2-4*A*b
^2+3*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(
3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.36 (sec) , antiderivative size = 1357, normalized size of antiderivative = 3.45

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3
/2)),x]
```

output
```
((-4*a*(a^4*A + 7*a^2*A*b^2 - 8*A*b^4 - 6*a^3*b*B + 6*a*b^3*B)*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - 4*a*(5*a^3*A*b - 8*a*A*b^3 - 3*a^4*B + 6*a^2*b^2*B)
*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
+ 2*(5*a^2*A*b^2 - 8*A*b^4 - 3*a^3*b*B + 6*a*b^3*B)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[...
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{Aa^2+3bBa-(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{Aa^2+3bBa-(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{Aa^2+3bBa-(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-4Ab^2+2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{2\int -\frac{-3Ba^3+5Aba^2+6b^2Ba-\left(Aa^2-3bBa+2Ab^2\right)\cos(c+dx)a-8Ab^3}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3Ba^3+5Aba^2+6b^2Ba-\left(Aa^2-3bBa+2Ab^2\right)\cos(c+dx)a-8Ab^3}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-3Ba^3+5Aba^2+6b^2Ba-\left(Aa^2-3bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-8Ab^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3477

$$\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^3B+5a^2Ab+6ab^2B-8Ab^3\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a+2b)(aA-3aB+}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^3B+5a^2Ab+6ab^2B-8Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a+2b)(a}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3295

$$\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^3B+5a^2Ab+6ab^2B-8Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a+}{}}{}}{a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$\downarrow$ 3473

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+$$

$$\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2(a-b)\sqrt{a+b}\left(-3a^3B+5a^2Ab+6ab^2B-8Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\text{arc}\right.}{a^2d}}{}}{a\left(\right.}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x
]
```

output
```
(2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]) + (-1/3*((2*(a - b)*Sqrt[a + b]*(5*a^2*A*b - 8*A*b^3 -
3*a^3*B + 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a
- b)*Sqrt[a + b]*(a + 2*b)*(a*A + 4*A*b - 3*a*B)*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(a*d))/a + (2*(a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))/(a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1931 vs. $2(357) = 714$.

Time = 40.19 (sec) , antiderivative size = 1932, normalized size of antiderivative = 4.92

method	result	size
parts	Expression too large to display	1932
default	Expression too large to display	1972

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*B/d*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
(a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*El
lipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(2+2*cos(d*x+c)^2+4*co
s(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+
1)/(a+b))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)+(2+2*cos(d*x+c)^2+4*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+co
s(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^3*EllipticE(cot(d*x+c)-csc(d*x+c
),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*EllipticF(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))
^(1/2)*a^2*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-2*cos
(d*x+c)^2-4*cos(d*x+c)-2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))-sin(d*x+c)*a^3-a^2*b*cos(d*x+c)*sin(d*x+c)+(1-cos(d*x+c)
)*sin(d*x+c)*b^2*a+2*b^3*cos(d*x+c)*sin(d*x+c))*(a+cos(d*x+c)*b)^(1/2)/(b*
cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)/cos(d*x+c)^(1/2)/a^2/(a-b)/(a+b)
-2/3*A/d*(((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(co...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)^{5/2}\,(a + b\,\cos(c + d\,x))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx + c)\,b + a}\,\sqrt{\cos(dx + c)}}{\cos(dx + c)^4\,b + \cos(dx + c)^3\,a}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*b + cos
(c + d*x)**3*a),x)
```

3.431 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 674

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B)\cot(c+dx)E\Big(\text{ar}}{3a(a-b)b^3(a}$$

$$-\frac{(6a^2Ab+2aAb^2-12Ab^3-15a^3B-5a^2bB+21ab^2B+3b^3B)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{3(a-b)b^3(a+b)^{3/2}d}$$

$$-\frac{\sqrt{a+b}(2Ab-5aB)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec}{a}}}{b^4d}$$

$$+\frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}$$

$$+\frac{2a(2a^2Ab-6Ab^3-5a^3B+9ab^2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

$$-\frac{(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^3(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}}$$

output
```
1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*cot(d*x+c)*Ellipt
icE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/
2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/
b^3/(a+b)^(3/2)/d-1/3*(6*A*a^2*b+2*A*a*b^2-12*A*b^3-15*B*a^3-5*B*a^2*b+21*
B*a*b^2+3*B*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d-(a+b)^(1/2)*(2*A*b-5*B*a
)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/b^4/d+2/3*a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a
^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^
2)*cos(d*x+c)^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-1/
3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*(a+b*cos(d*x+c))^(1
/2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.95 (sec) , antiderivative size = 1396, normalized size of antiderivative = 2.07

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(a^2*A*b*Sin[c + d*x])
 + a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(
-3*a^3*A*b*Sin[c + d*x] + 7*a*A*b^3*Sin[c + d*x] + 6*a^4*B*Sin[c + d*x] -
10*a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/
d + ((-4*a*(-2*a^3*A*b + 2*a*A*b^3 + 5*a^4*B - 8*a^2*b^2*B + 3*b^4*B)*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(2*a^2*A*b^2 + 6*A*b^4 + 4*a^3*b*B - 12*
a*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])) + 2*(-6*a^3*A*b + 14*a*A*b^3 + 15*a^4*B - 26*a^2*b^2*B + 3*b^4*...
```

Rubi [A] (verified)

Time = 3.46 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3468, 27, 3042, 3526, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+2Aba+3b^2B\right)\cos^2(c+dx)\right)-3b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+2Aba+3b^2B\right)\cos^2(c+dx)\right)-3b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5Ba^2-2Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3526

$$\frac{\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{-\left(\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)\cos^2(c+dx)\right)+b\left(2Ba^3+Aba^2-6b^2\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+d}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-\left(\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)\cos^2(c+dx)\right)+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)+a\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)}+\frac{2a(-5a}{}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(15Ba^4-6Aba^3-26b^2Ba^2+14Ab^3a+3b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}+\frac{2a(-}{}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3540

$$\frac{\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\cos^2(c+dx)+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\cos(c+dx)+a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(-15a^4B+6a^3Ab+\right.}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(-15a^4B+\right.}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3532

$$\frac{\frac{\frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+\int \frac{a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(-15a^4B\right.}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3288

$$\frac{\frac{\int \frac{a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^2-b^2\right)^2(2Ab-5aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c\ldots}{a+b}}}{}}{2b}}{b(a^2-b\ldots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3477

$$\frac{a(a-b)\left(15a^3B-a^2(6Ab-5bB)-ab^2(2A+21B)+3b^3(4A-B)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+a\left(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B\right)\int \frac{\cos(c\ldots}{\cos^{\frac{3}{2}}(c+dx)\ldots}}{2b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$a(a-b)\left(15a^3B-a^2(6Ab-5bB)-ab^2(2A+21B)+3b^3(4A-B)\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B\right)\int \frac{}{\sin(\ldots}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3295

$$a\left(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^2-b^2\right)^2(2Ab-5aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{bd}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3473}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{-\frac{6\sqrt{a+b}\left(a^2-b^2\right)^2(2Ab-5aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}(}{bd}}{}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x
]
```

output
```
(2*a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a +
b*Cos[c + d*x])^(3/2)) + ((2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
]) + (((2*(a - b)*Sqrt[a + b]*(6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*
b^2*B - 3*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*(a - b)*S
qrt[a + b]*(3*b^3*(4*A - B) + 15*a^3*B - a*b^2*(2*A + 21*B) - a^2*(6*A*b -
 5*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*Sqrt[a + b]*(a^2 - b^
2)^2*(2*A*b - 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d
))/(2*b) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/(b*(a^2 -
b^2)))/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4189 vs. $2(619) = 1238$.

Time = 32.50 (sec) , antiderivative size = 4190, normalized size of antiderivative = 6.22

method	result	size
default	Expression too large to display	4190
parts	Expression too large to display	4229

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/3/d*((-30*cos(d*x+c)^2-60*cos(d*x+c)-30)*B*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^4*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)
-6)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(
a+b))^(1/2)*a^5*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-
6*cos(d*x+c)^3-18*cos(d*x+c)^2-18*cos(d*x+c)-6)*A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*b^2*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^3+2*cos(d*x+c)^
2+22*cos(d*x+c)+14)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/
(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos
(d*x+c)+1)/(a+b))^(1/2)*b^6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*(3*cos(d*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c))+sin(d*x+c)*cos(d*x+c)
*(34*cos(d*x+c)-18)*B*a^3*b^3+sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)^2-20*cos
(d*x+c)-3)*B*a^2*b^4+(15*cos(d*x+c)^2+30*cos(d*x+c)+15)*B*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^6*Ellipt
icE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d
*x+c)-30)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c
)+1)/(a+b))^(1/2)*a^6*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))-3*B*b^6*cos(d*x+c)^3*sin(d*x+c)-15*B*a^6*cos(d*x+c)*sin(d*x+c)+(4...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x
)
```

output
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}}{\cos\left(dx+c\right)^{2}b^{2}+2\cos\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(cos(c +
 d*x)**2*b**2 + 2*cos(c + d*x)*a*b + a**2),x)
```

3.432 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 545

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(4Ab^3+3a^3B-7ab^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{3a(a-b)b^2(a+b)^{3/2}d}$$
$$+\frac{2(aAb^2-3Ab^3-3a^3B-a^2bB+6ab^2B)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\text{se}}{a}}}{3a(a-b)b^2(a+b)^{3/2}d}$$
$$-\frac{2\sqrt{a+b}B\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d}$$
$$+\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}$$
$$-\frac{2a(4Ab^3+3a^3B-7ab^2B)\sin(c+dx)}{3b^2(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*(4*A*b^3+3*B*a^3-7*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^2/(a+b)^(3/2)/d+2/3*(A*
a*b^2-3*A*b^3-3*B*a^3-B*a^2*b+6*B*a*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^2/(a+b)^(3/2)/
d-2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/3*a*(A*b-B*a)*cos(d*x+c)^(1/2)
*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-2/3*a*(4*A*b^3+3*B*a^3-7*
B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1
/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.48 (sec) , antiderivative size = 1342, normalized size of antiderivative = 2.46

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(a*A*b*Sin[c + d*x]) +
a^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(4*A*b
^3*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*x]))/(3*b*(-a
^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B
 - a*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Co
s[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(4*a*A*b^2 - a^2*b*B - 3
*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b)
, ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]])) + 2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b
*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]...
```

Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3468, 27, 3042, 3530, 3042, 3288, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{3\left(a^2-b^2\right)B\cos^2(c+dx)-3b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3\left(a^2-b^2\right)B\cos^2(c+dx)-3b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3\left(a^2-b^2\right)B\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3530}$$

$$\frac{\frac{\int \frac{ab(Ab-aB)-3\left(Ab^3+a\left(a^2-2b^2\right)B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{b}+\frac{3B\left(a^2-b^2\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{ab(Ab-aB)-3\left(Ab^3+a\left(a^2-2b^2\right)B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{3B\left(a^2-b^2\right)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3288}$$

$$\frac{\frac{\int \frac{ab(Ab-aB)-3\left(Ab^3+a\left(a^2-2b^2\right)B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}-\frac{6B\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\right.\right.}{b^2d}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3472}$$

$$\frac{\frac{\frac{\int \frac{a\left(3Ba^3-7b^2Ba+4Ab^3\right)+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}-\frac{6B\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{a\left(3Ba^3-7b^2Ba+4Ab^3\right)+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}-\frac{6B\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a}{}}}{}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3477

$$\frac{\frac{\frac{a\left(3a^3B-7ab^2B+4Ab^3\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(3a^3B+a^2bB-ab^2(A+6B)+3Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2a\left(3a^3B-7ab^2B+4\right.}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx}}}{b}}{3b\left(a^2-b\right.}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{a\left(3a^3B-7ab^2B+4Ab^3\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(3a^3B+a^2bB-ab^2(A+6B)+3Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{}{d(}}{b}}{3b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3295

$$\frac{\frac{a\left(3a^3B-7ab^2B+4Ab^3\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^3B+a^2bB-ab^2(A+6B)+3Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{ad}}{a^2-b^2}}{b}$$

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3473

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^3B-7ab^2B+4Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\left(3a^3B+a^2bB-ab^2(\right.}{}}{a^2-b^2}}{b}$$

input

```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x
]
```

output

```
(2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a +
b*Cos[c + d*x])^(3/2)) + ((-6*Sqrt[a + b]*(a^2 - b^2)*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (((2*(a - b)*Sqrt[a + b]*(4*A*b^3
 + 3*a^3*B - 7*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a
 - b)*Sqrt[a + b]*(3*A*b^3 + 3*a^3*B + a^2*b*B - a*b^2*(A + 6*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (2*a*(4*A*b^3 + 3*a^3*B
- 7*a*b^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]))/b)/(3*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3530
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b
*Sin[e + f*x]], x], x] + Simp[1/b   Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((
a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d,
e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2847 vs. $2(496) = 992$.

Time = 27.51 (sec) , antiderivative size = 2848, normalized size of antiderivative = 5.23

method	result	size
default	Expression too large to display	2848
parts	Expression too large to display	2901

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/3/d*((6*cos(d*x+c)^2+12*cos(d*x+c)+6)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a
+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*b*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^3+12*cos(d*x+c)^2+6*cos(d
*x+c))-4*A*b^5*cos(d*x+c)^2*sin(d*x+c)+3*B*a^5*cos(d*x+c)*sin(d*x+c)+(-3*c
os(d*x+c)^2-6*cos(d*x+c)-3)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)-1)*A*a^3*b^2+sin(d*
x+c)*cos(d*x+c)*(-3+5*cos(d*x+c))*A*a*b^4+sin(d*x+c)*cos(d*x+c)*(4*cos(d*x
+c)-2)*B*a^4*b+sin(d*x+c)*cos(d*x+c)*(-3*cos(d*x+c)-7)*B*a^3*b^2+sin(d*x+c
)*cos(d*x+c)*(-8*cos(d*x+c)+6)*B*a^2*b^3+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^5*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x
+c))+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*b^5*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*(6*cos(d*x+c)^3+12*cos(d*x+c)^2+6*cos(d*x+c))+A*((a+cos(d*x+c)*b)/(cos(d*
x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^5*EllipticE(cot(d
*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-4*cos(d*x+c)^3-8*cos(d*x+c)^2-4*c
os(d*x+c))+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x
)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}\,dx$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*
x)**2*b**2 + 2*cos(c + d*x)*a*b + a**2),x)
```

3.433 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 391

$$
\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = & -\frac{2(3a^2A+Ab^2-4abB)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} \\
& + \frac{2(3aA-Ab+aB-3bB)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} \\
& - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} \\
& + \frac{2(3a^2A+Ab^2-4abB)\sin(c+dx)}{3(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}
\end{aligned}
$$

output

```
-2/3*(3*A*a^2+A*b^2-4*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d+2/3*(3*A*a-A
*b+B*a-3*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(
d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d-2/3*(A*b-B*a)*cos(d*x+c)^(1/2
)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*(3*A*a^2+A*b^2-4*B*a*b
)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.40 (sec) , antiderivative size = 1335, normalized size of antiderivative = 3.41

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b*Sin[c + d*x]) + a*
B*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*A*b*Si
n[c + d*x] + A*b^3*Sin[c + d*x] - 4*a*b^2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2
)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B - a*b^2
*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3*A + a*A*b^2 - 4*a^2*b*B)*
((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
 2*(3*a^2*A*b + A*b^3 - 4*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
 d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)...
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3478, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3478}$$

$$-\frac{2\int \frac{Ab-aB-3(aA-bB)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{Ab-aB-3(aA-bB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{Ab-aB-3(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3472}$$

$$-\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(3a^2A-4abB+Ab^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(3a^2A-4abB+Ab^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3477

$$-\frac{\frac{(3a^2A-4abB+Ab^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a(3A+B)-b(A+3B))\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(3a^2A-4abB+Ab^2\cdots}{d(a^2-b^2)\sqrt{\cos(c+dx)}\cdots}}{3\left(a^2-b^2\right)}\cdots$$

$$\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{\frac{(3a^2A-4abB+Ab^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a(3A+B)-b(A+3B))\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(\cdots}{d(a^2-\cdots}}{3\left(a^2-b^2\right)}\cdots$$

$$\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3295

$$-\frac{\frac{(3a^2A-4abB+Ab^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a(3A+B)-b(A+3B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Ellip}\cdots}{ad}}{a^2-b^2}\cdots}{3\left(a^2-b^2\right)}\cdots$$

$$\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3473

$$-\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2A-4abB+Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a-b)\sqrt{a+b}(a(3A+B)-b(A+3B\cdots}{\cdots}}{a^2-b^2}\cdots}{3\left(a^2-b^2\cdots\right.}$$

$$\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x
]
```

output
```
(-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*C
os[c + d*x])^(3/2)) - (((2*(a - b)*Sqrt[a + b]*(3*a^2*A + A*b^2 - 4*a*b*B)
*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(a*
(3*A + B) - b*(A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2
- b^2) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2071 vs. $2(353) = 706$.

Time = 22.66 (sec) , antiderivative size = 2072, normalized size of antiderivative = 5.30

method	result	size
default	Expression too large to display	2072
parts	Expression too large to display	2133

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3/d*((-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*EllipticE(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))-A*b^4*cos(d*x+c)^2*sin(d*x+c)+(4*cos(d*x+c
)^3+9*cos(d*x+c)^2+6*cos(d*x+c)+1)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a
+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*EllipticF(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^3-6*cos(d*x+c)^2-9*cos(d*x+c)-
4)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a
+b))^(1/2)*a^3*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4
*cos(d*x+c)^3-11*cos(d*x+c)^2-10*cos(d*x+c)-3)*B*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*EllipticF(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)
*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*a^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d
*x+c)^2-2*cos(d*x+c)-1)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)
*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))+3*A*a^4*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)+1)*B*a^4+2*A*a*b^3*cos(d*x+c)^2*sin(d*x+c)-4*B*a^3*b*cos(d*x+c)*sin(
d*x+c)+sin(d*x+c)*cos(d*x+c)*(2*cos(d*x+c)-4)*A*a^3*b+sin(d*x+c)*cos(d*x+c
)*(-3*cos(d*x+c)+1)*A*a^2*b^2+sin(d*x+c)*cos(d*x+c)*(3-5*cos(d*x+c))*B*a^2
*b^2+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+...
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(5/
2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x
)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*b**2 +
2*cos(c + d*x)*a*b + a**2),x)
```

3.434 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 429

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\ldots\right.}{3a^3(a-b)(a+b)\ldots}$$
$$-\frac{2(2Ab^2-3a^2(A+B)+ab(3A+B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+\ldots}{a+b}}\ldots}{3a^2\sqrt{a+b}\,(a^2-b^2)\,d}$$
$$+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}$$
$$-\frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\sin(c+dx)}{3a\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d-2
/3*(2*A*b^2-3*a^2*(A+B)+a*b*(3*A+B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a+b)^(1/2)/(a^2-b^2)/d+
2/3*b*(A*b-B*a)*cos(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
^(3/2)-2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/
cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.20 (sec) , antiderivative size = 1384, normalized size of antiderivative = 3.23

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5
/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(A*b^2*Sin[c + d*x]) +
 a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-6*a^
2*A*b^2*Sin[c + d*x] + 2*A*b^4*Sin[c + d*x] + 3*a^3*b*B*Sin[c + d*x] + a*b
^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*
a*(3*a^4*A - 5*a^2*A*b^2 + 2*A*b^4 - a^3*b*B + a*b^3*B)*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]) - 4*a*(-6*a^3*A*b + 2*a*A*b^3 + 3*a^4*B + a^2*b^2*B)*((Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*
a^2*A*b^2 + 2*A*b^4 + 3*a^3*b*B + a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a ...
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3479, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{3Aa^2-bBa-3(Ab-aB)\cos(c+dx)a-2Ab^2}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Aa^2-bBa-3(Ab-aB)\cos(c+dx)a-2Ab^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3Aa^2-bBa-3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3472}$$

$$\frac{\frac{\int \frac{-3Ba^3+6Aba^2-b^2Ba+\left(3Aa^2-4bBa+Ab^2\right)\cos(c+dx)a-2Ab^3}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{-3Ba^3+6Aba^2-b^2Ba+\left(3Aa^2-4bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3477

$$\frac{\frac{\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(-3a^2(A+B)+ab(3A+B)+2Ab^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(\cdots}{d\cdots}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(-3a^2(A+B)+ab(3A+B)+2Ab^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+\cdots\right.}}\cdots}{a^2-b^2}\cdots}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3295

$$\frac{\frac{\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(-3a^2(A+B)+ab(3A+B)+2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\cdots}{ad}}{a^2-b^2}\cdots}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$\downarrow$ 3473

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a-b)\sqrt{a+b}\left(-3a^2(A+\cdots\right.}{\cdots}}{a^2-b^2}\cdots}{3a\cdots}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x
]
```

output
```
(2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a +
b*Cos[c + d*x])^(3/2)) + (((2*(a - b)*Sqrt[a + b]*(6*a^2*A*b - 2*A*b^3 - 3
*a^3*B - a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b
)*Sqrt[a + b]*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3*A + B))*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B
- a*b^2*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2441 vs. $2(391) = 782$.

Time = 29.33 (sec) , antiderivative size = 2442, normalized size of antiderivative = 5.69

method	result	size
default	Expression too large to display	2442
parts	Expression too large to display	2503

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3/d*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*b^5*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(
-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+2*B*a^2*b^3*cos(d*x+c)^2*sin(
d*x+c)+sin(d*x+c)*cos(d*x+c)*(-5*cos(d*x+c)+7)*A*a^3*b^2+sin(d*x+c)*cos(d*
x+c)*(6*cos(d*x+c)+2)*A*a^2*b^3+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)-3)*A*a*b
^4+sin(d*x+c)*cos(d*x+c)*(2*cos(d*x+c)-4)*B*a^4*b+sin(d*x+c)*cos(d*x+c)*(-
3*cos(d*x+c)+1)*B*a^3*b^2+(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*A*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^5*Ell
ipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^2+6*cos(d
*x+c)+3)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*a^5*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+
cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*b*EllipticE(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))+(6*cos(d*x+c)^3+18*cos(d*x+c)^2+18*cos(d*x+c)+
6)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a
+b))^(1/2)*a^3*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(
6*cos(d*x+c)^3+10*cos(d*x+c)^2+2*cos(d*x+c)-2)*A*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^3*EllipticE(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^3-6*cos(d*x+c)^2
-6*cos(d*x+c)-2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^
3*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
c))), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3\,b^2+2\cos(dx+c)^2\,ab+\cos(dx+c)\,a^2}\,dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*b**2 +
2*cos(c + d*x)**2*a*b + cos(c + d*x)*a**2),x)
```

3.435 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 456

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B)\cot(c+dx)E\Big(\text{ar}}{3a^4(a-b)(a}$$

$$+\frac{2(8Ab^3-3a^3(A-B)+2ab^2(3A-B)-3a^2b(3A+B))\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),\right.}{3a^3\sqrt{a+b}\,(a^2-b^2)\,d}$$

$$+\frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$+\frac{2b(8a^2Ab-4Ab^3-5a^3B+ab^2B)\sin(c+dx)}{3a^2\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4+6*B*a^3*b-2*B*a*b^3)*cot(d*x+c)*Elliptic
E((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2)
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/
(a+b)^(3/2)/d+2/3*(8*A*b^3-3*a^3*(A-B)+2*a*b^2*(3*A-B)-3*a^2*b*(3*A+B))*co
t(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(
a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a^3/(a+b)^(1/2)/(a^2-b^2)/d+2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/
d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3
+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(
1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.33 (sec) , antiderivative size = 1431, normalized size of antiderivative = 3.14

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5
/2)),x]
```

output
```
-1/3*((-4*a*(9*a^4*A*b - 17*a^2*A*b^3 + 8*A*b^5 - 3*a^5*B + 5*a^3*b^2*B -
2*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^5*A - 15*a^3*A*b^2 +
 8*a*A*b^4 + 6*a^4*b*B - 2*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^4*A*b - 15*a^2*A*b^3 + 8*A*b
^5 + 6*a^3*b^2*B - 2*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos
[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*...
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{3Aa^2+bBa-3(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-4Ab^2+2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{2\int \frac{3Aa^4+6bBa^3-15Ab^2a^2-2b^3Ba-\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)a+8Ab^4}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B-4Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3Aa^4+6bBa^3-15Ab^2a^2-2b^3Ba-\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)a+8Ab^4}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B-4Ab^3\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{3Aa^4+6bBa^3-15Ab^2a^2-2b^3Ba-\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+8Ab^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(-5a^3B+8a^2Ab+ab^2B-4Ab^3)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3477

$$\frac{\frac{(a-b)\left(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\left(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4\right)\int \frac{\text{co}}{\cos^{\frac{3}{2}}(c+d}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{(a-b)\left(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4\right)\int -}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3295

$$\frac{\frac{\left(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3\right)\text{cc}}{}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right.}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$\big\downarrow$ 3473

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{2b(-5a^3B+8a^2Ab+ab^2B-4Ab^3)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(a-b)\sqrt{a+b}\left(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+d}{a-b}}}{ad}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x
]
```

output
```
(2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(3/2)) + (((2*(a - b)*Sqrt[a + b]*(3*a^4*A - 15*a^2*A*b^2
+ 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a
^2*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 - 3*a^3*(A - B) + 2*a*b^2*(3*A - B
) - 3*a^2*b*(3*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a*(a^2
 - b^2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(a
*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b
^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2955 vs. $2(418) = 836$.

Time = 37.22 (sec) , antiderivative size = 2956, normalized size of antiderivative = 6.48

method	result	size
default	Expression too large to display	2956
parts	Expression too large to display	3009

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/3/d*(sin(d*x+c)*cos(d*x+c)*(4*cos(d*x+c)-12)*A*a*b^5-3*A*a^6*sin(d*x+c)
+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*b^6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-8*cos
(d*x+c)^3-16*cos(d*x+c)^2-8*cos(d*x+c))+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)
/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^5*EllipticF(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^3+16*cos(d*x+c)^2+8*cos(d*
x+c))+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*a^2*b^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2-2*cos(d*x+c))+(3*cos(d*x+c)^3+12*cos(d*x
+c)^2+15*cos(d*x+c)+6)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))+(6*cos(d*x+c)^3+13*cos(d*x+c)^2+8*cos(d*x+c)+1)*B*((a+cos
(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
^4*b^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^3
-3*cos(d*x+c)-2)*B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*a^3*b^3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))+(-3*cos(d*x+c)^3-9*cos(d*x+c)^2-9*cos(d*x+c)-3)*A*((a+cos(d*x
+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^5*b
*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-3*cos(d*x+c)^3+9*
cos(d*x+c)^2+27*cos(d*x+c)+15)*A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b)...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^3*cos(d*x + c)^5 + 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^
3*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{4}b^{2}+2\cos\left(dx+c\right)^{3}ab+\cos\left(dx+c\right)^{2}a^{2}}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*b**2 +
2*cos(c + d*x)**3*a*b + cos(c + d*x)**2*a**2),x)
```

3.436 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 567

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{3a^5(a-b)(a+b)^{3/2}d}$$

$$-\frac{2(16Ab^4-a^4(A-3B)+4ab^3(3A-2B)-9a^3b(A-B)-2a^2b^2(8A+3B))\cot(c+dx)\,\text{EllipticF}\left(\text{arcs}\right.}{3a^4\sqrt{a+b}\,(a^2-b^2)\,d}$$

$$+\frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$+\frac{2b(10a^2Ab-6Ab^3-7a^3B+3ab^2B)\sin(c+dx)}{3a^2\,(a^2-b^2)^2\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{2(a^4A-13a^2Ab^2+8Ab^4+8a^3bB-4ab^3B)\,\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3a^3\,(a^2-b^2)^2\,d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B*a^5+15*B*a^3*b^2-8*B*a*b^4)*cot(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a^5/(a-b)/(a+b)^(3/2)/d-2/3*(16*A*b^4-a^4*(A-3*B)+4*a*b^3*(3*A-2*B)-9
*a^3*b*(A-B)-2*a^2*b^2*(8*A+3*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a+b)^(1/2)/(a^2-b^2)/d+2/3
*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3
/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^
2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^4-13*A*a^2*b^2+8*A*b^
4+8*B*a^3*b-4*B*a*b^3)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/a^3/(a^2-b^2)^2/d
/cos(d*x+c)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.62 (sec) , antiderivative size = 1499, normalized size of antiderivative = 2.64

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5
/2)),x]
```

output
```
((-4*a*(a^6*A + 15*a^4*A*b^2 - 32*a^2*A*b^4 + 16*A*b^6 - 9*a^5*b*B + 17*a^
3*b^3*B - 8*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a^5*A*b - 28
*a^3*A*b^3 + 16*a*A*b^5 - 3*a^6*B + 15*a^4*b^2*B - 8*a^2*b^4*B)*((Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*
A*b^2 - 28*a^2*A*b^4 + 16*A*b^6 - 3*a^5*b*B + 15*a^3*b^3*B - 8*a*b^5*B)*((
I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a +...
```

Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{4b(Ab-aB)\cos^2(c+dx)-3a(Ab-aB)\cos(c+dx)+3(Aa^2+bBa-2Ab^2)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4b(Ab-aB)\cos^2(c+dx)-3a(Ab-aB)\cos(c+dx)+3(Aa^2+bBa-2Ab^2)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{4b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3(Aa^2+bBa-2Ab^2)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{2\int \frac{2b(-7Ba^3+10Aba^2+3b^2Ba-6Ab^3)\cos^2(c+dx)-a(-3Ba^3+6Aba^2-b^2Ba-2Ab^3)\cos(c+dx)+3(Aa^4+8bBa^3-13Ab^2a^2-4b^3Ba+8Ab^4)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(-7a^3 \ldots}{ad(a^2- \ldots}}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{2b(-7Ba^3+10Aba^2+3b^2Ba-6Ab^3)\cos^2(c+dx)-a(-3Ba^3+6Aba^2-b^2Ba-2Ab^3)\cos(c+dx)+3(Aa^4+8bBa^3-13Ab^2a^2-4b^3Ba+8Ab^4)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(-7a^3B \ldots}{ad(a^2- \ldots}}{3a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{2b\left(-7Ba^3+10Aba^2+3b^2Ba-6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(Aa^4+8bBa^3-13Ab^2a^2-4b^3Ba+8Ab^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(}{}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3534

$$\frac{\frac{\frac{2\int -\frac{3\left(-3Ba^5+8Aba^4+15b^2Ba^3-28Ab^3a^2-8b^4Ba-\left(Aa^4-6bBa^3+7Ab^2a^2+2b^3Ba-4Ab^4\right)\cos(c+dx)a+16Ab^5\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab\right.}{ad\cos^{\frac{3}{2}}(c+}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3Ba^5+8Aba^4+15b^2Ba^3-28Ab^3a^2-8b^4Ba-\left(Aa^4-6bBa^3+7Ab^2a^2+2b^3Ba-4Ab^4\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3Ba^5+8Aba^4+15b^2Ba^3-28Ab^3a^2-8b^4Ba-\left(Aa^4-6bBa^3+7Ab^2a^2+2b^3Ba-4Ab^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3477

$$\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(a-b)\left(-\left(a^4(A-3B)\right)-9a^3b(A-B)-2a^2b^2(8A+3B)+4ab^3(3A-2B)+16Ab^4\right)\int\frac{}{\sqrt{\cos(c+}}}{a(a^2-b^2)}}{3a}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(a-b)\left(-\left(a^4(A-3B)\right)-9a^3b(A-B)-2a^2b^2(8A+3B)+4ab^3(3A-2B)+16Ab^4\right)\int\frac{}{\sqrt{\sin\left(c-\right.}}}{a(a^2-b^2)}}{}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3295

$$\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^5B+8a^4Ab+15a^3b^2B-28a^2Ab^3-8ab^4B+16Ab^5\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c-\right.}}}{}}{}$$

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3473

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{\frac{2b(-7a^3B+10a^2Ab+3ab^2B-6Ab^3)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\left(-\left(a^4(A-3B)\right)-\right.}{}}{}}{}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x
]
```

output

```
(2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a +
b*Cos[c + d*x])^(3/2)) + ((2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B
)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x
]]) + (-(((2*(a - b)*Sqrt[a + b]*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*
a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^
2*d) + (2*(a - b)*Sqrt[a + b]*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*(3*A - 2
*B) - 9*a^3*b*(A - B) - 2*a^2*b^2*(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(a*d))/a) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a
*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(a*d*Cos[c + d*x]^(3/2)))/(
a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3719 vs. $2(523) = 1046$.

Time = 46.71 (sec) , antiderivative size = 3720, normalized size of antiderivative = 6.56

method	result	size
parts	Expression too large to display	3720
default	Expression too large to display	3801

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*B/d*((3*cos(d*x+c)^2+6*cos(d*x+c)+3)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/
(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^6*EllipticE(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))+(3*cos(d*x+c)^3+9*cos(d*x+c)^2+9*cos(d*x+c)
+3)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a^5*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(3*c
os(d*x+c)^3-9*cos(d*x+c)^2-27*cos(d*x+c)-15)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*b^2*EllipticE(cot(d
*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-15*cos(d*x+c)^3-45*cos(d*x+c)^2-4
5*cos(d*x+c)-15)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*a^3*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+
b))^(1/2))+(-15*cos(d*x+c)^3-22*cos(d*x+c)^2+cos(d*x+c)+8)*((a+cos(d*x+c)*
b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^4*E
llipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(8*cos(d*x+c)^3+24*co
s(d*x+c)^2+24*cos(d*x+c)+8)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^5*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))+((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*b^6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(8*cos(d*x+c)^3+16*cos(d*x+c)^2+8*cos(d*x+c))+(-3*cos(d*x+c)^2-6
*cos(d*x+c)-3)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a^6*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(b^3*cos(d*x + c)^6 + 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^
3*cos(d*x + c)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{5/2}\,(a+b\cos(c+dx))^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^5\,b^2+2\cos(dx+c)^4\,ab+\cos(dx+c)^3\,a^2}dx$$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**5*b**2 +
2*cos(c + d*x)**4*a*b + cos(c + d*x)**3*a**2),x)
```

3.437 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 419

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd} + \frac{\sqrt{a+b}B\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd} + \frac{a\sqrt{a+b}B\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} + \frac{aB\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
-(a-b)*(a+b)^(1/2)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+(a+b)^(1/2)*B*cot(d*x+c)*EllipticF((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+a*(a+b)^(1/
2)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/b^2/d+a*B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(a+b*cos(
d*x+c))^(1/2)+B*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 4.89 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.54

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{B\cos^{\frac{3}{2}}(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left((a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E(\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
])^(3/2),x]
```

output
```
(B*Cos[c + d*x]^(3/2)*Sec[(c + d*x)/2]^2*((a + b)*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - 2*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(2*b*d*(C
os[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2011, 3042, 3299, 3042, 3288, 3482, 27, 3042, 3472, 25, 27, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3299}$$

$$B\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$B\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)$$

$$\Big\downarrow \text{ 3288}$$

$$B\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\text{ar}\right.}{b^2d}\right.$$

$$\Big\downarrow \text{ 3482}$$

$$B\left(\frac{\frac{1}{2}\int \frac{2\left(\cos(c+dx)a^2+ba\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{b}\right.$$

$$\Big\downarrow \text{ 27}$$

$$B\left(\frac{\int \frac{\cos(c+dx)a^2+ba}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{b^2 d}\right.$$

$\downarrow$ 3042

$$B\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a^2+ba}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{}\right.$$

$\downarrow$ 3472

$$B\left(\frac{\frac{\int -\frac{a\left(a^2-b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}\right.$$

$\downarrow$ 25

$$B\left(\frac{-\frac{\int \frac{a\left(a^2-b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-s}{}}}{}\right.$$

$\downarrow$ 27

$$B\left(\frac{-a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{}}{}\right.$$

$\downarrow$ 3042

$$B\left(\frac{-a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+}{}\right.$$

$\downarrow$ 3280

$$B\left(\frac{-a\left(\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx}{d\sqrt{\cos(c+dx)}\sqrt{a+b}}}{2b}\right.$$

$\downarrow$ 3042

$$B\left(\frac{-a\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{}{d\sqrt{}}}{2b}\right.$$

$\downarrow$ 3295

$$B\left(\frac{-a\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{ad}\right.}{2b}\right.$$

$\downarrow$ 3473

$$B\left(\frac{-a\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{}\right.}{2b}\right.$$

input
```
Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/
2),x]
```

output
```
B*((a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d)
 + (-(a*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*
d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))) + (2*a*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (2*b*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]))/(2*b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3299
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> Simp[(-a)*(d/(2*b))   Int[Sqrt[d*Sin[e + f*x]]/Sqr
t[a + b*Sin[e + f*x]], x], x] + Simp[d/(2*b)   Int[Sqrt[d*Sin[e + f*x]]*((a
 + 2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e
, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

Maple [A] (verified)

Time = 22.11 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.93

method	result
default	$\frac{B\left(\left(2+2\cos(dx+c)^{2}+4\cos(dx+c)\right)\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)+\left(-\text{co}\right.\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
B/d*((2+2*cos(d*x+c)^2+4*cos(d*x+c))*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticPi(cot(d*x+c)-csc(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*Ellipt
icE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c
)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a
+b))^(1/2)*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+sin(d*x
+c)*cos(d*x+c)^2*b+cos(d*x+c)*sin(d*x+c)*a)*(a+cos(d*x+c)*b)^(1/2)/cos(d*x
+c)^(1/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)/b
```

Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
integral(B*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x
)
```

output
```
B*Integral(cos(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^{3/2}\,(B\,a+B\,b\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/
2),x)
```

output
```
int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\cos(dx+c)\,b+a}dx\right) b$$

input
```
int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x))*cos(c + d*x))/(cos(c + d*
x)*b + a),x)*b
```

3.438 $\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 117

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{a+b}B\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
-2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2B\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)-2\,\text{EllipticPi}\left(-1,\text{arcsin}\right.\right.}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x
])^(3/2),x]
```

output
```
(-2*B*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*Elliptic
Pi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2011, 3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3288}$$

$$-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/
2),x]
```

output
```
(-2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Maple [A] (verified)

Time = 15.70 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

method	result
default	$\frac{2B\sqrt{\cos(dx+c)}\left(\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)-2\operatorname{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{a+\cos(dx+c}{(\cos(dx+c)+1)(}}}{d\sqrt{a+\cos(dx+c)b}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2*B/d*cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2)*(EllipticF(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(
a+b))^(1/2)))*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)/(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(Bb\cos{(dx+c)}+Ba)\sqrt{\cos{(dx+c)}}}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
integral(B*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx=B\int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x
)
```

output
```
B*Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)
^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)
^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(B\,a+B\,b\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/
2),x)
```

output
```
int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\cos\left(dx+c\right)b+a}\,\sqrt{\cos\left(dx+c\right)}}{\cos\left(dx+c\right)b+a}dx\right)b$$

input
```
int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)*b + a),x)*
b
```

3.439 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 110

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}B\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a}{a}\cdots\right.}{ad}$$

output
```
2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{4(a+b)B\cos^{\frac{3}{2}}(c+dx)\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{a-b}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right.\right.}{ad\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}\right)^{3/2}}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]
)^(3/2)),x]
```

output
```
(-4*(a + b)*B*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a -
b))]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(
a - b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a))^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2011, 3042, 3295}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3295}$$

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2
)),x]
```

output
```
(2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Maple [A] (verified)

Time = 12.84 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2B(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}}{d\sqrt{\cos(dx+c)}\,\sqrt{a+\cos(dx+c)b}}$	112
parts	Expression too large to display	988

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-2*B/d*(cos(d*x+c)+1)/cos(d*x+c)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(
a+cos(d*x+c)*b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)
```

Fricas [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{Bb\cos\left(dx + c\right) + Ba}{\left(b\cos\left(dx + c\right) + a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx + c\right)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 +
 a*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a + b\cos\left(c + dx\right)}\sqrt{\cos\left(c + dx\right)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x
)
```

output
```
B*Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{3}{2}}\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*
x + c))), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{3}{2}}\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*
x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{\sqrt{\cos(c + d\,x)}\,(a + b\,\cos(c + d\,x))^{3/2}}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2
)),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2\,b+\cos(dx+c)\,a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*b + cos
(c + d*x)*a),x)*b
```

3.440 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 226

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}$$

$$-\frac{2\sqrt{a+b}B\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

output
```
2*(a-b)*(a+b)^(1/2)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*(a+b)^(1/2)*B*cot(d*x+c)*EllipticF(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 6.74 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.94

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2B\left(-\left((a+b)\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+d}}\right.\right.}{}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^(3/2)),x]
```

output
```
(2*B*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]) + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a
 + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2011, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 2011$$

$$B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3280}$$

$$B\left(\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx - \int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3295}$$

$$B\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Elli}}{ad}\right.$$

$$\Big\downarrow \text{3473}$$

$$B\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d} - \frac{2\sqrt{a+b}\text{co}}{}\right.$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2
)),x]
```

output
```
B*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Maple [A] (verified)

Time = 28.04 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.68

method	result
default	$-\frac{2B\left(\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\,a\,\mathrm{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+\left(-\mathrm{co}\right.}{}$
parts	Expression too large to display

input
```
int((B*a+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-2*B/d*((-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
(a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b*Elliptic
E(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1
)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))-b*cos(d*x+
c)*sin(d*x+c)-a*sin(d*x+c))*(a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)/(b*cos
(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)/a
```

Fricas [F]

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 +
 a*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a+b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x
)
```

output
```
B*Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{Bb\cos{(dx+c)} + Ba}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c
)^(3/2)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{Bb\cos{(dx+c)} + Ba}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c
)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\,a + B\,b\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2
)),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\cos(dx+c)\,b+a}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^{3}\,b+\cos(dx+c)^{2}\,a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(cos(c + d*x)*b + a)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*b + cos
(c + d*x)**2*a),x)*b
```

3.441 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 72

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = -\frac{\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{d}$$

output
```
-cot(d*x+c)*EllipticE(1/5*(2+3*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2),
5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]])
,x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]])
, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3\cos(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3473}$$

$$-\frac{\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}E\left(\arcsin\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{d}$$

input
```
Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

output
```
-((Cot[c + d*x]*EllipticE[ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Co
s[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(64) = 128$.

Time = 13.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.25

method	result
default	$\frac{(20+30\cos(dx+c))\sin(dx+c)+\left(5\cos(dx+c)^2+10\cos(dx+c)+5\right)\sqrt{10}\,\sqrt{2}\,\mathrm{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\frac{\sqrt{5}}{5}\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{}}{10d\sqrt{2+3\cos(dx+c)}\,\sqrt{co}}$
parts	$-\frac{\left((-30\cos(dx+c)-20)\sin(dx+c)+\left(2+2\cos(dx+c)^2+4\cos(dx+c)\right)\sqrt{10}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{2+3\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\cot(dx+c\right.\right.}{10d\sqrt{\cos(dx}}$

input
```
int((cos(d*x+c)+1)/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/10/d*((20+30*cos(d*x+c))*sin(d*x+c)+(5*cos(d*x+c)^2+10*cos(d*x+c)+5)*10^
(1/2)*2^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),1/5*5^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+(-4*cos(d*x+c)^
2-8*cos(d*x+c)-4)*10^(1/2)*2^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),1/5*5^(
1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2))/(2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)+1}{\sqrt{3\,\cos\left(dx+c\right)+2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
integral(sqrt(3*cos(d*x + c) + 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3
*cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+dx\right)+1}{\sqrt{3\cos\left(c+dx\right)+2}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) + 2)*cos(c + d*x)**(3/2))
, x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)+1}{\sqrt{3\,\cos\left(dx+c\right)+2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\sqrt{3\cos(c+d\,x)+2}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) + 2)^(1/2)),x)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{3\cos(dx+c)+2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3+2\cos(dx+c)^2}dx + \int \frac{\sqrt{3\cos(dx+c)+2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2+2\cos(dx+c)}dx$$

input
```
int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(3*cos(c + d*x) + 2)*sqrt(cos(c + d*x)))/(3*cos(c + d*x)**3 + 2*c
os(c + d*x)**2),x) + int((sqrt(3*cos(c + d*x) + 2)*sqrt(cos(c + d*x)))/(3*
cos(c + d*x)**2 + 2*cos(c + d*x)),x)
```

3.442 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 70

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = -\frac{\sqrt{5}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

output
```
-5^(1/2)*cot(d*x+c)*EllipticE((-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/5
*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]]
),x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]]
), x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3\cos(c+dx)-2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{ 3473}$$

$$-\frac{\sqrt{5}\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}E\left(\arcsin\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

input
```
Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

output
```
-((Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Co
s[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(61) = 122$.

Time = 11.94 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.09

method	result
default	$-\frac{(-2+3\cos(dx+c))\sin(dx+c)+\left(4\cos(dx+c)^2+8\cos(dx+c)+4\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)\sqrt{\frac{-2+3\cos}{\cos(dx}}}{d\sqrt{-2+3\cos(dx+c)}\ \sqrt{\cos(dx+c}}$
parts	$-\frac{\left((-2+3\cos(dx+c))\sin(dx+c)+\left(2+2\cos(dx+c)^2+4\cos(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \sqrt{\frac{-2+3\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}\left(\cot(dx+c)-\csc(dx\right.\right.}{d\sqrt{\cos(dx+c)}\ \left(3\cos\right.}$

input
```
int((cos(d*x+c)+1)/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-1/d*((-2+3*cos(d*x+c))*sin(d*x+c)+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))*((-2+3
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E(cot(d*x+c)-csc(d*x+c),5^(1/2)))/(-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
/(cos(d*x+c)+1)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

output
```
integral(sqrt(3*cos(d*x + c) - 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3
*cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\cos(c+dx)+1}{\sqrt{3\cos(c+dx)-2}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2))
, x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{3\cos(c+d\,x)-2}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) - 2)^(1/2)),x)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) - 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{3\cos(dx+c)-2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3-2\cos(dx+c)^2}dx + \int \frac{\sqrt{3\cos(dx+c)-2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2-2\cos(dx+c)}dx$$

input
```
int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(3*cos(c + d*x) - 2)*sqrt(cos(c + d*x)))/(3*cos(c + d*x)**3 - 2*c
os(c + d*x)**2),x) + int((sqrt(3*cos(c + d*x) - 2)*sqrt(cos(c + d*x)))/(3*
cos(c + d*x)**2 - 2*cos(c + d*x)),x)
```

3.443 $\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 93

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\text{se}}}{d}$$

output
```
5^(1/2)*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((2-3*cos
(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+s
ec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3042, 3474, 3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3474}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\cos(c+dx)+1}{\sqrt{2-3\cos(c+dx)}(-\cos(c+dx))^{3/2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}E\left(\arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

input
```
Int[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

output
```
(Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[Arc
Sin[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c +
d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3474
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]   Int[(A + B*Sin[e + f*x])/(((-b)*
Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
 f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(80) = 160$.

Time = 13.57 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.42

method	result
default	$\frac{\left(\left(-2+3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(4\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+4\right)\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\ \operatorname{EllipticF}\left(\cot\left(dx+c\right)-\csc\left(dx+c\right),\sqrt{5}\right)\sqrt{\frac{-2+3\cos(}{\cos(dx+}}}{d\sqrt{\cos\left(dx+c\right)}\left(3\cos(d\right.}$
parts	$\frac{\left(\left(-2+3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(2+2\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)\right)\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\ \sqrt{\frac{-2+3\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\ \operatorname{EllipticF}\left(\cot\left(dx+c\right)-\csc\left(dx+\right.\right.}{d\sqrt{\cos\left(dx+c\right)}\left(3\cos(d\right.}$

input
```
int((cos(d*x+c)+1)/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*((-2+3*cos(d*x+c))*sin(d*x+c)+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))*((-2+3*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
(cot(d*x+c)-csc(d*x+c),5^(1/2)))*(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(
3*cos(d*x+c)^2+cos(d*x+c)-2)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/
(3*cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(c+dx)+1}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((1+cos(d*x+c))/(2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(2 - 3*cos(c + d*x))*cos(c + d*x)**(3/2))
, x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)
), x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{2-3\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2 - 3*cos(c + d*x))^(1/2)),x)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2 - 3*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\left(\int \frac{\sqrt{-3\cos(dx+c)+2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3-2\cos(dx+c)^2}dx\right)$$
$$-\left(\int \frac{\sqrt{-3\cos(dx+c)+2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2-2\cos(dx+c)}dx\right)$$

input
```
int((1+cos(d*x+c))/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

output
```
 - (int((sqrt( - 3*cos(c + d*x) + 2)*sqrt(cos(c + d*x)))/(3*cos(c + d*x)**
3 - 2*cos(c + d*x)**2),x) + int((sqrt( - 3*cos(c + d*x) + 2)*sqrt(cos(c +
d*x)))/(3*cos(c + d*x)**2 - 2*cos(c + d*x)),x))
```

3.444 $\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{d}$$

output
```
(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE(1/5*(-2-3*cos(d*
x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^(1/2),5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-
sec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)
),x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)
), x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3042, 3474, 3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\sqrt{-3\cos(c+dx)-2}\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3474}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\cos(c+dx)+1}{\sqrt{-3\cos(c+dx)-2}(-\cos(c+dx))^{3/2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3473}$$

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}E\left(\arcsin\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{d}$$

input
```
Int[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

output
```
(Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[-2 - 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3474
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]   Int[(A + B*Sin[e + f*x])/(((-b)*
Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
 f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Maple [A] (verified)

Time = 13.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.68

method	result
default	$\frac{\left(\left(-30\cos\left(dx+c\right)-20\right)\sin\left(dx+c\right)+\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{10}\,\sqrt{5}\,\sqrt{2}\ \operatorname{EllipticE}\left(\frac{\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)\sqrt{5}}{5},\sqrt{5}\right)\sqrt{\frac{\cos(dx+c}{\cos\left(dx+c\right)}}}{10d\sqrt{\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+5\cos\left(dx+c\right)+2\right)}$
parts	$\frac{\left(\left(-30\cos\left(dx+c\right)-20\right)\sin\left(dx+c\right)+\left(2+2\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)\right)\sqrt{10}\,\sqrt{5}\,\sqrt{2}\ \operatorname{EllipticF}\left(\frac{\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)\sqrt{5}}{5},\sqrt{5}\right)\sqrt{\frac{\cos(dx}{\cos(dx+}}}{10d\sqrt{\cos(d}}$

input
```
int((cos(d*x+c)+1)/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/10/d*((-30*cos(d*x+c)-20)*sin(d*x+c)+(cos(d*x+c)^2+2*cos(d*x+c)+1)*10^(1
/2)*5^(1/2)*2^(1/2)*EllipticE(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
)*(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(3*cos(d*x+c)^2+5*cos(d*x+c)+2)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

output
```
integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/
(3*cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(c+dx)+1}{\sqrt{-3\cos(c+dx)-2}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(-3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2)
), x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)
), x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{-3\cos(c+d\,x)-2}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x
)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 3*cos(c + d*x) - 2)^(1/2)),
x)
```

Reduce [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\left(\int \frac{\sqrt{-3\cos(dx+c)-2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3+2\cos(dx+c)^2}dx\right)$$
$$-\left(\int \frac{\sqrt{-3\cos(dx+c)-2}\,\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2+2\cos(dx+c)}dx\right)$$

input
```
int((1+cos(d*x+c))/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

output
```
 - (int((sqrt( - 3*cos(c + d*x) - 2)*sqrt(cos(c + d*x)))/(3*cos(c + d*x)**
3 + 2*cos(c + d*x)**2),x) + int((sqrt( - 3*cos(c + d*x) - 2)*sqrt(cos(c +
d*x)))/(3*cos(c + d*x)**2 + 2*cos(c + d*x)),x))
```

3.445 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 72

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3d}$$

output
```
2/3*cot(d*x+c)*EllipticE(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/
2),I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]])
,x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]])
, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{ 3473}$$

$$\frac{2\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

input
```
Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

output
```
(2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Co
s[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(65) = 130$.

Time = 13.90 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.28

method	result
default	$\frac{(30+20\cos(dx+c))\sin(dx+c)+\left(-6\cos(dx+c)^2-12\cos(dx+c)-6\right)\sqrt{2}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\frac{i\sqrt{5}}{5}\right.}{15d\sqrt{3+2\cos(dx+c)}\,\sqrt{c}}$
parts	$-\frac{\left((-20\cos(dx+c)-30)\sin(dx+c)+\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sqrt{2}\,\sqrt{10}\ \text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\frac{i\sqrt{5}}{5}\right)\sqrt{\frac{\cos(dx+c}{\cos(dx+c)-}}\right.}{15d\sqrt{\cos(d}}$

input
```
int((cos(d*x+c)+1)/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/15/d*((30+20*cos(d*x+c))*sin(d*x+c)+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*2^
(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c
)-csc(d*x+c),1/5*I*5^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(5*cos(d*x+c
)^2+10*cos(d*x+c)+5)*2^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE(cot(d*x+c)-csc(d*x+c),1/5*I*5^(1/2))*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))/(3+2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
integral(sqrt(2*cos(d*x + c) + 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2
*cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\cos(c+dx)+1}{\sqrt{2\cos(c+dx)+3}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) + 3)*cos(c + d*x)**(3/2))
, x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{2\,\cos(c+d\,x)+3}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) + 3)^(1/2)),x)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{2\cos(dx+c)+3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^{3}+3\cos(dx+c)^{2}}dx+\int \frac{\sqrt{2\cos(dx+c)+3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^{2}+3\cos(dx+c)}dx$$

input
```
int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(2*cos(c + d*x) + 3)*sqrt(cos(c + d*x)))/(2*cos(c + d*x)**3 + 3*c
os(c + d*x)**2),x) + int((sqrt(2*cos(c + d*x) + 3)*sqrt(cos(c + d*x)))/(2*
cos(c + d*x)**2 + 3*cos(c + d*x)),x)
```

3.446 $\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 74

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{5}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3d}$$

output
```
2/3*5^(1/2)*cot(d*x+c)*EllipticE((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1
/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3473}$$

$$\frac{2\sqrt{5}\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

input
```
Int[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

output
```
(2*Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos
[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(64) = 128$.

Time = 12.61 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.27

method	result
default	$-\frac{\left((6-4\cos(dx+c))\sin(dx+c)+\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{2}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{\cos(dx+c)+1}}\ \text{EllipticE}\left(\cot(dx+c)-\csc(dx+c),i\sqrt{5}\right)\cdots\right.}{3d\sqrt{\cos(dx\cdots}}$
parts	$\frac{\left((-6+4\cos(dx+c))\sin(dx+c)+\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sqrt{2}\ \text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),i\sqrt{5}\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{-\frac{2(\cdots}{}}\right.}{3d\sqrt{\cos(dx+c\cdots}}$

input
```
int((cos(d*x+c)+1)/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/3/d*((6-4*cos(d*x+c))*sin(d*x+c)+(cos(d*x+c)^2+2*cos(d*x+c)+1)*2^(1/2)*
(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c
),I*5^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-6*cos(d*x+c)^2-12*cos(d*x
+c)-6)*2^(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d
*x+c)-csc(d*x+c),I*5^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(3-2*cos(d*
x+c))^(1/2)/cos(d*x+c)^(1/2)/(2*cos(d*x+c)^2-cos(d*x+c)-3)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/
(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(c+dx)+1}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((1+cos(d*x+c))/(3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(3 - 2*cos(c + d*x))*cos(c + d*x)**(3/2))
, x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)
), x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{3-2\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3 - 2*cos(c + d*x))^(1/2)),x)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx &= -\left(\int \frac{\sqrt{-2\cos(dx+c)+3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^{3}-3\cos(dx+c)^{2}}dx\right) \\ &\quad -\left(\int \frac{\sqrt{-2\cos(dx+c)+3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^{2}-3\cos(dx+c)}dx\right)\end{aligned}$$

input
```
int((1+cos(d*x+c))/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

output
```
 - (int((sqrt( - 2*cos(c + d*x) + 3)*sqrt(cos(c + d*x)))/(2*cos(c + d*x)**
3 - 3*cos(c + d*x)**2),x) + int((sqrt( - 2*cos(c + d*x) + 3)*sqrt(cos(c +
d*x)))/(2*cos(c + d*x)**2 - 3*cos(c + d*x)),x))
```

3.447 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1}}{3d}$$

output
```
-2/3*5^(1/2)*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((-3
+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/
2)*(1+sec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]]
),x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]]
), x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3042, 3474, 3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2\cos(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{3474}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\cos(c+dx)+1}{(-\cos(c+dx))^{3/2}\sqrt{2\cos(c+dx)-3}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$-\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\right.}{3d}$$

input
```
Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

output
```
(-2*Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[
ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[
c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3474
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]   Int[(A + B*Sin[e + f*x])/(((-b)*
Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
 f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Maple [A] (verified)

Time = 14.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.57

method	result
default	$\frac{(6-4\cos(dx+c))\sin(dx+c)+i\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{5}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{\cos(dx+c)+1}}\ \text{EllipticE}\left(i(\csc(dx+c\right.}{3d\sqrt{-3+2\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,(\cos(dx+c)+1)}$
parts	$-\frac{\left((20\cos(dx+c)-30)\sin(dx+c)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sqrt{5}\,\sqrt{2}\ \text{EllipticF}\left(i(\cot(dx+c)-\csc(dx+c))\sqrt{5},\frac{i\sqrt{5}}{5}\right)\sqrt{\frac{\cos(d}{\cos(dx}}\right.}{}$

input
```
int((cos(d*x+c)+1)/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/3/d*((6-4*cos(d*x+c))*sin(d*x+c)+I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*5^(1/2
)*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5*I*5^(1/2)))/
(-3+2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

output
```
integral(sqrt(2*cos(d*x + c) - 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2
*cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\cos(c+dx)+1}{\sqrt{2\cos(c+dx)-3}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2))
, x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{2\cos(c+d\,x)-3}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{2\cos(dx+c)-3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^3-3\cos(dx+c)^2}dx + \int \frac{\sqrt{2\cos(dx+c)-3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^2-3\cos(dx+c)}dx$$

input
```
int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

output
```
int((sqrt(2*cos(c + d*x) - 3)*sqrt(cos(c + d*x)))/(2*cos(c + d*x)**3 - 3*c
os(c + d*x)**2),x) + int((sqrt(2*cos(c + d*x) - 3)*sqrt(cos(c + d*x)))/(2*
cos(c + d*x)**2 - 3*cos(c + d*x)),x)
```

3.448 $\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 96

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+}}{3d}$$

output
```
-2/3*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE(1/5*(-3-2*c
os(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^(1/2),I*5^(1/2))*(1-sec(d*x+c))^(1/
2)*(1+sec(d*x+c))^(1/2)/d
```

Mathematica [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)
),x]
```

output
```
Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)
), x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3042, 3474, 3042, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+1}{\sqrt{-2\cos(c+dx)-3}\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3474}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\cos(c+dx)+1}{\sqrt{-2\cos(c+dx)-3}(-\cos(c+dx))^{3/2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sqrt{-\cos(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3473}$$

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}E\left(\arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-\right.}{3d}$$

input
```
Int[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

output

```
(-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[
c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3474

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt
[(-b)*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]   Int[(A + B*Sin[e + f*x])/(((-b)*
Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
 f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Maple [A] (verified)

Time = 14.02 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.73

method	result
default	$\frac{\left(\left(-20\cos(dx+c)-30\right)\sin(dx+c)+i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{5}\,\sqrt{10}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{EllipticE}\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5}\right.\right.}{15d\sqrt{\cos(dx+c)}\,\left(2\cos(dx+c)^2+5\cos(dx+c)+3\right)}$
parts	$-\frac{\left(\left(30+20\cos(dx+c)\right)\sin(dx+c)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sqrt{5}\,\sqrt{10}\,\sqrt{2}\ \operatorname{EllipticF}\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)\sqrt{\frac{\cos}{\cos(d}}\right.}{15d\sqrt{}}$

input
```
int((cos(d*x+c)+1)/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/15/d*((-20*cos(d*x+c)-30)*sin(d*x+c)+I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*5^(
1/2)*10^(1/2)*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(1/5*I*(c
sc(d*x+c)-cot(d*x+c))*5^(1/2),I*5^(1/2))*((3+2*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2))*(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(2*cos(d*x+c)^2+5*cos(d*x
+c)+3)
```

Fricas [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)+1}{\sqrt{-2\,\cos\left(dx+c\right)-3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

output
```
integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/
(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos\left(c+dx\right)+1}{\sqrt{-2\cos\left(c+dx\right)-3}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((cos(c + d*x) + 1)/(sqrt(-2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2)
), x)
```

Maxima [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)
), x)
```

Giac [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+cos(d*x+c))/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

output
```
integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{-2\cos(c+d\,x)-3}}\,dx$$

input
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x
)
```

output
```
int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 2*cos(c + d*x) - 3)^(1/2)),
x)
```

Reduce [F]

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\left(\int \frac{\sqrt{-2\cos(dx+c)-3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^{3}+3\cos(dx+c)^{2}}dx\right) - \left(\int \frac{\sqrt{-2\cos(dx+c)-3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^{2}+3\cos(dx+c)}dx\right)$$

input
```
int((1+cos(d*x+c))/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

output
```
- (int((sqrt( - 2*cos(c + d*x) - 3)*sqrt(cos(c + d*x)))/(2*cos(c + d*x)**
3 + 3*cos(c + d*x)**2),x) + int((sqrt( - 2*cos(c + d*x) - 3)*sqrt(cos(c +
d*x)))/(2*cos(c + d*x)**2 + 3*cos(c + d*x)),x))
```

3.449 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx = \text{Int}((c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx)),x)$$

output
```
Defer(Int)((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 26.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$

input
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x
]
```

output
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),
x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+B\cos(e+fx))(c\cos(e+fx))^m(a+b\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3486}$$

$$\int (A+B\cos(e+fx))(c\cos(e+fx))^m(a+b\cos(e+fx))^ndx$$

input
```
Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \left(c \cos \left(f x +e \right)\right)^{m} \left(a +\cos \left(f x +e \right) b \right)^{n} \left(A +B \cos \left(f x +e \right)\right)d x$$

input
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^n*(A+B*cos(f*x+e)),x)
```

output
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^n*(A+B*cos(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx = \int \left(B \cos \left(f x + e\right) + A\right)\left(b \cos \left(f x + e\right) + a\right)^{n}\left(c \cos \left(f x + e\right)\right)^{m} \,d x$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x
)
```

Sympy [N/A]

Not integrable

Time = 121.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx = \int (c\cos{(e+fx)})^m\,(A+B\cos{(e+fx)})\,(a+b\cos{(e+fx)})^n\,dx$$

input
```
integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**n*(A+B*cos(f*x+e)),x)
```

output
```
Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**n,
 x)
```

Maxima [N/A]

Not integrable

Time = 3.75 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx = \int (B\cos{(fx+e)}+A)(b\cos{(fx+e)}+a)^n(c\cos{(fx+e)})^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m,
x)
```

Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$
$$=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^n(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m,
x)
```

Mupad [N/A]

Not integrable

Time = 25.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$
$$=\int (c\,\cos(e+f\,x))^m\,(A+B\,\cos(e+f\,x))\,(a+b\,\cos(e+f\,x))^n\,dx$$

input
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n,x)
```

output
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$
$$= c^m\left(\left(\int (\cos(fx+e)\,b+a)^n\cos(fx+e)^m\cos(fx+e)\,dx\right)b + \left(\int (\cos(fx+e)\,b+a)^n\cos(fx+e)^m\,dx\right)a\right)$$

input
```
int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

output
```
c**m*(int((cos(e + f*x)*b + a)**n*cos(e + f*x)**m*cos(e + f*x),x)*b + int(
(cos(e + f*x)*b + a)**n*cos(e + f*x)**m,x)*a)
```

3.450 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 595

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx$$
$$= \frac{b(Ab^3(15+8m+m^2)+4ab^2B(15+8m+m^2)+2a^3B(28+10m+m^2)+a^2Ab(110+47m+5m^2))\,(c}{cf(2+m)(4+m)(5+m)}$$
$$+\frac{b^2(b^2B(4+m)^2+2aAb(5+m)^2+a^2B(36+11m+m^2))\cos(e+fx)(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(3+m)(4+m)(5+m)}$$
$$+\frac{b(Ab(5+m)+aB(8+m))(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^2\sin(e+fx)}{cf(4+m)(5+m)}$$
$$+\frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^3\sin(e+fx)}{cf(5+m)}$$
$$-\frac{(Ab^4(3+4m+m^2)+4ab^3B(3+4m+m^2)+6a^2Ab^2(4+5m+m^2)+4a^3bB(4+5m+m^2)+a^4A(}{cf(1+m)(2+m)(4+}$$
$$-\frac{(b^4B(8+6m+m^2)+4aAb^3(10+7m+m^2)+6a^2b^2B(10+7m+m^2)+4a^3Ab(15+8m+m^2)+a}{c^2f(2+m)(3+m)(5}$$

output
```
b*(A*b^3*(m^2+8*m+15)+4*a*b^2*B*(m^2+8*m+15)+2*a^3*B*(m^2+10*m+28)+a^2*A*b
*(5*m^2+47*m+110))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(2+m)/(4+m)/(5+m)+b
^2*(b^2*B*(4+m)^2+2*a*A*b*(5+m)^2+a^2*B*(m^2+11*m+36))*cos(f*x+e)*(c*cos(f
*x+e))^(1+m)*sin(f*x+e)/c/f/(3+m)/(4+m)/(5+m)+b*(A*b*(5+m)+a*B*(8+m))*(c*c
os(f*x+e))^(1+m)*(a+b*cos(f*x+e))^2*sin(f*x+e)/c/f/(4+m)/(5+m)+b*B*(c*cos(
f*x+e))^(1+m)*(a+b*cos(f*x+e))^3*sin(f*x+e)/c/f/(5+m)-(A*b^4*(m^2+4*m+3)+4
*a*b^3*B*(m^2+4*m+3)+6*a^2*A*b^2*(m^2+5*m+4)+4*a^3*b*B*(m^2+5*m+4)+a^4*A*(
m^2+6*m+8))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],co
s(f*x+e)^2)*sin(f*x+e)/c/f/(1+m)/(2+m)/(4+m)/(sin(f*x+e)^2)^(1/2)-(b^4*B*(
m^2+6*m+8)+4*a*A*b^3*(m^2+7*m+10)+6*a^2*b^2*B*(m^2+7*m+10)+4*a^3*A*b*(m^2+
8*m+15)+a^4*B*(m^2+8*m+15))*(c*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m],
[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(3+m)/(5+m)/(sin(f*x+e)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 6.24 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.81

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx =$$

$$-\frac{a^4A(c\cos(e+fx))^m\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt{\sin^2(e+fx)}}{f(1+m)}$$
$$-\frac{a^3(4Ab+aB)\cos(e+fx)(c\cos(e+fx))^m\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx\right.}{f(2+m)}$$
$$-\frac{2a^2b(3Ab+2aB)\cos^2(e+fx)(c\cos(e+fx))^m\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},\cos^2(e\right.}{f(3+m)}$$
$$-\frac{2ab^2(2Ab+3aB)\cos^3(e+fx)(c\cos(e+fx))^m\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},\cos^2(e\right.}{f(4+m)}$$
$$-\frac{b^3(Ab+4aB)\cos^4(e+fx)(c\cos(e+fx))^m\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},\cos^2(e+f\right.}{f(5+m)}$$
$$-\frac{b^4B\cos^5(e+fx)(c\cos(e+fx))^m\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6+m}{2},\frac{8+m}{2},\cos^2(e+fx)\right)\sqrt{\sin^2}}{f(6+m)}$$

input
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x
]
```

output
```
-((a^4*A*(c*Cos[e + f*x])^m*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*(1 + m))) - (a^3*(4*A
*b + a*B)*Cos[e + f*x]*(c*Cos[e + f*x])^m*Cot[e + f*x]*Hypergeometric2F1[1
/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*(2 + m)
) - (2*a^2*b*(3*A*b + 2*a*B)*Cos[e + f*x]^2*(c*Cos[e + f*x])^m*Cot[e + f*x
]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e
+ f*x]^2])/(f*(3 + m)) - (2*a*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^3*(c*Cos[e
+ f*x])^m*Cot[e + f*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e
+ f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*(4 + m)) - (b^3*(A*b + 4*a*B)*Cos[e + f
*x]^4*(c*Cos[e + f*x])^m*Cot[e + f*x]*Hypergeometric2F1[1/2, (5 + m)/2, (7
 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*(5 + m)) - (b^4*B*Cos[e
+ f*x]^5*(c*Cos[e + f*x])^m*Cot[e + f*x]*Hypergeometric2F1[1/2, (6 + m)/2,
 (8 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*(6 + m))
```

Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3469, 3042, 3528, 3042, 3512, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2\left(bc(Ab(m+5)+aB(m+8))\cos^2(e+fx)+c(B(m+4)b^2+a(2Ab+a}{c(m+5)}$$
$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2 \left(bc(Ab(m+5)+aB(m+8))\sin\left(e+fx+\frac{\pi}{2}\right)^2+c(B(m+4}{c(m+5)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\int(c\cos(e+fx))^m(a+b\cos(e+fx))\left(b\left(B\left(m^2+11m+36\right)a^2+2Ab(m+5)^2a+b^2B(m+4)^2\right)\cos^2(e+fx)c^2+a(a(m+4)(bB(m+1)+aA(m+5))+b(m}{c(m+4)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3042}$$

$$\int\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(b\left(B\left(m^2+11m+36\right)a^2+2Ab(m+5)^2a+b^2B(m+4)^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2c^2+a(a(m+4)(bB(m+1)+aA(r$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3512}$$

$$\int(c\cos(e+fx))^m\left(b(m+3)\left(2B\left(m^2+10m+28\right)a^3+Ab\left(5m^2+47m+110\right)a^2+4b^2B\left(m^2+8m+15\right)a+Ab^3\left(m^2+8m+15\right)\right)\cos^2(e+fx)c^3+a^2(m+3)(a(m+4)(bB(m$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3042}$$

$$\int\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(b(m+3)\left(2B\left(m^2+10m+28\right)a^3+Ab\left(5m^2+47m+110\right)a^2+4b^2B\left(m^2+8m+15\right)a+Ab^3\left(m^2+8m+15\right)\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2c^3+a^2(m+3)(a(m+$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3502}$$

$$\int (c\cos(e+fx))^m \left((m+3)\left(A\left(m^3+11m^2+38m+40\right)a^4+4bB\left(m^3+10m^2+29m+20\right)a^3+6Ab^2\left(m^3+10m^2+29m+20\right)a^2+4b^3B\left(m^3+9m^2+23m+15\right)a+Ab^4\left(m^3+\right.\right.\right.$$

$$c($$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left((m+3)\left(A\left(m^3+11m^2+38m+40\right)a^4+4bB\left(m^3+10m^2+29m+20\right)a^3+6Ab^2\left(m^3+10m^2+29m+20\right)a^2+4b^3B\left(m^3+9m^2+23m+15\right)a+Ab^4\right.\right.$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3227}$$

$$c^4(m+3)\left(a^4A\left(m^3+11m^2+38m+40\right)+4a^3bB\left(m^3+10m^2+29m+20\right)+6a^2Ab^2\left(m^3+10m^2+29m+20\right)+4ab^3B\left(m^3+9m^2+23m+15\right)+Ab^4\left(m^3+9m^2+23m+15\right)\right.$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3042}$$

$$c^4(m+3)\left(a^4A\left(m^3+11m^2+38m+40\right)+4a^3bB\left(m^3+10m^2+29m+20\right)+6a^2Ab^2\left(m^3+10m^2+29m+20\right)+4ab^3B\left(m^3+9m^2+23m+15\right)+Ab^4\left(m^3+9m^2+23m+15\right)\right.$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{b^2c\sin(e+fx)\cos(e+fx)\left(a^2B\left(m^2+11m+36\right)+2aAb(m+5)^2+b^2B(m+4)^2\right)(c\cos(e+fx))^{m+1}}{f(m+3)}+\frac{bc^2(m+3)\sin(e+fx)\left(2a^3B\left(m^2+10m+28\right)+a^2Ab\left(5m^2+47m\right.\right.}{}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^3(c\cos(e+fx))^{m+1}}{cf(m+5)}$$

input
```
Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x]
```

output
```
(b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^3*Sin[e + f*x])/(c*f*(5
 + m)) + ((b*(A*b*(5 + m) + a*B*(8 + m))*(c*Cos[e + f*x])^(1 + m)*(a + b*C
os[e + f*x])^2*Sin[e + f*x])/(f*(4 + m)) + ((b^2*c*(b^2*B*(4 + m)^2 + 2*a*
A*b*(5 + m)^2 + a^2*B*(36 + 11*m + m^2))*Cos[e + f*x]*(c*Cos[e + f*x])^(1
+ m)*Sin[e + f*x])/(f*(3 + m)) + ((b*c^2*(3 + m)*(A*b^3*(15 + 8*m + m^2) +
 4*a*b^2*B*(15 + 8*m + m^2) + 2*a^3*B*(28 + 10*m + m^2) + a^2*A*b*(110 + 4
7*m + 5*m^2))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(f*(2 + m)) + (-((c^3
*(3 + m)*(A*b^4*(15 + 23*m + 9*m^2 + m^3) + 4*a*b^3*B*(15 + 23*m + 9*m^2 +
 m^3) + 6*a^2*A*b^2*(20 + 29*m + 10*m^2 + m^3) + 4*a^3*b*B*(20 + 29*m + 10
*m^2 + m^3) + a^4*A*(40 + 38*m + 11*m^2 + m^3))*(c*Cos[e + f*x])^(1 + m)*H
ypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/
(f*(1 + m)*Sqrt[Sin[e + f*x]^2])) - (c^2*(4 + m)*(b^4*B*(8 + 6*m + m^2) +
4*a*A*b^3*(10 + 7*m + m^2) + 6*a^2*b^2*B*(10 + 7*m + m^2) + 4*a^3*A*b*(15
+ 8*m + m^2) + a^4*B*(15 + 8*m + m^2))*(c*Cos[e + f*x])^(2 + m)*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*Sqrt[S
in[e + f*x]^2]))/(c*(2 + m)))/(c*(3 + m)))/(c*(4 + m)))/(c*(5 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [F]

$$\int (c\cos(fx+e))^m (a+\cos(fx+e)b)^4 (A+B\cos(fx+e))\,dx$$

input
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^4*(A+B*cos(f*x+e)),x)
```

output
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^4*(A+B*cos(f*x+e)),x)
```

Fricas [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx$$
$$=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^4(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^
4 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^
2)*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*cos(f*x + e))^m,
x)
```

Sympy [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx=\text{Timed out}$$

input
```
integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**4*(A+B*cos(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^4(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m,
x)
```

Giac [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^4(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx = \int (c\,\cos(e+f\,x))^m\,(A+B\,\cos(e+f\,x))\,(a+b\,\cos(e+f\,x))^4\,dx$$

input
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4,x)
```

output
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4, x)
```

Reduce [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx = c^m\left(\left(\int \cos(fx+e)^m\,dx\right)a^5+5\left(\int \cos(fx+e)^m\cos(fx+e)\,dx\right)a^4b + \left(\int \cos(fx+e)^m\cos(fx+e)^5\,dx\right)b^5+5\left(\int \cos(fx+e)^m\cos(fx+e)^4\,dx\right)a\,b^4 + 10\left(\int \cos(fx+e)^m\cos(fx+e)^3\,dx\right)a^2b^3 + 10\left(\int \cos(fx+e)^m\cos(fx+e)^2\,dx\right)a^3b^2\right)$$

input
```
int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x)
```

output
```
c**m*(int(cos(e + f*x)**m,x)*a**5 + 5*int(cos(e + f*x)**m*cos(e + f*x),x)*
a**4*b + int(cos(e + f*x)**m*cos(e + f*x)**5,x)*b**5 + 5*int(cos(e + f*x)*
*m*cos(e + f*x)**4,x)*a*b**4 + 10*int(cos(e + f*x)**m*cos(e + f*x)**3,x)*a
**2*b**3 + 10*int(cos(e + f*x)**m*cos(e + f*x)**2,x)*a**3*b**2)
```

3.451 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 406

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$$
$$= \frac{b(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m))\,(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)(4+m)}$$
$$+ \frac{b^2(Ab(4+m)+aB(6+m))\cos(e+fx)(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(3+m)(4+m)}$$
$$+ \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^2\sin(e+fx)}{cf(4+m)}$$
$$- \frac{(a^2(2+m)(bB(1+m)+aA(4+m))+b(1+m)\,(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m)))\,(c\,\mathrm{co}}{cf(1+m)(2+m)(4+m)\sqrt{\sin^2(}}$$
$$- \frac{(Ab^3(2+m)+3ab^2B(2+m)+3a^2Ab(3+m)+a^3B(3+m))\,(c\cos(e+fx))^{2+m}\,\mathrm{Hypergeometric2F}}{c^2f(2+m)(3+m)\sqrt{\sin^2(e+fx)}}$$

output
```
b*(b^2*B*(3+m)+3*a*A*b*(4+m)+2*a^2*B*(5+m))*(c*cos(f*x+e))^(1+m)*sin(f*x+e
)/c/f/(2+m)/(4+m)+b^2*(A*b*(4+m)+a*B*(6+m))*cos(f*x+e)*(c*cos(f*x+e))^(1+m
)*sin(f*x+e)/c/f/(3+m)/(4+m)+b*B*(c*cos(f*x+e))^(1+m)*(a+b*cos(f*x+e))^2*s
in(f*x+e)/c/f/(4+m)-(a^2*(2+m)*(b*B*(1+m)+a*A*(4+m))+b*(1+m)*(b^2*B*(3+m)+
3*a*A*b*(4+m)+2*a^2*B*(5+m)))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2
*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c/f/(1+m)/(2+m)/(4+m)/(sin(f*x+e)
^2)^(1/2)-(A*b^3*(2+m)+3*a*b^2*B*(2+m)+3*a^2*A*b*(3+m)+a^3*B*(3+m))*(c*cos
(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)
/c^2/f/(2+m)/(3+m)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.65

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$$
$$= \frac{(c\cos(e+fx))^m\cot(e+fx)\left(-\frac{a^3A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)}{1+m}+\cos(e+fx)\left(-\frac{a^2(3Ab+aB)\,\text{Hyper}}{}\right.\right.}{}$$

input
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x
]
```

output
```
((c*Cos[e + f*x])^m*Cot[e + f*x]*(-((a^3*A*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a^2*(3*A*b + a*
B)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m))
+ b*Cos[e + f*x]*((-3*a*(A*b + a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 +
 m)/2, Cos[e + f*x]^2])/(3 + m) + b*Cos[e + f*x]*(-(((A*b + 3*a*B)*Hyperge
ometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m)) - (b*B*Cos
[e + f*x]*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Cos[e + f*x]^2])/(5
 + m)))))*Sqrt[Sin[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3469, 3042, 3512, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{\int (c\cos(e+fx))^m(a+b\cos(e+fx))\left(bc(Ab(m+4)+aB(m+6))\cos^2(e+fx)+c\left(B(m+3)b^2+a(2Ab+aB\cdots\right.\right.}{c(m+4)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^2(c\cos(e+fx))^{m+1}}{cf(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(bc(Ab(m+4)+aB(m+6))\sin\left(e+fx+\frac{\pi}{2}\right)^2+c(B(m+3)\cdots\right.}{c(m+4)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^2(c\cos(e+fx))^{m+1}}{cf(m+4)}$$

$$\Big\downarrow \text{3512}$$

$$\frac{\frac{\int (c\cos(e+fx))^m\left(b(m+3)\left(2B(m+5)a^2+3Ab(m+4)a+b^2B(m+3)\right)\cos^2(e+fx)c^2+a^2(m+3)(bB(m+1)+aA(m+4))c^2+(m+4)\left(B(m+3)a^3+3A\cdots\right.\right.}{c(m+3)}}{c(m+4)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^2(c\cos(e+fx))^{m+1}}{cf(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c \sin \left(e+f x+\frac{\pi}{2}\right)\right)^m \left(b(m+3)\left(2 B(m+5) a^2+3 A b(m+4) a+b^2 B(m+3)\right) \sin \left(e+f x+\frac{\pi}{2}\right)^2 c^2+a^2(m+3)(b B(m+1)+a A(m+4)) c^2+(m+4)(B(m+\ldots$$

$$c(m+3)$$

$$\frac{b B \sin (e+f x)(a+b \cos (e+f x))^2(c \cos (e+f x))^{m+1}}{c f(m+4)}$$

$$\Big\downarrow \ 3502$$

$$\int (c \cos (e+f x))^m \left((m+3)\left((m+2)(b B(m+1)+a A(m+4)) a^2+b(m+1)\left(2 B(m+5) a^2+3 A b(m+4) a+b^2 B(m+3)\right)\right) c^3+(m+2)(m+4)\left(B(m+3) a^3+3 A b(m+3) a^2+3 b^2 \ldots\right.$$

$$c(m+2)$$

$$c(m+3)$$

$$\frac{b B \sin (e+f x)(a+b \cos (e+f x))^2(c \cos (e+f x))^{m+1}}{c f(m+4)}$$

$$\Big\downarrow \ 3042$$

$$\int \left(c \sin \left(e+f x+\frac{\pi}{2}\right)\right)^m \left((m+3)\left((m+2)(b B(m+1)+a A(m+4)) a^2+b(m+1)\left(2 B(m+5) a^2+3 A b(m+4) a+b^2 B(m+3)\right)\right) c^3+(m+2)(m+4)\left(B(m+3) a^3+3 A b(m+3) a^2 \ldots\right.$$

$$c(m+2)$$

$$c(m+3)$$

$$\frac{b B \sin (e+f x)(a+b \cos (e+f x))^2(c \cos (e+f x))^{m+1}}{c f(m+4)}$$

$$\Big\downarrow \ 3227$$

$$c^3(m+3)\left(b(m+1)\left(2 a^2 B(m+5)+3 a A b(m+4)+b^2 B(m+3)\right)+a^2(m+2)(a A(m+4)+b B(m+1))\right) \int (c \cos (e+f x))^m dx+c^2(m+2)(m+4)\left(a^3 B(m+3)+3 a^2 A b(m+3)+\ldots\right.$$

$$c(m+2)$$

$$c(m+3)$$

$$\frac{b B \sin (e+f x)(a+b \cos (e+f x))^2(c \cos (e+f x))^{m+1}}{c f(m+4)}$$

$$\Big\downarrow \ 3042$$

$$c^3(m+3)\left(b(m+1)\left(2 a^2 B(m+5)+3 a A b(m+4)+b^2 B(m+3)\right)+a^2(m+2)(a A(m+4)+b B(m+1))\right) \int \left(c \sin \left(e+f x+\frac{\pi}{2}\right)\right)^m dx+c^2(m+2)(m+4)\left(a^3 B(m+3)+3 a^2 A b(m \ldots\right.$$

$$c(m+2)$$

$$c(m+3)$$

$$\frac{b B \sin (e+f x)(a+b \cos (e+f x))^2(c \cos (e+f x))^{m+1}}{c f(m+4)}$$

$$\Big\downarrow \ 3122$$

$$\frac{\frac{bc(m+3)\sin(e+fx)\left(2a^2B(m+5)+3aAb(m+4)+b^2B(m+3)\right)(c\cos(e+fx))^{m+1}}{f(m+2)}+\frac{-\frac{c^2(m+3)\sin(e+fx)\left(b(m+1)\left(2a^2B(m+5)+3aAb(m+4)+b^2B(m+3)\right)+a^2(m\cdots}{f\cdots}}{\cdots}}{\cdots}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))^2(c\cos(e+fx))^{m+1}}{cf(m+4)}$$

input
```
Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x]
```

output
```
(b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(4
 + m)) + ((b^2*(A*b*(4 + m) + a*B*(6 + m))*Cos[e + f*x]*(c*Cos[e + f*x])^(
1 + m)*Sin[e + f*x])/(f*(3 + m)) + ((b*c*(3 + m)*(b^2*B*(3 + m) + 3*a*A*b*
(4 + m) + 2*a^2*B*(5 + m))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(f*(2 +
m)) + (-((c^2*(3 + m)*(a^2*(2 + m)*(b*B*(1 + m) + a*A*(4 + m)) + b*(1 + m)
*(b^2*B*(3 + m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m)))*(c*Cos[e + f*x])^(1
+ m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e +
f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2])) - (c*(4 + m)*(A*b^3*(2 + m) + 3*a*
b^2*B*(2 + m) + 3*a^2*A*b*(3 + m) + a^3*B*(3 + m))*(c*Cos[e + f*x])^(2 + m
)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x
])/(f*Sqrt[Sin[e + f*x]^2]))/(c*(2 + m)))/(c*(3 + m)))/(c*(4 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (c\cos(fx+e))^m (a+\cos(fx+e)\,b)^3 (A+B\cos(fx+e))\,dx$$

input
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^3*(A+B*cos(f*x+e)),x)
```

output
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^3*(A+B*cos(f*x+e)),x)
```

Fricas [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^3(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^
3 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e
))*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**3*(A+B*cos(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^3(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m,
x)
```

Giac [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$$
$$=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^3(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$$
$$=\int (c\cos(e+fx))^m\,(A+B\cos(e+fx))\,(a+b\cos(e+fx))^3\,dx$$

input
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3,x)
```

output
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$$
$$= c^m\left(\left(\int \cos(fx+e)^m\,dx\right)a^4+4\left(\int \cos(fx+e)^m\cos(fx+e)\,dx\right)a^3b + \left(\int \cos(fx+e)^m\cos(fx+e)^4\,dx\right)b^4+4\left(\int \cos(fx+e)^m\cos(fx+e)^3\,dx\right)a\,b^3 + 6\left(\int \cos(fx+e)^m\cos(fx+e)^2\,dx\right)a^2b^2\right)$$

input
```
int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x)
```

output
```
c**m*(int(cos(e + f*x)**m,x)*a**4 + 4*int(cos(e + f*x)**m*cos(e + f*x),x)*
a**3*b + int(cos(e + f*x)**m*cos(e + f*x)**4,x)*b**4 + 4*int(cos(e + f*x)*
*m*cos(e + f*x)**3,x)*a*b**3 + 6*int(cos(e + f*x)**m*cos(e + f*x)**2,x)*a*
*2*b**2)
```

3.452 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 287

$$\begin{aligned}
&\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx \\
&= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)(3+m)} \\
&\quad + \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))\sin(e+fx)}{cf(3+m)} \\
&\quad - \frac{(Ab^2(1+m)+2abB(1+m)+a^2A(2+m))\,(c\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{cf(1+m)(2+m)\sqrt{\sin^2(e+fx)}} \\
&\quad - \frac{(b^2B(2+m)+a(2Ab+aB)(3+m))\,(c\cos(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{c^2f(2+m)(3+m)\sqrt{\sin^2(e+fx)}}
\end{aligned}$$

output

```
b*(A*b*(3+m)+a*B*(4+m))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(2+m)/(3+m)+b*
B*(c*cos(f*x+e))^(1+m)*(a+b*cos(f*x+e))*sin(f*x+e)/c/f/(3+m)-(A*b^2*(1+m)+
2*a*b*B*(1+m)+a^2*A*(2+m))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m]
,[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c/f/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)
-(b^2*B*(2+m)+a*(2*A*b+B*a)*(3+m))*(c*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+
1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(3+m)/(sin(f*x+e)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.74

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$$
$$= \frac{(c\cos(e+fx))^m\cot(e+fx)\left(-\frac{a^2A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)}{1+m}+\cos(e+fx)\left(-\frac{a(2Ab+aB)\,\text{Hyperg}}{}\right.\right.}{}$$

input
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x
]
```

output
```
((c*Cos[e + f*x])^m*Cot[e + f*x]*(-((a^2*A*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a*(2*A*b + a*B)
*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m)) +
b*Cos[e + f*x]*(-(((A*b + 2*a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)
/2, Cos[e + f*x]^2])/(3 + m)) - (b*B*Cos[e + f*x]*Hypergeometric2F1[1/2, (
4 + m)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m))))*Sqrt[Sin[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3469, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3469}$$

$$\frac{\int (c\cos(e+fx))^m \left(bc(Ab(m+3)+aB(m+4))\cos^2(e+fx)+c\left(B(m+2)b^2+a(2Ab+aB)(m+3)\right)\cos(e+\right.}{c(m+3)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

$\downarrow$ 3042

$$\frac{\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(bc(Ab(m+3)+aB(m+4))\sin\left(e+fx+\frac{\pi}{2}\right)^2+c\left(B(m+2)b^2+a(2Ab+aB)(m+3\right.\right.}{c(m+3)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

$\downarrow$ 3502

$$\frac{\frac{\int (c\cos(e+fx))^m((a(m+2)(bB(m+1)+aA(m+3))+b(m+1)(Ab(m+3)+aB(m+4)))c^2+(m+2)\left(B(m+2)b^2+a(2Ab+aB)(m+3)\right)\cos(e+fx)c^2)}{c(m+2)}}{c(m+3)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

$\downarrow$ 3042

$$\frac{\frac{\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m((a(m+2)(bB(m+1)+aA(m+3))+b(m+1)(Ab(m+3)+aB(m+4)))c^2+(m+2)\left(B(m+2)b^2+a(2Ab+aB)(m+3)\right)\sin(e+fx}{c(m+2)}}{c(m+3)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

$\downarrow$ 3227

$$\frac{\frac{c(m+2)\left(a(m+3)(aB+2Ab)+b^2B(m+2)\right)\int (c\cos(e+fx))^{m+1}dx+c^2(a(m+2)(aA(m+3)+bB(m+1))+b(m+1)(aB(m+4)+Ab(m+3)))\int (c\cos(}{c(m+2)}}{c(m+3)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

$\downarrow$ 3042

$$\frac{\frac{c(m+2)\left(a(m+3)(aB+2Ab)+b^2B(m+2)\right)\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx+c^2(a(m+2)(aA(m+3)+bB(m+1))+b(m+1)(aB(m+4)+Ab(m+3)))\int (c}{c(m+2)}}{c(m+3)}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

$\downarrow$ 3122

$$\frac{\frac{-\frac{\sin(e+fx)\left(a(m+3)(aB+2Ab)+b^2B(m+2)\right)(c\cos(e+fx))^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{c\sin(e+fx)(a(m+2)(aA(m+3)+bB(m+1))+}{}}{c(m+2)}}{c(m}$$

$$\frac{bB\sin(e+fx)(a+b\cos(e+fx))(c\cos(e+fx))^{m+1}}{cf(m+3)}$$

input
```
Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x]
```

output
```
(b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])*Sin[e + f*x])/(c*f*(3 +
 m)) + ((b*(A*b*(3 + m) + a*B*(4 + m))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*
x])/(f*(2 + m)) + (-((c*(a*(2 + m)*(b*B*(1 + m) + a*A*(3 + m)) + b*(1 + m)
*(A*b*(3 + m) + a*B*(4 + m)))*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1
/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 + m)*Sqrt[Si
n[e + f*x]^2])) - ((b^2*B*(2 + m) + a*(2*A*b + a*B)*(3 + m))*(c*Cos[e + f*
x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*S
in[e + f*x])/(f*Sqrt[Sin[e + f*x]^2]))/(c*(2 + m)))/(c*(3 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (c\cos(fx+e))^m (a+\cos(fx+e)\,b)^2 (A+B\cos(fx+e))\,dx$$

input
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^2*(A+B*cos(f*x+e)),x)
```

output
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^2*(A+B*cos(f*x+e)),x)
```

Fricas [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2
+ (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*cos(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**2*(A+B*cos(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m,
x)
```

Giac [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$$
$$= \int (c\cos(e+fx))^m\,(A+B\cos(e+fx))\,(a+b\cos(e+fx))^2\,dx$$

input
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2,x)
```

output
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$$
$$= c^m\left(\left(\int \cos(fx+e)^m\,dx\right)a^3+3\left(\int \cos(fx+e)^m\cos(fx+e)\,dx\right)a^2b + \left(\int \cos(fx+e)^m\cos(fx+e)^3\,dx\right)b^3+3\left(\int \cos(fx+e)^m\cos(fx+e)^2\,dx\right)a\,b^2\right)$$

input
```
int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x)
```

output
```
c**m*(int(cos(e + f*x)**m,x)*a**3 + 3*int(cos(e + f*x)**m*cos(e + f*x),x)*
a**2*b + int(cos(e + f*x)**m*cos(e + f*x)**3,x)*b**3 + 3*int(cos(e + f*x)*
*m*cos(e + f*x)**2,x)*a*b**2)
```

3.453 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 196

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$$
$$=\frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)}$$
$$-\frac{(bB(1+m)+aA(2+m))(c\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+}{cf(1+m)(2+m)\sqrt{\sin^2(e+fx)}}$$
$$-\frac{(Ab+aB)(c\cos(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{c^2f(2+m)\sqrt{\sin^2(e+fx)}}$$

output
```
b*B*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(2+m)-(b*B*(1+m)+a*A*(2+m))*(c*cos
(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*
x+e)/c/f/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-(A*b+B*a)*(c*cos(f*x+e))^(2+m)*h
ypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(si
n(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx =$$
$$-\frac{(c\cos(e+fx))^m\left(((bB(1+m)+aA(2+m))\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+\right.\right.}{}$$

input
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

output
```
-(((c*Cos[e + f*x])^m*((b*B*(1 + m) + a*A*(2 + m))*Cot[e + f*x]*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2] +
(A*b + a*B)*(1 + m)*Cos[e + f*x]*Cot[e + f*x]*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2] - (b*B*(1 + m)*Sin[2
*(e + f*x)])/2))/(f*(1 + m)*(2 + m)))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3447}$$

$$\int (c\cos(e+fx))^m\left((aB+Ab)\cos(e+fx)+aA+bB\cos^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left((aB+Ab)\sin\left(e+fx+\frac{\pi}{2}\right)+aA+bB\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$\downarrow$ 3502

$$\frac{\int (c\cos(e+fx))^m(c(bB(m+1)+aA(m+2))+(Ab+aB)c(m+2)\cos(e+fx))dx}{c(m+2)}+\frac{bB\sin(e+fx)(c\cos(e+fx))^{m+1}}{cf(m+2)}$$

$\downarrow$ 3042

$$\frac{\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(c(bB(m+1)+aA(m+2))+(Ab+aB)c(m+2)\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx}{c(m+2)}+\frac{bB\sin(e+fx)(c\cos(e+fx))^{m+1}}{cf(m+2)}$$

$\downarrow$ 3227

$$\frac{c(aA(m+2)+bB(m+1))\int (c\cos(e+fx))^m dx+(m+2)(aB+Ab)\int (c\cos(e+fx))^{m+1}dx}{c(m+2)}+\frac{bB\sin(e+fx)(c\cos(e+fx))^{m+1}}{cf(m+2)}$$

$\downarrow$ 3042

$$\frac{c(aA(m+2)+bB(m+1))\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx+(m+2)(aB+Ab)\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{c(m+2)}+\frac{bB\sin(e+fx)(c\cos(e+fx))^{m+1}}{cf(m+2)}$$

$\downarrow$ 3122

$$\frac{-\frac{\sin(e+fx)(aA(m+2)+bB(m+1))(c\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}}-\frac{(aB+Ab)\sin(e+fx)(c\cos(e+fx))^{m+}}{c}}{c(m+2)}+\frac{bB\sin(e+fx)(c\cos(e+fx))^{m+1}}{cf(m+2)}$$

input
```
Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

output
```
(b*B*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(2 + m)) + (-(((b*B*(1 +
m) + a*A*(2 + m))*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2
])) - ((A*b + a*B)*(c*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)
/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*Sqrt[Sin[e + f*x]^2]))/(
c*(2 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (c\cos(fx+e))^m (a+\cos(fx+e)\,b)(A+B\cos(fx+e))\,dx$$

input
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)*(A+B*cos(f*x+e)),x)
```

output
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)*(A+B*cos(f*x+e)),x)
```

Fricas [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*cos(f*x
+ e))^m, x)
```

Sympy [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m (A+B\cos(e+fx))(a+b\cos(e+fx))\,dx$$

input
```
integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

output
```
Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

Maxima [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$$
$$=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$$
$$=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$$
$$=\int (c\,\cos(e+f\,x))^m\,(A+B\,\cos(e+f\,x))\,(a+b\,\cos(e+f\,x))\,dx$$

input
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)),x)
```

output
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

Reduce [F]

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx = c^m\left(\left(\int \cos(fx+e)^m\,dx\right)a^2+2\left(\int \cos(fx+e)^m\cos(fx+e)\,dx\right)ab + \left(\int \cos(fx+e)^m\cos(fx+e)^2\,dx\right)b^2\right)$$

input
```
int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

output
```
c**m*(int(cos(e + f*x)**m,x)*a**2 + 2*int(cos(e + f*x)**m*cos(e + f*x),x)*
a*b + int(cos(e + f*x)**m*cos(e + f*x)**2,x)*b**2)
```

3.454 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 286

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \frac{a(Ab-aB)c\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},1,\frac{3}{2},\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)(c\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\text{si}}{b\left(a^2-b^2\right)f} - \frac{(Ab-aB)\,\text{AppellF1}\left(\frac{1}{2},-\frac{m}{2},1,\frac{3}{2},\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)(c\cos(e+fx))^m\cos^2(e+fx)^{-m/2}\sin(}{\left(a^2-b^2\right)f} - \frac{B(c\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{bcf(1+m)\sqrt{\sin^2(e+fx)}}$$

output
```
a*(A*b-B*a)*c*AppellF1(1/2,1/2-1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2/
(a^2-b^2))*(c*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*sin(f*x+e)/b/(
a^2-b^2)/f-(A*b-B*a)*AppellF1(1/2,-1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e
)^2/(a^2-b^2))*(c*cos(f*x+e))^m*sin(f*x+e)/(a^2-b^2)/f/((cos(f*x+e)^2)^(1/
2*m))-B*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*
x+e)^2)*sin(f*x+e)/b/c/f/(1+m)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10482 vs. 2(286) = 572.

Time = 35.00 (sec) , antiderivative size = 10482, normalized size of antiderivative = 36.65

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x
]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3481, 3042, 3122, 3302, 3042, 3668, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(e+fx))(c\cos(e+fx))^m}{a+b\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3481}$$

$$\frac{(Ab-aB)\int \frac{(c\cos(e+fx))^m}{a+b\cos(e+fx)}dx}{b}+\frac{B\int (c\cos(e+fx))^m dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{(Ab-aB)\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{b} - \frac{B\sin(e+fx)(c\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{bcf(m+1)\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3302}$$

$$\frac{(Ab-aB)\left(a\int \frac{(c\cos(e+fx))^m}{a^2-b^2\cos^2(e+fx)}dx - \frac{b\int \frac{(c\cos(e+fx))^{m+1}}{a^2-b^2\cos^2(e+fx)}dx}{c}\right)}{b} - \frac{B\sin(e+fx)(c\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{bcf(m+1)\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\left(a\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{a^2-b^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx - \frac{b\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}}{a^2-b^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx}{c}\right)}{b} - \frac{B\sin(e+fx)(c\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{bcf(m+1)\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3668}$$

$$\frac{(Ab-aB)\left(\frac{ac\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}\int \frac{\left(1-\sin^2(e+fx)\right)^{\frac{m-1}{2}}}{a^2-b^2+b^2\sin^2(e+fx)}d\sin(e+fx)}{f} - \frac{b\cos^2(e+fx)^{-m/2}(c\cos(e+fx))^m\int \frac{\left(1-\sin^2(e+f\right.}{a^2-b^2+b^2\sin}}{f}\right.}{b} \frac{B\sin(e+fx)(c\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{bcf(m+1)\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{(Ab-aB)\left(\frac{ac\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},1,\frac{3}{2},\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{f(a^2-b^2)} - \frac{b\sin(e+fx)\cos^2(e+\right.}{b} \frac{B\sin(e+fx)(c\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{bcf(m+1)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x]
```

output
```
-((B*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 Cos[e + f*x]^2]*Sin[e + f*x])/(b*c*f*(1 + m)*Sqrt[Sin[e + f*x]^2])) + ((A
*b - a*B)*((a*c*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Si
n[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1
 - m)/2)*Sin[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, -1/2*m, 1, 3/2,
Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x])^m*Si
n[e + f*x])/((a^2 - b^2)*f*(Cos[e + f*x]^2)^(m/2))))/b
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(c\cos(fx+e))^m (A+B\cos(fx+e))}{a+\cos(fx+e)\,b}dx$$

input
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+cos(f*x+e)*b),x)
```

output
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+cos(f*x+e)*b),x)
```

Fricas [F]

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \int \frac{(B\cos(fx+e)+A)(c\cos(fx+e))^m}{b\cos(fx+e)+a}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \int \frac{(B\cos(fx+e)+A)(c\cos(fx+e))^m}{b\cos(fx+e)+a}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0,0]%%%} / %%%{1,[0,0,1,0]%%%}+%%%{-1,[0,0,0,1]%%%
} Error:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \int \frac{(c\,\cos\left(e+f\,x\right))^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{a+b\,\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)),x)
```

output
```
int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = c^m\left(\int \cos\left(fx+e\right)^m dx\right)$$

input
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

output
```
c**m*int(cos(e + f*x)**m,x)
```

3.455 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx = \frac{2bB(c\cos(e+fx))^{1+m}\sqrt{a+b\cos(e+fx)}\sin(e+fx)}{cf(5+2m)} + \frac{2\mathrm{Int}\left(\frac{(c\cos(e+fx))^m\left(\frac{1}{2}ac\left(2bB(1+m)+2aA\left(\frac{5}{2}+m\right)\right)+\frac{1}{2}c\left(b^2B(3+2m)+a(2Ab+aB)(5+2m)\right)\cos(e+fx)+\frac{1}{2}bc(2aB(3+m)+Ab(5+2m))\cos^2}{\sqrt{a+b\cos(e+fx)}}\right.}{c(5+2m)}$$

output
```
2*b*B*(c*cos(f*x+e))^(1+m)*(a+b*cos(f*x+e))^(1/2)*sin(f*x+e)/c/f/(5+2*m)+2
*Defer(Int)((c*cos(f*x+e))^m*(1/2*a*c*(2*b*B*(1+m)+2*a*A*(5/2+m))+1/2*c*(b
^2*B*(3+2*m)+a*(2*A*b+B*a)*(5+2*m))*cos(f*x+e)+1/2*b*c*(2*a*B*(3+m)+A*b*(5
+2*m))*cos(f*x+e)^2)/(a+b*cos(f*x+e))^(1/2),x)/c/(5+2*m)
```

Mathematica [N/A]

Not integrable

Time = 107.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx=\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$$

input
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x
]),x]
```

output
```
Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x
]), x]
```

Rubi [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3469, 27, 3042, 3544}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{ 3469}$$

$$\frac{2\int \frac{(c\cos(e+fx))^m\left(bc(2aB(m+3)+Ab(2m+5))\cos^2(e+fx)+c\left(B(2m+3)b^2+a(2Ab+aB)(2m+5)\right)\cos(e+fx)+ac(2bB(m+1)+aA(2m+5))\right)}{2\sqrt{a+b\cos(e+fx)}}d}{c(2m+5)}$$

$$\frac{2bB\sin(e+fx)\sqrt{a+b\cos(e+fx)}(c\cos(e+fx))^{m+1}}{cf(2m+5)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(c\cos(e+fx))^m\left(bc(2aB(m+3)+Ab(2m+5))\cos^2(e+fx)+c\left(B(2m+3)b^2+a(2Ab+aB)(2m+5)\right)\cos(e+fx)+ac(2bB(m+1)+aA(2m+5))\right)}{\sqrt{a+b\cos(e+fx)}}dx}{c(2m+5)}$$

$$\frac{2bB\sin(e+fx)\sqrt{a+b\cos(e+fx)}(c\cos(e+fx))^{m+1}}{cf(2m+5)}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(bc(2aB(m+3)+Ab(2m+5))\sin\left(e+fx+\frac{\pi}{2}\right)^2+c\left(B(2m+3)b^2+a(2Ab+aB)(2m+5)\right)\sin\left(e+fx+\frac{\pi}{2}\right)+ac(2bB(m+1)+aA(2}{\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}}}{c(2m+5)}$$

$$\frac{2bB\sin(e+fx)\sqrt{a+b\cos(e+fx)}(c\cos(e+fx))^{m+1}}{cf(2m+5)}$$

$\Big\downarrow$ 3544

$$\frac{\int \frac{(c\cos(e+fx))^m\left(bc(2aB(m+3)+Ab(2m+5))\cos^2(e+fx)+c\left(B(2m+3)b^2+a(2Ab+aB)(2m+5)\right)\cos(e+fx)+ac(2bB(m+1)+aA(2m+5))\right)}{\sqrt{a+b\cos(e+fx)}}dx}{c(2m+5)}$$

$$\frac{2bB\sin(e+fx)\sqrt{a+b\cos(e+fx)}(c\cos(e+fx))^{m+1}}{cf(2m+5)}$$

input
```
Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3544
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d*
Sin[e + f*x])^n*(A + B*Sin[e + f*x] + C*Sin[e + f*x]^2), x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int (c\cos(fx+e))^m (a+\cos(fx+e)\,b)^{\frac{3}{2}} (A+B\cos(fx+e))\,dx$$

input
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^(3/2)*(A+B*cos(f*x+e)),x)
```

output
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^(3/2)*(A+B*cos(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^{\frac{3}{2}}(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(
f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

Sympy [N/A]

Not integrable

Time = 139.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m(A+B\cos(e+fx))(a+b\cos(e+fx))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e)),x)
```

output
```
Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**(3
/2), x)
```

Maxima [N/A]

Not integrable

Time = 3.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^{\frac{3}{2}}(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*cos(f*x + e))
^m, x)
```

Giac [N/A]

Not integrable

Time = 16.68 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^{\frac{3}{2}}(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*cos(f*x + e))
^m, x)
```

Mupad [N/A]

Not integrable

Time = 25.73 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx = \int (c\,\cos\left(e+f\,x\right))^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^{3/2}\,dx$$

input
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2),x)
```

output
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx = c^m\left(2\left(\int \sqrt{\cos\left(fx+e\right)b+a}\,\cos\left(fx+e\right)^m\cos\left(fx+e\right)dx\right)ab+\left(\int \sqrt{\cos\left(fx+e\right)b+a}\,\cos\left(fx+e\right)^m\cos\left(fx+e\right)^2dx\right)b^2+\left(\int \sqrt{\cos\left(fx+e\right)b+a}\,\cos\left(fx+e\right)^m dx\right)a^2\right)$$

input
```
int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x)
```

output
```
c**m*(2*int(sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**m*cos(e + f*x),x)*a*b +
 int(sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**m*cos(e + f*x)**2,x)*b**2 + in
t(sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**m,x)*a**2)
```

3.456 $\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx = \text{Int}\Big((c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx)),x\Big)$$

output
```
Defer(Int)((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 46.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$$

input
```
Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])
,x]
```

output
```
Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])
, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\,dx$$

$$\Big\downarrow \text{3486}$$

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\cos(e+fx))^m dx$$

input
```
Int[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \left(c \cos \left(f x +e \right)\right)^{m} \sqrt{a +\cos \left(f x +e \right) b}\, \left(A +B \cos \left(f x +e \right)\right)d x$$

input
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^(1/2)*(A+B*cos(f*x+e)),x)
```

output
```
int((c*cos(f*x+e))^m*(a+cos(f*x+e)*b)^(1/2)*(A+B*cos(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$$
$$=\int \left(B \cos \left(f x + e\right) + A\right) \sqrt{b \cos \left(f x + e\right) + a} \left(c \cos \left(f x + e\right)\right)^{m}\,{d x}$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
 x)
```

Sympy [N/A]

Not integrable

Time = 7.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx = \int (c\cos{(e+fx)})^m\,(A+B\cos{(e+fx)})\,\sqrt{a+b\cos{(e+fx)}}\,dx$$

input
```
integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**(1/2)*(A+B*cos(f*x+e)),x)
```

output
```
Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x))
, x)
```

Maxima [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx = \int (B\cos{(fx+e)}+A)\sqrt{b\cos{(fx+e)}+a}(c\cos{(fx+e)})^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m
, x)
```

Giac [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (c\cos(e+fx))^m \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$$
$$=\int (B\cos(fx+e)+A)\sqrt{b\cos(fx+e)+a}(c\cos(fx+e))^m\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m
, x)
```

Mupad [N/A]

Not integrable

Time = 25.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (c\cos(e+fx))^m \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx$$
$$=\int (c\,\cos(e+f\,x))^m\,(A+B\,\cos(e+f\,x))\,\sqrt{a+b\,\cos(e+f\,x)}\,dx$$

input
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2),x)
```

output
```
int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))\,dx = c^m\left(\left(\int\sqrt{\cos(fx+e)\,b+a}\,\cos(fx+e)^m\cos(fx+e)\,dx\right)b + \left(\int\sqrt{\cos(fx+e)\,b+a}\,\cos(fx+e)^m\,dx\right)a\right)$$

input
```
int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e)),x)
```

output
```
c**m*(int(sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**m*cos(e + f*x),x)*b + int
(sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**m,x)*a)
```

3.457 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \text{Int}\left(\frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}},x\right)$$

output
```
Defer(Int)((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 43.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

input
```
Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x
]],x]
```

output
```
Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x
]], x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(e+fx))(c\cos(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3486}$$

$$\int \frac{(A+B\cos(e+fx))(c\cos(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}dx$$

input
```
Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\left(c \cos \left(f x +e \right)\right)^{m} \left(A +B \cos \left(f x +e \right)\right)}{\sqrt{a +\cos \left(f x +e \right) b}}d x$$

input
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+cos(f*x+e)*b)^(1/2),x)
```

output
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+cos(f*x+e)*b)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(B\cos\left(fx+e\right)+A)(c\cos\left(fx+e\right))^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Sympy [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(c\cos{(e+fx)})^m\,(A+B\cos{(e+fx)})}{\sqrt{a+b\cos{(e+fx)}}}\,dx$$

input
```
integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(1/2),x)
```

output
```
Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x))
, x)
```

Maxima [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(B\cos{(fx+e)}+A)(c\cos{(fx+e)})^m}{\sqrt{b\cos{(fx+e)}+a}}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a)
, x)
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(B\cos(fx+e)+A)(c\cos(fx+e))^m}{\sqrt{b\cos(fx+e)+a}}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a)
, x)
```

Mupad [N/A]

Not integrable

Time = 25.74 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{{\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{\sqrt{a+b\,\cos\left(e+f\,x\right)}}\,dx$$

input
```
int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),x
)
```

output
```
int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),
x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx = c^m\left(\int \sqrt{\cos(fx+e)\,b+a}\,\cos(fx+e)^m\,dx\right)$$

input
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

output
```
c**m*int(sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**m,x)
```

3.458 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)(c\cos(e+fx))^{1+m}\sin(e+fx)}{a(a^2-b^2)\,cf\sqrt{a+b\cos(e+fx)}} + \frac{2\mathrm{Int}\left(\frac{(c\cos(e+fx))^m\left(\frac{1}{2}c\left(a(aA-bB)+2b(Ab-aB)\left(\frac{1}{2}+m\right)\right)-\frac{1}{2}a(Ab-aB)c\cos(e+fx)-\frac{1}{2}b(Ab-aB)c(3+2m)\cos^2(e+fx)\right)}{\sqrt{a+b\cos(e+fx)}},x\right)}{a(a^2-b^2)\,c}$$

output
```
2*b*(A*b-B*a)*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/a/(a^2-b^2)/c/f/(a+b*cos(f*x
+e))^(1/2)+2*Defer(Int)((c*cos(f*x+e))^m*(1/2*c*(a*(A*a-B*b)+2*b*(A*b-B*a)
*(1/2+m))-1/2*a*(A*b-B*a)*c*cos(f*x+e)-1/2*b*(A*b-B*a)*c*(3+2*m)*cos(f*x+e
)^2)/(a+b*cos(f*x+e))^(1/2),x)/a/(a^2-b^2)/c
```

Mathematica [N/A]

Not integrable

Time = 40.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx = \int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$$

input
```
Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(
3/2),x]
```

output
```
Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(
3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3479, 27, 3042, 3544}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(e+fx))(c\cos(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2\int \frac{(c\cos(e+fx))^m\left(-b(Ab-aB)c(2m+3)\cos^2(e+fx)-a(Ab-aB)c\cos(e+fx)+c\left(Aa^2-2bB(m+1)a+Ab^2(2m+1)\right)\right)}{2\sqrt{a+b\cos(e+fx)}}dx}{ac\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(e+fx)(c\cos(e+fx))^{m+1}}{acf\left(a^2-b^2\right)\sqrt{a+b\cos(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c\cos(e+fx))^m\left(-b(Ab-aB)c(2m+3)\cos^2(e+fx)-a(Ab-aB)c\cos(e+fx)+c\left(Aa^2-2bB(m+1)a+Ab^2(2m+1)\right)\right)}{\sqrt{a+b\cos(e+fx)}}dx}{ac\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(e+fx)(c\cos(e+fx))^{m+1}}{acf\left(a^2-b^2\right)\sqrt{a+b\cos(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(-b(Ab-aB)c(2m+3)\sin\left(e+fx+\frac{\pi}{2}\right)^2-a(Ab-aB)c\sin\left(e+fx+\frac{\pi}{2}\right)+c\left(Aa^2-2bB(m+1)a+Ab^2(2m+1)\right)\right)}{\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{ac\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(e+fx)(c\cos(e+fx))^{m+1}}{acf\left(a^2-b^2\right)\sqrt{a+b\cos(e+fx)}}$$

$$\Big\downarrow \text{3544}$$

$$\frac{\int \frac{(c\cos(e+fx))^m\left(-b(Ab-aB)c(2m+3)\cos^2(e+fx)-a(Ab-aB)c\cos(e+fx)+c\left(Aa^2-2bB(m+1)a+Ab^2(2m+1)\right)\right)}{\sqrt{a+b\cos(e+fx)}}dx}{ac\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(e+fx)(c\cos(e+fx))^{m+1}}{acf\left(a^2-b^2\right)\sqrt{a+b\cos(e+fx)}}$$

input
```
Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3/2),x
]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3544
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d*
Sin[e + f*x])^n*(A + B*Sin[e + f*x] + C*Sin[e + f*x]^2), x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(c\cos (fx+e))^m (A+B\cos (fx+e))}{(a+\cos (fx+e)\, b)^{\frac{3}{2}}}dx$$

input
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+cos(f*x+e)*b)^(3/2),x)
```

output
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+cos(f*x+e)*b)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{(B\cos(fx+e)+A)(c\cos(fx+e))^m}{(b\cos(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m/
(b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(3/2),x)
```

output
```
Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x))**(3
/2), x)
```

Maxima [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{(B\cos(fx+e)+A)(c\cos(fx+e))^m}{(b\cos(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/
2), x)
```

Giac [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{(B\cos(fx+e)+A)(c\cos(fx+e))^m}{(b\cos(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/
2), x)
```

Mupad [N/A]

Not integrable

Time = 26.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{{\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{{\left(a+b\,\cos\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),x
)
```

output
```
int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),
x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=c^m\left(\int \frac{\sqrt{\cos\left(fx+e\right)b+a}\,\cos\left(fx+e\right)^m}{\cos\left(fx+e\right)b+a}dx\right)$$

input
```
int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x)
```

output
```
c**m*int((sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**m)/(cos(e + f*x)*b + a),x
)
```

3.459 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 172

$$\begin{aligned}
&\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx \\
&= -\frac{2a(3A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{2a(A+B)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{2a(3A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2a(A+B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}
\end{aligned}$$

output
```
-2/5*a*(3*A+5*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*se
c(d*x+c)^(1/2)/d+2/3*a*(A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*a*(3*A+5*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/
d+2/3*a*(A+B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(5/2)*sin(d
*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.36 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.70

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{ae^{-ic}(-1+e^{2ic})(1+\cos(c+dx))\csc(c)\left(5A+5B-3Ae^{i(c+dx)}-15Be^{i(c+dx)}-24Ae^{3i(c+dx)}-30Be^{3i(\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(a*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])*Csc[c]*(5*A + 5*B - 3*A*E^(I*(c +
 d*x)) - 15*B*E^(I*(c + d*x)) - 24*A*E^((3*I)*(c + d*x)) - 30*B*E^((3*I)*(
c + d*x)) - 5*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 9*A*E^((5*
I)*(c + d*x)) - 15*B*E^((5*I)*(c + d*x)) - (5*I)*(A + B)*(1 + E^((2*I)*(c
+ d*x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (3*A + 5*B)*E^(I
*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/
4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(30*d*E^(
I*c)*(1 + E^((2*I)*(c + d*x)))^2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4485, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3439$$

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \; 4485$$

$$\frac{2}{5}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(a(3A+5B)+5a(A+B)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \sec^{\frac{3}{2}}(c+dx)(a(3A+5B)+5a(A+B)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a(3A+5B)+5a(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 4274$$

$$\frac{1}{5}\left(a(3A+5B)\int \sec^{\frac{3}{2}}(c+dx)dx+5a(A+B)\int \sec^{\frac{5}{2}}(c+dx)dx\right)+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(a(3A+5B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+5a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a(3A+5B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a(3A+5B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a(3A+5B)\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a(3A+\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a(3A+\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(5a(A+B)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+a(3A+5\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (a*(3*A + 5*B)*((-2*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/d) + 5*a*(A + B)*((2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(151) = 302$.

Time = 35.45 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.69

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\left(\frac{A\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\right.}}{}\right.}{}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/10*A/(8*
sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B
)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{-5i\sqrt{2}(A+B)a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*
B)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*B)*a*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*(3*(3*A + 5*B)*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c) + 3*A*
a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx$$
$$= \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)\sec(dx + c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}(a+a\cos(c+dx))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= a\Bigg(\left(\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a + int(sqrt(sec
(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*cos(
c + d*x)**2*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sec(c + d*x)**3,
x)*a)
```

3.460 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 135

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx &= -\frac{2a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a(A+3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2a(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
-2*a*(A+B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+
c)^(1/2)/d+2/3*a*(A+3*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))*sec(d*x+c)^(1/2)/d+2*a*(A+B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a*A*
sec(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.99 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.67

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{a(1+\cos(c+dx))\left((A+3B)\left(1+e^{2i(c+dx)}\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+i\left(A-3Ae^{i(c+dx)}\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*((A + 3*B)*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + I*(A - 3*A*E^(I*(c + d*x)) - 3*B*E^(I*(c +
 d*x)) - A*E^((2*I)*(c + d*x)) - 3*A*E^((3*I)*(c + d*x)) - 3*B*E^((3*I)*(c
 + d*x)) + (A + B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*Sec[(c + d*x)/2]^2*Sqrt[
Sec[c + d*x]])/(3*d*(1 + E^((2*I)*(c + d*x))))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4485, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3439}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \text{4485}$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{\sec(c+dx)}(a(A+3B)+3a(A+B)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{\sec(c+dx)}(a(A+3B)+3a(A+B)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(A+3B)+3a(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{3}\left(3a(A+B)\int \sec^{\frac{3}{2}}(c+dx)dx+a(A+3B)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a(A+3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a(A+B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{3}\left(a(A+3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a(A+B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)\right)+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a(A+3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a(A+B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a(A+B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a(A+B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 3119

$$\frac{1}{3}\left(a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a(A+B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3a(A+B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + ((2*a*(A + 3*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 3*a*(A + B)*((-
2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(122) = 244$.

Time = 35.01 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.96

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}+\frac{A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{6\cdots}\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{2Aa\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\cdots}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cdots\right.}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.39

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{-i\sqrt{2}(A+3B)a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}(A+3B)}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*B)*a*cos(d*x + c)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(A + B)*a*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) + 3*I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A +
B)*a*cos(d*x + c) + A*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx$$
$$= \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)\sec(dx + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx$$
$$= \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)\sec(dx + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+a\cos(c+dx))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx &= a\Bigg(\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a \\ &+ \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b \\ &+ \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b \\ &+ \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a\Bigg)\end{aligned}$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a + int(sqrt(sec
(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*cos(
c + d*x)**2*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,
x)*a)
```

3.461 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 106

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = -\frac{2a(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*a*(A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+
c)^(1/2)/d+2*a*(A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
))*sec(d*x+c)^(1/2)/d+2*a*A*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.48

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-3iA\cos(c+dx)+3iB\cos(c+dx)+3(A+B)\sqrt{\cos(c+dx)}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-3*I)*A*Cos[c + d*x] + (
3*I)*B*Cos[c + d*x] + 3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] + I*(A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x]))/(3*d*E^(I*d
*x))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 3439, 3042, 4485, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{3439}$$
$$\int \frac{(a\sec(c+dx)+a)(A\sec(c+dx)+B)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4485}$$

$$2\int -\frac{a(A-B)-a(A+B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{a(A-B)-a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{a(A-B)-a(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4274}$$

$$-a(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a(A+B)\int \sqrt{\sec(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-a(A-B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(A+B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4258}$$

$$a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3119}$$

$$a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} - \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(-2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(99) = 198$.

Time = 5.70 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.28

method	result
default	$\frac{2a\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\cdots\right.}\cdots\right.}{\cdots}$
parts	$-\frac{2Aa\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cdots}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
2*a*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\,dx$$
$$= \frac{-i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}(A+B)a\text{weierstrassPIn}}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - I*sqrt(2)*(A - B)*a*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - B)*a*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
2*A*a*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+a\cos(c+dx))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=a\left(\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)a+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a + int(sqrt(sec(c
+ d*x))*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*cos(c + d*
x)**2*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*sec(c + d*x),x)*a)
```

3.462 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,d$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\begin{aligned}\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx\\ &= \frac{2a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a(3A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2*a*(A+B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/d+2/3*a*(3*A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(
1/2))*sec(d*x+c)^(1/2)/d+2/3*a*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{2ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left((3A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-i(A+B)\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((3*A + B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] - I*(A + B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos
[c + d*x]*((3*I)*(A + B) + B*Sin[c + d*x])))/(3*d*E^(I*d*x))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {3042, 3439, 3042, 4484, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)(A\sec(c+dx)+B)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{4484}$$

$$\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{3a(A+B)+a(3A+B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3}\int \frac{3a(A+B)+a(3A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\int \frac{3a(A+B)+a(3A+B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{4274}$$

$$\frac{1}{3}\left(3a(A+B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a(3A+B)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\left(3a(A+B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(3A+B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)+$$
$$\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{1}{3}\left(a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(\right.$$
$$\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\left(a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\right.$$

$$\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 3119

$$\frac{1}{3}\left(a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E(\frac{}{}}{d}\right.$$

$$\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$\big\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E(}{d}\right.$$

$$\frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
((6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d)/3 + (2*a*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(99) = 198$.

Time = 6.52 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.92

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$
parts	$-\frac{2Aa\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}+\frac{2(Aa+Ba)\sqrt{}}{}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.29

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{2\,Ba\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-i\,\sqrt{2}(3\,A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/3*(2*B*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*(3*A + B)*a*weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(3*A + B)*a
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(
A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) - 3*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=a\left(\int A\sqrt{\sec(c+dx)}\,dx+\int A\cos(c+dx)\sqrt{\sec(c+dx)}\,dx\right.$$
$$\left.+\int B\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
a*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)*sqrt(sec(c
+ d*x)), x) + Integral(B*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(B*
cos(c + d*x)**2*sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}\,(a+a\cos(c+dx))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=a\left(\left(\int\sqrt{\sec(dx+c)}dx\right)a+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\,dx\right)a\right.$$
$$\left.+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x)),x)*a + int(sqrt(sec(c + d*x))*cos(c + d*x),x)*a
+ int(sqrt(sec(c + d*x))*cos(c + d*x),x)*b + int(sqrt(sec(c + d*x))*cos(c
+ d*x)**2,x)*b)
```

3.463 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2a(5A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
2/5*a*(5*A+3*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec
(d*x+c)^(1/2)/d+2/3*a*(A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*a*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*(
A+B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.80 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{a\sqrt{\sec(c+dx)}\Big(10(A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-2i(5A+3B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{H}}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x
]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(10*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2] - (2*I)*(5*A + 3*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((6*I)*
(5*A + 3*B) + 10*(A + B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(15*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4484, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3439$$

$$\int \frac{(a\sec(c+dx)+a)(A\sec(c+dx)+B)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4484}$$

$$\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5a(A+B)+a(5A+3B)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5a(A+B)+a(5A+3B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{5a(A+B)+a(5A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{5}\left(5a(A+B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+a(5A+3B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a(A+B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a(5A+3B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{5}\left(a(5A+3B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a(A+B)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(a(5A+3B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a(A+B)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 4258

$$\frac{1}{5}\left(a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+5a(A+B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{}}\right.\right.$$
$$\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(5A+3B)\sqrt{\mathrm{co}}\right.$$
$$\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3119

$$\frac{1}{5}\left(5a(A+B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a(5A+3B)\sqrt{\mathrm{c}}}{}\right.$$
$$\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5a(A+B)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.$$
$$\frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2*a*(5*A + 3*B)*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 5*a*(A + B)*(
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(124) = 248$.

Time = 10.06 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.52

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(20A+44B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-10A-16\right.}{}$
parts	$\frac{2Aa\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2(Aa+Ba)\sqrt{\left(2\right.}}{}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.20

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\sqrt{\sec(c + dx)}}\,dx$$

$$= \frac{-5i\sqrt{2}(A+B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}(A+B)a\text{weierstrassP}}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*B)*a*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*
A + 3*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) + 2*(3*B*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\sqrt{\sec(c + dx)}}\,dx = a\left(\int \frac{A}{\sqrt{\sec(c + dx)}}\,dx + \int \frac{A\cos(c + dx)}{\sqrt{\sec(c + dx)}}\,dx + \int \frac{B\cos(c + dx)}{\sqrt{\sec(c + dx)}}\,dx + \int \frac{B\cos^2(c + dx)}{\sqrt{\sec(c + dx)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)/sqrt(sec(c
+ d*x)), x) + Integral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(B*
cos(c + d*x)**2/sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)}{\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))(a+a\cos(c+dx))}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = a\left(\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)}{\sec(dx+c)}dx\right)a + \left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)}{\sec(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)^2}{\sec(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a + int((sqrt(sec(c + d*x))*cos(
c + d*x))/sec(c + d*x),x)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/sec(c
+ d*x),x)*b + int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*x),x)*b)
```

3.464 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 172

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{6a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
6/5*a*(A+B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x
+c)^(1/2)/d+2/21*a*(7*A+5*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/7*a*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*
(A+B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*a*(7*A+5*B)*sin(d*x+c)/d/sec(d*x+
c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.49 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(20(7A+5B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-84i(A-\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x
]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(7*A + 5*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] - (84*I)*(A + B)*E^(I*(c + d*x))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)] + Cos[c + d*x]*((252*I)*(A + B) + 5*(28*A + 23*B)*Sin[c + d*x] + 42*(A
+ B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4484, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)(A\sec(c+dx)+B)}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4484}$$

$$\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int -\frac{7a(A+B)+a(7A+5B)\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\int \frac{7a(A+B)+a(7A+5B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\int \frac{7a(A+B)+a(7A+5B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{1}{7}\left(7a(A+B)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+a(7A+5B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(7a(A+B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(7A+5B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{1}{7}\left(7a(A+B)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a(7A+5B)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+d}{3d\sqrt{\sec(c+}}\right.\right.$$
$$\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(7a(A+B)\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a(7A+5B)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\right.\right.$$
$$\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(7a(A+B)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a(7A+5B)\left(\frac{1}{3}\sqrt{\cos(c\right.\right.$$
$$\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(7a(A+B)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a(7A+5B)\left(\frac{1}{3}\right.\right.$$
$$\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(a(7A+5B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+7a(A+B)\left(\frac{2}{5d}\right.\right.$$
$$\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(a(7A+5B)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+7a(A+B)\left(\frac{2\,\mathrm{s}}{5d\,\mathrm{s}}\right.\right.$$
$$\frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (7*a*(A + B)*((6*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*(7*A + 5*B)*((2*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*
Sqrt[Sec[c + d*x]])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(151) = 302$.

Time = 15.07 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-168A-528B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(308A-\right.}{}$
parts	$-\frac{2Aa\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(
-112*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.09

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{-5i\sqrt{2}(7A+5B)a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5B)a\text{weiers}}{}$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/105*(-5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*B)*a*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt
(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))) + 2*(15*B*a*cos(d*x + c)^3 + 21*(A + B)*a*cos(d*x +
c)^2 + 5*(7*A + 5*B)*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\, dx$$
$$= a\left(\int \frac{A}{\sec^{\frac{3}{2}}(c + dx)}\, dx + \int \frac{A\cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)}\, dx + \int \frac{B\cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)}\, dx + \int \frac{B\cos^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)}\, dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A*cos(c + d*x)/sec(c + d*
x)**(3/2), x) + Integral(B*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral
(B*cos(c + d*x)**2/sec(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int \frac{(a + a\cos(c + dx))(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)}{\sec(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a\left(\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)}{\sec(dx+c)^2}dx\right)a + \left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)}{\sec(dx+c)^2}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)^2}{\sec(dx+c)^2}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
a*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a + int((sqrt(sec(c + d*x))*c
os(c + d*x))/sec(c + d*x)**2,x)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/
sec(c + d*x)**2,x)*b + int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*
x)**2,x)*b)
```

3.465 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 199

$$\begin{aligned}&\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx\\&=-\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\&\quad+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\&\quad+\frac{4a^2(4A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a^2(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}\\&\quad+\frac{2A\sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d}\end{aligned}$$

output
```
-4/5*a^2*(4*A+5*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
sec(d*x+c)^(1/2)/d+4/3*a^2*(A+2*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+4/5*a^2*(4*A+5*B)*sec(d*x+c)^(1/2)*sin
(d*x+c)/d+2/15*a^2*(7*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*A*sec(d*x+c
)^(3/2)*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.50

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx$$
$$= \frac{a^2e^{-ic}(-1 + e^{2ic})\,(1 + \cos(c + dx))^2\csc(c)\left(10A + 5B - 18Ae^{i(c+dx)} - 30Be^{i(c+dx)} - 54Ae^{3i(c+dx)} - 60B\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x
]
```

output
```
(a^2*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])^2*Csc[c]*(10*A + 5*B - 18*A*E^(
I*(c + d*x)) - 30*B*E^(I*(c + d*x)) - 54*A*E^((3*I)*(c + d*x)) - 60*B*E^((
3*I)*(c + d*x)) - 10*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 24*
A*E^((5*I)*(c + d*x)) - 30*B*E^((5*I)*(c + d*x)) - (10*I)*(A + 2*B)*(1 + E
^((2*I)*(c + d*x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(4*
A + 5*B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1
[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x
]])/(60*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3439, 3042, 4506, 27, 3042, 4485, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c + dx)(a\cos(c + dx) + a)^2(A + B\cos(c + dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \text{4506}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)(a(A+5B)+a(7A+5B)\sec(c+dx))dx+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)(a(A+5B)+a(7A+5B)\sec(c+dx))dx+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(A+5B)+a(7A+5B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \sqrt{\sec(c+dx)}\left(5(A+2B)a^2+3(4A+5B)\sec(c+dx)a^2\right)dx+\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(A+2B)a^2+3(4A+5B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx+\frac{2a^2(7A+5B)\sin(c+dx)s}{3d}\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(3a^2(4A+5B)\int \sec^{\frac{3}{2}}(c+dx)dx+5a^2(A+2B)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^2(4A+5B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{2a^2(7A+5B)\sin(}{}\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}d\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx\right.}}\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx}}{d}\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c}{d}\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2(4A+5B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c}{d}\right.\right.\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2}{3}\left(\frac{10a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{d}\right.\right.$$
$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(2*A*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d) + ((2
*a^2*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*((10*a^2*(A +
 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 3*a^2*(4*A + 5*B)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4506
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(d*(m + n))   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x]
)^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
 a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(178) = 356$.

Time = 62.87 (sec) , antiderivative size = 714, normalized size of antiderivative = 3.59

method	result	size
default	Expression too large to display	714
parts	Expression too large to display	916

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/20*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/2*B)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))+(1/2*A+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.20

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)\, dx =$$

$$-\frac{2\left(5i\sqrt{2}(A + 2B)a^2\cos(dx + c)^2 \text{ weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) - 5i\sqrt{2}(\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + 2*B)*a^2*cos(d*x + c)^2
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(
4*A + 5*B)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(4*A + 5*B)*a^2*cos(d*
x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - (6*(4*A + 5*B)*a^2*cos(d*x + c)^2 + 5*(2*A + B)*a^2*cos
(d*x + c) + 3*A*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)\, dx$$
$$= \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^2 \sec(dx + c)^{\frac{7}{2}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+a\cos(c+d\,x))^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= a^2\left(2\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a + 2\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
a**2*(2*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a + int(sqr
t(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))
*cos(c + d*x)**3*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x
)**2*sec(c + d*x)**3,x)*a + 2*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c
 + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*a)
```

3.466 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 160

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx &= -\frac{4a^2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &+\frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}\\ &+\frac{2A\sqrt{\sec(c+dx)}(a^2+a^2\sec(c+dx))\sin(c+dx)}{3d}\end{aligned}$$

output
```
-4*a^2*A*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/d+4/3*a^2*(2*A+3*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*a^2*(5*A+3*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/
d+2/3*A*sec(d*x+c)^(1/2)*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx =$$
$$-\frac{ia^2\sec^{\frac{3}{2}}(c+dx)\left(-6A-6A\cos(2(c+dx))+6Ae^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x
]
```

output
```
((-1/3*I)*a^2*Sec[c + d*x]^(3/2)*(-6*A - 6*A*Cos[2*(c + d*x)] + (6*A*(1 +
E^((2*I)*(c + d*x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/E^((2*I)*(c + d*x)) + (2*(2*A + 3*B)*(1 + E^((2*I)*(c + d*x)))^(
3/2)*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x
)) + (2*I)*A*Sin[c + d*x] + (6*I)*A*Sin[2*(c + d*x)] + (3*I)*B*Sin[2*(c +
d*x)]))/d
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4506, 27, 3042, 4485, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)^2(A\sec(c+dx)+B)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4506}$$

$$\frac{2}{3}\int -\frac{(\sec(c+dx)a+a)(a(A-3B)-a(5A+3B)\sec(c+dx))}{2\sqrt{\sec(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}-\frac{1}{3}\int \frac{(\sec(c+dx)a+a)(a(A-3B)-a(5A+3B)\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}-\frac{1}{3}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(A-3B)-a(5A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\int \frac{3a^2A-a^2(2A+3B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\int \frac{3a^2A-a^2(2A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\left(3a^2A\int\frac{1}{\sqrt{\sec(c+dx)}}dx-a^2(2A+3B)\int\sqrt{\sec(c+dx)}dx\right)\right.$$
$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\left(3a^2A\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2(2A+3B)\int\sqrt{\csc\left(c+dx\right.}\right.\right.$$
$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\left(3a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-a^2(2A+\right.\right.$$
$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\left(3a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\right.\right.$$
$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$\Big\downarrow$ 3119

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\left(\frac{6a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-a^2(2A+3\right.\right.$$
$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

$\Big\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\left(\frac{6a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a^2(2A+}{}\right.\right.$$
$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(2*A*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d) + (-2
*((6*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d - (2*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d) + (2*a^2*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4506
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(d*(m + n))   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x]
)^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
 a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(145) = 290$.

Time = 62.82 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.21

method	result
default	$-\frac{4\left(6\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,(2A+B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,(7A+3B)\sin\left(\frac{dx}{2}\right.\right.}{}$
parts	$-\frac{2a^2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))*a^2/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3
/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.26

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{2}(2A+3B)a^2\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}(2A\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*a^2*
cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*a^2*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*(2*A +
B)*a^2*cos(d*x + c) + A*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^2\sec(dx + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^2\sec(dx + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos{(c+d\,x)})\,\left(\frac{1}{\cos{(c+d\,x)}}\right)^{5/2}(a+a\,\cos{(c+d\,x)})^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=a^2\Bigg(2\left(\int\sqrt{\sec{(dx+c)}}\,\cos{(dx+c)}\sec{(dx+c)}^2\,dx\right)a$$
$$+\left(\int\sqrt{\sec{(dx+c)}}\,\cos{(dx+c)}\sec{(dx+c)}^2\,dx\right)b$$
$$+\left(\int\sqrt{\sec{(dx+c)}}\,\cos{(dx+c)}^3\sec{(dx+c)}^2\,dx\right)b$$
$$+\left(\int\sqrt{\sec{(dx+c)}}\,\cos{(dx+c)}^2\sec{(dx+c)}^2\,dx\right)a$$
$$+2\left(\int\sqrt{\sec{(dx+c)}}\,\cos{(dx+c)}^2\sec{(dx+c)}^2\,dx\right)b$$
$$+\left(\int\sqrt{\sec{(dx+c)}}\,\sec{(dx+c)}^2\,dx\right)a\Bigg)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
a**2*(2*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a + int(sqr
t(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))
*cos(c + d*x)**3*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x
)**2*sec(c + d*x)**2,x)*a + 2*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c
 + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*a)
```

3.467 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 160

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx\\ =\frac{4a^2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ +\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ +\frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2B(a^2+a^2\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
4*a^2*B*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^
(1/2)/d+4/3*a^2*(3*A+2*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))*sec(d*x+c)^(1/2)/d+2/3*a^2*(3*A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2
/3*B*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.95

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2a^2\left(12iB\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-4i(3A+2B)e^{i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.\right.}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x
]
```

output
```
(2*a^2*((12*I)*B*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
 (4*I)*(3*A + 2*B)*E^(I*(c + d*x))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2
*I)*(c + d*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*((-6*I)*B + B*Sin[c + d*x]
 + 3*A*Tan[c + d*x])))/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[Sec[c + d*x
]])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4505, 27, 3042, 4485, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)^2(A\sec(c+dx)+B)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4505$$

$$\frac{2}{3}\int \frac{(\sec(c+dx)a+a)(a(3A+5B)+a(3A-B)\sec(c+dx))}{2\sqrt{\sec(c+dx)}}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{(\sec(c+dx)a+a)(a(3A+5B)+a(3A-B)\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(3A+5B)+a(3A-B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4485$$

$$\frac{1}{3}\left(2\int \frac{3Ba^2+(3A+2B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(2\int \frac{3Ba^2+(3A+2B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4274$$

$$\frac{1}{3}\left(2\left(a^2(3A+2B)\int\sqrt{\sec(c+dx)}dx+3a^2B\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right)$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(2\left(a^2(3A+2B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^2B\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\mathrm{s}}}{d}\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4258

$$\frac{1}{3}\left(2\left(a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(}\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(2\left(a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3119

$$\frac{1}{3}\left(2\left(a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{d}\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+2\left(\frac{2a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{d}\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*
((6*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/d + (2*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d) + (2*a^2*(3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4505
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Sim
p[b/(a*d*n)   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Sim
p[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0
] && GtQ[m, 1/2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 7.58 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.52

method	result
default	$\frac{4a^2\left(-2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+3A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$
parts	$-\frac{2a^2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
4/3*a^2*(-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)\, dx =$$
$$-\frac{2\left(i\sqrt{2}(3A + 2B)a^2\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) - i\sqrt{2}(3A + 2B)a^2\text{we}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(2)*(3*A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - I*sqrt(2)*(3*A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*a^2*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*a^
2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))) - (B*a^2*cos(d*x + c) + 3*A*a^2)*sin(d*x + c)/sqrt(cos(d*x + c))
)/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=a^2\left(2\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)a\right.$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)b$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)b$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a$$
$$+2\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b$$
$$\left.+\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output

```
a**2*(2*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a + int(sqrt(s
ec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*cos(c
 + d*x)**3*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(
c + d*x),x)*a + 2*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*b
 + int(sqrt(sec(c + d*x))*sec(c + d*x),x)*a)
```

3.468 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}$

Optimal result

Integrand size = 33, antiderivative size = 166

$$\begin{aligned}\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &+ \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2B(a^2+a^2\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
4/5*a^2*(5*A+4*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/d+4/3*a^2*(2*A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/15*a^2*(5*A+7*B)*sin(d*x+c)/d/sec(d*x
+c)^(1/2)+2/5*B*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\sqrt{\sec(c + dx)}\, dx$$
$$= \frac{a^2\sqrt{\sec(c+dx)}\left(20(2A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - 4i(5A+4B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x
]
```

output
```
(a^2*Sqrt[Sec[c + d*x]]*(20*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] - (4*I)*(5*A + 4*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((6
0*I)*A + (48*I)*B + 10*(A + 2*B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(1
5*d)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3439, 3042, 4505, 27, 3042, 4484, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)^2(A\sec(c+dx)+B)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4505}$$

$$\frac{2}{5}\int \frac{(\sec(c+dx)a+a)(a(5A+7B)+a(5A+B)\sec(c+dx))}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(\sec(c+dx)a+a)(a(5A+7B)+a(5A+B)\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(5A+7B)+a(5A+B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4484}$$

$$\frac{1}{5}\left(\frac{2a^2(5A+7B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{3(5A+4B)a^2+5(2A+B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3(5A+4B)a^2+5(2A+B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2(5A+7B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\int\frac{3(5A+4B)a^2+5(2A+B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(5A+7B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 4274

$$\frac{1}{5}\left(\frac{2}{3}\left(3a^2(5A+4B)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^2(2A+B)\int\sqrt{\sec(c+dx)}dx\right)+\frac{2a^2(5A+7B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2}{3}\left(3a^2(5A+4B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^2(2A+B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2(5A+7B)\sin(c}{3d\sqrt{\sec(c+d}}\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 4258

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2(5A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2(5A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3119

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2(5A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\right.$$

$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{2a^2(5A+7B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\left(\frac{10a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2(5A\right.\right.$$

$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2
*((6*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + (10*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d))/3 + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(3*d*Sqr
t[Sec[c + d*x]]))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

rule 4505
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Sim
p[b/(a*d*n)   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Sim
p[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0
] && GtQ[m, 1/2] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(149) = 298.

Time = 10.60 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.15

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(10A+32B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-5A-13\cdots\right.}{\cdots}$
parts	$-\frac{2a^{2}A\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2(a^{2}A+2a^{2}\cdots}{\cdots}$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(2A+B)a^{2}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(2A+B)a^{2}\text{wei}\cdots\right.}{\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 5*I*sqrt(2)*(2*A + B)*a^2*weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 4*B)*a^2*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*s
qrt(2)*(5*A + 4*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) - (3*B*a^2*cos(d*x + c)^2 + 5*(A + 2*B)*a^2
*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= a^2\left(\int A\sqrt{\sec(c+dx)}\,dx+\int 2A\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int A\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int 2B\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos^3(c+dx)\sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(2*A*cos(c + d*x)*sqrt(s
ec(c + d*x)), x) + Integral(A*cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + Int
egral(B*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(2*B*cos(c + d*x)**2
*sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)**3*sqrt(sec(c + d*x)), x
))
```

Maxima [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = a^2\left(\left(\int\sqrt{\sec(dx+c)}dx\right)a+2\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\,dx\right)a + \left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^3\,dx\right)b + \left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^2\,dx\right)a+2\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
a**2*(int(sqrt(sec(c + d*x)),x)*a + 2*int(sqrt(sec(c + d*x))*cos(c + d*x),
x)*a + int(sqrt(sec(c + d*x))*cos(c + d*x),x)*b + int(sqrt(sec(c + d*x))*c
os(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x)**2,x)*a + 2*int(
sqrt(sec(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.469 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 201

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2B(a^2+a^2\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

output
```
4/5*a^2*(4*A+3*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/d+4/21*a^2*(7*A+6*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/35*a^2*(7*A+9*B)*sin(d*x+c)/d/sec(
d*x+c)^(3/2)+4/21*a^2*(7*A+6*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/7*B*(a^2+a
^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(40(7A+6B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-56i(4A\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]]
,x]
```

output
```
(a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(7*A + 6*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(4*A + 3*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
+ d*x))] + Cos[c + d*x]*((672*I)*A + (504*I)*B + 5*(56*A + 51*B)*Sin[c + d
*x] + 42*(A + 2*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I
*d*x))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4505, 27, 3042, 4484, 25, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3439$$

$$\int \frac{(a\sec(c+dx)+a)^2(A\sec(c+dx)+B)}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \ 4505$$

$$\frac{2}{7}\int \frac{(\sec(c+dx)a+a)(a(7A+9B)+a(7A+3B)\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{7}\int \frac{(\sec(c+dx)a+a)(a(7A+9B)+a(7A+3B)\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a(7A+9B)+a(7A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 4484$$

$$\frac{1}{7}\left(\frac{2a^2(7A+9B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5(7A+6B)a^2+7(4A+3B)\sec(c+dx)a^2}{\sec^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{5(7A+6B)a^2+7(4A+3B)\sec(c+dx)a^2}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(7A+9B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{5(7A+6B)a^2+7(4A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2(7A+9B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4274

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(7A+6B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+7a^2(4A+3B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2(7A+9B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(7A+6B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+7a^2(4A+3B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2(7A+9B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4256

$$\frac{1}{7}\left(\frac{2}{5}\left(7a^2(4A+3B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^2(7A+6B)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\left(7a^2(4A+3B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^2(7A+6B)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{2}{5}\left(7a^2(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+5a^2(7A+6B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(7A+6B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+7a^2(4A+\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3119

$$\frac{1}{7}\left(\frac{2}{5}\left(5a^2(7A+6B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{14a^2(4A\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3120

$$\frac{1}{7}\left(\frac{2a^2(7A+9B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\left(\frac{14a^2(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5a^2(7A+6B\right.\right.$$
$$\frac{2B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((2
*a^2*(7*A + 9*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*((14*a^2*(4*A
 + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + 5*a^2*(7*A + 6*B)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/5)/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

rule 4505
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Sim
p[b/(a*d*n)   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Sim
p[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0
] && GtQ[m, 1/2] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(180) = 360$.

Time = 15.77 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.92

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-84A-348B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(224A-\right.}{}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-91*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.05

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(7A+6B)a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(7A+6B)a^2\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 5*I*sqrt(2)*(7*A + 6*B)*a^2*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(4*A + 3*B)*a^2*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 21*I*sqrt(2)*(4*A + 3*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^2*cos(d*x + c)^3 + 21*(A +
 2*B)*a^2*cos(d*x + c)^2 + 10*(7*A + 6*B)*a^2*cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= a^2\left(\int \frac{A}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{2A\cos{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{A\cos^2{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{B\cos{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{2B\cos^2{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{B\cos^3{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(2*A*cos(c + d*x)/sqrt(s
ec(c + d*x)), x) + Integral(A*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Int
egral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(2*B*cos(c + d*x)**2
/sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)**3/sqrt(sec(c + d*x)), x
))
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)(a\cos{(dx+c)}+a)^2}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^2}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= a^2\left(\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)a+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a + 2*int((sqrt(sec(c + d*x))
*cos(c + d*x))/sec(c + d*x),x)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/s
ec(c + d*x),x)*b + int((sqrt(sec(c + d*x))*cos(c + d*x)**3)/sec(c + d*x),x
)*b + int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*x),x)*a + 2*int((
sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*x),x)*b)
```

3.470 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 244

$$\begin{aligned}
&\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx \\
&= -\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\
&\quad+\frac{4a^3(7A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{4a^3(41A+42B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} \\
&\quad+\frac{2aA\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d} \\
&\quad+\frac{2(11A+7B)\sec^{\frac{3}{2}}(c+dx)\left(a^3+a^3\sec(c+dx)\right)\sin(c+dx)}{35d}
\end{aligned}$$

output
```
-4/5*a^3*(7*A+9*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
sec(d*x+c)^(1/2)/d+4/21*a^3*(13*A+21*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+4/5*a^3*(7*A+9*B)*sec(d*x+c)^(1/2
)*sin(d*x+c)/d+4/105*a^3*(41*A+42*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*A
*sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+2/35*(11*A+7*B)*sec(d*x+
c)^(3/2)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.90 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.78

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{9}{2}}(c + dx)\, dx$$

$$= \frac{a^3 e^{-idx}(1+\cos(c+dx))^3 \csc(c) \sec^6\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}(7A+9B)e^{2idx}(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c}}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x
]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Csc[c]*Sec[(c + d*x)/2]^6*(7*Sqrt[2]*(7*A + 9*B)
*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))] - ((-1 + E^((2*I)*c))*(21*B*(-5 + 16*E^(I*(c + d*x)) -
5*E^((2*I)*(c + d*x)) + 54*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 5
6*E^((5*I)*(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 18*E^((7*I)*(c + d*x))) +
2*A*(-65 + 84*E^(I*(c + d*x)) - 95*E^((2*I)*(c + d*x)) + 441*E^((3*I)*(c +
 d*x)) + 95*E^((4*I)*(c + d*x)) + 504*E^((5*I)*(c + d*x)) + 65*E^((6*I)*(c
 + d*x)) + 147*E^((7*I)*(c + d*x))) + (10*I)*(13*A + 21*B)*(1 + E^((2*I)*(
c + d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*
x]])/(2*E^(I*(c - d*x))*(1 + E^((2*I)*(c + d*x)))^3)))/(420*d*E^(I*d*x))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3439, 3042, 4506, 27, 3042, 4506, 3042, 4485, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \text{4506}$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2(a(A+7B)+a(11A+7B)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2(a(A+7B)+a(11A+7B)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(A+7B)+a(11A+7B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{4506}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)\left((8A+21B)a^2+(41A+42B)\sec(c+dx)a^2\right)dx+\frac{2(11A+7B)\sin(c\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((8A+21B)a^2+(41A+42B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{2}{3}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(5(13A+21B)a^3+21(7A+9B)\sec(c+dx)a^3\right)dx+\frac{2a^3(41A+42B)\sin(c+dx)\text{se}}{3d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}\left(5(13A+21B)a^3+21(7A+9B)\sec(c+dx)a^3\right)dx+\frac{2a^3(41A+42B)\sin(c+dx)\sec}{3d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(13A+21B)a^3+21(7A+9B)\csc\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx+\frac{2a^3(41A+42B)\text{si}}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(21a^3(7A+9B)\int \sec^{\frac{3}{2}}(c+dx)dx+5a^3(13A+21B)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2a^3(41A+42B)\sin(c+}{3d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+21a^3(7A+9B)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{2a^3(41A+\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{}{\sqrt{\sec(}}\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{}{\sqrt{\csc}}\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx)\sqrt{se}}{d}\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx}{}\right.\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(13A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21a^3(7A+9B)\left(\frac{2\sin(c+dx\right.\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\frac{2(11A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{5d}+\frac{2}{5}\left(\frac{2a^3(41A+42B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output
```
(2*a*A*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d) + ((2
*(11*A + 7*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(5
*d) + (2*((2*a^3*(41*A + 42*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + ((
10*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + 21*a^3*(7*A + 9*B)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))
/3))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4506
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(d*(m + n))   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x]
)^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
 a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(219) = 438$.

Time = 184.51 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.70

method	result	size
default	Expression too large to display	902
parts	Expression too large to display	1171

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETURNV
ERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+(1/8*A+3/8*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/5*(3/8*A+1/8*B)/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(
1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.08

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{9}{2}}(c + dx)\, dx =$$

$$-\frac{2\left(5i\sqrt{2}(13A + 21B)a^3\cos(dx + c)^3\,\mathrm{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) - 5i\sqrt{}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d*x + c)^3*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(13*A + 21*B)*a^3*cos(d
*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I
*sqrt(2)*(7*A + 9*B)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(7*A + 9*B)
*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c))) - (42*(7*A + 9*B)*a^3*cos(d*x + c)^3 + 5*(26*A
 + 21*B)*a^3*cos(d*x + c)^2 + 21*(3*A + B)*a^3*cos(d*x + c) + 15*A*a^3)*si
n(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{9}{2}}(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{9}{2}}(c + dx) \, dx$$
$$= \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{9}{2}} \, dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{9/2}(a+a\cos(c+dx))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{9}{2}}(c + dx)\, dx$$
$$= a^3 \left(3\left(\int \sqrt{\sec(dx + c)} \cos(dx + c) \sec(dx + c)^4 \, dx \right) a + \left(\int \sqrt{\sec(dx + c)} \cos(dx + c) \sec(dx + c)^4 \, dx \right) b + \left(\int \sqrt{\sec(dx + c)} \cos(dx + c)^4 \sec(dx + c)^4 \, dx \right) b + \left(\int \sqrt{\sec(dx + c)} \cos(dx + c)^3 \sec(dx + c)^4 \, dx \right) a + 3\left(\int \sqrt{\sec(dx + c)} \cos(dx + c)^3 \sec(dx + c)^4 \, dx \right) b + 3\left(\int \sqrt{\sec(dx + c)} \cos(dx + c)^2 \sec(dx + c)^4 \, dx \right) a + 3\left(\int \sqrt{\sec(dx + c)} \cos(dx + c)^2 \sec(dx + c)^4 \, dx \right) b + \left(\int \sqrt{\sec(dx + c)} \sec(dx + c)^4 \, dx \right) a \right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
a**3*(3*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*a + int(sqr
t(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x))
*cos(c + d*x)**4*sec(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x
)**3*sec(c + d*x)**4,x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**3*sec(c
 + d*x)**4,x)*b + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**4
,x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**4,x)*b + in
t(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)*a)
```

3.471 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 211

$$\begin{aligned}
&\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx \\
&= -\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{4a^3(3A+5B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} \\
&\quad+\frac{2aA\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d} \\
&\quad+\frac{2(9A+5B)\sqrt{\sec(c+dx)}(a^3+a^3\sec(c+dx))\sin(c+dx)}{15d}
\end{aligned}$$

output

```
-4/5*a^3*(9*A+5*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
sec(d*x+c)^(1/2)/d+4/3*a^3*(3*A+5*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+4/15*a^3*(21*A+20*B)*sec(d*x+c)^(1/2
)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+2/
15*(9*A+5*B)*sec(d*x+c)^(1/2)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.98 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{a^3e^{-idx}\csc(c)\sec(c)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(2(9A+5B)e^{-i(c-dx)}(-1+e^{4ic})\sqrt{1+e^{2i(c+dx)}}\,\mathrm{H}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x
]
```

output
```
(a^3*Csc[c]*Sec[c]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((2*(9*A + 5
*B)*(-1 + E^((4*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c - d*x)) + (Sec[c + d*x]^2*Sin[2
*c]*((-18*I)*(9*A + 5*B)*Cos[c + d*x] - (54*I)*A*Cos[3*(c + d*x)] - (30*I)
*B*Cos[3*(c + d*x)] + 40*(3*A + 5*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x
)/2, 2] + 66*A*Sin[c + d*x] + 45*B*Sin[c + d*x] + 30*A*Sin[2*(c + d*x)] +
10*B*Sin[2*(c + d*x)] + 54*A*Sin[3*(c + d*x)] + 45*B*Sin[3*(c + d*x)]))/2)
)/(30*d*E^(I*d*x))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4506, 27, 3042, 4506, 3042, 4485, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)^3(A\sec(c+dx)+B)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4506}$$

$$\frac{2}{5}\int -\frac{(\sec(c+dx)a+a)^2(a(A-5B)-a(9A+5B)\sec(c+dx))}{2\sqrt{\sec(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}-\frac{1}{5}\int \frac{(\sec(c+dx)a+a)^2(a(A-5B)-a(9A+5B)\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}-\frac{1}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(A-5B)-a(9A+5B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4506}$$

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}(a^3\sec(c+dx)+a^3)}{3d}-\frac{2}{3}\int \frac{(\sec(c+dx)a+a)\left(a^2(6A-5B)-a^2(21\right.}{\sqrt{\sec(c+dx)}}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\int\frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(a^2(6A-5B)-\right.}{\sqrt{\csc\left(c+dx\right.}}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$\Big\downarrow$ 4485

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(2\int\frac{3a^3(9A+5B)-5a^3(3A+5B)\sec(c+}{2\sqrt{\sec(c+dx)}}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(\int\frac{3a^3(9A+5B)-5a^3(3A+5B)\sec(c+}{\sqrt{\sec(c+dx)}}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(\int\frac{3a^3(9A+5B)-5a^3(3A+5B)\csc\left(c+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$\Big\downarrow$ 4274

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(3a^3(9A+5B)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3(3\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(3a^3(9A+5B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(-5a^3(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(-5a^3(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(-5a^3(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}-\frac{2}{3}\left(-\frac{2a^3(21A+20B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((2
*(9*A + 5*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*
d) - (2*((6*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d - (10*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a^3*(21*A + 20*B)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/d))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4506
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(d*(m + n))   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x]
)^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
 a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(190) = 380$.

Time = 186.37 (sec) , antiderivative size = 916, normalized size of antiderivative = 4.34

method	result	size
default	Expression too large to display	916
parts	Expression too large to display	1061

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^3*(216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*A*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-108*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^4+180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-60*B*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^4-246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*A*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+108*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*sin(1/2*d*x+1/2*c)^2-190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+100
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+60*B*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1
5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)\, dx =$$

$$-\frac{2\left(5i\sqrt{2}(3A + 5B)a^3\cos(dx + c)^2 \,\mathrm{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) - 5i\sqrt{2}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(3*A + 5*B)*a^3*cos(d*x +
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(
2)*(9*A + 5*B)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 5*B)*a^3*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) - (9*(6*A + 5*B)*a^3*cos(d*x + c)^2 + 5*(3*A + B)*a^3
*cos(d*x + c) + 3*A*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)\, dx = \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)\, dx = \int (A + B\cos(c + dx)) \left(\frac{1}{\cos(c + dx)}\right)^{7/2} (a + a\cos(c + dx))^3\, dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= a^3\left(3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)a + 3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)b + 3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a + 3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
a**3*(3*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a + int(sqr
t(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))
*cos(c + d*x)**4*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x
)**3*sec(c + d*x)**3,x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**3*sec(c
 + d*x)**3,x)*b + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3
,x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*b + in
t(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*a)
```

3.472 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 199

$$\begin{aligned}
&\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx \\
&= -\frac{4a^3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad+\frac{20a^3(A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} \\
&\quad+\frac{2(A-B)\sqrt{\sec(c+dx)}(a^3+a^3\sec(c+dx))\sin(c+dx)}{3d}
\end{aligned}$$

output
```
-4*a^3*(A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*
x+c)^(1/2)/d+20/3*a^3*(A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))*sec(d*x+c)^(1/2)/d+4/3*a^3*(4*A+B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d
+2/3*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*(A-B)*sec(d*
x+c)^(1/2)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.58 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-12iA+12iB-12iA\cos(2(c+dx))+12iB\cos(2(c+dx))\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x
]
```

output
```
(a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((-12*I)*A + (12*I)*B - (1
2*I)*A*Cos[2*(c + d*x)] + (12*I)*B*Cos[2*(c + d*x)] + 40*(A + B)*Cos[c + d
*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + (4*I)*(A - B)*(1 + E^((2*I)*(c + d*x
)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 4*A*Sin
[c + d*x] + B*Sin[c + d*x] + 18*A*Sin[2*(c + d*x)] + 6*B*Sin[2*(c + d*x)]
 + B*Sin[3*(c + d*x)]))/(6*d*E^(I*d*x))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3439, 3042, 4505, 27, 3042, 4506, 27, 3042, 4485, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3439}$$

$$\int \frac{(a\sec(c+dx)+a)^3(A\sec(c+dx)+B)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4505}$$

$$\frac{2}{3}\int \frac{(\sec(c+dx)a+a)^2(a(3A+7B)+3a(A-B)\sec(c+dx))}{2\sqrt{\sec(c+dx)}}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(\sec(c+dx)a+a)^2(a(3A+7B)+3a(A-B)\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(3A+7B)+3a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4506}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{3(\sec(c+dx)a+a)\left((A+4B)a^2+(4A+B)\sec(c+dx)a^2\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}(a}{d}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(2\int \frac{(\sec(c+dx)a+a)\left((A+4B)a^2+(4A+B)\sec(c+dx)a^2\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}(a^3}{d}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((A+4B)a^2+(4A+B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(A-B)\sin(c+dx)\sqrt{\sec(}}{}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4485

$$\frac{1}{3}\left(2\left(2\int -\frac{3a^3(A-B)-5a^3(A+B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right)+\frac{2(A-B)\mathrm{si}}{}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(2\left(\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{3a^3(A-B)-5a^3(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2(A-B)\sin(c}{}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(2\left(\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{3a^3(A-B)-5a^3(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2(A-B)}{}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4274

$$\frac{1}{3}\left(2\left(-3a^3(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3(A+B)\int \sqrt{\sec(c+dx)}dx+\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(2\left(-3a^3(A-B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3(A+B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^3(4A+B)\sin(c+dx)}{d}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4258

$$\frac{1}{3}\left(2\left(5a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-3a^3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(2\left(5a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3119

$$\frac{1}{3}\left(2\left(5a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{d}+2\left(\frac{2a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{10a^3}{}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((2
*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/d + 2*(
(-6*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (10*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/d + (2*a^3*(4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4505
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Sim
p[b/(a*d*n)   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Sim
p[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0
] && GtQ[m, 1/2] && LtQ[n, -1]
```

rule 4506
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(d*(m + n))   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x]
)^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
 a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(180) = 360$.

Time = 185.51 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.29

method	result
default	$-\frac{4\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,(9A+5B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}{}$
parts	Expression too large to display

input

```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output

```
-4/3*(-4*B*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*
B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(A+B)a^3\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}(A+\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A - B)*a^
3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(A - B)*a^3*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (B
*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+a\cos(c+dx))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=a^3\Bigg(3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\sec(dx+c)^2\,dx\right)a$$
$$+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\sec(dx+c)^2\,dx\right)b$$
$$+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^4\sec(dx+c)^2\,dx\right)b$$
$$+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^3\sec(dx+c)^2\,dx\right)a$$
$$+3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b$$
$$+3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a$$
$$+3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b$$
$$+\left(\int\sqrt{\sec(dx+c)}\sec(dx+c)^2\,dx\right)a\Bigg)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
a**3*(3*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a + int(sqr
t(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))
*cos(c + d*x)**4*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x
)**3*sec(c + d*x)**2,x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**3*sec(c
 + d*x)**2,x)*b + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2
,x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*b + in
t(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*a)
```

3.473 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 211

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx &= \frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &+ \frac{4a^3(5A-6B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} \\ &+ \frac{2(5A+9B)\left(a^3+a^3\sec(c+dx)\right)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
4/5*a^3*(5*A+9*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/d+4/3*a^3*(5*A+3*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+4/15*a^3*(5*A-6*B)*sec(d*x+c)^(1/2)*s
in(d*x+c)/d+2/5*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*
(5*A+9*B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)\, dx$$
$$= \frac{a^3 e^{-idx} \sqrt{\sec(c + dx)} (\cos(dx) + i\sin(dx)) \left(120iA\cos(c + dx) + 216iB\cos(c + dx) + 40(5A + 3B)\sqrt{\cos\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x
]
```

output
```
(a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((120*I)*A*Cos[c + d*x] +
(216*I)*B*Cos[c + d*x] + 40*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2] - (8*I)*(5*A + 9*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 60*A*Sin[c + d
*x] + 3*B*Sin[c + d*x] + 10*A*Sin[2*(c + d*x)] + 30*B*Sin[2*(c + d*x)] + 3
*B*Sin[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4505, 27, 3042, 4505, 3042, 4485, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c + dx)(a\cos(c + dx) + a)^3 (A + B\cos(c + dx))\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c + dx + \frac{\pi}{2}\right)^{3/2} \left(a\sin\left(c + dx + \frac{\pi}{2}\right) + a\right)^3 \left(A + B\sin\left(c + dx + \frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)^3(A\sec(c+dx)+B)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4505}$$

$$\frac{2}{5}\int \frac{(\sec(c+dx)a+a)^2(a(5A+9B)+a(5A-B)\sec(c+dx))}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{(\sec(c+dx)a+a)^2(a(5A+9B)+a(5A-B)\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(5A+9B)+a(5A-B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4505}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{(\sec(c+dx)a+a)\left((20A+21B)a^2+(5A-6B)\sec(c+dx)a^2\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2(5A+9B)\sin(c+dx)\left(a^3\sec(c+\right.}{3d\sqrt{\sec(c+dx)}}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((20A+21B)a^2+(5A-6B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(5A+9B)\sin(c+d}{3d\sqrt{\sec}}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 4485$$

$$\frac{1}{5}\left(\frac{2}{3}\left(2\int \frac{3(5A+9B)a^3+5(5A+3B)\sec(c+dx)a^3}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right)+\frac{2(5A+\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\left(\frac{2}{3}\left(\int \frac{3(5A+9B)a^3+5(5A+3B)\sec(c+dx)a^3}{\sqrt{\sec(c+dx)}}dx+\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right)+\frac{2(5A+9\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{2}{3}\left(\int \frac{3(5A+9B)a^3+5(5A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right)+\frac{2(\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 4274$$

$$\frac{1}{5}\left(\frac{2}{3}\left(3a^3(5A+9B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3(5A+3B)\int \sqrt{\sec(c+dx)}dx+\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\sec}}{d}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{2}{3}\left(3a^3(5A+9B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3(5A+3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^3(5A-6B)\sin(c}{d}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 4258$$

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(5A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3(5A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(\frac{2}{3}\left(5a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{2(5A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\left(\frac{2a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{10a^3(5A+3B}{}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2
*(5*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]
]) + (2*((6*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d + (10*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(5*A - 6*B)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/d))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4505
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Sim
p[b/(a*d*n)   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Sim
p[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0
] && GtQ[m, 1/2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.60

method	result
default	$-\frac{4a^3\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+10A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+42B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*a^3*(-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+42*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2
0*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-18*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx =$$
$$-\frac{2\left(5i\sqrt{2}(5A+3B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(5A+3B)a^3\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*(5*A + 3*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*(5*A + 3*B)*a^3*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 9*B)*a^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3
*I*sqrt(2)*(5*A + 9*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - (3*B*a^3*cos(d*x + c)^2 + 5*(A + 3*B)
*a^3*cos(d*x + c) + 15*A*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)\, dx$$
$$= \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)\, dx$$
$$= \int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=a^3\left(3\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right)a\right.$$
$$+\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right)b$$
$$+\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)dx\right)b$$
$$+\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)dx\right)a$$
$$+3\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)dx\right)b$$
$$+3\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)dx\right)a$$
$$+3\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)dx\right)b$$
$$\left.+\left(\int\sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
a**3*(3*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a + int(sqrt(s
ec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*cos(c
 + d*x)**4*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x)**3*sec(
c + d*x),x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*b
 + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*a + 3*int(sqrt
(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*
sec(c + d*x),x)*a)
```

3.474 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 211

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} \\ &+ \frac{2(7A+11B)\left(a^3+a^3\sec(c+dx)\right)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
4/5*a^3*(9*A+7*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/d+4/21*a^3*(21*A+13*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/
2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+4/105*a^3*(42*A+41*B)*sin(d*x+c)/d
/sec(d*x+c)^(1/2)+2/7*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+2/35*(7*A+11*B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.61 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(40(21A+13B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-56i(\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x
]
```

output
```
(a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(21*A + 13*B)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(9*A + 7*B)*E^(I*(c + d*x))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))] + Cos[c + d*x]*((168*I)*(9*A + 7*B) + 5*(84*A + 107*B)*Sin[c +
d*x] + 42*(A + 3*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(
I*d*x))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4505, 27, 3042, 4505, 3042, 4484, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+a)^3(A\sec(c+dx)+B)}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4505}$$

$$\frac{2}{7}\int \frac{(\sec(c+dx)a+a)^2(a(7A+11B)+a(7A+B)\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(\sec(c+dx)a+a)^2(a(7A+11B)+a(7A+B)\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(7A+11B)+a(7A+B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4505}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{(\sec(c+dx)a+a)\left((42A+41B)a^2+(21A+8B)\sec(c+dx)a^2\right)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2(7A+11B)\sin(c+dx)\left(a^3\sec(\right.}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((42A+41B)a^2+(21A+8B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2(7A+11B)\sin(c+}{5d\sec}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4484}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{2a^3(42A+41B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{21(9A+7B)a^3+5(21A+13B)\sec(c+dx)a^3}{2\sqrt{\sec(c+dx)}}dx\right)+\frac{2(7A+11B\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{21(9A+7B)a^3+5(21A+13B)\sec(c+dx)a^3}{\sqrt{\sec(c+dx)}}dx+\frac{2a^3(42A+41B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2(7A+11B)\,s\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{21(9A+7B)a^3+5(21A+13B)\csc\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(42A+41B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2(7A+\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(21a^3(9A+7B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3(21A+13B)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2a^3(42A+41B)\sin(c}{3d\sqrt{\sec(c+dx)}}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(21a^3(9A+7B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3(21A+13B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^3(42A+}{3d\sqrt{}}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+21a^3(9A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+21a^3(9A+7B)\sqrt{\cos(c+dx)}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{7}\left(\frac{2}{5}\left(\frac{1}{3}\left(5a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{42a^3(9A+7B)\sqrt{\cos(c+dx)}}{}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{7}\left(\frac{2(7A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\left(\frac{2a^3(42A+41B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(\frac{10a^3(21A+13B)\sqrt{}}{}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((2
*(7*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/
2)) + (2*(((42*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d + (10*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/3 + (2*a^3*(42*A + 41*B)*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

rule 4505
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Sim
p[b/(a*d*n)   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Sim
p[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0
] && GtQ[m, 1/2] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(190) = 380$.

Time = 15.72 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.82

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, a^{3}\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-84A-432B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(294A-\right.}{}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-126*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}(21A+13B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(21A+13\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(21*A + 13*B)*a^3*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(9*A + 7*B)*a^3*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 21*I*sqrt(2)*(9*A + 7*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*B*a^3*cos(d*x + c)^3 + 21*
(A + 3*B)*a^3*cos(d*x + c)^2 + 5*(21*A + 26*B)*a^3*cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= a^3\left(\int A\sqrt{\sec(c+dx)}\,dx+\int 3A\cos(c+dx)\sqrt{\sec(c+dx)}\,dx\right.\\ &+\int 3A\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int A\cos^3(c+dx)\sqrt{\sec(c+dx)}\,dx\\ &+\int B\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int 3B\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx\\ &\left.+\int 3B\cos^3(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos^4(c+dx)\sqrt{\sec(c+dx)}\,dx\right)\end{aligned}$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
a**3*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(3*A*cos(c + d*x)*sqrt(s
ec(c + d*x)), x) + Integral(3*A*cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + I
ntegral(A*cos(c + d*x)**3*sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)
*sqrt(sec(c + d*x)), x) + Integral(3*B*cos(c + d*x)**2*sqrt(sec(c + d*x)),
 x) + Integral(3*B*cos(c + d*x)**3*sqrt(sec(c + d*x)), x) + Integral(B*cos
(c + d*x)**4*sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}(a+a\cos(c+dx))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=a^3\left(\left(\int\sqrt{\sec(dx+c)}dx\right)a+3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\,dx\right)a+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^4\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^3\,dx\right)a+3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^3\,dx\right)b+3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^2\,dx\right)a+3\left(\int\sqrt{\sec(dx+c)}\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
a**3*(int(sqrt(sec(c + d*x)),x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x),
x)*a + int(sqrt(sec(c + d*x))*cos(c + d*x),x)*b + int(sqrt(sec(c + d*x))*c
os(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x))*cos(c + d*x)**3,x)*a + 3*int(
sqrt(sec(c + d*x))*cos(c + d*x)**3,x)*b + 3*int(sqrt(sec(c + d*x))*cos(c +
 d*x)**2,x)*a + 3*int(sqrt(sec(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.475 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 244

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx &= \frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &+ \frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} \\ &+ \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} \\ &+ \frac{2(9A+13B)\left(a^3+a^3\sec(c+dx)\right)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output
```
4/15*a^3*(21*A+17*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*sec(d*x+c)^(1/2)/d+4/21*a^3*(13*A+11*B)*cos(d*x+c)^(1/2)*InverseJacobiAM
(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+4/105*a^3*(24*A+23*B)*sin(d*x+c
)/d/sec(d*x+c)^(3/2)+4/21*a^3*(13*A+11*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/
9*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/63*(9*A+13*B)*(a^
3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.80 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.80

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{a^3\sqrt{\sec(c+dx)}\left(240(13A+11B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-112i(21A+17B)e^{i(c+dx)}\sqrt{1-}\right.}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]]
,x]
```

output
```
(a^3*Sqrt[Sec[c + d*x]]*(240*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2] - (112*I)*(21*A + 17*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c +
 d*x]*((7056*I)*A + (5712*I)*B + 30*(107*A + 97*B)*Sin[c + d*x] + 14*(54*A
 + 73*B)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 270*B*Sin[3*(c + d*x)]
 + 35*B*Sin[4*(c + d*x)])))/(1260*d)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.02, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4505, 27, 3042, 4505, 27, 3042, 4484, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3439$$

$$\int \frac{(a\sec(c+dx)+a)^3(A\sec(c+dx)+B)}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \; 4505$$

$$\frac{2}{9}\int \frac{(\sec(c+dx)a+a)^2(a(9A+13B)+3a(3A+B)\sec(c+dx))}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\int \frac{(\sec(c+dx)a+a)^2(a(9A+13B)+3a(3A+B)\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(a(9A+13B)+3a(3A+B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4505$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{3(\sec(c+dx)a+a)\left((24A+23B)a^2+5(3A+2B)\sec(c+dx)a^2\right)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec\right.}{7d\sec^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{6}{7}\int \frac{(\sec(c+dx)a+a)\left((24A+23B)a^2+5(3A+2B)\sec(c+dx)a^2\right)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec(\right.}{7d\sec^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((24A+23B)a^2+5(3A+2B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2(9A+13B)\sin(c+\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4484

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{2a^3(24A+23B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{15(13A+11B)a^3+7(21A+17B)\sec(c+dx)a^3}{2\sec^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2(9A+13\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\int \frac{15(13A+11B)a^3+7(21A+17B)\sec(c+dx)a^3}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^3(24A+23B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2(9A+13B\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\int \frac{15(13A+11B)a^3+7(21A+17B)\csc\left(c+dx+\frac{\pi}{2}\right)a^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^3(24A+23B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2(9A\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4274

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(13A+11B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+7a^3(21A+17B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^3(24A+23B)\sin(}{5d\sec^{\frac{3}{2}}(c+dx}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(13A+11B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+7a^3(21A+17B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^3(24A}{5c}\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4256

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(7a^3(21A+17B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+15a^3(13A+11B)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+d}{3d\sqrt{\sec(c+}}\right.\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(7a^3(21A+17B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+15a^3(13A+11B)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin}{3d\sqrt{se}}\right.\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(7a^3(21A+17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+15a^3(13A+11B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\right.\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(13A+11B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+7a\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{9}\left(\frac{6}{7}\left(\frac{1}{5}\left(15a^3(13A+11B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{14}{}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3120

$$\frac{1}{9}\left(\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{6}{7}\left(\frac{2a^3(24A+23B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}\left(\frac{14a^3(21A+17B)\sqrt{}}{}\right.\right.\right.$$
$$\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((2
*(9*A + 13*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/
2)) + (6*((2*a^3*(24*A + 23*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((
14*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + 15*a^3*(13*A + 11*B)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
 d*x]])))/5))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

rule 4505
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Sim
p[b/(a*d*n)   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Sim
p[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0
] && GtQ[m, 1/2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 18.20 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(360A+2200B)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-1\right.}{}$
parts	Expression too large to display

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c
)^8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*
B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-441*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2\left(15i\sqrt{2}(13A+11B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}(13A+1\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-2/315*(15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(13*A + 11*B)*a^3*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(21*A + 17*B)*a^3*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 21*I*sqrt(2)*(21*A + 17*B)*a^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*B*a^3*cos(d*x + c)^4
 + 45*(A + 3*B)*a^3*cos(d*x + c)^3 + 7*(27*A + 34*B)*a^3*cos(d*x + c)^2 +
30*(13*A + 11*B)*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= a^3\left(\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{3A\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{3A\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{A\cos^3(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{3B\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{3B\cos^3(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos^4(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
a**3*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(3*A*cos(c + d*x)/sqrt(s
ec(c + d*x)), x) + Integral(3*A*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + I
ntegral(A*cos(c + d*x)**3/sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)
/sqrt(sec(c + d*x)), x) + Integral(3*B*cos(c + d*x)**2/sqrt(sec(c + d*x)),
 x) + Integral(3*B*cos(c + d*x)**3/sqrt(sec(c + d*x)), x) + Integral(B*cos
(c + d*x)**4/sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+a\,\cos\left(c+d\,x\right)\right)^3}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = a^3\left(\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a+3\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)a + \left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^4}{\sec(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right)a+3\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right)b + 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)a + 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)b\right)$$

input
```
int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a + 3*int((sqrt(sec(c + d*x))
*cos(c + d*x))/sec(c + d*x),x)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/s
ec(c + d*x),x)*b + int((sqrt(sec(c + d*x))*cos(c + d*x)**4)/sec(c + d*x),x
)*b + int((sqrt(sec(c + d*x))*cos(c + d*x)**3)/sec(c + d*x),x)*a + 3*int((
sqrt(sec(c + d*x))*cos(c + d*x)**3)/sec(c + d*x),x)*b + 3*int((sqrt(sec(c
+ d*x))*cos(c + d*x)**2)/sec(c + d*x),x)*a + 3*int((sqrt(sec(c + d*x))*cos
(c + d*x)**2)/sec(c + d*x),x)*b)
```

3.476 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 193

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &+ \frac{(5A-3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\ &- \frac{3(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} \\ &+ \frac{(5A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
3*(A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^
(1/2)/a/d+1/3*(5*A-3*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(
1/2))*sec(d*x+c)^(1/2)/a/d-3*(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d+1/3*(5*
A-3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d
/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.00 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.37

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,\text{Hypergeo}\right.}{\sqrt{2}d(a+a\cos(c+dx))}$$
$$+\frac{Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,\text{Hypergeo}\right.}{\sqrt{2}d(a+a\cos(c+dx))}$$
$$+\frac{5A\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}\sin(c)}{3d(a+a\cos(c+dx))}$$
$$-\frac{B\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}\sin(c)}{d(a+a\cos(c+dx))}$$
$$+\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{3(A-B)\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)}{d}+\frac{2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{d}+\frac{4A\sec(c)}{}\right.}{a+a\cos(c+dx)}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x
]
```

output
```
-((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] +
 E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x]))) + (B
*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*
x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(
(2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x])) + (5*A*Co
s[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])) - (B*Cos[c/2
 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/
2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])) + (Cos[c/2 + (d*x)/
2]^2*Sqrt[Sec[c + d*x]]*((-3*(A - B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (4*A*Sec[
c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (2*(2*A + 5*A*Cos[c] - 3*B*Cos[c])*Sec[c
]*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A\sec(c+dx)+B)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int -\frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(3a(A-B)-a(5A-3B)\sec(c+dx))dx}{a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \sec^{\frac{3}{2}}(c+dx)(3a(A-B)-a(5A-3B)\sec(c+dx))dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a(A-B)-a(5A-3B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{3a(A-B)\int \sec^{\frac{3}{2}}(c+dx)dx-a(5A-3B)\int \sec^{\frac{5}{2}}(c+dx)dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a(A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-a(5A-3B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{3a(A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-a(5A-3B)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right.}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a(A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-a(5A-3B)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\,\text{s}}{3\text{a}}\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\big\downarrow$ 4258

$$-\frac{3a(A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-a(5A-3B)\left(\frac{1}{3}\sqrt{\cos(c+\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\big\downarrow$ 3042

$$-\frac{3a(A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-a(5A-3B)\left(\frac{1}{3}\sqrt{\text{co}\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\big\downarrow$ 3119

$$-\frac{3a(A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-a(5A-3B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\big\downarrow$ 3120

$$-\frac{3a(A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-a(5A-3B)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2}{\cdots}\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```

output
```
-(((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - (3
*a*(A - B)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) - a*(5*A - 3*B)*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(178) = 356$.

Time = 7.35 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.41

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{(-2A+2B)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((-2*A+2*B)/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))+(A-B)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{\left(\sqrt{2}(-5i\,A+3i\,B)\cos{(dx+c)}^2+\sqrt{2}(-5i\,A+3i\,B)\cos{(dx+c)}\right)\text{weierstrassPInverse}(-4,0,\cos{(dx+}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/6*((sqrt(2)*(-5*I*A + 3*I*B)*cos(d*x + c)^2 + sqrt(2)*(-5*I*A + 3*I*B)*c
os(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (
sqrt(2)*(5*I*A - 3*I*B)*cos(d*x + c)^2 + sqrt(2)*(5*I*A - 3*I*B)*cos(d*x +
 c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2
)*(-I*A + I*B)*cos(d*x + c)^2 + sqrt(2)*(-I*A + I*B)*cos(d*x + c))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^2 + sqrt(2)*(I*A - I*B)*cos(d*x + c
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - 2*(9*(A - B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c) - 2*
A)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{(A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+a\cos(c+d\,x)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x)),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^2}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)+1}dx\right)a}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

output
```
(int((sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2)/(cos(c + d*x) + 1),
x)*b + int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(cos(c + d*x) + 1),x)*a)/a
```

3.477 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(3A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad -\frac{(A-B)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad +\frac{(3A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output

```
-(3*A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/a/d-(A-B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*s
ec(d*x+c)^(1/2)/a/d+(3*A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d-(A-B)*sec(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.45 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.52

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(6\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,\mathrm{Hyp}\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x
]
```

output
```
(Cos[(c + d*x)/2]^2*((6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))]))/E^(I*d*x) - (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^
((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/
2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 12*B*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 6*Sqrt[Sec[c + d*x]]*(2*(3*A -
B)*Cos[d*x]*Csc[c] + 2*(-A + B)*Tan[(c + d*x)/2])))/(6*a*d*(1 + Cos[c + d*
x]))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A\sec(c+dx)+B)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int -\frac{1}{2}\sqrt{\sec(c+dx)}(a(A-B)-a(3A-B)\sec(c+dx))dx}{a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \sqrt{\sec(c+dx)}(a(A-B)-a(3A-B)\sec(c+dx))dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(A-B)-a(3A-B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{a(A-B)\int \sqrt{\sec(c+dx)}dx-a(3A-B)\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a(A-B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-a(3A-B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ \textbf{4255}$$

$$-\frac{a(A-B)\int \sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}dx-a(3A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ \textbf{3042}$$

$$-\frac{a(A-B)\int \sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}dx-a(3A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ \textbf{4258}$$

$$-\frac{a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-a(3A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ \textbf{3042}$$

$$-\frac{a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx-a(3A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{s\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ \textbf{3119}$$

$$-\frac{a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx-a(3A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ \textbf{3120}$$

$$-\frac{\frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-a(3A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}\right.}{2a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```

output
```
-(((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - ((
2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d - a*(3*A - B)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(150) = 300$.

Time = 3.76 (sec), antiderivative size = 319, normalized size of antiderivative = 2.01

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1
/2*c)^2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.57

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\left(\sqrt{2}(i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}(i\,A-i\,B)\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/2*((sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-I*A + I*B)*cos(
d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + (sqrt(2)*(-3*I*A + I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A
+ I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) + (sqrt(2)*(3*I*A - I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - I
*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))) + 2*((3*A - B)*cos(d*x + c) + 2*A)*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x)),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\cos(dx+c)+1}dx\right)a}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

output
```
(int((sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x))/(cos(c + d*x) + 1),x)*
b + int((sqrt(sec(c + d*x))*sec(c + d*x))/(cos(c + d*x) + 1),x)*a)/a
```

3.478 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 123

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
(A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/a/d+(A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(
d*x+c)^(1/2)/a/d-(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.02 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = -\frac{e^{-\frac{1}{2}i(4c+dx)}(-1+e^{2ic})\left(3i(A+B)\left(1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A-B)\left(-\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x
]
```

output
```
-1/24*((-1 + E^((2*I)*c))*((3*I)*(A + B)*(1 + E^(I*(c + d*x)))*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + (A - B)*(-3*(1 + E^((2*I)*(c + d*x)))
+ E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*(Csc[c/2] + I*Sec[c/2
])*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/(a*d*E^((I/2)*(4*c + d*x)))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sqrt{\sec(c+dx)}(A\sec(c+dx)+B)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int \frac{a(A-B)+a(A+B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-B)+a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-B)+a(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{a(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a(A+B)\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A-B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(A+B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx\right.}}{2a^2}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3119

$$\frac{a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2} -$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2} -$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```

output
```
((2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/d)/(2*a^2) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(116) = 232.

Time = 3.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.98

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(A\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A\,\text{Elli}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\text{s}}}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2
*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.96

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = -\frac{2\,(A-B)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}(-i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}(-i\,A-i\,B)\right)\text{weierstrassP}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-I*A - I*B)*co
s(d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - (sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*
B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (sqrt(2)*(
I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - (sqrt(2)*(-I*A +
I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{a + a\cos(c + dx)}\,dx = \frac{\int \frac{A\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

output
```
(Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x) + Integral(B*cos(c +
 d*x)*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x))/a
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{a + a\cos(c + dx)}\,dx = \int \frac{(B\cos(dx + c) + A)\sqrt{\sec(dx + c)}}{a\cos(dx + c) + a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx=\int \frac{(A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}}{a+a\cos(c+dx)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x)),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)}{\cos(dx+c)+1}dx\right)b}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x) + 1),x)*a + int((sqrt(sec(c + d*x))*
cos(c + d*x))/(cos(c + d*x) + 1),x)*b)/a
```

3.479 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 125

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx &= -\frac{(A-3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}\\ &+\frac{(A-B)\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad}\\ &+\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
-(A-3*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/a/d+(A-B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*s
ec(d*x+c)^(1/2)/a/d+(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.64 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.38

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,\text{Hyp}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x
]
```

output
```
(Cos[(c + d*x)/2]^2*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))]))/E^(I*d*x) - (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^
((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/
2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*((A - 2*B)*Cos[(c - d*
x)/2] - B*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec
[c + d*x]] + 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]] - 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]]))/(6*a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3439, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4508}$$

$$\frac{\int -\frac{a(A-3B)-a(A-B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a(A-3B)-a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a(A-3B)-a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a(A-3B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a(A-B)\int \sqrt{\sec(c+dx)}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a(A-3B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a(A-B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx}{2a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

output
```
-1/2*((2*a*(A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d - (2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d)/a^2 + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*
(a + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(118) = 236$.

Time = 4.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.95

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+A\,\mathrm{Ell}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1
/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.90

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2\,(A-B)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}(-i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}(-i\,A+i\,B)\right)\text{weierstrassPIn}}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-I*A + I*B)*cos
(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + (sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B
))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-
I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 3*I*B))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(I*
A - 3*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 3*I*B))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c
) + a*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\int \frac{A}{\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}+\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{B\cos\left(c+dx\right)}{\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}+\sqrt{\sec\left(c+dx\right)}}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
(Integral(A/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x) + I
ntegral(B*cos(c + d*x)/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x
))), x))/a
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right)b}{a}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*a +
int((sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)*sec(c + d*x) + sec(c +
 d*x)),x)*b)/a
```

3.480 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 163

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} - \frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}$$

output
```
3*(A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^
(1/2)/a/d-1/3*(3*A-5*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(
1/2))*sec(d*x+c)^(1/2)/a/d-1/3*(3*A-5*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+(
A-B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.58 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.72

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-6\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\operatorname{Hy}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x
]
```

output
```
(Cos[(c + d*x)/2]^2*((-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E
^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 20*B*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - (Csc[c/2]*Sec[c/2]*Sec[(c + d*
x)/2]*((12*A - 13*B)*Cos[(c - d*x)/2] + (6*A - 5*B)*Cos[(3*c + d*x)/2] - 2
*B*Sin[c]*Sin[(3*(c + d*x))/2]))/Sqrt[Sec[c + d*x]]))/(6*a*d*(1 + Cos[c +
d*x]))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4508}$$

$$\frac{\int -\frac{a(3A-5B)-3a(A-B)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{\int \frac{a(3A-5B)-3a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{\int \frac{a(3A-5B)-3a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-3a(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-3a(A-B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \text{4256}$$

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a(A-B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$\big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a(A-B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$\big\downarrow$ 4258

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{2a^2}$$

$\big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{2a^2}$$

$\big\downarrow$ 3119

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{6a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d}}{2a^2}$$

$\big\downarrow$ 3120

$$\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{a(3A-5B)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{6a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

output
```
((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) - ((-6*
a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/d + a*(3*A - 5*B)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.61

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(3A\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9A\,\text{Elli}\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin}}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(
-3*A+7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.60

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}\,dx$$

$$= \frac{\left(\sqrt{2}(3i\,A - 5i\,B)\cos\left(dx + c\right) + \sqrt{2}(3i\,A - 5i\,B)\right)\mathrm{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right) + i\,\sin\left(dx + \right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/6*((sqrt(2)*(3*I*A - 5*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 5*I*B))*weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(-3*I*A +
 5*I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + 5*I*B))*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) +
sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt
(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*(2*B*cos(d*x + c)^2 - (3*A - 5*B)*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\int \frac{A}{\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B\cos(c+dx)}{\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
(Integral(A/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(3/2)), x) +
 Integral(B*cos(c + d*x)/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)*
*(3/2)), x))/a
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)\sec(dx+c)^2+\sec(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\cos(dx+c)\sec(dx+c)^2+\sec(dx+c)^2}dx\right)b}{a}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2),x
)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)*sec(c + d*x)**2
+ sec(c + d*x)**2),x)*b)/a
```

3.481 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 196

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = -\frac{3(5A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(A-B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} - \frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

output
```
-3/5*(5*A-7*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(
d*x+c)^(1/2)/a/d+5/3*(A-B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))*sec(d*x+c)^(1/2)/a/d-1/5*(5*A-7*B)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2
)+5/3*(A-B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+(A-B)*sin(d*x+c)/d/sec(d*x+c)^
(3/2)/(a+a*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.52 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.64

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(60\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,\mathrm{Hy}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x
]
```

output
```
(Cos[(c + d*x)/2]^2*((60*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))]))/E^(I*d*x) - (84*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1
+ E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 +
E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 200*A*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 200*B*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c + d*x]]*(3*(40*A
 - 51*B + (20*A - 33*B)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 40*(A - B)*
Cos[2*d*x]*Sin[2*c] + 12*B*Cos[3*d*x]*Sin[3*c] - 120*(A - B)*Sec[c/2]*Sec[
(c + d*x)/2]*Sin[(d*x)/2] - 12*(20*A - 33*B)*Cos[c]*Sin[d*x] + 40*(A - B)*
Cos[2*c]*Sin[2*d*x] + 12*B*Cos[3*c]*Sin[3*d*x] - 120*(A - B)*Tan[c/2])))/(
60*a*d*(1 + Cos[c + d*x]))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$

$\Big\downarrow$ 3042

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$\Big\downarrow$ 3439

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$\Big\downarrow$ 3042

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$\Big\downarrow$ 4508

$$\frac{\int -\frac{a(5A-7B)-5a(A-B)\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}+\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 27

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{\int \frac{a(5A-7B)-5a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{\int \frac{a(5A-7B)-5a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}$$

$\Big\downarrow$ 4274

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-5a(A-B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-5a(A-B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}$$

$\Big\downarrow$ 4256

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a(A-B)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a(A-B)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a(A-B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{s\right.}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a(A-B)\left(\frac{1}{3}\sqrt{\cos(c+a\right.}{2a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a(A-B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{2a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{a(5A-7B)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a(A-B)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{2a^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

output
```
((A - B)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])) - (a*(5
*A - 7*B)*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 5*a*(A - B)*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (
2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 7.02 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(25A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+45\right.\right.}{15}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1
/2*c)^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2
)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.42

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{25\left(\sqrt{2}(i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}(i\,A-i\,B)\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/30*(25*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*(sqrt(2)*(-I*A + I
*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) + 9*(sqrt(2)*(5*I*A - 7*I*B)*cos(d*x + c) + sqrt(
2)*(5*I*A - 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 9*(sqrt(2)*(-5*I*A + 7*I*B)*cos(d*x + c) + s
qrt(2)*(-5*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - 2*(6*B*cos(d*x + c)^3 + 2*(5*A - 2*B)*c
os(d*x + c)^2 + 25*(A - B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/
(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{A+B\cos(c+dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+a\cos(c+dx))}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)\sec(dx+c)^3+\sec(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)}{\cos(dx+c)\sec(dx+c)^3+\sec(dx+c)^3}dx\right)b}{a}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)*sec(c + d*x)**3 + sec(c + d*x)**3),x
)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)*sec(c + d*x)**3
+ sec(c + d*x)**3),x)*b)/a
```

3.482 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 208

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{(4A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad-\frac{(5A-2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ &\quad+\frac{(4A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} \\ &\quad-\frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-(4*A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/a^2/d-1/3*(5*A-2*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))*sec(d*x+c)^(1/2)/a^2/d+(4*A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d-
1/3*(5*A-2*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*s
ec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.77 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.46

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(29iA-5iB+2i(25A-7B)\cos(c+dx)+17iA\cos(2(c+dx))-}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2
,x]
```

output
```
-1/6*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*((29*I)*A - (5*I)*B + (2*I)*(25*
A - 7*B)*Cos[c + d*x] + (17*I)*A*Cos[2*(c + d*x)] - (5*I)*B*Cos[2*(c + d*x
)] - (I*(4*A - B)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 8*
(5*A - 2*B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) - 12*A*Sin[c + d*x] - 7*A*Sin[2*
(c + d*x)] + B*Sin[2*(c + d*x)])*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2
]))/(a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4507, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$\Big\downarrow$ 3439

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+a)^2}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$\Big\downarrow$ 4507

$$\frac{\int -\frac{\sec^{\frac{3}{2}}(c+dx)(3a(A-B)-a(7A-B)\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a(A-B)-a(7A-B)\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a(A-B)-a(7A-B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\int \sqrt{\sec(c+dx)}\left(a^2(5A-2B)-3a^2(4A-B)\sec(c+dx)\right)dx}{a^2}+\frac{2(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2(5A-2B)-3a^2(4A-B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{2(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{a^2(5A-2B)\int\sqrt{\sec(c+dx)}dx-3a^2(4A-B)\int\sec^{\frac{3}{2}}(c+dx)dx}{a^2}+\frac{2(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a^2(5A-2B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a^2(4A-B)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{a^2}+\frac{2(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4255

$$-\frac{\frac{a^2(5A-2B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a^2(4A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}+\frac{2(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a^2(5A-2B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a^2(4A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{2(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4258

$$-\frac{\frac{a^2(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-3a^2(4A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{a^2}+}{6a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a^2(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^2(4A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.}{a^2}}{6a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3119

$$-\frac{\frac{a^2(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^2(4A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{a^2}+2}{6a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3120

$$-\frac{\frac{\frac{2a^2(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-3a^2(4A-B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{a^2}+\frac{2(5A}{}}{6a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2)
- ((2*(5*A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(1 + Sec[c + d*x]))
+ ((2*a^2*(5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/d - 3*a^2*(4*A - B)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/a
^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(191) = 382$.

Time = 4.48 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(5A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\right.\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*
x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.76

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + a\cos(c + dx))^2}\,dx$$

$$= \frac{\left(\sqrt{2}(5i\,A - 2i\,B)\cos{(dx + c)}^2 - 2\sqrt{2}(-5i\,A + 2i\,B)\cos{(dx + c)} + \sqrt{2}(5i\,A - 2i\,B)\right)\text{weierstrassPInve}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/6*((sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-5*I*A + 2*I*B)*
cos(d*x + c) + sqrt(2)*(5*I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + (sqrt(2)*(-5*I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqr
t(2)*(5*I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 2*I*B))*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(4*I*A - I*B)*c
os(d*x + c)^2 + 2*sqrt(2)*(4*I*A - I*B)*cos(d*x + c) + sqrt(2)*(4*I*A - I*
B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*(sqrt(2)*(-4*I*A + I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*A
 + I*B)*cos(d*x + c) + sqrt(2)*(-4*I*A + I*B))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(4*A - B)*co
s(d*x + c)^2 + (19*A - 4*B)*cos(d*x + c) + 6*A)*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + a\cos(c + dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a+a\cos(c+dx))^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^2,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

output
```
(int((sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x))/(cos(c + d*x)**2 + 2*c
os(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sec(c + d*x))/(cos(c + d*x
)**2 + 2*cos(c + d*x) + 1),x)*a)/a**2
```

3.483 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx &= \frac{A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{(2A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ &\quad-\frac{A\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
A*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/
a^2/d+1/3*(2*A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*
sec(d*x+c)^(1/2)/a^2/d-A*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-
1/3*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.44 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.59

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-iAe^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2
,x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*A*(1 + E^(I*(c + d*x)))^3*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/E^(I*(c + d*x)) + 8*(2*A + B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I
)*Cos[c + d*x]*(7*A - B + (5*A + B)*Cos[c + d*x] + I*(A - B)*Sin[c + d*x])
)*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos
[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 3439}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 4507}$$

$$\frac{\int -\frac{\sqrt{\sec(c+dx)}(a(A-B)-a(5A+B)\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a(A-B)-a(5A+B)\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(A-B)-a(5A+B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4507}$$

$$-\frac{\frac{\int -\frac{3Aa^2+(2A+B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3Aa^2+(2A+B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3Aa^2+(2A+B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4274}$$

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{a^2(2A+B)\int\sqrt{\sec(c+dx)}dx+3a^2A\int\frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{a^2(2A+B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^2A\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4258

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{2a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2)
- (-(((6*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/d)/a^2) + (6*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 +
Sec[c + d*x])))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(148) = 296$.

Time = 4.40 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.17

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1
/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^3-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*
A*cos(1/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c
)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.02

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{\left(\sqrt{2}(-2i\,A-i\,B)\cos\left(dx+c\right)^2-2\,\sqrt{2}(2i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}(-2i\,A-i\,B)\right)\text{weierstrassPInvers}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/6*((sqrt(2)*(-2*I*A - I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A + I*B)*cos(
d*x + c) + sqrt(2)*(-2*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + (sqrt(2)*(2*I*A + I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-2
*I*A - I*B)*cos(d*x + c) + sqrt(2)*(2*I*A + I*B))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*A*cos(d*x + c)^2 - 2*I*s
qrt(2)*A*cos(d*x + c) - I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*A*cos(d*x + c)
^2 + 2*I*sqrt(2)*A*cos(d*x + c) + I*sqrt(2)*A)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*A*cos(d*x +
c)^2 + (4*A - B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{A\sqrt{\sec(c+dx)}}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sqrt{\sec(c+dx)}}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
(Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)
+ Integral(B*cos(c + d*x)*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c +
d*x) + 1), x))/a**2
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx=\int \frac{(A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}}{(a+a\cos(c+dx))^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^2,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a + int(
(sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x
)*b)/a**2
```

3.484 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 168

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = -\frac{B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A+2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{(A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output

```
-B*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)
/a^2/d+1/3*(A+2*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
*sec(d*x+c)^(1/2)/a^2/d+1/3*(A+2*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(1+s
ec(d*x+c))-1/3*(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.52

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(8(A+2B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
,x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(8*(A + 2*B)*Cos[(c + d*x)/2]^3*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x
)/2]) + I*((B*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 2*Cos[
c + d*x]*(-A - 5*B + (A - 7*B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x]))*(C
os[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c +
 d*x])^2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4508, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sqrt{\sec(c+dx)}(A\sec(c+dx)+B)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int \frac{a(A-B)+3a(A+B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-B)+3a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-B)+3a(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int -\frac{3a^2B-a^2(A+2B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2B-a^2(A+2B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2B-a^2(A+2B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2B\int\frac{1}{\sqrt{\sec(c+dx)}}dx-a^2(A+2B)\int\sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2B\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2(A+2B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4258

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

output
```
-1/3*((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2)
+ (-(((6*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d - (2*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/d)/a^2) + (2*(A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(d*(1 + Sec[c + d*x])))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(153) = 306$.

Time = 4.87 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.08

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+12}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3
*A*cos(1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*
c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.93

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{2}(-i\,A-2i\,B)\cos\left(dx+c\right)^{2}-2\,\sqrt{2}(i\,A+2i\,B)\cos\left(dx+c\right)+\sqrt{2}(-i\,A-2i\,B)\right)\text{weierstrassPInvers}}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
1/6*((sqrt(2)*(-I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(I*A + 2*I*B)*cos(
d*x + c) + sqrt(2)*(-I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + (sqrt(2)*(I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-I
*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 2*I*B))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*B*cos(d*x + c)^2 + 2*I*sq
rt(2)*B*cos(d*x + c) + I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*B*cos(d*x + c)
^2 - 2*I*sqrt(2)*B*cos(d*x + c) - I*sqrt(2)*B)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x +
c)^2 + (A + 2*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \frac{\int \frac{A}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\cos(c+dx)}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
(Integral(A/(cos(c + d*x)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(
c + d*x)) + sqrt(sec(c + d*x))), x) + Integral(B*cos(c + d*x)/(cos(c + d*x
)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c +
 d*x))), x))/a**2
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^2\sec(dx+c)+2\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^2\sec(dx+c)+2\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*sec(c + d*x) + 2*cos(c + d*x)*sec
(c + d*x) + sec(c + d*x)),x)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/(co
s(c + d*x)**2*sec(c + d*x) + 2*cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x
)*b)/a**2
```

3.485 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 176

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{(A-4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{(2A-5B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ &\quad+\frac{(2A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}+\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-(A-4*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)
^(1/2)/a^2/d+1/3*(2*A-5*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))*sec(d*x+c)^(1/2)/a^2/d+1/3*(2*A-5*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/
a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*sec(d*x+
c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.39 (sec) , antiderivative size = 732, normalized size of antiderivative = 4.16

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
,x]
```

output
```
(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) - (
4*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) +
(4*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/
2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) - (
10*B*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/
2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (
Cos[c/2 + (d*x)/2]^4*Sqrt[Sec[c + d*x]]*((-2*(-A + 3*B + B*Cos[2*c])*Cos[d
*x]*Csc[c/2]*Sec[c/2])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(4*A*Sin[(d*x)/2
] - 7*B*Sin[(d*x)/2]))/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2]))/(3*d) + (8*B*Cos[c]*Sin[d*x])/d - (4*(4*A - 7*B)*
Tan[c/2])/(3*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a
*Cos[c + d*x])^2
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4508, 27, 3042, 4508, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 4508}$$

$$\frac{\int -\frac{a(A-7B)-3a(A-B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{a(A-7B)-3a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{6a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{a(A-7B)-3a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}$$

$$\Big\downarrow \text{ 4508}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{3a^2(A-4B)-a^2(2A-5B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{3a^2(A-4B)-a^2(2A-5B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

$\Big\downarrow$ 4274

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^2(A-4B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a^2(2A-5B)\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^2(A-4B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2(2A-5B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

$\Big\downarrow$ 4258

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^2(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a^2(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^2(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a^2(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

$\Big\downarrow$ 3119

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{6a^2(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-a^2(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

$\Big\downarrow$ 3120

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{6a^2(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a^2(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{2(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

output
```
((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (
((6*a^2*(A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d - (2*a^2*(2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (2*(2*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
 - a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(161) = 322$.

Time = 5.82 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.39

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\ldots}\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(
1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B*cos(1/2*d*x+1/2*c)^3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2
*c)^4+9*A*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*
d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.06

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\left(\sqrt{2}(-2i\,A+5i\,B)\cos{(dx+c)}^2-2\sqrt{2}(2i\,A-5i\,B)\cos{(dx+c)}+\sqrt{2}(-2i\,A+5i\,B)\right)\text{weierstrassPInv}}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
1/6*((sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A - 5*I*B)*
cos(d*x + c) + sqrt(2)*(-2*I*A + 5*I*B))*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + (sqrt(2)*(2*I*A - 5*I*B)*cos(d*x + c)^2 - 2*sqr
t(2)*(-2*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*(2*I*A - 5*I*B))*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(I*A - 4*I*B)*c
os(d*x + c)^2 + 2*sqrt(2)*(I*A - 4*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 4*I*
B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*(sqrt(2)*(-I*A + 4*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-I*A +
 4*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 4*I*B))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A - 2*B)*co
s(d*x + c)^2 + (2*A - 5*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^2+2\cos(dx+c)\sec(dx+c)^2+\sec(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^2\sec(dx+c)^2+2\cos(dx+c)\sec(dx+c)^2+\sec(dx+}\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**2 + 2*cos(c + d*x)*
sec(c + d*x)**2 + sec(c + d*x)**2),x)*a + int((sqrt(sec(c + d*x))*cos(c +
d*x))/(cos(c + d*x)**2*sec(c + d*x)**2 + 2*cos(c + d*x)*sec(c + d*x)**2 +
sec(c + d*x)**2),x)*b)/a**2
```

3.486 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 206

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{(4A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\
&\quad - \frac{5(A-2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\
&\quad - \frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} \\
&\quad + \frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}
\end{aligned}$$

output

```
(4*A-7*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/a^2/d-5/3*(A-2*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))*sec(d*x+c)^(1/2)/a^2/d-5/3*(A-2*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/
2)+1/3*(4*A-7*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)/(1+sec(d*x+c))+1/3*(A-B
)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.79 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.77

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)}\, dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
,x]
```

output
```
(-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
 d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2)
+ (7*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c
+ d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/
4, -E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2)
 - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2)
 + (20*B*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2)
 + (Cos[c/2 + (d*x)/2]^4*Sqrt[Sec[c + d*x]]*((-2*(3*A - 5*B + A*Cos[2*c] -
 2*B*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (4*B*Cos[2*d*x]*Sin[2*c])/(
3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(7*A*Sin[(d*x)/2] - 10*B*Sin[(d*x)/2
]))/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)
/2]))/(3*d) + (8*(A - 2*B)*Cos[c]*Sin[d*x])/d + (4*B*Cos[2*c]*Sin[2*d*x])/
(3*d) + (4*(7*A - 10*B)*Tan[c/2])/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4508, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4508}$$

$$\frac{\int -\frac{3a(A-3B)-5a(A-B)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}+\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\int \frac{3a(A-3B)-5a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\int \frac{3a(A-3B)-5a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{3\left(5a^2(A-2B)-a^2(4A-7B)\sec(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{2(4A-7B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\int \frac{5a^2(A-2B)-a^2(4A-7B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{2(4A-7B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\int \frac{5a^2(A-2B)-a^2(4A-7B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{2(4A-7B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-a^2(4A-7B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}-\frac{2(4A-7B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-a^2(4A-7B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{2(4A-7B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-a^2(4A-7B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{2(4A-7B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}}{}$$

$\downarrow$ 3042

$$\frac{\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\left(\frac{1}{3}\int\sqrt{\csc(c+dx+\frac{\pi}{2})}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-a^2(4A-7B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{2(4A-7B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}}{}$$

$\downarrow$ 4258

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-a^2(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{a^2}-\frac{}{d\sqrt{}}}{6a^2}$$

$\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-a^2(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx\right)}{a^2}}{6a^2}$$

$\downarrow$ 3119

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{2a^2(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{a^2}-\frac{2(}{d\sqrt{se}}}{6a^2}$$

$\downarrow$ 3120

$$\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{\frac{3\left(5a^2(A-2B)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{2a^2(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{a^2}-\frac{2(4A-}{d\sqrt{\sec(c+}}}{6a^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

output
```
((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (
(-2*(4*A - 7*B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) +
(3*((-2*a^2*(4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + 5*a^2*(A - 2*B)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]]))))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(187) = 374$.

Time = 6.50 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.11

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-16B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{Ellipt}}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2
*d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*co
s(1/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12
*B*cos(1/2*d*x+1/2*c)^6-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)
^3-42*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1
/2*c)^4+48*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d
*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.83

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{5\left(\sqrt{2}(-i\,A+2i\,B)\cos\left(dx+c\right)^{2}+2\sqrt{2}(-i\,A+2i\,B)\cos\left(dx+c\right)+\sqrt{2}(-i\,A+2i\,B)\right)\text{weierstrassPI}}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-1/6*(5*(sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-I*A + 2*I*B)*
cos(d*x + c) + sqrt(2)*(-I*A + 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 5*(sqrt(2)*(I*A - 2*I*B)*cos(d*x + c)^2 + 2*sqrt(
2)*(I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 2*I*B))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-4*I*A + 7*I*B)*cos(d
*x + c)^2 + 2*sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x + c) + sqrt(2)*(-4*I*A + 7*
I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) + 3*(sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(4*I
*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(4*I*A - 7*I*B))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*B*cos(d
*x + c)^3 - (6*A - 13*B)*cos(d*x + c)^2 - 5*(A - 2*B)*cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{A+B\cos(c+dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+a\cos(c+dx))^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^3+2\cos(dx+c)\sec(dx+c)^3+\sec(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)}{\cos(dx+c)^2\sec(dx+c)^3+2\cos(dx+c)\sec(dx+c)^3+\sec(dx+}\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**3 + 2*cos(c + d*x)*
sec(c + d*x)**3 + sec(c + d*x)**3),x)*a + int((sqrt(sec(c + d*x))*cos(c +
d*x))/(cos(c + d*x)**2*sec(c + d*x)**3 + 2*cos(c + d*x)*sec(c + d*x)**3 +
sec(c + d*x)**3),x)*b)/a**2
```

3.487 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 261

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx \\
&= -\frac{(49A-9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad -\frac{(13A-3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} \\
&\quad +\frac{(49A-9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d}-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&\quad -\frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{(13A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}
\end{aligned}$$

output
```
-1/10*(49*A-9*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*se
c(d*x+c)^(1/2)/a^3/d-1/6*(13*A-3*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^3/d+1/10*(49*A-9*B)*sec(d*x+c)^(1/2)*
sin(d*x+c)/a^3/d-1/5*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^
3-1/15*(8*A-3*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/6*(1
3*A-3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.33 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-i(49A-9B)e^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeom}\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3
,x]
```

output
```
-1/120*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(49*A - 9*B)*(1 + E^(I*
(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 160*(13*A - 3*B)*Cos[(c +
d*x)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] -
 I*Sin[(c + d*x)/2]) + (2*I)*(642*A - 102*B + (1082*A - 207*B)*Cos[c + d*x
] + 6*(87*A - 17*B)*Cos[2*(c + d*x)] + 106*A*Cos[3*(c + d*x)] - 21*B*Cos[3
*(c + d*x)] + (161*I)*A*Sin[c + d*x] - (6*I)*B*Sin[c + d*x] + (148*I)*A*Si
n[2*(c + d*x)] - (18*I)*B*Sin[2*(c + d*x)] + (41*I)*A*Sin[3*(c + d*x)] - (
6*I)*B*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^
3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int -\frac{\sec^{\frac{5}{2}}(c+dx)(5a(A-B)-a(11A-B)\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a(A-B)-a(11A-B)\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a(A-B)-a(11A-B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2(8A-3B)-a^2(41A-6B)\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{2a(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2(8A-3B)-a^2(41A-6B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{2a(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(5a^3(13A-3B)-3a^3(49A-9B)\sec(c+dx)\right)dx}{a^2}+\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\int \sqrt{\sec(c+dx)}\left(5a^3(13A-3B)-3a^3(49A-9B)\sec(c+dx)\right)dx}{2a^2}+\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^3(13A-3B)-3a^3(49A-9B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{\frac{5a^3(13A-3B)\int \sqrt{\sec(c+dx)}dx-3a^3(49A-9B)\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}+\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{5a^3(13A-3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a^3(49A-9B)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}+\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(8A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4255

$$-\frac{\frac{\frac{5a^3(13A-3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a^3(49A-9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}+\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(8A-3B)\mathrm{s}\ldots}{3d(a\,\mathrm{s}\ldots}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{5a^3(13A-3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a^3(49A-9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}+\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(8A-\ldots}{3\ldots}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

$$-\frac{\frac{\frac{5a^3(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^3(49A-9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)}{2a^2}+\frac{5a^2(13A-3\ldots}{d(a\ldots}}{3a^2}\ldots}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{5a^3(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3(49A-9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}+5a^2\ldots}{3a^2}\ldots}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$-\frac{\frac{\frac{5a^3(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3(49A-9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}+\frac{5a^2(13\ldots}{}}{3a^2}\ldots}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3120

$$-\frac{\frac{\frac{5a^2(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{10a^3(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-3a^3(49A-9B)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+d}}{}\right.}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3)
- ((2*a*(8*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d
*x])^2) + ((5*a^2*(13*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*
Sec[c + d*x])) + ((10*a^3*(13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 3*a^3*(49*A - 9*B)*((-2*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d))/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(236) = 472$.

Time = 5.95 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.62

method	result	size
default	Expression too large to display	685

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(49*A-9*B)*sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*
c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.84

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + a\cos(c + dx))^3} dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
-1/60*(5*(sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A +
3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c) + sqrt(2)
*(-13*I*A + 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + 5*(sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 3*I
*B)*cos(d*x + c)^2 + 3*sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c) + sqrt(2)*(13
*I*A - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) +
 3*(sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(49*I*A - 9*I*B)*c
os(d*x + c)^2 + 3*sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c) + sqrt(2)*(49*I*A
- 9*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 3*(sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2
)*(-49*I*A + 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x +
 c) + sqrt(2)*(-49*I*A + 9*I*B))*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(49*A - 9*B)*cos(d*x + c)^
3 + 2*(188*A - 33*B)*cos(d*x + c)^2 + 5*(59*A - 9*B)*cos(d*x + c) + 60*A)*
sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
 c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + a\cos(c + dx))^3} dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + a\cos(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + a\cos(c + dx))^3}\,dx = \int \frac{(B\cos(dx + c) + A)\sec(dx + c)^{\frac{3}{2}}}{(a\cos(dx + c) + a)^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^3,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^3,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

output
```
(int((sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x))/(cos(c + d*x)**3 + 3*c
os(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sec(c
+ d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a)/a
**3
```

3.488 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 222

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{(9A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(9A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
1/10*(9*A+B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*
x+c)^(1/2)/a^3/d+1/6*(3*A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))*sec(d*x+c)^(1/2)/a^3/d-1/5*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/
(a+a*sec(d*x+c))^3-1/15*(6*A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d
*x+c))^2-1/10*(9*A+B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.61 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.57

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3
,x]
```

output
```
(-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
 d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3)
- (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3)
+ (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x
)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (
2*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2
, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (C
os[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(9*A + B)*Cos[d*x]*Csc[c/2]*Se
c[c/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(
d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + B*Sin
[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(3*A*Sin[(d*x)/2] + 2
*B*Sin[(d*x)/2]))/(15*d) + (4*(3*A + B)*Tan[c/2])/(3*d) + (4*(3*A + 2*B)*S
ec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan
[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int -\frac{\sec^{\frac{3}{2}}(c+dx)(3a(A-B)-a(9A+B)\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a(A-B)-a(9A+B)\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a(A-B)-a(9A+B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2(6A-B)-a^2(21A+4B)\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{2a(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2(6A-B)-a^2(21A+4B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{2a(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\frac{\int -\frac{3(9A+B)a^3+5(3A+B)\sec(c+dx)a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3(9A+B)a^3+5(3A+B)\sec(c+dx)a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}+\frac{2a(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3(9A+B)a^3+5(3A+B)\csc\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{2a(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3(9A+B)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3(3A+B)\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}+\frac{2a(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3(9A+B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3(3A+B)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{2a(6A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\big\downarrow$ 4258

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(9A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{2a^2}}{3a^2}+\frac{2a(6A}{}}{10a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\big\downarrow$ 3042

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3(9A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+}{10a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\big\downarrow$ 3119

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3(9A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{2a(}{}}{10a^2}$$
$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\big\downarrow$ 3120

$$-\frac{\frac{\frac{3a^2(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{10a^3(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3(9A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+2}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3)
- ((2*a*(6*A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x
])^2) + (-1/2*((6*a^3*(9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/d + (10*a^3*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (3*a^2*(9*A + B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(201) = 402$.

Time = 5.88 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.03

method	result
default	$\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(108A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*c
os(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-1
0*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)
^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+
1/2*c)^6+24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*
x+1/2*c)^2+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.14

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + a\cos(c + dx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
-1/60*(5*(sqrt(2)*(3*I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(3*I*A + I*B)*c
os(d*x + c)^2 + 3*sqrt(2)*(3*I*A + I*B)*cos(d*x + c) + sqrt(2)*(3*I*A + I*
B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)
*(-3*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-3*I*A - I*B)*cos(d*x + c)^2 +
 3*sqrt(2)*(-3*I*A - I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A - I*B))*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-9*I*A - I*
B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-
9*I*A - I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A - I*B))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(
9*I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9*I*A + I*B)*cos(d*x + c)^2 + 3*s
qrt(2)*(9*I*A + I*B)*cos(d*x + c) + sqrt(2)*(9*I*A + I*B))*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*
(9*A + B)*cos(d*x + c)^3 + 2*(33*A + 2*B)*cos(d*x + c)^2 + 5*(9*A - B)*cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{A\sqrt{\sec(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sqrt{\sec(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
(Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*co
s(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sqrt(sec(c + d*x))/(cos(c +
d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3,
x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{(A+B\cos(c+d\,x))\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\cos(c+d\,x))^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^3,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^3,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1),x)*a + int((sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)**3 + 3
*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b)/a**3
```

3.489 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx &= \frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad+\frac{(A+B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} \\
&\quad-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(4A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad+\frac{(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}
\end{aligned}$$

output
```
1/10*(A-B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+
c)^(1/2)/a^3/d+1/6*(A+B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))*sec(d*x+c)^(1/2)/a^3/d-1/5*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a
*sec(d*x+c))^3-1/15*(4*A+B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c
))^2+1/6*(A+B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.57 (sec) , antiderivative size = 792, normalized size of antiderivative = 3.67

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3\sqrt{\sec(c + dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]])
,x]
```

output
```
-1/15*(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(
c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4,
7/4, -E^((2*I)*(c + d*x))])*Sec[c/2])/(d*E^(I*d*x)*(a + a*Cos[c + d*x])^3)
 + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
 d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3)
 + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*
x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3)
+ (2*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x
)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) +
 (Cos[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(A - B)*Cos[d*x]*Csc[c/2]*S
ec[c/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] - 7*B*
Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] -
 B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] +
 B*Sin[(d*x)/2]))/(3*d) + (4*(A + B)*Tan[c/2])/(3*d) + (4*(2*A - 7*B)*Sec[
c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/
2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int -\frac{\sqrt{\sec(c+dx)}(a(A-B)-a(7A+3B)\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a(A-B)-a(7A+3B)\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(A-B)-a(7A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int -\frac{(4A+B)a^2+3(3A+2B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{(4A+B)a^2+3(3A+2B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{(4A+B)a^2+3(3A+2B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 4508}$$

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{3(A-B)a^3+5(A+B)\sec(c+dx)a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{3(A-B)a^3+5(A+B)\sec(c+dx)a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}+\frac{5a^2(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{3(A-B)a^3+5(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{5a^2(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3(A+B)\int \sqrt{\sec(c+dx)}dx}{2a^2}+\frac{5a^2(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3(A-B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3(A+B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4258

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{5a^2(A\cdots}{\cdots}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}\cdots}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3119

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{5a\cdots}{\cdots}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{2a(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^2(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\frac{10a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3(A-B)\sqrt{\cos(c}{\cdots}}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

output
```
-1/5*((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3)
- ((2*a*(4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x
])^2) - (((6*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d + (10*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) + (5*a^2*(A + B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(195) = 390$.

Time = 6.39 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.09

method	result
default	$\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*
c)^6+6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/
2*c)^2-17*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.19

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/60*(5*(sqrt(2)*(I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + I*B)*cos(d
*x + c)^2 + 3*sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*B))*weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-I*A -
 I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(
-I*A - I*B)*cos(d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-I*A + I*B)*cos(d*x + c)^3
+ 3*sqrt(2)*(-I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A + I*B)*cos(d*x +
 c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^
3 + 3*sqrt(2)*(I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A - I*B)*cos(d*x +
 c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A - B)*cos(d*x + c)^3 + 2*(2*A
- 7*B)*cos(d*x + c)^2 - 5*(A + B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sqrt{\sec(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sqrt{\sec(c + dx)}}\,dx = \int \frac{B\cos\left(dx + c\right) + A}{\left(a\cos\left(dx + c\right) + a\right)^3 \sqrt{\sec\left(dx + c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)))
, x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sqrt{\sec(c + dx)}}\,dx = \int \frac{B\cos\left(dx + c\right) + A}{\left(a\cos\left(dx + c\right) + a\right)^3 \sqrt{\sec\left(dx + c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sqrt{\sec(c + dx)}}\,dx = \int \frac{A + B\,\cos\left(c + d\,x\right)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a + a\,\cos\left(c + d\,x\right)\right)^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^3\sec(dx+c)+3\cos(dx+c)^2\sec(dx+c)+3\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\sec(}}{\cos(dx+c)^3\sec(dx+c)+3\cos(dx+c)}\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**3*sec(c + d*x) + 3*cos(c + d*x)**2*
sec(c + d*x) + 3*cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*a + int((sqr
t(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x) + 3*cos(c + d*
x)**2*sec(c + d*x) + 3*cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*b)/a**
3
```

3.490 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 222

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{(A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &\quad+\frac{(A+3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}\\ &\quad-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(2A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}\\ &\quad+\frac{(A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\,(a^3+a^3\sec(c+dx))}\end{aligned}$$

output
```
-1/10*(A+9*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d
*x+c)^(1/2)/a^3/d+1/6*(A+3*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2))*sec(d*x+c)^(1/2)/a^3/d-1/5*(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d
/(a+a*sec(d*x+c))^3+1/15*(2*A+3*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*se
c(d*x+c))^2+1/6*(A+3*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.45 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.57

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2))
,x]
```

output
```
(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) +
(3*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) +
 (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) +
(2*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/
2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (Co
s[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]]*((2*(A + 9*B)*Cos[d*x]*Csc[c/2]*Sec[
c/2])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(d*x)/2] - 12*B*Si
n[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 9*B
*Sin[(d*x)/2]))/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] -
 B*Sin[(d*x)/2]))/(5*d) + (4*(A - 9*B)*Tan[c/2])/(3*d) - (4*(7*A - 12*B)*S
ec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan
[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4507, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sqrt{\sec(c+dx)}(A\sec(c+dx)+B)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4507}$$

$$\frac{\int \frac{a(A-B)+5a(A+B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-B)+5a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-B)+5a(A+B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{(A-6B)a^2+3(2A+3B)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(A-6B)a^2+3(2A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\frac{\int -\frac{3a^3(A+9B)-5a^3(A+3B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3(A+9B)-5a^3(A+3B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}+\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3(A+9B)-5a^3(A+3B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3(A+9B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3(A+3B)\int \sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}+\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3(A+9B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3(A+3B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-5a^3(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}}{3a^2}+\frac{2a(2A+3\cdots}{3\cdots}}{10a^2}$$
$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\cdots}{10a^2}$$
$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{2a(2\cdots}{\cdots}}{10a^2}$$
$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{5a^2(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}}{3a^2}+\frac{2a(\cdots}{\cdots}}{10a^2}$$
$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

output
```
-1/5*((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3)
+ ((2*a*(2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d
*x])^2) + (-1/2*((6*a^3*(A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d - (10*a^3*(A + 3*B)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (5*a^2*(A + 3*B)*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(201) = 402$.

Time = 6.50 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/
2*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+3
0*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c
)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+
1/2*c)^6-24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2
*d*x+1/2*c)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.14

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-1/60*(5*(sqrt(2)*(I*A + 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 3*I*B)*c
os(d*x + c)^2 + 3*sqrt(2)*(I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 3*I*
B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)
*(-I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c)^2 +
 3*sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 3*I*B))*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(I*A + 9*I*B
)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A
 + 9*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 9*I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(-I*A
 - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c)^2 + 3*sqr
t(2)*(-I*A - 9*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 9*I*B))*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*
(A + 9*B)*cos(d*x + c)^3 + 2*(7*A + 18*B)*cos(d*x + c)^2 + 5*(A + 3*B)*cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^3\sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{A + B\cos(c + dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a + a\cos(c + dx))^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^2+3\cos(dx+c)^2\sec(dx+c)^2+3\cos(dx+c)\sec(dx+c)^2+\sec(dx+c)^2}dx\right)a+\left(\int \frac{}{\cos(dx+c)^3\sec(dx+c)^2+3\cos(a}\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**2 + 3*cos(c + d*x)*
*2*sec(c + d*x)**2 + 3*cos(c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2),x)*
a + int((sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**2
 + 3*cos(c + d*x)**2*sec(c + d*x)**2 + 3*cos(c + d*x)*sec(c + d*x)**2 + se
c(c + d*x)**2),x)*b)/a**3
```

3.491 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 228

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx
&= -\frac{(9A-49B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad+\frac{(3A-13B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} \\
&\quad+\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(3A-8B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad+\frac{(3A-13B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}
\end{aligned}$$

output

```
-1/10*(9*A-49*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*se
c(d*x+c)^(1/2)/a^3/d+1/6*(3*A-13*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^3/d+1/5*(A-B)*sec(d*x+c)^(1/2)*sin(d*
x+c)/d/(a+a*sec(d*x+c))^3+1/15*(3*A-8*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(
a+a*sec(d*x+c))^2+1/6*(3*A-13*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a^3+a^3*se
c(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 817, normalized size of antiderivative = 3.58

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
,x]
```

output
```
(3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sec[c/2]))/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) -
 (49*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c
+ d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/
4, -E^((2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3
) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) -
 (26*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x
)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) +
 (Cos[c/2 + (d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(-9*A + 39*B + 10*B*Cos[2*c
])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A
*Sin[(d*x)/2] - 23*B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]
^3*(12*A*Sin[(d*x)/2] - 17*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 +
 (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*B*Cos[c]*Sin[d*
x])/d - (4*(9*A - 23*B)*Tan[c/2])/(3*d) + (4*(12*A - 17*B)*Sec[c/2 + (d*x)
/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))
/(a + a*Cos[c + d*x])^3
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4508, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4508}$$

$$\frac{\int -\frac{a(A-11B)-5a(A-B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{a(A-11B)-5a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{a(A-11B)-5a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\int\frac{a^2(6A-41B)-3a^2(3A-8B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{2a(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\int\frac{a^2(6A-41B)-3a^2(3A-8B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$$\Big\downarrow \ 4508$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{\int\frac{3a^3(9A-49B)-5a^3(3A-13B)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{\int\frac{3a^3(9A-49B)-5a^3(3A-13B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{\int\frac{3a^3(9A-49B)-5a^3(3A-13B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$$\Big\downarrow \ 4274$$

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3a^3(9A-49B)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3(3A-13B)\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$\big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3a^3(9A-49B)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3(3A-13B)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a(3A-8B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$\big\downarrow$ 4258

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3a^3(9A-49B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-5a^3(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-2a}{10a^2}$$

$\big\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3a^3(9A-49B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$\big\downarrow$ 3119

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{\frac{6a^3(9A-49B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-}{10a^2}$$

$\big\downarrow$ 3120

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{\frac{6a^3(9A-49B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{5a^2(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

output
```
((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
(-2*a*(3*A - 8*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x
])^2) + (((6*a^3*(9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/d - (10*a^3*(3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (5*a^2*(3*A - 13*B)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(207) = 414$.

Time = 7.15 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.98

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(108A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1
/2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*
cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8
-130*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1
/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/
2*d*x+1/2*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*
cos(1/2*d*x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/
2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.10

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-1/60*(5*(sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(3*I*A - 13*
I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c) + sqrt(2)*(3
*I*A - 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
+ 5*(sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-3*I*A + 13*I*B
)*cos(d*x + c)^2 + 3*sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c) + sqrt(2)*(-3*
I*A + 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) +
 3*(sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9*I*A - 49*I*B)*c
os(d*x + c)^2 + 3*sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c) + sqrt(2)*(9*I*A -
 49*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 3*(sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2
)*(-9*I*A + 49*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x +
 c) + sqrt(2)*(-9*I*A + 49*I*B))*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(9*A - 29*B)*cos(d*x + c)^
3 + 2*(18*A - 73*B)*cos(d*x + c)^2 + 5*(3*A - 13*B)*cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2))
, x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)}\, dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}}}\, dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)}\, dx = \int \frac{A + B\cos(c + d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a + a\cos(c + d\,x))^3}\, dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^3+3\cos(dx+c)^2\sec(dx+c)^3+3\cos(dx+c)\sec(dx+c)^3+\sec(dx+c)^3}dx\right)a+\left(\int \frac{}{\cos(dx+c)^3\sec(dx+c)^3+3\cos(d}\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**3 + 3*cos(c + d*x)*
*2*sec(c + d*x)**3 + 3*cos(c + d*x)*sec(c + d*x)**3 + sec(c + d*x)**3),x)*
a + int((sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**3
 + 3*cos(c + d*x)**2*sec(c + d*x)**3 + 3*cos(c + d*x)*sec(c + d*x)**3 + se
c(c + d*x)**3),x)*b)/a**3
```

3.492 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 259

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{7(7A-17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &\quad-\frac{(13A-33B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}\\ &\quad-\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\\ &\quad+\frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}+\frac{7(7A-17B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,(a^3+a^3\sec(c+dx))}\end{aligned}$$

output
```
7/10*(7*A-17*B)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec
(d*x+c)^(1/2)/a^3/d-1/6*(13*A-33*B)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d
*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^3/d-1/6*(13*A-33*B)*sin(d*x+c)/a^3/d/
sec(d*x+c)^(1/2)+1/5*(A-B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^
3+1/3*(A-2*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2+7/30*(7*A
-17*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a^3+a^3*sec(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.86 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.27

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-98\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,\mathrm{H}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2))
,x]
```

output
```
(Cos[(c + d*x)/2]^6*((-98*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (238*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1
+ E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F
1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (((806*A - 1961*B)*Co
s[(c - d*x)/2] + (664*A - 1609*B)*Cos[(3*c + d*x)/2] + 470*A*Cos[(c + 3*d*
x)/2] - 1165*B*Cos[(c + 3*d*x)/2] + 265*A*Cos[(5*c + 3*d*x)/2] - 620*B*Cos
[(5*c + 3*d*x)/2] + 117*A*Cos[(3*c + 5*d*x)/2] - 292*B*Cos[(3*c + 5*d*x)/2
] + 30*A*Cos[(7*c + 5*d*x)/2] - 65*B*Cos[(7*c + 5*d*x)/2] - 5*B*Cos[(5*c +
 7*d*x)/2] + 5*B*Cos[(9*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^
5)/(8*Sqrt[Sec[c + d*x]]) - 260*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]] + 660*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3439, 3042, 4508, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4508}$$

$$\frac{\int -\frac{a(3A-13B)-7a(A-B)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\int \frac{a(3A-13B)-7a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\int \frac{a(3A-13B)-7a(A-B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\int \frac{3a^2(8A-23B)-25a^2(A-2B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\int \frac{3a^2(8A-23B)-25a^2(A-2B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}}{}$$

$\downarrow$ 4508

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{\int \frac{3\left(5a^3(13A-33B)-7a^3(7A-17B)\sec(c+dx)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 27

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\int \frac{5a^3(13A-33B)-7a^3(7A-17B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\int \frac{5a^3(13A-33B)-7a^3(7A-17B)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 4274

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-7a^3(7A-17B)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-7a^3(7A-17B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 4256

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^3(7A-17B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^3(7A-17B)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 4258

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^3(7A-17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 3042

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^3(7A-17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}-\frac{7a^2(7A-17B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{10a(A-2B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}$$

$\downarrow$ 3119

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{14a^3(7A-17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{7a^2(7A-\cdots}{d\sqrt{\sec(c+d\cdots}}}{3a^2}\cdots}{10a^2}$$

$\downarrow$ 3120

$$\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}-\frac{\frac{\frac{3\left(5a^3(13A-33B)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{14a^3(7A-17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{7a^2(7A\cdots}{d\sqrt{\sec(c+\cdots}}}{3a^2}\cdots}{10a^2}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

output
```
((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (
(-10*a*(A - 2*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]
)^2) + ((-7*a^2*(7*A - 17*B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Se
c[c + d*x])) + (3*((-14*a^3*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 5*a^3*(13*A - 33*B)*((2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x]
)/(3*d*Sqrt[Sec[c + d*x]]))))/(2*a^2))/(3*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 7.79 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.80

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-160B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+348A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+130A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\right.}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1
/2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-468*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*
x+1/2*c)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos
(1/2*d*x+1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-4
74*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c
)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.89

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-1/60*(5*(sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A +
 33*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c) + sqrt
(2)*(-13*I*A + 33*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 5*(sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A
- 33*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c) + sqrt
(2)*(13*I*A - 33*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 21*(sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-7*I*A
+ 17*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c) + sqrt
(2)*(-7*I*A + 17*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 21*(sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c)
^3 + 3*sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(7*I*A - 17*I*B
)*cos(d*x + c) + sqrt(2)*(7*I*A - 17*I*B))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20*B*cos(d*x + c)^
4 - 3*(29*A - 79*B)*cos(d*x + c)^3 - 2*(73*A - 188*B)*cos(d*x + c)^2 - 5*(
13*A - 33*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2))
, x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{A + B\cos(c + d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a + a\cos(c + d\,x))^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^3\sec(dx+c)^4+3\cos(dx+c)^2\sec(dx+c)^4+3\cos(dx+c)\sec(dx+c)^4+\sec(dx+c)^4}\,dx\right)a+\left(\int \frac{}{\cos(dx+c)^3\sec(dx+c)^4+3\cos(a\cdots}\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

output
```
(int(sqrt(sec(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**4 + 3*cos(c + d*x)*
*2*sec(c + d*x)**4 + 3*cos(c + d*x)*sec(c + d*x)**4 + sec(c + d*x)**4),x)*
a + int((sqrt(sec(c + d*x))*cos(c + d*x))/(cos(c + d*x)**3*sec(c + d*x)**4
 + 3*cos(c + d*x)**2*sec(c + d*x)**4 + 3*cos(c + d*x)*sec(c + d*x)**4 + se
c(c + d*x)**4),x)*b)/a**3
```

3.493 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 220

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx\\ =\frac{32a(8A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}+\frac{16a(8A+9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}}\\ +\frac{4a(8A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}\\ +\frac{2a(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output

```
32/315*a*(8*A+9*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16
/315*a*(8*A+9*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/10
5*a*(8*A+9*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a*
(8*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*a*A*sec
(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.56

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{a(1+\cos(c+dx))}(107A+81B+11(8A+9B)\cos(c+dx)+11(8A+9B)\cos(2(c+dx))+16A\cos$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2
),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(107*A + 81*B + 11*(8*A + 9*B)*Cos[c + d*x]
+ 11*(8*A + 9*B)*Cos[2*(c + d*x)] + 16*A*Cos[3*(c + d*x)] + 18*B*Cos[3*(c
+ d*x)] + 16*A*Cos[4*(c + d*x)] + 18*B*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2
)*Tan[(c + d*x)/2])/(315*d)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {3042, 3440, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{11}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2aA\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\left(\frac{6}{7}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\left(\frac{6}{7}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}\right.\right.$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}}\right.\right.\right.$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)}}\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c$$

$\downarrow$ 3250

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(8A+9B)\left(\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c\cdot}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sin[c + d*x])/(9*d*Cos[c + d
*x]^(9/2)*Sqrt[a + a*Cos[c + d*x]]) + ((8*A + 9*B)*((2*a*Sin[c + d*x])/(7*
d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (6*((2*a*Sin[c + d*x])/(5
*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(
3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/7))/9)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
 + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 21.13 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.66

method	result
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2048\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-3584\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+2496\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-752\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+107\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{315d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{4}}\right.$
parts	$\frac{2A\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2048\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-3584\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+2496\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-752\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+107\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\cdots}}{315d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{4}}$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
2^(1/2)*(2/315*A/d*tan(1/2*d*x+1/2*c)*(2048*cos(1/2*d*x+1/2*c)^8-3584*cos(
1/2*d*x+1/2*c)^6+2496*cos(1/2*d*x+1/2*c)^4-752*cos(1/2*d*x+1/2*c)^2+107)*(
a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^4-2/315*B/d*tan(1/2*d*x+1/2*c)*(2048*cos(1/2*d*x+1/2*c
)^8-3584*cos(1/2*d*x+1/2*c)^6+2496*cos(1/2*d*x+1/2*c)^4-752*cos(1/2*d*x+1/
2*c)^2+107)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^4+4/315*B/d*tan(1/2*d*x+1/2*c)*(2176*cos(
1/2*d*x+1/2*c)^8-3808*cos(1/2*d*x+1/2*c)^6+2652*cos(1/2*d*x+1/2*c)^4-799*c
os(1/2*d*x+1/2*c)^2+94)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1
/2*c)^2-1))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.55

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$$
$$= \frac{2\left(16\left(8A+9B\right)\cos\left(dx+c\right)^4+8\left(8A+9B\right)\cos\left(dx+c\right)^3+6\left(8A+9B\right)\cos\left(dx+c\right)^2+5\left(8A+9B\right)\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
2/315*(16*(8*A + 9*B)*cos(d*x + c)^4 + 8*(8*A + 9*B)*cos(d*x + c)^3 + 6*(8
*A + 9*B)*cos(d*x + c)^2 + 5*(8*A + 9*B)*cos(d*x + c) + 35*A)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(
d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(190) = 380$.

Time $= 0.24$ (sec) , antiderivative size $= 659$, normalized size of antiderivative $= 3.00$

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
2/315*(A*(315*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 735*sqrt(2
)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1302*sqrt(2)*sqrt(a)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 1206*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 107*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 + 1)) + 9*B*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(
cos(d*x + c) + 1) - 105*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 + 154*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 142*sqrt(2)*
sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 67*sqrt(2)*sqrt(a)*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 - 9*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x +
c) + 1)^11)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(co
s(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*
(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 32.97 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.18

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(1/2
),x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((a + a*(exp(-
 c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(256*A + 288*B)*1i)/(315*
d) - (exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2)*(256*A + 288*B)*1i)/(315*d) + (exp(c*2i + d*x*2i)*(a + a*(exp
(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(1152*A + 1296*B)*1i)/(
315*d) - (exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2))^(1/2)*(1152*A + 1296*B)*1i)/(315*d) + (exp(c*4i + d*x*4i)*(a +
a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2016*A + 1008*B)
*1i)/(315*d) - (exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*
1i + d*x*1i)/2))^(1/2)*(2016*A + 1008*B)*1i)/(315*d)))/(exp(c*1i + d*x*1i)
 + 4*exp(c*2i + d*x*2i) + 4*exp(c*3i + d*x*3i) + 6*exp(c*4i + d*x*4i) + 6*
exp(c*5i + d*x*5i) + 4*exp(c*6i + d*x*6i) + 4*exp(c*7i + d*x*7i) + exp(c*8
i + d*x*8i) + exp(c*9i + d*x*9i) + 1)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \sqrt{a}\left(\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^{5}\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^{5}\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

output
```
sqrt(a)*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c
+ d*x)**5,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x
)**5,x)*a)
```

3.494 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 175

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{16a(6A+7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{8a(6A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{2a(6A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}$$

output
```
16/105*a*(6*A+7*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+8/
105*a*(6*A+7*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35*
a*(6*A+7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*A*s
ec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{a(1+\cos(c+dx))}(27A+14B+9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos(3}{105d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2)
,x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] +
2*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d
*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3440, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(6A+7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(6A+7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(6A+7B)\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(6A+7B)\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}\right.\right.$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(6A+7B)\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(6A+7B)\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+}}\right.\right.\right.$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(6A+7B)\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c-}}\right.\right.\right.$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sin[c + d*x])/(7*d*Cos[c + d
*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + ((6*A + 7*B)*((2*a*Sin[c + d*x])/(5*
d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(151) = 302$.

Time = 18.29 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.87

method	result
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{\cos(dx+c)}}}{35d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{3}}-\frac{2B\tan\left(\frac{d}{2}\right.}{}\right.$
parts	$\frac{2A\sqrt{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{\cos(dx+c)}}}{35d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{3}}+B\sqrt{2}\left(-\frac{2}{}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
2^(1/2)*(2/35*A/d*tan(1/2*d*x+1/2*c)*(128*cos(1/2*d*x+1/2*c)^6-160*cos(1/2
*d*x+1/2*c)^4+76*cos(1/2*d*x+1/2*c)^2-9)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1
/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3-2/35*B/d*t
an(1/2*d*x+1/2*c)*(128*cos(1/2*d*x+1/2*c)^6-160*cos(1/2*d*x+1/2*c)^4+76*co
s(1/2*d*x+1/2*c)^2-9)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+4/105*B/d*tan(1/2*d*x+1/2*c)*
(416*cos(1/2*d*x+1/2*c)^6-520*cos(1/2*d*x+1/2*c)^4+247*cos(1/2*d*x+1/2*c)^
2-38)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\frac{2\left(8\left(6A+7B\right)\cos\left(dx+c\right)^3+4\left(6A+7B\right)\cos\left(dx+c\right)^2+3\left(6A+7B\right)\cos\left(dx+c\right)+15A\right)\sqrt{a\cos(}}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*
A + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*co
s(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(151) = 302.

Time = 0.26 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.25

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
2/105*(3*A*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2
)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 7*
B*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sqrt(2)*sqrt(a)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 24*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 7*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^{\frac{9}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 29.04 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.52

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(96\,A+112\,B)\,1i}{105\,d}-\frac{e^{c\,7i+d\,x\,7i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(96\,A+1}{105\,d}\right.}{e^{c\,1}}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(1/2)
,x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((a + a*(exp(-
 c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(96*A + 112*B)*1i)/(105*d
) - (exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2))^(1/2)*(96*A + 112*B)*1i)/(105*d) + (exp(c*2i + d*x*2i)*(a + a*(exp(-
 c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(336*A + 392*B)*1i)/(105*
d) - (exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2)*(336*A + 392*B)*1i)/(105*d) - (B*exp(c*3i + d*x*3i)*(a + a*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d) + (B*exp(c*
4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)
*8i)/(3*d)))/(exp(c*1i + d*x*1i) + 3*exp(c*2i + d*x*2i) + 3*exp(c*3i + d*x
*3i) + 3*exp(c*4i + d*x*4i) + 3*exp(c*5i + d*x*5i) + exp(c*6i + d*x*6i) +
exp(c*7i + d*x*7i) + 1)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)b\right.$$
$$\left.+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^4\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
sqrt(a)*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c
+ d*x)**4,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x
)**4,x)*a)
```

3.495 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 130

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{4a(4A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}$$

output
```
4/15*a*(4*A+5*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/15
*a*(4*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a*A*
sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{a(1+\cos(c+dx))}(7A+5B+(4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx)))\sec^{\frac{5}{2}}(c+dx)\tan$$
$$15d$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2)
,x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
 + 5*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3440, 3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\downarrow \text{3459}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(4A+5B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(4A+5B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(4A+5B)\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(4A+5B)\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}\right.\right.$$

$$\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(4A+5B)\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}}\right.\right.$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sin[c + d*x])/(5*d*Cos[c + d
*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + ((4*A + 5*B)*((2*a*Sin[c + d*x])/(3*
d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(112) = 224.

Time = 16.44 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.22

method	result
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+7\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{\cos(dx+c)}}}{15d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{2}}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}\right.$
parts	$\frac{2A\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+7\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{\cos(dx+c)}}}{15d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{2}}+B\sqrt{2}\left(-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(32\cos\right.}{}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2^(1/2)*(2/15*A/d*tan(1/2*d*x+1/2*c)*(32*cos(1/2*d*x+1/2*c)^4-24*cos(1/2*d
*x+1/2*c)^2+7)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1
))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-2/15*B/d*tan(1/2*d*x+1/2*c)*(32*cos(
1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)^2+7)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)
*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+4/5*B/d
*tan(1/2*d*x+1/2*c)*(12*cos(1/2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+2)*(a*
cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{2\left(2\left(4A+5B\right)\cos\left(dx+c\right)^{2}+\left(4A+5B\right)\cos\left(dx+c\right)+3A\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(
a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sq
rt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(112) = 224$.

Time = 0.22 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.65

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
2/15*(A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*s
qrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 5*B*(3*sqrt(2)*sqrt(a)*sin(d
*x + c)/(cos(d*x + c) + 1) - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 5*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - sqrt(2)
*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.51

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{4\sqrt{a}\,(\cos(c+d\,x)+1)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(14\,A\,\sin(c+d\,x)+10\,B\,\sin(c+d\,x)+8\,A\,\sin(2\,c+2\,d\,x)+18\,A}{15\,d\,(10\,\cos(c+d\,x)+8\,\mathrm{co}}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(1/2)
,x)
```

output
```
(4*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(14*A*sin(c + d*x)
+ 10*B*sin(c + d*x) + 8*A*sin(2*c + 2*d*x) + 18*A*sin(3*c + 3*d*x) + 4*A*s
in(4*c + 4*d*x) + 4*A*sin(5*c + 5*d*x) + 10*B*sin(2*c + 2*d*x) + 15*B*sin(
3*c + 3*d*x) + 5*B*sin(4*c + 4*d*x) + 5*B*sin(5*c + 5*d*x)))/(15*d*(10*cos
(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) +
 cos(5*c + 5*d*x) + 6))
```

Reduce [F]

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx$$
$$= \sqrt{a}\left(\left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}\,\sec(dx + c)^3\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c
+ d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x
)**3,x)*a)
```

3.496 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2a(2A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
2/3*a*(2*A+3*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a
*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(A+(2A+3B)\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2)
,x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Sec[c + d*x]^
(3/2)*Tan[(c + d*x)/2])/(3*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3440, 3042, 3459, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(2A+3B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(2A+3B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{ 3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(2*A + 3*B)*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(73) = 146$.

Time = 15.55 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.93

method	result
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{\cos(dx+c)}}}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sqrt{a\cos\left(\frac{dx}{2}-\right.}}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}\right.$
parts	$\frac{2A\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{\cos(dx+c)}}}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}+B\sqrt{2}\left(-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sqrt{a\,c}}{3d\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\right.}\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2^(1/2)*(2/3*A/d*tan(1/2*d*x+1/2*c)*(4*cos(1/2*d*x+1/2*c)^2-1)*(a*cos(1/2*
d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)-2/3*B/d*tan(1/2*d*x+1/2*c)*(4*cos(1/2*d*x+1/2*c)^2-1)*(a*cos(1/2
*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)+4/3*B/d*tan(1/2*d*x+1/2*c)*(5*cos(1/2*d*x+1/2*c)^2-2)*(a*cos(1/
2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2\left((2A+3B)\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(
(d*cos(d*x + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(73) = 146.

Time = 0.22 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.47

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2\left(\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{3B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\cdots\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\cdots}\cdots\right.}{3d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
2/3*(A*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt
(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x
 + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)
^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 1)) + 3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2
*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(2\,A\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(c+d\,x\right)+2\,A\,\sin\left(2\,c+2\,d\,x\right)+2\,A\,\mathrm{si}\right.}{3\,d\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(1/2)
,x)
```

output
```
(2*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(2*A*sin(c + d*x) +
 3*B*sin(c + d*x) + 2*A*sin(2*c + 2*d*x) + 2*A*sin(3*c + 3*d*x) + 3*B*sin(
3*c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x
) + 2))
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^2\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c
+ d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x
)**2,x)*a)
```

3.497 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 96

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{2\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a^(1/2)*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+2*a*A*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}B\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\right.}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2)
,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*B
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2
]))/d
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3440, 3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3459}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{2B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c
+ d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(82) = 164$.

Time = 15.22 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.32

method	result
parts	$\frac{2A\sqrt{2}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{\cos(dx+c)}}}{d}-\frac{2B\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{\cos(dx+c)}}\,\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}\cdots\right.\right.}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cdots}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\cdots\right.}}}\cdots\right.}{d}\cdots$
default	$\sqrt{2}\left(\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{\cos(dx+c)}}}{d}-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{\cos(dx+c)}}}{d}-\frac{B\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\cdots}{\cdots}\cdots\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2*A*2^(1/2)/d*tan(1/2*d*x+1/2*c)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(
1/2*d*x+1/2*c)^2-1))^(1/2)-2*B/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(
1/2*d*x+1/2*c)^2-1))^(1/2)*sec(1/2*d*x+1/2*c)*arctan(1/((2*cos(1/2*d*x+1/2
*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1
/2*c))*2^(1/2))*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2
)-2*B/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)*arctan(1/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(co
t(1/2*d*x+1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))*((2*cos(1/2*d*x+1/2*c)^2-1)/
(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = -\frac{2\left((B\cos(dx+c)+B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}A\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-2*((B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. 2(82) = 164.

Time = 0.48 (sec) , antiderivative size = 906, normalized size of antiderivative = 9.44

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*(B*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2...
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos{(c+d\,x)})\,\left(\frac{1}{\cos{(c+d\,x)}}\right)^{3/2}\,\sqrt{a+a\,\cos{(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)
, x)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\sqrt{a}\left(\left(\int \sqrt{\sec{(dx+c)}}\,\sqrt{\cos{(dx+c)}+1}\,\cos{(dx+c)}\sec{(dx+c)}\,dx\right)b + \left(\int \sqrt{\sec{(dx+c)}}\,\sqrt{\cos{(dx+c)}+1}\,\sec{(dx+c)}\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(c
+ d*x),x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x),x
)*a)
```

3.498 $\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+d$

Optimal result

Integrand size = 35, antiderivative size = 98

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \frac{\sqrt{a}(2A+B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
a^(1/2)*(2*A+B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d+a*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*
x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}(2A+B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{2d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
 + d*x]]*(Sqrt[2]*(2*A + B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Co
s[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3440, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3460}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}(2A+B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}(2A+B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{(2A+B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a}(2A+B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*
Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*B*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(84) = 168$.

Time = 14.91 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.66

method	result
parts	$-\frac{2A\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}}\right)\left(1+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+B\sqrt{2}\left(\right.$
default	$\sqrt{2}\left(-\frac{A\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\arctan\left(\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{\sqrt{\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}}\right)\left(1+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{d}+\right.$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*A/d*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*((2*cos(1/2*d*x+1/2*c)^2-1)/(c
os(1/2*d*x+1/2*c)+1)^2)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*arctan(1/((2*
cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c
)-csc(1/2*d*x+1/2*c))*2^(1/2))*(1+sec(1/2*d*x+1/2*c))+B*2^(1/2)*(1/d*(1/(2
*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1
/2*c)+1)^2)^(1/2)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*arctan(1/((2*cos(1/2*d*x+
1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*
x+1/2*c))*2^(1/2))*(1+sec(1/2*d*x+1/2*c))*2^(1/2)-1/2/d*(a*cos(1/2*d*x+1/2
*c)^2)^(1/2)*(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(tan(1/2*d*x+1/2*c)*(-4*
cos(1/2*d*x+1/2*c)^2+2)+2^(1/2)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1
/2*c)+1)^2)^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)
+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))*(3+3*sec(1/2
*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{a\cos(dx+c)+a}B\sqrt{\cos(dx+c)}\sin(dx+c) - ((2A+B)\cos(dx+c)+2A+B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos}}{}\right.}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A + B)*c
os(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \int \sqrt{a(\cos(c+dx)+1)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sqrt(sec(c + d*x)
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(84) = 168$.

Time = 0.35 (sec) , antiderivative size = 939, normalized size of antiderivative = 9.58

$$\int \sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))\sqrt{\sec(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((co...
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}\sqrt{a+a\cos(c+dx)}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)
, x)
```

Reduce [F]

$$\int \sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \sqrt{a}\left(\left(\int \sqrt{\sec(dx+c)}\sqrt{\cos(dx+c)+1}\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\sqrt{\cos(dx+c)+1}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*b +
 int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1),x)*a)
```

3.499 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 151

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{a}(4A+3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/4*a^(1/2)*(4*A+3*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/2*a*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)/sec(d*x+c)^(3/2)+1/4*a*(4*A+3*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/s
ec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}(4A+3B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{8d}$$

input
```
Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x
]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
 + d*x]]*(Sqrt[2]*(4*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Co
s[c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3440, 3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3440}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3460}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}(4A+3B)\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}(4A+3B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{aB\sin(c+dx)\,c}{2d\sqrt{a\cos(c+}}\right.$$

$$\Big\downarrow \text{3249}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}(4A+3B)\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin}{2d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}(4A+3B)\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}(4A+3B)\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right.\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}(4A+3B)\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\,\text{si}}{2d}\right.$$

input
```
Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + ((4*A + 3*B)*((Sqrt[a]*ArcSin[(Sqrt[a]
*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [F]

$$\int \frac{\sqrt{a+a\cos(dx+c)}\,(A+B\cos(dx+c))}{\sqrt{\sec(dx+c)}}dx$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = -\frac{((4A+3B)\cos(dx+c)+4A+3B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2B\cos(dx+c)^2+(4A+3B)\cos\right.}{\sqrt{\cos}}}{4\,(d\cos(dx+c)+d)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/4*(((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*cos(d*x + c)^
2 + (4*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}(A+B\cos\left(c+dx\right))}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(sec(c + d*x)
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. $2(127) = 254$.

Time $= 0.45$ (sec) , antiderivative size $= 1851$, normalized size of antiderivative $= 12.26$

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)...
```

Giac [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\sec(dx+c)}dx\right)b + \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}}{\sec(dx+c)}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/sec(
c + d*x),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/sec(c + d*
x),x)*a)
```

3.500 $\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 196

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}(6A+5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/8*a^(1/2)*(6*A+5*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/3*a*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)/sec(d*x+c)^(5/2)+1/12*a*(6*A+5*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/
sec(d*x+c)^(3/2)+1/8*a*(6*A+5*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d
*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(3\sqrt{2}(6A+5B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+\right.\right.}{48d}$$

input
```
Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/
2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
 + d*x]]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[
Cos[c + d*x]]*(18*A + 19*B + 2*(6*A + 5*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d
*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3440, 3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 3440}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}(A+B\cos(c+dx))dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 3460$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3249$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3249$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{aB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+}}\right.}{d}\right.\right.\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}(6A+5B)\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a}{\ }\right.\right.$$

input
```
Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + ((6*A + 5*B)*((a*Cos[c + d*x]^(3/2)*Si
n[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*
Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(166) = 332$.

Time = 16.97 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.78

method	result	size
default	Expression too large to display	740
parts	Expression too large to display	743

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2^(1/2)*(-1/8*A/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)+1)/(1
/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*
x+1/2*c)+1)^2)^(1/2)*(3*sec(1/2*d*x+1/2*c)*2^(1/2)*arctan(1/((2*cos(1/2*d*
x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*
d*x+1/2*c))*2^(1/2))+tan(1/2*d*x+1/2*c)*(-8*cos(1/2*d*x+1/2*c)^3-8*cos(1/2
*d*x+1/2*c)^2-2*cos(1/2*d*x+1/2*c)-2)*((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2
*d*x+1/2*c)+1)^2)^(1/2))+1/8*B/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d
*x+1/2*c)+1)/(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/((2*cos(1/2*d*x+1/2*c)^2
-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(3*sec(1/2*d*x+1/2*c)*2^(1/2)*arctan(1
/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(cot(1/2*d*x+
1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))+tan(1/2*d*x+1/2*c)*(-8*cos(1/2*d*x+1/2
*c)^3-8*cos(1/2*d*x+1/2*c)^2-2*cos(1/2*d*x+1/2*c)-2)*((2*cos(1/2*d*x+1/2*c
)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2))-1/48*B/d*(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)+1)/(1/(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/((2*cos(
1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2)*(33*sec(1/2*d*x+1/2*c)
*2^(1/2)*arctan(1/((2*cos(1/2*d*x+1/2*c)^2-1)/(cos(1/2*d*x+1/2*c)+1)^2)^(1
/2)*(cot(1/2*d*x+1/2*c)-csc(1/2*d*x+1/2*c))*2^(1/2))+tan(1/2*d*x+1/2*c)*(-
64*cos(1/2*d*x+1/2*c)^5-64*cos(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)^3-24
*cos(1/2*d*x+1/2*c)^2-38*cos(1/2*d*x+1/2*c)-38)*((2*cos(1/2*d*x+1/2*c)^2-1
)/(cos(1/2*d*x+1/2*c)+1)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{3\left((6A+5B)\cos(dx+c)+6A+5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8B\cos(dx+c)^3+2(6A+5B)\right.}{}}{24\left(d\cos(dx+c)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/24*(3*((6*A + 5*B)*cos(d*x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*cos(d*x +
c)^3 + 2*(6*A + 5*B)*cos(d*x + c)^2 + 3*(6*A + 5*B)*cos(d*x + c))*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{\sqrt{a\left(\cos\left(c + dx\right) + 1\right)}\left(A + B\cos\left(c + dx\right)\right)}{\sec^{\frac{3}{2}}\left(c + dx\right)}\, dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sec(c + d*x)**(3/
2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2981 vs. $2(166) = 332$.

Time = 0.75 (sec) , antiderivative size = 2981, normalized size of antiderivative = 15.21

$$\int \frac{\sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))...
```

Giac [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(A + B\,\cos(c + d\,x))\,\sqrt{a + a\,\cos(c + d\,x)}}{\left(\frac{1}{\cos(c + d\,x)}\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a + a\cos(c + dx)}(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}\,\cos(dx + c)}{\sec(dx + c)^2}dx\right)b + \left(\int \frac{\sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}}{\sec(dx + c)^2}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/sec(
c + d*x)**2,x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/sec(c +
 d*x)**2,x)*a)
```

3.501 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 275

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c&+dx)\,dx = \frac{32a^2(168A+187B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}}\\
&+\frac{16a^2(168A+187B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}}\\
&+\frac{4a^2(168A+187B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}}\\
&+\frac{2a^2(168A+187B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}\\
&+\frac{2a^2(12A+11B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}\\
&+\frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d}\end{aligned}$$

output
```
32/3465*a^2*(168*A+187*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(
1/2)+16/3465*a^2*(168*A+187*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+
c))^(1/2)+4/1155*a^2*(168*A+187*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(
d*x+c))^(1/2)+2/693*a^2*(168*A+187*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*c
os(d*x+c))^(1/2)+2/99*a^2*(12*A+11*B)*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*c
os(d*x+c))^(1/2)+2/11*a*A*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(11/2)*sin(d*x
+c)/d
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.53

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}(2478A+2057B+(6342A+6193B)\cos(c+dx)+13(168A+187B)\cos$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13
/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(2478*A + 2057*B + (6342*A + 6193*B)*Cos[c +
 d*x] + 13*(168*A + 187*B)*Cos[2*(c + d*x)] + 2184*A*Cos[3*(c + d*x)] + 24
31*B*Cos[3*(c + d*x)] + 336*A*Cos[4*(c + d*x)] + 374*B*Cos[4*(c + d*x)] +
336*A*Cos[5*(c + d*x)] + 374*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*Tan[(
c + d*x)/2])/(3465*d)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{13}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{13/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 3440$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{13/2}}dx$$

$$\Big\downarrow \ 3454$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{11}\int \frac{\sqrt{\cos(c+dx)a+a}(a(12A+11B)+a(8A+11B)\cos(c+dx))}{2\cos^{\frac{11}{2}}(c+dx)}dx+\frac{2aA\sin\right.$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\int \frac{\sqrt{\cos(c+dx)a+a}(a(12A+11B)+a(8A+11B)\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx+\frac{2aA\sin\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(12A+11B)+a(8A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx\right.$$

$$\Big\downarrow \ 3459$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2(12A+11B)\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)}}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2(12A+11B)\sin(c}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c}}\right.\right.$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\left(\frac{6}{7}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\left(\frac{6}{7}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a\sin(c+}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos}$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{}$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}a(168A+187B)\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a}{3d\cos^{\frac{3}{2}}(c+d}$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{2a^2(12A+11B)\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{1}{9}a(168A+187B)\left(\frac{2a\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + ((2*a^2*(12*A + 11*B)*Sin[c + d*x]
)/(9*d*Cos[c + d*x]^(9/2)*Sqrt[a + a*Cos[c + d*x]]) + (a*(168*A + 187*B)*(
(2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (6*
((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4
*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (
4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/
7))/9)/11)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 14.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.57

method	result
default	$\frac{2\sin(dx+c)\left(\left(2688\cos(dx+c)^5+1344\cos(dx+c)^4+1008\cos(dx+c)^3+840\cos(dx+c)^2+735\cos(dx+c)+315\right)A+\cos(dx+c)\left(2992\,\mathrm{c}\right.}{3465d(\cos(dx+}$
parts	$\frac{2A\sin(dx+c)\left(128\cos(dx+c)^5+64\cos(dx+c)^4+48\cos(dx+c)^3+40\cos(dx+c)^2+35\cos(dx+c)+15\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sec(dx+c)^{\frac{13}{2}}}{165d(\cos(dx+c)+1)}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/3465/d*sin(d*x+c)*((2688*cos(d*x+c)^5+1344*cos(d*x+c)^4+1008*cos(d*x+c)^
3+840*cos(d*x+c)^2+735*cos(d*x+c)+315)*A+cos(d*x+c)*(2992*cos(d*x+c)^4+149
6*cos(d*x+c)^3+1122*cos(d*x+c)^2+935*cos(d*x+c)+385)*B)*(a*cos(1/2*d*x+1/2
*c)^2)^(1/2)*cos(d*x+c)*sec(d*x+c)^(13/2)/(cos(d*x+c)+1)*a*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.52

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \frac{2\left(16\,(168\,A+187\,B)a\cos\left(dx+c\right)^5+8\,(168\,A+187\,B)a\cos\left(dx+c\right)^4+6\,(168\,A+187\,B)a\cos\cdots\right)}{3465\left(d\cos\left(\cdots\right.\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
2/3465*(16*(168*A + 187*B)*a*cos(d*x + c)^5 + 8*(168*A + 187*B)*a*cos(d*x
+ c)^4 + 6*(168*A + 187*B)*a*cos(d*x + c)^3 + 5*(168*A + 187*B)*a*cos(d*x
+ c)^2 + 35*(21*A + 11*B)*a*cos(d*x + c) + 315*A*a)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(239) = 478$.

Time = 0.24 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.59

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^{\frac{13}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
4/3465*(21*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 495*sqrt
(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1056*sqrt(2)*a^(3/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 299*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 46*sqrt(2)
*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 + 1)) + 11*(315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c
) + 1) - 1155*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2184*s
qrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2586*sqrt(2)*a^(3/2)*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1759*sqrt(2)*a^(3/2)*sin(d*x + c)^9/
(cos(d*x + c) + 1)^9 - 611*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) +
 1)^11 + 94*sqrt(2)*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*B*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 +...
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 28.64 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(-\frac{32\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,A+3\,B)\,\sqrt{a+a\,\cos(c+d\,x)}}{5\,d}+\frac{64\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d}{2}\right.}{3}\right.}{20\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+20\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+10\,e^{\frac{c\,11i}{2}+\frac{d\,x\,1}{2}}}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + a*cos(c + d*x))^(3/2
),x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((64*a*exp((c*1
1i)/2 + (d*x*11i)/2)*sin((3*c)/2 + (3*d*x)/2)*(21*A + 19*B)*(a + a*cos(c +
 d*x))^(1/2))/(35*d) - (32*a*exp((c*11i)/2 + (d*x*11i)/2)*sin(c/2 + (d*x)/
2)*(2*A + 3*B)*(a + a*cos(c + d*x))^(1/2))/(5*d) + (32*a*exp((c*11i)/2 + (
d*x*11i)/2)*sin((7*c)/2 + (7*d*x)/2)*(168*A + 187*B)*(a + a*cos(c + d*x))^
(1/2))/(315*d) + (64*a*exp((c*11i)/2 + (d*x*11i)/2)*sin((11*c)/2 + (11*d*x
)/2)*(168*A + 187*B)*(a + a*cos(c + d*x))^(1/2))/(3465*d)))/(20*exp((c*11i
)/2 + (d*x*11i)/2)*cos(c/2 + (d*x)/2) + 20*exp((c*11i)/2 + (d*x*11i)/2)*co
s((3*c)/2 + (3*d*x)/2) + 10*exp((c*11i)/2 + (d*x*11i)/2)*cos((5*c)/2 + (5*
d*x)/2) + 10*exp((c*11i)/2 + (d*x*11i)/2)*cos((7*c)/2 + (7*d*x)/2) + 2*exp
((c*11i)/2 + (d*x*11i)/2)*cos((9*c)/2 + (9*d*x)/2) + 2*exp((c*11i)/2 + (d*
x*11i)/2)*cos((11*c)/2 + (11*d*x)/2))
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^6\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^6\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^6\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^6\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**6,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d
*x)*sec(c + d*x)**6,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*c
os(c + d*x)**2*sec(c + d*x)**6,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c +
d*x) + 1)*sec(c + d*x)**6,x)*a)
```

3.502 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx &= \frac{16a^2(34A+39B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{8a^2(34A+39B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{2a^2(34A+39B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{2a^2(10A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
16/315*a^2*(34*A+39*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2
)+8/315*a^2*(34*A+39*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+2/105*a^2*(34*A+39*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)+2/63*a^2*(10*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+2/9*a*A*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.54

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}(376A+351B+(374A+324B)\cos(c+dx)+11(34A+39B)\cos(2(c+$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11
/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x
] + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3
*(c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Sec[c + d*x]^
(9/2)*Tan[(c + d*x)/2])/(315*d)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{11}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int \frac{\sqrt{\cos(c+dx)a+a}(a(10A+9B)+3a(2A+3B)\cos(c+dx))}{2\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{\sqrt{\cos(c+dx)a+a}(a(10A+9B)+3a(2A+3B)\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(c}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(10A+9B)+3a(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\right.$$

$\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2(10A+9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+c}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2(10A+9B)\sin(c+d}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+d}}\right.\right.$$

$\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c}}\right.\right.\right.$$

$\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{3}{7}a(34A+39B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\,\mathrm{co}}\right.\right.\right.\right.$$

$\downarrow$ 3250

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{2a^2(10A+9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3}{7}a(34A+39B)\left(\frac{2a\sin(c+d}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((2*a^2*(10*A + 9*B)*Sin[c + d*x])/(
7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (3*a*(34*A + 39*B)*((2*
a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2
*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*
Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/7)/9
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 13.93 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.60

method	result
default	$\frac{2\sin(dx+c)\left(\left(272\cos(dx+c)^4+136\cos(dx+c)^3+102\cos(dx+c)^2+85\cos(dx+c)+35\right)A+\cos(dx+c)\left(312\cos(dx+c)^3+156\cos(dx+\right.\right.}{315d(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(272\cos(dx+c)^4+136\cos(dx+c)^3+102\cos(dx+c)^2+85\cos(dx+c)+35\right)\sec(dx+c)^{\frac{11}{2}}\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos(dx+c)a\sqrt{}}{315d(\cos(dx+c)+1)}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/315/d*sin(d*x+c)*((272*cos(d*x+c)^4+136*cos(d*x+c)^3+102*cos(d*x+c)^2+85
*cos(d*x+c)+35)*A+cos(d*x+c)*(312*cos(d*x+c)^3+156*cos(d*x+c)^2+117*cos(d*
x+c)+45)*B)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*cos(d*x+c)*sec(d*x+c)^(11/2)/(c
os(d*x+c)+1)*a*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.55

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\frac{2\left(8\left(34A+39B\right)a\cos\left(dx+c\right)^4+4\left(34A+39B\right)a\cos\left(dx+c\right)^3+3\left(34A+39B\right)a\cos\left(dx+\right.\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
 + 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A
*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x +
 c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx)) \sec^{\frac{11}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(198) = 396$.

Time = 0.27 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.71

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx)) \sec^{\frac{11}{2}}(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
4/315*((315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 840*sqrt(2)*
a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)*a^(3/2)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
94*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 3*(105*sq
rt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 350*sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 518*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5 - 444*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 +
209*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 38*sqrt(2)*a^(3/
2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 31.61 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.39

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{96\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a+a\,\cos(c+d\,x)}\,(A+B)}{5\,d}-\frac{16\,B\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\sqrt{a\cdots}}{3\,d}\right.}{12\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}\cdots}}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(3/2
),x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((96*a*exp((c*9
i)/2 + (d*x*9i)/2)*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2)*(A + B))/
(5*d) - (16*B*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/2)*(a + a
*cos(c + d*x))^(1/2))/(3*d) + (16*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2
 + (5*d*x)/2)*(34*A + 39*B)*(a + a*cos(c + d*x))^(1/2))/(35*d) + (32*a*exp
((c*9i)/2 + (d*x*9i)/2)*sin((9*c)/2 + (9*d*x)/2)*(34*A + 39*B)*(a + a*cos(
c + d*x))^(1/2))/(315*d)))/(12*exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/
2) + 8*exp((c*9i)/2 + (d*x*9i)/2)*cos((3*c)/2 + (3*d*x)/2) + 8*exp((c*9i)/
2 + (d*x*9i)/2)*cos((5*c)/2 + (5*d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*co
s((7*c)/2 + (7*d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x
)/2))
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^{5}dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^{5}dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^{2}\sec(dx+c)^{5}dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^{5}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**5,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d
*x)*sec(c + d*x)**5,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*c
os(c + d*x)**2*sec(c + d*x)**5,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c +
d*x) + 1)*sec(c + d*x)**5,x)*a)
```

3.503 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{4a^2(52A+63B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(52A+63B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
4/105*a^2*(52*A+63*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
+2/105*a^2*(52*A+63*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2
)+2/35*a^2*(8*A+7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/7*a*A*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}(82A+63B+3(78A+77B)\cos(c+dx)+(52A+63B)\cos(2(c+dx))}{105d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/
2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x]
+ (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c +
 d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$\big\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{\sqrt{\cos(c+dx)a+a}(a(8A+7B)+a(4A+7B)\cos(c+dx))}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+}{7d}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{\cos(c+dx)a+a}(a(8A+7B)+a(4A+7B)\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(c+}{7d}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(8A+7B)+a(4A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2}{}\right.$$

$\big\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2(8A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2(8A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+d}}\right.\right.$$

$\big\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)}}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}a(52A+63B)\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c}}\right.\right.\right.$$

$\big\downarrow$ 3250

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{2a^2(8A+7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{1}{5}a(52A+63B)\left(\frac{2a\sin(c+d}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*a^2*(8*A + 7*B)*Sin[c + d*x])/(5
*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (a*(52*A + 63*B)*((2*a*S
in[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[
c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5)/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 14.00 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

method	result
default	$\frac{2\sin(dx+c)\left(\left(104\cos(dx+c)^3+52\cos(dx+c)^2+39\cos(dx+c)+15\right)A+\cos(dx+c)\left(126\cos(dx+c)^2+63\cos(dx+c)+21\right)B\right)\sqrt{a\cos(}}{105d(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(104\cos(dx+c)^3+52\cos(dx+c)^2+39\cos(dx+c)+15\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sec(dx+c)^{\frac{9}{2}}\cos(dx+c)a\sqrt{2}}{105d(\cos(dx+c)+1)}+\frac{2B\sin(dx+c}{}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105/d*sin(d*x+c)*((104*cos(d*x+c)^3+52*cos(d*x+c)^2+39*cos(d*x+c)+15)*A+
cos(d*x+c)*(126*cos(d*x+c)^2+63*cos(d*x+c)+21)*B)*(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)*cos(d*x+c)*sec(d*x+c)^(9/2)/(cos(d*x+c)+1)*a*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.59

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2\left(2\,(52\,A+63\,B)a\cos{(dx+c)}^3+(52\,A+63\,B)a\cos{(dx+c)}^2+3\,(13\,A+7\,B)a\cos{(dx+c)}\right.}{105\left(d\cos{(dx+c)}^4+d\cos{(dx+c)}^3\right)\sqrt{\cos{(dx+c)}}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
 3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(157) = 314$.

Time = 0.23 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.91

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^{\frac{9}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
4/105*((105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*
a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 + 1)) + 21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos
(d*x + c) + 1) - 15*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
17*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sqrt(2)*a^(3/2)
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^{\frac{9}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 32.11 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.43

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(-\frac{8\,a\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)(2\,A+3\,B)\,\sqrt{a+a\,\cos(c+d\,x)}}{3\,d}+\frac{16\,a\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)(1}{15\,d}\right.}{6\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+2\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((16*a*exp((c*7
i)/2 + (d*x*7i)/2)*sin((3*c)/2 + (3*d*x)/2)*(13*A + 12*B)*(a + a*cos(c + d
*x))^(1/2))/(15*d) - (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin(c/2 + (d*x)/2)*(2
*A + 3*B)*(a + a*cos(c + d*x))^(1/2))/(3*d) + (8*a*exp((c*7i)/2 + (d*x*7i)
/2)*sin((7*c)/2 + (7*d*x)/2)*(52*A + 63*B)*(a + a*cos(c + d*x))^(1/2))/(10
5*d)))/(6*exp((c*7i)/2 + (d*x*7i)/2)*cos(c/2 + (d*x)/2) + 6*exp((c*7i)/2 +
 (d*x*7i)/2)*cos((3*c)/2 + (3*d*x)/2) + 2*exp((c*7i)/2 + (d*x*7i)/2)*cos((
5*c)/2 + (5*d*x)/2) + 2*exp((c*7i)/2 + (d*x*7i)/2)*cos((7*c)/2 + (7*d*x)/2
))
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \sqrt{a}\,a\left(\left(\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^4\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**4,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d
*x)*sec(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*c
os(c + d*x)**2*sec(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c +
d*x) + 1)*sec(c + d*x)**4,x)*a)
```

3.504 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 134

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{2a^2(18A+25B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/15*a^2*(18*A+25*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/15*a^2*(6*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/
5*a*A*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}(24A+25B+2(9A+5B)\cos(c+dx)+(18A+25B)\cos(2(c+dx)))\,\mathrm{se}}{15d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/
2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] +
(18*A + 25*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d
)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.257$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3459, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{\sqrt{\cos(c+dx)a+a}(a(6A+5B)+a(2A+5B)\cos(c+dx))}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+}{5d}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{\cos(c+dx)a+a}(a(6A+5B)+a(2A+5B)\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+}{5d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(6A+5B)+a(2A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2}{}\right.$$

$\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}a(18A+25B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(6A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}a(18A+25B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2(6A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)}}\right.\right.$$

$\downarrow$ 3250

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2a^2(6A+5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((2*a^2*(6*A + 5*B)*Sin[c + d*x])/(3
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(18*A + 25*B)*Sin
[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 14.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

method	result
default	$\frac{2\sin(dx+c)\left(\left(12+9\cos(2dx+2c)+9\cos(dx+c)\right)A+\left(25\cos(dx+c)^{2}+5\cos(dx+c)\right)B\right)\sec(dx+c)^{\frac{7}{2}}\sqrt{a(\cos(dx+c)+1)}\,\cos(dx+c)a}{15d(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)\left(6\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sec(dx+c)^{\frac{7}{2}}\cos(dx+c)a\sqrt{2}}{5d(\cos(dx+c)+1)}+\frac{2B\sin(dx+c)(5\cos(dx+c)+1)\sqrt{a\cdots}}{3d(\cdots}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d*sin(d*x+c)*((12+9*cos(2*d*x+2*c)+9*cos(d*x+c))*A+(25*cos(d*x+c)^2+5
*cos(d*x+c))*B)*sec(d*x+c)^(7/2)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)/(cos(
d*x+c)+1)*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{2\left((18\,A+25\,B)a\cos\left(dx+c\right)^{2}+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)
^2)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(116) = 232$.

Time = 0.34 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.25

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{4\left(\frac{3\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)A\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(\ldots}{\cos(dx+c\ldots}\right.}{\ldots}\right.}{15\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
4/15*(3*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^
(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
1)) + 5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sqrt(2)*a^(
3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1
)))/d
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(48\,A\,\sin\left(c+d\,x\right)+50\,B\,\sin\left(c+d\,x\right)+36\,A\,\sin\left(2\,c+2\right.\right.}{15\,d\,\left(10\,\cos\left(c\right.\right.}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

output
```
(2*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(48*A*sin(c + d*x
) + 50*B*sin(c + d*x) + 36*A*sin(2*c + 2*d*x) + 66*A*sin(3*c + 3*d*x) + 18
*A*sin(4*c + 4*d*x) + 18*A*sin(5*c + 5*d*x) + 20*B*sin(2*c + 2*d*x) + 75*B
*sin(3*c + 3*d*x) + 10*B*sin(4*c + 4*d*x) + 25*B*sin(5*c + 5*d*x)))/(15*d*
(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4
*d*x) + cos(5*c + 5*d*x) + 6))
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\sqrt{a}\,a\left(\left(\left(\int\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}dx\right)a+\left(\int\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}dx\right)b+\left(\int\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}dx\right)b+\left(\int\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)+1}\,\sec\left(dx+c\right)^{3}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**3,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d
*x)*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*c
os(c + d*x)**2*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c +
d*x) + 1)*sec(c + d*x)**3,x)*a)
```

3.505 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 145

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2*a^(3/2)*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+2/3*a^2*(4*A+3*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a
+a*cos(d*x+c))^(1/2)+2/3*a*A*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(3\sqrt{2}B\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c-\right.}{3d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/
2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt[
2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3
*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+3B)+3aB\cos(c+dx))}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a}}{3d\cos^{\frac{3}{2}}(c}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+3B)+3aB\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{a}}{3d\cos^{\frac{3}{2}}(c}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(4A+3B)+3aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2aA\sin(c}{}\right.$$

$\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3aB\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA\sin}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3aB\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2}{}\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{6aB\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right.\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{6a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2aA}{}\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((6*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c
+ d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*a^2*(4*A + 3*B)*Sin[c + d*x])/(d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 14.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

method	result
default	$\frac{2\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(3\cos(dx+c)^3+3\cos(dx+c)^2\right)+\cos(dx+c)\left(\cdots\right.\right.}{3d(\cos(dx+c)+1)}$
parts	$\frac{2A\sin(dx+c)(5\cos(dx+c)+1)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sec(dx+c)^{\frac{5}{2}}\cos(dx+c)a\sqrt{2}}{3d(\cos(dx+c)+1)}+\frac{2B\cos(dx+c)^2\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(\left(\cdots\right.\right.}{\cdots}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3/d*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)*(B*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*(3*cos(d*x+c)^3+3*cos(d*x+c)^2)+cos(d*x+c)*(5*cos(d*x+c)+1)*sin(d*x+c
)*A+3*cos(d*x+c)^2*B*sin(d*x+c))*a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = -\frac{2\left(3\left(Ba\cos\left(dx+c\right)^2+Ba\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{((5A+3B)a\cos(dx+c)+Aa)\sqrt{c}}{\sqrt{\cos(dx+}}}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
-2/3*(3*(B*a*cos(d*x + c)^2 + B*a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((5*A + 3*B)*a*
cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. 2(123) = 246.

Time = 0.37 (sec) , antiderivative size = 1462, normalized size of antiderivative = 10.08

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
1/6*(3*(6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(3/4)*a^(3/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*((2*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) - a*sin(2*d*x + 2*c) - 2*(a*cos(2*d*x + 2*c) + a)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + (2*a*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + a*cos(2*d*x + 2*c) + 2*(a*cos(2*d*x + 2*c) + a)*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + ((a*cos(2*d*x + 2*c)^2 +
a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a*cos(
2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arcta...
```

Giac [F(-1)]

Timed out.

$$\int (a + a\cos(c+dx))^{3/2}(A + B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c+dx))^{3/2}(A + B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (A + B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a + a\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^2\,dx\right)a + \left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}\,\cos(dx + c)\sec(dx + c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}\,\cos(dx + c)^2\sec(dx + c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c) + 1}\,\sec(dx + c)^2\,dx\right)a\right)$$

input

```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output

```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**2,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d
*x)*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*c
os(c + d*x)**2*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c +
d*x) + 1)*sec(c + d*x)**2,x)*a)
```

3.506 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 146

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{a^{3/2}(2A+3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
a^(3/2)*(2*A+3*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d-a^2*(2*A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)/sec(d*x+c)^(1/2)+2*a*A*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x
+c)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}(2A+3B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/
2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]
*(2*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A
+ B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(2*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\int \frac{\sqrt{\cos(c+dx)a+a}(a(2A+B)-a(2A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx}{d\sqrt{c}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sqrt{\cos(c+dx)a+a}(a(2A+B)-a(2A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx)}{d\sqrt{\cos}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(2A+B)-a(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aA\,\mathrm{s}}{}\right.$$

$\downarrow$ 3460

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(2A+3B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(2A+3B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+d}}{d\sqrt{a\cos(c+dx)+a}}\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a(2A+3B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\mathrm{co}}}{d\sqrt{a\cos(c+dx)+}}\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^{3/2}(2A+3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a
]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a^2*(2*A - B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.62 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.35

method	result
default	$\frac{\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{3}{2}}\left(B\cos(dx+c)^{2}\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2A\cos(dx+c)^{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+A\sqrt{\cdots}\right.}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$
parts	$\frac{2A\cos(dx+c)\sec(dx+c)^{\frac{3}{2}}\sqrt{a(\cos(dx+c)+1)}\left((\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sin(dx+c)\right)a}{d(\cos(dx+c)+1)}+$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/d*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)+2*A*cos(d*x+c)^2*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(2*d*x+2*c)+3*B*cos(d*x+c)^2*ar
ctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))*a
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx =$$

$$-\frac{((2A+3B)a\cos(dx+c)+(2A+3B)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(Ba\cos(dx+c)+2Aa)\sqrt{a\cos(}}{\sqrt{\cos(dx+c}}}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-(((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (B*a*cos(d*x + c
) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*co
s(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. 2(128) = 256.

Time = 0.51 (sec) , antiderivative size = 1801, normalized size of antiderivative = 12.34

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2...
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(a+a\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x),x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c +
 d*x)**2*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)
*sec(c + d*x),x)*a)
```

3.507 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}$

Optimal result

Integrand size = 35, antiderivative size = 153

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\frac{a^{3/2}(12A+7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d}+\frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}$$

output
```
1/4*a^(3/2)*(12*A+7*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/4*a^2*(4*A+5*B)*sin(d*x+c)/d/(a+a*cos
(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*a*B*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c)/
d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}(\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x
]],x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec
[c + d*x]]*(Sqrt[2]*(12*A + 7*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt
[Cos[c + d*x]]*(4*A + 7*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3440, 3042, 3455, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\downarrow$ 3455

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+B)+a(4A+5B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)a+a}(a(4A+B)+a(4A+5B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c+dx}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(4A+B)+a(4A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{aB}{}\right.$$

$\downarrow$ 3460

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}a(12A+7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+a}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}a(12A+7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{a(12A+7B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{}{\sqrt{}}\right.}{d}\right.\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{a^{3/2}(12A+7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]
*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a^2*(4*A + 5*B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.78 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.24

method	result
default	$\frac{\left(12A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+7B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+4A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\mathrm{si}}{4d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$
parts	$\frac{A\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+3\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\right)\sqrt{a(\cos(dx+c)+1)}\,\sqrt{\sec(dx+c)}\,\cos(dx+c)a}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}+\frac{B\left(7\arctan\left(\mathrm{t}\right.\right.}{}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*(12*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7*B*arcta
n(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*A*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(sin(2*d*x+2*c)+7
*sin(d*x+c))*B)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sec(d*x+c)^(1/2)/(cos(
d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{((12\,A+7\,B)a\cos{(dx+c)}+(12\,A+7\,B)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2\,Ba\cos(dx+c)^2+(4\,A+7\right.}{}}{4\,(d\cos{(dx+c)}+d)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/4*(((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*a*cos
(d*x + c)^2 + (4*A + 7*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1884 vs. 2(129) = 258.

Time = 0.61 (sec) , antiderivative size = 1884, normalized size of antiderivative = 12.31

$$\int (a + a\cos(c+dx))^{3/2}(A + B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x ...
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (A + B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*a
 + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*b + int(s
qrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2,x)*b + int(sqrt(s
ec(c + d*x))*sqrt(cos(c + d*x) + 1),x)*a)
```

3.508 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 200

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^{3/2}(14A+11B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{8d} + \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(14A+11B)\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/8*a^(3/2)*(14*A+11*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/12*a^2*(6*A+7*B)*sin(d*x+c)/d/(a+a*c
os(d*x+c))^(1/2)/sec(d*x+c)^(3/2)+1/3*a*B*(a+a*cos(d*x+c))^(1/2)*sin(d*x+c
)/d/sec(d*x+c)^(3/2)+1/8*a^2*(14*A+11*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{a\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{}}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d
*x]],x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec
[c + d*x]]*(3*Sqrt[2]*(14*A + 11*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*S
qrt[Cos[c + d*x]]*(42*A + 37*B + 2*(6*A + 11*B)*Cos[c + d*x] + 4*B*Cos[2*(
c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3440, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\big\downarrow$ 3455

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}(3a(2A+B)+a(6A+7B)\cos(c+dx))dx\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}(3a(2A+B)+a(6A+7B)\cos(c+dx))dx+\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3a(2A+B)+a(6A+7B)\sin\right.\right.$$

$\big\downarrow$ 3460

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a^2(6A+7B)\sin(c+}{2d\sqrt{a\cos(c+}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2(6A+}{2}\right.\right.$$

$\big\downarrow$ 3249

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+d}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.\right.$$

$\big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{3}{4}a(14A+11B)\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+}{\sqrt{\cos(c+d}}\right.}{d}\right.\right.\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{3}{4}a(14A+11B)\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(}{\sqrt{a\cos(c}}\right.}{d}\right.\right.\right.$$

input
```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((a^2*(6*A + 7*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*a*(14*A + 11*B)*((Sqrt[a]
*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4)/6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 18.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(42A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+33B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+(12\cos(dx+c)+42)\sin(dx+c)A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{24d(\cos(dx+c)+1)\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$
parts	$\frac{A\left(7\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\sin(2dx+2c)+7\sin(dx+c))\right)\sqrt{a(\cos(dx+c)+1)}\,a}{4d(\cos(dx+c)+1)\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}+\frac{B\left(33\arctan\left(\tan(d\right.\right.}{}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*(42*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+33*B*arc
tan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(12*cos(d*x+c)+42)*sin(d
*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(8*cos(d*x+c)^2+22*cos(d*x+c)+33
)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{3\left((14\,A+11\,B)a\cos\left(dx+c\right)+(14\,A+11\,B)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,Ba\cos(dx+c)^3+2\right.}{}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/24*(3*((14*A + 11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*
a*cos(d*x + c)^3 + 2*(6*A + 11*B)*a*cos(d*x + c)^2 + 3*(14*A + 11*B)*a*cos
(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*co
s(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3023 vs. $2(170) = 340$.

Time = 0.67 (sec) , antiderivative size = 3023, normalized size of antiderivative = 15.12

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1)
 - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos...
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)(a\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \sqrt{a}\,a\left(\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\sec(dx+c)}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}}{\sec(dx+c)}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/se
c(c + d*x),x)*a + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d
*x))/sec(c + d*x),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*co
s(c + d*x)**2)/sec(c + d*x),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*
x) + 1))/sec(c + d*x),x)*a)
```

3.509 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 247

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^{3/2}(88A+75B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{64d} + \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(88A+75B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(88A+75B)\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/64*a^(3/2)*(88*A+75*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/24*a^2*(8*A+9*B)*sin(d*x+c)/d/(a+a*
cos(d*x+c))^(1/2)/sec(d*x+c)^(5/2)+1/4*a*B*(a+a*cos(d*x+c))^(1/2)*sin(d*x+
c)/d/sec(d*x+c)^(5/2)+1/96*a^2*(88*A+75*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(
1/2)/sec(d*x+c)^(3/2)+1/64*a^2*(88*A+75*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(
1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{}}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(
3/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec
[c + d*x]]*(3*Sqrt[2]*(88*A + 75*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*S
qrt[Cos[c + d*x]]*(296*A + 285*B + 2*(88*A + 93*B)*Cos[c + d*x] + 4*(8*A +
 15*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(384*d
)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 3440$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}(A+B\cos(c+dx))dx$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\big\downarrow$ 3455

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}(a(8A+5B)+a(8A+9B)\cos(c+dx))dx+\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}(a(8A+5B)+a(8A+9B)\cos(c+dx))dx+\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(8A+5B)+a(8A+9B)\sin\right.\right.$$

$\big\downarrow$ 3460

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a^2(8A+9B)\sin(c+d}{3d\sqrt{a\cos(c+}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2(8A+}{3}\right.\right.$$

$\big\downarrow$ 3249

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}}{2d\sqrt{a\cos(c+d}}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(}{2d}\right.\right.\right.$$

$\big\downarrow$ 3249

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin}{\sqrt{\cos}}\right.}{d}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{1}{6}a(88A+75B)\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a}}\right.}{d}\right.\right.\right.\right.$$

input
```
Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Cos[c + d*x]^(5/2)*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(4*d) + ((a^2*(8*A + 9*B)*Cos[c + d*x]^(5/2)*S
in[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(88*A + 75*B)*((a*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*Ar
cSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6)/8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 18.61 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(264A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+225B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(64\cos(dx+c)^2+176\cos(dx+c)+264\right)\sin(d}{192d\sqrt{\sec(dx+c)}\,(\cos(d}$
parts	$\frac{A\sqrt{a(\cos(dx+c)+1)}\left(33\sec(dx+c)\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\tan(dx+c)(37+4\cos(2dx+2c)+22\cos(dx+c))\sqrt{\frac{\cos(dx+c}{\cos(dx+c)}}\right.}{24d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d/sec(d*x+c)^(1/2)*(264*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))+225*B*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(64*
cos(d*x+c)^2+176*cos(d*x+c)+264)*sin(d*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+(48*cos(d*x+c)^3+120*cos(d*x+c)^2+150*cos(d*x+c)+225)*sin(d*x+c)*B*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+
c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.69

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{3\left((88\,A+75\,B)a\cos\left(dx+c\right)+(88\,A+75\,B)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,Ba\cos(dx+c)^{4}+8\cdots\right.}{\cdots}}{192\left(d\cos\left(dx+c\right)+d\cdots\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/192*(3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*B)*a)*sqrt(a)*arctan(
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*
B*a*cos(d*x + c)^4 + 8*(8*A + 15*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*c
os(d*x + c)^2 + 3*(88*A + 75*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8901 vs. $2(211) = 422$.

Time = 1.12 (sec) , antiderivative size = 8901, normalized size of antiderivative = 36.04

$$\int \frac{(a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos...
```

Giac [F]

$$\int \frac{(a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{(B\cos(dx + c) + A)(a\cos(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))}{\sec^{\frac{3}{2}}(c + dx)}\, dx = \int \frac{(A + B\cos(c + d\,x))\,(a + a\cos(c + d\,x))^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\, dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\sqrt{a}\,a\left(\left(\left(\int \frac{\sqrt{\sec (dx+c)}\,\sqrt{\cos (dx+c)+1}\,\cos (dx+\right.\right.\right.}{\sec (dx+c)^2}$$

$$+\left(\int \frac{\sqrt{\sec (dx+c)}\,\sqrt{\cos (dx+c)+1}\,\cos (dx+c)}{\sec (dx+c)^2}dx\right)b$$

$$+\left(\int \frac{\sqrt{\sec (dx+c)}\,\sqrt{\cos (dx+c)+1}\,\cos (dx+c)^2}{\sec (dx+c)^2}dx\right)b$$

$$\left.+\left(\int \frac{\sqrt{\sec (dx+c)}\,\sqrt{\cos (dx+c)+1}}{\sec (dx+c)^2}dx\right)a\right)$$

input

```
int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output

```
sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/se
c(c + d*x)**2,x)*a + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c
+ d*x))/sec(c + d*x)**2,x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**2)/sec(c + d*x)**2,x)*b + int((sqrt(sec(c + d*x))*sqrt(c
os(c + d*x) + 1))/sec(c + d*x)**2,x)*a)
```

3.510 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 322

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx = \frac{32a^3(4184A+4615B)\sqrt{\sec(c+dx)}\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}} + \frac{16a^3(4184A+4615B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}} + \frac{4a^3(4184A+4615B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15015d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(4184A+4615B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(280A+299B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{1287d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(16A+13B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{143d} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{13}{2}}(c+dx)\sin(c+dx)}{13d}$$

output
```
32/45045*a^3*(4184*A+4615*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(1/2)+16/45045*a^3*(4184*A+4615*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*co
s(d*x+c))^(1/2)+4/15015*a^3*(4184*A+4615*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/
(a+a*cos(d*x+c))^(1/2)+2/9009*a^3*(4184*A+4615*B)*sec(d*x+c)^(7/2)*sin(d*x
+c)/d/(a+a*cos(d*x+c))^(1/2)+2/1287*a^3*(280*A+299*B)*sec(d*x+c)^(9/2)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/143*a^2*(16*A+13*B)*(a+a*cos(d*x+c))^(1
/2)*sec(d*x+c)^(11/2)*sin(d*x+c)/d+2/13*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x
+c)^(13/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.53

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}(171806A+162955B+35(5552A+5083B)\cos(c+dx)+14(15167A+\cdots}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15
/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(171806*A + 162955*B + 35*(5552*A + 5083*B
)*Cos[c + d*x] + 14*(15167*A + 15925*B)*Cos[2*(c + d*x)] + 62760*A*Cos[3*(
c + d*x)] + 69225*B*Cos[3*(c + d*x)] + 62760*A*Cos[4*(c + d*x)] + 69225*B*
Cos[4*(c + d*x)] + 8368*A*Cos[5*(c + d*x)] + 9230*B*Cos[5*(c + d*x)] + 836
8*A*Cos[6*(c + d*x)] + 9230*B*Cos[6*(c + d*x)])*Sec[c + d*x]^(13/2)*Tan[(c
 + d*x)/2])/(90090*d)
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.514$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{15}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{15/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{15/2}}dx$$

$$\Big\downarrow \text{3454}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{13}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(16A+13B)+a(8A+13B)\cos(c+dx))}{2\cos^{\frac{13}{2}}(c+dx)}dx+\frac{2aA\text{s}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(16A+13B)+a(8A+13B)\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}dx+\frac{2aA\text{s}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(16A+13B)+a(8A+13B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{13/2}}\right.$$

$$\Big\downarrow \text{3454}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{2}{11}\int \frac{\sqrt{\cos(c+dx)a+a}\left((280A+299B)a^2+(216A+247B)\cos(c+dx)a^2\right)}{2\cos^{\frac{11}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\int \frac{\sqrt{\cos(c+dx)a+a}\left((280A+299B)a^2+(216A+247B)\cos(c+dx)a^2\right)}{\cos^{\frac{11}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}((280A+299B)a^2+(216A+247B)\sin\left(c+d\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}\right.\right.$$

$\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2a^3(280A+299B)\text{si}}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^3(280A+29}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{}}\right.\right.\right.$$

$\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\left(\frac{6}{7}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a}}\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\left(\frac{6}{7}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a}{7d\cos^{\frac{7}{2}}(c+d}\right.\right.\right.\right.$$

$\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin}{5d\cos^{\frac{5}{2}}(c+dx)}\right.\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{}{5d\cos^{\frac{5}{2}}(c}\right.\right.\right.\right.\right.$$

$\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2}{3d\cos^{\frac{3}{2}}(c+}\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{9}a^2(4184A+4615B)\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{}{3d\cos}\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 3250$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{13}\left(\frac{2a^2(16A+13B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{11d\cos^{\frac{11}{2}}(c+dx)}+\frac{1}{11}\left(\frac{2a^3(280A+299B)s}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos}}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(13*d*Cos[c + d*x]^(13/2)) + ((2*a^2*(16*A + 13*B)*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + ((2*a^3*(280*A +
 299*B)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)*Sqrt[a + a*Cos[c + d*x]]) +
(a^2*(4184*A + 4615*B)*((2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a
+ a*Cos[c + d*x]]) + (6*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a
 + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])))/5))/7))/9)/11)/13)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.53

$$\frac{2\sin\left(dx+c\right)\left(\left(66944\cos\left(dx+c\right)^{6}+33472\cos\left(dx+c\right)^{5}+25104\cos\left(dx+c\right)^{4}+20920\cos\left(dx+c\right)^{3}+18\right.\right.}{}$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x)
```

output
```
2/45045/d*sin(d*x+c)*((66944*cos(d*x+c)^6+33472*cos(d*x+c)^5+25104*cos(d*x
+c)^4+20920*cos(d*x+c)^3+18305*cos(d*x+c)^2+11970*cos(d*x+c)+3465)*A+cos(d
*x+c)*(73840*cos(d*x+c)^5+36920*cos(d*x+c)^4+27690*cos(d*x+c)^3+23075*cos(
d*x+c)^2+14560*cos(d*x+c)+4095)*B)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sec
(d*x+c)^(15/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.55

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx = \frac{2\left(16\,(4184\,A+4615\,B)a^2\cos\left(dx+c\right)^6+8\,(4184\,A+4615\,B)a^2\cos\left(dx+c\right)^5+6\,(4184\,A+46\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, alg
orithm="fricas")
```

output
```
2/45045*(16*(4184*A + 4615*B)*a^2*cos(d*x + c)^6 + 8*(4184*A + 4615*B)*a^2
*cos(d*x + c)^5 + 6*(4184*A + 4615*B)*a^2*cos(d*x + c)^4 + 5*(4184*A + 461
5*B)*a^2*cos(d*x + c)^3 + 35*(523*A + 416*B)*a^2*cos(d*x + c)^2 + 315*(38*
A + 13*B)*a^2*cos(d*x + c) + 3465*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/((d*cos(d*x + c)^7 + d*cos(d*x + c)^6)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{15}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(15/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(280) = 560$.

Time = 0.20 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.37

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{15}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, alg
orithm="maxima")
```

output
```
8/45045*((45045*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 165165*s
qrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*sqrt(2)*a^(5/2
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 - 289185*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
 + 88980*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11864*sqr
t(2)*a^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + 1)) + 65*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x
 + c) + 1) - 3003*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 69
30*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10098*sqrt(2)*a^(
5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9053*sqrt(2)*a^(5/2)*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9 - 4875*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 1500*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13
- 200*sqrt(2)*a^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*B*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(1
5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(c...
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 30.29 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.45

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{15}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(15/2)*(a + a*cos(c + d*x))^(5/2
),x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B)*32i
)/(45045*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp
(c*1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*16i)/(5*d) + (a^2*exp(c*8i + d*x*8i)
*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)
*16i)/(5*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp
(c*1i + d*x*1i)/2))^(1/2)*(116*A + 115*B)*16i)/(35*d) - (a^2*exp(c*7i + d*
x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(116*A
 + 115*B)*16i)/(35*d) + (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1
i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(1046*A + 1075*B)*16i)/(315*d) - (a^2*
exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))
^(1/2)*(1046*A + 1075*B)*16i)/(315*d) + (a^2*exp(c*2i + d*x*2i)*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B)*16i)
/(3465*d) - (a^2*exp(c*11i + d*x*11i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp
(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B)*16i)/(3465*d) - (a^2*exp(c*13i
 + d*x*13i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*
(4184*A + 4615*B)*32i)/(45045*d)))/(exp(c*1i + d*x*1i) + 6*exp(c*2i + d*x*
2i) + 6*exp(c*3i + d*x*3i) + 15*exp(c*4i + d*x*4i) + 15*exp(c*5i + d*x*5i)
 + 20*exp(c*6i + d*x*6i) + 20*exp(c*7i + d*x*7i) + 15*exp(c*8i + d*x*8i) +
 15*exp(c*9i + d*x*9i) + 6*exp(c*10i + d*x*10i) + 6*exp(c*11i + d*x*11i...
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^7\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^7\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^7\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^7\,dx\right)a + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^7\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^7\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x)**7,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(
c + d*x)*sec(c + d*x)**7,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**3*sec(c + d*x)**7,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos
(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**7,x)*a + 2*int(sqrt(sec(c + d
*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**7,x)*b + int(sqr
t(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**7,x)*a)
```

3.511 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 275

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \frac{16a^3(710A+803B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \frac{8a^3(710A+803B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A+803B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(194A+209B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(14A+11B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d}$$

output
```
16/3465*a^3*(710*A+803*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(
1/2)+8/3465*a^3*(710*A+803*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+2/1155*a^3*(710*A+803*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(1/2)+2/693*a^3*(194*A+209*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*co
s(d*x+c))^(1/2)+2/99*a^2*(14*A+11*B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(9/
2)*sin(d*x+c)/d+2/11*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(11/2)*sin(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.53

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(9070A+7678B+(25070A+24827B)\cos(c+dx)+(9230A+9284B$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13
/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos
[c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] +
 10439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*
x)] + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/
2)*Tan[(c + d*x)/2])/(6930*d)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{13}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{13/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{13/2}}dx$$

$$\Big\downarrow \text{ 3454}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{11}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(14A+11B)+a(6A+11B)\cos(c+dx))}{2\cos^{\frac{11}{2}}(c+dx)}dx+\frac{2aA\,\mathrm{s}}{}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(14A+11B)+a(6A+11B)\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx+\frac{2aA\,\mathrm{s}}{}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(14A+11B)+a(6A+11B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}\right.$$

$$\Big\downarrow \text{ 3454}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{2}{9}\int \frac{\sqrt{\cos(c+dx)a+a}\left((194A+209B)a^2+3(46A+55B)\cos(c+dx)a^2\right)}{2\cos^{\frac{9}{2}}(c+dx)}dx\right.\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\int \frac{\sqrt{\cos(c+dx)a+a}\left((194A+209B)a^2+3(46A+55B)\cos(c+dx)a^2\right)}{\cos^{\frac{9}{2}}(c+dx)}dx\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((194A+209B)a^2+3(46A+55B)\sin\left(c+dx\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}\right.\right.$$

$$\Big\downarrow 3459$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^3(194A+209B)\sin(c}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+}}\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^3(194A+209B}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\,c}}\right.\right.\right.$$

$$\Big\downarrow 3251$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+d}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos}}\right.\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{}}\right.\right.\right.\right.$$

$$\Big\downarrow 3251$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{3}{7}a^2(710A+803B)\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a}{3d\cos^{\frac{3}{2}}(c+d}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 3250$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{2a^2(14A+11B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2a^3(194A+209B)\sin}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(}}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + ((2*a^2*(14*A + 11*B)*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((2*a^3*(194*A + 2
09*B)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (3
*a^2*(710*A + 803*B)*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a +
a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a +
 a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])))/5))/7)/9)/11)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.55

$$\frac{2\sin(dx+c)\left(\left(5680\cos(dx+c)^5+2840\cos(dx+c)^4+2130\cos(dx+c)^3+1775\cos(dx+c)^2+1120\,\mathrm{co}\right.\right.}{}$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

output
```
2/3465/d*sin(d*x+c)*((5680*cos(d*x+c)^5+2840*cos(d*x+c)^4+2130*cos(d*x+c)^
3+1775*cos(d*x+c)^2+1120*cos(d*x+c)+315)*A+cos(d*x+c)*(6424*cos(d*x+c)^4+3
212*cos(d*x+c)^3+2409*cos(d*x+c)^2+1430*cos(d*x+c)+385)*B)*(a*(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)*sec(d*x+c)^(13/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.57

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \frac{2\left(8\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^5+4\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^4+3\,(710\,A+803\,B)a^2\cdots\right)}{3465\,\left(d\cos\cdots\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d
*x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*c
os(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos
(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(239) = 478$.

Time = 0.25 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.44

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{13}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
8/3465*(5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt
(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 - 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt
(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 11*(315*sqrt(2)*
a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 1260*sqrt(2)*a^(5/2)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 2394*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 2736*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 185
9*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 676*sqrt(2)*a^(5/2
)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 104*sqrt(2)*a^(5/2)*sin(d*x + c)
^13/(cos(d*x + c) + 1)^13)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/(
(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x +
c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx)) \sec^{\frac{13}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 31.30 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.73

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx)) \sec^{\frac{13}{2}}(c + dx)\, dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + a*cos(c + d*x))^(5/2
),x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(710*A + 803*B)*16i)/
(3465*d) - (B*a^2*exp(c*3i + d*x*3i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(
c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d) + (B*a^2*exp(c*8i + d*x*8i)*(a + a*(exp
(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d) - (a^2*exp(c*
5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)
*(30*A + 41*B)*8i)/(15*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(30*A + 41*B)*8i)/(15*d) + (a^2*ex
p(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(
1/2)*(160*A + 157*B)*8i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(a + a*(exp(- c*
1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(160*A + 157*B)*8i)/(35*d) +
 (a^2*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2)*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*9i + d*x*9i)*(a + a*
(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(710*A + 803*B)*8i)
/(315*d) - (a^2*exp(c*11i + d*x*11i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(
c*1i + d*x*1i)/2))^(1/2)*(710*A + 803*B)*16i)/(3465*d)))/(exp(c*1i + d*x*1
i) + 5*exp(c*2i + d*x*2i) + 5*exp(c*3i + d*x*3i) + 10*exp(c*4i + d*x*4i) +
 10*exp(c*5i + d*x*5i) + 10*exp(c*6i + d*x*6i) + 10*exp(c*7i + d*x*7i) + 5
*exp(c*8i + d*x*8i) + 5*exp(c*9i + d*x*9i) + exp(c*10i + d*x*10i) + exp(c*
11i + d*x*11i) + 1)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^6\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^6\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^6\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^6\,dx\right)a + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^6\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^6\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x)**6,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(
c + d*x)*sec(c + d*x)**6,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**3*sec(c + d*x)**6,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos
(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**6,x)*a + 2*int(sqrt(sec(c + d
*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**6,x)*b + int(sqr
t(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**6,x)*a)
```

3.512 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{4a^3(292A+345B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(292A+345B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(124A+135B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}$$

output

```
4/315*a^3*(292*A+345*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+2/315*a^3*(292*A+345*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^
(1/2)+2/315*a^3*(124*A+135*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+2/21*a^2*(4*A+3*B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)*sin(d*
x+c)/d+2/9*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.55

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(1454A+1395B+(1396A+1215B)\cos(c+dx)+2(803A+870B)\cos\ldots}{\ldots}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11
/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
 + d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 34
5*B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Se
c[c + d*x]^(9/2)*Tan[(c + d*x)/2])/(630*d)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{11}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(4A+3B)+a(4A+9B)\cos(c+dx))}{2\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(4A+3B)+a(4A+9B)\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2aA\sin(\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(3a(4A+3B)+a(4A+9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx\right.$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{2}{7}\int \frac{\sqrt{\cos(c+dx)a+a}\left((124A+135B)a^2+(76A+99B)\cos(c+dx)a^2\right)}{2\cos^{\frac{7}{2}}(c+dx)}dx+\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\int \frac{\sqrt{\cos(c+dx)a+a}\left((124A+135B)a^2+(76A+99B)\cos(c+dx)a^2\right)}{\cos^{\frac{7}{2}}(c+dx)}dx+\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((124A+135B)a^2+(76A+99B)\sin\left(c+dx+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}\right.\right.$$

$\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^3(124A+135B)\sin(c-}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^3(124A+135B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos}}\right.\right.\right.$$

$\Big\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(}}\right.\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{3}{5}a^2(292A+345B)\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}\right.\right.\right.\right.$$

$\Big\downarrow$ 3250

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{6a^2(4A+3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2a^3(124A+135B)\sin(c}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((6*a^2*(4*A + 3*B)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*a^3*(124*A + 135*B
)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (3*a^2
*(292*A + 345*B)*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Co
s[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])))/5)/7)/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.58

$$\frac{2\sin\left(dx+c\right)\left(\left(584\cos\left(dx+c\right)^{4}+292\cos\left(dx+c\right)^{3}+219\cos\left(dx+c\right)^{2}+130\cos\left(dx+c\right)+35\right)A+\cos}{3}$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

output
```
2/315/d*sin(d*x+c)*((584*cos(d*x+c)^4+292*cos(d*x+c)^3+219*cos(d*x+c)^2+13
0*cos(d*x+c)+35)*A+cos(d*x+c)*(690*cos(d*x+c)^3+345*cos(d*x+c)^2+180*cos(d
*x+c)+45)*B)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sec(d*x+c)^(11/2)/(cos(d*
x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.59

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)\,dx = \frac{2\left(2\,(292\,A + 345\,B)a^2\cos\left(dx + c\right)^4 + (292\,A + 345\,B)a^2\cos\left(dx + c\right)^3 + 3\,(73\,A + 60\,B)a^2\cos\right.}{315\left(d\cos\left(dx + c\right)^5 + d\cos\left(dx\right.\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x
+ c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x +
 c) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 +
 d*cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(198) = 396$.

Time = 0.23 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.54

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
8/315*((315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 1)) + 15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c
) + 1) - 77*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 119*sqrt
(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*sqrt(2)*a^(5/2)*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 + 44*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B
*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 54.25 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.71

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(5/2
),x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(292*A + 345*B)*4i)/(
315*d) - (a^2*exp(c*3i + d*x*3i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
 + d*x*1i)/2))^(1/2)*(2*A + 5*B)*4i)/(3*d) + (a^2*exp(c*6i + d*x*6i)*(a +
a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*4i)/(
3*d) + (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
 d*x*1i)/2))^(1/2)*(24*A + 25*B)*4i)/(5*d) - (a^2*exp(c*5i + d*x*5i)*(a +
a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(24*A + 25*B)*4i)
/(5*d) + (a^2*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
 + d*x*1i)/2))^(1/2)*(146*A + 155*B)*4i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*
(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(146*A + 155
*B)*4i)/(35*d) - (a^2*exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 +
exp(c*1i + d*x*1i)/2))^(1/2)*(292*A + 345*B)*4i)/(315*d)))/(exp(c*1i + d*x
*1i) + 4*exp(c*2i + d*x*2i) + 4*exp(c*3i + d*x*3i) + 6*exp(c*4i + d*x*4i)
+ 6*exp(c*5i + d*x*5i) + 4*exp(c*6i + d*x*6i) + 4*exp(c*7i + d*x*7i) + exp
(c*8i + d*x*8i) + exp(c*9i + d*x*9i) + 1)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\sqrt{a}\,a^2\left(2\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^5\,dx\right)a+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^5\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^5\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^5\,dx\right)a+2\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^5\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^5\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x)**5,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(
c + d*x)*sec(c + d*x)**5,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**3*sec(c + d*x)**5,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos
(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**5,x)*a + 2*int(sqrt(sec(c + d
*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**5,x)*b + int(sqr
t(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**5,x)*a)
```

3.513 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2a^3(230A+301B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(10A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
2/105*a^3*(230*A+301*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+2/15*a^3*(10*A+11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+2/35*a^2*(10*A+7*B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+2/7*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.57

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(290A+196B+(930A+987B)\cos(c+dx)+2(115A+98B)\cos(2(c-}{210d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/
2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d
*x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*C
os[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(210*d)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(10A+7B)+a(2A+7B)\cos(c+dx))}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(10A+7B)+a(2A+7B)\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2aA\sin(\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(10A+7B)+a(2A+7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right.$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{2}{5}\int \frac{\sqrt{\cos(c+dx)a+a}\left(7(10A+11B)a^2+(30A+49B)\cos(c+dx)a^2\right)}{2\cos^{\frac{5}{2}}(c+dx)}dx+\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{\sqrt{\cos(c+dx)a+a}\left(7(10A+11B)a^2+(30A+49B)\cos(c+dx)a^2\right)}{\cos^{\frac{5}{2}}(c+dx)}dx+\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(7(10A+11B)a^2+(30A+49B)\sin\left(c+dx+\frac{\pi}{2}\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}\right.\right.$$

$\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}a^2(230A+301B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{14a^3(10A+11B)\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}a^2(230A+301B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{14a^3(10A+11B)\,\mathrm{s}}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\,\mathrm{co}}}\right.\right.\right.$$

$\downarrow$ 3250

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{14a^3(10A+11B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+301B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*a^2*(10*A + 7*B)*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((14*a^3*(10*A + 11*B
)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3
*(230*A + 301*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]))/5)/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.62

$$\frac{2\sin(dx+c)\left(\left(230\cos(dx+c)^3+115\cos(dx+c)^2+60\cos(dx+c)+15\right)A+\cos(dx+c)\left(301\cos(dx\right.\right.}{105d\left(\cos(dx+c)+1\right.}$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
2/105/d*sin(d*x+c)*((230*cos(d*x+c)^3+115*cos(d*x+c)^2+60*cos(d*x+c)+15)*A
+cos(d*x+c)*(301*cos(d*x+c)^2+98*cos(d*x+c)+21)*B)*(a*(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)*sec(d*x+c)^(9/2)/(cos(d*x+c)+1)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2\left((230\,A+301\,B)a^2\cos(dx+c)^3+(115\,A+98\,B)a^2\cos(dx+c)^2+3\,(20\,A+7\,B)a^2\cos(dx\right.}{105\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)\sqrt{\cos(dx+}}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c
)^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(157) = 314$.

Time = 0.23 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.70

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{9}{2}}(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
8/105*(5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 7*(15*sq
rt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 50*sqrt(2)*a^(5/2)*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*s
qrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

Giac [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{9}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 50.33 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.20

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{9}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

output
```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(230*A + 301*B)*2i)/(
105*d) - (B*a^2*exp(c*1i + d*x*1i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*
1i + d*x*1i)/2))^(1/2)*2i)/d + (B*a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1
i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*2i)/d - (a^2*exp(c*3i + d*x*3
i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(10*A + 1
7*B)*2i)/(3*d) + (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 +
exp(c*1i + d*x*1i)/2))^(1/2)*(10*A + 17*B)*2i)/(3*d) + (a^2*exp(c*2i + d*x
*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(100*A
+ 113*B)*2i)/(15*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)
/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(100*A + 113*B)*2i)/(15*d) - (a^2*exp(c*
7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)
*(230*A + 301*B)*2i)/(105*d)))/(exp(c*1i + d*x*1i) + 3*exp(c*2i + d*x*2i)
+ 3*exp(c*3i + d*x*3i) + 3*exp(c*4i + d*x*4i) + 3*exp(c*5i + d*x*5i) + exp
(c*6i + d*x*6i) + exp(c*7i + d*x*7i) + 1)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)a + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^4\,dx\right)a\right)$$

input

```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output

```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x)**4,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(
c + d*x)*sec(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**3*sec(c + d*x)**4,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos
(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**4,x)*a + 2*int(sqrt(sec(c + d
*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**4,x)*b + int(sqr
t(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**4,x)*a)
```

3.514 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 192

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output

```
2*a^(5/2)*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+2/15*a^3*(32*A+35*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d
/(a+a*cos(d*x+c))^(1/2)+2/15*a^2*(8*A+5*B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+
c)^(3/2)*sin(d*x+c)/d+2/5*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2)*sin(
d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(30\sqrt{2}B\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{5}{2}}(c+dx)\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/
2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(30*Sq
rt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40
*B + 2*(14*A + 5*B)*Cos[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c
+ d*x)/2]))/(30*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3454, 27, 3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(8A+5B)+5aB\cos(c+dx))}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(c}{5d\cos}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(8A+5B)+5aB\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2aA\sin(c+dx)(c}{5d\cos}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(8A+5B)+5aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2aA\sin}{}\right.$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}\left((32A+35B)a^2+15B\cos(c+dx)a^2\right)}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(8A+}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\sqrt{\cos(c+dx)a+a}\left((32A+35B)a^2+15B\cos(c+dx)a^2\right)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2(8A+}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((32A+35B)a^2+15B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\right.\right.$$

$\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(15a^2B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^3(32A+35B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(15a^2B\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(32A+35B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right.\right.\right.$$

$\Big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^3(32A+35B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{30a^2B\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+}{\sqrt{\cos(c+d}}\right.}{d}\right.\right.\right.$$

$\Big\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{30a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+}{\sqrt{a\cos(c+d}}\right.}{d}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((30*a^(5/2)*B*ArcSin[
(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*a^3*(32*A + 35*B)
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/3)/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\frac{2\sqrt{a\left(\cos\left(dx+c\right)+1\right)}\,\sec\left(dx+c\right)^{\frac{7}{2}}\left(B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(15\cos\left(dx+c\right)^{4}\right.\right.}{}$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
2/15/d*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)+1)*(B*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(15*cos(d*x+c)^4+15*cos(d*x+c)^3)+cos(d*x+c)*(43*cos(d*x+c)^2+14*cos
(d*x+c)+3)*sin(d*x+c)*A+cos(d*x+c)^2*sin(d*x+c)*(40*cos(d*x+c)+5)*B)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.84

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(15\left(Ba^{2}\cos\left(dx+c\right)^{3}+Ba^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left((43\,A+40\,B)a^{2}\cos(dx+c\right.}{}}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
-2/15*(15*(B*a^2*cos(d*x + c)^3 + B*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((43*A
+ 40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + 3*A*a^2)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 +
d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. $2(164) = 328$.

Time = 0.41 (sec) , antiderivative size = 1713, normalized size of antiderivative = 8.92

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
1/30*(5*(10*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*a^(5/2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x +
2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*
c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin...
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}(a+a\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\sqrt{a}\,a^2\Bigg(2\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a+2\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^3\,dx\right)a\Bigg)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x)**3,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(
c + d*x)*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**3*sec(c + d*x)**3,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos
(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**3,x)*a + 2*int(sqrt(sec(c + d
*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**3,x)*b + int(sqr
t(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**3,x)*a)
```

3.515 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 193

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{a^{5/2}(2A+5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output

```
a^(5/2)*(2*A+5*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d-1/3*a^3*(14*A+3*B)*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(1/2)/sec(d*x+c)^(1/2)+2*a^2*(2*A+B)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c
)^(1/2)*sin(d*x+c)/d+2/3*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2)*sin(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(3\sqrt{2}(2A+5B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{6d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/
2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqr
t[2]*(2*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*
A + 3*B + 4*(8*A + 3*B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)
/2]))/(6*d)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.371$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3454, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$\big\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(2A+B)-a(2A-3B)\cos(c+dx))}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{(\cos(c+dx)a+a)^{3/2}(3a(2A+B)-a(2A-3B)\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2aA\sin(c\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(3a(2A+B)-a(2A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\right.$$

$\big\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(2\int \frac{\sqrt{\cos(c+dx)a+a}\,(a^2(10A+9B)-a^2(14A+3B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{6a^2(\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{\sqrt{\cos(c+dx)a+a}\,(a^2(10A+9B)-a^2(14A+3B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{6a^2(2\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\,\left(a^2(10A+9B)-a^2(14A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}a\right.\right.$$

$\big\downarrow$ 3460

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{3}{2}a^2(2A+5B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{3}{2}a^2(2A+5B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\text{co}}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$\Big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(-\frac{3a^2(2A+5B)\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+}{d\sqrt{a\cos(c+}}\right.\right.$$

$\Big\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{3a^{5/2}(2A+5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((3*a^(5/2)*(2*A + 5*B)*ArcSin[(Sq
rt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a^3*(14*A + 3*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (6*a^2*(2*A + B
)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.21

$$\frac{\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sec\left(dx+c\right)^{\frac{5}{2}}\left(A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\tan\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(6\cos\left(dx+c\right)^{3}+6\cos\right.\right.}{}$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
1/3/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)*(A*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*(6*cos(d*x+c)^3+6*cos(d*x+c)^2)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(15*cos(d*x+c)^3
+15*cos(d*x+c)^2)+cos(d*x+c)*(16*cos(d*x+c)+2)*sin(d*x+c)*A+cos(d*x+c)^2*s
in(d*x+c)*(3*cos(d*x+c)+6)*B)*a^2*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx =$$

$$-\frac{3\left((2A+5B)a^{2}\cos\left(dx+c\right)^{2}+(2A+5B)a^{2}\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(3Ba^{2}\right.}{}}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
-1/3*(3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sq
rt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
+ c))) - (3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^
2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2780 vs. $2(167) = 334$.

Time = 0.53 (sec) , antiderivative size = 2780, normalized size of antiderivative = 14.40

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
1/12*(2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*si
n(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 +
4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2
*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(...
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+a\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^2\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x)**2,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(
c + d*x)*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
 1)*cos(c + d*x)**3*sec(c + d*x)**2,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos
(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**2,x)*a + 2*int(sqrt(sec(c + d
*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x)**2,x)*b + int(sqr
t(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**2,x)*a)
```

3.516 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 198

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{a^{5/2}(20A+19B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d}-\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{a^2(4A-B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}+\frac{2aA(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
1/4*a^(5/2)*(20*A+19*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d-1/4*a^3*(4*A-9*B)*sin(d*x+c)/d/(a+a*co
s(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-1/2*a^2*(4*A-B)*(a+a*cos(d*x+c))^(1/2)*si
n(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2)*
sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}(20A+19B)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{8}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/
2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[
2]*(20*A + 19*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(
8*A + B + (4*A + 11*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]
))/(8*d)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3454, 27, 3042, 3455, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$\downarrow$ 3454

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\int \frac{(\cos(c+dx)a+a)^{3/2}(a(4A+B)-a(4A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+}{d}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{(\cos(c+dx)a+a)^{3/2}(a(4A+B)-a(4A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{2aA\sin(c+dx}{d\sqrt{}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(4A+B)-a(4A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2a\right.$$

$\downarrow$ 3455

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}\left(a^2(12A+5B)-a^2(4A-9B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)}}dx-\frac{a^2(4A-}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)a+a}\left(a^2(12A+5B)-a^2(4A-9B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx-\frac{a^2(4A-}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a^2(12A+5B)-a^2(4A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$

$\downarrow$ 3460

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}a^2(20A+19B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}a^2(20A+19B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{c}}{d\sqrt{a\cos(c+dx)+}}\right.\right.$$

$\big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(-\frac{a^2(20A+19B)\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^3(4A-9B)\sin(c+}{d\sqrt{a\cos(c+}}\right.\right.$$

$\big\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}\left(\frac{a^{5/2}(20A+19B)}{}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(a^2*(4*A - B)*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/d + (2*a*A*(a + a*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((a^(5/2)*(20*A + 19*B)*ArcS
in[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a^3*(4*A - 9*B)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 21.98 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

method	result
default	$\frac{\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{3}{2}}\left(20A\cos(dx+c)^2\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+19B\cos(dx+c)^2\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(d}{\cos(dx}}\right.\right.}{4d(\cos(dx+c)+1)\sqrt{}}$
parts	$\frac{A\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{3}{2}}\left(5\cos(dx+c)^2\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\cos(dx+c)(\cos(dx+c)+2)\sin(dx+c)\sqrt{\frac{\cos(d}{\cos(dx}}\right.}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(20*A*cos(d*x+c)^2*arctan(tan(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))+19*B*cos(d*x+c)^2*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))+cos(d*x+c)*(4*cos(d*x+c)+8)*sin(d*x+c)*A*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+cos(d*x+c)^2*sin(d*x+c)*(2*cos(d*x+c)+11)*B*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*a^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.74

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx =$$

$$-\frac{\left((20\,A+19\,B)a^2\cos\left(dx+c\right)+(20\,A+19\,B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2\,Ba^2\cos(dx+c)^2+\cdots\right.}{\cdots}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/4*(((20*A + 19*B)*a^2*cos(d*x + c) + (20*A + 19*B)*a^2)*sqrt(a)*arctan(
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B
*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (A + B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+a\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)\,dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
*sec(c + d*x),x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c +
 d*x)*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*co
s(c + d*x)**3*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)
 + 1)*cos(c + d*x)**2*sec(c + d*x),x)*a + 2*int(sqrt(sec(c + d*x))*sqrt(co
s(c + d*x) + 1)*cos(c + d*x)**2*sec(c + d*x),x)*b + int(sqrt(sec(c + d*x))
*sqrt(cos(c + d*x) + 1)*sec(c + d*x),x)*a)
```

3.517 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 200

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\frac{a^{5/2}(38A+25B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}+\frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{a^2(2A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
1/8*a^(5/2)*(38*A+25*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/24*a^3*(54*A+49*B)*sin(d*x+c)/d/(a+a
*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/4*a^2*(2*A+3*B)*(a+a*cos(d*x+c))^(1/
2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/3*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)
/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\frac{a^2\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3\sqrt{\right.}}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x
]],x]
```

output
```
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[S
ec[c + d*x]]*(3*Sqrt[2]*(38*A + 25*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2
*Sqrt[Cos[c + d*x]]*(66*A + 79*B + 2*(6*A + 17*B)*Cos[c + d*x] + 4*B*Cos[2
*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3455}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(6A+B)+3a(2A+3B)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c-$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{(\cos(c+dx)a+a)^{3/2}(a(6A+B)+3a(2A+3B)\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx+\frac{aB\sin(c-$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(6A+B)+3a(2A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-$$

$$\Big\downarrow \text{3455}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}\left((30A+13B)a^2+(54A+49B)\cos(c+dx)a^2\right)}{2\sqrt{\cos(c+dx)}}dx+\frac{3a}{}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)a+a}\left((30A+13B)a^2+(54A+49B)\cos(c+dx)a^2\right)}{\sqrt{\cos(c+dx)}}dx+\frac{3a}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left((30A+13B)a^2+(54A+49B)\sin\left(c+dx+\frac{\pi}{2}\right)\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$$\Big\downarrow \text{3460}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}a^2(38A+25B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{c}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}a^2(38A+25B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^3(54A+49B)\sin(c+c}{d\sqrt{a\cos(c+d}}\right.\right.\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{3a^2(38A+25B)\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+}{\cos(c+dx)}}}}{d}\right.\right.\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{3a^2(2A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}\left(\frac{3a^{5/2}(38A+}{}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + ((3*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((3*a^(5/2)*(38*A + 25*B
)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a^3*(54*A
+ 49*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/4)/
6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 22.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

method	result
default	$\frac{\left(114A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+75B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+(12\cos(dx+c)+66)\sin(dx+c)A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{24d(\cos(dx+c)+1)\sqrt{\frac{\cos}{\cos(}}}$
parts	$\frac{A\left(19\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\sin(2dx+2c)+11\sin(dx+c)\right)\right)\sqrt{a(\cos(dx+c)+1)}\cos(dx+c)\sqrt{\sec(dx+c)}}{4d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input

```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
1/24/d*(114*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+75*B*ar
ctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(12*cos(d*x+c)+66)*sin(
d*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(8*cos(d*x+c)^2+34*cos(d*x+c)+7
5)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(a*cos(1/2*d*x+1/2*c)^2
)^(1/2)*cos(d*x+c)*sec(d*x+c)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*2^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{3\left((38\,A+25\,B)a^2\cos\left(dx+c\right)+(38\,A+25\,B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,Ba^2\cos(dx+c)^3\right.}{}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

input

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/24*(3*((38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arct
an(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (
8*B*a^2*cos(d*x + c)^3 + 2*(6*A + 17*B)*a^2*cos(d*x + c)^2 + 3*(22*A + 25*
B)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sqrt{\sec(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3071 vs. $2(170) = 340$.

Time = 0.67 (sec) , antiderivative size = 3071, normalized size of antiderivative = 15.36

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sqrt{\sec(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2
*d*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*
cos(2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) +
 19*(a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc...
```

Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (A + B\cos(c+d\,x))\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\cos(c+d\,x))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \sqrt{a}\,a^2\left(2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)a + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a**2*(2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)
,x)*a + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x),x)*b +
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**3,x)*b + int(s
qrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2,x)*a + 2*int(sqrt
(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2,x)*b + int(sqrt(sec(
c + d*x))*sqrt(cos(c + d*x) + 1),x)*a)
```

3.518 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 247

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^{5/2}(200A+163B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{64d}$$
$$+\frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}+\frac{a^2(8A+11B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)}$$
$$+\frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3(200A+163B)\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/64*a^(5/2)*(200*A+163*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/96*a^3*(104*A+95*B)*sin(d*x+c)/d/
(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2)+1/24*a^2*(8*A+11*B)*(a+a*cos(d*x+c
))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+1/4*a*B*(a+a*cos(d*x+c))^(3/2)*sin(
d*x+c)/d/sec(d*x+c)^(3/2)+1/64*a^3*(200*A+163*B)*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^2\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{}}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d
*x]],x]
```

output
```
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[S
ec[c + d*x]]*(3*Sqrt[2]*(200*A + 163*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] +
 2*Sqrt[Cos[c + d*x]]*(632*A + 581*B + (272*A + 362*B)*Cos[c + d*x] + 4*(8
*A + 23*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(3
84*d)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3455}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{1}{2}\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}(a(8A+3B)+a(8A+11B)\cos(c+dx)\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\int\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}(a(8A+3B)+a(8A+11B)\cos(c+dx))d\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(a(8A+3B)+a(8A+11B\right.\right.$$

$$\Big\downarrow \text{3455}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{3}\int\frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}(3(24A+17B)a^2+(104A+95B)\cos(c\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}(3(24A+17B)a^2+(104A+95B)\cos(c-\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3(24A+17B)a^2+(104A+\right.\right.\right.$$

$$\Big\downarrow \text{3460}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a^3(104A+95I}{2d\sqrt{}}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^3}{}\right.\right.\right.$$

$\Big\downarrow$ 3249

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+}}{d\sqrt{a\cos(c+dx)+a}}\right.\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos}}{d\sqrt{a\cos(c+d}}\right.\right.\right.\right.$$

$\Big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}a^2(200A+163B)\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{}{\sqrt{}}\right.}{d}\right.\right.\right.\right.$$

$\Big\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{a^2(8A+11B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{1}{6}\left(\frac{a^3(104A+95}{2d\sqrt{}}\right.\right.\right.$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Cos[c + d*x]^(3/2)*(a + a*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + ((a^2*(8*A + 11*B)*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((a^3*(104*A + 95*B)*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*a^2*(200*
A + 163*B)*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))
/4)/6)/8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 18.71 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(600A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+489B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(64\cos(dx+c)^2+272\cos(dx+c)+600\right)\sin(d}{192d(\cos(dx+c)+1)\sqrt{s}}$
parts	$\frac{A\left(75\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(8\cos(dx+c)^2+34\cos(dx+c)+75\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{2}\,a^2}{24d(\cos(dx+c)+1)\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*(600*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+489*B*
arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(64*cos(d*x+c)^2+272*
cos(d*x+c)+600)*sin(d*x+c)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(48*cos(d*x
+c)^3+184*cos(d*x+c)^2+326*cos(d*x+c)+489)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^
(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.74

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{3\left((200\,A+163\,B)a^2\cos\left(dx+c\right)+(200\,A+163\,B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,Ba^2\cos(\right.}{}}{192\left(d\cos\left(dx+\right.\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/192*(3*((200*A + 163*B)*a^2*cos(d*x + c) + (200*A + 163*B)*a^2)*sqrt(a)
*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
) - (48*B*a^2*cos(d*x + c)^4 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^3 + 2*(136*
A + 163*B)*a^2*cos(d*x + c)^2 + 3*(200*A + 163*B)*a^2*cos(d*x + c))*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9390 vs. $2(211) = 422$.

Time = 1.05 (sec) , antiderivative size = 9390, normalized size of antiderivative = 38.02

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*si
n(3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*
(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x...
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)(a\cos{(dx+c)}+a)^{\frac{5}{2}}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \sqrt{a}\,a^2\left(2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx\right.\right.$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\sec(dx+c)}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)a$$
$$+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)b$$
$$\left.+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}}{\sec(dx+c)}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a**2*(2*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x
))/sec(c + d*x),x)*a + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(
c + d*x))/sec(c + d*x),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) +
1)*cos(c + d*x)**3)/sec(c + d*x),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c
 + d*x) + 1)*cos(c + d*x)**2)/sec(c + d*x),x)*a + 2*int((sqrt(sec(c + d*x)
)*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2)/sec(c + d*x),x)*b + int((sqrt(se
c(c + d*x))*sqrt(cos(c + d*x) + 1))/sec(c + d*x),x)*a)
```

3.519 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 294

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^{5/2}(326A+283B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{128d}$$
$$+ \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}$$
$$+ \frac{a^2(10A+13B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{40d\sec^{\frac{5}{2}}(c+dx)} + \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{5}{2}}(c+dx)}$$
$$+ \frac{a^3(326A+283B)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^3(326A+283B)\sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/128*a^(5/2)*(326*A+283*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/240*a^3*(170*A+157*B)*sin(d*x+c)
/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(5/2)+1/40*a^2*(10*A+13*B)*(a+a*cos(d
*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+1/5*a*B*(a+a*cos(d*x+c))^(3/2)*
sin(d*x+c)/d/sec(d*x+c)^(5/2)+1/192*a^3*(326*A+283*B)*sin(d*x+c)/d/(a+a*co
s(d*x+c))^(1/2)/sec(d*x+c)^(3/2)+1/128*a^3*(326*A+283*B)*sin(d*x+c)/d/(a+a
*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.61

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{}}{}$$

input
```
Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(
3/2),x]
```

output
```
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[S
ec[c + d*x]]*(15*Sqrt[2]*(326*A + 283*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]
+ 2*Sqrt[Cos[c + d*x]]*(5810*A + 5521*B + (3620*A + 3874*B)*Cos[c + d*x] +
 4*(230*A + 331*B)*Cos[2*(c + d*x)] + 120*A*Cos[3*(c + d*x)] + 348*B*Cos[3
*(c + d*x)] + 48*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(3840*d)
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {3042, 3440, 3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}(A+B\cos(c+dx))dx$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow \text{3455}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}(5a(2A+B)+a(10A+13B)\cos(c+dx)\right.$$

$$\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}(5a(2A+B)+a(10A+13B)\cos(c+dx)\right)$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(5a(2A+B)+a(10A+13\right.\right.$$

$$\downarrow \text{3455}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{4}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}(5(26A+21B)a^2+(170A+157B)\cos\right.\right.$$

$$\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}(5(26A+21B)a^2+(170A+157B)\cos(c\right.\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5(26A+21B)a^2+(170A\right.\right.\right.$$

$$\downarrow \text{3460}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a^3(170A+157}{3d\sqrt{}}\right.\right.\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\right.\right.\right.$$

$\big\downarrow$ 3249

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+}{2d\sqrt{a\,c}}\right.\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right.\right.\right.\right.$$

$\big\downarrow$ 3249

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(}}{d\sqrt{a\cos(c+dx)}}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)}{d\sqrt{a\cos(}}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{6}a^2(326A+283B)\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d}{d}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{10}\left(\frac{a^2(10A+13B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{1}{8}\left(\frac{a^3(170A+}{}\right.\right.\right.$$

input
```
Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*B*Cos[c + d*x]^(5/2)*(a + a*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + ((a^2*(10*A + 13*B)*Cos[c + d*x]^(5/
2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + ((a^3*(170*A + 157*B)*Co
s[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (5*a^2*(32
6*A + 283*B)*((a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c +
d*x]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))
)/4))/6)/8)/10)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 18.75 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(4890A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+4245B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(480\cos(dx+c)^3+1840\cos(dx+c)^2+3260\right.}{}$
parts	$\frac{A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(489\sec(dx+c)\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\tan(dx+c)\left(48\cos(dx+c)^3+184\cos(dx+c)^2+326\cos(dx+\right.\right.}{192d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/1920/d/sec(d*x+c)^(1/2)*(4890*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))+4245*B*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(
480*cos(d*x+c)^3+1840*cos(d*x+c)^2+3260*cos(d*x+c)+4890)*sin(d*x+c)*A*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+(384*cos(d*x+c)^4+1392*cos(d*x+c)^3+2264*cos
(d*x+c)^2+2830*cos(d*x+c)+4245)*sin(d*x+c)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*2^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.69

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{15\left((326\,A+283\,B)a^2\cos\left(dx+c\right)+(326\,A+283\,B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(384\,Ba^2\,c\cdots\right.}{\cdots}}{19\cdots}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/1920*(15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sqrt(
a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c
))) - (384*B*a^2*cos(d*x + c)^5 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^4 + 8*
(230*A + 283*B)*a^2*cos(d*x + c)^3 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^2
 + 15*(326*A + 283*B)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10042 vs. $2(252) = 504$.

Time = 0.91 (sec) , antiderivative size = 10042, normalized size of antiderivative = 34.16

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/7680*((10*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + sin(
2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 + 2*cos(2/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(3/4)*((75*a^2*sin(4/5*arctan2(sin(
5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*sin(3/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c))) + 75*a^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))))*cos(3/2*arctan2(sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)) - (75*a^2
*cos(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 88*a^2*cos(3/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) - 75*a^2*cos(1/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c))) - 88*a^2)*sin(3/2*arctan2(sin(2/5*arctan2(si
n(5*d*x + 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(
5*d*x + 5*c))) + 1)))*sqrt(a) + 6*(cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5
*d*x + 5*c)))^2 + sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2 +
 2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 1)^(1/4)*(8*(a^2
*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))^2*sin(5*d*x + 5*c) +
 a^2*sin(5*d*x + 5*c)*sin(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))
^2 + 2*a^2*cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))*sin(5*d*x
+ 5*c) + a^2*sin(5*d*x + 5*c))*cos(5/2*arctan2(sin(2/5*arctan2(sin(5*d*x +
 5*c), cos(5*d*x + 5*c))), cos(2/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c))) + 1)) - 5*(15*a^2*sin(4/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5...
```

Giac [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(B\cos{(dx+c)}+A)(a\cos{(dx+c)}+a)^{\frac{5}{2}}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(A+B\cos(c+d\,x))\,(a+a\cos(c+d\,x))^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\sqrt{a}\,a^2\left(2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx\right.\right.$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\sec(dx+c)^2}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^3}{\sec(dx+c)^2}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\sec(dx+c)^2}dx\right)a$$
$$+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\sec(dx+c)^2}dx\right)b$$
$$\left.+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}}{\sec(dx+c)^2}dx\right)a\right)$$

input
```
int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
sqrt(a)*a**2*(2*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x
))/sec(c + d*x)**2,x)*a + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*c
os(c + d*x))/sec(c + d*x)**2,x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d
*x) + 1)*cos(c + d*x)**3)/sec(c + d*x)**2,x)*b + int((sqrt(sec(c + d*x))*s
qrt(cos(c + d*x) + 1)*cos(c + d*x)**2)/sec(c + d*x)**2,x)*a + 2*int((sqrt(
sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)**2)/sec(c + d*x)**2,x)*b
 + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/sec(c + d*x)**2,x)*a)
```

3.520 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 295

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2(257A-129B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(19A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}}$$

output

```
-2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+2/315*(257
*A-129*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/315*(29*A
-93*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/105*(19*A-3*
B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/63*(A-9*B)*sec(d
*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*A*sec(d*x+c)^(9/2)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.77 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.92

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(-315i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}(-127\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*
x]],x]
```

output
```
(2*Cos[(c + d*x)/2]*((-315*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-1279*A + 423*B + (214*A - 918*
B)*Cos[c + d*x] - 8*(157*A - 69*B)*Cos[2*(c + d*x)] + 58*A*Cos[3*(c + d*x)
] - 186*B*Cos[3*(c + d*x)] - 257*A*Cos[4*(c + d*x)] + 129*B*Cos[4*(c + d*x
)])*Sec[c + d*x]^(9/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*
x)/2])/4))/(315*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.14, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3440, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{11}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{ 3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{a(A-9B)-8aA\cos(c+dx)}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{9a}+\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-9B)-8aA\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{9a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-9B)-8aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{9a}\right)$$

$$\Big\downarrow \text{ 3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{3\left(a^2(19A-3B)-2a^2(A-9B)\cos(c+dx)\right)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{7a}+\frac{2}{7d\cos}\cdots}{9a}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\int\frac{a^2(19A-3B)-2a}{\cos^{\frac{7}{2}}(c+dx)}}{}}{9a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\int\frac{a^2(19A-3B)-2a}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}}{}}{9a}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2\int-\frac{a^3(29A-93}{2\cos^{\frac{5}{2}}}}{}\right.}{}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{}}\right.}{}}{9}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}\right.}{}}{}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}\right.}{}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}\right.}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}} - \frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}} - \frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}\right.}{}}{}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}} - \frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}} - \frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}\right.}{}}{}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-9B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{2a^2(19A-3B)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{c}}\right.}{}}{}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sin[c + d*x])/(9*d*Cos[c + d*x
]^(9/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a*(A - 9*B)*Sin[c + d*x])/(7*d*Cos
[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) - (3*((2*a^2*(19*A - 3*B)*Sin[c
+ d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a^3*(29*A
- 93*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) -
((-315*Sqrt[2]*a^(7/2)*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^4*(257*A - 129*B)*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a)))/
(7*a))/(9*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.90

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{11}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(315\cos(dx+c)^{6}+315\cos(dx+c)^{5}\right)+B\arcsin\right.}{}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{11}{2}}\left(\cos(dx+c)\sin(dx+c)\left(257\cos(dx+c)^{4}-29\cos(dx+c)^{3}+57\cos(dx+c)^{2}-5\cos(dx+c)+35\right)\right.}{315d(\cos(dx+c)+1)a}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/315/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(11/2)/(cos(d*x+c)+1)*
(A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(315*co
s(d*x+c)^6+315*cos(d*x+c)^5)+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(-315*cos(d*x+c)^6-315*cos(d*x+c)^5)+cos(d*x+c)*sin(d
*x+c)*(257*cos(d*x+c)^4-29*cos(d*x+c)^3+57*cos(d*x+c)^2-5*cos(d*x+c)+35)*2
^(1/2)*A+cos(d*x+c)^2*sin(d*x+c)*(-129*cos(d*x+c)^3+93*cos(d*x+c)^2-9*cos(
d*x+c)+45)*2^(1/2)*B)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.67

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\frac{315\sqrt{2}\left((A-B)a\cos(dx+c)^5+(A-B)a\cos(dx+c)^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}} + \frac{2\left((257A-129B)\cos(dx+c)^4-(29A-93B)\right.}{}}{315\left(ad\cos(dx+c)^5+ad\cos(dx+\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="fricas")
```

output
```
1/315*(315*sqrt(2)*((A - B)*a*cos(d*x + c)^5 + (A - B)*a*cos(d*x + c)^4)*a
rctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
 + c)))/sqrt(a) + 2*((257*A - 129*B)*cos(d*x + c)^4 - (29*A - 93*B)*cos(d*
x + c)^3 + 3*(19*A - 3*B)*cos(d*x + c)^2 - 5*(A - 9*B)*cos(d*x + c) + 35*A
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
 c)^5 + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(11/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{(B\cos(dx + c) + A)\sec(dx + c)^{\frac{11}{2}}}{\sqrt{a\cos(dx + c) + a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(11/2)/sqrt(a*cos(d*x + c) + a
), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{(B\cos(dx + c) + A)\sec(dx + c)^{\frac{11}{2}}}{\sqrt{a\cos(dx + c) + a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, alg
orithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(11/2)/sqrt(a*cos(d*x + c) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2))/(a + a*cos(c + d*x))^(1
/2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2))/(a + a*cos(c + d*x))^(1
/2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^5}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^5}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**5)/(cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c
+ d*x) + 1)*sec(c + d*x)**5)/(cos(c + d*x) + 1),x)*a))/a
```

3.521 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 250

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx \\
&= \frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \\
&\quad - \frac{2(43A-91B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

output
```
2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d-2/105*(43*A
-91*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/105*(31*A-7*
B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/35*(A-7*B)*sec(d
*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*A*sec(d*x+c)^(7/2)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.06 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(105i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{2}(-122A}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x
]],x]
```

output
```
(2*Cos[(c + d*x)/2]*((105*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sq
rt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-122*A + 14*B + 3*(47*A - 119*B)
*Cos[c + d*x] + (-62*A + 14*B)*Cos[2*(c + d*x)] + 43*A*Cos[3*(c + d*x)] -
91*B*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d
*x)/2])*Sin[(c + d*x)/2])/2))/(105*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c
 + d*x])])
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {3042, 3440, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{a(A-7B)-6aA\cos(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{7a}+\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-7B)-6aA\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{7a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-7B)-6aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{7a}\right)$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{a^2(31A-7B)-4a^2(A-7B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2a(A}{5d\cos^{\frac{5}{2}}(c}}{7a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(31A-7B)-4a^2(}{\cos^{\frac{5}{2}}(c+dx)\sqrt{}}}{5}}{7a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(31A-7B)-4a^2(}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}}{}}{7a}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{a^3(43A-91B)-}{2\cos^{\frac{3}{2}}(c+}}{}}{}}{7}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(31A-7B)\sin}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos}}}{}}{7a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(31A-7B)\sin}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos}}}{}}{7}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(31A-7B)\sin}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos}}}{}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(31A-7B)\sin}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos}}}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(31A-7B)\sin}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos}}}{}}{}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(31A-7B)\sin\ldots}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos\ldots}}\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-7B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(31A-7B)\sin\ldots}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos\ldots}}\ldots}{\ldots}}{\ldots}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sin[c + d*x])/(7*d*Cos[c + d*x
]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a*(A - 7*B)*Sin[c + d*x])/(5*d*Cos
[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a^2*(31*A - 7*B)*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-105*Sqrt[2]*a^
(5/2)*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/d + (2*a^3*(43*A - 91*B)*Sin[c + d*x])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a))/(7*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 13.98 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{9}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(105\cos(dx+c)^{5}+105\cos(dx+c)^{4}\right)+B\,\mathrm{arcs}\right.}{}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{9}{2}}\left(\cos(dx+c)\sin(dx+c)\left(43\cos(dx+c)^{3}-31\cos(dx+c)^{2}+3\cos(dx+c)-15\right)\sqrt{2}+\sqrt{\frac{\cos(dx+c}{\cos(dx+c)}}\right.}{105d(\cos(dx+c)+1)a}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/105/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(9/2)/(cos(d*x+c)+1)*
(A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(105*co
s(d*x+c)^5+105*cos(d*x+c)^4)+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(-105*cos(d*x+c)^5-105*cos(d*x+c)^4)+cos(d*x+c)*sin(d
*x+c)*(43*cos(d*x+c)^3-31*cos(d*x+c)^2+3*cos(d*x+c)-15)*2^(1/2)*A+cos(d*x+
c)^2*sin(d*x+c)*(-91*cos(d*x+c)^2+7*cos(d*x+c)-21)*2^(1/2)*B)/a
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.72

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\frac{105\sqrt{2}\left((A-B)a\cos(dx+c)^4+(A-B)a\cos(dx+c)^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}} + \frac{2\left((43A-91B)\cos(dx+c)^3-(31A-7B)\cos\ldots\right.}{\ldots}}{105\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
-1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*
arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c)))/sqrt(a) + 2*((43*A - 91*B)*cos(d*x + c)^3 - (31*A - 7*B)*cos(d*x
+ c)^2 + 3*(A - 7*B)*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{9}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{9}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^4}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**4)/(cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c
+ d*x) + 1)*sec(c + d*x)**4)/(cos(c + d*x) + 1),x)*a))/a
```

3.522 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 207

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}$$

output

```
-2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+2/15*(13*A
-5*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/15*(A-5*B)*se
c(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*A*sec(d*x+c)^(5/2)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.22 (sec) , antiderivative size = 1719, normalized size of antiderivative = 8.30

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x
]],x]
```

output
```
(2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*S
in[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(5*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(5/2)) + (8*B*Sin[c/2 + (d*x)/2])/(15*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3
/2)) + (16*B*Sin[c/2 + (d*x)/2])/(15*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - (
(A - B)*Csc[c/2 + (d*x)/2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 +
(d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 6
55812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos
[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*S
in[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c
/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 +
(d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*...
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{a(A-5B)-4aA\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-5B)-4aA\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a(A-5B)-4aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)$$

$\big\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{a^2(13A-5B)-2a^2(A-5B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a(\cdots}{3d\cos^{\frac{3}{2}}(c\cdots}}{5a}\cdots\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(13A-5B)-2a^2(\cdots}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cdots}}\cdots}{3\cdots}}{5a}\cdots\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^2(13A-5B)-2a^2(\cdots}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\cdots}\cdots}{\cdots}}{5a}\cdots\right.$$

$\big\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2\int-\frac{15a^3(\cdots}{2\sqrt{\cos(c+dx)}\sqrt{\cdots}}\cdots}{a\cdots}}{5a}\cdots\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(13A-5B)\sin(}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(}}}{}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(13A-5B)\sin(}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(}}}{}}{}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{30a^3(A-B)\int\frac{}{\frac{\sin(c+}{\text{co}}}}{}}{}}{}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-5B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(13A-5B)\sin(}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(}}}{}}{}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sin[c + d*x])/(5*d*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a*(A - 5*B)*Sin[c + d*x])/(3*d*Cos
[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-15*Sqrt[2]*a^(3/2)*(A - B)*
ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])])/d + (2*a^2*(13*A - 5*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]]))/(3*a))/(5*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 13.91 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{7}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(15\cos(dx+c)^4+15\cos(dx+c)^3\right)+B\arcsin(c\right.}{}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{7}{2}}\left(\cos(dx+c)\sin(dx+c)\left(13\cos(dx+c)^2-\cos(dx+c)+3\right)\sqrt{2}+\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\right.}{15d(\cos(dx+c)+1)a}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/15/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)+1)*(A
*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(15*cos(d
*x+c)^4+15*cos(d*x+c)^3)+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(-15*cos(d*x+c)^4-15*cos(d*x+c)^3)+cos(d*x+c)*sin(d*x+c)*
(13*cos(d*x+c)^2-cos(d*x+c)+3)*2^(1/2)*A+cos(d*x+c)^2*sin(d*x+c)*(-5*cos(d
*x+c)+5)*2^(1/2)*B)/a
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\frac{15\sqrt{2}\left((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}} + \frac{2\left((13A-5B)\cos(dx+c)^2-(A-5B)\cos(dx+c\right.}{\sqrt{\cos(dx+c}}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*arc
tan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c)))/sqrt(a) + 2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) +
3*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*
x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{a+a\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^3}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**3)/(cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c
+ d*x) + 1)*sec(c + d*x)**3)/(cos(c + d*x) + 1),x)*a))/a
```

3.523 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 162

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d-2/3*(A-3*B)
*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*A*sec(d*x+c)^(3/
2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.03 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.81

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(\frac{2B\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3/2}}+\frac{4B\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}+\frac{(A-B)\csc^5(}{}\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x
]],x]
```

output
```
(2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*S
in[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(3*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(3/2)) + (4*B*Sin[c/2 + (d*x)/2])/(3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]
) + ((A - B)*Csc[c/2 + (d*x)/2]^5*(-12*Cos[(c + d*x)/2]^4*HypergeometricPF
Q[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^
2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[c/2 +
(d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[
c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2]^2
/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*Si
n[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2)
*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sq
rt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 +
 (d*x)/2]^2))))/(63*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))))/(d*Sqrt[a*(1 + C
os[c + d*x])])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3042, 3440, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{a(A-3B)-2aA\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a(A-3B)-2aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a(A-3B)-2aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}\right)$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{3a^2(A-B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a(A-3B)}{d\sqrt{\cos(c+dx)}\sqrt{}}}{3a}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a(A-B)\int\frac{}{\sqrt{c}}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a(A-B)\int\frac{}{\sqrt{s}}}{3a}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^2(A-B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin}{\sqrt{\cos(c+dx)}}\right.}{d}}{3a}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}(A-B)\operatorname{arcta}}{}}{3a}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-3*Sqrt[2]*Sqrt[a]*(A - B)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
])/d + (2*a*(A - 3*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]))/(3*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 13.92 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(3\cos(dx+c)^{3}+3\cos(dx+c)^{2}\right)+B\arcsin(c\ldots\right.}{3\ldots}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(\cos(dx+c)\sin(dx+c)(\cos(dx+c)-1)\sqrt{2}+\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(3\ldots\right.\right.}{3d(\cos(dx+c)+1)a}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/3/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)*(A
*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(3*cos(d*
x+c)^3+3*cos(d*x+c)^2)+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(-3*cos(d*x+c)^3-3*cos(d*x+c)^2)+cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)-1)*2^(1/2)*A-3*cos(d*x+c)^2*B*sin(d*x+c)*2^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((A-3B)\cos(dx+c)-A\right)\sqrt{a\cos(dx+c)+a}\,\mathrm{s}}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
-1/3*(3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan
(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)
))/sqrt(a) + 2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^{2}}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^{2}}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**2)/(cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c
+ d*x) + 1)*sec(c + d*x)**2)/(cos(c + d*x) + 1),x)*a))/a
```

3.524 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 119

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
-2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+2*A*sec(d*
x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.79 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.71

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\left(10B-(A-B)\sec(c+dx)\left(-\frac{5}{4}(1+4\cos(c+dx)+\cos\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x
]],x]
```

output
```
(2*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sin[(c + d*x)/2]*(10*B - (A - B)*Se
c[c + d*x]*((-5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4
*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos
[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -
(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Sq
rt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3440, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{a(A-B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx\right.}}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(A-B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2A\,\mathrm{si}}{d\sqrt{\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}}\right.}{\sqrt{a}d}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqr
t[a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 13.87 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.49

method	result
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\cos(dx+c)\sec(dx+c)^{\frac{3}{2}}\left(\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+1)\arcsin(\cot(dx+c)-\csc(dx+c))+\sin(dx+c)\sqrt{2}\right)}{d(\cos(dx+c)+1)a} -$
default	$\frac{\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(2A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\csc(dx+c)-\cot(dx+c))+\left(-\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)A\arcsin(\cot(dx+c}{d\left(\csc(dx+c)^2(1-\cos(dx+c))^2\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
A/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sec(d*x+c)^(3/2)*((cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)*arcsin(cot(d*x+c)-csc(d*x+c))+sin
(d*x+c)*2^(1/2))/(cos(d*x+c)+1)/a-B/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)*se
c(d*x+c)^(3/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}A\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos(dx+c)+ad}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*
cos(d*x + c) + a)*A*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a
*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{a+a\cos(c+dx)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{a}\left(\left(\int\frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)+1}dx\right)b+\left(\int\frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x))/(cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d
*x) + 1)*sec(c + d*x))/(cos(c + d*x) + 1),x)*a))/a
```

3.525 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 140

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(1/2)/d+2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2
)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\left(\sqrt{2}B\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+(A-B)\arctan\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x
]],x]
```

output
```
(2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3440, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3461}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{a}\right.$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}}{ad}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+}{\sqrt{a\cos(c+dx}}\right.}{\sqrt{a}d}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2a(A-B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+}{\sqrt{\cos(c+dx)}\sqrt{co}}\right.}{d}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
 + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d))*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 13.90 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

method	result
default	$-\frac{\left(-B\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+A\arcsin(\cot(dx+c)-\csc(dx+c))-B\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{\sec(dx+c)}\,\cdots}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}$
parts	$-\frac{A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\sec(dx+c)}\,\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\cos(dx+c)}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}+\frac{B\left(\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\cdots\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/d*(-B*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+A*ar
csin(cot(d*x+c)-csc(d*x+c))-B*arcsin(cot(d*x+c)-csc(d*x+c)))*sec(d*x+c)^(1
/2)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)/(cos(d*x+c)+1)/(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\,B\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
-(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*B*sqrt(a)*arctan(sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sqrt{\sec{(c+dx)}}}{\sqrt{a(\cos{(c+dx)}+1)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1)
), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 1221, normalized size of antiderivative = 8.72

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
(sqrt(2)*A*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x
 + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(
I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1
/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (
abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x +
 c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c)
+ 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(
cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c)
+ 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*
x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*ar
ctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e
^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) +
 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a
)*abs(e^(I*d*x + I*c) + 1)))/sqrt(a) - (sqrt(2)*sqrt(a)*arctan2(((abs(2*e^
(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x
+ c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2
 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x +
c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x + c) + 16)^(1/4)*sin(1/2*arctan2(8*(
cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}}{\cos(dx+c)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(co
s(c + d*x) + 1),x)*a))/a
```

3.526 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{(2A-B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
(2*A-B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a^(1/2)/d-2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^
(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a^(1/2)/d+B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.88 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.58

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\left(B-Be^{i(c+dx)}+Be^{2i(c+dx)}-Be^{3i(c+dx)}-(2A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arcsin}\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]),x]
```

output
```
((I/4)*(1 + E^(I*(c + d*x)))*(B - B*E^(I*(c + d*x)) + B*E^((2*I)*(c + d*x)
) - B*E^((3*I)*(c + d*x)) - (2*A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^
((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I
)*(c + d*x))])] + 2*Sqrt[2]*A*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))
]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
- Sqrt[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(-1 + E^
(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 2*A*E^(I*(c + d*
x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] -
 B*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[
c + d*x])])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3440, 3042, 3462, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \textbf{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \textbf{3462}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{aB+a(2A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{aB+a(2A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{aB+a(2A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \textbf{3461}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(2A-B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-2a(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{B\sin(c-}{d\sqrt{a}}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(2A-B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-2a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-2a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2(2A-B)\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin}{\sqrt{\cos(}}\right.}{d}}{2a}\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sqrt{a}(2A-B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-2a(A-B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{4a^2(A-B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2\sqrt{a}(2A-B)\arcsin\left(\frac{\sqrt{a}\sin}{\sqrt{a\cos(}}\right.}{d}}{2a}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sqrt{a}(2A-B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2\sqrt{2}\sqrt{a}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{B\,s}{}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*Sqrt[a]*(2*A - B)*ArcSin[(Sqrt[
a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (2*Sqrt[2]*Sqrt[a]*(A - B)
*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])])/d)/(2*a) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*
Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 18.01 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08

method	result
default	$\frac{\left(B\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2A\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)-B\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+2A\arcsin(\text{}}{2d(\cos(dx+c)+1)\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)}{d(\cos(dx+c)+1)\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a}+\frac{B\sqrt{2}\left(\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{si}}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2/d*(B*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*A*2^(1/2)*
arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-B*2^(1/2)*arctan(tan(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*A*arcsin(cot(d*x+c)-csc(d*x+c)
)-2*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos
(d*x+c)+1)/sec(d*x+c)^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.93

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{a\cos(dx+c)+a}B\sqrt{\cos(dx+c)}\sin(dx+c) - ((2A-B)\cos(dx+c)+2A-B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos}}{}\right.}{ad\cos(dx+c)+ad}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*c
os(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B
)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt{a(\cos(c+dx)+1)}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x
))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found %i
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + a\cos(c + dx)}\sqrt{\sec(c + dx)}}\,dx = \int \frac{A + B\cos(c + dx)}{\sqrt{\frac{1}{\cos(c+dx)}}\sqrt{a + a\cos(c + dx)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\sqrt{\cos(dx+c)+1}\cos(dx+c)}{\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\sqrt{\cos(dx+c)+1}}{\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*b + int((sqrt(sec(c + d*x))*sqr
t(cos(c + d*x) + 1))/(cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*a))/a
```

3.527 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 230

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} \\ &\quad+\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \\ &\quad+\frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}+\frac{(4A-B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-1/4*(4*A-7*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sin(d*
x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(1/2)/d+1/2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+
c)^(3/2)+1/4*(4*A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.79

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{ie^{-3i(c+dx)}\left(1+e^{i(c+dx)}\right)\left(-B-4Ae^{i(c+dx)}+2Be^{i(c+dx)}+4Ae^{2i(c+dx)}-3Be^{2i(c+dx)}-4Ae^{3i(c+dx)}+3B\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2
)),x]
```

output
```
((-1/16*I)*(1 + E^(I*(c + d*x)))*(-B - 4*A*E^(I*(c + d*x)) + 2*B*E^(I*(c +
 d*x)) + 4*A*E^((2*I)*(c + d*x)) - 3*B*E^((2*I)*(c + d*x)) - 4*A*E^((3*I)*
(c + d*x)) + 3*B*E^((3*I)*(c + d*x)) + 4*A*E^((4*I)*(c + d*x)) - 2*B*E^((4
*I)*(c + d*x)) + B*E^((5*I)*(c + d*x)) - (4*A - 7*B)*E^((2*I)*(c + d*x))*S
qrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] - 8*Sqrt[2]*(A - B)*
E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d
*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 4*A*E^((2*I)*(c + d*x))*S
qrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 7*B*
E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x))*Sqrt[a*(1 + Cos
[c + d*x])])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3462, 27, 3042, 3462, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3462}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3aB+a(4A-B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3aB+a(4A-B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3aB+a(4A-B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3462}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(4A-B)-a^2(4A-7B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(4A-B)-a^2(4A-7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c}{2d\sqrt{a\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(4A-B)-a^2(4A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}}{2d\sqrt{a\cos(c+d}}\right.$$

$$\Big\downarrow \text{3461}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-a(4A-7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{2a}+\frac{a(4A-B)\sin(c+dx)\sqrt{\cos(}}{d\sqrt{a\cos(c+dx)+a}}}{4a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-a(4A-7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a(4A-B)\sin(}{d\sqrt{a\,co}}}{4a}\right.$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2a(4A-7B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}}{4a}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{8a^2(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2a^{3/2}(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{a(4A-B)s}{d\sqrt{}}}{4a}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{16a^3(A-B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2a^{3/2}(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c}{\sqrt{a\cos(c+}}\right.}{d}}{2a}}{4a}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{8\sqrt{2}a^{3/2}(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^{3/2}(4A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{a(4A-}{d}}{4a}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (((-2*a^(3/2)*(4*A - 7*B)*ArcSin[(Sqrt[a
]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (8*Sqrt[2]*a^(3/2)*(A - B)*
ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])])/d)/(2*a) + (a*(4*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sq
rt[a + a*Cos[c + d*x]]))/(4*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 17.96 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(-4A\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+7B\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+4A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)+\sqrt{2}\,\sqrt{}\right.}{8d\sqrt{\sec(dx+c)}\,(\cos(}$
parts	$\frac{A\sqrt{2}\left(\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)-2\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(\cos(dx+c)+1)}}{2d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+1)\sqrt{\sec(dx+c)}\,a}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/d/sec(d*x+c)^(1/2)*(-4*A*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+7*B*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))+4*A*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(sin(2*d*x+2*c)-sin(d*x+c))*B-8*A*arcsin(c
ot(d*x+c)-csc(d*x+c))+8*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(cos(d
*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.84

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{((4A-7B)\cos(dx+c)+4A-7B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \frac{4\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)}{}}{4\,(ad\cos(dx+c)+ad}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
1/4*(((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2)*((A - B)*
a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + (2*B*cos(d*x + c)^2 + (4*A
- B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)
))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt{a(\cos(c+dx)+1)}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**(3
/2)), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2
)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{A+B\cos(c+dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}\sqrt{a+a\cos(c+dx)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)\sec(dx+c)^2+\sec(dx+c)^2}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}}{\cos(dx+c)\sec(dx+c)^2+\sec(dx+c)^2}dx\right)a\right)}{a}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2),x)*b + int((sqrt(sec(c + d*x
))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2
),x)*a))/a
```

3.528 $\int \frac{(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 54, antiderivative size = 192

$$\int \frac{(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{(2Ab+2aB-bB)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}(a-b)(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{bB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
(2*A*b+2*B*a-B*b)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+2^(1/2)*(a-b)*(A-B)*arctan(1/2*a^(1/
2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/a^(1/2)/d+b*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/s
ec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.74

$$\int \frac{(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c + dx)\right)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\left(\sqrt{2}(2Ab + 2aB - bB)\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c + dx)\right)\right) + 2(a - \right.}{d\sqrt{a(1 + \cos(c + dx))}}$$

input
```
Integrate[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*(2*A*b +
2*a*B - b*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(a - b)*(A - B)*ArcTan[S
in[(c + d*x)/2]/Sqrt[Cos[c + d*x]]] + 2*b*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*
x)/2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4709, 3042, 3524, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c + dx)}((aB + Ab)\cos(c + dx) + aA + bB\cos^2(c + dx))}{\sqrt{a\cos(c + dx) + a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sec(c + dx)}((aB + Ab)\cos(c + dx) + aA + bB\cos(c + dx)^2)}{\sqrt{a\cos(c + dx) + a}}dx$$

$$\Big\downarrow \text{4709}$$

$$\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\int \frac{bB\cos^2(c + dx) + (Ab + aB)\cos(c + dx) + aA}{\sqrt{\cos(c + dx)}\sqrt{\cos(c + dx)a + a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{bB\sin\left(c+dx+\frac{\pi}{2}\right)^2+(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+aA}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3524}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(2aA+bB)+a(2Ab-Bb+2aB)\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(2aA+bB)+a(2Ab-Bb+2aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(2aA+bB)+a(2Ab-Bb+2aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3461}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(a-b)(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+(2aB+2Ab-bB)\int \frac{\sqrt{\cos(c+dx)a+}}{\sqrt{\cos(c+dx)}}}{2a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(a-b)(A-B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+(2aB+2Ab-bB)\int \frac{\sqrt{\sin(c}}{\sqrt{\sin}}}{2a}\right.$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(a-b)(A-B)\int \frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}\sqrt{\sin(c+dx+\frac{\pi}{2})a+a}}dx-\frac{2(2aB+2Ab-bB)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+}{\cos(c+dx)}}}}{d}}{2a}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(a-b)(A-B)\int \frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}\sqrt{\sin(c+dx+\frac{\pi}{2})a+a}}dx+\frac{2\sqrt{a}(2aB+2Ab-bB)\arcsin\left(\frac{\sqrt{a}\,s}{\sqrt{a\cos}}\right.}{d}}{2a}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sqrt{a}(2aB+2Ab-bB)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^2(a-b)(A-B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{}{\sqrt{co}}\right.}{d}}{2a}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sqrt{a}(2aB+2Ab-bB)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2\sqrt{2}\sqrt{a}(a-b)(A-B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx}}\right.}{d}}{2a}\right.$$

input
```
Int[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c + d*
x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*Sqrt[a]*(2*A*b + 2*a*B - b*B)*A
rcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*Sqrt[2]*Sqr
t[a]*(a - b)*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(2*a) + (b*B*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3524
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(b*d*(m +
 n + 2))   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m
+ n + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}
, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !Lt
Q[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

rule 4709
```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m   Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Maple [A] (verified)

Time = 24.22 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.46

method	result
default	$\frac{\left(B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,b\sin(dx+c)+2A\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)b+2B\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)a-B\sqrt{2}\right.}{}$
parts	$-\frac{A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\sec(dx+c)}\,\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\cos(dx+c)}{d(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}+\frac{(Ab+Ba)\left(\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.\right.}{}$

input
```
int((A*a+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(
d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(B*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*sin(d*x+c)+2*A*2^(1/2
)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*b+2*B*2^(1/2)*arcta
n(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*a-B*2^(1/2)*arctan(tan(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*b-2*A*arcsin(cot(d*x+c)-csc(d*x+c))
*a+2*A*arcsin(cot(d*x+c)-csc(d*x+c))*b+2*B*arcsin(cot(d*x+c)-csc(d*x+c))*a
-2*B*arcsin(cot(d*x+c)-csc(d*x+c))*b)*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec
(d*x+c)^(1/2)*cos(d*x+c)/(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/
a
```

Fricas [A] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.08

$$\int \frac{(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx$$

$$= \frac{\sqrt{a\cos(dx + c) + a}Bb\sqrt{\cos(dx + c)}\sin(dx + c) - (2\,Ba + (2\,A - B)b + (2\,Ba + (2\,A - B)b)\cos(dx}{ad}$$

input
```
integrate((A*a+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a*cos(d*x + c) + a)*B*b*sqrt(cos(d*x + c))*sin(d*x + c) - (2*B*a + (
2*A - B)*b + (2*B*a + (2*A - B)*b)*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A -
B)*a^2 - (A - B)*a*b + ((A - B)*a^2 - (A - B)*a*b)*cos(d*x + c))*arctan(sq
rt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/
sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{(A + B\cos(c + dx))(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

input
```
integrate((A*a+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(
a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt
(a*(cos(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{\left(Bb\cos\left(dx + c\right)^2 + Aa + \left(Ba + Ab\right)\cos\left(dx + c\right)\right)\sqrt{\sec\left(dx + c\right)}}{\sqrt{a\cos\left(dx + c\right) + a}}\,dx$$

input
```
integrate((A*a+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d
*x + c))/sqrt(a*cos(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A*a+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aA + (Ab + aB)\cos(c + dx) + bB\cos^2(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(B\,b\cos\left(c + d\,x\right)^2 + \left(A\,b + B\,a\right)\,\cos\left(c + d\,x\right) + A\,a\right)}{\sqrt{a + a\,\cos\left(c + d\,x\right)}}\,dx$$

input
```
int(((1/cos(c + d*x))^(1/2)*(A*a + cos(c + d*x)*(A*b + B*a) + B*b*cos(c +
d*x)^2))/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(((1/cos(c + d*x))^(1/2)*(A*a + cos(c + d*x)*(A*b + B*a) + B*b*cos(c +
d*x)^2))/(a + a*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{a}\left(2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)+1}dx\right)ab+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^2}{\cos(dx+c)+1}dx\right)b^2+\left(\int \frac{\sqrt{\sec(dx+c)}}{\cos(}\right.\right.}{a}$$

input
```
int((A*a+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(
d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(2*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(
cos(c + d*x) + 1),x)*a*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*
cos(c + d*x)**2)/(cos(c + d*x) + 1),x)*b**2 + int((sqrt(sec(c + d*x))*sqrt
(cos(c + d*x) + 1))/(cos(c + d*x) + 1),x)*a**2))/a
```

3.529 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 317

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(19A-15B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{2\sqrt{2}a^{3/2}d}$$

$$-\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}}$$

$$+\frac{(397A-273B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}}-\frac{(67A-63B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(11A-7B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{14ad\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*(19*A-15*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1
/210*(1201*A-1029*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2
)+1/210*(397*A-273*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/
2)-1/70*(67*A-63*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
-1/2*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/14*(11*A
-7*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 2966, normalized size of antiderivative = 9.36

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(
3/2),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2
*Sin[c/2 + (d*x)/2]^2]*(-1/28*((A - B)*(1 - 2*Sin[c/2 + (d*x)/2]))/((1 + S
in[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) + ((A - B)*(1 + 2*S
in[c/2 + (d*x)/2]))/(28*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]
^2)^(7/2)) - ((A - B)*(315*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]] + (5 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2]
)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - (11 + 17*Sin[c/2 + (d*x)/2])/((1 -
 Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 + 71*Sin[c/
2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])
+ (193*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/70 + (
(A - B)*(315*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/
2]^2]] + (5 - 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c
/2 + (d*x)/2]^2)^(5/2)) - (11 - 17*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*
x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 - 71*Sin[c/2 + (d*x)/2])/
((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sqrt[1
- 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/70 - ((-A - 3*B)*Csc
[c/2 + (d*x)/2]^9*(363825*Sin[c/2 + (d*x)/2]^2 - 4729725*Sin[c/2 + (d*x)/2
]^4 + 26785605*Sin[c/2 + (d*x)/2]^6 - 86790165*Sin[c/2 + (d*x)/2]^8 + 1776
77808*Sin[c/2 + (d*x)/2]^10 - 239283044*Sin[c/2 + (d*x)/2]^12 + 52080*Hype
rgeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*...
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.09, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A-7B)-8a(A-B)\cos(c+dx)}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A-7B)-8a(A-B)\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A-7B)-8a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$\big\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{a^2(67A-63B)-6a^2(11A-7B)\cos(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{7a}+\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-}{2d\cos^{\frac{7}{2}}(c+dx}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(67A-63B)-6a^2(11A-7B)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{7a}}{4a^2}-\frac{(A-B)}{2d\cos^{\frac{7}{2}}(c+dx)(a}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(67A-63B)-6a^2(11A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{7a}}{4a^2}-\frac{(A-}{2d\cos^{\frac{7}{2}}(c+dx}\right.$$

$\big\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{a^3(397A-273B)-4a^3(67A-63B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2a^2(67A-63B)\sin}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos}}}{7a}}{4a^2}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(397A-273B)-4a^3(67A-63B)\cos(c+}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}}{5a}}{7a}}{4a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(397A-273B)-4a^3(67A-63B)\sin\left(c+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)c}}}{5a}}{7a}}{4a^2}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{a^4(1201A-1029B)-2a^4(397A-273B}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a}}}{3a}}{}}{7a}}{4a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(397A-273B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int a^4(12}{}}{}}{7a}}{4a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(397A-273B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^4(12}{}}{}}{7a}}{4a^2}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(397A-273B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2\int-\frac{}{2}}{}}{7a}}{4a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(397A-273B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(12}{d\sqrt{\cos(}}}{}}{7}}{4a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(397A-273B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(120\cdots}{d\sqrt{\cos(\cdots}}}{\cdots}}{\cdots}}{4a^2\cdots}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(397A-273B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{210a^5(\cdots}{\cdots}}{\cdots}}{4a^2\cdots}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(11A-7B)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(67A-63B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(397A-273B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(120\cdots}{d\sqrt{\cos(\cdots}}}{\cdots}}{\cdots}}{4a^2\cdots}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((A - B)*Sin[c + d*x])/(d*Cos[
c + d*x]^(7/2)*(a + a*Cos[c + d*x])^(3/2)) + ((2*a*(11*A - 7*B)*Sin[c + d*
x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a^2*(67*A - 63
*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*
a^3*(397*A - 273*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
 + d*x]]) - ((-105*Sqrt[2]*a^(7/2)*(19*A - 15*B)*ArcTan[(Sqrt[a]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^4*(1
201*A - 1029*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]]))/(3*a))/(5*a))/(7*a))/(4*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.48 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{9}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(1995\cos(dx+c)^{6}+3990\cos(dx+c)^{5}+1995\right.\right.}{}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{9}{2}}\left(\cos(dx+c)\sin(dx+c)\left(1201\cos(dx+c)^{4}+804\cos(dx+c)^{3}-196\cos(dx+c)^{2}+36\cos(dx+c)\right.\right.}{420d(\cos(dx+c)+1}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/420/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(9/2)/(cos(d*x+c)+1)^
2*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1995
*cos(d*x+c)^6+3990*cos(d*x+c)^5+1995*cos(d*x+c)^4)+B*arcsin(cot(d*x+c)-csc
(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-1575*cos(d*x+c)^6-3150*cos(d*
x+c)^5-1575*cos(d*x+c)^4)+cos(d*x+c)*sin(d*x+c)*(1201*cos(d*x+c)^4+804*cos
(d*x+c)^3-196*cos(d*x+c)^2+36*cos(d*x+c)-60)*2^(1/2)*A+cos(d*x+c)^2*sin(d*
x+c)*(-1029*cos(d*x+c)^3-756*cos(d*x+c)^2+84*cos(d*x+c)-84)*2^(1/2)*B)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.75

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{105\sqrt{2}\left((19A-15B)\cos(dx+c)^5+2(19A-15B)\cos(dx+c)^4+(19A-15B)\cos(dx+c)^3\right)\sqrt{a}\,\mathrm{ar}}{420\left(a^2d\cos\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/420*(105*sqrt(2)*((19*A - 15*B)*cos(d*x + c)^5 + 2*(19*A - 15*B)*cos(d*
x + c)^4 + (19*A - 15*B)*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1201*A - 1
029*B)*cos(d*x + c)^4 + 12*(67*A - 63*B)*cos(d*x + c)^3 - 28*(7*A - 3*B)*c
os(d*x + c)^2 + 12*(3*A - 7*B)*cos(d*x + c) - 60*A)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*
x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{9/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^4}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(\cdots}}{\cdots}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**4)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(sec(c
 + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**4)/(cos(c + d*x)**2 + 2*cos(
c + d*x) + 1),x)*a))/a**2
```

3.530 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 270

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(15A-11B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} + \frac{(147A-95B)\sqrt{\sec(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}}$$

output

```
-1/4*(15*A-11*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a
+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d+
1/30*(147*A-95*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1
/30*(39*A-35*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/2
*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/10*(9*A-5*B)
*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.26 (sec) , antiderivative size = 2166, normalized size of antiderivative = 8.02

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(
3/2),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2
*Sin[c/2 + (d*x)/2]^2]*(-1/20*((A - B)*(1 - 2*Sin[c/2 + (d*x)/2]))/((1 + S
in[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + ((A - B)*(1 + 2*S
in[c/2 + (d*x)/2]))/(20*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]
^2)^(5/2)) - ((A - B)*(-105*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*S
in[c/2 + (d*x)/2]^2]] + (4 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2
])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) - (19 + 29*Sin[c/2 + (d*x)/2])/((1
- Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) - (67*Sqrt[1 - 2*S
in[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/30 + ((A - B)*(-105*ArcTa
n[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (4 - 3*Si
n[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(
3/2)) - (19 - 29*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]) - (67*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 + Sin[c/
2 + (d*x)/2])))/30 + ((-A - 3*B)*Csc[c/2 + (d*x)/2]^7*(4725*Sin[c/2 + (d*x
)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630
*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2
F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c
/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2},
 {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2
 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[...
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(9A-5B)-6a(A-B)\cos(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(9A-5B)-6a(A-B)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(9A-5B)-6a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{a^2(39A-35B)-4a^2(9A-5B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B}{2d\cos^{\frac{5}{2}}(c+dx)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(39A-35B)-4a^2(9A-5B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}}{4a^2}-\frac{(A-B)s}{2d\cos^{\frac{5}{2}}(c+dx)(a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(39A-35B)-4a^2(9A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}}{4a^2}-\frac{(A-B}{2d\cos^{\frac{5}{2}}(c+dx)}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{a^3(147A-95B)-2a^3(39A-35B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a^2(39A-35B)\sin(}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(}}}{5a}}{4a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(39A-35B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(147A-95B)-2a^3(39A-35B)\cos(c+\cdots}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}\cdots}{3a}}{5a}}{4a^2}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(39A-35B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(147A-95B)-2a^3(39A-35B)\sin\left(c+\cdots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a\cdots}}\cdots}{3a}}{5a}}{4a^2}\cdots\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(39A-35B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{15a^4(15A-11B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{\cdots}{d\cdots}}{3a}}{5a}}{4a^2}\cdots\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(39A-35B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(147A-95B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^3(15A\cdots}{3a}}{5a}}{4a^2}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(39A-35B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(147A-95B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^3(15A\cdots}{\cdots}}{5a}}{4a^2}\right.$$

$$\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(39A-35B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{30a^4(15A-11B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^{3}\cdots}{\cos(c+dx)a+a}}\cdots}{\cdots}}{5a\cdots}}{4a^2}\right.$$

$$\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(9A-5B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(39A-35B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(147A-95B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{15\sqrt{2}a^{5/2}\cdots}{\cdots}}{5a}}{4a^2}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((A - B)*Sin[c + d*x])/(d*Cos[
c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)) + ((2*a*(9*A - 5*B)*Sin[c + d*x
])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a^2*(39*A - 35*
B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-15
*Sqrt[2]*a^(5/2)*(15*A - 11*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^3*(147*A - 95*B)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a))/(4
*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.98

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{7}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(225\cos(dx+c)^5+450\cos(dx+c)^4+225\cos(d\right.\right.}{}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{7}{2}}\left(\cos(dx+c)\sin(dx+c)\left(49\cos(dx+c)^3+36\cos(dx+c)^2-4\cos(dx+c)+4\right)\sqrt{2}+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{20d(\cos(dx+c)+1)^2a^2}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/60/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)+1)^2*
(A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(225*co
s(d*x+c)^5+450*cos(d*x+c)^4+225*cos(d*x+c)^3)+B*arcsin(cot(d*x+c)-csc(d*x+
c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-165*cos(d*x+c)^5-330*cos(d*x+c)^4-
165*cos(d*x+c)^3)+cos(d*x+c)*sin(d*x+c)*(147*cos(d*x+c)^3+108*cos(d*x+c)^2
-12*cos(d*x+c)+12)*2^(1/2)*A+cos(d*x+c)^2*sin(d*x+c)*(-95*cos(d*x+c)^2-60*
cos(d*x+c)+20)*2^(1/2)*B)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.81

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{15\sqrt{2}\big((15A-11B)\cos(dx+c)^4+2(15A-11B)\cos(dx+c)\ldots}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^4 + 2*(15*A - 11*B)*cos(d*x +
 c)^3 + (15*A - 11*B)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((147*A - 95*B)
*cos(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 - 4*(3*A - 5*B)*cos(d*x +
c) + 12*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*
d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{(a+a\cos(c+dx))^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(d}}{c}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**3)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(sec(c
 + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**3)/(cos(c + d*x)**2 + 2*cos(
c + d*x) + 1),x)*a))/a**2
```

3.531 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 223

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(11A-7B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{}}{2\sqrt{2}a^{3/2}d}$$
$$-\frac{(19A-15B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}}$$
$$-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(7A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}}$$

output
```
1/4*(11*A-7*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1/
6*(19*A-15*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/2*(
A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/6*(7*A-3*B)*se
c(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.09 (sec) , antiderivative size = 981, normalized size of antiderivative = 4.40

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(
3/2),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2
*Sin[c/2 + (d*x)/2]^2]*(-1/12*((A - B)*(1 - 2*Sin[c/2 + (d*x)/2]))/((1 + S
in[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + ((A - B)*(1 + 2*S
in[c/2 + (d*x)/2]))/(12*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]
^2)^(3/2)) - ((A - B)*(5*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[
c/2 + (d*x)/2]^2]] + (1 + Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sq
rt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1
- Sin[c/2 + (d*x)/2])))/2 + ((A - B)*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/
Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2
 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (
d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/2 + ((A + 3*B)*Csc[c/2 + (d*x)/2]^5
*(-12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/
2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hy
pergeometric2F1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)
/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d
*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(
1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 +
(d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 -
 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*
Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(126*(1 - 2*Si...
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(7A-3B)-4a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(7A-3B)-4a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(7A-3B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{a^2(19A-15B)-2a^2(7A-3B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B\cdots}{2d\cos^{\frac{3}{2}}(c+dx)\cdots}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(19A-15B)-2a^2(7A-3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{(A-B)\,\mathrm{s}\cdots}{2d\cos^{\frac{3}{2}}(c+dx)(a\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^2(19A-15B)-2a^2(7A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{(A-B\cdots}{2d\cos^{\frac{3}{2}}(c+dx)\cdots}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{3a^3(11A-7B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\cdots\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^2(11A-7B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a}}}{3a}}{4a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^2(11A-7B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+\right.}}}{3a}}{4a^2}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^3(11A-7B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a}}\right.}{d}}{3a}}{4a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(7A-3B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^2(19A-15B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}a^{3/2}(11A-7B)\arctan\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{\cos(c+dx)}}\right.}{d}}{3a}}{4a^2}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((A - B)*Sin[c + d*x])/(d*Cos[
c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((2*a*(7*A - 3*B)*Sin[c + d*x
])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-3*Sqrt[2]*a^(3/2
)*(11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/d + (2*a^2*(19*A - 15*B)*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\left(33\cos(dx+c)^{4}+66\cos(dx+c)^{3}+33\cos(dx\right.\right.}{}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(\cos(dx+c)\sin(dx+c)\left(19\cos(dx+c)^{2}+12\cos(dx+c)-4\right)\sqrt{2}+\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arcsin(\cot(d\right.}{12d(\cos(dx+c)+1)^{2}a^{2}}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/12/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)^2
*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(33*co
s(d*x+c)^4+66*cos(d*x+c)^3+33*cos(d*x+c)^2)+B*arcsin(cot(d*x+c)-csc(d*x+c)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-21*cos(d*x+c)^4-42*cos(d*x+c)^3-21*c
os(d*x+c)^2)+cos(d*x+c)*sin(d*x+c)*(19*cos(d*x+c)^2+12*cos(d*x+c)-4)*2^(1/
2)*A+cos(d*x+c)^2*sin(d*x+c)*(-15*cos(d*x+c)-12)*2^(1/2)*B)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{3\sqrt{2}\big((11A-7B)\cos{(dx+c)}^3+2\,(11A-7B)\cos{(dx+c)}^2+(11A-7B)\cos{(dx+c)}\big)\sqrt{a}\arctan\Big(}{12\,\big(a^2d\cos{(dx+c)}^3+2\,a^2d\cos{(dx-}}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^3 + 2*(11*A - 7*B)*cos(d*x + c
)^2 + (11*A - 7*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((19*A - 15*B)*cos(d
*x + c)^2 + 12*(A - B)*cos(d*x + c) - 4*A)*sqrt(a*cos(d*x + c) + a)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2
+ a^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(\ldots}}{\ldots}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**2)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(sec(c
 + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**2)/(cos(c + d*x)**2 + 2*cos(
c + d*x) + 1),x)*a))/a**2
```

3.532 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 176

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(7A-3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/4*(7*A-3*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1/
2*(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/2*(5*A-B)*s
ec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.16 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.52

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(30(A-B)\arctan\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(
3/2),x]
```

output
```
(Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(30*(A - B)*ArcT
an[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] - 30*(A - B)*ArcTan[(1 + 2
*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] - (20*(A - B)*Sqrt[Cos[c + d*x]])/(
-1 + Sin[(c + d*x)/2]) - (20*(A - B)*Sqrt[Cos[c + d*x]])/(1 + Sin[(c + d*x
)/2]) + (5*(A - B)*(-1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(Cos[(c
+ d*x)/4] + Sin[(c + d*x)/4])^2) - (5*(A - B)*(1 + 2*Sin[(c + d*x)/2]))/(S
qrt[Cos[c + d*x]]*(-1 + Sin[(c + d*x)/2])) + ((A + 3*B)*Csc[(c + d*x)/2]^3
*(5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(1 - Cos[c + d*x] + ArcTanh[Sq
rt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*
x]]) - 2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)
]*Sin[(c + d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))/(2*Cos[c + d*x]^(3/2))))/
(10*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(5A-B)-2a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(5A-B)-2a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(5A-B)-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{a^2(7A-3B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a(7A-3B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{(A}{2d\sqrt{\cos(c}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a(7A-3B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{}{2d\sqrt{}}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a^2(7A-3B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}}}{4a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a(5A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{a}(7A-3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{}{2d\sqrt{\cos}}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((A - B)*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + (-((Sqrt[2]*Sqrt[a]*(7*A - 3*
B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/d) + (2*a*(5*A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]]))/(4*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.13

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\cos(dx+c)\left(\frac{7(3+\cos(2dx+2c)+4\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,A\arcsin(\cot(dx+c)-\csc(dx+c))}{2}-\frac{3(3+\cos(2dx+2c)+4\cos}{}\right.}{4d(\cos(dx+c)+1}$
parts	$\frac{A\cos(dx+c)\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sin(dx+c)(5\cos(dx+c)+4)\sqrt{2}+\frac{7(3+\cos(2dx+2c)+4\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arcsin(\cot(dx+c)-c}{2}\right.}{4d(\cos(dx+c)+1)^2a^2}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*(7/2*(3+cos(2*d*x+2*c)+4
*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*A*arcsin(cot(d*x+c)-csc(d*x
+c))-3/2*(3+cos(2*d*x+2*c)+4*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*B*arcsin(cot(d*x+c)-csc(d*x+c))+sin(d*x+c)*(5*cos(d*x+c)+4)*2^(1/2)*A-1/2
*2^(1/2)*sin(2*d*x+2*c)*B)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left((7\,A-3\,B)\cos\left(dx+c\right)^2+2\,(7\,A-3\,B)\cos\left(dx+c\right)+7\,A\right.}{4\,\left(a^2d\cos\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*
A - 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) + 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*
x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos\left(dx+c\right)+A)\sec\left(dx+c\right)^{\frac{3}{2}}}{(a\cos\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{(A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx}}{\mathrm{co}}\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(sec(c +
d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*
x) + 1),x)*a))/a**2
```

3.533 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 127

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(3A+B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/4*(3*A+B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(3/2)/d-1/2*(
A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.50

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3A\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+d\ldots\right.}{\ldots}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(
3/2),x]
```

output
```
-1/2*(Cot[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*A*ArcTanh[Sqrt[-(Sec[c + d*x]
*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Sqrt[2 - 2*Sec[c +
d*x]] - 2*(B*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Cos[(c + d*
x)/2]^2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2] + (-A + B)*Cos[c + d*x]*Sin[(c
 + d*x)/2]^2)))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(3A+B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(3A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(3A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{(3A+B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(3A+B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((3*A + B)*ArcTan[(Sqrt[a]*Sin[c +
d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a
^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c +
d*x])^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 14.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.51

method	result
default	$-\frac{\sqrt{\sec(dx+c)}\left(\csc(dx+c)^2(1-\cos(dx+c))^2-1\right)\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(-A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\csc(dx+c)-\cot(dx+c))+B\sqrt{2}\,\sqrt{\right.}}{8d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^2}$
parts	$\frac{A\sqrt{\sec(dx+c)}\left(\csc(dx+c)^2(1-\cos(dx+c))^2-1\right)\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\csc(dx+c)-\cot(dx+c))+3\arcsin(\cot(\right.}{8d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^2}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/8/d*sec(d*x+c)^(1/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*2^(1/2)*(a*(cos(
d*x+c)+1))^(1/2)*(-A*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(csc(d*x+c)
-cot(d*x+c))+B*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(csc(d*x+c)-cot(d
*x+c))-3*A*arcsin(cot(d*x+c)-csc(d*x+c))-B*arcsin(cot(d*x+c)-csc(d*x+c)))/
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right.}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
 B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sq
rt(a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sqrt{\sec{(c+dx)}}}{(a(\cos{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a*(cos(c + d*x) + 1))**(
3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sqrt{\sec{(dx+c)}}}{(a\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\mathrm{co}\cdots}}{\cos(dx+c)^2+2\cos\cdots}\right.\cdots\right)}{a^2}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(co
s(c + d*x) + 1))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1),x)*a))/a**2
```

3.534 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 185

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx=\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d}+\frac{(A-5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(3/2)/d+1/4*(A-5*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/
cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
*2^(1/2)/a^(3/2)/d+1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.31

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*
x]]),x]
```

output
```
(Cos[(c + d*x)/2]^3*(((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(4*B*ArcSinh[E^(I*(c + d*x))] - Sqr
t[2]*(A - 5*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))])] - 4*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d
*x)) + (A - B)*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] +
Sin[(3*(c + d*x))/2])))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$\Big\downarrow$ 3456

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+4aB\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+4aB\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+4aB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3461

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a(A-5B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+4B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{(A-B)\sin(c-}{2d(a\cos(c}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a(A-5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+4B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{(A-}{2}\right.$$

$\Big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a(A-5B)\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}\sqrt{\sin(c+dx+\frac{\pi}{2})a+a}}dx-\frac{8B\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right.}{d}}{4a^2}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a(A-5B)\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}\sqrt{\sin(c+dx+\frac{\pi}{2})a+a}}dx+\frac{8\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{(A-}{2}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{8\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^2(A-5B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a}}\right.}{d}}{4a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\sqrt{2}\sqrt{a}(A-5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{8\sqrt{a}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{(A-B)}{2d(}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((8*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c
 + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (Sqrt[2]*Sqrt[a]*(A - 5*B)*ArcTan[
(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]
])])/d)/(4*a^2) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Co
s[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 10.75 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\sec(dx+c)}\,(\cos(dx+c)+1)\left(\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)\left(\csc(dx+c)^2(1-\cos(dx+c))^2-1\right)\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(A\sqrt{2}\,\sqrt{\right.}}{}$
parts	$\frac{A\left((-\cos(dx+c)-1)\arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)\right)\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}}{4d(\cos(dx+c)+1)^2\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^2}+\frac{B\sqrt{\sec(dx+c)}\,(\cos}{}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/16/d*sec(d*x+c)^(1/2)*(cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*2^(1/2
)*(a*(cos(d*x+c)+1))^(1/2)*(A*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(c
sc(d*x+c)-cot(d*x+c))+4*B*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))-B*2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(csc(d*x+c)-cot(d
*x+c))-A*arcsin(cot(d*x+c)-csc(d*x+c))+5*B*arcsin(cot(d*x+c)-csc(d*x+c)))/
a^2
```

Fricas [A] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx =$$
$$-\frac{\sqrt{2}\left((A-5\,B)\cos\left(dx+c\right)^2+2\,(A-5\,B)\cos\left(dx+c\right)+A-5\,B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{4\,(a^2}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5
*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sq
rt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c
))*sin(d*x + c) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*arct
an(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(sec(c +
d*x))), x)
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}\sqrt{\sec(c + dx)}}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^{\frac{3}{2}}\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}\sqrt{\sec(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^2\sec(dx+c)+2\cos(dx+c)\sec(dx+c)+\sec(dx+c)}dx\right)b+\left(\right.\right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**2*sec(c + d*x) + 2*cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x
)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)**2*sec
(c + d*x) + 2*cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*a))/a**2
```

3.535 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 237

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{(2A-3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d}$$
$$-\frac{(5A-9B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}$$
$$+\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}-\frac{(A-3B)\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
(2*A-3*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^(3/2)/d-1/4*(5*A-9*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*
2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)*2^(1/2)/a^(3/2)/d+1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/s
ec(d*x+c)^(3/2)-1/2*(A-3*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+
c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.21 (sec) , antiderivative size = 836, normalized size of antiderivative = 3.53

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3
/2)),x]
```

output
```
((-I)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
d*x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]
))^(3/2)) + ((3*I)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[
1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E
^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 +
 Cos[c + d*x]))^(3/2)) + ((2*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*
I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] +
 Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*
x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*
E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2)) - ((3*I)*Sqrt[2]*B*Sqrt[
E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(
-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]
)*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2
)) + (Cos[c/2 + (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((-2*A*Cos[(d*x)/2]*Sin[c/2]
)/d + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[c/2] - B*Sin[c/2]))/d + (2*B*Cos
[(3*d*x)/2]*Sin[(3*c)/2])/d - (2*A*Cos[c/2]*Sin[(d*x)/2])/d + (Sec[c/2]*Se
c[c/2 + (d*x)/2]^2*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (2*B*Cos[(3*c)/2
]*Sin[(3*d*x)/2])/d))/(a*(1 + Cos[c + d*x]))^(3/2)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)-2a(A-3B)\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)-2a(A-3B)\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)-2a(A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+a}{2d(a\cos(c+dx)+a)^{3/2}}\right.$$

$\big\downarrow$ 3462

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{a^2(A-3B)-2a^2(2A-3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+d}{2d(a\cos(c+c}\right.$$

$\big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int \frac{a^2(A-3B)-2a^2(2A-3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c+d}{2d(a\cos(c+}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int \frac{a^2(A-3B)-2a^2(2A-3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{(A-B)\sin(c}{2d(a\cos(c}\right.$$

$\big\downarrow$ 3461

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-2a(2A-3B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2a(A-3B)\sin(c+dx)}{d\sqrt{a\cos(c+dx}}}{4a^2}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a(2A-3B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2a(A-3B}{d}}{4a^2}\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a(2A-3B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right.}{d}}{a}}{4a^2}\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{a^2(5A-9B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}(2A-3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a(A-3}{}}{4a^2}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{-\frac{2a^3(5A-9B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^{3/2}(2A-3B)\arcsin\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{a}\,\mathrm{co}}\right.}{d}}{a}}{4a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\frac{\sqrt{2}a^{3/2}(5A-9B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}(2A-3B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a(}{}}{4a^2}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (-(((-4*a^(3/2)*(2*A - 3*B)*ArcS
in[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (Sqrt[2]*a^(3/2)*
(5*A - 9*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/d)/a) - (2*a*(A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 18.25 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\,A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)(-4-4\sec(dx+c))+\sqrt{2}\,B\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)(6}{}$
parts	$\frac{A\sqrt{\sec(dx+c)}\,(\cos(dx+c)+1)^2\left(\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)^2\left(\csc(dx+c)^2(1-\cos(dx+c))^2-1\right)\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(-4\sqrt{2}\right.}{32d\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)^2+2*cos(d*x+c)+1)/sec(
d*x+c)^(3/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2^(1/2)*A*arctan(tan(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-4-4*sec(d*x+c))+2^(1/2)*B*arctan(ta
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(6+6*sec(d*x+c))+tan(d*x+c)*A*
2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(2*sin(d*x+c)+3*tan(d*x+c))*B+A*arcsin(cot(d*x+c)-csc(d*x+c))*(-
5-5*sec(d*x+c))+B*arcsin(cot(d*x+c)-csc(d*x+c))*(9+9*sec(d*x+c)))/a^2
```

Fricas [A] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \frac{\sqrt{2}\left((5\,A - 9\,B)\cos\left(dx + c\right)^2 + 2\,(5\,A - 9\,B)\cos\left(dx + c\right) + 5\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*
A - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*
cos(d*x + c) + 2*A - 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(2*B*cos(d*x + c)^2 - (A - 3*B)*cos
(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*
d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{A+B\cos(c+dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+a\cos(c+dx))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^2\sec(dx+c)^2+2\cos(dx+c)\sec(dx+c)^2+\sec(dx+c)^2}dx\right)b + \right.}{a^2}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**2*sec(c + d*x)**2 + 2*cos(c + d*x)*sec(c + d*x)**2 + sec(c + d
*x)**2),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*
x)**2*sec(c + d*x)**2 + 2*cos(c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2),
x)*a))/a**2
```

3.536 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 317

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(283A-163B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(157A-85B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{80a^2d\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/32*(283*A-163*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)
/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)
/d+1/240*(2671*A-1495*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c)
)^(1/2)-1/240*(787*A-475*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x
+c))^(1/2)-1/4*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-
1/16*(21*A-13*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/
80*(157*A-85*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.44 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-240i(283A-163B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{}\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(
5/2),x]
```

output
```
(Cos[(c + d*x)/2]^5*(((-240*I)*(283*A - 163*B)*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c +
d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (15
053*A - 7685*B + 10*(2605*A - 1381*B)*Cos[c + d*x] + 108*(157*A - 85*B)*Co
s[2*(c + d*x)] + 9110*A*Cos[3*(c + d*x)] - 5030*B*Cos[3*(c + d*x)] + 2671*
A*Cos[4*(c + d*x)] - 1495*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^3*Sec[c + d
*x]^(5/2)*Tan[(c + d*x)/2]))/(960*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.09, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$\Big\downarrow$ 3440

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$\Big\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(13A-5B)-8a(A-B)\cos(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(13A-5B)-8a(A-B)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(13A-5B)-8a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(157A-85B)-6a^2(21A-13B)\cos(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(21A-13B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-}{4d\cos^{\frac{5}{2}}(c+dx}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(157A-85B)-6a^2(21A-13B)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(21A-13B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-}{4d\cos^{\frac{5}{2}}(c+dx}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(157A-85B)-6a^2(21A-13B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(21A-13B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A}{4d\cos^{\frac{5}{2}}(c+}\right.$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int -\frac{a^3(787A-475B)-4a^3(157A-85B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(21A-13B)\sin}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(}}{8a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^3(787A-475B)-4a^3(157A-85B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}}{4a^2}-\frac{a(21A-13B)\sin(c-}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+}}{8a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(787A-475B)-4a^3(157A-85B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}}{4a^2}-\frac{a(21A-13B)\mathrm{si}}{2d\cos^{\frac{5}{2}}(c+dx)(a\cos}}{8a^2}\right.$$

$\Big\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{a^4(2671A-1495B)-2a^4(787A-475B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a^3(787A-475B)\sin(}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c}}}{5a}}{4a^2}}{8a^2}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(787A-475B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^4(2671A-1495B)-2a^4(787A-475B)\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}}{3a}}{5a}}{4a^2}}{8a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(787A-475B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^4(2671A-1495B)-2a^4(787A-475B)\sin\left(c+d\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}}{3a}}{5a}}{4a^2}}{8a^2}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(787A-475B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{15a^5(283A-163B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^4(267\ldots}{d\sqrt{\cos(\ldots}}}{3a}}{5a}}{4a^2}}{8a^2}\ldots\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(787A-475B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(2671A-1495B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^4(283A-163\ldots}{3a}}{5a}}{4a^2}}{8a^2}\ldots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(787A-475B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(2671A-1495B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^4(283A-163\ldots}{3a}}{5a}}{4a^2}}{8a^2}\ldots\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(787A-475B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{30a^5(283A-163B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2\ldots}}{d}\ldots}{5a}}{4a^2}}{8a\ldots}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(157A-85B)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(787A-475B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(2671A-1495B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{15\sqrt{2}a^{7/2}(283A\ldots}{\ldots}}{\ldots}}{5a}}{4a^2}}{8a^2}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*((A - B)*Sin[c + d*x])/(d*Cos[
c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)) + (-1/2*(a*(21*A - 13*B)*Sin[c
+ d*x])/(d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)) + ((2*a^2*(157*A
 - 85*B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) -
 ((2*a^3*(787*A - 475*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) - ((-15*Sqrt[2]*a^(7/2)*(283*A - 163*B)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*
a^4*(2671*A - 1495*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]))/(3*a))/(5*a))/(4*a^2))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.80 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.03

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{7}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(4245\cos(dx+c)^6+12735\cos(dx+c)^5+12735\right.\right.}{}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{7}{2}}\left(\cos(dx+c)\sin(dx+c)\left(2671\cos(dx+c)^4+4555\cos(dx+c)^3+1568\cos(dx+c)^2-160\cos(dx+c)\right.\right.}{480d(\cos(dx+c)+1)\left(\cos\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/480/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)+1)/(
cos(d*x+c)^2+2*cos(d*x+c)+1)*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(4245*cos(d*x+c)^6+12735*cos(d*x+c)^5+12735*cos(d*x+
c)^4+4245*cos(d*x+c)^3)+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(-2445*cos(d*x+c)^6-7335*cos(d*x+c)^5-7335*cos(d*x+c)^4-24
45*cos(d*x+c)^3)+cos(d*x+c)*sin(d*x+c)*(2671*cos(d*x+c)^4+4555*cos(d*x+c)^
3+1568*cos(d*x+c)^2-160*cos(d*x+c)+96)*2^(1/2)*A+cos(d*x+c)^2*sin(d*x+c)*(
-1495*cos(d*x+c)^3-2515*cos(d*x+c)^2-800*cos(d*x+c)+160)*2^(1/2)*B)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{15\sqrt{2}\big((283\,A-163\,B)\cos\left(dx+c\right)^5+3\,(283\,A-163\,B)\cos\left(d\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^5 + 3*(283*A - 163*B)*cos(
d*x + c)^4 + 3*(283*A - 163*B)*cos(d*x + c)^3 + (283*A - 163*B)*cos(d*x +
c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
sqrt(a)*sin(d*x + c))) + 2*((2671*A - 1495*B)*cos(d*x + c)^4 + 5*(911*A -
503*B)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 - 160*(A - B)*cos(
d*x + c) + 96*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
 a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(5/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^3}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(}}{\cos(dx}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**3)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)
*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**3)/(cos(
c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a))/a**3
```

3.537 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 270

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(163A-75B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{16\sqrt{2}a^{5/2}d}$$
$$-\frac{(299A-147B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}$$
$$-\frac{(17A-9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{(95A-39B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/32*(163*A-75*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d
-1/48*(299*A-147*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/
2)-1/4*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(17
*A-9*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/48*(95*A-
39*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.01 (sec) , antiderivative size = 1152, normalized size of antiderivative = 4.27

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(
5/2),x]
```

output
```
(2*B*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((1 - 2*Si
n[c/2 + (d*x)/2]^2)^(-1))^(3/2)*((8*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{
2, 2, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (Csc[c/
2 + (d*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-343 + 1465*Sin[c/2 + (d*x)/
2]^2 - 2021*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d*x)/2]^6) + Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145 + 33980*Sin[c/2 + (d*
x)/2]^2 - 87764*Sin[c/2 + (d*x)/2]^4 + 109737*Sin[c/2 + (d*x)/2]^6 - 66122
*Sin[c/2 + (d*x)/2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))/120))/(d*(a*(1 + Co
s[c + d*x]))^(5/2)) - (A*Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c
+ d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(7/2)*(640*Cos[(c + d*x)/2
]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 - 1280*Cos[(c +
d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2
+ (d*x)/2]^2) + 33*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-10935 + 72902*Sin[c...
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A-3B)-6a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A-3B)-6a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A-3B)-6a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(95A-39B)-4a^2(17A-9B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B}{4d\cos^{\frac{3}{2}}(c+dx)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(95A-39B)-4a^2(17A-9B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B}{4d\cos^{\frac{3}{2}}(c+dx)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(95A-39B)-4a^2(17A-9B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(17A-9B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-}{4d\cos^{\frac{3}{2}}(c+}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int -\frac{a^3(299A-147B)-2a^3(95A-39B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(17A-9B)\sin(}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c}}{8a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(299A-147B)-2a^3(95A-39B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{a(17A-9B)\sin(c+}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+d}}{8a^2}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^3(299A-147B)-2a^3(95A-39B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{a(17A-9B)\sin(}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(}}{8a^2}\right.$$

$$\Big\downarrow \text{ 3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{3a^4(163A-75B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{}{2d\cos}}{8a^2}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^3(163A-75B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}}{8a^2}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^3(163A-75B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{3a}}{4a^2}}{8a^2}\right.$$

$\Big\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^4(163A-75B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right.}{d}}{3a}}{4a^2}}{8a^2}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(95A-39B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^3(299A-147B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}a^{5/2}(163A-75B)\arctan\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.}{d}}{3a}}{4a^2}}{8a^2}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*((A - B)*Sin[c + d*x])/(d*Cos[
c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) + (-1/2*(a*(17*A - 9*B)*Sin[c +
 d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((2*a^2*(95*A -
 39*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (
(-3*Sqrt[2]*a^(5/2)*(163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^3*(299*A - 147*B)*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(4*a
^2))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.65 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(489\cos(dx+c)^{5}+1467\cos(dx+c)^{4}+1467\,\mathrm{c}\right.\right.}{}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(\cos(dx+c)\sin(dx+c)\left(299\cos(dx+c)^{3}+503\cos(dx+c)^{2}+160\cos(dx+c)-32\right)\sqrt{2}+\sqrt{\frac{\cos(}{\cos(d}}\right.}{96d(\cos(dx+c)+1)\left(\cos(dx+c)^{2}+\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/96/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)/(
cos(d*x+c)^2+2*cos(d*x+c)+1)*(A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(489*cos(d*x+c)^5+1467*cos(d*x+c)^4+1467*cos(d*x+c)^
3+489*cos(d*x+c)^2)+B*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(-225*cos(d*x+c)^5-675*cos(d*x+c)^4-675*cos(d*x+c)^3-225*cos(d
*x+c)^2)+cos(d*x+c)*sin(d*x+c)*(299*cos(d*x+c)^3+503*cos(d*x+c)^2+160*cos(
d*x+c)-32)*2^(1/2)*A+cos(d*x+c)^2*sin(d*x+c)*(-147*cos(d*x+c)^2-255*cos(d*
x+c)-96)*2^(1/2)*B)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((163A-75B)\cos(dx+c)^4+3(163A-75B)\cos(dx+c)^3+3(163A-75B)\cos(dx+c)^2+(\cdots\right.}{96\left(a^3d\cos(dx\cdots\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^4 + 3*(163*A - 75*B)*cos(d*x
 + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + (163*A - 75*B)*cos(d*x + c))*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) + 2*((299*A - 147*B)*cos(d*x + c)^3 + (503*A - 255*B)*cos(
d*x + c)^2 + 32*(5*A - 3*B)*cos(d*x + c) - 32*A)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(5/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)^2}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^2}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)a\right)}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**2)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)
*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x)**2)/(cos(
c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a))/a**3
```

3.538 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 223

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(75A-19B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(49A-9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/32*(75*A-19*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d
-1/4*(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(13*A
-5*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/16*(49*A-9*
B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.40 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.28

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(
5/2),x]
```

output
```
-1/4*(B*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*Sqrt[(1
 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*(11 - 31*Sin[c/2 + (d*x)/2]^2 + 18*Sin[c/
2 + (d*x)/2]^4 - (19*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 +
(d*x)/2]^2))]]*Cos[(c + d*x)/2]^4)/Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[
c/2 + (d*x)/2]^2))]))/(d*(a*(1 + Cos[c + d*x]))^(5/2)) + (2*A*Cos[c/2 + (d
*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c/2 + (d*x)/2]^
2)^(-1))^(3/2)*((8*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 5/2}, {1
, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 +
(d*x)/2]^2)/(315*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (Csc[c/2 + (d*x)/2]^8*(1
 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Cos[(c + d*x)/2]^4*(-343 + 1465*Sin[c/2 + (d*x)/2]^2 - 2021*Sin[c
/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145 + 33980*Sin[c/2 + (d*x)/2]^2 - 87764*S
in[c/2 + (d*x)/2]^4 + 109737*Sin[c/2 + (d*x)/2]^6 - 66122*Sin[c/2 + (d*x)/
2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))/120))/(d*(a*(1 + Cos[c + d*x]))^(5/2
))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(9A-B)-4a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(9A-B)-4a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(9A-B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \textbf{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(49A-9B)-2a^2(13A-5B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B}{4d\sqrt{\cos(c+dx)}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(49A-9B)-2a^2(13A-5B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B}{4d\sqrt{\cos(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(49A-9B)-2a^2(13A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-}{4d\sqrt{\cos(c+}}\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int -\frac{a^3(75A-19B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a(13A-5B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a^2(75A-19B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a(13A-5B)\sin(c+}{2d\sqrt{\cos(c+dx)}(a\cos(c+d}}{8a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-a^2(75A-19B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(13A-5B)}{2d\sqrt{\cos(c+dx)}(a\,c}}{8a^2}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^3(75A-19B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}}{8a^2}\right.$$

$$\Big\downarrow \text{ 218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2a^2(49A-9B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}a^{3/2}(75A-19B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{a(13A-5}{2d\sqrt{\cos(c+dx)}(}}{8a^2}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*((A - B)*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) + (-1/2*(a*(13*A - 5*B)*Sin[c +
 d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + (-((Sqrt[2]*a^(
3/2)*(75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/d) + (2*a^2*(49*A - 9*B)*Sin[c + d*x])/(d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.17

method	result
default	$\frac{\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\cos(dx+c)\left(\left(75\cos(dx+c)^{3}+225\cos(dx+c)^{2}+225\cos(dx+c)+75\right)A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.}{}$
parts	$\frac{A\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\cos(dx+c)\left(\sin(dx+c)\left(49\cos(dx+c)^{2}+85\cos(dx+c)+32\right)\sqrt{2}+\left(75\cos(dx+c)^{3}+225\cos(dx+c)^{2}+225\cos(dx+\right.\right.}{16d(\cos(dx+c)+1)\left(\cos(dx+c)^{2}+2\cos(dx+c)+1\right)a^{3}}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/16/d*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*cos(d*x+c)*((75*cos(d*x+c)^3+225*cos
(d*x+c)^2+225*cos(d*x+c)+75)*A*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+(-19*cos(d*x+c)^3-57*cos(d*x+c)^2-57*cos(d*x+c)-19)*B
*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c
)*(49*cos(d*x+c)^2+85*cos(d*x+c)+32)*2^(1/2)*A+sin(d*x+c)*cos(d*x+c)*(-9*c
os(d*x+c)-13)*2^(1/2)*B)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(cos(d*x+c)^2+2*c
os(d*x+c)+1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left((75\,A-19\,B)\cos\left(dx+c\right)^{3}+3\,(75\,A-19\,B)\cos\left(dx+c\right)^{2}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)
^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*arctan(sqrt(2)*sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((49
*A - 9*B)*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a+a\cos(c+dx))^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(5/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\sqrt{\cos(dx+c)+1}\cos(dx+c)\sec(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(}{\cos(dx+}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b
+ int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c + d*x))/(cos(c + d*
x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*a))/a**3
```

3.539 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 176

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(19A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\ldots}}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output

```
1/32*(19*A+5*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d-1
/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/16*(9*A-B)
*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(4(19A+5B)\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\ldots\right.}\right.\right.}{\ldots}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(
5/2),x]
```

output
```
(Sec[(c + d*x)/2]^2*(4*(19*A + 5*B)*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d
*x)/2]^2)]]*Cos[(c + d*x)/2]^4 + Cos[c + d*x]*(-13*A + 5*B + (-9*A + B)*Co
s[c + d*x])*Sqrt[2 - 2*Sec[c + d*x]])*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])
/(32*Sqrt[2]*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(7A+B)-2a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(7A+B)-2a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(7A+B)-2a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(19A+5B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos}}{4d(a\cos(c+dx)+a}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}(19A+5B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)}{4d(}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}(19A+5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A}{}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{a(19A+5B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos}}{2d(a\cos(c+dx)+a)^{3}}}{8a^2}\right.$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(19A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{a(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\mathrm{si}}{4d(a\,\mathrm{c}}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*((A - B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) + (((19*A + 5*B)*ArcTan[(Sqrt[a
]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2
*Sqrt[2]*Sqrt[a]*d) - (a*(9*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(
a + a*Cos[c + d*x])^(3/2)))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 14.45 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\left(\sin(dx+c)(9\cos(dx+c)+13)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)(-\cos(dx+c)-5)\sqrt{2}\,B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(19\cos(dx+c)^2+38\cos(dx\right.\right.}{32d\left(\cos(dx+c)^3+\right.}$
parts	$-\frac{A\left(\sin(dx+c)(9\cos(dx+c)+13)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(19\cos(dx+c)^2+38\cos(dx+c)+19\right)\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{2}\,\sqrt{a(}}{32d\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^3}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/32/d*(sin(d*x+c)*(9*cos(d*x+c)+13)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)+sin(d*x+c)*(-cos(d*x+c)-5)*2^(1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+(19*cos(d*x+c)^2+38*cos(d*x+c)+19)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(5
*cos(d*x+c)^2+10*cos(d*x+c)+5)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a
*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sec(d*x+c)^(1/2)/(cos(d*x+c)^3+3*cos(d*x
+c)^2+3*cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right.}{32\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx-\right.\right.}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^
2 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((9*A -
 B)*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}}{(a+a\cos(c+dx))^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(5/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\sqrt{\cos(dx+c)+1}\cos(dx+c)}{\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1}dx\right)b+\left(\int \frac{\sqrt{\sec(\ldots}}{\cos(dx+c)^3+3\ldots}\right.\right.}{a^3}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1),x)*b + int((sqrt(s
ec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2
 + 3*cos(c + d*x) + 1),x)*a))/a**3
```

3.540 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 174

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \frac{(5A+3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{16\sqrt{2}a^{5/2}d}$$
$$+\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}+\frac{(A+7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/32*(5*A+3*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d+1/
4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/16*(A+7*B)*
sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 474 vs. $2(174) = 348$.

Time = 6.48 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.72

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \frac{A\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-\sin^2\left(\frac{c}{2}+\right.\right.}{4d(a(1-}$$

$$+\frac{B\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(3\arcsin\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\frac{5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sec^2\left(\frac{1}{2}(c+\right.}}{\sqrt{\cos^2\left(\frac{1}{2}(c+\right.}}\right.}{4d(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*
x]]),x]
```

output
```
(A*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*(3 - Sin[c/2
 + (d*x)/2]^2 - 5*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*
x)/2]^2))]]*Cos[(c + d*x)/2]^4*Csc[c/2 + (d*x)/2]^2*Sqrt[-(Sin[c/2 + (d*x)
/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)*S
qrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]) + (B*Cos[c/2 + (d*x)/2]^5*Sqrt[(1
- 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(3*ArcSin
[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]] + (5*Sin[c/2 + (d*x)/2]*Sqrt
[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/Sqrt[Cos[(c + d*x)/2]^2] -
(2*Sin[c/2 + (d*x)/2]^3*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])
/(Cos[(c + d*x)/2]^2)^(3/2)))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}\,dx$$

$\downarrow$ 3042

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$\downarrow$ 3440

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$\downarrow$ 3456

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+2a(A+3B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+2a(A+3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+2a(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(5A+3B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}(5A+3B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\,\mathrm{s}}{4d(a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}(5A+3B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A}{}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{a(5A+3B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a}}\right.}{2d}}{8a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(5A+3B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}+\frac{a(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin}{4d(a\cos}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (((5*A + 3*B)*ArcTan[(Sqrt[a]*Si
n[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqr
t[2]*Sqrt[a]*d) + (a*(A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a +
a*Cos[c + d*x])^(3/2)))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 10.82 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.37

method	result
default	$-\frac{\left(\sin(dx+c)(-\cos(dx+c)-5)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)(-7\cos(dx+c)-3)\sqrt{2}\,B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(5\cos(dx+c)^2+10\cos(dx\right.\right.}{32d\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos\right.}$
parts	$\frac{A\left(\sqrt{2}\,\sin(dx+c)(\cos(dx+c)+5)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(-5\cos(dx+c)^2-10\cos(dx+c)-5\right)\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{2}\,\sqrt{a(\cos(d}}{32d\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^3}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/32/d*(sin(d*x+c)*(-cos(d*x+c)-5)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+sin(d*x+c)*(-7*cos(d*x+c)-3)*2^(1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+(5*cos(d*x+c)^2+10*cos(d*x+c)+5)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(3*c
os(d*x+c)^2+6*cos(d*x+c)+3)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(c
os(d*x+c)+1))^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/sec(d*x+c
)^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{5/2}\sqrt{\sec(c + dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left((5\,A + 3\,B)\cos\left(dx + c\right)^{3} + 3\,(5\,A + 3\,B)\cos\left(dx + c\right)^{2} + 3\,(5\,A + 3\,B)\cos\left(dx + c\right) + 5\,A + 3\,B\right)\sqrt{a}}{32\,\left(a^{3}d\cos\left(dx + c\right)^{3} + 3\,a^{3}d\cos\left(dx + c\right.\right.}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2
+ 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((A + 7*B)*
cos(d*x + c)^2 + (5*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{5/2}\sqrt{\sec(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^3\sec(dx+c)+3\cos(dx+c)^2\sec(dx+c)+3\cos(dx+c)\sec(dx+c)}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**3*sec(c + d*x) + 3*cos(c + d*x)**2*sec(c + d*x) + 3*cos(c + d*
x)*sec(c + d*x) + sec(c + d*x)),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c
+ d*x) + 1))/(cos(c + d*x)**3*sec(c + d*x) + 3*cos(c + d*x)**2*sec(c + d*x
) + 3*cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*a))/a**3
```

3.541 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 234

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}+\frac{(3A-43B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}+\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}+\frac{(3A-11B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(5/2)/d+1/32*(3*A-43*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1
/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)*2^(1/2)/a^(5/2)/d+1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d
*x+c)^(3/2)+1/16*(3*A-11*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+
c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.44 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.13

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(3\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3
/2)),x]
```

output
```
(Cos[(c + d*x)/2]^5*(((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(32*B*ArcSinh[E^(I*(c + d*x))] - Sq
rt[2]*(3*A - 43*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I
)*(c + d*x))])] - 32*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(
c + d*x)) + ((3*A - 11*B + (7*A - 15*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*S
qrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/2))/(8*d*(a*
(1 + Cos[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 3440$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+8aB\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+8aB\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)+8aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(3A-11B)a^2+32B\cos(c+dx)a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\,\text{c}}{4d(a\cos(c+dx)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(3A-11B)a^2+32B\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+dx)\,\text{c}}{4d(a\cos(c+dx)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(3A-11B)a^2+32B\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(c+d}{4d(a\cos(c+}\right.$$

$\downarrow$ 3461

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+32aB\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{a(3A-11B)\sin(c+dx)\sqrt{\cos(}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+32aB\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{a(3A-11B)\sin(c}{2d(a\cos(c+}}{8a^2}\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64aB\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+}{8a^2}\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{a(3A-11B)\sin}{2d(a\cos}}{8a^2}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{64a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^3(3A-43B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+\cdots}}\right.}{d}}{4a^2}}{8a^2}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\sqrt{2}a^{3/2}(3A-43B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{64a^{3/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{a(3A-11B)\cdots}{2d(a\cdots}}{8a^2}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (((64*a^(3/2)*B*ArcSin[(Sqrt[a]*
Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (Sqrt[2]*a^(3/2)*(3*A - 43*B)
*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])])/d)/(4*a^2) + (a*(3*A - 11*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 10.82 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(-\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(7\sin(dx+c)+3\tan(dx+c))A+\sqrt{2}\,B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(15\sin(dx+c)+11\tan(dx+c))-\right.}{}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(7\sin(dx+c)+3\tan(dx+c))+\arcsin(\cot(dx+c)-\csc(dx+c))(-3\cos(dx+c)-6-3\sec(\right.}{32d\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sec(dx+c)^{\frac{3}{2}}a^3}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/32/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*co
s(d*x+c)+1)/sec(d*x+c)^(3/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-2^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(7*sin(d*x+c)+3*tan(d*x+c))*A+2^(1/2)*B*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(15*sin(d*x+c)+11*tan(d*x+c))-2^(1/2)*(32
*cos(d*x+c)+64+32*sec(d*x+c))*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*B+A*arcsin(cot(d*x+c)-csc(d*x+c))*(3*cos(d*x+c)+6+3*sec(d*x+c))+B
*arcsin(cot(d*x+c)-csc(d*x+c))*(-43*cos(d*x+c)-86-43*sec(d*x+c)))/a^3
```

Fricas [A] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.18

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{\sqrt{2}\left((3\,A-43\,B)\cos\left(dx+c\right)^{3}+3\,(3\,A-43\,B)\cos\left(dx+c\right)^{2}+3\,(3\,A-43\,B)\cos\left(dx+c\right)+3\,A-43\,B\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^
2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 64*(B*cos
(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((7*
A - 15*B)*cos(d*x + c)^2 + (3*A - 11*B)*cos(d*x + c))*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{5/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{B\cos(dx + c) + A}{(a\cos(dx + c) + a)^{\frac{5}{2}}\sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{5/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + a\cos(c + dx))^{5/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{A + B\,\cos(c + d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a + a\,\cos(c + d\,x))^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^3\sec(dx+c)^2+3\cos(dx+c)^2\sec(dx+c)^2+3\cos(dx+c)\sec(dx+c}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**3*sec(c + d*x)**2 + 3*cos(c + d*x)**2*sec(c + d*x)**2 + 3*cos(
c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2),x)*b + int((sqrt(sec(c + d*x))
*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)**3*sec(c + d*x)**2 + 3*cos(c + d*x)
**2*sec(c + d*x)**2 + 3*cos(c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2),x)
*a))/a**3
```

3.542 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 286

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(2A-5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}$$
$$-\frac{(43A-115B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}$$
$$+\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}+\frac{(7A-15B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}$$
$$-\frac{(11A-35B)\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
(2*A-5*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^(5/2)/d-1/32*(43*A-115*B)*arctan(1/2*a^(1/2)*sin(d*x
+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)*2^(1/2)/a^(5/2)/d+1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/
2)/sec(d*x+c)^(5/2)+1/16*(7*A-15*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/
sec(d*x+c)^(3/2)-1/16*(11*A-35*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)/
sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.58 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.25

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5
/2)),x]
```

output
```
(((-11*I)/4)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I
)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c
 + d*x]))^(5/2)) + (((35*I)/4)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d
*x))*(a*(1 + Cos[c + d*x]))^(5/2)) + ((4*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))
/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(
c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((
2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*
x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2)) - ((10*I)*Sq
rt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d
*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c
+ d*x]))^(5/2)) + (Cos[c/2 + (d*x)/2]^5*Sqrt[Sec[c + d*x]]*((15*(-A + B)*C
os[(d*x)/2]*Sin[c/2])/(2*d) + (4*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d - (15*(A
 - B)*Cos[c/2]*Sin[(d*x)/2])/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(19*A*
Sin[(d*x)/2] - 27*B*Sin[(d*x)/2]))/(4*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*
(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(2*d) + (4*B*Cos[(3*c)/2]*Sin[(3*...
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-2a(A-5B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)-2a(A-5B)\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a(A-B)-2a(A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+}{4d(a\cos(c+dx)+a)^{5/2}}\right.$$

$\big\downarrow$ 3456

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2(7A-15B)-2a^2(11A-35B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-}{}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2(7A-15B)-2a^2(11A-35B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{(A-}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a^2(7A-15B)-2a^2(11A-35B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(7A-15B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}\right.$$

$\big\downarrow$ 3462

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int -\frac{a^3(11A-35B)-16a^3(2A-5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B)\sin(c+dx}{2d(a\cos(c+dx)}}{8a^2}\right.$$

$\big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{\int\frac{a^3(11A-35B)-16a^3(2A-5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B)\sin(c+dx}{2d(a\cos(c+dx)}}{8a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{\int\frac{a^3(11A-35B)-16a^3(2A-5B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{a(7A-15B)\sin(c}{2d(a\cos(c+}}{8a^2}\right.$$

$\downarrow$ 3461

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{a^3(43A-115B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-16a^2(2A-5B)\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2a^2(11A-35B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)-}}}{4a^2}}{8a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{a^3(43A-115B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2(2A-5B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2a^2(11A-35}{d\sqrt{}}}{4a^2}}{8a^2}\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{-\frac{a^3(43A-115B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2(2A-5B)\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx}{\sqrt{\cos(c+dx)a}}\right.}{d}}{a}}{4a^2}}{8a^2}\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\frac{a^3(43A-115B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}(2A-5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a^2(11A}{}}{4a^2}}{8a^2}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{-\frac{2a^4(43A-115B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{32a^{5/2}(2A-5B)\arcsin\left(\frac{}{\sqrt{}}\right.}{d}}{a}}{4a^2}}{8a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{-\frac{\sqrt{2}a^{5/2}(43A-115B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}(2A-5B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a^2}{}}{4a^2}}{8a^2}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((a*(7*A - 15*B)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (-(((-32*a^(5/2)*(2*A
 - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (Sqrt
[2]*a^(5/2)*(43*A - 115*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) - (2*a^2*(11*A - 35*B)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2))/(8*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 18.58 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.24

method	result
default	$\frac{\left(\sin(dx+c)(-15\cos(dx+c)-11)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)(43+8\cos(2dx+2c)+55\cos(dx+c))\sqrt{2}\,B\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+(48+16\,\text{c}\cdots}{\cdots}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-32-64\sec(dx+c)-32\sec(dx+c)^2\right)+\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(15\,\text{t}\cdots}{32d\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\cdots}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/32/d/sec(d*x+c)^(1/2)*(sin(d*x+c)*(-15*cos(d*x+c)-11)*2^(1/2)*A*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(43+8*cos(2*d*x+2*c)+55*cos(d*x+c))*2
^(1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(48+16*cos(2*d*x+2*c)+64*cos(d*
x+c))*2^(1/2)*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-40*(3
+cos(2*d*x+2*c)+4*cos(d*x+c))*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*B+(43*cos(d*x+c)^2+86*cos(d*x+c)+43)*A*arcsin(cot(d*x+c)-
csc(d*x+c))+(-115*cos(d*x+c)^2-230*cos(d*x+c)-115)*B*arcsin(cot(d*x+c)-csc
(d*x+c)))*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)^3+3*cos(d*x+c)^2+3*
cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 7.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\left((43\,A-115\,B)\cos\left(dx+c\right)^{3}+3\,(43\,A-115\,B)\cos\left(dx+\right.\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x +
c)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 32
*((2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B
)*cos(d*x + c) + 2*A - 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(16*B*cos(d*x + c)^3 - 5*(3*A - 1
1*B)*cos(d*x + c)^2 - (11*A - 35*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^3\sec(dx+c)^3+3\cos(dx+c)^2\sec(dx+c)^3+3\cos(dx+c)\sec(dx+c}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**3*sec(c + d*x)**3 + 3*cos(c + d*x)**2*sec(c + d*x)**3 + 3*cos(
c + d*x)*sec(c + d*x)**3 + sec(c + d*x)**3),x)*b + int((sqrt(sec(c + d*x))
*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)**3*sec(c + d*x)**3 + 3*cos(c + d*x)
**2*sec(c + d*x)**3 + 3*cos(c + d*x)*sec(c + d*x)**3 + sec(c + d*x)**3),x)
*a))/a**3
```

3.543 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 317

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(1015A-363B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+d}}{64\sqrt{2}a^{7/2}d}$$
$$-\frac{(1887A-691B)\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}}-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}$$
$$-\frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}}-\frac{(109A-41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}}$$
$$+\frac{(579A-199B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}}$$

output
```
1/128*(1015*A-363*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2
)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(7/2
)/d-1/192*(1887*A-691*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c)
)^(1/2)-1/6*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-1/4
8*(23*A-11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-1/64*
(109*A-41*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)+1/19
2*(579*A-199*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.33 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{i(1015A-363B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\mathrm{arc}}{d}\right.}{}$$

input

```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(
7/2),x]
```

output

```
(Cos[(c + d*x)/2]^7*((I*(1015*A - 363*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))
/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(d*E^((I/2)*(c + d*x))) - ((216
41*A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos
[2*(c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*
A*Cos[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Sec[c + d*
x]^(3/2)*Tan[(c + d*x)/2])/(96*d)))/(8*(a*(1 + Cos[c + d*x]))^(7/2))
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.09, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \ 3440$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \ 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a(5A-B)-8a(A-B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a(5A-B)-8a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a(5A-B)-8a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3\left(a^2(63A-19B)-2a^2(23A-11B)\cos(c+dx)\right)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A}{6d\cos^{\frac{3}{2}}(c+}\right.$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int \frac{a^2(63A-19B)-2a^2(23A-11B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-}{6d\cos^{\frac{3}{2}}(c+d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int \frac{a^2(63A-19B)-2a^2(23A-11B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{a(23A-11B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A}{6d\cos^{\frac{3}{2}}(c-}\right.$$

$\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\int \frac{a^3(579A-199B)-4a^3(109A-41B)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11}{4d\cos^{\frac{3}{2}}(c+dx)(a}}{12a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\int \frac{a^3(579A-199B)-4a^3(109A-41B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-11}{4d\cos^{\frac{3}{2}}(c+dx)(a}}{12a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\int \frac{a^3(579A-199B)-4a^3(109A-41B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a^2(109A-41B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{a(23A-\cdots}{4d\cos^{\frac{3}{2}}(c+d\cdots}}{12a^2}\cdots\right.$$

$\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2\int -\frac{a^4(1887A-691B)-2a^4(579A-199B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a^2(109A-41B)\cdots}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos\cdots}\right.}{8a^2}\cdots}{12a^2}\cdots\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a^4(1887A-691B)-2a^4(579A-199B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{a^2(109A-41B)\sin(\cdots}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+\cdots}\right.}{8a^2}\cdots}{12a^2}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{a^4(1887A-691B)-2a^4(579A-199B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{a^2(109A-41B\cdots}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos\cdots}\right.\cdots}{8a^2}\cdots}{12a^2}\cdots\right.$$

$$\Big\downarrow \; 3463$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{3a^5(1015A-363B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{\cdots}{2d\cos\cdots}\right.\cdots}{8a^2}\cdots}{12a^2}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^4(1015A-363B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a\cdots}}\cdots}{3a}}{4a^2}\cdots\right.}{8a^2}\cdots}{12a^2}\cdots\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a^4(1015A-363B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+\right.}}}{3a}}{4a^2}\right)}{8a^2}}{12a^2}\right.$$

$\big\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^5(1015A-363B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx}}\right.}{d}}{3a}}{4a^2}\right)}{8a^2}}{}\right.$$

$\big\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2a^3(579A-199B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a^4(1887A-691B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}a^{7/2}(1015A-363B)\arctan\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{2}\sqrt{\cos(c+dx}}\right.}{d}}{3a}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(7/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*((A - B)*Sin[c + d*x])/(d*Cos[
c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)) + (-1/4*(a*(23*A - 11*B)*Sin[c
+ d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) + (3*(-1/2*(a^2*
(109*A - 41*B)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3
/2)) + ((2*a^3*(579*A - 199*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Cos[c + d*x]]) - ((-3*Sqrt[2]*a^(7/2)*(1015*A - 363*B)*ArcTan[(Sqrt[
a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d
 + (2*a^4*(1887*A - 691*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]]))/(3*a))/(4*a^2)))/(8*a^2))/(12*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.84 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.12

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(A\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(3045\cos(dx+c)^{6}+12180\cos(dx+c)^{5}+182\right.\right.}{}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\sec(dx+c)^{\frac{5}{2}}\left(\cos(dx+c)\sin(dx+c)\left(1887\cos(dx+c)^{4}+5082\cos(dx+c)^{3}+4251\cos(dx+c)^{2}+896\cos(dx-\right.\right.}{384d(\cos(dx+c)+1}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/384/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)/
(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)*(A*arcsin(cot(d*x+c)-csc(d*x+
c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(3045*cos(d*x+c)^6+12180*cos(d*x+c)^
5+18270*cos(d*x+c)^4+12180*cos(d*x+c)^3+3045*cos(d*x+c)^2)+B*arcsin(cot(d*
x+c)-csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-1089*cos(d*x+c)^6-435
6*cos(d*x+c)^5-6534*cos(d*x+c)^4-4356*cos(d*x+c)^3-1089*cos(d*x+c)^2)+cos(
d*x+c)*sin(d*x+c)*(1887*cos(d*x+c)^4+5082*cos(d*x+c)^3+4251*cos(d*x+c)^2+8
96*cos(d*x+c)-128)*2^(1/2)*A+cos(d*x+c)^2*sin(d*x+c)*(-691*cos(d*x+c)^3-18
74*cos(d*x+c)^2-1599*cos(d*x+c)-384)*2^(1/2)*B)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((1015\,A-363\,B)\cos{(dx+c)}^{5}+4\,(1015\,A-363\,B)\cos{(dx+c)}^{4}+6\,(1015\,A-363\,B)\cos{(dx+\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^5 + 4*(1015*A - 363*B)*co
s(d*x + c)^4 + 6*(1015*A - 363*B)*cos(d*x + c)^3 + 4*(1015*A - 363*B)*cos(
d*x + c)^2 + (1015*A - 363*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1887*A
- 691*B)*cos(d*x + c)^4 + 2*(2541*A - 937*B)*cos(d*x + c)^3 + 39*(109*A -
41*B)*cos(d*x + c)^2 + 128*(7*A - 3*B)*cos(d*x + c) - 128*A)*sqrt(a*cos(d*
x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5 + 4*a^4
*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*
d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a+a\cos(c+dx))^{7/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(7/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(7/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\sqrt{\cos(dx+c)+1}\cos(dx+c)\sec(dx+c)^2}{\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}dx\right)b+\left(\int \right.\right.}{a^4}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x)**2)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*
cos(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*se
c(c + d*x)**2)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 +
4*cos(c + d*x) + 1),x)*a))/a**4
```

3.544 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 270

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{3(121A-21B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{(199A-43B)\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{(691A-103B)\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}}$$

output
```
-3/128*(121*A-21*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)
/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(7/2)
/d-1/6*(A-B)*sec(d*x+c)^(1/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-1/48*(19
*A-7*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-1/192*(199*
A-43*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)+1/192*(69
1*A-103*B)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.02 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.97

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(
7/2),x]
```

output
```
-1/24*(B*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*Sqrt[(
1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*(141 - 518*Sin[c/2 + (d*x)/2]^2 + 575*Si
n[c/2 + (d*x)/2]^4 - 206*Sin[c/2 + (d*x)/2]^6 - (189*ArcTanh[Sqrt[-(Sin[c/
2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*Cos[(c + d*x)/2]^6)/Sqrt[-(
Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]))/(d*(a*(1 + Cos[c + d
*x]))^(7/2)) + (2*A*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x
)/2]*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(3/2)*((16*Cos[(c + d*x)/2]^8*Hyp
ergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)) - (Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTanh[Sqrt[Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(2187
 - 12908*Sin[c/2 + (d*x)/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380*Sin[c/2
 + (d*x)/2]^6 + 8752*Sin[c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2 - 8267
707*Sin[c/2 + (d*x)/2]^4 + 17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*Sin[c/
2 + (d*x)/2]^8 + 16548816*Sin[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d*x)/
2]^12 + 1144608*Sin[c/2 + (d*x)/2]^14)))/1680))/(d*(a*(1 + Cos[c + d*x]))^
(7/2))
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(13A-B)-6a(A-B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(13A-B)-6a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\frac{a(13A-B)-6a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}\right)$$

$\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^2(41A-5B)-4a^2(19A-7B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-}{6d\sqrt{\cos(c+dx}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^2(41A-5B)-4a^2(19A-7B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-}{6d\sqrt{\cos(c+dx}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^2(41A-5B)-4a^2(19A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{a(19A-7B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A}{6d\sqrt{\cos(c+}}\right.$$

$\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int\frac{a^3(691A-103B)-2a^3(199A-43B)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(}{4d\sqrt{\cos(c+dx)}(a\cos(}}{12a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{a^3(691A-103B)-2a^3(199A-43B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)\sin(}{4d\sqrt{\cos(c+dx)}(a\cos(}}{12a^2}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{a^3(691A-103B)-2a^3(199A-43B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{a(19A-7B)}{4d\sqrt{\cos(c+dx)}(a\,c}}{12a^2}\right.$$

$\big\downarrow$ 3463

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{2\int -\frac{9a^4(121A-21B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{}{4d\sqrt{}}}{12a^2}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-9a^3(121A-21B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3}}}{8a^2}}{12a^2}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-9a^3(121A-21B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a^2(199A-43B)\sin}{2d\sqrt{\cos(c+dx)}(a\cos(c-}}{8a^2}}{12a^2}\right.$$

$\Big\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{18a^4(121A-21B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}}{8a^2}}{12a^2}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{2a^3(691A-103B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{9\sqrt{2}a^{5/2}(121A-21B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{a^2(199A-43}{2d\sqrt{\cos(c+dx)}(a}}{8a^2}}{12a^2}\right.$$

input

```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(7/2),x
]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*((A - B)*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)) + (-1/4*(a*(19*A - 7*B)*Sin[c +
 d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) + (-1/2*(a^2*(199
*A - 43*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2))
 + ((-9*Sqrt[2]*a^(5/2)*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (2*a^3*(691*A - 103
*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)
)/(8*a^2))/(12*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [A] (verified)

Time = 14.58 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.15

method	result
default	$\frac{\cos(dx+c)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\left(1089\cos(dx+c)^4+4356\cos(dx+c)^3+6534\cos(dx+c)^2+4356\cos(dx+c)+1089\right)A\arcsin(\cot(dx+c)-\right.}{}$
parts	$\frac{A\cos(dx+c)\sqrt{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\sin(dx+c)\left(691\cos(dx+c)^3+1874\cos(dx+c)^2+1599\cos(dx+c)+384\right)\sqrt{2}+\left(1089\cos(dx+c)^4+435\right.\right.}{192d(\cos(dx+c)+1)\left(\cos(dx+c)^3+3\cos(\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d*cos(d*x+c)*(a*cos(1/2*d*x+1/2*c)^2)^(1/2)*((1089*cos(d*x+c)^4+4356
*cos(d*x+c)^3+6534*cos(d*x+c)^2+4356*cos(d*x+c)+1089)*A*arcsin(cot(d*x+c)-
csc(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-189*cos(d*x+c)^4-756*cos(d
*x+c)^3-1134*cos(d*x+c)^2-756*cos(d*x+c)-189)*B*arcsin(cot(d*x+c)-csc(d*x+
c))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(691*cos(d*x+c)^3+1874*co
s(d*x+c)^2+1599*cos(d*x+c)+384)*2^(1/2)*A+sin(d*x+c)*cos(d*x+c)*(-103*cos(
d*x+c)^2-266*cos(d*x+c)-195)*2^(1/2)*B)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(c
os(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{9\sqrt{2}\big((121\,A-21\,B)\cos{(dx+c)}^4+4\,(121\,A-21\,B)\cos{(dx+}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^4 + 4*(121*A - 21*B)*cos(d*x
 + c)^3 + 6*(121*A - 21*B)*cos(d*x + c)^2 + 4*(121*A - 21*B)*cos(d*x + c)
+ 121*A - 21*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(
937*A - 133*B)*cos(d*x + c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^
4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c)
 + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(7/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(7/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)\sec(dx+c)}{\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}dx\right)b+\left(\int \cdots\right.\right.}{a^4}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x)*sec(
c + d*x))/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos
(c + d*x) + 1),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*sec(c
 + d*x))/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(
c + d*x) + 1),x)*a))/a**4
```

3.545 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 223

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(63A+13B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{64\sqrt{2}a^{7/2}d}$$
$$-\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}-\frac{(5A-B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$
$$-\frac{(103A+5B)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/128*(63*A+13*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(7/2)/d
-1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)-1/16*(5*A-
B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/192*(103*A+5*B
)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.81

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx =$$
$$-\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-48(63A+13B)\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)+\cos(c\right.}{1536\sqrt{2}a^3d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(
7/2),x]
```

output
```
-1/1536*(Sec[(c + d*x)/2]^4*(-48*(63*A + 13*B)*ArcTanh[Sqrt[-(Sec[c + d*x]
*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^6 + Cos[c + d*x]*(493*A - 73*B + (
532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(c + d*x)])*Sqrt[2 - 2*Sec
[c + d*x]])*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/(Sqrt[2]*a^3*d*Sqrt[a*(1
+ Cos[c + d*x])]*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3457, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a\cos(c+dx)+a)^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$
$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{7/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$\Big\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A+B)-4a(A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A+B)-4a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(11A+B)-4a(A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$\Big\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(73A+11B)-6a^2(5A-B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+d}{6d(a\cos(c+}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(73A+11B)-6a^2(5A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c+d}{6d(a\cos(c+}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a^2(73A+11B)-6a^2(5A-B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{(A-B)\sin(c-}{6d(a\cos(c}\right.$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3a^3(63A+13B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3}{4}a(63A+13B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin(c+da}{4d(a\cos(c+dx}}{12a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3}{4}a(63A+13B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)s}{4d(a\,c}}{12a^2}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{3a^2(63A+13B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\sqrt{a}(63A+13B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{a^2(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{3a(5A-B)\sin}{4d(a\cos}}{12a^2}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(7/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*((A - B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(d*(a + a*Cos[c + d*x])^(7/2)) + ((-3*a*(5*A - B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*Sqrt[a]*(63*A
+ 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (a^2*(103*A + 5*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 14.66 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(\sin(dx+c)\left(103\cos(dx+c)^2+266\cos(dx+c)+195\right)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)\left(5\cos(dx+c)^2-2\cos(dx+c)-39\right)\sqrt{2}\,B\sqrt{\frac{c}{co}}\right.}{}$
parts	$-\frac{A\left(\sin(dx+c)\left(103\cos(dx+c)^2+266\cos(dx+c)+195\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(189\cos(dx+c)^3+567\cos(dx+c)^2+567\cos(dx+c)+18\right.\right.}{384d\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/384/d*(sin(d*x+c)*(103*cos(d*x+c)^2+266*cos(d*x+c)+195)*2^(1/2)*A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(5*cos(d*x+c)^2-2*cos(d*x+c)-39)*2
^(1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(189*cos(d*x+c)^3+567*cos(d*x+c
)^2+567*cos(d*x+c)+189)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(39*cos(d*x+c)^3+1
17*cos(d*x+c)^2+117*cos(d*x+c)+39)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2
)*(a*(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sec(d*x+c)^(1/2)/(cos(d*x+c)^4+4*cos
(d*x+c)^3+6*cos(d*x+c)^2+4*cos(d*x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.15

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\big((63\,A+13\,B)\cos{(dx+c)}^4+4\,(63\,A+13\,B)\cos{(dx+c)}^3+6\,(63\,A+13\,B)\cos{(dx+c)}^2+4\,(63}{384\,\big(a^4d\cos{(dx+}}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x
+ c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 6
3*A + 13*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) + 2*((103*A + 5*B)*cos(d*x + c)^3 + 2*(133*A
- B)*cos(d*x + c)^2 + 39*(5*A - B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x +
 c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(7/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(7/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+c)}{\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}dx\right)b+\left(\int\right.\right.}{a^4}$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1
),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)**4
+ 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1),x)*a))/a**4
```

3.546 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \frac{(13A+7B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}}{64\sqrt{2}a^{7/2}d} + \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(5A-17B)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/128*(13*A+7*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a
+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(7/2)/d+
1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+1/16*(A+3*B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/192*(5*A-17*B)*
sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.81

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(48(13A+7B)\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*
x]]),x]
```

output
```
(Sec[(c + d*x)/2]^4*(48*(13*A + 7*B)*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c +
d*x)/2]^2)]]*Cos[(c + d*x)/2]^6 + Cos[c + d*x]*(73*A + 59*B + 4*(A + 35*B)
*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*
Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/(1536*Sqrt[2]*a^3*d*Sqrt[a*(1 + Cos[c
 + d*x])]*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$\big\downarrow$ 3456

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+4a(A+2B)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+4a(A+2B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a(A-B)+4a(A+2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$\big\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(11A+B)a^2+6(A+3B)\cos(c+dx)a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)}{6d(a\cos(c+dx}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(11A+B)a^2+6(A+3B)\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx)}{6d(a\cos(c+dx}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(11A+B)a^2+6(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+}{6d(a\cos(c}\right.$$

$\downarrow$ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3a^3(13A+7B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3}{4}a(13A+7B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(c+dx)}{4d(a\cos(c+dx)+}}{12a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3}{4}a(13A+7B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin}{4d(a\cos}}{12a^2}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{3a^2(13A+7B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c-}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}}{12a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\sqrt{a}(13A+7B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{a^2(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(A+3B)\sin(\dots}{4d(a\cos(\dots}}{12a^2}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*a*(A + 3*B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*Sqrt[a]*(13*A + 7*
B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*d) - (a^2*(5*A - 17*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 10.75 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.30

method	result
default	$-\frac{\left(\sin(dx+c)\left(5\cos(dx+c)^2-2\cos(dx+c)-39\right)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)\left(-17\cos(dx+c)^2-70\cos(dx+c)-21\right)\sqrt{2}\,B\sqrt{\frac{\cos}{\cos(}}\right.}{384d\left(\mathrm{co}\right.}$
parts	$-\frac{A\left(\sin(dx+c)\left(5\cos(dx+c)^2-2\cos(dx+c)-39\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\left(39\cos(dx+c)^3+117\cos(dx+c)^2+117\cos(dx+c)+39\right)\mathrm{arcsi}\right.}{384d\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(d}{\cos(dx}}}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/384/d*(sin(d*x+c)*(5*cos(d*x+c)^2-2*cos(d*x+c)-39)*2^(1/2)*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*(-17*cos(d*x+c)^2-70*cos(d*x+c)-21)*2^(
1/2)*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(39*cos(d*x+c)^3+117*cos(d*x+c)^2
+117*cos(d*x+c)+39)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(21*cos(d*x+c)^3+63*co
s(d*x+c)^2+63*cos(d*x+c)+21)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)*(a*(
cos(d*x+c)+1))^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*cos(d*x+c)^2+4*cos(d*x
+c)+1)/sec(d*x+c)^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.15

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{3\sqrt{2}\big((13\,A+7\,B)\cos{(dx+c)}^4+4\,(13\,A+7\,B)\cos{(dx+c)}^3+6\,(13\,A+7\,B)\cos{(dx+c)}^2+4\,(13\,A+}{384\,\big(a^4d\cos{(dx+c)}^4}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x +
c)^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A
+ 7*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) + 2*((5*A - 17*B)*cos(d*x + c)^3 - 2*(A + 35*B)*co
s(d*x + c)^2 - 3*(13*A + 7*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x +
c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx+}{\cos(dx+c)^4\sec(dx+c)+4\cos(dx+c)^3\sec(dx+c)+6\cos(dx+c)^2\sec(dx+c}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**4*sec(c + d*x) + 4*cos(c + d*x)**3*sec(c + d*x) + 6*cos(c + d*
x)**2*sec(c + d*x) + 4*cos(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*b + in
t((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)**4*sec(c + d*x
) + 4*cos(c + d*x)**3*sec(c + d*x) + 6*cos(c + d*x)**2*sec(c + d*x) + 4*co
s(c + d*x)*sec(c + d*x) + sec(c + d*x)),x)*a))/a**4
```

3.547 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{(7A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{}}{64\sqrt{2}a^{7/2}d}$$
$$+\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}+\frac{(A-13B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$
$$+\frac{(17A+67B)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/128*(7*A+5*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/a^(7/2)/d+1
/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2)+1/48*(A-13*B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/192*(17*A+67*B)
*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 561 vs. $2(221) = 442$.

Time = 6.77 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.54

$$\int \frac{A + B\cos(c+dx)}{(a + a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{A\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\left(\right.}{}$$
$$+\frac{B\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(15\arcsin\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\frac{33\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sec^2\left(\frac{1}{2}(c\right.}}{\sqrt{\cos^2\left(\frac{1}{2}(c\right.}}\right.}{24d(a(1+\cos(c}$$

input

```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3
/2)),x]
```

output

```
(A*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*Sqrt[(1 - 2*
Sin[c/2 + (d*x)/2]^2)^(-1)]*(27 - 106*Sin[c/2 + (d*x)/2]^2 + 121*Sin[c/2 +
 (d*x)/2]^4 - 34*Sin[c/2 + (d*x)/2]^6 + (21*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)
/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*Cos[(c + d*x)/2]^6)/Sqrt[-(Sin[c/2 +
 (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]))/(24*d*(a*(1 + Cos[c + d*x]))^
(7/2)) + (B*Cos[c/2 + (d*x)/2]^7*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*S
qrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(15*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c
+ d*x)/2]^2]] + (33*Sin[c/2 + (d*x)/2]*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2
 + (d*x)/2]^2])/Sqrt[Cos[(c + d*x)/2]^2] - (26*Sin[c/2 + (d*x)/2]^3*Sqrt[1
 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(Cos[(c + d*x)/2]^2)^(3/2) +
(8*Sin[c/2 + (d*x)/2]^5*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])
/(Cos[(c + d*x)/2]^2)^(5/2)))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+2a(A+5B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(3a(A-B)+2a(A+5B)\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a(A-B)+2a(A+5B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+a}{6d(a\cos(c+dx)+a)^{7/2}}\right.$$

↓ 3456

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(A-13B)a^2+18(A+3B)\cos(c+dx)a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx}{6d(a\cos(c+d}\right.$$

↓ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(A-13B)a^2+18(A+3B)\cos(c+dx)a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+dx}{6d(a\cos(c+d}\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{(A-13B)a^2+18(A+3B)\sin\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\sin(c+}{6d(a\cos(c-}\right.$$

↓ 3457

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3a^3(7A+5B)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}\right.$$

↓ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3}{4}a(7A+5B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{a(A-13B)\sin(c+dx)\sqrt{}}{4d(a\cos(c+dx)+}}{12a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3}{4}a(7A+5B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{a(A-13B)\sin}{4d(a\cos(}}{12a^2}\right.$$

$\downarrow$ 3261

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{3a^2(7A+5B)\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right.}{2d}}{8a^2}}{12a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{a^2(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\sqrt{a}(7A+5B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}}{8a^2}+\frac{a(A-13B)\sin(}{4d(a\cos(}}{12a^2}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((a*(A - 13*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*Sqrt[a]*(7*A + 5*B)
*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])])/(2*Sqrt[2]*d) + (a^2*(17*A + 67*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
 + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 10.69 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.29

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\tan(dx+c)\left(17\cos(dx+c)^{2}+70\cos(dx+c)+21\right)\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\tan(dx+c)\left(67\cos(dx+c)^{2}+50\cos(dx\right.\right.}{384d\left(\cos(\right.}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\tan(dx+c)\left(17\cos(dx+c)^{2}+70\cos(dx+c)+21\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\arcsin(\cot(dx+c)-\csc(dx+c))\left(-21\,c\right.\right.}{384d\left(\cos(dx+c)^{4}+4\cos(dx+c)^{3}+6\cos(dx+c)^{2}+4\cos(dx+c)+1\right)\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+}{\cos(dx+c}}}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/384/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*co
s(d*x+c)^2+4*cos(d*x+c)+1)/sec(d*x+c)^(3/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(tan(d*x+c)*(17*cos(d*x+c)^2+70*cos(d*x+c)+21)*2^(1/2)*A*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+tan(d*x+c)*(67*cos(d*x+c)^2+50*cos(d*x+c)+15)*2^(1/2)
*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+A*arcsin(cot(d*x+c)-csc(d*x+c))*(-21*
cos(d*x+c)^2-63*cos(d*x+c)-63-21*sec(d*x+c))+B*arcsin(cot(d*x+c)-csc(d*x+c
))*(-15*cos(d*x+c)^2-45*cos(d*x+c)-45-15*sec(d*x+c)))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.16

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{3\sqrt{2}\left((7A+5B)\cos\left(dx+c\right)^4+4\left(7A+5B\right)\cos\left(dx+c\right)^3+6\left(7A+5B\right)\cos\left(dx+c\right)^2+4\left(7A+5B\right.\right.}{384\left(a^4d\cos\left(dx+c\right)^4+\right.}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)
^3 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B
)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*((17*A + 67*B)*cos(d*x + c)^3 + 10*(7*A + 5*B)*cos(
d*x + c)^2 + 3*(7*A + 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 +
6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx}{\cos(dx+c)^4\sec(dx+c)^2+4\cos(dx+c)^3\sec(dx+c)^2+6\cos(dx+c)^2\sec(dx+}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**4*sec(c + d*x)**2 + 4*cos(c + d*x)**3*sec(c + d*x)**2 + 6*cos(
c + d*x)**2*sec(c + d*x)**2 + 4*cos(c + d*x)*sec(c + d*x)**2 + sec(c + d*x
)**2),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)
**4*sec(c + d*x)**2 + 4*cos(c + d*x)**3*sec(c + d*x)**2 + 6*cos(c + d*x)**
2*sec(c + d*x)**2 + 4*cos(c + d*x)*sec(c + d*x)**2 + sec(c + d*x)**2),x)*a
))/a**4
```

3.548 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 281

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d}$$
$$+\frac{(5A-177B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}$$
$$+\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}+\frac{(5A-17B)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}$$
$$+\frac{(5A-49B)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
2*B*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(7/2)/d+1/128*(5*A-177*B)*arctan(1/2*a^(1/2)*sin(d*x+c)*2^
(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)*2^(1/2)/a^(7/2)/d+1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec
(d*x+c)^(5/2)+1/48*(5*A-17*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*
x+c)^(3/2)+1/64*(5*A-49*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-3i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5
/2)),x]
```

output
```
(Cos[(c + d*x)/2]^7*(((-3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(128*B*ArcSinh[E^(I*(c + d*x))] -
 Sqrt[2]*(5*A - 177*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^(
(2*I)*(c + d*x))])] - 128*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I
/2)*(c + d*x)) + ((97*A - 541*B + 4*(25*A - 181*B)*Cos[c + d*x] + (67*A -
247*B)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(-Sin[(c +
d*x)/2] + Sin[(3*(c + d*x))/2]))/8))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)+12aB\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a(A-B)+12aB\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a(A-B)+12aB\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left((5A-17B)a^2+32B\cos(c+dx)a^2\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\,\mathrm{si}}{6d(a\,\mathrm{c}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int \frac{\sqrt{\cos(c+dx)}\left((5A-17B)a^2+32B\cos(c+dx)a^2\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-B)\,\mathrm{si}}{6d(a\,\mathrm{c}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left((5A-17B)a^2+32B\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-\right.$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\int \frac{(5A-49B)a^3+128B\cos(c+dx)a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}}{4d(a\cos(c+dx)+a)^{5/}}}{12a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\int \frac{(5A-49B)a^3+128B\cos(c+dx)a^3}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(c+dx)\cos^{\frac{3}{2}}}{4d(a\cos(c+dx)+a)^{5/}}}{12a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\int \frac{(5A-49B)a^3+128B\sin\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{a(5A-17B)\sin(c+dx)}{4d(a\cos(c+dx)+}}{12a^2}\right.$$

$$\Big\downarrow \text{3461}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{a^3(5A-177B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+128a^2B\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right.}{8a^2}}{12a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{a^3(5A-177B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+128a^2B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{a^2(5A-49B)\sin(c}{2d(a\cos(c+}\right.}{8a^2}}{12a^2}\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{a^3(5A-177B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{256a^2B\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

$\downarrow$ 223

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{a^3(5A-177B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{256a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{a^2(5A-49B)\sin\ldots}{2d(a\cos\ldots}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{256a^{5/2}B\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2a^4(5A-177B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+\ldots}}\right.}{d}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{a^2(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{\frac{\sqrt{2}a^{5/2}(5A-177B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{256a^{5/2}B\ldots}{\ldots}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((a*(5*A - 17*B)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*(((256*a^(5/2)*B*A
rcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (Sqrt[2]*a^(5/
2)*(5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/d)/(4*a^2) + (a^2*(5*A - 49*B)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))))/(8*a^2))/(12*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 10.82 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.25

method	result
default	$\frac{\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\sqrt{2}\,A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(67\sin(dx+c)+50\tan(dx+c)+15\sec(dx+c)\tan(dx+c))-\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(147\sec(}{}$
parts	$\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\,\left(\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(67\sin(dx+c)+50\tan(dx+c)+15\sec(dx+c)\tan(dx+c))+\arcsin(\cot(dx+c)-\csc(dx+}{384d\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sec(dx+c)^{\frac{5}{2}}\sqrt{\frac{\cos}{\cos(}}}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/384/d*2^(1/2)*(a*(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*co
s(d*x+c)^2+4*cos(d*x+c)+1)/sec(d*x+c)^(5/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(67*sin(d*x+c)+50*tan(d*x
+c)+15*sec(d*x+c)*tan(d*x+c))-2^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
47*sec(d*x+c)*tan(d*x+c)+247*sin(d*x+c)+362*tan(d*x+c))*B+384*2^(1/2)*(3+c
os(d*x+c)+3*sec(d*x+c)+sec(d*x+c)^2)*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*B+A*arcsin(cot(d*x+c)-csc(d*x+c))*(-15*cos(d*x+c)-45-45*se
c(d*x+c)-15*sec(d*x+c)^2)+B*arcsin(cot(d*x+c)-csc(d*x+c))*(531*cos(d*x+c)+
1593+1593*sec(d*x+c)+531*sec(d*x+c)^2))/a^4
```

Fricas [A] (verification not implemented)

Time = 7.40 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.20

$$\int \frac{A + B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{3\sqrt{2}\left((5A-177B)\cos(dx+c)^4+4(5A-177B)\cos(dx+c)^3+6(5A-177B)\cos(dx+c)^2+4(5\ldots\right.}{}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x
+ c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5
*A - 177*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) + 768*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3
+ 6*B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((67*A - 247*B)*
cos(d*x + c)^3 + 2*(25*A - 181*B)*cos(d*x + c)^2 + 3*(5*A - 49*B)*cos(d*x
+ c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos
(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*co
s(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx}{\cos(dx+c)^4\sec(dx+c)^3+4\cos(dx+c)^3\sec(dx+c)^3+6\cos(dx+c)^2\sec(dx+}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**4*sec(c + d*x)**3 + 4*cos(c + d*x)**3*sec(c + d*x)**3 + 6*cos(
c + d*x)**2*sec(c + d*x)**3 + 4*cos(c + d*x)*sec(c + d*x)**3 + sec(c + d*x
)**3),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)
**4*sec(c + d*x)**3 + 4*cos(c + d*x)**3*sec(c + d*x)**3 + 6*cos(c + d*x)**
2*sec(c + d*x)**3 + 4*cos(c + d*x)*sec(c + d*x)**3 + sec(c + d*x)**3),x)*a
))/a**4
```

3.549 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 333

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(2A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d}$$
$$-\frac{(177A-637B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}$$
$$+\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}+\frac{(3A-7B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}$$
$$+\frac{(79A-259B)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}-\frac{7(7A-27B)\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
(2*A-7*B)*arcsin(a^(1/2)*sin(d*x+c)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^(7/2)/d-1/128*(177*A-637*B)*arctan(1/2*a^(1/2)*sin(d
*x+c)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)*2^(1/2)/a^(7/2)/d+1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(
7/2)/sec(d*x+c)^(7/2)+1/16*(3*A-7*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)
/sec(d*x+c)^(5/2)+1/192*(79*A-259*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/
2)/sec(d*x+c)^(3/2)-7/64*(7*A-27*B)*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2
)/sec(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.33 (sec) , antiderivative size = 1017, normalized size of antiderivative = 3.05

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7
/2)),x]
```

output
```
(((-49*I)/8)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I
)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c
 + d*x]))^(7/2)) + (((189*I)/8)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c +
d*x))*(a*(1 + Cos[c + d*x]))^(7/2)) + ((8*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x)
)/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*
(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^(
(2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d
*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(7/2)) - ((28*I)*S
qrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c +
d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c
 + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*(((-247*A + 42
7*B)*Cos[(d*x)/2]*Sin[c/2])/(12*d) + (8*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d -
 ((247*A - 427*B)*Cos[c/2]*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x
)/2]^2*(379*A*Sin[(d*x)/2] - 703*B*Sin[(d*x)/2]))/(24*d) + (Sec[c/2]*Sec[c
/2 + (d*x)/2]^6*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (Sec[c/2]*Se...
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.02, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3440, 3042, 3456, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-2a(A-7B)\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a(A-B)-2a(A-7B)\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(7a(A-B)-2a(A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+}{6d(a\cos(c+dx)+a)^{7/2}}\right.$$

$\big\downarrow$ 3456

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2(3A-7B)-2a^2(17A-77B)\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-}{}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2(3A-7B)-2a^2(17A-77B)\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{(A-}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(15a^2(3A-7B)-2a^2(17A-77B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a(3A-7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}\right.$$

$\big\downarrow$ 3456

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left(a^3(79A-259B)-14a^3(7A-27B)\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A-}{4}}{12a^2}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\int \frac{\sqrt{\cos(c+dx)}\left(a^3(79A-259B)-14a^3(7A-27B)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a(3A\cdots}{\cdots}}{12a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^3(79A-259B)-14a^3(7A-27B)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\cdots}{12a^2}\right.$$

$\Big\downarrow$ 3462

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(\frac{\int -\frac{7a^4(7A-27B)-64a^4(2A-7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-259B)\sin(c+\cdots}{2d(a\cos(c+dx\cdots}}{8a^2}\cdots}{12a^2}\right.$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(-\frac{\int \frac{7a^4(7A-27B)-64a^4(2A-7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-259B)\sin(c+\cdots}{2d(a\cos(c+dx\cdots}}{8a^2}\cdots}{12a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(-\frac{\int \frac{7a^4(7A-27B)-64a^4(2A-7B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{a^2(79A-259B)\sin \ldots}{2d(a\cos(\ldots}\ldots}{8a^2}\ldots}{12a^2}\ldots\right.$$

$\downarrow$ 3461

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(-\frac{a^4(177A-637B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-64a^3(2A-7B)\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{14a^3(7A-27B)\sin(c+\ldots}{d\sqrt{a\cos(c+\ldots}}\ldots\right.}{4a^2}\ldots}{8a^2}\ldots}{12a^2}\ldots\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(-\frac{a^4(177A-637B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-64a^3(2A-7B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{14a^3(7A\ldots}{\ldots}\ldots\right.}{4a^2}\ldots}{8a^2}\ldots}{12a\ldots}\ldots\right.$$

$\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(-\frac{a^4(177A-637B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{128a^3(2A-7B)\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(}{\sqrt{\cos(c-}}\right.}{d}}{a}\right.}{4a^2}}{8a^2}\right.$$

$$\Big\downarrow \; 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(-\frac{a^4(177A-637B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{128a^{7/2}(2A-7B)\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{14a}{}\right.}{4a^2}}{8a^2}\right.$$

1

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(-\frac{-\frac{2a^5(177A-637B)\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{128a^{7/2}(2A-7B)\,\text{arcsi}}{a}\right.}{4a^2}}{8a^2}\right.$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{a^2(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\left(-\frac{\frac{\sqrt{2}a^{7/2}(177A-637B)\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{128a\cdots}{\cdots}}{a}\cdots\right.}{\cdots}}{8a^2}\cdots\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((A - B)*Cos[c + d*x]^(7/2)*Sin[c +
 d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*a*(3*A - 7*B)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((a^2*(79*A - 259*B)
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (3*(-
(((-128*a^(7/2)*(2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c
 + d*x]]])/d + (Sqrt[2]*a^(7/2)*(177*A - 637*B)*ArcTan[(Sqrt[a]*Sin[c + d*
x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) - (14*a^
3*(7*A - 27*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]
])))/(4*a^2))/(8*a^2))/(12*a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [A] (verified)

Time = 18.94 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(\left(384\cos(dx+c)^3+1152\cos(dx+c)^2+1152\cos(dx+c)+384\right)\sqrt{2}\,A\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\left(-1344\cos(dx+c)^3-4032\right.}{}$
parts	$-\frac{A\sqrt{2}\,\sqrt{a(\cos(dx+c)+1)}\left(\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-384-1152\sec(dx+c)-1152\sec(dx+c)^2-384\sec(dx+c)^3\right)+\right.}{384d\left(\cos(dx+c)^4+4\cos(d\right.}$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d/sec(d*x+c)^(1/2)*((384*cos(d*x+c)^3+1152*cos(d*x+c)^2+1152*cos(d*x
+c)+384)*2^(1/2)*A*arctan(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(-
1344*cos(d*x+c)^3-4032*cos(d*x+c)^2-4032*cos(d*x+c)-1344)*2^(1/2)*B*arctan
(tan(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*(-247*cos(d*x+c)
^2-362*cos(d*x+c)-147)*2^(1/2)*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x
+c)*(192*cos(d*x+c)^3+1099*cos(d*x+c)^2+1442*cos(d*x+c)+567)*2^(1/2)*B*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+(531*cos(d*x+c)^3+1593*cos(d*x+c)^2+1593*co
s(d*x+c)+531)*A*arcsin(cot(d*x+c)-csc(d*x+c))+(-1911*cos(d*x+c)^3-5733*cos
(d*x+c)^2-5733*cos(d*x+c)-1911)*B*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*cos(1/
2*d*x+1/2*c)^2)^(1/2)/(cos(d*x+c)^4+4*cos(d*x+c)^3+6*cos(d*x+c)^2+4*cos(d*
x+c)+1)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.14

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{3\sqrt{2}\big((177\,A-637\,B)\cos{(dx+c)}^4+4\,(177\,A-637\,B)\cos{(a}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(d
*x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x +
 c) + 177*A - 637*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 384*((2*A - 7*B)*cos(d*x + c)^4 +
4*(2*A - 7*B)*cos(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 + 4*(2*A - 7*B
)*cos(d*x + c) + 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(192*B*cos(d*x + c)^4 - (247*A -
1099*B)*cos(d*x + c)^3 - 2*(181*A - 721*B)*cos(d*x + c)^2 - 21*(7*A - 27*B
)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/
(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 +
4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)+1}\,\cos(dx}{\cos(dx+c)^4\sec(dx+c)^4+4\cos(dx+c)^3\sec(dx+c)^4+6\cos(dx+c)^2\sec(dx+}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1)*cos(c + d*x))/(co
s(c + d*x)**4*sec(c + d*x)**4 + 4*cos(c + d*x)**3*sec(c + d*x)**4 + 6*cos(
c + d*x)**2*sec(c + d*x)**4 + 4*cos(c + d*x)*sec(c + d*x)**4 + sec(c + d*x
)**4),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x) + 1))/(cos(c + d*x)
**4*sec(c + d*x)**4 + 4*cos(c + d*x)**3*sec(c + d*x)**4 + 6*cos(c + d*x)**
2*sec(c + d*x)**4 + 4*cos(c + d*x)*sec(c + d*x)**4 + sec(c + d*x)**4),x)*a
))/a**4
```

3.550 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 180

$$\begin{aligned}
&\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx \\
&= -\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{2(3aA+5bB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2(Ab+aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}
\end{aligned}$$

output
```
-2/5*(3*A*a+5*B*b)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
sec(d*x+c)^(1/2)/d+2/3*(A*b+B*a)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*(3*A*a+5*B*b)*sec(d*x+c)^(1/2)*sin(d
*x+c)/d+2/3*(A*b+B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(5/
2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(3aA+5bB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(Ab+aB)\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(\right.\right.}{30}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*(-12*(3*a*A + 5*b*B)*Cos[c + d*x]^(5/2)*EllipticE[(c +
 d*x)/2, 2] + 20*(A*b + a*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2]
+ 2*(15*(a*A + b*B) + 10*(A*b + a*B)*Cos[c + d*x] + 3*(3*a*A + 5*b*B)*Cos[
2*(c + d*x)])*Sin[c + d*x]))/(30*d)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4485, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \text{ 4485}$$

$$\frac{2}{5}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(3aA+5bB+5(Ab+aB)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \sec^{\frac{3}{2}}(c+dx)(3aA+5bB+5(Ab+aB)\sec(c+dx))dx+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3aA+5bB+5(Ab+aB)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{1}{5}\left((3aA+5bB)\int \sec^{\frac{3}{2}}(c+dx)dx+5(aB+Ab)\int \sec^{\frac{5}{2}}(c+dx)dx\right)+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left((3aA+5bB)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+5(aB+Ab)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+(3aA+5bB)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+d}}{d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+(3aA+5bB)\left(\frac{2\sin(c+dx)\sqrt{\mathrm{sec}}}{d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+(3aA+5bB\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+(3aA\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+(3aA\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+(3aA+\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + ((3*a*A + 5*b*B)*((-2*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/d) + 5*(A*b + a*B)*((2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(159) = 318$.

Time = 104.36 (sec) , antiderivative size = 636, normalized size of antiderivative = 3.53

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2(Ab+Ba)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{2}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos(\cdots}}{3\sqrt{-2\sin(\cdots}}\right.\right.}{\cdots}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-
1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*A*a/(8*sin(1/2*d*x+1/2*c)^6-
12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*B*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.31

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx =$$

$$-\frac{5\sqrt{2}(i\,Ba+i\,Ab)\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-i\,B}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
-1/15*(5*sqrt(2)*(I*B*a + I*A*b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3
*I*A*a + 5*I*B*b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a - 5*I*B*b)*
cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*(3*(3*A*a + 5*B*b)*cos(d*x + c)^2 + 3*A*a + 5*(
B*a + A*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}(a+b\cos(c+dx))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=2\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)ab$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)b^2$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a*b + int(sqrt(se
c(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*b**2 + int(sqrt(sec(c + d*x
))*sec(c + d*x)**3,x)*a**2
```

3.551 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 143

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = -\frac{2(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(aA+3bB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2(Ab+aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
-2*(A*b+B*a)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*
x+c)^(1/2)/d+2/3*(A*a+3*B*b)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))*sec(d*x+c)^(1/2)/d+2*(A*b+B*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/
3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$=\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-3(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(aA+3bB)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{(c}{}\right.}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B)*EllipticE[(c + d*
x)/2, 2] + (a*A + 3*b*B)*EllipticF[(c + d*x)/2, 2] + ((a*A + 3*(A*b + a*B)
*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4485, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+b)(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)} \left(a \csc \left(c+dx+\frac{\pi }{2}\right)+b\right) \left(A \csc \left(c+dx+\frac{\pi }{2}\right)+B\right) dx$$

$\big\downarrow$ 4485

$$\frac{2}{3} \int \frac{1}{2} \sqrt{\sec (c+dx)} (aA+3bB+3 (Ab+aB) \sec (c+dx))dx+\frac{2 a A \sin (c+dx) \sec ^{\frac{3}{2}}(c+dx)}{3 d}$$

$\big\downarrow$ 27

$$\frac{1}{3} \int \sqrt{\sec (c+dx)} (aA+3bB+3 (Ab+aB) \sec (c+dx))dx+\frac{2 a A \sin (c+dx) \sec ^{\frac{3}{2}}(c+dx)}{3 d}$$

$\big\downarrow$ 3042

$$\frac{1}{3} \int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)} \left(aA+3bB+3 (Ab+aB) \csc \left(c+dx+\frac{\pi }{2}\right)\right) dx+\frac{2 a A \sin (c+dx) \sec ^{\frac{3}{2}}(c+dx)}{3 d}$$

$\big\downarrow$ 4274

$$\frac{1}{3} \left(3 (aB+Ab) \int \sec ^{\frac{3}{2}}(c+dx)dx+(aA+3bB) \int \sqrt{\sec (c+dx)}dx\right)+\frac{2 a A \sin (c+dx) \sec ^{\frac{3}{2}}(c+dx)}{3 d}$$

$\big\downarrow$ 3042

$$\frac{1}{3} \left((aA+3bB) \int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}dx+3 (aB+Ab) \int \csc \left(c+dx+\frac{\pi }{2}\right)^{3/2}dx\right)+\frac{2 a A \sin (c+dx) \sec ^{\frac{3}{2}}(c+dx)}{3 d}$$

$\big\downarrow$ 4255

$$\frac{1}{3} \left((aA+3bB) \int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}dx+3 (aB+Ab) \left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\int \frac{1}{\sqrt{\sec (c+dx)}}dx\right)\right)+\frac{2 a A \sin (c+dx) \sec ^{\frac{3}{2}}(c+dx)}{3 d}$$

$\big\downarrow$ 3042

$$\frac{1}{3} \left((aA+3bB) \int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}dx+3 (aB+Ab) \left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}}dx\right)\right)+\frac{2 a A \sin (c+dx) \sec ^{\frac{3}{2}}(c+dx)}{3 d}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{1}{3}\left((aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3(aB+Ab)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left((aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3(aB+Ab)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{1}{3}\left((aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3(aB+Ab)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{1}{3}\left(\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3(aB+Ab)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + ((2*(a*A + 3*b*B)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 3*(A*b + a*B)
*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(130) = 260$.

Time = 103.41 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.80

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}+\frac{2(Ab+Ba)\sqrt{-2\sin\cdots}}{\cdots}\right.}{\cdots}$
parts	$-\frac{2Aa\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\cos\cdots}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cdots\right.}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
(A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.43

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{\sqrt{2}(-iAa-3iBb)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}(iAa+3}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
1/3*(sqrt(2)*(-I*A*a - 3*I*B*b)*cos(d*x + c)*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a + 3*I*B*b)*cos(d*x + c)*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a +
 I*A*b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*(A*a + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos(c+d\,x))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= 2\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)ab$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b^2$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a*b + int(sqrt(se
c(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*b**2 + int(sqrt(sec(c + d*x
))*sec(c + d*x)**2,x)*a**2
```

3.552 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 111

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = -\frac{2(aA-bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*(A*a-B*b)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*
x+c)^(1/2)/d+2*(A*b+B*a)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))*sec(d*x+c)^(1/2)/d+2*a*A*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{\sec(c+dx)}\left(-\left((aA-bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(Ab+aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.\right.}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-((a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]) + (A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*A*
Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {3042, 3439, 3042, 4485, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+b)(A\sec(c+dx)+B)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4485}$$

$$2\int -\frac{aA-bB-(Ab+aB)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{aA-bB-(Ab+aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{aA-bB+(-Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4274}$$

$$-(aA-bB)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+(aB+Ab)\int \sqrt{\sec(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-(aA-bB)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+(aB+Ab)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4258}$$

$$(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3119}$$

$$(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$\big\downarrow$ 3120

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} - \frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(-2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(104) = 208$.

Time = 7.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.22

method	result
default	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2Ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(}}{}$
parts	$-\frac{2Aa\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a-A*b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.38

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{\frac{2\,Aa\sin(dx+c)}{\sqrt{\cos(dx+c)}} + \sqrt{2}(-i\,Ba - i\,Ab)\text{weierstrassPInverse}(-4,0,\cos(dx+c) + i\,\sin(dx+c)) + \sqrt{2}(i\,Ba + i}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
(2*A*a*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(-I*B*a - I*A*b)*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*B*a + I*A*b
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*
A*a + I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) + sqrt(2)*(I*A*a - I*B*b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\,dx$$
$$= \int (B\cos(dx + c) + A)(b\cos(dx + c) + a)\sec(dx + c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\,dx$$
$$= \int (B\cos(dx + c) + A)(b\cos(dx + c) + a)\sec(dx + c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(a+b\,\cos(c+d\,x))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=2\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)ab+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b^2+\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a*b + int(sqrt(sec(c
 + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*b**2 + int(sqrt(sec(c + d*x))*sec
(c + d*x),x)*a**2
```

3.553 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,d$

Optimal result

Integrand size = 31, antiderivative size = 115

$$\begin{aligned}\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2*(A*b+B*a)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x
+c)^(1/2)/d+2/3*(3*A*a+B*b)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*b*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{\sqrt{\sec(c+dx)}\left(6(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3aA+bB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2] + 2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*B
*Sin[2*(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {3042, 3439, 3042, 4484, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+b)(A\sec(c+dx)+B)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4484}$$

$$\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{3(Ab+aB)+(3aA+bB)\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{3(Ab+aB)+(3aA+bB)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{3(Ab+aB)+(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{1}{3}\left(3(aB+Ab)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+(3aA+bB)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(3(aB+Ab)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+(3aA+bB)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{1}{3}\left((3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos\right.$$
$$\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left((3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.$$
$$\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left((3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$

$$\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$

$$\frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
((6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d)/3 + (2*b*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(104) = 208$.

Time = 8.53 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.83

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+3Aa\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\text{co}\right.}{}$
parts	$-\frac{2Aa\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}+\frac{2(Ab+Ba)\sqrt{}}{}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b+3*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*b-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+B*b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \frac{2\,Bb\sqrt{\cos{(dx+c)}}\sin{(dx+c)}+\sqrt{2}(-3i\,Aa-i\,Bb)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/3*(2*B*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-3*I*A*a - I*B*b)*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A*a
 + I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sq
rt(2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sqrt{\sec(c + dx)}\,dx$$
$$= \int (A + B\cos(c + dx))\,(a + b\cos(c + dx))\,\sqrt{\sec(c + dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sqrt{\sec(c + dx)}\,dx$$
$$= \int (B\cos(dx + c) + A)(b\cos(dx + c) + a)\sqrt{\sec(dx + c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sqrt{\sec(c + dx)}\,dx$$
$$= \int (B\cos(dx + c) + A)(b\cos(dx + c) + a)\sqrt{\sec(dx + c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\,\cos(c+d\,x))\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\int \sqrt{\sec(dx+c)}dx\right)a^2 + 2\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\,dx\right)b^2$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)),x)*a**2 + 2*int(sqrt(sec(c + d*x))*cos(c + d*x),x)*
a*b + int(sqrt(sec(c + d*x))*cos(c + d*x)**2,x)*b**2
```

3.554 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2(5aA+3bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output

```
2/5*(5*A*a+3*B*b)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*s
ec(d*x+c)^(1/2)/d+2/3*(A*b+B*a)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*b*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3
*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(6(5aA+3bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(Ab+aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.}{15d}$$

input
```
Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x
]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
(5*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4484, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3439}$$
$$\int \frac{(a\sec(c+dx)+b)(A\sec(c+dx)+B)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a \csc \left(c+dx+\frac{\pi}{2}\right)+b\right) \left(A \csc \left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc \left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4484$$

$$\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5(Ab+aB)+(5aA+3bB)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \frac{5(Ab+aB)+(5aA+3bB)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{5(Ab+aB)+(5aA+3bB)\csc \left(c+dx+\frac{\pi}{2}\right)}{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4274$$

$$\frac{1}{5}\left(5(aB+Ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+(5aA+3bB)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(5(aB+Ab)\int \frac{1}{\csc \left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+(5aA+3bB)\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4256$$

$$\frac{1}{5}\left((5aA+3bB)\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+5(aB+Ab)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left((5aA+3bB)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5(aB+Ab)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+$$
$$\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4258

$$\frac{1}{5}\left((5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+5(aB+Ab)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.$$
$$\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+(5aA+3bB)\sqrt{\right.$$
$$\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3119

$$\frac{1}{5}\left(5(aB+Ab)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2(5aA+3bB)\sqrt{}}{}\right.$$
$$\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5(aB+Ab)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx}}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2*(5*a*A + 3*b*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 5*(A*b + a*
B)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(131) = 262$.

Time = 11.54 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.51

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6 b+(20Ab+20Ba+24Bb)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(\right.}{}$
parts	$\frac{2Aa\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2(Ab+Ba)\sqrt{\left(2\right.}}{}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a+5*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{5\sqrt{2}(iBa+iAb)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-iBa-iAb)\text{weie}}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
-1/15*(5*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-5*I*A*a - 3*I*B*b)*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3
*sqrt(2)*(5*I*A*a + 3*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*B*b*cos(d*x + c)^2 + 5*(B*a +
A*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\,(a+b\,\cos(c+d\,x))}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^2 + 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)ab + \left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)b^2$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a**2 + 2*int((sqrt(sec(c + d*x))*co
s(c + d*x))/sec(c + d*x),x)*a*b + int((sqrt(sec(c + d*x))*cos(c + d*x)**2)
/sec(c + d*x),x)*b**2
```

3.555 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 180

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{6(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
6/5*(A*b+B*a)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d
*x+c)^(1/2)/d+2/21*(7*A*a+5*B*b)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/7*b*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/
5*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*(7*A*a+5*B*b)*sin(d*x+c)/d/
sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(252(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(7aA+5bB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\right.\right.}{210d}$$

input
```
Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x
]
```

output
```
(Sqrt[Sec[c + d*x]]*(252*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2] + 20*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ (70*a*A + 65*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*b*B*Cos[2*(c + d*x)]
)*Sin[2*(c + d*x)]))/(210*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 3439, 3042, 4484, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 3439$$
$$\int \frac{(a\sec(c+dx)+b)(A\sec(c+dx)+B)}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4484}$$

$$\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int -\frac{7(Ab+aB)+(7aA+5bB)\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{7(Ab+aB)+(7aA+5bB)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{7(Ab+aB)+(7aA+5bB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{7}\left(7(aB+Ab)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+(7aA+5bB)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(7(aB+Ab)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+(7aA+5bB)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{7}\left(7(aB+Ab)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+(7aA+5bB)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+}{3d\sqrt{\sec(c}}\right.\right.$$
$$\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(7(aB+Ab)\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+(7aA+5bB)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right.\right.$$
$$\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(7(aB+Ab)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+(7aA+5bB)\left(\frac{1}{3}\sqrt{\cos}\right.\right.$$
$$\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(7(aB+Ab)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+(7aA+5bB)\left(\right.\right.$$
$$\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left((7aA+5bB)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+7(aB+Ab)\left(\right.\right.$$
$$\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left((7aA+5bB)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+7(aB+Ab)\left(\frac{2}{5d}\right.\right.$$
$$\frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

output
```
(2*b*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (7*(A*b + a*B)*((6*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (7*a*A + 5*b*B)*((2*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(
3*d*Sqrt[Sec[c + d*x]])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4484
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Simp[1/(d*n)   Int[(d*Csc[e + f*x])^(
n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(159) = 318$.

Time = 14.92 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.29

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}b+(-168Ab-168Ba-360Bb)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos(\frac{dx}{2}+\frac{c}{2}}{}$
parts	$-\frac{2Aa\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+cos(d*x+c)*b)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*b+25*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(7i\,Aa+5i\,Bb)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-7i\,Aa-5i\,Bb)}{}$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
-1/105*(5*sqrt(2)*(7*I*A*a + 5*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A*a - 5*I*B*b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*sqrt(2)*(-I*B*a - I*A*b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 63*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*B*b*cos(d*x + c)^3 + 21*(B
*a + A*b)*cos(d*x + c)^2 + 5*(7*A*a + 5*B*b)*cos(d*x + c))*sin(d*x + c)/sq
rt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{2}}dx\right)a^{2}+2\left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{2}}dx\right)ab + \left(\int \frac{\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}}{\sec\left(dx+c\right)^{2}}dx\right)b^{2}$$

input
```
int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a**2 + 2*int((sqrt(sec(c + d*x))
*cos(c + d*x))/sec(c + d*x)**2,x)*a*b + int((sqrt(sec(c + d*x))*cos(c + d*
x)**2)/sec(c + d*x)**2,x)*b**2
```

3.556 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 221

$$\begin{aligned}
&\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx \\
&= -\frac{2(3a^2A+5b(Ab+2aB))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{2(2aAb+a^2B+3b^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{2(3a^2A+5b(Ab+2aB))\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2a(7Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} \\
&\quad+\frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d}
\end{aligned}$$

output
```
-2/5*(3*A*a^2+5*b*(A*b+2*B*a))*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*cos(d*x+c)^(1/2
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*(3*A*a^2+5
*b*(A*b+2*B*a))*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/15*a*(7*A*b+5*B*a)*sec(d*x
+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(3/2)*(b+a*sec(d*x+c))*sin(d*x+c
)/d
```

Mathematica [A] (verified)

Time = 4.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.77

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(3a^2A+5Ab^2+10abB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(2aAb+a^2B+3b^2B)\cos\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x
]
```

output
```
(Sec[c + d*x]^(5/2)*(-12*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(5/2)
*EllipticE[(c + d*x)/2, 2] + 20*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(
5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a^2*A + A*b^2 + 2*a*b*B) + 10*a*(2
*A*b + a*B)*Cos[c + d*x] + 3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[2*(c + d*x
)])*Sin[c + d*x]))/(30*d)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 3439, 3042, 4514, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \; 4514$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(a(7Ab+5aB)\sec^2(c+dx)+\left(3Aa^2+5b(Ab+2aB)\right)\sec(c+dx)+b(aA+5bB)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \sqrt{\sec(c+dx)}\left(a(7Ab+5aB)\sec^2(c+dx)+\left(3Aa^2+5b(Ab+2aB)\right)\sec(c+dx)+b(aA+5bB)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(7Ab+5aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+5b(Ab+2aB)\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b(aA\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{5}\left(\left(3a^2A+5b(2aB+Ab)\right)\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(a(7Ab+5aB)\sec^2(c+dx)+b(aA+5bB)\right)d\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\left(3a^2A+5b(2aB+Ab)\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(7Ab+5aB)\csc\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{5}\left(\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(7\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a(7Ab+5aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+b(aA+5bB)\right)dx+\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2}{-}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$\downarrow$ 4534

$$\frac{1}{5}\left(\frac{5}{3}\left(a^2B+2aAb+3b^2B\right)\int\sqrt{\sec(c+dx)}dx+\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\text{co}}}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{5}{3}\left(a^2B+2aAb+3b^2B\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\frac{5}{3}\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2\sin(c}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{5}{3}\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{10\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\left(3a^2A+5b(2aB+Ab)\right)\left(\frac{2}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d) + ((10*
(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(7*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*d) + (3*a^2*A + 5*b*(A*b + 2*a*B))*((-2*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4514
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(m + n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*
Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1)
)*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x]
, x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2
- b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(200) = 400$.

Time = 516.69 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.27

method	result	size
default	Expression too large to display	723
parts	Expression too large to display	914

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*a*(2*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*a^2*A/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(
1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(A*b+2*B*a)/sin(1/2*d*x+1/2*c)^2/(2*si
n(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.29

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx =$$

$$-\frac{5\sqrt{2}(i\,Ba^2+2i\,Aab+3i\,Bb^2)\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})\cdots}{\cdots}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-1/15*(5*sqrt(2)*(I*B*a^2 + 2*I*A*a*b + 3*I*B*b^2)*cos(d*x + c)^2*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a^2 -
2*I*A*a*b - 3*I*B*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a^2 + 10*I*B*a*b + 5*I*A*b^2)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + 3*sqrt(2)*(-3*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(3*A*a^2 + 3*(3*A*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x
 + c)^2 + 5*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+b\cos(c+d\,x))^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= 3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a^2b$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)b^3$$
$$+3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a\,b^2$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a**2*b + int(sqrt
(sec(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**3,x)*b**3 + 3*int(sqrt(sec(c
+ d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*a*b**2 + int(sqrt(sec(c + d*x))
*sec(c + d*x)**3,x)*a**3
```

3.557 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 177

$$\begin{aligned}
&\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx \\
&= -\frac{2(2aAb+a^2B-b^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad+\frac{2(a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{2a(5Ab+3aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} \\
&\quad+\frac{2aA\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\sin(c+dx)}{3d}
\end{aligned}$$

output
```
-2*(2*A*a*b+B*a^2-B*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*sec(d*x+c)^(1/2)/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*cos(d*x+c)^(1/2)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*a*(5*A*b+3*B*a)*
sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a*A*sec(d*x+c)^(1/2)*(b+a*sec(d*x+c))*si
n(d*x+c)/d
```

Mathematica [A] (verified)

Time = 7.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.71

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(-3(2aAb+a^2B-b^2B)\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+(a^2A+3Ab^2+6abB)\,\text{Elliptic}}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x
]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(2*a*A*b + a^2*B - b^2*B)*Ell
ipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2
, 2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^
(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3439, 3042, 4514, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+b)^2(A\sec(c+dx)+B)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a \csc \left(c+dx+\frac{\pi}{2}\right)+b\right)^2 \left(A \csc \left(c+dx+\frac{\pi}{2}\right)+B\right)}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4514$$

$$\frac{2}{3} \int -\frac{-a(5Ab+3aB) \sec^2(c+dx)-\left(Aa^2+6bBa+3Ab^2\right) \sec(c+dx)+b(aA-3bB)}{2\sqrt{\sec(c+dx)}}dx+\frac{2aA \sin(c+dx)\sqrt{\sec(c+dx)}(a \sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{2aA \sin(c+dx)\sqrt{\sec(c+dx)}(a \sec(c+dx)+b)}{3d}-\frac{1}{3} \int \frac{-a(5Ab+3aB) \sec^2(c+dx)-\left(Aa^2+6bBa+3Ab^2\right) \sec(c+dx)+b(aA-3bB)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{2aA \sin(c+dx)\sqrt{\sec(c+dx)}(a \sec(c+dx)+b)}{3d}-\frac{1}{3} \int \frac{-a(5Ab+3aB) \csc \left(c+dx+\frac{\pi}{2}\right)^2+\left(-Aa^2-6bBa-3Ab^2\right) \csc \left(c+dx+\frac{\pi}{2}\right)+b(aA-3bB)}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4535$$

$$\frac{1}{3} \left(\left(a^2A+6abB+3Ab^2\right) \int \sqrt{\sec(c+dx)}dx-\int \frac{b(aA-3bB)-a(5Ab+3aB) \sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2aA \sin(c+dx)\sqrt{\sec(c+dx)}(a \sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3} \left(\left(a^2A+6abB+3Ab^2\right) \int \sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}dx-\int \frac{b(aA-3bB)-a(5Ab+3aB) \csc \left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2aA \sin(c+dx)\sqrt{\sec(c+dx)}(a \sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \ 4258$$

$$\frac{1}{3}\left(\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-\int\frac{b(aA-3bB)-a(5Ab+3aB)\operatorname{cs}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right.}}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{b(aA-3bB)-a(5Ab+3a}{\sqrt{\csc\left(c+dx\right.}}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\int\frac{b(aA-3bB)-a(5Ab+3a}{\sqrt{\csc\left(c+dx\right.}}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}\left(-3\left(a^2B+2aAb-b^2B\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}}{d}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-3\left(a^2B+2aAb-b^2B\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{Ellip}}{d}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(-3\left(a^2B+2aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+}}{}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-3\left(a^2B+2aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\text{co}}}{}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left(\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6\left(a^2B+2aAb-b^2B\right)\sqrt{\cos(}}{}\right.$$
$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d) + ((-6*
(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*(5*A*b + 3*a*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4514
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(m + n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*
Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1)
)*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x]
, x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2
- b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(162) = 324$.

Time = 527.68 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.67

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2Ab^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}+\frac{2a(2Ab+Ba)\sqrt{-2}}{}\right)}{}$
parts	$-\frac{2a^2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-}\right)}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*a*(2*A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1
/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))-2*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))+4*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.40

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\frac{\sqrt{2}(-i\,Aa^2-6i\,Bab-3i\,Ab^2)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(-I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*cos(d*x + c)*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^2 + 6*I*B*a*
b + 3*I*A*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 3*sqrt(2)*(I*B*a^2 + 2*I*A*a*b - I*B*b^2)*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) - 3*sqrt(2)*(-I*B*a^2 - 2*I*A*a*b + I*B*b^2)*cos(d*x + c)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(
A*a^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a^2b$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b^3$$
$$+3\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a\,b^2$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a**2*b + int(sqrt
(sec(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*b**3 + 3*int(sqrt(sec(c
+ d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*a*b**2 + int(sqrt(sec(c + d*x))
*sec(c + d*x)**2,x)*a**3
```

3.558 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\begin{aligned}&\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx\\&=-\frac{2(a^2A-b(Ab+2aB))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\&\quad+\frac{2(6aAb+3a^2B+b^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\&\quad+\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a^2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
-2*(A*a^2-b*(A*b+2*B*a))*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*sec(d*x+c)^(1/2)/d+2/3*(6*A*a*b+3*B*a^2+B*b^2)*cos(d*x+c)^(1/2)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*b^2*B*sin(d*x+c)
/d/sec(d*x+c)^(1/2)+2*a^2*A*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 6.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Big((-6a^2A+6Ab^2+12abB)\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+2(6aAb+3a^2B+b^2B)\,\text{Ellip}}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x
]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-6*a^2*A + 6*A*b^2 + 12*a*b*B)*El
lipticE[(c + d*x)/2, 2] + 2*(6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x
)/2, 2] + (2*(3*a^2*A + b^2*B*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d*x
]]))/(3*d)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {3042, 3439, 3042, 4512, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3439$$

$$\int \frac{(a\sec(c+dx)+b)^2(A\sec(c+dx)+B)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4512}$$

$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{3a^2A\sec^2(c+dx)+\left(3Ba^2+6Aba+b^2B\right)\sec(c+dx)+3b(Ab+2aB)}{2\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2A\sec^2(c+dx)+\left(3Ba^2+6Aba+b^2B\right)\sec(c+dx)+3b(Ab+2aB)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Ba^2+6Aba+b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b(Ab+2aB)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{3}\left(\left(3a^2B+6aAb+b^2B\right)\int \sqrt{\sec(c+dx)}dx+\int \frac{3a^2A\sec^2(c+dx)+3b(Ab+2aB)}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2B+6aAb+b^2B\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{3a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b(Ab+2aB)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\int\frac{3a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b(Ab}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{3a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+3}{\sqrt{\csc\left(c+dx+\right.}}\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3120

$$\frac{1}{3}\left(\int\frac{3a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b(Ab+2aB)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{Elliptic}}{d}\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4534

$$\frac{1}{3}\left(-3\left(a^2A-2abB-Ab^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}}{d}\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(-3\left(a^2A-2abB-Ab^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{Ellip}}{d}\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4258

$$\frac{1}{3}\left(-3\left(a^2A-2abB-Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

↓ 3042

$$\frac{1}{3}\left(-3\left(a^2A-2abB-Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\mathrm{co}\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

↓ 3119

$$\frac{1}{3}\left(\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6\left(a^2A-2abB-Ab^2\right)\sqrt{\cos(\right.$$
$$\frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*b^2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((-6*(a^2*A - A*b^2 - 2*
a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (6*a^2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4512
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Simp[1/(d*n)   Int[(d*Csc
[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)
))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(148) = 296$.

Time = 9.13 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.52

method	result
default	$\frac{-\frac{8B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b^{2}}{3}+4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a^{2}-4Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
parts	$-\frac{2a^{2}A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input

```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output

```
2/3*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2+6*A*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*a^2-6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+2*B*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2*b^2-3*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*b^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{\sqrt{2}(-3i\,Ba^2-6i\,Aab-i\,Bb^2)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(3i\,Ba^2+}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(-3*I*B*a^2 - 6*I*A*a*b - I*B*b^2)*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*B*a^2 + 6*I*A*a*b + I*B*b^2
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I
*A*a^2 - 2*I*B*a*b - I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*A*a^2 + 2*I*B*a*b +
I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(B*b^2*cos(d*x + c) + 3*A*a^2)*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=3\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)a^2b$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)b^3$$
$$+3\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a\,b^2$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a**2*b + int(sqrt(se
c(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*b**3 + 3*int(sqrt(sec(c + d*x)
)*cos(c + d*x)**2*sec(c + d*x),x)*a*b**2 + int(sqrt(sec(c + d*x))*sec(c +
d*x),x)*a**3
```

3.559 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}$

Optimal result

Integrand size = 33, antiderivative size = 171

$$\begin{aligned}\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \frac{2(10aAb+5a^2B+3b^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{2(3a^2A+Ab^2+2abB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(Ab+2aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/5*(10*A*a*b+5*B*a^2+3*B*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*cos(d*x+c)^(1/2
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/5*b^2*B*sin(
d*x+c)/d/sec(d*x+c)^(3/2)+2/3*b*(A*b+2*B*a)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 6.72 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{\sqrt{\sec(c+dx)}\left(6(10aAb+5a^2B+3b^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(3a^2A+Ab^2+2abB)\sqrt{\cos(c+dx)}\right.}{15d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x
]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] + 10*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + b*(5*A*b + 10*a*B + 3*b*B*Cos[c + d*x])*Sin
[2*(c + d*x)]))/(15*d)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3439, 3042, 4512, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{ 3439}$$
$$\int \frac{(a\sec(c+dx)+b)^2(A\sec(c+dx)+B)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4512}$$

$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5a^2A\sec^2(c+dx)+\left(5Ba^2+10Aba+3b^2B\right)\sec(c+dx)+5b(Ab+2aB)}{2\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5a^2A\sec^2(c+dx)+\left(5Ba^2+10Aba+3b^2B\right)\sec(c+dx)+5b(Ab+2aB)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{5a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Ba^2+10Aba+3b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+5b(Ab+2aB)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{5}\left(\left(5a^2B+10aAb+3b^2B\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\int \frac{5a^2A\sec^2(c+dx)+5b(Ab+2aB)}{\sec^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2B+10aAb+3b^2B\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{5a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b(Ab+2aB)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\int\frac{5a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b(A\right.$$
$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{5a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+}{\csc\left(c+dx+\right.}\right.$$
$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(\int\frac{5a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b(Ab+2aB)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}\right.$$
$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{5}\left(\frac{5}{3}\left(3a^2A+2abB+Ab^2\right)\int\sqrt{\sec(c+dx)}dx+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}\right.$$
$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{5}{3}\left(3a^2A+2abB+Ab^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$
$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(\frac{5}{3}\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c\right.}{}\right.$$
$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{5}{3}\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{}\right.$$

$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{10\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{}\right.$$

$$\frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2*(10*a*A*b + 5*a^2*B
+ 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d + (10*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (10*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4512
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Simp[1/(d*n)   Int[(d*Csc
[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)
))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(154) = 308$.

Time = 12.23 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.85

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}b^{2}+\left(20A\,b^{2}+40Bab+24B\,b^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
parts	$-\frac{2a^{2}A\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2\left(A\,b^{2}+2Ba\right.}{}$

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.32

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(3i\,Aa^2+2i\,Bab+i\,Ab^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-3i\,A$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/15*(5*sqrt(2)*(3*I*A*a^2 + 2*I*B*a*b + I*A*b^2)*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-3*I*A*a^2 - 2*I*B*a*b - I
*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt
(2)*(-5*I*B*a^2 - 10*I*A*a*b - 3*I*B*b^2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(5*I*B*a^2 +
 10*I*A*a*b + 3*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - 2*(3*B*b^2*cos(d*x + c)^2 + 5*(2*B*a*b
+ A*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)),
x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\left(\int\sqrt{\sec(dx+c)}dx\right)a^3+3\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\,dx\right)a^2b$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^3\,dx\right)b^3+3\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^2\,dx\right)a\,b^2$$

input
```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)),x)*a**3 + 3*int(sqrt(sec(c + d*x))*cos(c + d*x),x)*
a**2*b + int(sqrt(sec(c + d*x))*cos(c + d*x)**3,x)*b**3 + 3*int(sqrt(sec(c
 + d*x))*cos(c + d*x)**2,x)*a*b**2
```

3.560 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{2(14aAb+7a^2B+5b^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\ &\quad+\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b(Ab+2aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\\ &\quad+\frac{2(14aAb+7a^2B+5b^2B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))*sec(d*x+c)^(1/2)/d+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*cos(d*x+c)^(
1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/d+2/7*b^2*B*s
in(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*b*(A*b+2*B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/
2)+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 7.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(84(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|2\big)+20(14aAb+7a^2B+5b^2B)\sqrt{}}{}$$

input
```
Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]]
,x]
```

output
```
(Sqrt[Sec[c + d*x]]*(84*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] + 20*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] + (42*b*(A*b + 2*a*B)*Cos[c + d*x] + 5*(28
*a*A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]
))/(210*d)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3439, 3042, 4512, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+b)^2(A\sec(c+dx)+B)}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4512}$$

$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int -\frac{7a^2A\sec^2(c+dx)+\left(7Ba^2+14Aba+5b^2B\right)\sec(c+dx)+7b(Ab+2aB)}{2\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\int \frac{7a^2A\sec^2(c+dx)+\left(7Ba^2+14Aba+5b^2B\right)\sec(c+dx)+7b(Ab+2aB)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\int \frac{7a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Ba^2+14Aba+5b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+7b(Ab+2aB)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4535}$$

$$\frac{1}{7}\left(\left(7a^2B+14aAb+5b^2B\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{7a^2A\sec^2(c+dx)+7b(Ab+2aB)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\left(7a^2B+14aAb+5b^2B\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{7a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b(Ab+2aB)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)+\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{1}{7}\left(\left(7a^2B+14aAb+5b^2B\right)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\int\frac{7a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b(Ab-\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2B+14aAb+5b^2B\right)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\int\frac{7a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+7\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\left(7a^2B+14aAb+5b^2B\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\int\frac{7a^2A\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2B+14aAb+5b^2B\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\int\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\int\frac{7a^2A\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b(Ab+2aB)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\left(7a^2B+14aAb+5b^2B\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{7}\left(\frac{7}{5}\left(5a^2A+6abB+3Ab^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\left(7a^2B+14aAb+5b^2B\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{7}{5}\left(5a^2A+6abB+3Ab^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(7a^2B+14aAb+5b^2B\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c}{}\right.\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{7}{5}\left(5a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\left(7a^2B+14aAb+5b^2B\right)\left(\frac{2\sin}{3d\sqrt{\mathrm{se}}}\right.\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{7}{5}\left(5a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\left(7a^2B+14aAb+5b^2B\right)\left(\right.\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{7}\left(\frac{14\left(5a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\left(7a^2B+14aAb+5b^2B\right)\left(\frac{2\sin(c}{3d\sqrt{\sec}}\right.\right.$$
$$\frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((14*(5*a^2*A + 3*A*b^2
+ 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(5*d) + (14*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (14
*a*A*b + 7*a^2*B + 5*b^2*B)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))
/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4512
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Simp[1/(d*n)   Int[(d*Csc
[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)
))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(192) = 384$.

Time = 16.74 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.57

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8b^2+\left(-168A\,b^2-336Bab-360B\,b^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^2+(-168*A*b^2-336*B*a*b-360*B*b^2)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*
B*a*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*
b^2-70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70
*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*b^2+35*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*b^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{5\sqrt{2}(7i\,Ba^2+14i\,Aab+5i\,Bb^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-7i$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/105*(5*sqrt(2)*(7*I*B*a^2 + 14*I*A*a*b + 5*I*B*b^2)*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*B*a^2 - 14*I*A*a*
b - 5*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) +
 21*sqrt(2)*(-5*I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(5*I
*A*a^2 + 6*I*B*a*b + 3*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*B*b^2*cos(d*x + c)^3 + 21*
(2*B*a*b + A*b^2)*cos(d*x + c)^2 + 5*(7*B*a^2 + 14*A*a*b + 5*B*b^2)*cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)),
x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$=\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^2}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a + b\cos(c + dx))^2(A + B\cos(c + dx))}{\sqrt{\sec(c + dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^3 + 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)a^2b + \left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right)b^3 + 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)a\,b^2$$

input

```
int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output

```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a**3 + 3*int((sqrt(sec(c + d*x))*co
s(c + d*x))/sec(c + d*x),x)*a**2*b + int((sqrt(sec(c + d*x))*cos(c + d*x)*
*3)/sec(c + d*x),x)*b**3 + 3*int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(
c + d*x),x)*a*b**2
```

3.561 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 295

$$\begin{aligned}
&\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx \\
&= -\frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{2(5a^3A+21aAb^2+21a^2bB+21b^3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\
&\quad+\frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2a(5a^2A+18Ab^2+21abB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\
&\quad+\frac{2a^2(11Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} \\
&\quad+\frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2\sin(c+dx)}{7d}
\end{aligned}$$

output
```
-2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/21*(5*A*a^3+21*A*a*b^2+21*B*
a^2*b+21*B*b^3)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*se
c(d*x+c)^(1/2)/d+2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*sec(d*x+c)^(1/
2)*sin(d*x+c)/d+2/21*a*(5*A*a^2+18*A*b^2+21*B*a*b)*sec(d*x+c)^(3/2)*sin(d*
x+c)/d+2/35*a^2*(11*A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a*A*sec(d
*x+c)^(3/2)*(b+a*sec(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 6.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.76

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{\sec(c+dx)}\left(-21(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5a^3A+21aA\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x
]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-21*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*a^3*A + 21*a*A*b^2 + 21*
a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(9*a
^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 5*a*(5*a^2*A + 21*
A*b^2 + 21*a*b*B)*Tan[c + d*x] + 21*a^2*(3*A*b + a*B)*Sec[c + d*x]*Tan[c +
 d*x] + 15*a^3*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.94, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3439, 3042, 4514, 27, 3042, 4564, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^3(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \text{4514}$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\left(a(11Ab+7aB)\sec^2(c+dx)+\left(5Aa^2+7b(Ab+2aB)\right)\sec(c+dx)+b(aA+7bB)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \sqrt{\sec(c+dx)}(b+a\sec(c+dx))\left(a(11Ab+7aB)\sec^2(c+dx)+\left(5Aa^2+7b(Ab+2aB)\right)\sec(c+dx)+b(aA+7bB)\right)dx+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(11Ab+7aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Aa^2+7b(Ab+2aB\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{4564}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\sec^2(c+dx)+7\left(3Ba^3+9Aba^2+15b^2B\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\sec(c+dx)}(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\sec^2(c+dx)+7\left(3Ba^3+9Aba^2+15b^2Ba\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7\left(3Ba^3+9Ab\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\sec(c+dx)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\sec^2(c+dx)\right)dx+7\left(3a^3B+9a^2Ab+15\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+7\left(3a^3B\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+7\left(3a^3B\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+7\left(3a^3B\right.\right.\right.$$

$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+7\left(3a^3B\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+7\left(3a^3B\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(aA+7bB)b^2+5a\left(5Aa^2+21bBa+18Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+7\left(3a^3B\right.\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^3A+21a^2bB+21aAb^2+21b^3B\right)\int \sqrt{\sec(c+dx)}dx+\frac{10a\left(5a^2A+21abB+18Ab^2\right)\sin(c+dx)\text{se}}{3d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^3A+21a^2bB+21aAb^2+21b^3B\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{10a\left(5a^2A+21abB+18Ab^2\right)\sin(c+}{3d}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^3A+21a^2bB+21aAb^2+21b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{10a\left(5a^2A+21\right.}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^3A+21a^2bB+21aAb^2+21b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10a\left(5a^2A\right.}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{1}{7}\left(\frac{2a^2(7aB+11Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{1}{5}\left(\frac{10a\left(5a^2A+21abB+18Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{10}{}\right.\right.$$
$$\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}{7d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output
```
(2*a*A*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d) + ((2
*a^2*(11*A*b + 7*a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + ((10*(5*a^3
*A + 21*a*A*b^2 + 21*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (10*a*(5*a^2*A + 18*A*b^2 + 21*a*b
*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + 7*(9*a^2*A*b + 5*A*b^3 + 3*a^
3*B + 15*a*b^2*B)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4514
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(m + n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*
Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1)
)*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x]
, x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2
- b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(270) = 540$.

Time = 2342.57 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.11

method	result	size
default	Expression too large to display	917
parts	Expression too large to display	1175

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2/5*a^2*(3*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(
1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^3*A*(-1/56*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*
(A*b+3*B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.23

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx =$$

$$-\frac{5\sqrt{2}(5i\,Aa^3+21i\,Ba^2b+21i\,Aab^2+21i\,Bb^3)\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorith
m="fricas")
```

output
```
-1/105*(5*sqrt(2)*(5*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 21*I*B*b^3)*c
os(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
5*sqrt(2)*(-5*I*A*a^3 - 21*I*B*a^2*b - 21*I*A*a*b^2 - 21*I*B*b^3)*cos(d*x
+ c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt
(2)*(3*I*B*a^3 + 9*I*A*a^2*b + 15*I*B*a*b^2 + 5*I*A*b^3)*cos(d*x + c)^3*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c))) + 21*sqrt(2)*(-3*I*B*a^3 - 9*I*A*a^2*b - 15*I*B*a*b^2 - 5*I*A*b^3)*c
os(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))) - 2*(15*A*a^3 + 21*(3*B*a^3 + 9*A*a^2*b + 15*B*a*b^2
 + 5*A*b^3)*cos(d*x + c)^3 + 5*(5*A*a^3 + 21*B*a^2*b + 21*A*a*b^2)*cos(d*x
 + c)^2 + 21*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{9/2}(a+b\cos(c+dx))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$= 4\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)a^3b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^4\,dx\right)b^4 + 4\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^4\,dx\right)a\,b^3 + 6\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)a^2b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)a^4$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
4*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*a**3*b + int(sqrt
(sec(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**4,x)*b**4 + 4*int(sqrt(sec(c
+ d*x))*cos(c + d*x)**3*sec(c + d*x)**4,x)*a*b**3 + 6*int(sqrt(sec(c + d*x
))*cos(c + d*x)**2*sec(c + d*x)**4,x)*a**2*b**2 + int(sqrt(sec(c + d*x))*s
ec(c + d*x)**4,x)*a**4
```

3.562 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 244

$$\begin{aligned}
&\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx \\
&= -\frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{2(3a^2Ab+3Ab^3+a^3B+9ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{2a(3a^2A+14Ab^2+15abB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2a^2(9Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} \\
&\quad+\frac{2aA\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2\sin(c+dx)}{5d}
\end{aligned}$$

output

```
-2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*cos(d*x+c)^(1/2)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*(3*A*a^2*b+3*A*b^3+B*a^3+
9*B*a*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/d+2/5*a*(3*A*a^2+14*A*b^2+15*B*a*b)*sec(d*x+c)^(1/2)*sin(d*x+c)/
d+2/15*a^2*(9*A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^
(1/2)*(b+a*sec(d*x+c))^2*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 12.90 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.79

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-3(3a^3A+15aAb^2+15a^2bB-5b^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(3a^2Ab+3Ab\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x
]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(3*a^3*A + 15*a*A*b^2 + 15*a^
2*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*A*b + 3*A*b^3 + a^3*
B + 9*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (a*(15*(a^2*A + 3*A*b^2 + 3*a*b
*B) + 10*a*(3*A*b + a*B)*Cos[c + d*x] + 9*(a^2*A + 5*A*b^2 + 5*a*b*B)*Cos[
2*(c + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(5/2))))/(15*d)
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3439, 3042, 4514, 27, 3042, 4564, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+b)^3(A\sec(c+dx)+B)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4514}$$

$$\frac{2}{5}\int -\frac{(b+a\sec(c+dx))\left(-a(9Ab+5aB)\sec^2(c+dx)-\left(3Aa^2+5b(Ab+2aB)\right)\sec(c+dx)+b(aA-5bB)\right)}{2\sqrt{\sec(c+dx)}}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}-$$

$$\frac{1}{5}\int \frac{(b+a\sec(c+dx))\left(-a(9Ab+5aB)\sec^2(c+dx)-\left(3Aa^2+5b(Ab+2aB)\right)\sec(c+dx)+b(aA-5bB)\right)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}-$$

$$\frac{1}{5}\int \frac{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(-a(9Ab+5aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(-3Aa^2-5b(Ab+2aB)\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}$$

$$\Big\downarrow \text{ 4564}$$

$$\frac{1}{5}\left(\frac{2a^2(5aB+9Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{2}{3}\int \frac{3(aA-5bB)b^2-3a\left(3Aa^2+15bBa+14Ab^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+d}}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{2a^2(5aB+9Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{1}{3}\int \frac{3(aA-5bB)b^2-3a\left(3Aa^2+15bBa+14Ab^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+d}}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{2a^2(5aB+9Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{1}{3}\int\frac{3(aA-5bB)b^2-3a\left(3Aa^2+15bBa+14Ab^2\right)\csc\left(c+dx\right.}{\sqrt{\csc\left(c+\right.}}\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\int\sqrt{\sec(c+dx)}dx-\int\frac{3b^2(aA-5bB)-3a\left(3Aa^2+15bBa+14Ab^2\right)}{\sqrt{\sec(c+dx)}}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\int\frac{3b^2(aA-5bB)-3a\left(3Aa^2+15bBa+1\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right.}}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-\int\frac{3b^2(aA-5bB)-}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(5\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{3b^2(aA-5b}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{10\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\int\frac{3b^2(aA-5b\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \; 4534$$

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{6a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)\sqrt{s}}{d}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(3a^2A+15abB+14Ab^2\right)\sin(c+d}{d}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{6a\left(3a^2A+15a\right.}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a\left(3a^2A\right.}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{5}\left(\frac{2a^2(5aB+9Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{1}{3}\left(\frac{6a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{10(}{}\right.\right.$$

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((2
*a^2*(9*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + ((-6*(3*a^3*
A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b
^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (6*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/
3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4514
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(m + n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*
Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1)
)*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x]
, x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2
- b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(223) = 446$.

Time = 2639.66 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.98

method	result	size
default	Expression too large to display	970
parts	Expression too large to display	1065

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*b^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*a^2*(3*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*a^3*A/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/si
n(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b^3*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.34

$$\int (a + b\cos(c + dx))^3(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx =$$

$$-\frac{5\sqrt{2}(iBa^3 + 3iAa^2b + 9iBab^2 + 3iAb^3)\cos(dx + c)^2\,\mathrm{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-1/15*(5*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b + 9*I*B*a*b^2 + 3*I*A*b^3)*cos(d*x
 + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt
(2)*(-I*B*a^3 - 3*I*A*a^2*b - 9*I*B*a*b^2 - 3*I*A*b^3)*cos(d*x + c)^2*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(3*I*A*a
^3 + 15*I*B*a^2*b + 15*I*A*a*b^2 - 5*I*B*b^3)*cos(d*x + c)^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*s
qrt(2)*(-3*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b^2 + 5*I*B*b^3)*cos(d*x + c)
^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - 2*(3*A*a^3 + 9*(A*a^3 + 5*B*a^2*b + 5*A*a*b^2)*cos(d*x + c)^2
 + 5*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^3(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= 4\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a^3b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^3\,dx\right)b^4 + 4\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)a\,b^3 + 6\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a^2b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)a^4$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
4*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a**3*b + int(sqrt
(sec(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**3,x)*b**4 + 4*int(sqrt(sec(c
+ d*x))*cos(c + d*x)**3*sec(c + d*x)**3,x)*a*b**3 + 6*int(sqrt(sec(c + d*x
))*cos(c + d*x)**2*sec(c + d*x)**3,x)*a**2*b**2 + int(sqrt(sec(c + d*x))*s
ec(c + d*x)**3,x)*a**4
```

3.563 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 239

$$\begin{aligned}
&\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx \\
&= -\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad+\frac{2(a^3A+9aAb^2+9a^2bB+b^3B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{2a(9aAb+3a^2B-2b^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} \\
&\quad+\frac{2a^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}
\end{aligned}$$

output
```
-2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*(A*a^3+9*A*a*b^2+9*B*a^2*b+B*b^3)
*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/
d+2/3*a*(9*A*a*b+3*B*a^2-2*B*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a^2*(A
*a-B*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*b*B*(b+a*sec(d*x+c))^2*sin(d*x+c
)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.69

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(-6(3a^2Ab-Ab^3+a^3B-3ab^2B)\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+2(a^3A+9aAb^2+9a}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x
]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*
a*b^2*B)*EllipticE[(c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^
3*B)*EllipticF[(c + d*x)/2, 2] + ((2*a^3*A + b^3*B + 6*a^2*(3*A*b + a*B)*C
os[c + d*x] + b^3*B*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(
3*d)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.576, Rules used = {3042, 3439, 3042, 4513, 27, 3042, 4564, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3439$$

$$\int \frac{(a\sec(c+dx)+b)^3(A\sec(c+dx)+B)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4513}$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{(b+a\sec(c+dx))\left(3a(aA-bB)\sec^2(c+dx)+\left(3Ba^2+6Aba+b^2B\right)\sec(c+dx)+b(3Ab+7aB)\right)}{2\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{(b+a\sec(c+dx))\left(3a(aA-bB)\sec^2(c+dx)+\left(3Ba^2+6Aba+b^2B\right)\sec(c+dx)+b(3Ab+7aB)\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a(aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Ba^2+6Aba+b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b(3A\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4564}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{3\left((3Ab+7aB)b^2+a\left(3Ba^2+9Aba-2b^2B\right)\sec^2(c+dx)+\left(Aa^3+9bBa^2+9Ab^2a+b^3B\right)\sec(c+dx)\right.}{2\sqrt{\sec(c+dx)}}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\int \frac{(3Ab+7aB)b^2+a\left(3Ba^2+9Aba-2b^2B\right)\sec^2(c+dx)+\left(Aa^3+9bBa^2+9Ab^2a+b^3B\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{(3Ab+7aB)b^2+a\left(3Ba^2+9Aba-2b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^3+9bBa^2+9Ab^2a+b^3B\right)\csc\left(c+dx\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4535

$$\frac{1}{3}\left(\int \frac{(3Ab+7aB)b^2+a\left(3Ba^2+9Aba-2b^2B\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\int\sqrt{\sec(c+\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{(3Ab+7aB)b^2+a\left(3Ba^2+9Aba-2b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\int\sqrt{\csc\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(\int \frac{(3Ab+7aB)b^2+a\left(3Ba^2+9Aba-2b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{(3Ab+7aB)b^2+a\left(3Ba^2+9Aba-2b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3120

$$\frac{1}{3}\left(\int \frac{(3Ab+7aB)b^2+a(3Ba^2+9Aba-2b^2B)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(aA-bB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4534

$$\frac{1}{3}\left(-3(a^3B+3a^2Ab-3ab^2B-Ab^3)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a(3a^2B+9aAb-2b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-3(a^3B+3a^2Ab-3ab^2B-Ab^3)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(3a^2B+9aAb-2b^2B)\sin(c+dx)\sqrt{\sec(c+}}{d}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(-3(a^3B+3a^2Ab-3ab^2B-Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2a(3a^2B+9aAb-2b^2}{}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-3(a^3B+3a^2Ab-3ab^2B-Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a(3a^2B+9aAb}{}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3119

$$\frac{1}{3}\left(\frac{2a(3a^2B+9aAb-2b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^2(aA-bB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{2(a^3A+9a}{}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((-
6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b
^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a*(9*a*A*b + 3*a^2*B - 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (
2*a^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4513
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Sim
p[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[
a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
 f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &
& LeQ[n, -1]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(220) = 440$.

Time = 2609.82 (sec) , antiderivative size = 886, normalized size of antiderivative = 3.71

method	result	size
parts	Expression too large to display	886
default	Expression too large to display	1210

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*a^3*A*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*((2*cos(1/2*d*x+1
/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d+2*
(A*b^3+3*B*a*b^2)*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2*(3*A*a*b^2+3
*B*a^2*b)*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2*(3*A*a^2*b+B*a^3)*(-
2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d-2/3*b^3*B*((2*
cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*sin(1/2*d*x+1/2*...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.25

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{\sqrt{2}(-i\,Aa^3 - 9i\,Ba^2b - 9i\,Aab^2 - i\,Bb^3)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx\cdots}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(-I*A*a^3 - 9*I*B*a^2*b - 9*I*A*a*b^2 - I*B*b^3)*cos(d*x + c)
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*
a^3 + 9*I*B*a^2*b + 9*I*A*a*b^2 + I*B*b^3)*cos(d*x + c)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b
 - 3*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a^3 - 3*I
*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(B*b^3*cos(d*x
+ c)^2 + A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\cos(c + dx))^3 (A + B\cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)\, dx = \int (B\cos(dx + c) + A)(b\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}}\, dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+b\cos(c+d\,x))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= 4\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a^3b + \left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^2\,dx\right)b^4 + 4\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)a\,b^3 + 6\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a^2b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)a^4$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
4*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a**3*b + int(sqrt
(sec(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x)*b**4 + 4*int(sqrt(sec(c
+ d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*a*b**3 + 6*int(sqrt(sec(c + d*x
))*cos(c + d*x)**2*sec(c + d*x)**2,x)*a**2*b**2 + int(sqrt(sec(c + d*x))*s
ec(c + d*x)**2,x)*a**4
```

3.564 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 237

$$\begin{aligned}
&\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx \\
&= -\frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{2(9a^2Ab+Ab^3+3a^3B+3ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad+\frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2a^2(5aA-bB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

output
```
-2/5*(5*A*a^3-15*A*a*b^2-15*B*a^2*b-3*B*b^3)*cos(d*x+c)^(1/2)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/3*(9*A*a^2*b+A*b^3+3*B*a^3+
3*B*a*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x
+c)^(1/2)/d+2/15*b^2*(5*A*b+9*B*a)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*a^2*(
5*A*a-B*b)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/5*b*B*(b+a*sec(d*x+c))^2*sin(d*
x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 10.85 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(12(-5a^3A+15aAb^2+15a^2bB+3b^3B)\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+20(9a^2Ab+Ab}{30}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x
]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(12*(-5*a^3*A + 15*a*A*b^2 + 15*a^2
*b*B + 3*b^3*B)*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*A*b + A*b^3 + 3*a^3*
B + 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (2*(10*b^2*(A*b + 3*a*B)*Cos[c
+ d*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/Sqrt
[Cos[c + d*x]]))/(30*d)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3439, 3042, 4513, 27, 3042, 4562, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3439$$

$$\int \frac{(a\sec(c+dx)+b)^3(A\sec(c+dx)+B)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$\big\downarrow$ 3042

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$\big\downarrow$ 4513

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{(b+a\sec(c+dx))\left(a(5aA-bB)\sec^2(c+dx)+\left(5Ba^2+10Aba+3b^2B\right)\sec(c+dx)+b(5Ab+9aB)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx$$

$\big\downarrow$ 27

$$\frac{1}{5}\int \frac{(b+a\sec(c+dx))\left(a(5aA-bB)\sec^2(c+dx)+\left(5Ba^2+10Aba+3b^2B\right)\sec(c+dx)+b(5Ab+9aB)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\int \frac{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(5aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Ba^2+10Aba+3b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b(5\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 4562

$$\frac{1}{5}\left(\frac{2b^2(9aB+5Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{3a^2(5aA-bB)\sec^2(c+dx)+5\left(3Ba^3+9Aba^2+3b^2Ba+Ab^3\right)\sec(c\right.}{2\sqrt{\sec(c+dx)}}$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3a^2(5aA-bB)\sec^2(c+dx)+5\left(3Ba^3+9Aba^2+3b^2Ba+Ab^3\right)\sec(c+dx)+3b\left(14Ba^2+15Aba+3b\right.}{\sqrt{\sec(c+dx)}}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3a^2(5aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+5\left(3Ba^3+9Aba^2+3b^2Ba+Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(14Ba^2+1\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4535

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(5aA-bB)\sec^2(c+dx)+3b\left(14Ba^2+15Aba+3b^2B\right)}{\sqrt{\sec(c+dx)}}dx+5\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)\int\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(5aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(14Ba^2+15Aba+3b^2B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5\left(3a^3B+9a^2Ab+3ab^2B+Ab\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4258

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(5aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(14Ba^2+15Aba+3b^2B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5\left(3a^3B+9a^2Ab+3ab^2B+Ab\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(5aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(14Ba^2+15Aba+3b^2B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5\left(3a^3B+9a^2Ab+3ab^2B+Ab\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2(5aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(14Ba^2+15Aba+3b^2B\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10\left(3a^3B+9a^2Ab+3ab^2B+A\right.}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4534

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{6a^2(5aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2(5aA-bB)\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{6a^2(5aA-bB)}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(-3\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a^2(5aA}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{6a^2(5aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{10\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2
*b^2*(5*A*b + 9*a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((-6*(5*a^3*
A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b
^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (6*a^2*(5*a*A - b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4513
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Sim
p[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[
a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
 f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &
& LeQ[n, -1]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(216) = 432$.

Time = 13.83 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.70

method	result
default	$-\frac{2\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}b^{3}+20A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b^{3}+60B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a\,b^{2}+24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/15*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+20*A*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3+60*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4*a*b^2+24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-30*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3-10*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2*b^3+45*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b
^2-30*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-6*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*b^3+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*a*b^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-9*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.14

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx =$$

$$-\frac{5\sqrt{2}(3i\,Ba^3+9i\,Aa^2b+3i\,Bab^2+i\,Ab^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-1/15*(5*sqrt(2)*(3*I*B*a^3 + 9*I*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-3*I*B*a^3
 - 9*I*A*a^2*b - 3*I*B*a*b^2 - I*A*b^3)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b
^2 - 3*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*A*a^3 + 15*I*B*a^2*b + 15*I*A*a*
b^2 + 3*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - 2*(3*B*b^3*cos(d*x + c)^2 + 15*A*a^3 + 5*(3*B*a
*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=4\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)a^3b$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^4\sec(dx+c)\,dx\right)b^4$$
$$+4\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)a\,b^3$$
$$+6\left(\int\sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a^2b^2$$
$$+\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)a^4$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
4*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a**3*b + int(sqrt(se
c(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x)*b**4 + 4*int(sqrt(sec(c + d*x)
)*cos(c + d*x)**3*sec(c + d*x),x)*a*b**3 + 6*int(sqrt(sec(c + d*x))*cos(c
+ d*x)**2*sec(c + d*x),x)*a**2*b**2 + int(sqrt(sec(c + d*x))*sec(c + d*x),
x)*a**4
```

3.565 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 245

$$\begin{aligned}&\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx\\&=\frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\&\quad+\frac{2(21a^3A+21aAb^2+21a^2bB+5b^3B)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\&\quad+\frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(21aAb+18a^2B+5b^2B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\\&\quad+\frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output

```
2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^3+21*A*a*b^2+21*B*
a^2*b+5*B*b^3)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec
(d*x+c)^(1/2)/d+2/35*b^2*(7*A*b+11*B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21
*b*(21*A*a*b+18*B*a^2+5*B*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/7*b*B*(b+a*
sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 10.79 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{\sqrt{\sec(c+dx)}\Big(84(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|2\big)+20(21a^3A+21aA}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x
]
```

output
```
(Sqrt[Sec[c + d*x]]*(84*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(21*a^3*A + 21*a*A*b^2 + 21*a
^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(42*b*(
A*b + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b + 42*a^2*B + 13*b^2*B + 3*b^2*B*Co
s[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.576, Rules used = {3042, 3439, 3042, 4513, 27, 3042, 4562, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+b)^3(A\sec(c+dx)+B)}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4513}$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}-$$

$$\frac{2}{7}\int -\frac{(b+a\sec(c+dx))\left(a(7aA+bB)\sec^2(c+dx)+\left(7Ba^2+14Aba+5b^2B\right)\sec(c+dx)+b(7Ab+11aB)\right)}{2\sec^{\frac{5}{2}}(c+dx)}d$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(b+a\sec(c+dx))\left(a(7aA+bB)\sec^2(c+dx)+\left(7Ba^2+14Aba+5b^2B\right)\sec(c+dx)+b(7Ab+11aB)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(7aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Ba^2+14Aba+5b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b(7\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4562}$$

$$\frac{1}{7}\left(\frac{2b^2(11aB+7Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5a^2(7aA+bB)\sec^2(c+dx)+7\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\text{se}}{2\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5a^2(7aA+bB)\sec^2(c+dx)+7\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\sec(c+dx)+5b\left(18Ba^2+21Aba+\right.}{\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5a^2(7aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7\left(5Ba^3+15Aba^2+9b^2Ba+3Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)+5b\left(18Ba^2+\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4535

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5a^2(7aA+bB)\sec^2(c+dx)+5b\left(18Ba^2+21Aba+5b^2B\right)}{\sec^{\frac{3}{2}}(c+dx)}dx+7\left(5a^3B+15a^2Ab+9ab^2B+3Ab^3\right)\int\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5a^2(7aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\left(18Ba^2+21Aba+5b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+7\left(5a^3B+15a^2Ab+9ab^2B+3A\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5a^2(7aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\left(18Ba^2+21Aba+5b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+7\left(5a^3B+15a^2Ab+9ab^2B+3A\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5a^2(7aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\left(18Ba^2+21Aba+5b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+7\left(5a^3B+15a^2Ab+9ab^2B+3A\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5a^2(7aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\left(18Ba^2+21Aba+5b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{14\left(5a^3B+15a^2Ab+9ab^2B+3\right.}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4533

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\int\sqrt{\sec(c+dx)}dx+\frac{10b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{10b\left(18a^2B+21aAb+5b^2B\right)\sin(c+\right.\right.$$
$$3d\sqrt{\sec(c+dx)}$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4258

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{10b\left(18a^2B+2\right.}{3d}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10b\left(18a^2\right.}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3120

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{10b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{10\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{3d}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((2
*b^2*(7*A*b + 11*a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((14*(15*a^
2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B
+ 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*d) + (10*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sin[c + d*x])/(3*d*Sqrt[Se
c[c + d*x]]))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4513
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Sim
p[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[
a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
 f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &
& LeQ[n, -1]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(224) = 448$.

Time = 20.32 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.71

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}b^{3}+\left(-168A\,b^{3}-504Ba\,b^{2}-360b^{3}B\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-168*A*b^3-504*B*a*b^2-360*B*b^3)
*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*
b+504*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a
*b^2-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)+105*a^3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*a*b^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^3+105*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.20

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(21i\,Aa^3+21i\,Ba^2b+21i\,Aab^2+5i\,Bb^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/105*(5*sqrt(2)*(21*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 5*I*B*b^3)*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*
I*A*a^3 - 21*I*B*a^2*b - 21*I*A*a*b^2 - 5*I*B*b^3)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-5*I*B*a^3 - 15*I*A*a^2*b
 - 9*I*B*a*b^2 - 3*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(5*I*B*a^3 + 15*I*A*a^2*b
 + 9*I*B*a*b^2 + 3*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*B*b^3*cos(d*x + c)^3 + 21*(3*B
*a*b^2 + A*b^3)*cos(d*x + c)^2 + 5*(21*B*a^2*b + 21*A*a*b^2 + 5*B*b^3)*cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^3\,\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sqrt(sec(c + d*x)),
x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}\,(a+b\cos(c+dx))^3\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\left(\int \sqrt{\sec(dx+c)}dx\right)a^4+4\left(\int \sqrt{\sec(dx+c)}\cos(dx+c)\,dx\right)a^3b$$
$$+\left(\int \sqrt{\sec(dx+c)}\cos(dx+c)^4\,dx\right)b^4+4\left(\int \sqrt{\sec(dx+c)}\cos(dx+c)^3\,dx\right)a\,b^3$$
$$+6\left(\int \sqrt{\sec(dx+c)}\cos(dx+c)^2\,dx\right)a^2b^2$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x)),x)*a**4 + 4*int(sqrt(sec(c + d*x))*cos(c + d*x),x)*
a**3*b + int(sqrt(sec(c + d*x))*cos(c + d*x)**4,x)*b**4 + 4*int(sqrt(sec(c
 + d*x))*cos(c + d*x)**3,x)*a*b**3 + 6*int(sqrt(sec(c + d*x))*cos(c + d*x)
**2,x)*a**2*b**2
```

3.566 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 295

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(27aAb+22a^2B+7b^2B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

output

```
2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*cos(d*x+c)^(1/2)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^2*b+5*A*b^3+7*B
*a^3+15*B*a*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*s
ec(d*x+c)^(1/2)/d+2/63*b^2*(9*A*b+13*B*a)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/
45*b*(27*A*a*b+22*B*a^2+7*B*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*(21*A*
a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/9*b*B*(b
+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 11.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(168(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+120(21a^2Ab+$$

input
```
Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]]
,x]
```

output
```
(Sqrt[Sec[c + d*x]]*(168*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(21*a^2*A*b + 5*A*b^3 + 7
*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*b*(
108*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(252*a^2*A*b + 78*A*b^3
 + 84*a^3*B + 234*a*b^2*B + 18*b^2*(A*b + 3*a*B)*Cos[2*(c + d*x)] + 7*b^3*
B*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 3439, 3042, 4513, 27, 3042, 4562, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(a\sec(c+dx)+b)^3(A\sec(c+dx)+B)}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$\downarrow$ 4513

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}-$$
$$\frac{2}{9}\int -\frac{(b+a\sec(c+dx))\left(3a(3aA+bB)\sec^2(c+dx)+\left(9Ba^2+18Aba+7b^2B\right)\sec(c+dx)+b(9Ab+13aB)\right)}{2\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\int \frac{(b+a\sec(c+dx))\left(3a(3aA+bB)\sec^2(c+dx)+\left(9Ba^2+18Aba+7b^2B\right)\sec(c+dx)+b(9Ab+13aB)\right)}{\sec^{\frac{7}{2}}(c+dx)}d$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\int \frac{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(9Ba^2+18Aba+7b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b(\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4562

$$\frac{1}{9}\left(\frac{2b^2(13aB+9Ab)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int -\frac{21a^2(3aA+bB)\sec^2(c+dx)+9\left(7Ba^3+21Aba^2+15b^2Ba+5Ab^3\right)}{2\sec^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{21a^2(3aA+bB)\sec^2(c+dx)+9\left(7Ba^3+21Aba^2+15b^2Ba+5Ab^3\right)\sec(c+dx)+7b\left(22Ba^2+27Aba\right.}{\sec^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{21a^2(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+9\left(7Ba^3+21Aba^2+15b^2Ba+5Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)+7b\left(22Ba^2\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4535

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{21a^2(3aA+bB)\sec^2(c+dx)+7b\left(22Ba^2+27Aba+7b^2B\right)}{\sec^{\frac{5}{2}}(c+dx)}dx+9\left(7a^3B+21a^2Ab+15ab^2B+5Ab^3\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{21a^2(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\left(22Ba^2+27Aba+7b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+9\left(7a^3B+21a^2Ab+15ab^2B+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4256

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{21a^2(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\left(22Ba^2+27Aba+7b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+9\left(7a^3B+21a^2Ab+15ab^2B+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{21a^2(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\left(22Ba^2+27Aba+7b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+9\left(7a^3B+21a^2Ab+15ab^2B+\right.\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{21a^2(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\left(22Ba^2+27Aba+7b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+9\left(7a^3B+21a^2Ab+15ab^2B+\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{21a^2(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\left(22Ba^2+27Aba+7b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+9\left(7a^3B+21a^2Ab+15ab^2B+\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{21a^2(3aA+bB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\left(22Ba^2+27Aba+7b^2B\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+9\left(7a^3B+21a^2Ab+15ab^2B+\right.\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{14b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{14b\left(22a^2B+27aAb+7b^2B\right)\sin(c+}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{14b(22a^2B+2}{5d}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{14b(22a^2}{}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{14b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{42\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{5d}\right.\right.$$
$$\frac{2bB\sin(c+dx)(a\sec(c+dx)+b)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((2
*b^2*(9*A*b + 13*a*B)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((42*(15*a^
3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (14*b*(27*a*A*b + 22*a^2*B + 7*b^2
*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 9*(21*a^2*A*b + 5*A*b^3 + 7*a
^3*B + 15*a*b^2*B)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4513
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Sim
p[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[
a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
 f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &
& LeQ[n, -1]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(270) = 540$.

Time = 24.90 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}b^{3}+\left(720A\,b^{3}+2160Ba\,b^{2}+2240b^{3}B\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(720*A*b^3+2160*B*a*b^2+2240*B*
b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-151
2*B*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
+(1260*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+
952*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b
^2-240*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b^2+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{15\sqrt{2}(7i\,Ba^3+21i\,Aa^2b+15i\,Bab^2+5i\,Ab^3)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/315*(15*sqrt(2)*(7*I*B*a^3 + 21*I*A*a^2*b + 15*I*B*a*b^2 + 5*I*A*b^3)*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-7*
I*B*a^3 - 21*I*A*a^2*b - 15*I*B*a*b^2 - 5*I*A*b^3)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-15*I*A*a^3 - 27*I*B*a^2*
b - 27*I*A*a*b^2 - 7*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(15*I*A*a^3 + 27*I*B*a^
2*b + 27*I*A*a*b^2 + 7*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*B*b^3*cos(d*x + c)^4 + 45*
(3*B*a*b^2 + A*b^3)*cos(d*x + c)^3 + 7*(27*B*a^2*b + 27*A*a*b^2 + 7*B*b^3)
*cos(d*x + c)^2 + 15*(7*B*a^3 + 21*A*a^2*b + 15*B*a*b^2 + 5*A*b^3)*cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\,(a+b\cos{(c+dx)})^3}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3/sqrt(sec(c + d*x)),
x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x
)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a^4 + 4\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)a^3b + \left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^4}{\sec(dx+c)}dx\right)b^4 + 4\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right)a\,b^3 + 6\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)a^2b^2$$

input
```
int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output

```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a**4 + 4*int((sqrt(sec(c + d*x))*co
s(c + d*x))/sec(c + d*x),x)*a**3*b + int((sqrt(sec(c + d*x))*cos(c + d*x)*
*4)/sec(c + d*x),x)*b**4 + 4*int((sqrt(sec(c + d*x))*cos(c + d*x)**3)/sec(
c + d*x),x)*a*b**3 + 6*int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*
x),x)*a**2*b**2
```

3.567 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 210

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx \\
&= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\
&\quad+\frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\
&\quad+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} \\
&\quad-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}
\end{aligned}$$

output
```
2*(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x
+c)^(1/2)/a^2/d+2/3*A*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/
2))*sec(d*x+c)^(1/2)/a/d+2*b*(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2
*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a+b)/d-2*(A*b-B*a)*se
c(d*x+c)^(1/2)*sin(d*x+c)/a^2/d+2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 13.65 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.07

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx =$$
$$-\frac{\cot(c+dx)\left(-a^2A\sec^{\frac{5}{2}}(c+dx)+a^2A\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)-6a(-Ab+aB)E\left(\arcsin\left(\sqrt{\sec\ldots}\right.\right.\right.}{\ldots}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x
]
```

output
```
-1/3*(Cot[c + d*x]*(-(a^2*A*Sec[c + d*x]^(5/2)) + a^2*A*Cos[2*(c + d*x)]*S
ec[c + d*x]^(5/2) - 6*a*(-(A*b) + a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[-Tan[c + d*x]^2] - 2*(3*A*b^2 + a^2*(A - 3*B) + 3*a*b*(A - B))
*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*A*b^2
*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
- 6*a*b*B*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c +
 d*x]^2]))/(a^3*d)
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4521, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A\sec(c+dx)+B)}{a\sec(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{4521}$$

$$\frac{2\int \frac{\sqrt{\sec(c+dx)}(-3(Ab-aB)\sec^2(c+dx)+aA\sec(c+dx)+Ab)}{2(b+a\sec(c+dx))}dx}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(-3(Ab-aB)\sec^2(c+dx)+aA\sec(c+dx)+Ab)}{b+a\sec(c+dx)}dx}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+aA\csc\left(c+dx+\frac{\pi}{2}\right)+Ab\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{4590}$$

$$\frac{\frac{2\int \frac{\left(Aa^2-3bBa+3Ab^2\right)\sec^2(c+dx)+a(4Ab-3aB)\sec(c+dx)+3b(Ab-aB)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(Aa^2-3bBa+3Ab^2\right)\sec^2(c+dx)+a(4Ab-3aB)\sec(c+dx)+3b(Ab-aB)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\left(Aa^2-3bBa+3Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(4Ab-3aB)\csc\left(c+dx+\frac{\pi}{2}\right)+3b(Ab-aB)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{6(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 4594

$$\frac{\frac{\frac{\int \frac{3(Ab-aB)b^2+aA\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}+3b(Ab-aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{a}-\frac{6(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{3(Ab-aB)b^2+aA\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+3b(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{6(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{3b^2(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+aAb^2\int \sqrt{\sec(c+dx)}dx}{b^2}+3b(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{6(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3b^2(Ab-aB)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+aAb^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+3b(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{6(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{3b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+aAb^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}+3b(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-6(Ab-aB)}{3a}$$

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+aAb^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+3b(Ab-aB)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-6(}{3a}$$

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{aAb^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}+3b(Ab-aB)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-6(Ab-}{3a}$$

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3120

$$\frac{\frac{3b(Ab-aB)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aAb^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}}{a}-6(Ab}{3a}$$

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 4336

$$\frac{\frac{3b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{\frac{6b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aAb^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}}{b^2}}{a}}{3a}$$

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3042

$$\frac{\frac{3b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{6b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aAb^2\sqrt{\cos(c+dx}}{}}{b^2}}{a}}{3a}$$

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3284

$$\frac{\frac{\frac{\frac{6b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aAb^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}+\frac{6b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{}{a}\right.}{d(a+b)}}{a}}{3a}$$

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

output
```
(2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((((6*b^2*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*A*b^
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2
+ (6*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/a - (6*(A*b - a*B)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(a*d))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4521
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Simp[d^2/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
 n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B
, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m +
n, 0] &&  !IGtQ[m, 1]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(195) = 390$.

Time = 15.18 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.10

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\cdots}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\cdots}}\cdots\right.}{a}\cdots}{\cdots}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)/a^2/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
-4*b^2*(A*b-B*a)/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x)),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \int \sqrt{\sec(dx + c)}\,\sec(dx + c)^2\,dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)
```

3.568 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 126

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx &= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a+b)d} \\ &\quad + \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}\end{aligned}$$

output
```
-2*A*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/a/d-2*(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b
),2^(1/2))*sec(d*x+c)^(1/2)/a/(a+b)/d+2*A*sec(d*x+c)^(1/2)*sin(d*x+c)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx =$$
$$-\frac{2\cos(2(c+dx))\csc(c+dx)\left(aAE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-(aA+Ab-aB)\operatorname{EllipticF}\left(\operatorname{arcsi}\right.\right.}{a^2d\,(-}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x
]
```

output
```
(-2*Cos[2*(c + d*x)]*Csc[c + d*x]*(a*A*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]
], -1] - (a*A + A*b - a*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*
b - a*B)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]*
Sqrt[-Tan[c + d*x]^2])/(a^2*d*(-2 + Sec[c + d*x]^2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3439, 3042, 4521, 27, 3042, 4594, 27, 3042, 4258, 3042, 3119, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3439$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A\sec(c+dx)+B)}{a\sec(c+dx)+b}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$\Big\downarrow$ 4521

$$\frac{2\int -\frac{(Ab-aB)\sec^2(c+dx)+aA\sec(c+dx)+Ab}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}$$

$\Big\downarrow$ 27

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{(Ab-aB)\sec^2(c+dx)+aA\sec(c+dx)+Ab}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+aA\csc\left(c+dx+\frac{\pi}{2}\right)+Ab}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$\Big\downarrow$ 4594

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(Ab-aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+\frac{\int \frac{Ab^2}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}$$

$\Big\downarrow$ 27

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(Ab-aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+A\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+A\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$\Big\downarrow$ 4258

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-}{\frac{(Ab-aB)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(Ab-aB)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

$$\Big\downarrow \text{ 4336}$$

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

output
```
-(((2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/a) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4521
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Simp[d^2/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
 n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B
, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m +
n, 0] &&  !IGtQ[m, 1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(120) = 240$.

Time = 5.15 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.38

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2A\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin(\cdots}\right.}{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos(\cdots}}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*A/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+4*(A*b-B*a)/a/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**(3/2)/(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x)),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \int \sqrt{\sec(dx + c)}\,\sec(dx + c)\,dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x),x)
```

3.569 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\mathrm{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b(a+b)d}$$

output
```
2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/
2)/b/d+2*(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b
),2^(1/2))*sec(d*x+c)^(1/2)/b/(a+b)/d
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \frac{2\cot(c+dx)\left(Ab\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)+(-Ab+aB)\,\mathrm{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(\right.\right.\right.}{abd}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x
]
```

output
```
(2*Cot[c + d*x]*(A*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (-(A*b) +
 a*B)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*
x]^2])/(a*b*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3439, 3042, 4526, 3042, 4258, 3042, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sqrt{\sec(c+dx)}(A\sec(c+dx)+B)}{a\sec(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{4526}$$

$$\frac{(Ab-aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b}+\frac{B\int \sqrt{\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}$$

$$\Big\downarrow \text{4336}$$

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

output
```
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d
) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4526
```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[A/a   Int[
(d*Csc[e + f*x])^n, x], x] - Simp[(A*b - a*B)/(a*d)   Int[(d*Csc[e + f*x])^
(n + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x]
&& NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(96) = 192$.

Time = 4.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.15

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(A\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)b+B\,\mathrm{Ellipt}}{(a-b)b\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))*b+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))*a)/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{(A+B\cos{(c+dx)})\sqrt{\sec{(c+dx)}}}{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sqrt{\sec{(dx+c)}}}{b\cos{(dx+c)}+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sqrt{\sec{(dx+c)}}}{b\cos{(dx+c)}+a}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x)),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \sqrt{\sec\left(dx+c\right)}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x)),x)
```

3.570 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2d} - \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d}$$

output
```
2*B*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2
)/b/d+2*(A*b-B*a)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*
sec(d*x+c)^(1/2)/b^2/d-2*a*(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d
*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Mathematica [A] (warning: unable to verify)

Time = 21.56 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.48

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\cot(c+dx)\left(-bB\sec^{\frac{3}{2}}(c+dx)-bB\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+bB\sec^{\frac{7}{2}}(c+dx)+bB\cos(2(c+dx)\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x
]
```

output
```
(Cot[c + d*x]*(-(b*B*Sec[c + d*x]^(3/2)) - b*B*Cos[2*(c + d*x)]*Sec[c + d*
x]^(3/2) + b*B*Sec[c + d*x]^(7/2) + b*B*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2
) - 2*b*B*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
+ 2*b*B*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] +
2*A*b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x
]^2] - 2*a*B*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2]))/(b^2*d)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4526, 3042, 4258, 3042, 3119, 4335, 3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \text{4526}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\sec(c+dx)}}{b+a\sec(c+dx)}dx}{b}+\frac{B\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$$\Big\downarrow \text{4335}$$

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$\Big\downarrow$ 3282

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}\right)}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$\Big\downarrow$ 3042

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$\Big\downarrow$ 3120

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

$\Big\downarrow$ 3284

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}\right)}{b}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d
) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]*((2*EllipticF[(c + d*x)/2, 2])/(b*d) -
 (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))*Sqrt[Sec[c
 + d*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4335
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d)   Int[
Sqrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

rule 4526
```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[A/a   Int[
(d*Csc[e + f*x])^n, x], x] - Simp[(A*b - a*B)/(a*d)   Int[(d*Csc[e + f*x])^
(n + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x]
&& NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(142) = 284$.

Time = 6.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.98

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab-A\,\mathrm{EllipticF}\left(c\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)/sec(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-A*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+B
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\, dx = \int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\, dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x
)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sec\left(dx+c\right)}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)
```

3.571 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 197

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx
&= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} \\
&\quad - \frac{2(3aAb-3a^2B-b^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3b^3d} \\
&\quad + \frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d} \\
&\quad + \frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}
\end{aligned}$$

output
```
2*(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x
+c)^(1/2)/b^2/d-2/3*(3*A*a*b-3*B*a^2-B*b^2)*cos(d*x+c)^(1/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b^3/d+2*a^2*(A*b-B*a)*cos(d*x+c
)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/
b^3/(a+b)/d+2/3*B*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 542 vs. $2(197) = 394$.

Time = 7.80 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.75

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\frac{2(-3Ab+aB)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+}}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$

$$+\frac{B\sqrt{\sec(c+dx)}\sin(2(c+dx))}{3bd}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x
]
```

output
```
-1/6*((2*(-3*A*b + a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)) - (4*B*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d
*x])/((a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-3*A*b + 3*a*B)*Cos[2
*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2
*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[
c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(b*d) + (B*Sqrt[Sec[
c + d*x]]*Sin[2*(c + d*x)])/(3*b*d)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4522, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \text{ 4522}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{aB\sec^2(c+dx)+bB\sec(c+dx)+3(Ab-aB)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{3b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{aB\sec^2(c+dx)+bB\sec(c+dx)+3(Ab-aB)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{3b}+\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{aB\csc\left(c+dx+\frac{\pi}{2}\right)^2+bB\csc\left(c+dx+\frac{\pi}{2}\right)+3(Ab-aB)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}+\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4594}$$

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int \frac{3b(Ab-aB)-\left(-3Ba^2+3Aba-b^2B\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}}{3b}+\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int \frac{3b(Ab-aB)+\left(3Ba^2-3Aba+b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{3b}+\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4274

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-\left(-3a^2B+3aAb-b^2B\right)\int \sqrt{\sec(c+dx)}dx}{b^2}}{3b}+\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b(Ab-aB)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(-3a^2B+3aAb-b^2B\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{3b}+\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4258

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{b^2}}{3b}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{b^2}}{3b}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3119

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{\sin(}}}{b^2}}{3b}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3120

$$\frac{\frac{3a^2(Ab-aB)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(\right.}{d}}{b^2}}{3b}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4336

$$\frac{\frac{3a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{\frac{6b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2\left(-3a^2B+3aAb-b^2B\right)}{}}{b^2}}{3b}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{3a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{\frac{6b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2\left(-3a^2B+3a\right.}{}}{b^2}}{3b}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3284

$$\frac{\frac{6a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)}+\frac{\frac{6b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2\left(-3a^2B+3aAb-b^2B\right)\sqrt{c}}{}}{b^2}}{3b}$$

$$\frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

output
```
(((6*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d - (2*(3*a*A*b - 3*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*a^2*(A*b - a*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^
2*(a + b)*d))/(3*b) + (2*B*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4522
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Sim
p[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*
n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
 && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(184) = 368$.

Time = 8.74 (sec) , antiderivative size = 822, normalized size of antiderivative = 4.17

method	result	size
default	Expression too large to display	822

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)/sec(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2+4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4*b^3+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^3-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b+2*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*a*b^2-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
b^3-3*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
B*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*B*(si...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\cos{(c+dx)}}{(a+b\cos{(c+dx)})\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))*sec(c + d*x)**(3/2)),
x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos{(dx+c)}+A}{(b\cos{(dx+c)}+a)\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\cos(c+dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+b\cos(c+dx))}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)
```

3.572 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 405

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx \\
&= \frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)d} \\
&+ \frac{(2a^2A-5Ab^2+3abB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2(a^2-b^2)d} \\
&+ \frac{b(7a^2Ab-5Ab^3-5a^3B+3ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} \\
&- \frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3(a^2-b^2)d} \\
&+ \frac{(2a^2A-5Ab^2+3abB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d} \\
&+ \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
(4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*A*a^2-5*A*b^2+3
*B*a*b)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)
^(1/2)/a^2/(a^2-b^2)/d+b*(7*A*a^2*b-5*A*b^3-5*B*a^3+3*B*a*b^2)*cos(d*x+c)^
(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^
3/(a-b)/(a+b)^2/d-(4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*sec(d*x+c)^(1/2)*s
in(d*x+c)/a^3/(a^2-b^2)/d+1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*sec(d*x+c)^(3/2)*s
in(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b
^2)/d/(b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.57 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.81

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{2(-4a^4A-44a^2Ab^2+45Ab^4+30a^3bB-27ab^3B)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right.}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(\frac{(-4a^2Ab+5Ab^3+2a^3B-3ab^2B)\sin(c+dx)}{a^3(a^2-b^2)}+\frac{-Ab^3\sin(c+dx)+ab^2B\sin(c+dx)}{a^2(a^2-b^2)(a+b\cos(c+dx))}+\frac{2A\tan(c+dx)}{3a^2}\right)}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2
,x]
```

output
```
((2*(-4*a^4*A - 44*a^2*A*b^2 + 45*A*b^4 + 30*a^3*b*B - 27*a*b^3*B)*Cos[c +
 d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), Ar
cSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^
2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-28*a
^3*A*b + 40*a*A*b^3 + 12*a^4*B - 24*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-
(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c
 + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (
(-12*a^2*A*b^2 + 15*A*b^4 + 6*a^3*b*B - 9*a*b^3*B)*Cos[2*(c + d*x)]*(b + a
*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a
 - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[S
ec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Se
c[c + d*x]]*(((-4*a^2*A*b + 5*A*b^3 + 2*a^3*B - 3*a*b^2*B)*Sin[c + d*x])/(
a^3*(a^2 - b^2)) + (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x])/(a^2*(a^
2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^2)))/d
```

Rubi [A] (verified)

Time = 3.16 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.96, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 3439, 3042, 4517, 27, 3042, 4590, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \ 4517$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int -\frac{\sec^{\frac{3}{2}}(c+dx)\left(\left(2Aa^2+3bBa-5Ab^2\right)\sec^2(c+dx)-2a(Ab-aB)\sec(c+dx)+3b(Ab-aB)\right)}{2(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\left(2Aa^2+3bBa-5Ab^2\right)\sec^2(c+dx)-2a(Ab-aB)\sec(c+dx)+3b(Ab-aB)\right)}{b+a\sec(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\left(2Aa^2+3bBa-5Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)+3b(Ab-aB)\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4590$$

$$\frac{\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)\sec^2(c+dx)+2a\left(Aa^2-3bBa+2Ab^2\right)\sec(c+dx)+b\left(2Aa^2+3bBa-5Ab^2\right)\right)}{2(b+a\sec(c+dx))}dx}{3a}+\frac{2\left(2a^2A+3abB-5Ab^2\right)}{3a}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)\sec^2(c+dx)+2a\left(Aa^2-3bBa+2Ab^2\right)\sec(c+dx)+b\left(2Aa^2+3bBa-5Ab^2\right)\right)}{b+a\sec(c+dx)}dx}{3a}+\frac{2\left(2a^2A+3abB-5Ab^2\right)\mathrm{si}}{3ad}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(Aa^2-3bBa+2Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(2Aa^2+3bBa-5Ab^2\right)\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2(2a^2A+3a}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 4590$$

$$\frac{\frac{\frac{2\int \frac{\left(2Aa^4-12bBa^3+16Ab^2a^2+9b^3Ba-15Ab^4\right)\sec^2(c+dx)+2a\left(-3Ba^3+7Aba^2+6b^2Ba-10Ab^3\right)\sec(c+dx)+3b\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)}{2\sqrt{\sec(c+dx)}\left(b+a\sec(c+dx)\right)}dx}{a}-\frac{6\left(-2a^3B\right.}{}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{\left(2Aa^4-12bBa^3+16Ab^2a^2+9b^3Ba-15Ab^4\right)\sec^2(c+dx)+2a\left(-3Ba^3+7Aba^2+6b^2Ba-10Ab^3\right)\sec(c+dx)+3b\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)}{\sqrt{\sec(c+dx)}\left(b+a\sec(c+dx)\right)}dx}{a}-\frac{6\left(-2a^3B+\right.}{}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{\left(2Aa^4-12bBa^3+16Ab^2a^2+9b^3Ba-15Ab^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(-3Ba^3+7Aba^2+6b^2Ba-10Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{6}{}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 4594$$

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+\frac{\int \frac{3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)b^2+a\left(2Aa^2+3bBa-5Ab^2\right)\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}-\frac{6\left(-2a^3B+4a^2Ab+3\right.}{}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3\left(-2Ba^3+4Aba^2+3b^2Ba-5Ab^3\right)b^2+a\left(2Aa^2+3bBa-5Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}-\frac{6\left(-2a^3B+\right.}{}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2A+3abB-5Ab^2\right)\int \sqrt{\sec(c+dx)}dx+3b^2\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{ab^2\left(2a^2A+3abB-5Ab^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b^2\left(-2a^3B+4a^2Ab+3ab^2B-5Ab^3\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right.}{b+a\csc\left(c+d\right.}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$\Big\downarrow$ 4258

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2A+3abB-5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3b^2\left(-2a^3B+4a^2Ab+3ab^2B-\right.}{b^2}}{a}}{3a}}{2a\left(a\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2A+3abB-5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2\left(-2a^3B+4a^2Ab+3ab\right.}{b^2}}{a}}{3a}}{2a}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2A+3abB-5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^2\left(-2a^3B+4a^2Ab+3a\right.}{}}{b^2}}{a}}{3a}}{2a(}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2ab^2\left(2a^2A+3abB-5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^2\left(-2a^3B+4a^2Ab+\right.}{}}{b^2}}{a}}{3a}}{2a}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 4336$$

$$\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{2ab^2\left(2a^2A+3abB-5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(\right.}{d}}{a}}{3a}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ab^2\left(2a^2A+3abB-5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elli}}{d}}{a}}{3a}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$\big\downarrow$ 3284

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+$$

$$\frac{\frac{2\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{\frac{\frac{6b\left(-5a^3B+7a^2Ab+3ab^2B-5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2ab^2\left(2a^2A+\right.}{}}{}}{}}{}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Se
c[c + d*x])) + ((2*(2*a^2*A - 5*A*b^2 + 3*a*b*B)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*a*d) + ((((6*b^2*(4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*b^
2*(2*a^2*A - 5*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*
b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/((a + b)*d))/a - (6*(4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d))/(3*a))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4517
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[d/(b*(m + 1)*(a^2 - b^2))   Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(
n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2
*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[
n, 1]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(388) = 776$.

Time = 42.84 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.48

method	result	size
default	Expression too large to display	1005

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(2*A*b-B*a)/a^3/sin(1/2*d*x+1/2*c)
^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))-4*b^2*(2*A*b-B*a)/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(A*b-B*a)*b/
a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b
/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^2,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^2}{\cos\left(dx+c\right)b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(cos(c + d*x)*b + a),x)
```

3.573 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 316

$$\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx
&= -\frac{(2a^2A-3Ab^2+abB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)d} \\
&\quad+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a^2-b^2)d} \\
&\quad-\frac{(5a^2Ab-3Ab^3-3a^3B+ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} \\
&\quad+\frac{(2a^2A-3Ab^2+abB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2(a^2-b^2)d} \\
&\quad+\frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
-(2*A*a^2-3*A*b^2+B*a*b)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+(A*b-B*a)*cos(d*x+c)^(1/2)*InverseJ
acobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(5*A*a^2*b-3
*A*b^3-3*B*a^3+B*a*b^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b
/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a-b)/(a+b)^2/d+(2*A*a^2-3*A*b^2+B*a*
b)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*sec(d*x+c)^(3/2
)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 681 vs. $2(316) = 632$.

Time = 7.32 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.16

$$\int \frac{(A + B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{\frac{2(10a^2Ab - 9Ab^3 - 4a^3B + 3ab^2B)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right) - \text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{} + \frac{\sqrt{\sec(c+dx)}\left(\frac{(2a^2A - 3Ab^2 + abB)\sin(c+dx)}{a^2(a^2-b^2)} + \frac{Ab^2\sin(c+dx) - abB\sin(c+dx)}{a(a^2-b^2)(a+b\cos(c+dx))}\right)}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2
,x]
```

output
```
-1/4*((2*(10*a^2*A*b - 9*A*b^3 - 4*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^2*(Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec
[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*
x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*a^3*A - 8*a*A*b^
2 + 4*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]
], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a +
 b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((2*a^2*A*b - 3*A*b^3 + a*b^2*B)*
Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a
*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 -
 Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(a^2*(a - b)*(a
 + b)*d) + (Sqrt[Sec[c + d*x]]*(((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])
/(a^2*(a^2 - b^2)) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x])/(a*(a^2 - b
^2)*(a + b*Cos[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.97, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4517, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \; 4517$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(\left(2Aa^2+bBa-3Ab^2\right)\sec^2(c+dx)-2a(Ab-aB)\sec(c+dx)+b(Ab-aB)\right)}{2(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(2Aa^2+bBa-3Ab^2\right)\sec^2(c+dx)-2a(Ab-aB)\sec(c+dx)+b(Ab-aB)\right)}{b+a\sec(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(2Aa^2+bBa-3Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)+b(Ab-aB)\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 4590$$

$$\frac{\frac{2\int -\frac{\left(-2Ba^3+4Aba^2+b^2Ba-3Ab^3\right)\sec^2(c+dx)+2a\left(Aa^2+bBa-2Ab^2\right)\sec(c+dx)+b\left(2Aa^2+bBa-3Ab^2\right)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}+\frac{2\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)\sqrt{\sec(c-}}{ad}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{\left(-2Ba^3+4Aba^2+b^2Ba-3Ab^3\right)\sec^2(c+dx)+2a\left(Aa^2+bBa-2Ab^2\right)\sec(c+dx)+b\left(2Aa^2+bBa-3Ab^2\right)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}d}{a}}{2a\left(a^2-b^2\right)}$$
$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{\left(-2Ba^3+4Aba^2+b^2Ba-3Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(Aa^2+bBa-2Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(2Aa^2+bBa-\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 4594}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{\int \frac{b^2\left(2Aa^2+bBa-3Ab^2\right)-ab^2(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}+\left(-3a^3B+5a^2Ab+ab^2B-3Ab^3\right)\int \frac{\sec^{\frac{3}{2}}(c+dx}{b+a\sec(c+}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{\int \frac{b^2\left(2Aa^2+bBa-3Ab^2\right)-ab^2(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+\left(-3a^3B+5a^2Ab+ab^2B-3Ab^3\right)\int \frac{\csc\left(c+\right.}{b+a\csc}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2\left(2a^2A+abB-3Ab^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-ab^2(Ab-aB)\int \sqrt{\sec(c+dx)}dx}{b^2}+\left(-3a^3B+5a^2Ab+ab^2B-3A\right.}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2\left(2a^2A+abB-3Ab^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab^2(Ab-aB)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\left(-3a^3B+5a^2Ab+a\right.}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2(2a^2A+abB-3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-ab^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{b^2}}{a}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2(2a^2A+abB-3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-ab^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{b^2}}{a}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$\downarrow$ 3119

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{\frac{2b^2(2a^2A+abB-3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-ab^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{b^2}}{a}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$\downarrow$ 3120

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b^2(2a^2A+abB-3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}}{a}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$\downarrow$ 4336

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{2b^2(}{}}{2a(a^2-b^2)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}d}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}+$$

$$\frac{\frac{2(2a^2A+abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{\frac{2b^2\left(2a^2A+abB-3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2ab^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{b^2}}{}}{2a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Se
c[c + d*x])) + (-((((2*b^2*(2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*b^2*(A*b - a*B)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (2*
(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/a) + (2*(2*a
^2*A - 3*A*b^2 + a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d))/(2*a*(a^2
- b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4517
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[d/(b*(m + 1)*(a^2 - b^2))   Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(
n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2
*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[
n, 1]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(305) = 610$.

Time = 7.31 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.71

method	result	size
default	Expression too large to display	856

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2/sin(1/
2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))-2*(A*b-B*a)/a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a
-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/
(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2)))+4*A*b^2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^2,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\cos(dx+c)\,b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(cos(c + d*x)*b + a),x)
```

3.574 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 260

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\,(a^2-b^2)\,d} \\ &\quad -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b\,(a^2-b^2)\,d} \\ &\quad +\frac{(3a^2Ab-Ab^3-a^3B-ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a-b)b(a+b)^2d} \\ &\quad +\frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\,(a^2-b^2)\,d(b+a\sec(c+dx))}\end{aligned}$$

output
```
-(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+
c)^(1/2)/a/(a^2-b^2)/d-(A*b-B*a)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(3*A*a^2*b-A*b^3-B*a^3-B*a*b
^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(
d*x+c)^(1/2)/a/(a-b)/b/(a+b)^2/d+b*(A*b-B*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/a
/(a^2-b^2)/d/(b+a*sec(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 639 vs. $2(260) = 520$.

Time = 7.23 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.46

$$\int \frac{(A + B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{2(-4a^2A+3Ab^2+abB)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\text{se}}}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+ \frac{\sqrt{\sec(c+dx)}\left(-\frac{(-Ab+aB)\sin(c+dx)}{a(a^2-b^2)}+\frac{-Ab\sin(c+dx)+aB\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}\right)}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2
,x]
```

output
```
((2*(-4*a^2*A + 3*A*b^2 + a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b
+ a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c +
 d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*a*A*b - 4*a^2*B)*Cos[c + d*x]^2*Ellip
ticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)) + ((A*b^2 - a*b*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a
*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*
EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a
 + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*
x]^2)))/(4*a*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-(((-(A*b) + a*B)*
Sin[c + d*x])/(a*(a^2 - b^2))) + (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/
((a^2 - b^2)*(a + b*Cos[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4517, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{4517}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int \frac{-\left(\left(2Aa^2-bBa-Ab^2\right)\sec^2(c+dx)\right)+2a(Ab-aB)\sec(c+dx)+b(Ab-aB)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int \frac{-\left(\left(2Aa^2-bBa-Ab^2\right)\sec^2(c+dx)\right)+2a(Ab-aB)\sec(c+dx)+b(Ab-aB)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\int \frac{\left(-2Aa^2+bBa+Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)+b(Ab-aB)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\int \frac{(Ab-aB)b^2+a(Ab-aB)\sec(c+dx)b}{\sqrt{\sec(c+dx)}}dx}{b^2}-\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b}}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\int \frac{(Ab-aB)b^2+a(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{b^2(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+ab(Ab-aB)\int \sqrt{\sec(c+dx)}dx}{b^2}-\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{b^2(Ab-aB)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(Ab-aB)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+ab(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}-\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right.}{b}}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\frac{(a^3(-B)+3a^2Ab-ab}{}}{2a(a^2-b^2)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{ab(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{(a^3(-B)+3a^2Ab-ab^2B-Ab^3)\int}{b}}{2a(a^2-b^2)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-\frac{(a^3(-B)+3a^2Ab-ab^2B-Ab^3)\int\frac{\text{cs}}{b+}}{b}}{2a(a^2-b^2)}$$

$$\Big\downarrow \text{4336}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-\frac{(a^3(-B)+3a^2Ab-ab^2B-Ab^3)\sqrt{\text{co}}}{}}{2a(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-\frac{(a^3(-B)+3a^2Ab-ab^2B-Ab^3)\sqrt{\text{co}}}{}}{2a(a^2-b^2)}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-\frac{2\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\sqrt{c}}{}}{2a\left(a^2-b^2\right)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,x]
```

output
```
-1/2*(((2*b^2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d + (2*a*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 - (2*(3*a^2*A*b - A*b^3 - a^3*B - a*
b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(b*(a + b)*d))/(a*(a^2 - b^2)) + (b*(A*b - a*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4517
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[d/(b*(m + 1)*(a^2 - b^2))   Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(
n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2
*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[
n, 1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(251) = 502$.

Time = 6.51 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.77

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-\frac{4B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}+\frac{2(Ab-Ba)\cdots}{\cdots}\right.}{\cdots}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2
*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2
*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+
2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2,
x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2,
x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^2,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^2,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\cos\left(dx+c\right)b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)*b + a),x)
```

3.575 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 258

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)\,d} \\
&\quad+\frac{(aAb+a^2B-2b^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)\,d} \\
&\quad-\frac{(a^2Ab+Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} \\
&\quad-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
(A*b-B*a)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c
)^(1/2)/b/(a^2-b^2)/d+(A*a*b+B*a^2-2*B*b^2)*cos(d*x+c)^(1/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-(A*a^2*b+A*b^3+
B*a^3-3*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),
2^(1/2))*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d-(A*b-B*a)*sec(d*x+c)^(1/2)*s
in(d*x+c)/(a^2-b^2)/d/(b+a*sec(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 626 vs. $2(258) = 516$.

Time = 7.19 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.43

$$\int \frac{A + B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\frac{2(-Ab+aB)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\,\text{s}}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+ \frac{\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\sin(c+dx)}{b(-a^2+b^2)} + \frac{-aAb\sin(c+dx)+a^2B\sin(c+dx)}{b(-a^2+b^2)(a+b\cos(c+dx))}\right)}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
,x]
```

output
```
((2*(-(A*b) + a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*
x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Co
s[c + d*x]^2)) + (2*(4*a*A - 4*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcS
in[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((A*b - a*B)
*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*
a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*
Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1
- Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a
+ b)*d) + (Sqrt[Sec[c + d*x]]*(((A*b - a*B)*Sin[c + d*x])/(b*(-a^2 + b^2))
 + (-(a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos
[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4515, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sqrt{\sec(c+dx)}(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{4515}$$

$$\frac{\int \frac{-\left((Ab-aB)\sec^2(c+dx)\right)+2(aA-bB)\sec(c+dx)+Ab-aB}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a^2-b^2}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\left((Ab-aB)\sec^2(c+dx)\right)+2(aA-bB)\sec(c+dx)+Ab-aB}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(aB-Ab)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2(aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)+Ab-aB}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{\frac{\int \frac{b(Ab-aB)+\left(Ba^2+Aba-2b^2B\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{b(Ab-aB)+\left(Ba^2+Aba-2b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4274$$

$$\frac{\frac{\left(a^2B+aAb-2b^2B\right)\int \sqrt{\sec(c+dx)}dx+b(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\left(a^2B+aAb-2b^2B\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b(Ab-aB)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4258$$

$$\frac{\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+\right.}{}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{\left(a^3B+a^2Ab-3\right.}{}}{2\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$\downarrow$ 3119

$$\frac{\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right.}{b^2}}{2\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{2\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\int}{b^2}}{2\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$\downarrow$ 4336

$$\frac{\frac{\frac{2\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\sqrt{}}{}}{2\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\sqrt{}}{}}{2\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$\downarrow$ 3284

$$\frac{\frac{\frac{2\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{2\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\sqrt{}}{}}{2\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

output
```
(((2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*(a*A*b + a^2*B - 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 - (2*(a^2*A*b + A*b^3 + a^3*B - 3
*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(b^2*(a + b)*d))/(2*(a^2 - b^2)) - ((A*b - a*B)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4515
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B)
+ d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
 NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(249) = 498$.

Time = 6.85 (sec) , antiderivative size = 808, normalized size of antiderivative = 3.13

method	result	size
default	Expression too large to display	808

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
4/b*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-b^2/a
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^
(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^2 \sqrt{\sec(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\cos{(c+dx)}}{\left(a+b\cos{(c+dx)}\right)^2\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)))
, x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos{(dx+c)}+A}{\left(b\cos{(dx+c)}+a\right)^2\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos{(dx+c)}+A}{\left(b\cos{(dx+c)}+a\right)^2\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$
$$=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\cos\left(dx+c\right)\sec\left(dx+c\right)b+\sec\left(dx+c\right)a}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)*sec(c + d*x)*b + sec(c + d*x)*a),x)
```

3.576 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{(aAb-3a^2B+2b^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} + \frac{(a^2Ab-2Ab^3-3a^3B+4ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)d} - \frac{a(a^2Ab-3Ab^3-3a^3B+5ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} + \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(b+a\sec(c+dx))}$$

output
```
-(A*a*b-3*B*a^2+2*B*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+(A*a^2*b-2*A*b^3-3*B*a^3+4*B*a*b^2)
*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/
b^3/(a^2-b^2)/d-a*(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^2)*cos(d*x+c)^(1/2)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/(a-b)/b^3/(
a+b)^2/d+a*(A*b-B*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)/d/(b+a*sec(d*
x+c))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 655 vs. $2(284) = 568$.

Time = 7.40 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.31

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\frac{2(-aAb-a^2B+2b^2B)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec}}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(-\frac{a(-Ab+aB)\sin(c+dx)}{b^2(a^2-b^2)}+\frac{a^2Ab\sin(c+dx)-a^3B\sin(c+dx)}{b^2(-a^2+b^2)(a+b\cos(c+dx))}\right)}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
,x]
```

output
```
((2*(-(a*A*b) - a^2*B + 2*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b
+ a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c +
 d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*A*b^2 - 4*a*b*B)*Cos[c + d*x]^2*Ellip
ticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)) + ((a*A*b - 3*a^2*B + 2*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(
-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x
])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
- Sec[c + d*x]^2)))/(4*b*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((a*(
-(A*b) + a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2))) + (a^2*A*b*Sin[c + d*x] - a
^3*B*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3439, 3042, 4518, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{4518}$$

$$\frac{\int -\frac{-3Ba^2-(Ab-aB)\sec^2(c+dx)a+Aba+2b^2B+2b(Ab-aB)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b(a^2-b^2)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{-3Ba^2-(Ab-aB)\sec^2(c+dx)a+Aba+2b^2B+2b(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2b(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\int\frac{-3Ba^2-(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+Aba+2b^2B+2b(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4594

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int\frac{b\left(-3Ba^2+Aba+2b^2B\right)-\left(-3Ba^3+Aba^2+4b^2Ba-2Ab^3\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int\frac{b\left(-3Ba^2+Aba+2b^2B\right)+\left(3Ba^3-Aba^2-4b^2Ba+2Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4274

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\left(-3a^2B+aAb+2b^2B\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\left(-3a^3B+a^2Ab+4ab^2B-2Ab^3\right)\int\sqrt{\sec(c+}}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{b\left(-3a^2B+aAb+2b^2B\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(-3a^3B+a^2Ab+4ab^2B-2Ab^3\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\csc(c}{b+a\,c}}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4258

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\left(-3a^3B+a^2Ab\right.}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(-3a^3B+a^2\right.}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3119

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{2b\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\left(-3a^3B+a^2Ab+4ab^2B\right.}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3120

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{2b\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2\left(-3a^3B+a^2Ab+4ab^2B-\right.}{}}{b^2}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4336

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{2b\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{2b\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{d}}{2b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3284

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)} - \frac{\frac{2a\left(-3a^3B+a^2Ab+5ab^2B-3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)} + \frac{2b\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}}{2b\left(a^2-b^2\right)}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

output
```
-1/2*(((2*b*(a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(a^2*A*b - 2*A*b^3 - 3*a^3*B + 4*a*b
^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
b^2 + (2*a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d
))/(b*(a^2 - b^2)) + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a
^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4518
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[
e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
 + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)
*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*
b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && IL
tQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(275) = 550$.

Time = 9.05 (sec) , antiderivative size = 849, normalized size of antiderivative = 2.99

method	result	size
default	Expression too large to display	849

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*a^2*(A*b-B*a)/b^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)
^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b
^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2)))+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\cos\left(dx+c\right)\sec\left(dx+c\right)^2 b+\sec\left(dx+c\right)^2 a}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)*sec(c + d*x)**2*b + sec(c + d*x)**2*a
),x)
```

3.577 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 363

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)d} \\
&\quad-\frac{(9a^3Ab-12aAb^3-15a^4B+16a^2b^2B+2b^4B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3b^4(a^2-b^2)d} \\
&\quad+\frac{a^2(3a^2Ab-5Ab^3-5a^3B+7ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^4(a+b)^2d} \\
&\quad-\frac{(3aAb-5a^2B+2b^2B)\sin(c+dx)}{3b^2(a^2-b^2)d\sqrt{\sec(c+dx)}}+\frac{a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)d\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}
\end{aligned}$$

output
```
(3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d-1/3*(9*A*a^3*b-12*A*a*
b^3-15*B*a^4+16*B*a^2*b^2+2*B*b^4)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b^4/(a^2-b^2)/d+a^2*(3*A*a^2*b-5*A*b^3-5
*B*a^3+7*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b)
,2^(1/2))*sec(d*x+c)^(1/2)/(a-b)/b^4/(a+b)^2/d-1/3*(3*A*a*b-5*B*a^2+2*B*b^
2)*sin(d*x+c)/b^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+a*(A*b-B*a)*sin(d*x+c)/b/(a
^2-b^2)/d/sec(d*x+c)^(1/2)/(b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.51 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.93

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{\frac{2(-3a^2Ab+6Ab^3+5a^3B-8ab^2B)\cos^2(c+dx)\left(\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\operatorname{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$

$$+\frac{\sqrt{\sec(c+dx)}\left(\frac{a^2(-Ab+aB)\sin(c+dx)}{b^3(a^2-b^2)}-\frac{a^3Ab\sin(c+dx)-a^4B\sin(c+dx)}{b^3(-a^2+b^2)(a+b\cos(c+dx))}+\frac{B\sin(2(c+dx))}{3b^2}\right)}{d}$$

input

```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
,x]
```

output

```
-1/12*((2*(-3*a^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Cos[c + d*x]^2*(Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Se
c[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d
*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-12*a*A*b^2 + 8*a
^2*b*B + 4*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*
x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(
a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b + 6*A*b^3 + 15*a^
3*B - 12*a*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Se
c[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*Ellip
ticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)
))/((a - b)*b^2*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((a^2*(-(A*b) + a*B)*Sin[
c + d*x])/(b^3*(a^2 - b^2)) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d*x])/
(b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x])) + (B*Sin[2*(c + d*x)])/(3*b^2)))/d
```

Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4518, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{4518}$$

$$\frac{\int -\frac{-5Ba^2-3(Ab-aB)\sec^2(c+dx)a+3Aba+2b^2B+2b(Ab-aB)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}dx}{b(a^2-b^2)}+\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\int \frac{-5Ba^2-3(Ab-aB)\sec^2(c+dx)a+3Aba+2b^2B+2b(Ab-aB)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}dx}{2b(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\int\frac{-5Ba^2-3(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+3Aba+2b^2B+2b(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}$$

$\big\downarrow$ 4592

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-$$
$$\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{2\int\frac{-a\left(-5Ba^2+3Aba+2b^2B\right)\sec^2(c+dx)+2b\left(-2Ba^2+3Aba-b^2B\right)\sec(c+dx)+3\left(-5Ba^3+3Aba^2+4b^2Ba-2Ab^3\right)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}}{3b}}{2b\left(a^2-b^2\right)}$$

$\big\downarrow$ 27

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-$$
$$\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\int\frac{-a\left(-5Ba^2+3Aba+2b^2B\right)\sec^2(c+dx)+2b\left(-2Ba^2+3Aba-b^2B\right)\sec(c+dx)+3\left(-5Ba^3+3Aba^2+4b^2Ba-2Ab^3\right)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{3b}}{2b\left(a^2-b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-$$
$$\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\int\frac{-a\left(-5Ba^2+3Aba+2b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(-2Ba^2+3Aba-b^2B\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(-5Ba^3+3Aba^2+4b^2Ba-\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{3b}}{2b\left(a^2-b^2\right)}$$

$\big\downarrow$ 4594

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-$$
$$\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int\frac{3b\left(-5Ba^3+3Aba^2+4b^2Ba-2Ab^3\right)-\left(-15Ba^4+9Aba^3\right)}{\sqrt{\sec(c+dx)}}}{b^2}}{3b}}{2b\left(a^2-b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int\frac{3b\left(-5Ba^3+3Aba^2+4b^2Ba-2Ab^3\right)+\left(15Ba^4-9A\right.}{\sqrt{\csc\left(c+\right.}}}{b^2}}{3b}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 4274

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+3b\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\int\frac{1}{\sqrt{\sec(c+dx)}}d}{3b}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+3b\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\right.}}}{3b}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 4258

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{}}{3b}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{}}{}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 3119

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6b\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{se}}{d}}{}}{}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 3120

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6b\left(-5a^3B+3a^2Ab+4ab^2B-2Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{se}}{d}}{}}{}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 4336

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{6b\left(-5a^3B+3a^2A\right.}{}}{}}{2b\left(a^2-\right.}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{6b\left(-5a\right.}{}}{}}{2b\left(a\right.}$$

$\downarrow$ 3284

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}-\frac{\frac{2\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{6a^2\left(-5a^3B+3a^2Ab+7ab^2B-5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)}+\frac{6b\left(-5a^3B+3a^2Ab+4a\right.}{}}{}}{2b\left(a^2-b\right.}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

output
```
(a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a*Se
c[c + d*x])) - (-1/3*(((6*b*(3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(9*a
^3*A*b - 12*a*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b^4*B)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*a^2*(3*a^2*A*b
 - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d))/b + (2*(3*a*A*b
- 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]]))/(2*b*(a^2 -
 b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4518
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[
e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
 + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)
*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*
b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && IL
tQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(348) = 696$.

Time = 11.37 (sec) , antiderivative size = 1066, normalized size of antiderivative = 2.94

method	result	size
default	Expression too large to display	1066

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^4*b^2+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-9*a^2*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)-2*a^3*(A*b-B*a)/b^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a
/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)\sec(dx+c)^3 b+\sec(dx+c)^3 a}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)*sec(c + d*x)**3*b + sec(c + d*x)**3*a
),x)
```

3.578 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 480

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = &-\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a^2-b^2)^2d}\\
&+\frac{(11a^2Ab-5Ab^3-7a^3B+ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2d}\\
&-\frac{(35a^4Ab-38a^2Ab^3+15Ab^5-15a^5B+6a^3b^2B-3ab^4B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^3(a-b)^2(a+b)^3d}\\
&+\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3(a^2-b^2)^2d}\\
&+\frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a(a^2-b^2)d(b+a\sec(c+dx))^2}\\
&+\frac{b(11a^2Ab-5Ab^3-7a^3B+ab^2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4a^2(a^2-b^2)^2d(b+a\sec(c+dx))}\end{aligned}$$

output
```
-1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*cos(d*x+c)^(1/2)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1
/4*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*(35*A*a^4*b-3
8*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)*cos(d*x+c)^(1/2)*Elli
pticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^3/(a-b)^2/
(a+b)^3/d+1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*sec(d*x+
c)^(1/2)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*sec(d*x+c)^(5/2)*sin
(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3
+B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.83 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.76

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\left(56a^4Ab-95a^2Ab^3+45Ab^5-16a^5B+19a^3b^2B-9ab^4B\right)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c\right.\right.\right.}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$

$$+\frac{\sqrt{\sec(c+dx)}\left(\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\sin(c+dx)}{4a^3(a^2-b^2)^2}+\frac{Ab^2\sin(c+dx)-abB\sin(c+dx)}{2a(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{11a^2Ab^2\sin(c+dx)-5Ab}{4a^2}\right.}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3
,x]
```

output
```
-1/16*((2*(56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B
- 9*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - E
llipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sq
rt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c +
d*x]^2)) + (2*(16*a^5*A - 80*a^3*A*b^2 + 40*a*A*b^4 + 32*a^4*b*B - 8*a^2*b
^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b
 + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c
+ d*x])*(1 - Cos[c + d*x]^2)) + ((8*a^4*A*b - 29*a^2*A*b^3 + 15*A*b^5 + 9*
a^3*b^2*B - 3*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a
*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*
EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a
 + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*
x]^2)))/(a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((8*a^4*A - 29*
a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a^2 -
b^2)^2) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x])/(2*a*(a^2 - b^2)*(a +
b*Cos[c + d*x])^2) + (11*a^2*A*b^2*Sin[c + d*x] - 5*A*b^4*Sin[c + d*x] ...
```

Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.99, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 3439, 3042, 4517, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4517}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int -\frac{\sec^{\frac{3}{2}}(c+dx)\left(\left(4Aa^2+bBa-5Ab^2\right)\sec^2(c+dx)-4a(Ab-aB)\sec(c+dx)+3b(Ab-aB)\right)}{2(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\left(4Aa^2+bBa-5Ab^2\right)\sec^2(c+dx)-4a(Ab-aB)\sec(c+dx)+3b(Ab-aB)\right)}{(b+a\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\left(4Aa^2+bBa-5Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4a(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)+3b(Ab-aB)\right)}{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4586}$$

$$\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)\sec^2(c+dx)-4a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sec(c+dx)+b\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\right)}{2(b+a\sec(c+dx))}dx}{a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)\sec^2(c+dx)-4a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sec(c+dx)+b\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\right)}{b+a\sec(c+dx)}dx}{2a(a^2-b^2)}+}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(-7Ba^3+11Aba^2+b^2Ba-5\right.\right.}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4590}$$

$$\frac{\frac{\frac{2\int -\frac{\left(-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2-3b^4Ba+15Ab^5\right)\sec^2(c+dx)+4a\left(2Aa^4+4bBa^3-10Ab^2a^2-b^3Ba+5Ab^4\right)\sec(c+dx)+b\left(8Aa^4+9bBa^3-29Ab^2a^2-3b\right.}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{\left(-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2-3b^4Ba+15Ab^5\right)\sec^2(c+dx)+4a\left(2Aa^4+4bBa^3-\right.}{\sqrt{\sec(c+dx)}(b+a\,\mathrm{se}}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{\left(-8Ba^5+24Aba^4+5b^2Ba^3-33Ab^3a^2-3b^4Ba+15Ab^5\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(2Aa^4+4bB\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b\right.}}{}}{2a(a^2-b^2)}}{4a\left(a^2\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4594}$$

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{\int\frac{b^2\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)-ab^2\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\sec(}{\sqrt{\sec(c+dx)}}}{b^2}}{a}}{2a(a^2-b^2)}}{4a\,(a^2-}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\,(a^2-b^2)\,(a\sec(c+dx)+b)^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{\int\frac{b^2\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)-ab^2\left(-7Ba^3+11Aba^2+b^2Ba-5Ab^3\right)\csc(}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{b^2}}{}}{2a(a^2-b^2)}}{4a\,(}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\,(a^2-b^2)\,(a\sec(c+dx)+b)^2}$$

$\big\downarrow$ 4274

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-ab^2\left(-7a^3B+11a^2Ab+ab^2\right.}{b^2}}{}}{2a(a^2-b^2)}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\,(a^2-b^2)\,(a\sec(c+dx)+b)^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab^2\left(-7a^3B+11a^2Ab\right.}{b^2}}{}}{2a(a^2-b^2)}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\,(a^2-b^2)\,(a\sec(c+dx)+b)^2}$$

$\big\downarrow$ 4258

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-ab}{b^2}}{}}{2a(a^2}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{b^2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{}}{2a}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{\frac{2b^2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-a}{b^2}}{}}{2a(}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b^2(8a}{}}{}}{2a}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\Big\downarrow$ 4336

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\cos(c}}}{}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin(c}}}{}}{}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$\downarrow$ 3284

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+$$

$$\frac{\frac{b\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\frac{2\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2b^2\left(8a^4A+9a^3bB-29\right.}{}}{}}{}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*
Sec[c + d*x])^2) + ((b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])) + (-((((2*
b^2*(8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*b^2*(11*a^
2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (2*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*
b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/a) + (2*(8*a
^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(a*d))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4517
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[d/(b*(m + 1)*(a^2 - b^2))   Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(
n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2
*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[
n, 1]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1974 vs. $2(455) = 910$.

Time = 9.56 (sec) , antiderivative size = 1975, normalized size of antiderivative = 4.11

method	result	size
default	Expression too large to display	1975

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3/sin(1/
2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))-2*(A*b-B*a)/a*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)
^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)
-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^
2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^3}\,dx = \int \frac{(B\cos(dx + c) + A)\sec(dx + c)^{\frac{3}{2}}}{(b\cos(dx + c) + a)^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^3,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^3,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^2 b^2+2\cos\left(dx+c\right)ab+a^2}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
int((sqrt(sec(c + d*x))*sec(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(c + d*
x)*a*b + a**2),x)
```

3.579 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 405

$$\begin{aligned}
&\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2d} \\
&\quad-\frac{(7a^2Ab-Ab^3-3a^3B-3ab^2B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4ab(a^2-b^2)^2d} \\
&\quad+\frac{(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{s}}{4a^2(a-b)^2b(a+b)^3d} \\
&\quad+\frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a(a^2-b^2)d(b+a\sec(c+dx))^2} \\
&\quad+\frac{b(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2(a^2-b^2)^2d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
-1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*(7*A*a^2*b-A*
b^3-3*B*a^3-3*B*a*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1
/2))*sec(d*x+c)^(1/2)/a/b/(a^2-b^2)^2/d+1/4*(15*A*a^4*b-6*A*a^2*b^3+3*A*b^
5-3*B*a^5-10*B*a^3*b^2+B*a*b^4)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/
2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a^2/(a-b)^2/b/(a+b)^3/d+1/2*b*(A*
b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*b*
(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/(a^2-b
^2)^2/d/(b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.39 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.97

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{2(16a^4A-19a^2Ab^2+9Ab^4-9a^3bB+3ab^3B)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(-\frac{(-9a^2Ab+3Ab^3+5a^3B+ab^2B)\sin(c+dx)}{4a^2(a^2-b^2)^2}+\frac{-Ab\sin(c+dx)+aB\sin(c+dx)}{2(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{-7a^2Ab\sin(c+dx)+Ab^3\sin(c+dx}{4a(a^2-b^2)^2}\right.}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3
,x]
```

output
```
((2*(16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4 - 9*a^3*b*B + 3*a*b^3*B)*Cos[c + d*
x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSi
n[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*
A*b + 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-(a/b)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*
x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a
^2*A*b^2 + 3*A*b^4 + 5*a^3*b*B + a*b^3*B)*Cos[2*(c + d*x)]*(b + a*Sec[c +
d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*E
llipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[
c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*
x]]*(2 - Sec[c + d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c +
d*x]]*(-1/4*((-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(a^2
*(a^2 - b^2)^2) + (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/(2*(a^2 - b^2)*
(a + b*Cos[c + d*x])^2) + (-7*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] +
3*a^3*B*Sin[c + d*x] + 3*a*b^2*B*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*(a + ...
```

Rubi [A] (verified)

Time = 2.82 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4517, 27, 3042, 4586, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4517}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(\left(4Aa^2-bBa-3Ab^2\right)\sec^2(c+dx)-4a(Ab-aB)\sec(c+dx)+b(Ab-aB)\right)}{2(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(4Aa^2-bBa-3Ab^2\right)\sec^2(c+dx)-4a(Ab-aB)\sec(c+dx)+b(Ab-aB)\right)}{(b+a\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(4Aa^2-bBa-3Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4a(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)+b(Ab-aB)\right)}{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4586}$$

$$\frac{\frac{\int -\frac{-\left(\left(8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba+3Ab^4\right)\sec^2(c+dx)\right)+4a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)\sec(c+dx)+b\left(-5Ba^3+9Aba^2-b^2Ba-3Ab^3\right)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b\left(-5a^3B+\cdots\right.}{\cdots}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int\frac{-\left(\left(8Aa^4-7bBa^3-5Ab^2a^2+b^3Ba+3Ab^4\right)\sec^2(c+dx)\right)+4a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int\frac{\left(-8Aa^4+7bBa^3+5Ab^2a^2-b^3Ba-3Ab^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(-2Ba^3+4Aba^2-b^2Ba-Ab^3\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 4594

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{\left(-5Ba^3+9Aba^2-b^2Ba-3Ab^3\right)b^2+a\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}}dx}{b^2}-\frac{(-3}{}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{\left(-5Ba^3+9Aba^2-b^2Ba-3Ab^3\right)b^2+a\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 4274

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+ab\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\int\sqrt{s}}{b^2}}{2a(a^2}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)} - \frac{\frac{b^2\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)}{b^2}}{2a}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 4258

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)} - \frac{\frac{b^2\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\left(-3a^3B+7a^2\right.}{b^2}}{}}{4a\left(a^2-b^2\right)}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)} - \frac{\frac{b^2\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\left(-3a^3B\right.}{b^2}}{}}{4a\left(a^2-b\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 3119

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)} - \frac{\frac{ab\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\left(-5a^3\right.}{}}{b^2}}{}}{4a\left(a^2-b^2\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\downarrow$ 3120

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)} - \frac{\frac{\frac{2ab\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\left(-5a\right.}{}}{b^2}}{}}{4a\left(a^2-b\right.}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 4336$$

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2ab\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2(-5a\ldots}{\ldots}}{b^2}}{\ldots}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2ab\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2(-5a\ldots}{\ldots}}{b^2}}{\ldots}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+$$

$$\frac{\frac{b(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2ab\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2(-5a\ldots}{\ldots}}{b^2}}{\ldots}$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*
Sec[c + d*x])^2) + (-1/2*(((2*b^2*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B
)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*
a*b*(7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 - (2*(15*a^4*A*b - 6*a^2*A*b^3
 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d))/(a*(a
^2 - b^2)) + (b*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])))/(4*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4517
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[d/(b*(m + 1)*(a^2 - b^2))   Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(
n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2
*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[
n, 1]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. 2(384) = 768.

Time = 7.69 (sec) , antiderivative size = 1744, normalized size of antiderivative = 4.31

method	result	size
default	Expression too large to display	1744

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-b^2/a/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(A*b-B*a)/b*(-
1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2
/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(A+B\cos{(c+dx)})\sqrt{\sec{(c+dx)}}}{(a+b\cos{(c+dx)})^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**3,
x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^3}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^3,x
)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^3,
x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^2 b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(c + d*x)*a*b + a**2),
x)
```

3.580 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 402

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx &= \frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4ab(a^2-b^2)^2d} \\ &+ \frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d} \\ &- \frac{(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b^2(a+b)^3d} \\ &+ \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a(a^2-b^2)d(b+a\sec(c+dx))^2} \\ &- \frac{(7a^2Ab-Ab^3-3a^3B-3ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a(a^2-b^2)^2d(b+a\sec(c+dx))}\end{aligned}$$

output
```
1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/a/b/(a^2-b^2)^2/d+1/4*(3*A*a^2*b+3*A*b
^3+B*a^3-7*B*a*b^2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)
)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d-1/4*(3*A*a^4*b+10*A*a^2*b^3-A*b^5+B*a
^5-10*B*a^3*b^2-3*B*a*b^4)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),
2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/a/(a-b)^2/b^2/(a+b)^3/d+1/2*b*(A*b-B*a
)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2-1/4*(7*A*a^
2*b-A*b^3-3*B*a^3-3*B*a*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)^2/d/(
b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.45 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.95

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \frac{\frac{2(-9a^2Ab+3Ab^3+5a^3B+ab^2B)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+dx}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{} + \frac{\sqrt{\sec(c+dx)}\left(\frac{(-5a^2Ab-Ab^3+a^3B+5ab^2B)\sin(c+dx)}{4ab(a^2-b^2)^2}-\frac{aAb\sin(c+dx)-a^2B\sin(c+dx)}{2b(-a^2+b^2)(a+b\cos(c+dx))^2}+\frac{3a^2Ab\sin(c+dx)+3Ab^3\sin(c+dx)+c}{4b(-a^2+b^2)^2(a+}\right.}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]])
,x]
```

output
```
((2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x]^2*(EllipticF[A
rcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*
x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*
(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^3*A + 8*a*A*b^2 - 24
*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)) + ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*
B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 -
4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a*(a - b
)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((-5*a^2*A*b - A*b^3 + a^3*B + 5*a
*b^2*B)*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2) - (a*A*b*Sin[c + d*x] - a^2*B*
Sin[c + d*x])/(2*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (3*a^2*A*b*Sin[c
 + d*x] + 3*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*
x])/(4*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.99, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4517, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{ 4517}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int \frac{-\left(\left(4Aa^2-3bBa-Ab^2\right)\sec^2(c+dx)\right)+4a(Ab-aB)\sec(c+dx)+b(Ab-aB)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int \frac{-\left(\left(4Aa^2-3bBa-Ab^2\right)\sec^2(c+dx)\right)+4a(Ab-aB)\sec(c+dx)+b(Ab-aB)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int \frac{\left(-4Aa^2+3bBa+Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)+b(Ab-aB)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4588}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\int -\frac{-b\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\sec^2(c+dx)+4ab\left(2Aa^2-3bBa+Ab^2\right)\sec(c+dx)+b\left(-Ba^3+5Aba^2-5b^2Ba+Ab^3\right)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b\left(a^2-b^2\right)}+\frac{\left(-3a^3B+7a^2Ab-3ab^2B-A\right.}{d\left(a^2-b^2\right)(a\,\mathrm{se}}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int \frac{-b\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\sec^2(c+dx)+4ab\left(2Aa^2-3bBa+Ab^2\right)\sec(c+dx)+b\left(-Ba\right.}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{-b\left(-3Ba^3+7Aba^2-3b^2Ba-Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4ab\left(2Aa^2-3bBa+Ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 4594

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int \frac{\left(-Ba^3+5Aba^2-5b^2Ba+Ab^3\right)b^2+a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}}dx}{b^2}-\frac{\left(a^5B+3\right)}{}}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int \frac{\left(-Ba^3+5Aba^2-5b^2Ba+Ab^3\right)b^2+a\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\frac{\left(a^5\right)}{}}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 4274

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+ab\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\int \sqrt{\sec}}{b^2}}{2b(a^2-b}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\int}{b^2}}{2b(a}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 4258

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-$$

$$\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{b^2\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\left(a^3B+3a^2Ab\right.}{b^2}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-$$

$$\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{b^2\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\left(a^3B+3a\right.}{b^2}}{4a\left(a^2-b^2\right.}$$

$\downarrow$ 3119

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-$$

$$\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{ab\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\left(a^3(-B)\right.}{}}{b^2}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 3120

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-$$

$$\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2ab\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\left(a^3(-\right.}{}}{b^2}}{4a\left(a^2-b^2\right)}$$

$\downarrow$ 4336

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-$$

$$\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2ab\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\left(a^3(-\right.}{}}{b^2}$$

$\downarrow$ 3042

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-$$

$$\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2ab\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\left(a^3(-B\right.}{}}{b^2}}{}$$

$\downarrow$ 3284

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-$$

$$\frac{\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2ab\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\left(a^3(-B\right.}{}}{b^2}}{4a}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

output
```
(b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*
Sec[c + d*x])^2) - (-1/2*(((2*b^2*(5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*
b*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 - (2*(3*a^4*A*b + 10*a^2*A*b^3 -
 A*b^5 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d))/(b*(a^2 -
 b^2)) + ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x])))/(4*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4517
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[d/(b*(m + 1)*(a^2 - b^2))   Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(
n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2
*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[
n, 1]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1849 vs. $2(381) = 762$.

Time = 8.79 (sec) , antiderivative size = 1850, normalized size of antiderivative = 4.60

method	result	size
default	Expression too large to display	1850

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/b/(-2*a*b
+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d
*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-
2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a*(A*b-B*a)/b^2*(-1/2*b^2/a/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*co...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3 \sqrt{\sec(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3\sqrt{\sec(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3\sqrt{\sec(c + dx)}}\,dx = \int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)^3\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)))
, x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3\sqrt{\sec(c + dx)}}\,dx = \int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)^3\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^2\sec(dx+c)\,b^2+2\cos(dx+c)\sec(dx+c)\,ab+\sec(dx+c)\,a^2}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)*b**2 + 2*cos(c + d*x)
*sec(c + d*x)*a*b + sec(c + d*x)*a**2),x)
```

3.581 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 400

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d}$$

$$+\frac{(a^3Ab-7aAb^3+3a^4B-5a^2b^2B+8b^4B)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b^3(a^2-b^2)^2d}$$

$$-\frac{(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\mathrm{s}\cdots}}{4(a-b)^2b^3(a+b)^3d}$$

$$-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2}$$

$$+\frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b(a^2-b^2)^2d(b+a\sec(c+dx))}$$

output
```
-1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*cos(d*x+c)^(1/2)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+1/4*(A*a^3*b-7*A*
a*b^3+3*B*a^4-5*B*a^2*b^2+8*B*b^4)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d-1/4*(A*a^4*b-10*A*a^2*
b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*B*a*b^4)*cos(d*x+c)^(1/2)*EllipticPi(si
n(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
-1/2*(A*b-B*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+
1/4*(3*A*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b/(a^2
-b^2)^2/d/(b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.57 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.96

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{\frac{2(-5a^2Ab-Ab^3+a^3B+5ab^2B)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\sin(c+dx)}{4b^2(a^2-b^2)^2}-\frac{-a^2Ab\sin(c+dx)+a^3B\sin(c+dx)}{2b^2(-a^2+b^2)(a+b\cos(c+dx))^2}+\frac{a^3Ab\sin(c+dx)-7aAb^3\sin(c+dx)}{4b^2(-a^2+b^2)}\right.}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2))
,x]
```

output
```
-1/16*((2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x]^2*(Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
+ d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])
/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(24*a*A*b^2 - 8*a^2*b*
B - 16*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a +
b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*
a*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x
]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a
/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a -
b)^2*b*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((a^2*A*b + 5*A*b^3 + 3*a^3*B -
 9*a*b^2*B)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2) - (-(a^2*A*b*Sin[c + d*x])
 + a^3*B*Sin[c + d*x])/(2*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (a^3*
A*b*Sin[c + d*x] - 7*a*A*b^3*Sin[c + d*x] - 5*a^4*B*Sin[c + d*x] + 11*a^2*
b^2*B*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 2.79 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.99, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4515, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{\sqrt{\sec(c+dx)}(A\sec(c+dx)+B)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4515}$$

$$\frac{\int \frac{-3(Ab-aB)\sec^2(c+dx)+4(aA-bB)\sec(c+dx)+Ab-aB}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-3(Ab-aB)\sec^2(c+dx)+4(aA-bB)\sec(c+dx)+Ab-aB}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-3(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4(aA-bB)\csc\left(c+dx+\frac{\pi}{2}\right)+Ab-aB}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{\int -\frac{3Ba^3+Aba^2-9b^2Ba+5Ab^3-\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\sec^2(c+dx)+4b\left(-Ba^2+3Aba-2b^2B\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b(a^2-b^2)}+\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c\text{-}}{bd(a^2-b^2)(a\sec(c+dx)}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{3Ba^3+Aba^2-9b^2Ba+5Ab^3-\left(Ba^3+3Aba^2-7b^2Ba+3Ab^3\right)\sec^2(c+dx)+4b\left(-Ba^2+3Aba-2b^2\right.}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int\frac{3Ba^3+Aba^2-9b^2Ba+5Ab^3+\left(-Ba^3-3Aba^2+7b^2Ba-3Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\left(-Ba^2+3A\ldots\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$
$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 4594

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{b\left(3Ba^3+Aba^2-9b^2Ba+5Ab^3\right)-\left(3Ba^4+Aba^3-5b^2Ba^2-7Ab^3a+8b^4B\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}+\frac{\left(3a^5\ldots\right.}{}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$
$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{b\left(3Ba^3+Aba^2-9b^2Ba+5Ab^3\right)+\left(-3Ba^4-Aba^3+5b^2Ba^2+7Ab^3a-8b^4B\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+\ldots}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$
$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\left(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B\right)\int\sqrt{\text{se}\ldots}}{b^2}}{2b(a^2-\ldots}}{4\left(a^2-b^2\right)}$$
$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(3a^4B+a^3Ab-5a^2b^2B-7aAb^3+8b^4B\right)\int\ldots}{b^2}}{2b(\ldots}}{4\left(a^2-b^2\right)}$$
$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 4258

$$\frac{\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b(3a^3B+a^2Ab-9ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-(3a^4B+a^3Ab-5a^2b^2}{b^2}}{}}{4(a^2-b^2)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b(3a^3B+a^2Ab-9ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-(3a^4B+a^3Ab-5a}{b^2}}{}}{4(a^2-b^2}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2b(3a^3B+a^2Ab-9ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-(3a^4B+a^3Ab-5a^2b^2}{b^2}}{}}{4(a^2-b^2)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{2b(3a^3B+a^2Ab-}{}}{}}{4(a^2-b^2)}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a\sec(c+dx)+b)^2}$$

$\Big\downarrow$ 4336

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\left(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b}}{b^2}}{}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\left(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}(}}{b^2}}{}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$\Big\downarrow$ 3284

$$\frac{\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{2b\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^4B+a^3Ab-5a^2b\right.}{}}{b^2}}{}$$

$$\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

input

```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

output

```
-1/2*((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*S
ec[c + d*x])^2) + (-1/2*(((2*b*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(a^
3*A*b - 7*a*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4*B)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (2*(a^4*A*b - 10*a^2*
A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*
d))/(b*(a^2 - b^2)) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x])))/(4*(a^2 -
b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4515
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B)
+ d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
 NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1936 vs. $2(379) = 758$.

Time = 6.89 (sec) , antiderivative size = 1937, normalized size of antiderivative = 4.84

method	result	size
default	Expression too large to display	1937

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
4/b^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-B*a)/b^3*(-1
/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+
b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \int \frac{\sqrt{\sec\left(dx+c\right)}}{\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2b^2+2\cos\left(dx+c\right)\sec\left(dx+c\right)^2ab+\sec\left(dx+c\right)^2a^2}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

output

```
int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**2*b**2 + 2*cos(c + d
*x)*sec(c + d*x)**2*a*b + sec(c + d*x)**2*a**2),x)
```

3.582 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 427

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3(a^2-b^2)^2d}$$
$$+\frac{(3a^4Ab-5a^2Ab^3+8Ab^5-15a^5B+33a^3b^2B-24ab^4B)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec}}{4b^4(a^2-b^2)^2d}$$
$$-\frac{a(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right.}{4(a-b)^2b^4(a+b)^3d}$$
$$+\frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b(a^2-b^2)d(b+a\sec(c+dx))^2}$$
$$+\frac{a(a^2Ab-7Ab^3-5a^3B+11ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2(a^2-b^2)^2d(b+a\sec(c+dx))}$$

output
```
-1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*cos(d*x+c)^(1/2)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d+1
/4*(3*A*a^4*b-5*A*a^2*b^3+8*A*b^5-15*B*a^5+33*B*a^3*b^2-24*B*a*b^4)*cos(d*
x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/2)/b^4/(a^
2-b^2)^2/d-1/4*a*(3*A*a^4*b-6*A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b^2-35*
B*a*b^4)*cos(d*x+c)^(1/2)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))
*sec(d*x+c)^(1/2)/(a-b)^2/b^4/(a+b)^3/d+1/2*a*(A*b-B*a)*sec(d*x+c)^(1/2)*s
in(d*x+c)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+
11*B*a*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.84 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.92

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\frac{2\left(-a^3Ab-5aAb^3+5a^4B-7a^2b^2B+8b^4B\right)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(-\frac{a\left(-3a^2Ab+9Ab^3+7a^3B-13ab^2B\right)\sin(c+dx)}{4b^3(a^2-b^2)^2}-\frac{a^3Ab\sin(c+dx)-a^4B\sin(c+dx)}{2b^3(-a^2+b^2)(a+b\cos(c+dx))^2}+\frac{-5a^4Ab\sin(c+dx)+11a^2Ab}{4b^3(}\right.}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
,x]
```

output
```
((2*(-(a^3*A*b) - 5*a*A*b^3 + 5*a^4*B - 7*a^2*b^2*B + 8*b^4*B)*Cos[c + d*x
]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin
[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*S
in[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^2*A*b
^2 + 16*A*b^4 + 8*a^3*b*B - 32*a*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b),
ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]
^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-3*a^3
*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*Cos[2*(c + d*x)]*(b
+ a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(
2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a
/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqr
t[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (S
qrt[Sec[c + d*x]]*(-1/4*(a*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*S
in[c + d*x])/(b^3*(a^2 - b^2)^2) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d
*x])/(2*b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (-5*a^4*A*b*Sin[c + d*x
] + 11*a^2*A*b^3*Sin[c + d*x] + 9*a^5*B*Sin[c + d*x] - 15*a^3*b^2*B*Sin...
```

Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 3439, 3042, 4518, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{ 4518}$$

$$\frac{\int -\frac{-5Ba^2-3(Ab-aB)\sec^2(c+dx)a+Aba+4b^2B+4b(Ab-aB)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int \frac{-5Ba^2-3(Ab-aB)\sec^2(c+dx)a+Aba+4b^2B+4b(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int \frac{-5Ba^2-3(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+Aba+4b^2B+4b(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4588}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\int \frac{-15Ba^4+3Aba^3+29b^2Ba^2-9Ab^3a-\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)\sec^2(c+dx)a-8b^4B-4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b(a^2-b^2)}-\frac{a\left(-5a^3B+a^2A\right.}{}}{4b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\int \frac{-15Ba^4+3Aba^3+29b^2Ba^2-9Ab^3a-\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)\sec^2(c+dx)a-8b^4B-4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2b(a^2-b^2)}-\frac{a\left(-5a^3B+a^2A\right.}{}}{4b\left(a^2-b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\int \frac{-15Ba^4+3Aba^3+29b^2Ba^2-9Ab^3a-\left(-5Ba^3+Aba^2+11b^2Ba-7Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a-8b^4B-4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}-\frac{a(-5a}{}}{4b\left(a^2-b^2\right)}$$

$\big\downarrow$ 4594

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int \frac{b\left(-15Ba^4+3Aba^3+29b^2Ba^2-9Ab^3a-8b^4B\right)-\left(-15Ba^5+3Aba^4+33b^2Ba^3-\right.}{\sqrt{\sec(c+dx)}}}{b^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int \frac{b\left(-15Ba^4+3Aba^3+29b^2Ba^2-9Ab^3a-8b^4B\right)+\left(15Ba^5-3Aba^4-33b^2Ba\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{b^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$\big\downarrow$ 4274

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-\left(-15a^5B\right.}{b^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(-15a^5B+3a^4Ab+33a^3b^2B-5a^2Ab^3-24ab^4B+8Ab^5\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{a\left(-15a^5B\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$\big\downarrow$ 4258

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cdots}}{\cdots}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cdots}}{\cdots}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 3119

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{2b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\cdots\right.}{d}}{\cdots}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 3120

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{2b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\cdots\right.}{d}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 4336

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{2b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-\cdots\right.}{\cdots}}{2b\left(a^2-b^{\cdots}\right.}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{2b\left(-15a^4B+3a^3Ab+29a^2b^2B-\right.}{}}{2b(a}}{}$$

$$\Big\downarrow \text{3284}$$

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{2a\left(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)}+\frac{\frac{2b\left(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B\right.}{d}}{}}{2b(a^2-b^2)}}{}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

output
```
(a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*
Sec[c + d*x])^2) - ((((2*b*(3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*
B - 8*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d - (2*(3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5 - 15*a^5*B + 33*a^3*b^2*B -
24*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d)/b^2 + (2*a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b
^2*B - 35*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d))/(2*b*(a^2 - b^2)) - (a*(a^2*A*b
 - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^
2 - b^2)*d*(b + a*Sec[c + d*x]))))/(4*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4518
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[
e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
 + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)
*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*
b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && IL
tQ[n, 0])
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(406) = 812$.

Time = 11.18 (sec) , antiderivative size = 1977, normalized size of antiderivative = 4.63

method	result	size
default	Expression too large to display	1977

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*b*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*a^2/b^4*(3*A*b-4*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/
2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b
+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2)))+12/b^3*a*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2))
, x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3b^2+2\cos\left(dx+c\right)\sec\left(dx+c\right)^3ab+\sec\left(dx+c\right)^3a^2}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**3*b**2 + 2*cos(c + d
*x)*sec(c + d*x)**3*a*b + sec(c + d*x)**3*a**2),x)
```

3.583 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 521

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^4(a^2-b^2)^2d}$$

$$-\frac{(45a^5Ab-99a^3Ab^3+72aAb^5-105a^6B+223a^4b^2B-128a^2b^4B-8b^6B)\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(\right.}{12b^5(a^2-b^2)^2d}$$

$$+\frac{a^2(15a^4Ab-38a^2Ab^3+35Ab^5-35a^5B+86a^3b^2B-63ab^4B)\sqrt{\cos(c+dx)}\,\mathrm{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx\right.}{4(a-b)^2b^5(a+b)^3d}$$

$$-\frac{(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B)\sin(c+dx)}{12b^3(a^2-b^2)^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{a(Ab-aB)\sin(c+dx)}{2b(a^2-b^2)d\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}$$

$$+\frac{a(3a^2Ab-9Ab^3-7a^3B+13ab^2B)\sin(c+dx)}{4b^2(a^2-b^2)^2d\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}$$

output
```
1/4*(15*A*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*a^5+65*B*a^3*b^2-24*B*a*b^4)*cos
(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2)/b^4/(
a^2-b^2)^2/d-1/12*(45*A*a^5*b-99*A*a^3*b^3+72*A*a*b^5-105*B*a^6+223*B*a^4*
b^2-128*B*a^2*b^4-8*B*b^6)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))*sec(d*x+c)^(1/2)/b^5/(a^2-b^2)^2/d+1/4*a^2*(15*A*a^4*b-38*A*a^2*b
^3+35*A*b^5-35*B*a^5+86*B*a^3*b^2-63*B*a*b^4)*cos(d*x+c)^(1/2)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*sec(d*x+c)^(1/2)/(a-b)^2/b^5/(a+b)^3
/d-1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)*sin(d*x+c)/b
^3/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d
/sec(d*x+c)^(1/2)/(b+a*sec(d*x+c))^2+1/4*a*(3*A*a^2*b-9*A*b^3-7*B*a^3+13*B
*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)/(b+a*sec(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 8.03 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.66

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = -\frac{\frac{2(-15a^4Ab+21a^2Ab^3-24Ab^5+35a^5B-73a^3b^2B+56ab^4B)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\text{se}}\right.\right.\right.}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{} + \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2(-7a^2Ab+13Ab^3+11a^3B-17ab^2B)\sin(c+dx)}{4b^4(a^2-b^2)^2} - \frac{-a^4Ab\sin(c+dx)+a^5B\sin(c+dx)}{2b^4(-a^2+b^2)(a+b\cos(c+dx))^2} + \frac{9a^5Ab\sin(c+dx)-15a^3A}{4b}\right.}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2))
,x]
```

output
```
-1/48*((2*(-15*a^4*A*b + 21*a^2*A*b^3 - 24*A*b^5 + 35*a^5*B - 73*a^3*b^2*B
 + 56*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] -
 EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*
Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c
+ d*x]^2)) + (2*(-24*a^3*A*b^2 + 96*a*A*b^4 + 56*a^4*b*B - 112*a^2*b^3*B -
 16*b^5*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -
1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*C
os[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-45*a^4*A*b + 87*a^2*A*b^3 - 24*A*b
^5 + 105*a^5*B - 195*a^3*b^2*B + 72*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c
 + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*
b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Se
c[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*S
in[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c +
 d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c +
d*x]]*((a^2*(-7*a^2*A*b + 13*A*b^3 + 11*a^3*B - 17*a*b^2*B)*Sin[c + d*x])/
(4*b^4*(a^2 - b^2)^2) - (-(a^4*A*b*Sin[c + d*x]) + a^5*B*Sin[c + d*x])/(2*
b^4*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (9*a^5*A*b*Sin[c + d*x] - 15...
```

Rubi [A] (verified)

Time = 3.89 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 3439, 3042, 4518, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4518}$$

$$\frac{\int -\frac{-7Ba^2-5(Ab-aB)\sec^2(c+dx)a+3Aba+4b^2B+4b(Ab-aB)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\int \frac{-7Ba^2-5(Ab-aB)\sec^2(c+dx)a+3Aba+4b^2B+4b(Ab-aB)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\int \frac{-7Ba^2-5(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+3Aba+4b^2B+4b(Ab-aB)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4588}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\frac{\int \frac{-35Ba^4+15Aba^3+61b^2Ba^2-33Ab^3a-3\left(-7Ba^3+3Aba^2+13b^2Ba-9Ab^3\right)\sec^2(c+dx)a-8b^4B-4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}dx}{b(a^2-b^2)}-\frac{a\left(-7a^3B-\right.}{bd(a^2-}}{4b\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\frac{\int \frac{-35Ba^4+15Aba^3+61b^2Ba^2-33Ab^3a-3\left(-7Ba^3+3Aba^2+13b^2Ba-9Ab^3\right)\sec^2(c+dx)a-8b^4B-4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}dx}{2b(a^2-b^2)}-\frac{a\left(-7a^3B-\right.}{bd(a^2-}}{4b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\frac{\int \frac{-35Ba^4+15Aba^3+61b^2Ba^2-33Ab^3a-3\left(-7Ba^3+3Aba^2+13b^2Ba-9Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a-8b^4B-4b\left(Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}-\frac{a(}{}}{4b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4592

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-a\left(-35Ba^4+15Aba^3+61b^2Ba^2-33Ab^3a-8b^4B\right)\sec^2(c+dx)+4b\left(-7Ba^4+3Aba^3+14b^2Ba^2-\right.}{2\sqrt{\sec(c+dx)}(b+a\,s}}{3b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right.}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\int \frac{-a\left(-35Ba^4+15Aba^3+61b^2Ba^2-33Ab^3a-8b^4B\right)\sec^2(c+dx)+4b\left(-7Ba^4+3Aba^3+14b^2Ba^2-1\right.}{\sqrt{\sec(c+dx)}(b+a\sec}}{3b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\int \frac{-a\left(-35Ba^4+15Aba^3+61b^2Ba^2-33Ab^3a-8b^4B\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\left(-7Ba^4+3Aba^3+14b^2Ba\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+\right.}}{3b}}{2b(a^2-b^2)}}{4b\left(a^2-\right.}$$

$\Big\downarrow$ 4594

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\int \frac{3b\left(-35Ba^5\right.}{}}{}}{2b(a^2-b^2)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$
$$\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int\frac{3b\left(-3\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{2b\left(a^2-b^2\right)}$$

$\downarrow$ 4274

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$
$$\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b\left(-35a\cdots\right.}{\cdots}}{\cdots}}{2b\left(a^2\cdots\right.}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$
$$\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b\left(-35a\cdots\right.}{\cdots}}{\cdots}}{2\cdots}$$

$\downarrow$ 4258

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$
$$\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b\left(-35a\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$
$$\frac{\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3b\left(-35a\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3119

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$

$$\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{6b\left(-35a\right.}{}$$

$\downarrow$ 3120

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$

$$\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{6b\left(-35a\right.}{}$$

$\downarrow$ 4336

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$

$$\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\cos(c+}}}{b^2}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$

$$\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{3a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin\left(c+\right.}}}{b^2}$$

$\downarrow$ 3284

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}-$$

$$\frac{2\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{6a^2\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\right.}{b^2d(a+b)}$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

output
```
(a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a*
Sec[c + d*x])^2) - (-((a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[
c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x]))) + (-1
/3*(((6*b*(15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B -
 24*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d - (2*(45*a^5*A*b - 99*a^3*A*b^3 + 72*a*A*b^5 - 105*a^6*B + 223*a^4*b
^2*B - 128*a^2*b^4*B - 8*b^6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*a^2*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b
^5 - 35*a^5*B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d))/b + (2*
(15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*Sin[c + d*x]
)/(3*b*d*Sqrt[Sec[c + d*x]]))/(2*b*(a^2 - b^2)))/(4*b*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4518
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[
e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
 + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)
*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*
b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && IL
tQ[n, 0])
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2194 vs. $2(496) = 992$.

Time = 13.55 (sec) , antiderivative size = 2195, normalized size of antiderivative = 4.21

method	result	size
default	Expression too large to display	2195

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^5/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^4*b^2+9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-18*a^2*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b)+2*a^4*(A*b-B*a)/b^5*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d
*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*
c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b
)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)}\, dx = \int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}}}\, dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2))
, x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)}\, dx = \int \frac{B\cos(dx + c) + A}{(b\cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}}}\, dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),x
)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),
x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{\sqrt{\sec(dx+c)}}{\cos(dx+c)^2\sec(dx+c)^4b^2+2\cos(dx+c)\sec(dx+c)^4ab+\sec(dx+c)^4a^2}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x))/(cos(c + d*x)**2*sec(c + d*x)**4*b**2 + 2*cos(c + d
*x)*sec(c + d*x)**4*a*b + sec(c + d*x)**4*a**2),x)
```

3.584 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 64

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(
1/2)/d+2/3*B*sec(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2B\sec^{\frac{3}{2}}(c+dx)\left(\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{3d}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x
]),x]
```

output
```
(2*B*Sec[c + d*x]^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Si
n[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2011, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \sec^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{ 4255}$$

$$B\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$B\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$B\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

output
```
B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(55) = 110$.

Time = 2.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.34

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}$

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*B*((2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int\frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$=\frac{-i\sqrt{2}B\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}B\cos(dx+c)\,\text{weie}}{3\,d\cos(dx+c)}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
1/3*(-I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 2*B*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \int \frac{(Bb\cos(dx + c) + Ba)\sec(dx + c)^{\frac{5}{2}}}{b\cos(dx + c) + a}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \int \frac{(Bb\cos{(dx + c)} + Ba)\sec{(dx + c)}^{\frac{5}{2}}}{b\cos{(dx + c)} + a}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(B\,a + B\,b\,\cos{(c + d\,x)})}{a + b\,\cos{(c + d\,x)}}\,dx$$

input
```
int(((1/cos(c + d*x))^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
,x)
```

output
```
int(((1/cos(c + d*x))^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
, x)
```

Reduce [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*b
```

3.585 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 60

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*B*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/
2)/d+2*B*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x
]),x]
```

output
```
(2*B*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
 Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2011, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \sec^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

output
```
B*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(56) = 112$.

Time = 1.88 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.05

method	result
default	$-\frac{2B\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
-2*B*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.27

$$\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}B\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + i\sqrt{2}B\text{weiers}}{d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
(-I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*B*sin(d*x + c)/sqrt(cos(d*
x + c)))/d
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
B*Integral(sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{(Bb\cos(dx+c) + Ba)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \int \frac{(Bb\cos(dx + c) + Ba)\sec(dx + c)^{\frac{3}{2}}}{b\cos(dx + c) + a}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(B\,a + B\,b\,\cos(c + d\,x))}{a + b\,\cos(c + d\,x)}\,dx$$

input
```
int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
,x)
```

output
```
int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
, x)
```

Reduce [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{a + b\cos(c + dx)}\,dx = \left(\int \sqrt{\sec(dx + c)}\,\sec(dx + c)\,dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x))*sec(c + d*x),x)*b
```

3.586 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 37

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}$$

output
```
2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(1/
2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x
]),x]
```

output
```
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2011, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4258}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

output

```
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(34) = 68$.

Time = 1.74 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.62

method	result	size
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\, d}$	134

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETUR
NVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{(aB + bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos}{d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
(-I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = B\int \sqrt{\sec(c+dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
B*Integral(sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(aB + bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(aB + bB\cos(c + dx))\sqrt{\sec(c + dx)}}{a + b\cos(c + dx)}\,dx = \int \frac{(Bb\cos(dx + c) + Ba)\sqrt{\sec(dx + c)}}{b\cos(dx + c) + a}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c + dx))\sqrt{\sec(c + dx)}}{a + b\cos(c + dx)}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(B\,a + B\,b\,\cos(c + d\,x))}{a + b\,\cos(c + d\,x)}\,dx$$

input
```
int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
,x)
```

output
```
int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
, x)
```

Reduce [F]

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \left(\int \sqrt{\sec\left(dx+c\right)}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
int(sqrt(sec(c + d*x)),x)*b
```

3.587 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 37

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

output
```
2*B*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1/2
)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]
]),x]
```

output
```
(2*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2011, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(35) = 70$.

Time = 2.83 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.62

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
risch	$-\frac{i\sqrt{2}\,B}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}+1\right)}{\sqrt{\left(e^{2i(dx+c)}+1\right)e^{i(dx+c)}}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\left(-2i\,\operatorname{EllipticE}\left(\sqrt{-i\left(e^{i(dx+c)}\right.}\right.\right.}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\right.}$

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)/sec(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx$$
$$= \frac{i\sqrt{2}B\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))) - i\sqrt{2}B\text{weierstr}}{d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
(I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx = B\int \frac{1}{\sqrt{\sec(c + dx)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
B*Integral(1/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a + b\,\cos(c + d\,x))}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)))
,x)
```

output
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)))
, x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sqrt{\sec(c + dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx + c)}}{\sec(dx + c)}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x),x)*b
```

3.588 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 64

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*sec(d*x+c)^(
1/2)/d+2/3*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{B\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2
)),x]
```

output
```
(B*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Si
n[2*(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2011, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$B\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$B\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

output
```
B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(55) = 110$.

Time = 4.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.81

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, B\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{d}{}\right.}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)/sec(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\,B\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right) - i\,\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)) + i\,\sqrt{2}B}{3\,d}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
1/3*(2*B*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*B*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}\,dx = B\int \frac{1}{\sec^{\frac{3}{2}}(c + dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
B*Integral(sec(c + d*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)\sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\,a + B\,b\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)))
,x)
```

output
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)))
, x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right) b$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*b
```

3.589 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 64

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
6/5*B*cos(d*x+c)^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*sec(d*x+c)^(1
/2)/d+2/5*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{B\sqrt{\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2
)),x]
```

output
```
(B*Sqrt[Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + S
in[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2011, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$B\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$B\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$B\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$B\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

output
```
B*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(56) = 112.

Time = 5.86 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.17

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,B\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)/sec(d*x+c)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)}\,dx$$
$$= \frac{2\,B\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right) + 3i\,\sqrt{2}B\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos\left(dx+c\right) +$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
1/5*(2*B*cos(d*x + c)^(3/2)*sin(d*x + c) + 3*I*sqrt(2)*B*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqr
t(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2
)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\,a + B\,b\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)))
,x)
```

output
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)))
, x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right) b$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*b
```

3.590 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 473

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}(19a^2Ab+8Ab^3+63a^3B-14ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{105a^4d\sqrt{\sec(c+dx)}}$$
$$+ \frac{2(a-b)\sqrt{a+b}(8Ab^2+a^2(25A-63B)+2ab(3A-7B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\right.}{105a^3d\sqrt{\sec(c+dx)}}$$
$$+ \frac{2(25a^2A-4Ab^2+7abB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105a^2d}$$
$$+ \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35ad}$$
$$+ \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
2/105*(a-b)*(a+b)^(1/2)*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*cos(d*x+c
)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+
c))/(a-b))^(1/2)/a^4/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(a+b)^(1/2)*(8*A*b^2+a
^2*(25*A-63*B)+2*a*b*(3*A-7*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)
^(1/2)+2/105*(25*A*a^2-4*A*b^2+7*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^
(3/2)*sin(d*x+c)/a^2/d+2/35*(A*b+7*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^
(5/2)*sin(d*x+c)/a/d+2/7*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)*sin(d*x
+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3321 vs. $2(473) = 946$.

Time = 25.96 (sec) , antiderivative size = 3321, normalized size of antiderivative = 7.02

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2)
,x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(19*a^2*A*b + 8*A*b^3 + 6
3*a^3*B - 14*a*b^2*B)*Sin[c + d*x])/(105*a^3) + (2*Sec[c + d*x]^2*(A*b*Sin
[c + d*x] + 7*a*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]*(25*a^2*A*Sin[c
+ d*x] - 4*A*b^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]))/(105*a^2) + (2*A*Se
c[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-19*A*b)/(105*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (3*a*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
 + (2*b^2*B)/(15*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*A*S
qrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (17*A*b^2*Sqrt[Sec[c +
d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(10
5*a^3*Sqrt[a + b*Cos[c + d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a +
b*Cos[c + d*x]]) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c +
 d*x]]) - (19*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b
*Cos[c + d*x]]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^3*S
qrt[a + b*Cos[c + d*x]]) - (3*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*
Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/
(15*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(
-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)...
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3478, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$\Big\downarrow$ 3478

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{4Ab\cos^2(c+dx)+(5aA+7bB)\cos(c+dx)+Ab+7aB}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)}{7d\cos\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4Ab\cos^2(c+dx)+(5aA+7bB)\cos(c+dx)+Ab+7aB}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)}{7d\cos\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+(5aA+7bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab+7aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\,s}{\right.$$

$\Big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{2\int \frac{25Aa^2+7bBa+(23Ab+21aB)\cos(c+dx)a-4Ab^2+2b(Ab+7aB)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(7aB+A}{\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{25Aa^2+7bBa+(23Ab+21aB)\cos(c+dx)a-4Ab^2+2b(Ab+7aB)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2(7aB+Ab}{\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{25Aa^2+7bBa+(23Ab+21aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-4Ab^2+2b(Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2(7aB}{\right.\right.$$

$\Big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2\int\frac{63Ba^3+19Aba^2-14b^2Ba+\left(25Aa^2+49bBa+2Ab^2\right)\cos(c+dx)a+8Ab^3}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin}{3ad\cos^{\frac{3}{2}}(}}{5a}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int\frac{63Ba^3+19Aba^2-14b^2Ba+\left(25Aa^2+49bBa+2Ab^2\right)\cos(c+dx)a+8Ab^3}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c}{3ad\cos^{\frac{3}{2}}(c}}{5a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int\frac{63Ba^3+19Aba^2-14b^2Ba+\left(25Aa^2+49bBa+2Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+8Ab^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2A+7abB-4Ab^2\right)\mathrm{s}}{3ad\cos}}{5a}\right.\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{(a-b)\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\left(63a^3B+19a^2Ab-14ab\right.}{3a}}{5a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{(a-b)\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(63a^3B+19a^2A\right.}{3a}}{}\right.\right.$$

$\Big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\left(63a^3B+19a^2Ab-14ab^2B+8Ab^3\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{ 3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(a^2(25A-63B)+2ab(3A-7B)+8Ab^2\right.}{}}{}\right.\right.$$

input

```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(
19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + a^2*(25*A - 63*B) + 2
*a*b*(3*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(3*a) + (2*(
25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d*Cos[c + d*x]^(3/2)))/(5*a))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(421) = 842$.

Time = 56.48 (sec) , antiderivative size = 2095, normalized size of antiderivative = 4.43

method	result	size
default	Expression too large to display	2095
parts	Expression too large to display	2123

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105/d*(a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(9/2)/(b*cos(d*x+c)^2+a*cos(d*x+
c)+cos(d*x+c)*b+a)*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/
(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*(19*cos(d*x+c)^6+38*cos(d*x+c)^5+19*cos(d*x+c)^4)+A*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^
2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(19*cos(d*x+c)
^6+38*cos(d*x+c)^5+19*cos(d*x+c)^4)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((
a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^3*EllipticE(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^6+16*cos(d*x+c)^5+8*cos(d*x+c)
^4)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(
a+b))^(1/2)*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(8*c
os(d*x+c)^6+16*cos(d*x+c)^5+8*cos(d*x+c)^4)+B*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*EllipticE(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(63*cos(d*x+c)^6+126*cos(d*x+c)^5+63*c
os(d*x+c)^4)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*
x+c)+1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*(63*cos(d*x+c)^6+126*cos(d*x+c)^5+63*cos(d*x+c)^4)+B*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b^2*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-14*cos(d*x+c)^6-28
*cos(d*x+c)^5-14*cos(d*x+c)^4)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+...
```

Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2)
, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{9/2}\sqrt{a+b\cos(c+dx)}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(1/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(1/2)
, x)
```

Reduce [F]

$$
\begin{aligned}
&\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx \\
&= \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)b \\
&\quad+\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^4\,dx\right)a
\end{aligned}
$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)*
*4,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**4,
x)*a
```

3.591 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 390

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}(9a^2A-2Ab^2+5abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(\cdots}{}}}{15a^3d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(a-b)\sqrt{a+b}(9aA+2Ab-5aB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+\cdots}{a-\cdots}\cdots\right)}{15a^2d\sqrt{\sec(c+dx)}}$$
$$+\frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad}$$
$$+\frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2-2*A*b^2+5*B*a*b)*cos(d*x+c)^(1/2)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(
a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/a^3/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(a+b)^(1/2)*(9*A*a+2*A*b-5*B*a)*cos(d*x
+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/15*(A*b+5*B*a)*(a+b*cos(d*x+c)
)^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d+2/5*A*(a+b*cos(d*x+c))^(1/2)*sec(d
*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2899 vs. $2(390) = 780$.

Time = 22.66 (sec) , antiderivative size = 2899, normalized size of antiderivative = 7.43

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2)
,x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A - 2*A*b^2 + 5*a*
b*B)*Sin[c + d*x])/(15*a^2) + (2*Sec[c + d*x]*(A*b*Sin[c + d*x] + 5*a*B*Si
n[c + d*x]))/(15*a) + (2*A*Sec[c + d*x]*Tan[c + d*x])/5))/d + (2*((-3*a*A)
/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^2)/(15*a*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b*B)/(3*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) - (2*A*b*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*
x]]) + (2*A*b^3*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]) + (a
*B*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) - (b^2*B*Sqrt[Sec[c +
d*x]])/(3*a*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c
 + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^3*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]) - (b^2*B*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(3*a*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(9*
a*A - 2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)] - (9*a^2*A - 2*A*b^2 + 5*a*b*B)*Cos[c + d*x]*(a + b*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a^2*d*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c ...
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3478, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$\Big\downarrow$ 3478

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{2Ab\cos^2(c+dx)+(3aA+5bB)\cos(c+dx)+Ab+5aB}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)}{5d\cos\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2Ab\cos^2(c+dx)+(3aA+5bB)\cos(c+dx)+Ab+5aB}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)}{5d\cos\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+(3aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)+Ab+5aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\,\mathrm{s}}{}\right.$$

$\Big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2\int \frac{9Aa^2+5bBa+(7Ab+5aB)\cos(c+dx)a-2Ab^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b\,\mathrm{c}}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9Aa^2+5bBa+(7Ab+5aB)\cos(c+dx)a-2Ab^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b\cos}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9Aa^2+5bBa+(7Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{(9a^2A+5abB-2Ab^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(9aA-5aB+2A}{3a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{(9a^2A+5abB-2Ab^2)\int\frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx-(a-b)(9aA-5a}{3a}\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{(9a^2A+5abB-2Ab^2)\int\frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx-\frac{2(a-b)\sqrt{a+b}(9aA-5}{}}{3}\right.\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}(9a^2A+5abB-2Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{a^2d}}{}\right.\right.$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*
A*b^2 + 5*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)
*Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(a*d))/(3*a) + (2*(A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*
a*d*Cos[c + d*x]^(3/2)))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1514 vs. $2(344) = 688$.

Time = 44.34 (sec) , antiderivative size = 1515, normalized size of antiderivative = 3.88

method	result	size
default	Expression too large to display	1515
parts	Expression too large to display	1535

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d*(a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(7/2)/(b*cos(d*x+c)^2+a*cos(d*x+c
)+cos(d*x+c)*b+a)*(A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+
b))^(1/2))*(9*cos(d*x+c)^5+18*cos(d*x+c)^4+9*cos(d*x+c)^3)+A*((a+cos(d*x+c
)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*E
llipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(9*cos(d*x+c)^5+18*co
s(d*x+c)^4+9*cos(d*x+c)^3)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)^5-4*cos(d*x+c)^4-2*cos(d*x+c)^3)+A*((a
+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-2*cos(d*x+c
)^5-4*cos(d*x+c)^4-2*cos(d*x+c)^3)+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))*(5*cos(d*x+c)^5+10*cos(d*x+c)^4+5*cos(d*x+c)^
3)+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(5*
cos(d*x+c)^5+10*cos(d*x+c)^4+5*cos(d*x+c)^3)+A*((a+cos(d*x+c)*b)/(cos(d*x+
c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*EllipticF(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-9*cos(d*x+c)^5-18*cos(d*x+c)^4-9*co
s(d*x+c)^3)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)...
```

Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2)
, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2)
, x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}dx\right)b$$
$$+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{3}dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)*
*3,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**3,
x)*a
```

3.592 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 324

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}(Ab+3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\ldots}}{3a^2d\sqrt{\sec(c+dx)}}$$
$$+ \frac{2(a-b)\sqrt{a+b}(A-3B)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\ldots}{\ldots}}}{3ad\sqrt{\sec(c+dx)}}$$
$$+ \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*(a-b)*(a+b)^(1/2)*(A*b+3*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a
+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a
*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+
c)^(1/2)+2/3*(a-b)*(a+b)^(1/2)*(A-3*B)*cos(d*x+c)^(1/2)*csc(d*x+c)*Ellipti
cF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2
))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d
*x+c)^(1/2)+2/3*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 16.25 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.07

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)(Ab+3aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(\right.\right.\right.}{}$$
$$+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(Ab+3aB)\sin(c+dx)}{3a}+\frac{2}{3}A\tan(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2)
,x]
```

output
```
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(A*b + 3*a*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
a + b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c
 + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b + 3
*a*B)*Sin[c + d*x])/(3*a) + (2*A*Tan[c + d*x])/3))/d
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3440, 3042, 3478, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3478}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{Ab+3aB+(aA+3bB)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{Ab+3aB+(aA+3bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{Ab+3aB+(aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left((3aB+Ab)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)(A-3B)\int \frac{}{\sqrt{\cos(}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left((a-b)(A-3B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+(3aB+A\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left((3aB+Ab)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+\right.\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\right.}{a^2d}\right.\right.$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*Sqrt[a + b]*(A*b + 3*a*
B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(
A - 3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/3 + (2*A*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(284) = 568$.

Time = 34.80 (sec) , antiderivative size = 1036, normalized size of antiderivative = 3.20

method	result	size
parts	Expression too large to display	1036
default	Expression too large to display	1041

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*A/d*((cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(
cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticE(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/
2)*a^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-cos(d*x+c)^
3-2*cos(d*x+c)^2-cos(d*x+c))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x
+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(
a-b)/(a+b))^(1/2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(cos(d*x+c)+1
)*sin(d*x+c)*a^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)+2)*a*b+b^2*cos(d*x+c)^2
*sin(d*x+c))*(a+cos(d*x+c)*b)^(1/2)*cos(d*x+c)*sec(d*x+c)^(5/2)/(b*cos(d*x
+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)/a+2*B/d*((cos(d*x+c)^2+2*cos(d*x+c)+1)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^
2+2*cos(d*x+c)+1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticF(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*((a+cos(d*x+c)*...
```

Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos{(c+d\,x)})\,\left(\frac{1}{\cos{(c+d\,x)}}\right)^{5/2}\,\sqrt{a+b\,\cos{(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)
, x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\left(\int \sqrt{\sec{(dx+c)}}\,\sqrt{\cos{(dx+c)}\,b+a}\,\cos{(dx+c)}\sec{(dx+c)}^{2}\,dx\right)b$$
$$+\left(\int \sqrt{\sec{(dx+c)}}\,\sqrt{\cos{(dx+c)}\,b+a}\,\sec{(dx+c)}^{2}\,dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x)*
*2,x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2,
x)*a
```

3.593 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 411

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$

$$= \frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

$$+ \frac{2\sqrt{a+b}(Ab-a(A-B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{ad\sqrt{\sec(c+dx)}}$$

$$- \frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{d\sqrt{\sec(c+dx)}}$$

output
```
2*A*(a-b)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2*(a+
b)^(1/2)*(A*b-a*(A-B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*(a
+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 18.16 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.55

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$
$$+ \frac{2\Big(aA\tan\left(\frac{1}{2}(c+dx)\right)+Ab\tan\left(\frac{1}{2}(c+dx)\right)-2Ab\tan^3\left(\frac{1}{2}(c+dx)\right)-aA\tan^5\left(\frac{1}{2}(c+dx)\right)+Ab\tan^5}{}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2)
,x]
```

output
```
(2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a*A
*Tan[(c + d*x)/2] + A*b*Tan[(c + d*x)/2] - 2*A*b*Tan[(c + d*x)/2]^3 - a*A*
Tan[(c + d*x)/2]^5 + A*b*Tan[(c + d*x)/2]^5 - 2*b*B*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*B*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] + A*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (b*(A - B)
+ a*(A + B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[(1 - Tan[(c + d*x)/2]^2
)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)
])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3440, 3042, 3470, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$\downarrow$ 3470

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{aA+(Ab+aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+bB\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{aA+(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+bB\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right.}}\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{aA+(Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\mathrm{se}}{}}}{}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((Ab-a(A-B))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+aA\int \frac{\cos(c+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((Ab-a(A-B))\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+aA\int \frac{}{\sin(c+}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(aA\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(Ab-a(A-B))}{}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b}(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{arc}\right.}{ad}\right.$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*(a - b)*Sqrt[a + b]*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt[a + b]*(A*b - a*(A - B))*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3470
```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[B*(d
/b^2)   Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*
c + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*
x]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [A] (verified)

Time = 27.33 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.62

method	result
parts	$\frac{2A\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a\,\mathrm{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+\left(\cos(dx+c\right.\right.}{}$
default	$-\frac{2\sec(dx+c)^{\frac{3}{2}}\left(\csc(dx+c)^2(1-\cos(dx+c))^2-1\right)\left(\left(\csc(dx+c)^3(1-\cos(dx+c))^3-\cot(dx+c)+\csc(dx+c)\right)Aa+\left(-\csc(dx+c)^3(1-\right.\right.}{}$

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2*A/d*((cos(d*x+c)^2+2*cos(d*x+c)+1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*((a+cos(d*x+c)*b)/(
cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b*EllipticE(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*
((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)
^2-2*cos(d*x+c)-1)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*b*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))+b*cos(d*x+c)*sin(d*x+c)+a*sin(d*x+c))*(a+cos(d*x+c)*b)^(1/2)*cos(d*
x+c)*sec(d*x+c)^(3/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)-2*B/d*s
ec(d*x+c)^(3/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(a+cos(d*x+c)*b)^(1/2)*(
EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*b*EllipticPi(cot(d*x+c)-csc(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2)))*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(
cos(d*x+c)^2+cos(d*x+c))
```

Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)
, x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)\,dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x),
x)*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x),x)*a
```

3.594 $\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+d}$

Optimal result

Integrand size = 35, antiderivative size = 445

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c}{a-}}}{ad\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(2A+B)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(2Ab+aB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{bd\sqrt{\sec(c+dx)}}$$

$$+\frac{B\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-(a-b)*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+(a+b)^
(1/2)*(2*A+B)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)
/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-(a+b)^(1/2)*(2*
A*b+B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+B*(a+b*co
s(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 20.17 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.77

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
,x]
```

output
```
(-(a*B*Tan[(c + d*x)/2]) - b*B*Tan[(c + d*x)/2] + 2*b*B*Tan[(c + d*x)/2]^3
 + a*B*Tan[(c + d*x)/2]^5 - b*B*Tan[(c + d*x)/2]^5 - 4*A*b*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*B
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/
(a + b)] - 4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
 d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*B*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] - (a + b)*B*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(A*b + a*(-A + B))*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)])/(d*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)
/2]^2)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Ta
n[(c + d*x)/2]^2)]*(-1 + Tan[(c + d*x)/2]^4))
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {3042, 3440, 3042, 3482, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3482}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int -\frac{-\left((2Ab+aB)\cos^2(c+dx)\right)-2aA\cos(c+dx)+aB}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{B\sin(c+dx)\sqrt{c}}{d\sqrt{\cos(}}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int \frac{-\left((2Ab+aB)\cos^2(c+dx)\right)-2aA\cos(c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{B\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int \frac{(-2Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2aA\sin}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c\right.}}\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left((aB+2Ab)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx-\int \frac{aB-2aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\right)\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left((aB+2Ab)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{aB-2aA\sin\left(c+dx\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin}}\right.\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(-\int \frac{aB-2aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(aB+2Ab)\cot}{}\right.\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(a(2A+B)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-aB\int\frac{\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(a(2A+B)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-aB\int\frac{}{\sin\left(c+d\right.}\right.\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(-aB\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(2A+B)\,\mathrm{co}}{}\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2\sqrt{a+b}(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\right.}{d}\right.\right.$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((-2*(a - b)*Sqrt[a + b]*B*Cot[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt[a + b]*(2*A + B)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/d - (2*Sqrt[a + b]*(2*A*b + a*B)*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/2 + (B*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Elliptи
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 21.93 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.50

method	result
parts	$-\frac{2A\sqrt{\sec(dx+c)}\left(\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)a-\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)b+2b\operatorname{EllipticPi}\left(\cot\right.\right.}{d\sqrt{a+\cos(dx+c)b}}$
default	$-\frac{\left(\left(4\cos(dx+c)^2+8\cos(dx+c)+4\right)A\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{a+\cos(dx+c)b}{(\cos(dx+c)+1)(a+b)}}\,b\operatorname{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)+\left(2+\right.\right.}{}$

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*A/d*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2)*(EllipticF(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*a-EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*b+2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))
*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1)+B/d*((-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*Ellipt
icPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d
*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1
)/(a+b))^(1/2)*a*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+(-c
os(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+
c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))+(2+2*cos(d*x+c)^2+4*cos(d*x+c))*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)^2*b+cos(d*x+c)*s
in(d*x+c)*a)*(a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(1/2)/(b*cos(d*x+c)^2+a*cos
(d*x+c)+cos(d*x+c)*b+a)
```

Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \int (A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}\sqrt{\sec{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)),
 x)
```

Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
, x)
```

Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)
, x)
```

Reduce [F]

$$\int \sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$=\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)b$$
$$+\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x),x)*b + int(sq
rt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a),x)*a
```

3.595 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 533

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(4Ab+aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4abd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(4Ab+(a+2b)B)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{4bd\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(4aAb-a^2B+4b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+}{a-}\right.}{4b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}$$

$$+\frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4bd}$$

output
```
-1/4*(a-b)*(a+b)^(1/2)*(4*A*b+B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x
+c)^(1/2)+1/4*(a+b)^(1/2)*(4*A*b+(a+2*b)*B)*cos(d*x+c)^(1/2)*csc(d*x+c)*El
lipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))
^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/
sec(d*x+c)^(1/2)-1/4*(a+b)^(1/2)*(4*A*a*b-B*a^2+4*B*b^2)*cos(d*x+c)^(1/2)*
csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+1/2*B*(a+b*cos(d*x+c))^(1/2)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c
)^(1/2)*sin(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1121 vs. 2(533) = 1066.

Time = 20.04 (sec) , antiderivative size = 1121, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x
]],x]
```

output

```
(B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*Ta
n[(c + d*x)/2] + a^2*B*Tan[(c + d*x)/2] + a*b*B*Tan[(c + d*x)/2] - 8*A*b^2
*Tan[(c + d*x)/2]^3 - 2*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c + d*x)/2
]^5 + 4*A*b^2*Tan[(c + d*x)/2]^5 - a^2*B*Tan[(c + d*x)/2]^5 + a*b*B*Tan[(c
 + d*x)/2]^5 + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a*A
*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2
*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(4*A*b + a*B)*EllipticE[ArcSin[...
```

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 492, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3482, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3482}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{(4Ab+aB)\cos^2(c+dx)+2(2aA+bB)\cos(c+dx)+aB}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{B\sin(c+dx)}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{(4Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2(2aA+bB)\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\,\mathrm{s}}{}\right.$$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int-\frac{-((-Ba^2+4Aba+4b^2B)\cos^2(c+dx))-2abB\cos(c+dx)+a(4Ab+aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(aB+4Ab)\,\mathrm{si}}{b}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{-((-Ba^2+4Aba+4b^2B)\cos^2(c+dx))-}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{2b}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{(Ba^2-4Aba-4b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+}}}{2b}\right.\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab+aB)-2abB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab+aB)-2abB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab+aB)-2abB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(aB+4Ab)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{-a(B(a+2b)+4Ab)\int\frac{}{\sqrt{\sin(c+d}}}{}\right.\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(aB+4Ab)\int\frac{\sin(c+dx+}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a-}}}{}\right.\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{2\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)}{}\right.\right.$$

input
```
Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*Sqrt[a + b]*(4*A*b + a
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*(4*A*b + (
a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*Sqrt[a + b]*(4*a*A
*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*
d))/b + ((4*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Co
s[c + d*x]]))/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(470) = 940$.

Time = 20.66 (sec) , antiderivative size = 1145, normalized size of antiderivative = 2.15

method	result	size
default	Expression too large to display	1145
parts	Expression too large to display	1178

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*(a+cos(d*x+c)*b)^(1/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)/
sec(d*x+c)^(1/2)*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(c
os(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*(-8*cos(d*x+c)-16-8*sec(d*x+c))+B*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^2*EllipticPi(cot(d
*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(2*cos(d*x+c)+4+2*sec(d*x+c))+B*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^
(1/2)*b^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-8*co
s(d*x+c)-16-8*sec(d*x+c))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+
c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*(-4*cos(d*x+c)-8-4*sec(d*x+c))+A*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^2*EllipticE(cot
(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-4*cos(d*x+c)-8-4*sec(d*x+c))+B*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^
(1/2)*a^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-cos(d*x+
c)-2-sec(d*x+c))+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(co
s(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(-cos(d*x+c)-2-sec(d*x+c))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((
a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b*EllipticF(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)+16+8*sec(d*x+c))+B*(cos(d*x+c...
```

Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sqrt{a+b\cos{(c+dx)}}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}+a}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c))
, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\sec(dx+c)}dx\right)b$$
$$+ \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\sec(dx+c)}dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/sec(c + d*x
),x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/sec(c + d*x),x)
*a
```

3.596 $\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 620

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(6aAb-3a^2B+16b^2B)\,\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{24ab^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(a+2b)(6Ab-3aB+8bB)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+}{a-}\right.}{24b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(2a^2Ab-8Ab^3-a^3B-4ab^2B)\,\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{8b^3d\sqrt{\sec(c+dx)}}$$

$$+\frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}}+\frac{B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$$+\frac{(6aAb-3a^2B+16b^2B)\,\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{24b^2d}$$

output
```
-1/24*(a-b)*(a+b)^(1/2)*(6*A*a*b-3*B*a^2+16*B*b^2)*cos(d*x+c)^(1/2)*csc(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)
/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a/b^2/d/sec(d*x+c)^(1/2)+1/24*(a+b)^(1/2)*(a+2*b)*(6*A*b-3*B*a+8*B*b)*c
os(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+1/8*(a+b)^(1/2)*(2*A*a^2*b
-8*A*b^3-B*a^3-4*B*a*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x
+c)^(1/2)+1/4*(2*A*b-B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d/sec(d*x+c)
^(1/2)+1/3*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/24*(
6*A*a*b-3*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+
c)/b^2/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1533 vs. $2(620) = 1240$.

Time = 16.49 (sec) , antiderivative size = 1533, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/
2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sin[c + d*x])/12 + ((6*A*
b + a*B)*Sin[2*(c + d*x)])/(24*b) + (B*Sin[3*(c + d*x)])/12))/d + (Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^
2)]*(6*a^2*A*b*Tan[(c + d*x)/2] + 6*a*A*b^2*Tan[(c + d*x)/2] - 3*a^3*B*Tan
[(c + d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + 16*a*b^2*B*Tan[(c + d*x)/2] +
 16*b^3*B*Tan[(c + d*x)/2] - 12*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a^2*b*B*Tan
[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3 - 6*a^2*A*b*Tan[(c + d*x)/2]
^5 + 6*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B
*Tan[(c + d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d*x)/2]^5 + 16*b^3*B*Tan[(c + d*
x)/2]^5 - 12*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B*EllipticPi[-1, A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*b
^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] - 12*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b ...
```

Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\cos(c+dx)}(3(2Ab-aB)\cos^2(c+dx)+4bB\cos(c+dx)+aB)}{2\sqrt{\cos(c+dx)}}dx}{3b}+\frac{B\sin(c+dx)\sqrt{\cos(c+}}{3}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\cos(c+dx)}(3(2Ab-aB)\cos^2(c+dx)+4bB\cos(c+dx)+aB)}{\sqrt{\cos(c+dx)}}dx}{6b}+\frac{B\sin(c+dx)\sqrt{\cos(c+}}{3}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3(2Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+4bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6b}+\frac{B\sin(c+dx)}{}\right.$$

$$\Big\downarrow \text{3528}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{2}\int \frac{(-3Ba^2+6Aba+16b^2B)\cos^2(c+dx)+2b(6Ab+7aB)\cos(c+dx)+a(6Ab+aB)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3(2Ab-aB)\sin(}{}}{6b}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\int \frac{(-3Ba^2+6Aba+16b^2B)\cos^2(c+dx)+2b(6Ab+7aB)\cos(c+dx)+a(6Ab+aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3(2Ab-aB)\sin(}{}}{6b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\int \frac{\left(-3Ba^2+6Aba+16b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(6Ab+aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3(2Ab-}{}}{6b}\right.$$

$\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\int -\frac{3\left(-Ba^3+2Aba^2-4b^2Ba-8Ab^3\right)\cos^2(c+dx)-2ab(6Ab+aB)\cos(c+dx)+a\left(-3Ba^2+6Aba+16b^2B\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\right.}{}\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{3\left(-Ba^3+2Aba^2-4b^2Ba-8Ab^3\right)\cos^2(c+dx)-}{\cos^{\frac{3}{2}}(c+dx)}}{}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{3\left(-Ba^3+2Aba^2-4b^2Ba-8Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right.}}{}\right.}{}\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-3Ba^2+6Aba+16b^2B\right)-2ab(6Ab+aB)\cos(c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-3Ba^2+6Aba+16b^2B\right)-2ab(6Ab+aB)\sin\left(c\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.}{}\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-3Ba^2+6Aba+16b^2B\right)-2ab(6Ab+aB)\sin\left(c\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.}{}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-3a^2B+6aAb+16b^2B\right)\int\frac{\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(-3a^2B+6aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(-3a^2B+6aAb+16b^2B)\int\frac{\sin(c+}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{}\right.}{}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(-3a^2B+6aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(-3a^2B+6aAb+16b^2B)\int\frac{\sin(c+}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{}\right.}{}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(-3a^2B+6aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{2(a-b)\sqrt{a+b}\left(-3a^2B+6aAb+16b^2B\right)\cot(c+dx)\sqrt{}}{}\right.}{}\right.$$

input
```
Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*b*d) + ((3*(2*A*b - a*B)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*Sqrt[a +
 b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a
*d) - (2*Sqrt[a + b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/d - (6*Sqrt[a + b]*(2*a^2*A*b - 8*A*b^3 - a^3*B - 4*a*b^
2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/b + ((6*a*A*b
 - 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Co
s[c + d*x]]))/4)/(6*b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1655 vs. 2(551) = 1102.

Time = 27.44 (sec) , antiderivative size = 1656, normalized size of antiderivative = 2.67

method	result	size
default	Expression too large to display	1656
parts	Expression too large to display	1687

input
```
int((a+cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*(a+cos(d*x+c)*b)^(1/2)/(b*cos(d*x+c)^2+a*cos(d*x+c)+cos(d*x+c)*b+a)
/sec(d*x+c)^(3/2)*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(
cos(d*x+c)+1)/(a+b))^(1/2)*a^2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*(12+24*sec(d*x+c)+12*sec(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*b^3*EllipticPi
(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-48-96*sec(d*x+c)-48*sec(
d*x+c)^2)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c
)+1)/(a+b))^(1/2)*a^3*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*(-6-12*sec(d*x+c)-6*sec(d*x+c)^2)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticPi(cot(d*x+c
)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-24-48*sec(d*x+c)-24*sec(d*x+c)^2)+
A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b)
)^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-6-12
*sec(d*x+c)-6*sec(d*x+c)^2)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*
x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*(-6-12*sec(d*x+c)-6*sec(d*x+c)^2)+B*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*Ellipt
icE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(3+6*sec(d*x+c)+3*sec(d*x+
c)^2)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)
/(a+b))^(1/2)*a^2*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sec(c + d*x)**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\sec(dx+c)^2}dx\right)b + \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\sec(dx+c)^2}dx\right)a$$

input
```
int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/sec(c + d*x
)**2,x)*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/sec(c + d*x)
**2,x)*a
```

3.597 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 562

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB-18ab^3B)\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(}{315a^4d\sqrt{\sec(c+dx)}} + \frac{2(a-b)\sqrt{a+b}(8Ab^3-a^3(147A-75B)+3a^2b(13A-57B)+6ab^2(A-3B))\sqrt{\cos(c+dx)}\csc(c+dx}{315a^3d\sqrt{\sec(c+dx)}} + \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315a^2d} + \frac{2(49a^2A+3Ab^2+72abB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315ad} + \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}$$

output
```
2/315*(a-b)*(a+b)^(1/2)*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B*a^3*b-18*B*a
*b^3)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x+c)^(1/2)+2/315*(a-b)*(a+b)^(
1/2)*(8*A*b^3-a^3*(147*A-75*B)+3*a^2*b*(13*A-57*B)+6*a*b^2*(A-3*B))*cos(d*
x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/315*(88*A*a^2*b-4*A*b^3+75*B*
a^3+9*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d+2/
315*(49*A*a^2+3*A*b^2+72*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(5/2)*si
n(d*x+c)/a/d+2/63*(10*A*b+9*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)*s
in(d*x+c)/d+2/9*a*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3739 vs. $2(562) = 1124$.

Time = 28.81 (sec) , antiderivative size = 3739, normalized size of antiderivative = 6.65

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11
/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 33*a^2*A*b^2
 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sin[c + d*x])/(315*a^3) + (2*Sec[c
+ d*x]^3*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^
2*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 72*a*b*B*Sin[c + d*x]))/
(315*a) + (2*Sec[c + d*x]*(88*a^2*A*b*Sin[c + d*x] - 4*A*b^3*Sin[c + d*x]
+ 75*a^3*B*Sin[c + d*x] + 9*a*b^2*B*Sin[c + d*x]))/(315*a^2) + (2*a*A*Sec[
c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^2*A)/(15*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (11*A*b^2)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (8*A*b^4)/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (82*a*b*B)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^
3*B)/(35*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (13*a*A*b*Sqrt[S
ec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Sqrt[Sec[c + d*x]
])/(315*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(315*a^
3*Sqrt[a + b*Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*
Cos[c + d*x]]) - (31*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x
]]) + (2*b^4*B*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) - (7*
a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) -
 (11*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c +
d*x]]) - (8*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b
*Cos[c + d*x]]) - (82*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*S...
```

Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{11}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int \frac{3b(2aA+3bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos(c+dx)+a(10Ab}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{3b(2aA+3bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos(c+dx)+a(10Ab}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{3b(2aA+3bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Aa^2+18bBa+9Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{2\int \frac{4ab(10Ab+9aB)\cos^2(c+dx)+a\left(45Ba^2+92Aba+63b^2B\right)\cos(c+dx)+a\left(49Aa^2+72bBa+3Ab^2\right)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}d}{7a}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int \frac{4ab(10Ab+9aB)\cos^2(c+dx)+a\left(45Ba^2+92Aba+63b^2B\right)\cos(c+dx)+a\left(49Aa^2+72bBa+3Ab^2\right)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int \frac{4ab(10Ab+9aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\left(45Ba^2+92Aba+63b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(49Aa^2+72bBa+3A\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{7a}\right.\right.$$

$\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{2\int\frac{\left(147Aa^2+396bBa+209Ab^2\right)\cos(c+dx)a^2+2b\left(49Aa^2+72bBa+3Ab^2\right)\cos^2(c+dx)a+3\left(75Ba^3+88Aba^2+9b^2\right.}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}}{7a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{\left(147Aa^2+396bBa+209Ab^2\right)\cos(c+dx)a^2+2b\left(49Aa^2+72bBa+3Ab^2\right)\cos^2(c+dx)a+3\left(75Ba^3+88Aba^2+9b^2B\right.}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}}{7a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{\left(147Aa^2+396bBa+209Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(49Aa^2+72bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+3\left(75Ba^3+88Aba\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{5a}}{7a}\right.\right.$$

$\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2\int\frac{3\left(\left(75Ba^3+186Aba^2+153b^2Ba+2Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+246bBa^3+33Ab^2a^2-18b^3Ba+8Ab^4\right)a\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+}{5a}}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{\left(75Ba^3+186Aba^2+153b^2Ba+2Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+246bBa^3+33Ab^2a^2-18b^3Ba+8Ab^4\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2(75}{}}{5a}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{\left(75Ba^3+186Aba^2+153b^2Ba+2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(147Aa^4+246bBa^3+33Ab^2a^2-18b^3Ba+8Ab^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+}{5a}}{}\right.\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{a(a-b)\left(-3a^3(49A-25B)+a^2(39Ab-171bB)+6ab^2(A-3B)+8Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+a\left(147a^4\right.}{a}}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{a(a-b)\left(-3a^3(49A-25B)+a^2(39Ab-171bB)+6ab^2(A-3B)+8Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+}{a}}{}}{}\right.\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{a\left(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+8Ab^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(-3\right.}{}}{}}{}\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{2(49a^2A+72abB+3Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(75a^3B+88a^2Ab+9ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{a}}{d\cos^{\frac{3}{2}}(c+dx)}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((2*(10*A*b + 9*a*B)*Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*(49*a^2*A + 3*A*b^2
 + 72*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (((2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^
3*b*B - 18*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*(a - b
)*Sqrt[a + b]*(8*A*b^3 - 3*a^3*(49*A - 25*B) + 6*a*b^2*(A - 3*B) + a^2*(39
*A*b - 171*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/a + (2*(88*a^2*A*
b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)))/(5*a))/(7*a))/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2692 vs. $2(504) = 1008$.

Time = 70.53 (sec) , antiderivative size = 2693, normalized size of antiderivative = 4.79

method	result	size
default	Expression too large to display	2693
parts	Expression too large to display	2717

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
2/315/d*(a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(11/2)/(b*cos(d*x+c)^2+a*cos(d*x
+c)+cos(d*x+c)*b+a)*(8*cos(d*x+c)^6*A*b^5*sin(d*x+c)-18*cos(d*x+c)^6*B*a*b
^4*sin(d*x+c)+cos(d*x+c)*(147*cos(d*x+c)^4+49*cos(d*x+c)^3+49*cos(d*x+c)^2
+35*cos(d*x+c)+35)*sin(d*x+c)*A*a^5+cos(d*x+c)^2*sin(d*x+c)*(75*cos(d*x+c)
^3+75*cos(d*x+c)^2+45*cos(d*x+c)+45)*B*a^5+A*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^4*b*EllipticE(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(147*cos(d*x+c)^7+294*cos(d*x+c)^6+14
7*cos(d*x+c)^5)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a
+b))^(1/2))*(33*cos(d*x+c)^7+66*cos(d*x+c)^6+33*cos(d*x+c)^5)+A*((a+cos(d*
x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*
b^3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(33*cos(d*x+c)^7
+66*cos(d*x+c)^6+33*cos(d*x+c)^5)+A*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^4*EllipticE(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^7+16*cos(d*x+c)^6+8*cos(d*x+c)^5
)+B*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a^4*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(246
*cos(d*x+c)^7+492*cos(d*x+c)^6+246*cos(d*x+c)^5)+B*((a+cos(d*x+c)*b)/(cos(
d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b^2*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(246*cos(d*x+c)^7+492*cos(...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11
/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11
/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{11/2}(a+b\cos(c+dx))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(3/2
),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(3/2
), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^5\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^5\,dx\right)b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^5\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**5,x)*a*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)
**2*sec(c + d*x)**5,x)*b**2 + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b +
 a)*sec(c + d*x)**5,x)*a**2
```

3.598 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 473

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\frac{2(a-b)\sqrt{a+b}(82a^2Ab-6Ab^3+63a^3B+21ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sqrt{}}}\right.\right.}{105a^3d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(a-b)\sqrt{a+b}(6Ab^2-a^2(25A-63B)+3ab(19A-7B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{105a^2d\sqrt{\sec(c+dx)}}$$
$$+\frac{2(25a^2A+3Ab^2+42abB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105ad}$$
$$+\frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}$$
$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
2/105*(a-b)*(a+b)^(1/2)*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*cos(d*x+c
)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+
c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/105*(a-b)*(a+b)^(1/2)*(6*A*b^2-a
^2*(25*A-63*B)+3*a*b*(19*A-7*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c
)^(1/2)+2/105*(25*A*a^2+3*A*b^2+42*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c
)^(3/2)*sin(d*x+c)/a/d+2/35*(8*A*b+7*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c
)^(5/2)*sin(d*x+c)/d+2/7*a*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)*sin(d
*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3318 vs. $2(473) = 946$.

Time = 26.48 (sec) , antiderivative size = 3318, normalized size of antiderivative = 7.01

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/
2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-82*a^2*A*b + 6*A*b^3 -
 63*a^3*B - 21*a*b^2*B)*Sin[c + d*x])/(105*a^2) + (2*Sec[c + d*x]^2*(8*A*b
*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Sin[c
+ d*x] + 3*A*b^2*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x]))/(105*a) + (2*a*A*S
ec[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-82*a*A*b)/(105*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/(35*a*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (3*a^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) - (b^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*A*Sq
rt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^2*Sqrt[Sec[c + d
*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c + d*x]])/(35*a^
2*Sqrt[a + b*Cos[c + d*x]]) + (a*b*B*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos
[c + d*x]]) - (b^3*B*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) -
(82*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x
]]) + (2*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos
[c + d*x]]) - (3*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*
Cos[c + d*x]]) - (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a +
 b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(82*a
^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*A*b^2 + 3*a...
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$\Big\downarrow$ 3468

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{b(4aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos(c+dx)+a(8Ab+}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{b(4aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos(c+dx)+a(8Ab+}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{b(4aA+7bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Aa^2+14bBa+7Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)-}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\Big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{2\int \frac{2ab(8Ab+7aB)\cos^2(c+dx)+a\left(21Ba^2+44Aba+35b^2B\right)\cos(c+dx)+a\left(25Aa^2+42bBa+3Ab^2\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{2ab(8Ab+7aB)\cos^2(c+dx)+a\left(21Ba^2+44Aba+35b^2B\right)\cos(c+dx)+a\left(25Aa^2+42bBa+3Ab^2\right)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{2ab(8Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\left(21Ba^2+44Aba+35b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(25Aa^2+42bBa+3Ab\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{5a}\right.\right.$$

$\Big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2\int\frac{\left(25Aa^2+84bBa+51Ab^2\right)\cos(c+dx)a^2+\left(63Ba^3+82Aba^2+21b^2Ba-6Ab^3\right)a}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+42abB+3A\right.}{3d}}{5a}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int\frac{\left(25Aa^2+84bBa+51Ab^2\right)\cos(c+dx)a^2+\left(63Ba^3+82Aba^2+21b^2Ba-6Ab^3\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2A+42abB+3Ab^2\right.}{3d\,\mathrm{co}}}{5a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int\frac{\left(25Aa^2+84bBa+51Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(63Ba^3+82Aba^2+21b^2Ba-6Ab^3\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2A+42abB+\right.}{3}}{5a}\right.\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(-\left(a^2(25A-63B)\right)+a(5\right.\right.}{3a}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(-\left(a^2(25A-63\right.\right.\right.}{3a}}{}\right.\right.$$

$\Big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(-\left(a^2(25A\right.\right.}{}}{}}{}\right.\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(63a^3B+82a^2Ab+21ab^2B-6Ab^3\right)}{}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*(8*A*b + 7*a*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]
*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cot[c + d*x]*EllipticE[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2 - a^2*(25*A - 63*B) + a
*(57*A*b - 21*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(3*a) + (2*(25
*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)))/(5*a))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(421) = 842$.

Time = 60.62 (sec) , antiderivative size = 2095, normalized size of antiderivative = 4.43

method	result	size
default	Expression too large to display	2095
parts	Expression too large to display	2114

input
```
int((a+cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/105/d*(a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(9/2)/(b*cos(d*x+c)^2+a*cos(d*x
+c)+cos(d*x+c)*b+a)*(A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)
/(cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))*(-82*cos(d*x+c)^6-164*cos(d*x+c)^5-82*cos(d*x+c)^4)+A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)
*a^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-82*cos(d*
x+c)^6-164*cos(d*x+c)^5-82*cos(d*x+c)^4)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a*b^3*EllipticE(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(6*cos(d*x+c)^6+12*cos(d*x+c)^5+6*cos(d
*x+c)^4)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*b^4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*(6*cos(d*x+c)^6+12*cos(d*x+c)^5+6*cos(d*x+c)^4)+B*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a^4*EllipticE(cot
(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-63*cos(d*x+c)^6-126*cos(d*x+c)^
5-63*cos(d*x+c)^4)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(
cos(d*x+c)+1)/(a+b))^(1/2)*a^3*b*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*(-63*cos(d*x+c)^6-126*cos(d*x+c)^5-63*cos(d*x+c)^4)+B*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*((a+cos(d*x+c)*b)/(cos(d*x+c)+1)/(a+b))^(1/2)*a
^2*b^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(-21*cos(d*x+
c)^6-42*cos(d*x+c)^5-21*cos(d*x+c)^4)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{9/2}(a+b\cos(c+dx))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^4\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**4,x)*a*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)
**2*sec(c + d*x)**4,x)*b**2 + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b +
 a)*sec(c + d*x)**4,x)*a**2
```

3.599 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 393

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(9a^2A+3Ab^2+20abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{15a^2d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(a-b)\sqrt{a+b}(9aA-3Ab-5aB+15bB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{15ad\sqrt{\sec(c+dx)}}$$
$$+\frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}$$
$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2+3*A*b^2+20*B*a*b)*cos(d*x+c)^(1/2)*csc(d*x
+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/
(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/a^2/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(a+b)^(1/2)*(9*A*a-3*A*b-5*B*a+15*B*b)
*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/15*(6*A*b+5*B*a)*(a+b*co
s(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*(a+b*cos(d*x+c))^(1/
2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2931 vs. 2(393) = 786.

Time = 23.95 (sec) , antiderivative size = 2931, normalized size of antiderivative = 7.46

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/
2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 3*A*b^2 + 20*a
*b*B)*Sin[c + d*x])/(15*a) + (2*Sec[c + d*x]*(6*A*b*Sin[c + d*x] + 5*a*B*S
in[c + d*x]))/15 + (2*a*A*Sec[c + d*x]*Tan[c + d*x])/5))/d + (2*((-3*a^2*A
)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^2)/(5*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*a*b*B)/(3*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) + (a*A*b*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x
]]) - (A*b^3*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) + (a^2*B*S
qrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) - (b^2*B*Sqrt[Sec[c + d*x]
])/(3*Sqrt[a + b*Cos[c + d*x]]) - (3*a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d
*x]])/(5*Sqrt[a + b*Cos[c + d*x]]) - (A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c +
d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (4*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec
[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*(-2*(a + b)*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(3*b*(A + 5
*B) + a*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)] - (9*a^2*A + 3*A*b^2 + 20*a*b*B)*Cos[c + d*x]*(a + b*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a*d*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +...
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$\Big\downarrow$ 3468

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{b(2aA+5bB)\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(6Ab+}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{b(2aA+5bB)\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)+a(6Ab+}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{b(2aA+5bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+10bBa+5Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)-}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\Big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2\int \frac{a\left(9Aa^2+20bBa+3Ab^2\right)+a\left(5Ba^2+12Aba+15b^2B\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+6Ab)\sin(c-}{3d\cos}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{a\left(9Aa^2+20bBa+3Ab^2\right)+a\left(5Ba^2+12Aba+15b^2B\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(5aB+6Ab)\sin(c+}{3d\cos^{\frac{3}{2}}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{a\left(9Aa^2+20bBa+3Ab^2\right)+a\left(5Ba^2+12Aba+15b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(5aB+6Ab)\sin(c}{3d\,c}\right.\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a\left(9a^2A+20abB+3Ab^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(9aA-5aB+\right.}{3a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a\left(9a^2A+20abB+3Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)(9aA-\right.}{3a}\right.\right.$$

$\Big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a\left(9a^2A+20abB+3Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(9aA}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{ad}}{}\right.\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(9*a^2*A +
3*A*b^2 + 20*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b
)*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B + 15*b*B)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/d)/(3*a) + (2*(6*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1630 vs. $2(347) = 694$.

Time = 43.64 (sec) , antiderivative size = 1631, normalized size of antiderivative = 4.15

method	result	size
default	Expression too large to display	1631
parts	Expression too large to display	1635

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/15/d/a*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)^2*b+a*cos(d*
x+c)+b*cos(d*x+c)+a)*(A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-
b)/(a+b))^(1/2))*(-9*cos(d*x+c)^5-18*cos(d*x+c)^4-9*cos(d*x+c)^3)+A*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*a^2*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-9*cos(d*x
+c)^5-18*cos(d*x+c)^4-9*cos(d*x+c)^3)+A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(-csc(d*x+c
)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-3*cos(d*x+c)^5-6*cos(d*x+c)^4-3*cos(d
*x+c)^3)+A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*b^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/
2))*(-3*cos(d*x+c)^5-6*cos(d*x+c)^4-3*cos(d*x+c)^3)+B*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b*Ellipt
icE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-20*cos(d*x+c)^5-40*cos(
d*x+c)^4-20*cos(d*x+c)^3)+B*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(-20*cos(d*x+c)^5-40*cos(d*x+c)^4-20*cos(d*x+c)^3)+
A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*a^3*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(9*co
s(d*x+c)^5+18*cos(d*x+c)^4+9*cos(d*x+c)^3)+A*(1/(a+b)*(a+b*cos(d*x+c))/...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=2\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}dx\right)ab+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}dx\right)b^{2}+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{3}dx\right)a^{2}$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**3,x)*a*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)
**2*sec(c + d*x)**3,x)*b**2 + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b +
 a)*sec(c + d*x)**3,x)*a**2
```

3.600 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 479

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(4Ab+3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{3ad\sqrt{\sec(c+dx)}}$$
$$+\frac{2\sqrt{a+b}(3Ab^2+a^2(A-3B)-a(4Ab-6bB))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+d}}\right.\right.}{3ad\sqrt{\sec(c+dx)}}$$
$$-\frac{2b\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{d\sqrt{\sec(c+dx)}}$$
$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*(a-b)*(a+b)^(1/2)*(4*A*b+3*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+
c)^(1/2)+2/3*(a+b)^(1/2)*(3*A*b^2+a^2*(A-3*B)-a*(4*A*b-6*B*b))*cos(d*x+c)^
(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*b*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc
(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+
b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+2/3*a*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c
)^(3/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5981 vs. $2(479) = 958$.

Time = 26.43 (sec) , antiderivative size = 5981, normalized size of antiderivative = 12.49

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/
2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{3b^2B\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(4Ab+3aB)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{3b^2B\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)+a(4Ab+3aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{3b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(4Ab+3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{a(4Ab+3aB)+\left(Aa^2+6bBa+3Ab^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3b^2B\int \frac{\sqrt{\cos(c}}{\sqrt{a+b\cos}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{a(4Ab+3aB)+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int \frac{\sqrt{s}}{\sqrt{a+}}\right.\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{a(4Ab+3aB)+\left(Aa^2+6bBa+3Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6bB\sqrt{a+b}\text{co}}{}\right.\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx\right.}}\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a(3aB+4Ab)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a^2}{}\right.\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(}{}}}{ad}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*Sqrt[a + b]*(4*A*b + 3*
a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt[a + b]*(3*A*b^2
+ a^2*(A - 3*B) - a*(4*A*b - 6*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(a*d) - (6*b*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/d)/3 + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1370 vs. $2(424) = 848$.

Time = 15.35 (sec) , antiderivative size = 1371, normalized size of antiderivative = 2.86

method	result	size
parts	Expression too large to display	1371
default	Expression too large to display	1381

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*A/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(4*cos(d*x+c)^3+8*cos(d*x+c)^2+4*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(-csc(
d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(4*cos(d*x+c)^3+8*cos(d*x+c)^2+4*c
os(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a^2*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/
2))*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(-csc(d
*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-4*cos(d*x+c)^3-8*cos(d*x+c)^2-4*c
os(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*b^2*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/
2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+(1+cos(d*x+c))*sin(d*x+c
)*a^2+sin(d*x+c)*cos(d*x+c)*(5+cos(d*x+c))*a*b+4*b^2*cos(d*x+c)^2*sin(d*x+
c))*(a+b*cos(d*x+c))^(1/2)*cos(d*x+c)*sec(d*x+c)^(5/2)/(cos(d*x+c)^2*b+a*c
os(d*x+c)+b*cos(d*x+c)+a)+2*B/d*(((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)
-cot(d*x+c))*a^2+(-(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))*a*
b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=2\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}dx\right)ab+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}dx\right)b^{2}+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}dx\right)a^{2}$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**2,x)*a*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)
**2*sec(c + d*x)**2,x)*b**2 + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b +
 a)*sec(c + d*x)**2,x)*a**2
```

3.601 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 509

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{(a-b)\sqrt{a+b}(2aA-bB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec\cdots}{a-\cdots}}}{ad\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(2a(A-B)-b(4A+B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\cdots}}{d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(2Ab+3aB)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(\cdots}{a+\cdots}}}{d\sqrt{\sec(c+dx)}}$$

$$+\frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

$$-\frac{(2aA-bB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
(a-b)*(a+b)^(1/2)*(2*A*a-B*b)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*c
os(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/
2)-(a+b)^(1/2)*(2*a*(A-B)-b*(4*A+B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c
)^(1/2)-(a+b)^(1/2)*(2*A*b+3*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^
(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec
(d*x+c)^(1/2)+2*a*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d-(
2*A*a-B*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 17.65 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.82

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/
2),x]
```

output
```
(2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt
[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a^2*A*Tan[(c + d*x)/2] - 2*a*A*b*Tan[(
c + d*x)/2] + a*b*B*Tan[(c + d*x)/2] + b^2*B*Tan[(c + d*x)/2] + 4*a*A*b*Ta
n[(c + d*x)/2]^3 - 2*b^2*B*Tan[(c + d*x)/2]^3 + 2*a^2*A*Tan[(c + d*x)/2]^5
 - 2*a*A*b*Tan[(c + d*x)/2]^5 - a*b*B*Tan[(c + d*x)/2]^5 + b^2*B*Tan[(c +
d*x)/2]^5 + 4*A*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*T
an[(c + d*x)/2]^2)/(a + b)] + 6*a*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*A*b^2*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/
(a + b)] + 6*a*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(2*a*A - b*B)*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + 2*(-(A*b^2) + 2*a*b*(A - B) + a^2*(A + B))*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan...
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$\downarrow$ 3468

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\int \frac{-b(2aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(2Ab+aB}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{-b(2aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+dx)+a(2Ab+aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{-b(2aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-Aa^2+2bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a(}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b^2(2Ab+3aB)\cos^2(c+dx)+2ab(2Ab+aB)\cos(c+dx)+ab(2aA-bB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(2aA-bB)\sin(c+d}{d\sqrt{\cos(}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b^2(2Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab(2Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(2aA-bB)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(2aA-bB)\sin(}{d}\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2(3aB+2Ab)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+\int \frac{ab(2aA-bB)+2ab(2Ab+aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(2aA-}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2(3aB+2Ab)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{ab(2aA-bB)+2ab(2Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\right.$$

$\Big\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\frac{ab(2aA-bB)+2ab(2Ab+aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}(3aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{}}{2b}\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{ab(2aA-bB)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(2aA-2aB-4Ab-bB)\int\frac{}{\sqrt{\cos(c}}}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-ab(2aA-2aB-4Ab-bB)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab(2aA-bB)}{}\right.$$

$\Big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{ab(2aA-bB)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}(2aA-2aB-4Ab-bB)\cot(c+}{}}{}\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2b\sqrt{a+b}(2aA-2aB-4Ab-bB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\,\text{co}}}{\sqrt{a+b}\sqrt{\text{c}}}\right.\right.}{d}\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*b*Sqrt[a + b]*(2*a*A -
b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(2*a*A
- 4*A*b - 2*a*B - b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b*Sqrt[
a + b]*(2*A*b + 3*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)
/(2*b) + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]) - ((2*a*A - b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c
+ d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(456) = 912$.

Time = 14.68 (sec) , antiderivative size = 1245, normalized size of antiderivative = 2.45

method	result	size
default	Expression too large to display	1245
parts	Expression too large to display	1295

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/d*((-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticPi(-csc(d*x+c)
+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^2-12*cos(d*x+c)-6)*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*a*b*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(2*c
os(d*x+c)^2+4*cos(d*x+c)+2)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticE(-csc(d*x+c)+cot(d*x+c)
,(-(a-b)/(a+b))^(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*Ellip
ticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*x
+c)-1)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2)
)+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(-csc(d*x+c)+cot(d
*x+c),(-(a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^2-4*cos(d*x+c)-2)*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2
*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8
*cos(d*x+c)-4)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b
))^(1/2))+(2*cos(d*x+c)^2+4*cos(d*x+c)+2)*A*(cos(d*x+c)/(1+cos(d*x+c)))...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=2\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right)ab+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)dx\right)b^{2}+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)a^{2}$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
),x)*a*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2
*sec(c + d*x),x)*b**2 + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*se
c(c + d*x),x)*a**2
```

3.602 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx}$

Optimal result

Integrand size = 35, antiderivative size = 532

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(4Ab+5aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{4ad\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(8aA+4Ab+5aB+2bB)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{}}{4d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(12aAb+3a^2B+4b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{4bd\sqrt{\sec(c+dx)}}$$

$$+\frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}$$

$$+\frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
-1/4*(a-b)*(a+b)^(1/2)*(4*A*b+5*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x
+c)^(1/2)+1/4*(a+b)^(1/2)*(8*A*a+4*A*b+5*B*a+2*B*b)*cos(d*x+c)^(1/2)*csc(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/d/sec(d*x+c)^(1/2)-1/4*(a+b)^(1/2)*(12*A*a*b+3*B*a^2+4*B*b^2)*cos(d*x+
c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/2*b*B*(a+b*cos(d*x+c))^(
1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+5*B*a)*(a+b*cos(d*x+c))^(1/2
)*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1134 vs. 2(532) = 1064.

Time = 19.17 (sec) , antiderivative size = 1134, normalized size of antiderivative = 2.13

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x
]],x]
```

output
```
(b*B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) +
 (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*
Tan[(c + d*x)/2] + 5*a^2*B*Tan[(c + d*x)/2] + 5*a*b*B*Tan[(c + d*x)/2] - 8
*A*b^2*Tan[(c + d*x)/2]^3 - 10*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c +
 d*x)/2]^5 + 4*A*b^2*Tan[(c + d*x)/2]^5 - 5*a^2*B*Tan[(c + d*x)/2]^5 + 5*a
*b*B*Tan[(c + d*x)/2]^5 + 24*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 24*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 8*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(4*A*b + 5*a*B)*El...
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 3469

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{b(4Ab+5aB)\cos^2(c+dx)+2\left(2Ba^2+4Aba+b^2B\right)\cos(c+dx)+a(4aA+b}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{b(4Ab+5aB)\cos^2(c+dx)+2\left(2Ba^2+4Aba+b^2B\right)\cos(c+dx)+a(4aA+b}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{b(4Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2Ba^2+4Aba+b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)+}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\Big\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int -\frac{-b\left(3Ba^2+12Aba+4b^2B\right)\cos^2(c+dx)-2ab(4aA+bB)\cos(c+dx)+ab(4Ab+5aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(5aB}{}\right.\right.$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-b\left(3Ba^2+12Aba+4b^2B\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-b\left(3Ba^2+12Aba+4b^2B\right)\sin(c+dx+}{\sin(c+dx+\frac{\pi}{2}}}{}\right.\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab(4Ab+5aB)-2ab(4aA+bB)\cos(c+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab(4Ab+5aB)-2ab(4aA+bB)\sin(c+}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin(c+dx-}}}{}\right.\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab(4Ab+5aB)-2ab(4aA+bB)\sin(c+}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin(c+dx-}}}{}\right.\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(5aB+4Ab)\int\frac{\cos(c+dx}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab(8aA+5aB+4Ab+2bB)}{}\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(5aB+4Ab)\int\frac{\sin(c+}{\sin(c+dx+\frac{\pi}{2})^{3/2}}}{}\right.\right.\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2\sqrt{a+b}(3a^2B+12aAb+4b^2B)\cot(c+dx}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b*Sqrt[a + b]*(4*A*b
 + 5*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(8
*a*A + 4*A*b + 5*a*B + 2*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*
Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)
/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/d)/b + ((4*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(d*Sqrt[Cos[c + d*x]]))/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. 2(469) = 938.

Time = 12.40 (sec) , antiderivative size = 1389, normalized size of antiderivative = 2.61

method	result	size
default	Expression too large to display	1389
parts	Expression too large to display	1428

input

```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/4/d*((24*cos(d*x+c)^2+48*cos(d*x+c)+24)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticPi(-csc(d
*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(6*cos(d*x+c)^2+12*cos(d*x+c)+6)
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*a^2*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+
(8*cos(d*x+c)^2+16*cos(d*x+c)+8)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticPi(-csc(d*x+c)+cot(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
a*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(4*cos(d*x+c)^2
+8*cos(d*x+c)+4)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a
+b))^(1/2))+(5*cos(d*x+c)^2+10*cos(d*x+c)+5)*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticE(-csc(
d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(5*cos(d*x+c)^2+10*cos(d*x+c)+5)*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*a*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(8*cos
(d*x+c)^2+16*cos(d*x+c)+8)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(-csc(d*x+c)+cot(d*x+c),
(-(a-b)/(a+b))^(1/2))+(-16*cos(d*x+c)^2-32*cos(d*x+c)-16)*A*(cos(d*x+c)...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
)), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}(a+b\cos(c+dx))^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = 2\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)ab + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
2*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x),x)*a*b + in
t(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2,x)*b**2 + in
t(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a),x)*a**2
```

3.603 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 626

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(30aAb+3a^2B+16b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\cdots}}{24abd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(30aAb+12Ab^2+3a^2B+14abB+16b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos\cdots}}{\sqrt{a+b}\sqrt{\cos\cdots}}\cdots\right.\right.}{24bd\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(6a^2Ab+8Ab^3-a^3B+12ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\cdots\right.\right.}{8b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)}+\frac{(6Ab+7aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d\sqrt{\sec(c+dx)}}$$

$$+\frac{(30aAb+3a^2B+16b^2B)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{24bd}$$

output
```
-1/24*(a-b)*(a+b)^(1/2)*(30*A*a*b+3*B*a^2+16*B*b^2)*cos(d*x+c)^(1/2)*csc(d
*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a/b/d/sec(d*x+c)^(1/2)+1/24*(a+b)^(1/2)*(30*A*a*b+12*A*b^2+3*B*a^2+14*
B*a*b+16*B*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/8*(a+b)^(
1/2)*(6*A*a^2*b+8*A*b^3-B*a^3+12*B*a*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*Elli
pticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)
/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/b^2/d/sec(d*x+c)^(1/2)+1/3*b*B*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/sec(
d*x+c)^(3/2)+1/12*(6*A*b+7*B*a)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*
x+c)^(1/2)+1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x
+c)^(1/2)*sin(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1489 vs. $2(626) = 1252$.

Time = 20.98 (sec) , antiderivative size = 1489, normalized size of antiderivative = 2.38

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d
*x]],x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Sin[c + d*x])/12 + ((6*
A*b + 7*a*B)*Sin[2*(c + d*x)])/24 + (b*B*Sin[3*(c + d*x)])/12))/d + (Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*(30*a^2*A*b*Tan[(c + d*x)/2] + 30*a*A*b^2*T
an[(c + d*x)/2] + 3*a^3*B*Tan[(c + d*x)/2] + 3*a^2*b*B*Tan[(c + d*x)/2] +
16*a*b^2*B*Tan[(c + d*x)/2] + 16*b^3*B*Tan[(c + d*x)/2] - 60*a*A*b^2*Tan[(
c + d*x)/2]^3 - 6*a^2*b*B*Tan[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3
 - 30*a^2*A*b*Tan[(c + d*x)/2]^5 + 30*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^3*B
*Tan[(c + d*x)/2]^5 + 3*a^2*b*B*Tan[(c + d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d
*x)/2]^5 + 16*b^3*B*Tan[(c + d*x)/2]^5 + 36*a^2*A*b*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] - 6*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 72*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
 d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 36*a^2*A*b*EllipticPi[-1, Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]...
```

Rubi [A] (verified)

Time = 3.18 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3469}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{\sqrt{\cos(c+dx)}(b(6Ab+7aB)\cos^2(c+dx)+2(3Ba^2+6Aba+2b^2B)\cos(c+}{2\sqrt{a+b\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int\frac{\sqrt{\cos(c+dx)}(b(6Ab+7aB)\cos^2(c+dx)+2(3Ba^2+6Aba+2b^2B)\cos(c+}{\sqrt{a+b\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2(3Ba^2+6Aba+2b^2I\right.}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{3528}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int\frac{b(3Ba^2+30Aba+16b^2B)\cos^2(c+dx)+2b(12Aa^2+13bBa+6Ab^2)\cos(c+dx)+ab(6Ab+7aB)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int\frac{b(3Ba^2+30Aba+16b^2B)\cos^2(c+dx)+2b(12Aa^2+13bBa+6Ab^2)\cos(c+dx)+ab(6Ab+7aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int \frac{b\left(3Ba^2+30Aba+16b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(12Aa^2+13bBa+6Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(6Ab+7aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{4b}\right.\right.$$

$\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{\int -\frac{-2a(6Ab+7aB)\cos(c+dx)b^2-3\left(-Ba^3+6Aba^2+12b^2Ba+8Ab^3\right)\cos^2(c+dx)b+a\left(3Ba^2+30Aba+16b^2B\right)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}\right.\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-2a(6Ab+7aB)\cos(c+dx)b^2-3\left(-Ba^3+6Aba^2\right.}{\cos^{\frac{3}{2}}(c+}}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-2a(6Ab+7aB)\sin\left(c+dx+\frac{\pi}{2}\right)b^2-3\left(-Ba^3+6A\right.}{\sin\left(c+dx\right.}}}{4b}\right.\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3Ba^2+30Aba+16b^2B\right)-2ab^2(6Ab+7aB)\cos}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3Ba^2+30Aba+16b^2B\right)-2ab^2(6Ab+7aB)\sin}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right.}}}{}}{4b}\right.\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3Ba^2+30Aba+16b^2B\right)-2ab^2(6Ab+7aB)\sin}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right.}}}{}}{}\right.\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(3a^2B+30aAb+16b^2B)\int\frac{\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab(3a^2B+30aAb+14abB+12Ab^2+16b^2B)}{}}{}\right.\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(3a^2B+30aAb+16b^2B)\int\frac{\sin\left(c+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{}}{}\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b\sqrt{a+b}\left(3a^2B+30aAb+14abB+12Ab^2+16b^2B\right)\cdots}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Cos[c + d*x]^(3/2)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (((6*A*b + 7*a*B)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b*Sqrt[a +
 b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d) - (2*b*Sqrt[a + b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14*a*b*B + 16*b^2
*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (6*Sqrt[a + b]*(6*a^2*A*b + 8
*A*b^3 - a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d)/b + ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]]))/(4*b))/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1738 vs. $2(557) = 1114$.

Time = 11.61 (sec) , antiderivative size = 1739, normalized size of antiderivative = 2.78

method	result	size
default	Expression too large to display	1739
parts	Expression too large to display	1766

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d/b*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*cos(d*x+c)+
a)/sec(d*x+c)^(1/2)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*(-36*cos(d*x+c)-72-36*sec(d*x+c))+A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3*Elli
pticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-48*cos(d*x+c)-96-
48*sec(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a^3*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*(6*cos(d*x+c)+12+6*sec(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticPi(-c
sc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-72*cos(d*x+c)-144-72*sec(d
*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a^2*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/
2))*(-30*cos(d*x+c)-60-30*sec(d*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(-csc(d*x+c
)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-30*cos(d*x+c)-60-30*sec(d*x+c))+B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*a^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-3*cos(d*
x+c)-6-3*sec(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(
d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c
)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\sec(dx+c)}dx\right.$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)b^2$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\sec(dx+c)}dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
2*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/sec(c + d
*x),x)*a*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)
**2)/sec(c + d*x),x)*b**2 + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b +
a))/sec(c + d*x),x)*a**2
```

3.604 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 730

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{192ab^2d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(9a^3B-6a^2b(4A+B)-8b^3(16A+9B)-4ab^2(28A+39B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{Elliptic}}{192b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(8a^3Ab-96aAb^3-3a^4B-24a^2b^2B-48b^4B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\right.\right.}{64b^3d\sqrt{\sec(c+dx)}}$$

$$+\frac{(8aAb-3a^2B+12b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}}$$

$$+\frac{(8Ab-3aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24bd\sqrt{\sec(c+dx)}}+\frac{B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}}$$

$$+\frac{(24a^2Ab+128Ab^3-9a^3B+156ab^2B)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{192b^2d}$$

output
```
-1/192*(a-b)*(a+b)^(1/2)*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B*a*b^2)*cos(d*
x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)-1/192*(a+b)^(1/2)*(9*B*a^3-6*
a^2*b*(4*A+B)-8*b^3*(16*A+9*B)-4*a*b^2*(28*A+39*B))*cos(d*x+c)^(1/2)*csc(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/b^2/d/sec(d*x+c)^(1/2)+1/64*(a+b)^(1/2)*(8*A*a^3*b-96*A*a*b^3-3*B*a^4-
24*B*a^2*b^2-48*B*b^4)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)
^(1/2)+1/32*(8*A*a*b-3*B*a^2+12*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/b
/d/sec(d*x+c)^(1/2)+1/24*(8*A*b-3*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b
/d/sec(d*x+c)^(1/2)+1/4*B*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d/sec(d*x+c)
^(1/2)+1/192*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B*a*b^2)*(a+b*cos(d*x+c))^(
1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1888 vs. $2(730) = 1460$.

Time = 22.34 (sec) , antiderivative size = 1888, normalized size of antiderivative = 2.59

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(
3/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((8*A*b + 9*a*B)*Sin[c + d*x
])/96 + ((56*a*A*b + 3*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/(192*b) + ((8*A
*b + 9*a*B)*Sin[3*(c + d*x)])/96 + (b*B*Sin[4*(c + d*x)])/32))/d - (Sqrt[(
1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(24*a^3*A*b*Tan[(c + d*x)/2] + 24
*a^2*A*b^2*Tan[(c + d*x)/2] + 128*a*A*b^3*Tan[(c + d*x)/2] + 128*A*b^4*Tan
[(c + d*x)/2] - 9*a^4*B*Tan[(c + d*x)/2] - 9*a^3*b*B*Tan[(c + d*x)/2] + 15
6*a^2*b^2*B*Tan[(c + d*x)/2] + 156*a*b^3*B*Tan[(c + d*x)/2] - 48*a^2*A*b^2
*Tan[(c + d*x)/2]^3 - 256*A*b^4*Tan[(c + d*x)/2]^3 + 18*a^3*b*B*Tan[(c + d
*x)/2]^3 - 312*a*b^3*B*Tan[(c + d*x)/2]^3 - 24*a^3*A*b*Tan[(c + d*x)/2]^5
+ 24*a^2*A*b^2*Tan[(c + d*x)/2]^5 - 128*a*A*b^3*Tan[(c + d*x)/2]^5 + 128*A
*b^4*Tan[(c + d*x)/2]^5 + 9*a^4*B*Tan[(c + d*x)/2]^5 - 9*a^3*b*B*Tan[(c +
d*x)/2]^5 - 156*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 156*a*b^3*B*Tan[(c + d*x)/2
]^5 - 48*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)] + 576*a*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 18*a^4*B*EllipticPi[-1, A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 144...
```

Rubi [A] (verified)

Time = 3.73 (sec) , antiderivative size = 696, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.629$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3469

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{(a+b\cos(c+dx))^{3/2}((8Ab-3aB)\cos^2(c+dx)+6bB\cos(c+dx)+aB)}{2\sqrt{\cos(c+dx)}}dx}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{(a+b\cos(c+dx))^{3/2}((8Ab-3aB)\cos^2(c+dx)+6bB\cos(c+dx)+aB)}{\sqrt{\cos(c+dx)}}dx}{8b}+\frac{B\sin(c+dx)\sqrt{\cos(c\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left((8Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+6bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{8b}+\frac{B\sin(c+dx\right.$$

$\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{3}\int \frac{\sqrt{a+b\cos(c+dx)}(3(-3Ba^2+8Aba+12b^2B)\cos^2(c+dx)+2b(16Ab+15aB)\cos(c+dx)+a(8Ab+3aB))}{2\sqrt{\cos(c+dx)}}}{8b}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\int \frac{\sqrt{a+b\cos(c+dx)}(3(-3Ba^2+8Aba+12b^2B)\cos^2(c+dx)+2b(16Ab+15aB)\cos(c+dx)+a(8Ab+3aB))}{\sqrt{\cos(c+dx)}}}{8b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\int\frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3\left(-3Ba^2+8Aba+12b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b(16Ab+15aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{8b}\right.$$

$\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(57Ba^2+104Aba+36b^2B\right)\cos(c+dx)+a(3}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\int\frac{\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(57Ba^2+104Aba+36b^2B\right)\cos(c+dx)+a(3}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\int\frac{\left(-9Ba^3+24Aba^2+156b^2Ba+128Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(57Ba^2+104Aba+36b^2B\right)\sin\left(c+dx+\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.}{}\right.$$

$\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int-\frac{3\left(-3Ba^4+8Aba^3-24b^2Ba^2-96Ab^3a-48b^4B\right)\cos^2(c+dx)-2ab\left(3Ba^2+56Aba+36b^2B\right)\cos(c+dx)+a\left(\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.\right.}{}\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{3\left(-3Ba^4+8Aba^3-24b^2B\ldots\right.}{}}{}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{3\left(-3Ba^4+8Aba^3-24b^2B\ldots\right.}{}}{}\right.\right.}{}\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-9Ba^3+24Aba^2+156b^2\ldots\right.}{\ldots}}{}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-9Ba^3+24Aba^2+156b^2\ldots\right.}{\sin(\ldots}}{}\right.\right.}{}\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a\left(-9Ba^3+24Aba^2+156b^2\cdots\right.}{\sin(\cdots}}{\cdots}\right.\right.}{\cdots}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-9a^3B+24a^2Ab+156a\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(9a^3B-6a^2b(4A+B)-4\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a\left(-9a^3B+24a^2Ab+156a\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow \; 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{6}\left(\frac{3\left(-3a^2B+8aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{1}{4}\left(\frac{\left(-9a^3B+24a^2Ab+156ab^2\right.}{}\right.\right.}{}\right.$$

input
```
Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(4*b*d) + (((8*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]
*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + ((3*(8*a*A*b - 3*a^2*B +
 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
 + (-1/2*((2*(a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a
*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt[a + b]*(9*a^3
*B - 6*a^2*b*(4*A + B) - 8*b^3*(16*A + 9*B) - 4*a*b^2*(28*A + 39*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3
 - 3*a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(b*d))/b + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/4)/6)/(8*
b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. 2(655) = 1310.

Time = 13.08 (sec) , antiderivative size = 2280, normalized size of antiderivative = 3.12

method	result	size
default	Expression too large to display	2280
parts	Expression too large to display	2303

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d/b^2*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*cos(d*x+
c)+a)/sec(d*x+c)^(3/2)*(-9*B*a^4*tan(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticF(-c
sc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(228+456*sec(d*x+c)+228*sec(d*x
+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a*b^3*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/
2))*(-72-144*sec(d*x+c)-72*sec(d*x+c)^2)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticPi(-csc(d
*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(48+96*sec(d*x+c)+48*sec(d*x+c)^
2)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a*b^3*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*(-576-1152*sec(d*x+c)-576*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticPi(-
csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-144-288*sec(d*x+c)-144*se
c(d*x+c)^2)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(-24-48*sec(d*x+c)-24*sec(d*x+c)^2)+A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticE(-
csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-24-48*sec(d*x+c)-24*sec(d*x+
c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\sec(dx+c)^2}dx\right.$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2}{\sec(dx+c)^2}dx\right)b^2$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\sec(dx+c)^2}dx\right)a^2$$

input
```
int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
2*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/sec(c + d
*x)**2,x)*a*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d
*x)**2)/sec(c + d*x)**2,x)*b**2 + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x
)*b + a))/sec(c + d*x)**2,x)*a**2
```

3.605 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 662

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(3705a^4Ab+255a^2Ab^3+40Ab^5+1617a^5B+3069a^3b^2B-110ab^4B)\sqrt{\cos(c+}}{3465a^4d\sqrt{\sec(c+}}$$
$$+\frac{2(a-b)\sqrt{a+b}(40Ab^4+3a^4(225A-539B)-6a^3b(505A-209B)+15a^2b^2(19A-121B)+10ab^3(3A}{3465a^3d\sqrt{}}$$
$$+\frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465a^2d}$$
$$+\frac{2(1145a^2Ab+15Ab^3+539a^3B+825ab^2B)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3465ad}$$
$$+\frac{2(81a^2A+113Ab^2+209abB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{693d}$$
$$+\frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d}$$
$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx)}{11d}$$

output
```
2/3465*(a-b)*(a+b)^(1/2)*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+3
069*B*a^3*b^2-110*B*a*b^4)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x+c)^(1/2
)+2/3465*(a-b)*(a+b)^(1/2)*(40*A*b^4+3*a^4*(225*A-539*B)-6*a^3*b*(505*A-20
9*B)+15*a^2*b^2*(19*A-121*B)+10*a*b^3*(3*A-11*B))*cos(d*x+c)^(1/2)*csc(d*x
+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/
(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/a^3/d/sec(d*x+c)^(1/2)+2/3465*(675*A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*
a^3*b+55*B*a*b^3)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d
+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^3+825*B*a*b^2)*(a+b*cos(d*x+c))^(1/
2)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d+2/693*(81*A*a^2+113*A*b^2+209*B*a*b)*(a
+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/99*a*(14*A*b+11*B*a)*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d+2/11*a*A*(a+b*cos(d*x
+c))^(3/2)*sec(d*x+c)^(11/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4198 vs. $2(662) = 1324$.

Time = 30.01 (sec) , antiderivative size = 4198, normalized size of antiderivative = 6.34

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13
/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3705*a^4*A*b + 255*a^2*A
*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sin[c + d*x])
/(3465*a^3) + (2*Sec[c + d*x]^4*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c +
d*x]))/99 + (2*Sec[c + d*x]^3*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d
*x] + 209*a*b*B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(1145*a^2*A*b*Sin[c
 + d*x] + 15*A*b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin
[c + d*x]))/(3465*a) + (2*Sec[c + d*x]*(675*a^4*A*Sin[c + d*x] + 1025*a^2*
A*b^2*Sin[c + d*x] - 20*A*b^4*Sin[c + d*x] + 1793*a^3*b*B*Sin[c + d*x] + 5
5*a*b^3*B*Sin[c + d*x]))/(3465*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x]
)/11))/d + (2*((-247*a^2*A*b)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]]) - (17*A*b^3)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*
A*b^5)/(693*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*a^3*B)/(
15*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a*b^2*B)/(35*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^4*B)/(63*a*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (15*a^3*A*Sqrt[Sec[c + d*x]])/(77*Sqrt[a + b*
Cos[c + d*x]]) - (26*a*A*b^2*Sqrt[Sec[c + d*x]])/(231*Sqrt[a + b*Cos[c + d
*x]]) - (7*A*b^4*Sqrt[Sec[c + d*x]])/(99*a*Sqrt[a + b*Cos[c + d*x]]) - (8*
A*b^6*Sqrt[Sec[c + d*x]])/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) + (38*a^2*b*B
*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*b^3*B*Sqrt[Sec[
c + d*x]])/(315*Sqrt[a + b*Cos[c + d*x]]) + (2*b^5*B*Sqrt[Sec[c + d*x]]...
```

Rubi [A] (verified)

Time = 3.68 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.629$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{13}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{13/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{13/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{11}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(6aA+11bB)\cos^2(c+dx)+\left(9Aa^2+22bBa+11Ab^2\right)\right.}{2\cos^{\frac{11}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(6aA+11bB)\cos^2(c+dx)+\left(9Aa^2+22bBa+11Ab^2\right)\right.}{\cos^{\frac{11}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(6aA+11bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(9Aa^2+22bB\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}\right.$$

$$\Big\downarrow \text{3526}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{2}{9}\int \frac{3b\left(22Ba^2+46Aba+33b^2B\right)\cos^2(c+dx)+\left(77Ba^3+233Aba^2+297b^2\right.}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\int \frac{3b\left(22Ba^2+46Aba+33b^2B\right)\cos^2(c+dx)+\left(77Ba^3+233Aba^2+297b^2\right.}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\int \frac{3b\left(22Ba^2+46Aba+33b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(77Ba^3+233Aba^2+2\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+}}\right.\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\int\frac{4ab(81Aa^2+209bBa+113Ab^2)\cos^2(c+dx)+a(405Aa^3+1507bBa^2+1531Ab^2a+693b^3B)\,c}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{7a}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\int\frac{4ab(81Aa^2+209bBa+113Ab^2)\cos^2(c+dx)+a(405Aa^3+1507bBa^2+1531Ab^2a+693b^3B)\cos}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{7a}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\int\frac{4ab(81Aa^2+209bBa+113Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a(405Aa^3+1507bBa^2+1531Ab^2a+693b^3B)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin(c+dx+}}}{7a}\right.\right.\right.$$

$\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{2\int\frac{(1617Ba^3+5055Aba^2+6655b^2Ba+2305Ab^3)\cos(c+dx)a^2+2b(539Ba^3+1145Aba^2+825b^2Ba+15Ab^3)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}}{}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{(1617Ba^3+5055Aba^2+6655b^2Ba+2305Ab^3)\cos(c+dx)a^2+2b(539Ba^3+1145Aba^2+825b^2Ba+15Ab^3)\,c}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{(1617Ba^3+5055Aba^2+6655b^2Ba+2305Ab^3)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b(539Ba^3+1145Aba^2+825b^2Ba+15A}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin(c+d}}}{5a}}{}\right.\right.\right.$$

$\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{2\int \frac{3\left(\left(675Aa^4+2871bBa^3+3315Ab^2a^2+1705b^3Ba+10Ab^4\right)\cos(c+dx)a^2+\left(1617Ba^5+3705Aba^4+3069b^2\right.}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a}}{}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(675Aa^4+2871bBa^3+3315Ab^2a^2+1705b^3Ba+10Ab^4\right)\cos(c+dx)a^2+\left(1617Ba^5+3705Aba^4+3069b^2Ba^3\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{a}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(675Aa^4+2871bBa^3+3315Ab^2a^2+1705b^3Ba+10Ab^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(1617Ba^5+3705Aba^4+3069b^2\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{}\right.\right.\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{a(a-b)\left(3a^4(225A-539B)-6a^3b(505A-209B)+15a^2b^2(19A-121B)+10ab^3(3A-11B)+40Ab^4\right)\int \frac{}{\sqrt{\cos(c\cdots}}}{}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\frac{a(a-b)\left(3a^4(225A-539B)-6a^3b(505A-209B)+15a^2b^2(19A-121B)+10ab^3(3A-11B)+40Ab^4\right)\int\frac{}{\sqrt{\sin\left(c\right.}}}{}}{}}{}\right.\right.\right.$$

$$\Big\downarrow \; 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{\frac{\frac{a\left(1617a^5B+3705a^4Ab+3069a^3b^2B+255a^2Ab^3-110ab^4B+40Ab^5\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\right.}}}{}}{}}{}\right.\right.\right.$$

$$\Big\downarrow \; 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\left(539a^3B+\right.}{}\right.\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + b*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + ((2*a*(14*A*b + 11*a*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((2*(81*a^2*A +
113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
 d*x]^(7/2)) + ((2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b
)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069
*a^3*b^2*B - 110*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*
(a - b)*Sqrt[a + b]*(40*A*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b*(505*A - 2
09*B) + 15*a^2*b^2*(19*A - 121*B) + 10*a*b^3*(3*A - 11*B))*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/d)/a + (2*(675*a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1
793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c
+ d*x]^(3/2)))/(5*a))/(7*a))/9)/11)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

output
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{13}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(1
3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{13}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(13
/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{13}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(13
/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{13/2}(a+b\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + b*cos(c + d*x))^(5/2
),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + b*cos(c + d*x))^(5/2
), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = 3\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^6\,dx\right)a^2b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\sec(dx+c)^6\,dx\right)b^3 + 3\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^6\,dx\right)a\,b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^6\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**6,x)*a**2*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d
*x)**3*sec(c + d*x)**6,x)*b**3 + 3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x
)*b + a)*cos(c + d*x)**2*sec(c + d*x)**6,x)*a*b**2 + int(sqrt(sec(c + d*x)
)*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**6,x)*a**3
```

3.606 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 562

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(147a^4A+279a^2Ab^2-10Ab^4+435a^3bB+45ab^3B)\sqrt{\cos(c+dx)}\csc(c+dx)E}{315a^3d\sqrt{\sec(c+dx)}}$$

$$-\frac{2(a-b)\sqrt{a+b}(10Ab^3-6a^2b(19A-60B)+3a^3(49A-25B)+15ab^2(11A-3B))\sqrt{\cos(c+dx)}\csc(c}{315a^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315ad}$$

$$+\frac{2(49a^2A+75Ab^2+135abB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d}$$

$$+\frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{21d}$$

$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}$$

output
```
2/315*(a-b)*(a+b)^(1/2)*(147*A*a^4+279*A*a^2*b^2-10*A*b^4+435*B*a^3*b+45*B
*a*b^3)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/315*(a-b)*(a+b)
^(1/2)*(10*A*b^3-6*a^2*b*(19*A-60*B)+3*a^3*(49*A-25*B)+15*a*b^2*(11*A-3*B)
)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/315*(163*A*a^2*b+5*A*
b^3+75*B*a^3+135*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+
c)/a/d+2/315*(49*A*a^2+75*A*b^2+135*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+
c)^(5/2)*sin(d*x+c)/d+2/21*a*(4*A*b+3*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+
c)^(7/2)*sin(d*x+c)/d+2/9*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2)*sin(
d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3755 vs. 2(562) = 1124.

Time = 28.41 (sec) , antiderivative size = 3755, normalized size of antiderivative = 6.68

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11
/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 279*a^2*A*b^
2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sin[c + d*x])/(315*a^2) + (2*Sec[
c + d*x]^3*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin[c + d*x]))/63 + (2*Sec[c +
 d*x]^2*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*x] + 135*a*b*B*Sin[c +
 d*x]))/315 + (2*Sec[c + d*x]*(163*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c +
d*x] + 75*a^3*B*Sin[c + d*x] + 135*a*b^2*B*Sin[c + d*x]))/(315*a) + (2*a^2
*A*Sec[c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^3*A)/(15*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a*A*b^2)/(35*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) + (2*A*b^4)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]) - (29*a^2*b*B)/(21*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (b^3*B)/(7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (38*a^2*A*b*Sqr
t[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*A*b^3*Sqrt[Sec[c +
d*x]])/(315*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Sqrt[Sec[c + d*x]])/(63*a
^2*Sqrt[a + b*Cos[c + d*x]]) + (5*a^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b
*Cos[c + d*x]]) - (2*a*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*
x]]) - (b^4*B*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*
A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (
31*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]
) + (2*A*b^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c
 + d*x]]) - (29*a*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a...
```

Rubi [A] (verified)

Time = 2.94 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{11}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$\downarrow$ 3468

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(4aA+9bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos\right.}{2\cos^{\frac{9}{2}}(c+dx)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(4aA+9bB)\cos^2(c+dx)+\left(7Aa^2+18bBa+9Ab^2\right)\cos\right.}{\cos^{\frac{9}{2}}(c+dx)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(4aA+9bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(7Aa^2+18bBa+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}\right.$$

$\downarrow$ 3526

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{2}{7}\int \frac{b\left(36Ba^2+76Aba+63b^2B\right)\cos^2(c+dx)+\left(45Ba^3+137Aba^2+189b^2Ba\right.}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(}}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\int \frac{b\left(36Ba^2+76Aba+63b^2B\right)\cos^2(c+dx)+\left(45Ba^3+137Aba^2+189b^2Ba\right.}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\int \frac{b\left(36Ba^2+76Aba+63b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(45Ba^3+137Aba^2+189\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+}}\right.\right.$$

$\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{2\int \frac{2ab(49Aa^2+135bBa+75Ab^2)\cos^2(c+dx)+a(147Aa^3+585bBa^2+605Ab^2a+315b^3B)\cos(c+}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{\int \frac{2ab(49Aa^2+135bBa+75Ab^2)\cos^2(c+dx)+a(147Aa^3+585bBa^2+605Ab^2a+315b^3B)\cos(c+d}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{\int \frac{2ab(49Aa^2+135bBa+75Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2+a(147Aa^3+585bBa^2+605Ab^2a+315b^3B)\sin(}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{5a}\right.\right.\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{2\int \frac{3\left(\left(75Ba^3+261Aba^2+405b^2Ba+155Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+435bBa^3+279Ab^2a^2+45b^3Ba-10A\right.\right.}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a}}{5a}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(75Ba^3+261Aba^2+405b^2Ba+155Ab^3\right)\cos(c+dx)a^2+\left(147Aa^4+435bBa^3+279Ab^2a^2+45b^3Ba-10Ab^4\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{a}}{5a}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(75Ba^3+261Aba^2+405b^2Ba+155Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(147Aa^4+435bBa^3+279Ab^2a^2+45b^3Ba-10A\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{5a}\right.\right.\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{\frac{a\left(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(3a^3\right.}{a}}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{a\left(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(\right.\right.\right.$$

$\Big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{a\left(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(}{}\right.\right.\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{1}{7}\left(\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(75a^3B+163\right.}{}\right.\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + b*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((6*a*(4*A*b + 3*a*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*(49*a^2*A + 75*A
*b^2 + 135*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]
^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 +
 435*a^3*b*B + 45*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2
*(a - b)*Sqrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A - 60*B) + 3*a^3*(49*A - 25*
B) + 15*a*b^2*(11*A - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/a + (2
*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)))/(5*a))/7)/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

output
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11
/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11
/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{11/2}(a+b\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2
),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2
), x)
```

Reduce [F]

$$\int (a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{11}{2}}(c + dx)\,dx = 3\left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c)\,b + a}\,\cos(dx + c)\sec(dx + c)^5\,dx\right)a^2 b + \left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c)\,b + a}\,\cos(dx + c)^3\sec(dx + c)^5\,dx\right)b^3 + 3\left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c)\,b + a}\,\cos(dx + c)^2\sec(dx + c)^5\,dx\right)a\,b^2 + \left(\int \sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c)\,b + a}\,\sec(dx + c)^5\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**5,x)*a**2*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d
*x)**3*sec(c + d*x)**5,x)*b**3 + 3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x
)*b + a)*cos(c + d*x)**2*sec(c + d*x)**5,x)*a*b**2 + int(sqrt(sec(c + d*x)
)*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**5,x)*a**3
```

3.607 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 474

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(145a^2Ab+15Ab^3+63a^3B+161ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{105a^2d\sqrt{\sec(c+dx)}}$$
$$+\frac{2(a-b)\sqrt{a+b}(a^2(25A-63B)+15b^2(A-7B)-8ab(15A-7B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\,(}{105ad\sqrt{\sec(c+dx)}}$$
$$+\frac{2(25a^2A+45Ab^2+77abB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d}$$
$$+\frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}$$
$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
2/105*(a-b)*(a+b)^(1/2)*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*cos(d*
x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(a+b)^(1/2)*(a^2*(2
5*A-63*B)+15*b^2*(A-7*B)-8*a*b*(15*A-7*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*Ell
ipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^
(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/s
ec(d*x+c)^(1/2)+2/105*(25*A*a^2+45*A*b^2+77*B*a*b)*(a+b*cos(d*x+c))^(1/2)*
sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/35*a*(10*A*b+7*B*a)*(a+b*cos(d*x+c))^(1/2)
*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(
7/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3348 vs. $2(474) = 948$.

Time = 26.75 (sec) , antiderivative size = 3348, normalized size of antiderivative = 7.06

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/
2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(145*a^2*A*b + 15*A*b^3 +
 63*a^3*B + 161*a*b^2*B)*Sin[c + d*x])/(105*a) + (2*Sec[c + d*x]^2*(15*a*A
*b*Sin[c + d*x] + 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Si
n[c + d*x] + 45*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x]))/105 + (2*a^2*
A*Sec[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-29*a^2*A*b)/(21*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^3)/(7*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (3*a^3*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
) - (23*a*b^2*B)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^3
*A*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (2*a*A*b^2*Sqrt[Sec
[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (A*b^4*Sqrt[Sec[c + d*x]])/(7*
a*Sqrt[a + b*Cos[c + d*x]]) + (8*a^2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a +
b*Cos[c + d*x]]) - (8*b^3*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x
]]) - (29*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[
c + d*x]]) - (A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*C
os[c + d*x]]) - (3*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a
+ b*Cos[c + d*x]]) - (23*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sq
rt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)
*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^...
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3526, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$\big\downarrow$ 3468

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(2aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos\right.}{2\cos^{\frac{7}{2}}(c+dx)}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{a+b\cos(c+dx)}\left(b(2aA+7bB)\cos^2(c+dx)+\left(5Aa^2+14bBa+7Ab^2\right)\cos\right.}{\cos^{\frac{7}{2}}(c+dx)}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(2aA+7bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5Aa^2+14bBa+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}\right.$$

$\big\downarrow$ 3526

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{2}{5}\int \frac{b\left(14Ba^2+30Aba+35b^2B\right)\cos^2(c+dx)+\left(21Ba^3+65Aba^2+105b^2Ba\right.}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c}}\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{b\left(14Ba^2+30Aba+35b^2B\right)\cos^2(c+dx)+\left(21Ba^3+65Aba^2+105b^2Ba\right.}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c-}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{b\left(14Ba^2+30Aba+35b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(21Ba^3+65Aba^2+105b\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b}}\right.\right.$$

$\big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{2\int \frac{a(63Ba^3+145Aba^2+161b^2Ba+15Ab^3)+a(25Aa^3+119bBa^2+135Ab^2a+105b^3B)\cos(c+dx}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a}$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{\int \frac{a(63Ba^3+145Aba^2+161b^2Ba+15Ab^3)+a(25Aa^3+119bBa^2+135Ab^2a+105b^3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}c}{3a}$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{\int \frac{a(63Ba^3+145Aba^2+161b^2Ba+15Ab^3)+a(25Aa^3+119bBa^2+135Ab^2a+105b^3B)\sin(c+dx+}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}}{3a}$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{a(a-b)\left(a^2(25A-63B)-8ab(15A-7B)+15b^2(A-7B)\right)\int \frac{}{\sqrt{\cos(c+dx)}\sqrt{}}}{}$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{a(a-b)\left(a^2(25A-63B)-8ab(15A-7B)+15b^2(A-7B)\right)\int \frac{}{\sqrt{\sin(c+dx+\frac{\pi}{2}}}}{}$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{a(63a^3B+145a^2Ab+161ab^2B+15Ab^3)\int \frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}}{}$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2(25a^2A+77abB+45Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}(a\right.\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + b*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*a*(10*A*b + 7*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a
 + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a*d) + (2*(a - b)*Sqrt[a + b]*(a^2*(25*A - 63*B) + 15*
b^2*(A - 7*B) - 8*a*b*(15*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
)/(3*a) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/5)/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9
/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{9/2}(a+b\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = 3\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)a^2b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\sec(dx+c)^4\,dx\right)b^3 + 3\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)a\,b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^4\,dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**4,x)*a**2*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d
*x)**3*sec(c + d*x)**4,x)*b**3 + 3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x
)*b + a)*cos(c + d*x)**2*sec(c + d*x)**4,x)*a*b**2 + int(sqrt(sec(c + d*x)
)*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**4,x)*a**3
```

3.608 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 553

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(9a^2A+23Ab^2+35abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{15ad\sqrt{\sec(c+dx)}}$$
$$+\frac{2\sqrt{a+b}(15Ab^3-ab^2(23A-45B)+a^2b(17A-35B)-a^3(9A-5B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{Elliptic}}{15ad\sqrt{\sec(c+dx)}}$$
$$-\frac{2b^2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{d\sqrt{\sec(c+dx)}}$$
$$+\frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}$$
$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2+23*A*b^2+35*B*a*b)*cos(d*x+c)^(1/2)*csc(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)
/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a/d/sec(d*x+c)^(1/2)+2/15*(a+b)^(1/2)*(15*A*b^3-a*b^2*(23*A-45*B)+a^2*b
*(17*A-35*B)-a^3*(9*A-5*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
-2*b^2*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
+2/15*a*(8*A*b+5*B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d
+2/5*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7032 vs. 2(553) = 1106.

Time = 27.65 (sec) , antiderivative size = 7032, normalized size of antiderivative = 12.72

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/
2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.94, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.457, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3526, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3468}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{\sqrt{a+b\cos(c+dx)}\left(5b^2B\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)\right.}{2\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{a+b\cos(c+dx)}\left(5b^2B\cos^2(c+dx)+\left(3Aa^2+10bBa+5Ab^2\right)\cos(c+dx)\right.}{\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(5b^2B\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3Aa^2+10bBa+5Ab^2\right)\mathrm{s}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}\right.$$

$$\Big\downarrow \text{ 3526}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{15B\cos^2(c+dx)b^3+a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45\right.}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{15B\cos^2(c+dx)b^3+a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{15B\sin\left(c+dx+\frac{\pi}{2}\right)^2 b^3+a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\right.}}\right.\right.$$

$$\Big\downarrow \; 3532$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\cos(c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\sin\left(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$

$$\Big\downarrow \; 3288$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{a\left(9Aa^2+35bBa+23Ab^2\right)+\left(5Ba^3+17Aba^2+45b^2Ba+15Ab^3\right)\sin\left(\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$

$$\Big\downarrow \; 3477$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(9a^2A+35abB+23Ab^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\left(-\left(a^3\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(9a^2A+35abB+23Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$

$$\Big\downarrow \; 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(9a^2A+35abB+23Ab^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(9a^2A+35abB+23Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{ad}\right.\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + b*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(9*a^2*A
+ 23*A*b^2 + 35*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqr
t[a + b]*(15*A*b^3 - a*b^2*(23*A - 45*B) + a^2*b*(17*A - 35*B) - a^3*(9*A
- 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (30*b^2*Sqrt[a + b]*B
*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/3 + (2*a*(8*A*b + 5*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

Fricas [F]

$$\int (a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \int (B\cos(dx + c) + A)(b\cos(dx + c) + a)^{\frac{5}{2}}\sec(dx + c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7
/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}(a+b\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=3\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^{3}dx\right)a^{2}b+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^{3}\sec(dx+c)^{3}dx\right)b^{3}+3\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^{2}\sec(dx+c)^{3}dx\right)a\,b^{2}+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^{3}dx\right)a^{3}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**3,x)*a**2*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d
*x)**3*sec(c + d*x)**3,x)*b**3 + 3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x
)*b + a)*cos(c + d*x)**2*sec(c + d*x)**3,x)*a*b**2 + int(sqrt(sec(c + d*x)
)*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**3,x)*a**3
```

3.609 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 596

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{(a-b)\sqrt{a+b}(14aAb+6a^2B-3b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{3ad\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(2ab(7A-9B)-2a^2(A-3B)-3b^2(6A+B))\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{3d\sqrt{\sec(c+dx)}}$$

$$-\frac{b\sqrt{a+b}(2Ab+5aB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a-}}}{d\sqrt{\sec(c+dx)}}$$

$$+\frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

$$-\frac{(14aAb+6a^2B-3b^2B)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}$$

$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
1/3*(a-b)*(a+b)^(1/2)*(14*A*a*b+6*B*a^2-3*B*b^2)*cos(d*x+c)^(1/2)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(
a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/a/d/sec(d*x+c)^(1/2)-1/3*(a+b)^(1/2)*(2*a*b*(7*A-9*B)-2*a^2*(A-3*B)-3*b^2
*(6*A+B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-b*(a+b)^(1/2)*(2*A*
b+5*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+2*a*(2*A*b+
B*a)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d-1/3*(14*A*a*b+6*
B*a^2-3*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d+2/3*a*
A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7700 vs. $2(596) = 1192$.

Time = 28.06 (sec) , antiderivative size = 7700, normalized size of antiderivative = 12.92

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/
2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.08 (sec) , antiderivative size = 568, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3526, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(2aA-3bB)\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos\right.}{2\cos^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(2aA-3bB)\cos^2(c+dx)+\left(Aa^2+6bBa+3Ab^2\right)\cos\right.}{\cos^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-b(2aA-3bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(Aa^2+6bBa+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}\right.$$

$$\Big\downarrow \text{3526}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(2\int \frac{-b\left(6Ba^2+14Aba-3b^2B\right)\cos^2(c+dx)-\left(3Ba^3+7Aba^2-9b^2Ba-3Ab\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{-b\left(6Ba^2+14Aba-3b^2B\right)\cos^2(c+dx)-\left(3Ba^3+7Aba^2-9b^2Ba-3Ab^3\right)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{-b\left(6Ba^2+14Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-3Ba^3-7Aba^2+9b^2Ba\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+\right.}}\right.\right.$$

$\big\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\int \frac{3(2Ab+5aB)\cos^2(c+dx)b^3+a\left(6Ba^2+14Aba-3b^2B\right)b+2a\left(Aa^2+9bBa+9Ab^2\right)\cos(c+dx)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\int \frac{3(2Ab+5aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+a\left(6Ba^2+14Aba-3b^2B\right)b+2a\left(Aa^2+9bBa+9Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}\right.\right.$$

$\big\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\int \frac{ab\left(6Ba^2+14Aba-3b^2B\right)+2ab\left(Aa^2+9bBa+9Ab^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3b^3(5aB+2Ab)\int \frac{\sqrt{c}}{\sqrt{a+}}}{2b}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\int \frac{ab\left(6Ba^2+14Aba-3b^2B\right)+2ab\left(Aa^2+9bBa+9Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3(5aB+2Ab)\int -}{2b}\right.\right.$$

$\big\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\int \frac{ab(6Ba^2+14Aba-3b^2B)+2ab(Aa^2+9bBa+9Ab^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{6b^2\sqrt{a+b}(5aB+2Ab)\cot(}{}}{2b}\right.\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{ab\left(6a^2B+14aAb-3b^2B\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx - ab\left(-2a^2(A-3B)+\right.}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{-ab\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B)\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin(c}}}{}\right.\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{ab\left(6a^2B+14aAb-3b^2B\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx - \frac{2b\sqrt{a+b}\left(-2a^2(A\right.}{}}{}\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{-\frac{2b\sqrt{a+b}\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{d}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*A*(a + b*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (((2*(a - b)*b*Sqrt[a + b]*(14*a*A
*b + 6*a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*
Sqrt[a + b]*(2*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3*b^2*(6*A + B))*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/d - (6*b^2*Sqrt[a + b]*(2*A*b + 5*a*B)*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(2*b) + (6*a*(2*A*b + a*B)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((14*a*A*
b + 6*a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+b\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx &= 3\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a^2b \\ &+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b^3 \\ &+3\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a\,b^2 \\ &+\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)^2\,dx\right)a^3\end{aligned}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
)**2,x)*a**2*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d
*x)**3*sec(c + d*x)**2,x)*b**3 + 3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x
)*b + a)*cos(c + d*x)**2*sec(c + d*x)**2,x)*a*b**2 + int(sqrt(sec(c + d*x)
)*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2,x)*a**3
```

3.610 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 607

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{(a-b)\sqrt{a+b}(8a^2A-4Ab^2-9abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{4ad\sqrt{\sec(c+dx)}}$$
$$-\frac{\sqrt{a+b}(8a^2(A-B)-2b^2(2A+B)-3ab(8A+3B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{4d\sqrt{\sec(c+dx)}}$$
$$-\frac{\sqrt{a+b}(20aAb+15a^2B+4b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a-}{a-}\right.}{4d\sqrt{\sec(c+dx)}}$$
$$-\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}$$
$$-\frac{(8a^2A-4Ab^2-9abB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d}$$
$$+\frac{2aA(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
1/4*(a-b)*(a+b)^(1/2)*(8*A*a^2-4*A*b^2-9*B*a*b)*cos(d*x+c)^(1/2)*csc(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a
-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
a/d/sec(d*x+c)^(1/2)-1/4*(a+b)^(1/2)*(8*a^2*(A-B)-2*b^2*(2*A+B)-3*a*b*(8*A
+3*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/4*(a+b)^(1/2)*(20*A*
a*b+15*B*a^2+4*B*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2
)-1/2*b*(4*A*a-B*b)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)-1
/4*(8*A*a^2-4*A*b^2-9*B*a*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d
*x+c)/d+2*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1278 vs. $2(607) = 1214$.

Time = 20.93 (sec) , antiderivative size = 1278, normalized size of antiderivative = 2.11

$$\int (a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/
2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(2*a^2*A*Sin[c + d*x] + (b^2*
B*Sin[2*(c + d*x)])/4))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-8*a^3*A
*Tan[(c + d*x)/2] - 8*a^2*A*b*Tan[(c + d*x)/2] + 4*a*A*b^2*Tan[(c + d*x)/2
] + 4*A*b^3*Tan[(c + d*x)/2] + 9*a^2*b*B*Tan[(c + d*x)/2] + 9*a*b^2*B*Tan[
(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2]^3 - 8*A*b^3*Tan[(c + d*x)/2]^3
- 18*a*b^2*B*Tan[(c + d*x)/2]^3 + 8*a^3*A*Tan[(c + d*x)/2]^5 - 8*a^2*A*b*T
an[(c + d*x)/2]^5 - 4*a*A*b^2*Tan[(c + d*x)/2]^5 + 4*A*b^3*Tan[(c + d*x)/2
]^5 - 9*a^2*b*B*Tan[(c + d*x)/2]^5 + 9*a*b^2*B*Tan[(c + d*x)/2]^5 + 40*a*A
*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] + 30*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^3*B*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 40*a*A*b^2*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + 30*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^3*B*Ell...
```

Rubi [A] (verified)

Time = 3.10 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3528, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$\downarrow$ 3468

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(4aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c\right.}{2\sqrt{\cos(c+dx)}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sqrt{a+b\cos(c+dx)}\left(-b(4aA-bB)\cos^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\cos(c+\right.}{\sqrt{\cos(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-b(4aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(-Aa^2+2bBa+\right.\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{-b\left(8Aa^2-9bBa-4Ab^2\right)\cos^2(c+dx)-2\left(2Aa^3-6bBa^2-6Ab^2a-b^3B\right)\mathrm{co}}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{-b\left(8Aa^2-9bBa-4Ab^2\right)\cos^2(c+dx)-2\left(2Aa^3-6bBa^2-6Ab^2a-b^3B\right)\mathrm{co}}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{-b\left(8Aa^2-9bBa-4Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2\left(2Aa^3-6bBa^2-6Ab^2a-b^3\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+d\right.}}\right.$$

$\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int \frac{b^2(15Ba^2+20Aba+4b^2B)\cos^2(c+dx)+2ab(4Ba^2+12Aba+b^2B)\cos(c+dx)+ab(8Aa^2-9bBa-4A}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int \frac{b^2(15Ba^2+20Aba+4b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab(4Ba^2+12Aba+b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)+ab(8Aa^2-9b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}\right.\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int \frac{ab(8Aa^2-9bBa-4Ab^2)+2ab(4Ba^2+12Aba+b^2B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+b^2(15a^2B+20aAb+4}{2b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{b^2(15a^2B+20aAb+4b^2B)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{ab(8Aa^2-9bBa-4Ab^2)+2ab(4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{2b}\right.\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int \frac{ab(8Aa^2-9bBa-4Ab^2)+2ab(4Ba^2+12Aba+b^2B)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}(15a^2B+20aAb+}{}}{}\right.\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{ab(8a^2A-9abB-4Ab^2)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(8a^2(A-B)-3ab}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{-ab\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx\right.}}}{}\right.\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{ab\left(8a^2A-9abB-4Ab^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\left(8a^2(A-B\right.}{}}{}\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(-\frac{2b\sqrt{a+b}\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{Ell}}{d}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(b*(4*a*A - b*B)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/d + (2*a*A*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (((2*(a - b)*b*Sqrt[a + b]
*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
- (2*b*Sqrt[a + b]*(8*a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*(8*A + 3*B))*C
ot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b*Sqrt[a + b]*(20*a*A*b + 15*a^
2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(2*b) -
 ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]))/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1853 vs. $2(540) = 1080$.

Time = 20.54 (sec) , antiderivative size = 1854, normalized size of antiderivative = 3.05

method	result	size
default	Expression too large to display	1854
parts	Expression too large to display	1893

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d*((-40*cos(d*x+c)^2-80*cos(d*x+c)-40)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticPi(-csc
(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c
)-30)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*a^2*b*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))+(-8*cos(d*x+c)^2-16*cos(d*x+c)-8)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^3*EllipticPi(-csc(d*x
+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(8*cos(d*x+c)^2+16*cos(d*x+c)+8)*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*a^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(8*cos
(d*x+c)^2+16*cos(d*x+c)+8)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(-csc(d*x+c)+cot(d*x+c
),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*E
llipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-4*cos(d*x+c)^2-8*c
os(d*x+c)-4)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*b^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))
^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(-csc(d
*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-9*cos(d*x+c)^2-18*cos(d*x+c)-9...
```

Fricas [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=3\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right)a^{2}b+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)dx\right)b^{3}+3\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)dx\right)a\,b^{2}+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)dx\right)a^{3}$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)*sec(c + d*x
),x)*a**2*b + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)
**3*sec(c + d*x),x)*b**3 + 3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b +
a)*cos(c + d*x)**2*sec(c + d*x),x)*a*b**2 + int(sqrt(sec(c + d*x))*sqrt(co
s(c + d*x)*b + a)*sec(c + d*x),x)*a**3
```

3.611 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx}$

Optimal result

Integrand size = 35, antiderivative size = 624

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(54aAb+33a^2B+16b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{24ad\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(4b^2(3A+4B)+a^2(48A+33B)+a(54Ab+26bB))\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\right.}{24d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(30a^2Ab+8Ab^3+5a^3B+20ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{8bd\sqrt{\sec(c+dx)}}$$

$$+\frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$+\frac{(54aAb+33a^2B+16b^2B)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{24d}$$

output
```
-1/24*(a-b)*(a+b)^(1/2)*(54*A*a*b+33*B*a^2+16*B*b^2)*cos(d*x+c)^(1/2)*csc(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a/d/sec(d*x+c)^(1/2)+1/24*(a+b)^(1/2)*(4*b^2*(3*A+4*B)+a^2*(48*A+33*B
)+a*(54*A*b+26*B*b))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/8*(a+b
)^(1/2)*(30*A*a^2*b+8*A*b^3+5*B*a^3+20*B*a*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c
)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(
-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*b*(2*A*b+3*B*a)*(a+b*cos(d*x+c))^(1/2)*s
in(d*x+c)/d/sec(d*x+c)^(1/2)+1/3*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/s
ec(d*x+c)^(1/2)+1/24*(54*A*a*b+33*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^(1/2)*s
ec(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1504 vs. 2(624) = 1248.

Time = 22.73 (sec) , antiderivative size = 1504, normalized size of antiderivative = 2.41

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x
]],x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*B*Sin[c + d*x])/12 + (b
*(6*A*b + 13*a*B)*Sin[2*(c + d*x)])/24 + (b^2*B*Sin[3*(c + d*x)])/12))/d +
 (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(54*a^2*A*b*Tan[(c + d*x)/2] + 54*a*
A*b^2*Tan[(c + d*x)/2] + 33*a^3*B*Tan[(c + d*x)/2] + 33*a^2*b*B*Tan[(c + d
*x)/2] + 16*a*b^2*B*Tan[(c + d*x)/2] + 16*b^3*B*Tan[(c + d*x)/2] - 108*a*A
*b^2*Tan[(c + d*x)/2]^3 - 66*a^2*b*B*Tan[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c
+ d*x)/2]^3 - 54*a^2*A*b*Tan[(c + d*x)/2]^5 + 54*a*A*b^2*Tan[(c + d*x)/2]^
5 - 33*a^3*B*Tan[(c + d*x)/2]^5 + 33*a^2*b*B*Tan[(c + d*x)/2]^5 - 16*a*b^2
*B*Tan[(c + d*x)/2]^5 + 16*b^3*B*Tan[(c + d*x)/2]^5 + 180*a^2*A*b*Elliptic
Pi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
+ 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d
*x)/2]^2)/(a + b)] + 30*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*B*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a^2*A*b*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2
]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b...
```

Rubi [A] (verified)

Time = 3.24 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 3469

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(3b(2Ab+3aB)\cos^2(c+dx)+2\left(3Ba^2+6Aba+2b^2B\right)c\right.}{2\sqrt{\cos(c+dx)}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{\sqrt{a+b\cos(c+dx)}\left(3b(2Ab+3aB)\cos^2(c+dx)+2\left(3Ba^2+6Aba+2b^2B\right)c\right.}{\sqrt{\cos(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3b(2Ab+3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(3Ba^2+6Aba\right.\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\Big\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{b\left(33Ba^2+54Aba+16b^2B\right)\cos^2(c+dx)+2\left(12Ba^3+36Aba^2+19b^2Ba\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c\right.}}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\int \frac{b\left(33Ba^2+54Aba+16b^2B\right)\cos^2(c+dx)+2\left(12Ba^3+36Aba^2+19b^2Ba\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+\right.}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\int \frac{b\left(33Ba^2+54Aba+16b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(12Ba^3+36Aba^2+19b\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\,\mathrm{si}}}\right.\right.$$

$\Big\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int -\frac{-3b(5Ba^3+30Aba^2+20b^2Ba+8Ab^3)\cos^2(c+dx)-2ab(24Aa^2+13bBa+6Ab^2)\cos(c+dx)+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.\right.\right.$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-3b(5Ba^3+30}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-3b(5Ba^3+30}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab(33Ba^2+54}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab(33Ba^2+54}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab(33Ba^2+54}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(33a^2B+5}{}\right.\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab(a^2(48A}{}\right.\right.\right.$$

$$\Big\downarrow \ 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(33a^2B+5}{}\right.\right.\right.$$

$$\Big\downarrow \ 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b\sqrt{a+b}(a^2(48}{}}{}\right.\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + ((3*b*(2*A*b + 3*a*B)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b*Sq
rt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(a*d) - (2*b*Sqrt[a + b]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*B) + a*
(54*A*b + 26*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (6*Sqrt[a + b]
*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/d)/b + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/4)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2003 vs. $2(555) = 1110$.

Time = 12.48 (sec) , antiderivative size = 2004, normalized size of antiderivative = 3.21

method	result	size
default	Expression too large to display	2004
parts	Expression too large to display	2039

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/24/d*((-180*cos(d*x+c)^2-360*cos(d*x+c)-180)*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticPi(
-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-48*cos(d*x+c)^2-96*cos(d
*x+c)-48)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*b^3*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))+(-30*cos(d*x+c)^2-60*cos(d*x+c)-30)*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*EllipticPi(-csc
(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+(-120*cos(d*x+c)^2-240*cos(d*x
+c)-120)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*a*b^2*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))+(-54*cos(d*x+c)^2-108*cos(d*x+c)-54)*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(-
csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-54*cos(d*x+c)^2-108*cos(d*x+
c)-54)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*a*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/
2))+(-33*cos(d*x+c)^2-66*cos(d*x+c)-33)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(-csc(d*x+c
)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-33*cos(d*x+c)^2-66*cos(d*x+c)-33)*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*a^2*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-1...
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
)), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}(a+b\cos(c+dx))^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = 3\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)\,dx\right)a^2b + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2\,dx\right)a\,b^2 + \left(\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

output
```
3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x),x)*a**2*b +
 int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**3,x)*b**3 +
 3*int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x)**2,x)*a*b*
*2 + int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a),x)*a**3
```

3.612 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 724

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx =$$
$$-\frac{(a-b)\sqrt{a+b}(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{192abd\sqrt{\sec(c+dx)}}$$
$$+\frac{\sqrt{a+b}(15a^3B+8b^3(16A+9B)+2a^2b(132A+59B)+4ab^2(52A+71B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{El}}{192bd\sqrt{\sec(c+dx)}}$$
$$-\frac{\sqrt{a+b}(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\mathrm{arcs}\right.}{64b^2d\sqrt{\sec(c+dx)}}$$
$$+\frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)}$$
$$+\frac{(24aAb+5a^2B+12b^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{32d\sqrt{\sec(c+dx)}}$$
$$+\frac{(8Ab+11aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24d\sqrt{\sec(c+dx)}}$$
$$+\frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{192bd}$$

output
```
-1/192*(a-b)*(a+b)^(1/2)*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*cos(
d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/192*(a+b)^(1/2)*(15*B*a^3+8
*b^3*(16*A+9*B)+2*a^2*b*(132*A+59*B)+4*a*b^2*(52*A+71*B))*cos(d*x+c)^(1/2)
*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-
b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/64*(a+b)^(1/2)*(40*A*a^3*b+160*A*a*b^3-5*
B*a^4+120*B*a^2*b^2+48*B*b^4)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2
))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec
(d*x+c)^(1/2)+1/4*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+1/32*(24*A*a*b+5*B*a^2+12*B*b^2)*(a+b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/sec(
d*x+c)^(1/2)+1/24*(8*A*b+11*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d
*x+c)^(1/2)+1/192*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*(a+b*cos(d*
x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1857 vs. $2(724) = 1448$.

Time = 21.84 (sec) , antiderivative size = 1857, normalized size of antiderivative = 2.56

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d
*x]],x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*(8*A*b + 17*a*B)*Sin[c +
d*x])/96 + ((104*a*A*b + 59*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/192 + (b*(
8*A*b + 17*a*B)*Sin[3*(c + d*x)])/96 + (b^2*B*Sin[4*(c + d*x)])/32))/d + (
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(264*a^3*A*b*Tan[(c + d*x)/2] + 264*a^
2*A*b^2*Tan[(c + d*x)/2] + 128*a*A*b^3*Tan[(c + d*x)/2] + 128*A*b^4*Tan[(c
 + d*x)/2] + 15*a^4*B*Tan[(c + d*x)/2] + 15*a^3*b*B*Tan[(c + d*x)/2] + 284
*a^2*b^2*B*Tan[(c + d*x)/2] + 284*a*b^3*B*Tan[(c + d*x)/2] - 528*a^2*A*b^2
*Tan[(c + d*x)/2]^3 - 256*A*b^4*Tan[(c + d*x)/2]^3 - 30*a^3*b*B*Tan[(c + d
*x)/2]^3 - 568*a*b^3*B*Tan[(c + d*x)/2]^3 - 264*a^3*A*b*Tan[(c + d*x)/2]^5
 + 264*a^2*A*b^2*Tan[(c + d*x)/2]^5 - 128*a*A*b^3*Tan[(c + d*x)/2]^5 + 128
*A*b^4*Tan[(c + d*x)/2]^5 - 15*a^4*B*Tan[(c + d*x)/2]^5 + 15*a^3*b*B*Tan[(
c + d*x)/2]^5 - 284*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 284*a*b^3*B*Tan[(c + d*
x)/2]^5 + 240*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] + 960*a*A*b^3*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*B*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 7
20*a^2*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]...
```

Rubi [A] (verified)

Time = 3.88 (sec) , antiderivative size = 693, normalized size of antiderivative = 0.96, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.629$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 3440

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\Big\downarrow$ 3469

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(b(8Ab+11aB)\cos^2(c+dx)+2\left(4Ba^2+8\right.\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\int\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(b(8Ab+11aB)\cos^2(c+dx)+2\left(4Ba^2+8A\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(8Ab+11aB)\sin\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$

$\Big\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\int\frac{\sqrt{a+b\cos(c+dx)}\left(3b\left(5Ba^2+24Aba+12b^2B\right)\cos^2(c+dx)+2b\left(24Aa^2+31bBa+16Ab^2\right)\cos(c+dx)+\right.}{2\sqrt{\cos(c+dx)}}}{3b}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\int\frac{\sqrt{a+b\cos(c+dx)}\left(3b\left(5Ba^2+24Aba+12b^2B\right)\cos^2(c+dx)+2b\left(24Aa^2+31bBa+16Ab^2\right)\cos(c+dx)+\right.}{\sqrt{\cos(c+dx)}}}{6b}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3b\left(5Ba^2+24Aba+12b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(24Aa^2+31bBa+16Ab^2\right)\sin\left(c\right.\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{6b}\right.\right.$$

$\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{2}\int \frac{b\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(96Aa^3+161bBa^2+152Ab^2a+36b^3B\right)\cos}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\int \frac{b\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)\cos^2(c+dx)+2b\left(96Aa^3+161bBa^2+152Ab^2a+36b^3B\right)\cos}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\int \frac{b\left(15Ba^3+264Aba^2+284b^2Ba+128Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(96Aa^3+161bBa^2+152Ab^2a+36b^3B\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.\right.$$

$\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{\int -\frac{-2a\left(59Ba^2+104Aba+36b^2B\right)\cos(c+dx)b^2-3\left(-5Ba^4+40Aba^3+120b^2Ba^2+160Ab^3a+48b^4B\right)\cos^2(c-}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.}{}\right.\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-2a\left(59Ba^2+104Aba+36\ldots\right.}{\ldots}}{\ldots}\right.}{\ldots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-2a\left(59Ba^2+104Aba+36\ldots\right.}{\ldots}}{\ldots}\right.}{\ldots}\right.\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(15Ba^3+264Aba^2+284\ldots\right.}{\ldots}}{\ldots}\right.}{\ldots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(15Ba^3+264Aba^2+284\ldots\right.}{\ldots}}{\ldots}\right.}{\ldots}\right.\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(15Ba^3+264Aba^2+284\right.}{}}{}\right.}{}\right.\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(15a^3B+264a^2Ab+28\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab(15a^3B+2a^2b(132A}{}\right.}{}\right.\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(15a^3B+264a^2Ab+28\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{3b(5a^2B+24aAb+12b^2B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(15a^3B+264a^2Ab+284ab\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

input

```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + (((8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]
]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + ((3*b*(24*a*A*b + 5*a^2
*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
2*d) + (-1/2*((2*(a - b)*b*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B
 + 284*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a +
 b]*(15*a^3*B + 8*b^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59*B) + 4*a*b^2*(52*
A + 71*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (6*Sqrt[a + b]*(40*a^3
*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*
B + 284*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]))/4)/(6*b))/8)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. $2(649) = 1298$.

Time = 11.88 (sec) , antiderivative size = 2353, normalized size of antiderivative = 3.25

method	result	size
default	Expression too large to display	2353
parts	Expression too large to display	2373

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/192/d/b*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*cos(d*x+c)
+a)/sec(d*x+c)^(1/2)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^4*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-
b)/(a+b))^(1/2))*(-128*cos(d*x+c)-256-128*sec(d*x+c))+B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*Ellipt
icE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-15*cos(d*x+c)-30-15*sec
(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*b^4*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/
2))*(144*cos(d*x+c)+288+144*sec(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*EllipticPi(-csc(d*x+
c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(30*cos(d*x+c)+60+30*sec(d*x+c))+B*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*b^4*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-2
88*cos(d*x+c)-576-288*sec(d*x+c))+sin(d*x+c)*(272*cos(d*x+c)^2+272*cos(d*x
+c)+128)*A*a*b^3+sin(d*x+c)*(133*cos(d*x+c)+118)*B*a^3*b+sin(d*x+c)*(254*c
os(d*x+c)^2+254*cos(d*x+c)+284)*B*a^2*b^2+sin(d*x+c)*(184*cos(d*x+c)^3+184
*cos(d*x+c)^2+356*cos(d*x+c)+72)*B*a*b^3+264*A*a^3*b*sin(d*x+c)+B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
a^3*b*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-118*cos(d*x
+c)-236-118*sec(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a...
```

Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c
)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\sec(dx+c)}dx\right) a^2 b + \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right) b^3 + 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right) a\,b^2 + \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\sec(dx+c)}dx\right) a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

output
```
3*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/sec(c + d
*x),x)*a**2*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d
*x)**3)/sec(c + d*x),x)*b**3 + 3*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)
*b + a)*cos(c + d*x)**2)/sec(c + d*x),x)*a*b**2 + int((sqrt(sec(c + d*x))*
sqrt(cos(c + d*x)*b + a))/sec(c + d*x),x)*a**3
```

3.613 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 839

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B)\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(\text{arc}}{1920ab^2d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(45a^4B-30a^3b(5A+B)-16b^4(45A+64B)-8ab^3(355A+193B)-4a^2b^2(295A+423B))\sqrt{c}}{1920b^2d\sqrt{\sec(c}}$$

$$+\frac{\sqrt{a+b}(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5B-40a^3b^2B-240ab^4B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticP}}{128b^3d\sqrt{\sec(c+dx)}}$$

$$+\frac{(50a^2Ab+120Ab^3-15a^3B+172ab^2B)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{320bd\sqrt{\sec(c+dx)}}$$

$$+\frac{(50aAb-15a^2B+64b^2B)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{240bd\sqrt{\sec(c+dx)}}$$

$$+\frac{(10Ab-3aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd\sqrt{\sec(c+dx)}}+\frac{B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd\sqrt{\sec(c+dx)}}$$

$$+\frac{(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{1920b^2d}$$

output
```
-1/1920*(a-b)*(a+b)^(1/2)*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^
2+1024*B*b^4)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)
/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)-1/1920*(a
+b)^(1/2)*(45*B*a^4-30*a^3*b*(5*A+B)-16*b^4*(45*A+64*B)-8*a*b^3*(355*A+193
*B)-4*a^2*b^2*(295*A+423*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1
/2)+1/128*(a+b)^(1/2)*(10*A*a^4*b-240*A*a^2*b^3-96*A*b^5-3*B*a^5-40*B*a^3*
b^2-240*B*a*b^4)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
+1/320*(50*A*a^2*b+120*A*b^3-15*B*a^3+172*B*a*b^2)*(a+b*cos(d*x+c))^(1/2)*
sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/240*(50*A*a*b-15*B*a^2+64*B*b^2)*(a+b*co
s(d*x+c))^(3/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/40*(10*A*b-3*B*a)*(a+b*c
os(d*x+c))^(5/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/5*B*(a+b*cos(d*x+c))^(7
/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/1920*(150*A*a^3*b+2840*A*a*b^3-45*B*
a^4+1692*B*a^2*b^2+1024*B*b^4)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin
(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 18.08 (sec) , antiderivative size = 703, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{960}(170aAb+93a^2B\right.}{}$$

$$-\frac{-b(a+b)\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1024b^4B\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\text{s}}{}$$

input
```
Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(
3/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((170*a*A*b + 93*a^2*B + 88*
b^2*B)*Sin[c + d*x])/960 + ((590*a^2*A*b + 480*A*b^3 + 15*a^3*B + 1024*a*b
^2*B)*Sin[2*(c + d*x)])/(1920*b) + ((170*a*A*b + 93*a^2*B + 100*b^2*B)*Sin
[3*(c + d*x)])/960 + (b*(10*A*b + 21*a*B)*Sin[4*(c + d*x)])/320 + (b^2*B*S
in[5*(c + d*x)])/80))/d - (-(b*(a + b)*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^
4*B + 1692*a^2*b^2*B + 1024*b^4*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)]) + a*(a + b)*(45*a^4*B - 30*a^3*b*(5*A + 3*B) + 60*a^2*b^2
*(5*A + 11*B) + 16*b^4*(45*A + 64*B) + 8*a*b^3*(265*A + 129*B))*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 15*(10*a^4*A*b - 240*a^2*A*
b^3 - 96*A*b^5 - 3*a^5*B - 40*a^3*b^2*B - 240*a*b^4*B)*((a - b)*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*b*EllipticPi[-1, ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(150*a^3*A*b + 2840*a*A*b^3 - 45*
a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Se
c[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2])/(1920*b^3*d*Sqrt[a
+ b*Cos[c + d*x]]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(3/
2))
```

Rubi [A] (verified)

Time = 4.53 (sec) , antiderivative size = 807, normalized size of antiderivative = 0.96, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3469

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{(a+b\cos(c+dx))^{5/2}((10Ab-3aB)\cos^2(c+dx)+8bB\cos(c+dx)+aB)}{2\sqrt{\cos(c+dx)}}dx}{5b}+\frac{B\sin(c+dx)\sqrt{\cos(c\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{(a+b\cos(c+dx))^{5/2}((10Ab-3aB)\cos^2(c+dx)+8bB\cos(c+dx)+aB)}{\sqrt{\cos(c+dx)}}dx}{10b}+\frac{B\sin(c+dx)\sqrt{\cos(c\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left((10Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+8bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{10b}+\frac{B\sin(c+d\right.$$

$\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\int \frac{(a+b\cos(c+dx))^{3/2}((-15Ba^2+50Aba+64b^2B)\cos^2(c+dx)+6b(10Ab+9aB)\cos(c+dx)+5a(2Ab+aI}{2\sqrt{\cos(c+dx)}}}{10b}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\int \frac{(a+b\cos(c+dx))^{3/2}((-15Ba^2+50Aba+64b^2B)\cos^2(c+dx)+6b(10Ab+9aB)\cos(c+dx)+5a(2Ab+aI}{\sqrt{\cos(c+dx)}}}{10b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(\left(-15Ba^2+50Aba+64b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+6b(10Ab+9aB)\sin\left(c+dx+\frac{\pi}{2}\right)+\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{10b}\right.$$

$\big\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{3}\int \frac{\sqrt{a+b\cos(c+dx)}\left(3\left(-15Ba^3+50Aba^2+172b^2Ba+120Ab^3\right)\cos^2(c+dx)+2b\left(147Ba^2+310Aba+1\right.\right.}{2\sqrt{\cos(c+dx)}}}{}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{\sqrt{a+b\cos(c+dx)}\left(3\left(-15Ba^3+50Aba^2+172b^2Ba+120Ab^3\right)\cos^2(c+dx)+2b\left(147Ba^2+310Aba+1\right.\right.}{\sqrt{\cos(c+dx)}}}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3\left(-15Ba^3+50Aba^2+172b^2Ba+120Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(147Ba^2+31\right.\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.$$

$\big\downarrow$ 3528

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3a+1024b^4B\right)\cos^2(c+dx)+2b\left(573Ba^3+1610Aba^2\right.}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}\right.\right.}{}\right.$$

$\big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\int \frac{\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3a+1024b^4B\right)\cos^2(c+dx)+2b\left(573Ba^3+1610Aba^2\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}\right.\right.}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\int\frac{\left(-45Ba^4+150Aba^3+1692b^2Ba^2+2840Ab^3a+1024b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(573Ba^3+1610A\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b}}\right.\right.}{}\right.$$

$\Big\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\int-\frac{15\left(-3Ba^5+10Aba^4-40b^2Ba^3-240Ab^3a^2-240b^4Ba-96Ab^5\right)\cos^2(c+dx)-2ab\left(15Ba^3+590Aba^2\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{2b}\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{15}{}}{}\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{15}{}}{}\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a}{\ }}{\ }\right.\right.\right.}{\ }\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a}{\ }}{\ }\right.\right.\right.}{\ }\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a}{\ }}{\ }\right.\right.\right.}{\ }\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(-}{\ }\right.\right.\right.}{\ }\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(4\ldots}{\ldots}\right.\right.\right.}{\ldots}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{(-45a^4B+150a^3Ab+1692a^2b^2B+2840aAb^3+1024b^4B)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(-\ldots}{\ldots}\right.\right.\right.}{\ldots}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd}+\frac{\frac{(10Ab-3aB)\sqrt{\cos(c+dx)}\sin(c+dx)(a\ldots}{4d}}{\ldots}\right.$$

input
```
Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c
+ d*x])^(7/2)*Sin[c + d*x])/(5*b*d) + (((10*A*b - 3*a*B)*Sqrt[Cos[c + d*x]
]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*d) + (((50*a*A*b - 15*a^2*B
+ 64*b^2*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d) + ((3*(50*a^2*A*b + 120*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*Sqrt
[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B + 1024*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt[a + b]*(45*a^4*B
- 30*a^3*b*(5*A + B) - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355*A + 193*B) - 4*
a^2*b^2*(295*A + 423*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (30*Sqrt
[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a^5*B - 40*a^3*b^2*B -
240*a*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/b + (
(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/4)/6)/8)/(...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2923 vs. $2(758) = 1516$.

Time = 15.78 (sec) , antiderivative size = 2924, normalized size of antiderivative = 3.49

method	result	size
default	Expression too large to display	2924
parts	Expression too large to display	2942

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/1920/d/b^2*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*cos(d*
x+c)+a)/sec(d*x+c)^(3/2)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^5*EllipticF(-csc(d*x+c)+cot(d*x+c),(
-(a-b)/(a+b))^(1/2))*(-1440-2880*sec(d*x+c)-1440*sec(d*x+c)^2)+A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b
^5*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(2880+5760*s
ec(d*x+c)+2880*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5*EllipticPi(-csc(d*x+c)+cot(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*(90+180*sec(d*x+c)+90*sec(d*x+c)^2)+45*B*a^5*t
an(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a^2*b^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(1692+3384*sec(d*x+c)+1692*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^4*Elliptic
E(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(1024+2048*sec(d*x+c)+1024*
sec(d*x+c)^2)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*a^3*b^2*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(
a+b))^(1/2))*(1180+2360*sec(d*x+c)+1180*sec(d*x+c)^2)+(-1392*cos(d*x+c)^4-
1392*cos(d*x+c)^3-2056*cos(d*x+c)^2-2056*cos(d*x+c)-1024)*B*a*b^4*tan(d*x+
c)+(-2540*cos(d*x+c)^2-2540*cos(d*x+c)-2840)*A*a^2*b^3*tan(d*x+c)+(-1840*c
os(d*x+c)^3-1840*cos(d*x+c)^2-3560*cos(d*x+c)-720)*A*a*b^4*tan(d*x+c)+(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx=3\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)}{\sec(dx+c)^2}dx\right.$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^3}{\sec(dx+c)^2}dx\right)b^3$$
$$+3\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\cos(dx+c)^2}{\sec(dx+c)^2}dx\right)a\,b^2$$
$$+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\sec(dx+c)^2}dx\right)a^3$$

input
```
int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

output
```
3*int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c + d*x))/sec(c + d
*x)**2,x)*a**2*b + int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*cos(c
+ d*x)**3)/sec(c + d*x)**2,x)*b**3 + 3*int((sqrt(sec(c + d*x))*sqrt(cos(c
+ d*x)*b + a)*cos(c + d*x)**2)/sec(c + d*x)**2,x)*a*b**2 + int((sqrt(sec(c
 + d*x))*sqrt(cos(c + d*x)*b + a))/sec(c + d*x)**2,x)*a**3
```

3.614 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 403

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}(9a^2A+8Ab^2-10abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{15a^4d\sqrt{\sec(c+dx)}}$$
$$-\frac{2\sqrt{a+b}(8Ab^2+a^2(9A-5B)-2ab(A+5B))\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d}$$
$$+\frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(9*A*a^2+8*A*b^2-10*B*a*b)*cos(d*x+c)^(1/2)*csc(d*x
+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/
(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/a^4/d/sec(d*x+c)^(1/2)-2/15*(a+b)^(1/2)*(8*A*b^2+a^2*(9*A-5*B)-2*a*b*(A+
5*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/15*(4*A*b-5*B*a)*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d+2/5*A*(a+b*cos(d*
x+c))^(1/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2987 vs. $2(403) = 806$.

Time = 23.59 (sec) , antiderivative size = 2987, normalized size of antiderivative = 7.41

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x
]],x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 8*A*b^2 - 10*a
*b*B)*Sin[c + d*x])/(15*a^3) + (2*Sec[c + d*x]*(-4*A*b*Sin[c + d*x] + 5*a*
B*Sin[c + d*x]))/(15*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(5*a)))/d + (2
*((-3*A)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^2)/(15*a
^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*B)/(3*a*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*A*b*Sqrt[Sec[c + d*x]])/(15*a*Sqrt[
a + b*Cos[c + d*x]]) - (8*A*b^3*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos
[c + d*x]]) + (B*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2
*B*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*Cos[2*(c
+ d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^3*Cos[
2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^
2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]))
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10
*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 8
*A*b^2 - 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(15*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/...
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3479}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{-2Ab\cos^2(c+dx)-3aA\cos(c+dx)+4Ab-5aB}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\int \frac{-2Ab\cos^2(c+dx)-3aA\cos(c+dx)+4Ab-5aB}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\int \frac{-2Ab\sin\left(c+dx+\frac{\pi}{2}\right)^2-3aA\sin\left(c+dx+\frac{\pi}{2}\right)+4Ab-5aB}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}d}{5a}\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2\int -\frac{9Aa^2-10bBa+(2Ab+5aB)\cos(c+dx)a+8Ab^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(4A}{}}{5a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{9Aa^2-10bI}{\cos}}{}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{9Aa^2-10bB}{\sin(c+}}{}}{5a}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(9a^2A-10ab}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(9a^2A-10ab}{}}{}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(9a^2A-10ab}{}}{}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2(a-b)\sqrt{a+b}(}{}}{}}{}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (-1/3*((2*(a - b)*Sqrt[a + b]*(9*a^2
*A + 8*A*b^2 - 10*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2
*Sqrt[a + b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*b*(A + 5*B))*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(a*d))/a + (2*(4*A*b - 5*a*B)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))/(5*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1516 vs. $2(357) = 714$.

Time = 32.02 (sec) , antiderivative size = 1517, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	1517
parts	Expression too large to display	1539

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15/d/a^3*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)^2*b+a*cos(d
*x+c)+b*cos(d*x+c)+a)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a
-b)/(a+b))^(1/2))*(9*cos(d*x+c)^5+18*cos(d*x+c)^4+9*cos(d*x+c)^3)+A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*a^2*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(9*cos(d*x+
c)^5+18*cos(d*x+c)^4+9*cos(d*x+c)^3)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(-csc(d*x+c)
+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^5+16*cos(d*x+c)^4+8*cos(d*
x+c)^3)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*b^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2
))*(8*cos(d*x+c)^5+16*cos(d*x+c)^4+8*cos(d*x+c)^3)+B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*Ellipti
cE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-10*cos(d*x+c)^5-20*cos(d
*x+c)^4-10*cos(d*x+c)^3)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c),
(-(a-b)/(a+b))^(1/2))*(-10*cos(d*x+c)^5-20*cos(d*x+c)^4-10*cos(d*x+c)^3)+A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*a^3*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-9*co
s(d*x+c)^5-18*cos(d*x+c)^4-9*cos(d*x+c)^3)+A*(cos(d*x+c)/(1+cos(d*x+c))...
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{3}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**3,x)
```

3.615 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 330

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{3a^3d\sqrt{\sec(c+dx)}} + \frac{2\sqrt{a+b}(2Ab+a(A-3B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{3a^2d\sqrt{\sec(c+dx)}} + \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}$$

output
```
-2/3*(a-b)*(a+b)^(1/2)*(2*A*b-3*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d
*x+c)^(1/2)+2/3*(a+b)^(1/2)*(2*A*b+a*(A-3*B))*cos(d*x+c)^(1/2)*csc(d*x+c)*
EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b
))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
2/d/sec(d*x+c)^(1/2)+2/3*A*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x
+c)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 17.17 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.08

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(a+b)(-2Ab+3aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\right.\right.\right.}{}$$
$$+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(-2Ab+3aB)\sin(c+dx)}{3a^2}+\frac{2A\tan(c+dx)}{3a}\right)}{d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x
]],x]
```

output
```
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*
a*(-2*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*
((2*(-2*A*b + 3*a*B)*Sin[c + d*x])/(3*a^2) + (2*A*Tan[c + d*x])/(3*a)))/d
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3479}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{2Ab-3aB-aA\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2Ab-3aB-aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2Ab-3aB-aA\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}\right)$$

$$\Big\downarrow \text{ 3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(2Ab-3aB)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(2Ab-3aB)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.$$

$\Big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(2Ab-3aB)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(}{}}}{a^2d}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*((2*(a - b)*Sqrt[a + b]*(2*A*b
 - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*(2
*A*b + a*(A - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a + (2*A*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(290) = 580$.

Time = 15.31 (sec) , antiderivative size = 940, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	940
parts	Expression too large to display	953

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/3/d/a^2*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^
(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))+A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*Elliptic
E(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c
)^2+2*cos(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(
a+b))^(1/2))*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*E
llipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-3*cos(d*x+c)^3-6*c
os(d*x+c)^2-3*cos(d*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(-csc(d*x+c)+cot(d*x+c),(
-(a-b)/(a+b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))+A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*
b*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)^3-
4*cos(d*x+c)^2-2*cos(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticF(-csc(d*x+c)+cot(d*x+c
),(-(a-b)/(a+b))^(1/2))*(3*cos(d*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c))+(-cos
(d*x+c)-1)*sin(d*x+c)*A*a^2+sin(d*x+c)*cos(d*x+c)*(1-cos(d*x+c))*A*a*b+2*A
*b^2*cos(d*x+c)^2*sin(d*x+c)-3*B*a*b*cos(d*x+c)^2*sin(d*x+c)-3*B*a^2*co...
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^2dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2,x)
```

3.616 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 270

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{a^2d\sqrt{\sec(c+dx)}}$$
$$-\frac{2\sqrt{a+b}(A-B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$

output
```
2*A*(a-b)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*(
a+b)^(1/2)*(A-B)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 16.22 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}$$
$$- \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2A(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x
]],x]
```

output
```
(2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*A*(a + b)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(A + B)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*Cos[c
 + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sq
rt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3440, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(A\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c-}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(A\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(A-B)\int \frac{}{\sqrt{\sin\left(c+dx+\right.}}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(A\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(A-B)\cot(c+dx}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{a^2d}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*(a - b)*Sqrt[a + b]*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*(A - B)*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(a*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(240) = 480$.

Time = 10.22 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.80

method	result
default	$\frac{2\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,a\,\text{EllipticE}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+\left(\cos(dx\right.\right.}{}$
parts	$\frac{2A\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,a\,\text{EllipticE}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+\left(\cos(dx\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/d/a*((cos(d*x+c)^2+2*cos(d*x+c)+1)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*EllipticE(-csc(d*x+c)+cot
(d*x+c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b*El
lipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(
d*x+c)-1)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*a*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2
))+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*EllipticF(-csc(d*x+c)+cot(d*
x+c),(-(a-b)/(a+b))^(1/2))+A*sin(d*x+c)*b*cos(d*x+c)+A*sin(d*x+c)*a)*(a+b*
cos(d*x+c))^(1/2)*cos(d*x+c)*sec(d*x+c)^(3/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)
+b*cos(d*x+c)+a)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{(A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)\,dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x),x)
```

3.617 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 268

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

$$-\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{bd\sqrt{\sec(c+dx)}}$$

output
```
2*A*(a+b)^(1/2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*(a+b)^(1/
2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.86 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.59

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((A-B)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)+2B\,\text{EllipticPi}\left(-1,\arcsin\left(\tan\right.\right.\right.}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec^2\left(\frac{1}{2}(c+d\right.}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x
]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((A - B)*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*B*EllipticPi[-1, ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]
^2]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3440, 3042, 3485, 3042, 3288, 3295}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\downarrow$ 3485

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(A\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(A\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right.}}\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(A\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{ad}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*A*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3485
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[B/d   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Sim
p[(B*c - A*d)/d   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 7.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\left(A\,\mathrm{EllipticF}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)-B\,\mathrm{EllipticF}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+2B\,\mathrm{EllipticPi}\left(-\csc(dx+\right.\right.}{d\sqrt{a+b\cos(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$
parts	$-\frac{2A(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\mathrm{EllipticF}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\sec(dx+c)}}{d\sqrt{a+b\cos(dx+c)}}-\frac{2B\sqrt{\sec(}}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/d*(A*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))-B*EllipticF
(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+2*B*EllipticPi(-csc(d*x+c)+c
ot(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos\left(dx+c\right)+A)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + b\cos(c + dx)}}\, dx = \int \frac{(A + B\cos{(c + dx)})\sqrt{\sec{(c + dx)}}}{\sqrt{a + b\cos{(c + dx)}}}\, dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + b\cos(c + dx)}}\, dx = \int \frac{(B\cos{(dx + c)} + A)\sqrt{\sec{(dx + c)}}}{\sqrt{b\cos{(dx + c)} + a}}\, dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(1/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a),x)
```

3.618 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 487

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c}{a-b}}}{abd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec}{a-}}}{bd\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(2Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\text{se}}{a}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{aB\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{a+b\cos(c+dx)}}$$

output
```
-(a-b)*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+(a+b
)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-(a+b)^(1/2)*(2*A*
b-B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+B*sin(d*x
+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*B*sec(d*x+c)^(1/2)*sin(d*x
+c)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 20.71 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.67

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]),x]
```

output
```
(Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
d*x)/2]^2)]*(a*B*Tan[(c + d*x)/2] + b*B*Tan[(c + d*x)/2] - 2*b*B*Tan[(c +
d*x)/2]^3 - a*B*Tan[(c + d*x)/2]^5 + b*B*Tan[(c + d*x)/2]^5 + 4*A*b*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] - 2*a*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + 4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*B*EllipticPi[-1
, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + (a + b)*B*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*A*b*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)]))/(b*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2
]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(
c + d*x)/2]^2)))
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.94, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3482, 3042, 3530, 3042, 3288, 3472, 25, 27, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 3482

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\frac{(2Ab-aB)\cos^2(c+dx)+2aA\cos(c+dx)+aB}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{B\sin(c+dx)\sqrt{\cos(c-}{d\sqrt{a+b\cos(c+dx}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\frac{(2Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2aA\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\sin(c+dx}{d\sqrt{a+b}}\right.$$

$\Big\downarrow$ 3530

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\int\frac{B\cos(c+dx)a^2+bBa}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{b}+\frac{(2Ab-aB)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{B\sin(c+}{d\sqrt{a}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\int\frac{B\sin\left(c+dx+\frac{\pi}{2}\right)a^2+bBa}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{(2Ab-aB)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\right.$$

$\Big\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\int\frac{B\sin\left(c+dx+\frac{\pi}{2}\right)a^2+bBa}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}\right.\right.$$

$\Big\downarrow$ 3472

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{\int -\frac{a(a^2-b^2)B}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\,c\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 25$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{a(a^2-b^2)B}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}}{b}-\frac{2\sqrt{a+b}(2Ab-aB)\cot\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\sqrt{a+b}(2Ab-\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\sqrt{a+b}(\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 3280$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a\cdots}}\cdots\right.}{b}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\cdots}{\sqrt{\sin(c\cdots}}\cdots\right.}{b}\right.\right.$$

$$\Big\downarrow \; 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\int\frac{\sin(c+dx+\frac{\pi}{2})+1}{\sin(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx-\frac{2\sqrt{a+b}\text{co}}{}\right.}{b}\right.\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-aB\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{a^2d}\right.}{}\right.\right.$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + b*Cos[c + d*x]]) + ((-2*Sqrt[a + b]*(2*A*b - a*B)*Cot[c + d*x
]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (-(a*B*((2*(a - b)*Sqrt[a
+ b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(a*d))) + (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/b)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 3530
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b
*Sin[e + f*x]], x], x] + Simp[1/b   Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((
a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d,
e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 9.12 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.04

method	result
parts	$\frac{2A\left(\operatorname{EllipticF}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)-2\operatorname{EllipticPi}\left(-\csc(dx+c)+\cot(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}}{d\sqrt{a+b\cos(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\sec(dx+c)}} -$
default	$-\frac{\sqrt{a+b\cos(dx+c)}\,\left(A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,b\operatorname{EllipticPi}\left(-\csc(dx+c)+\cot(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)(4\cos(dx+c)+8+4\right.}{}$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2*A/d/(a+b*cos(d*x+c))^(1/2)*(EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+
b))^(1/2))-2*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/sec(d*x+c)^(1/2)-B/d/b*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos
(d*x+c)+b*cos(d*x+c)+a)/sec(d*x+c)^(1/2)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*EllipticPi(-csc(d*x+c)
+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)-4-2*sec(d*x+c))+(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*a*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)+2+s
ec(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)+2+sec(d*x+c))-cos(d*x+c)*sin(d*x+c)*b-sin(d*x+c)*a)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\cos\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))
), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}}{\sec\left(dx+c\right)}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/sec(c + d*x),x)
```

3.619 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 539

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{(a-b)\sqrt{a+b}(4Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\cdots}{4ab^2d\sqrt{\sec(c+dx)}}$$
$$+\frac{\sqrt{a+b}(4Ab-3aB+2bB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1\cdots}{\cdots}}}{4b^2d\sqrt{\sec(c+dx)}}$$
$$+\frac{\sqrt{a+b}(4aAb-3a^2B-4b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a\cdots}{a\cdots}\cdots\right.}{4b^3d\sqrt{\sec(c+dx)}}$$
$$+\frac{B\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}}$$
$$+\frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2d}$$

output
```
-1/4*(a-b)*(a+b)^(1/2)*(4*A*b-3*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec
(d*x+c)^(1/2)+1/4*(a+b)^(1/2)*(4*A*b-3*B*a+2*B*b)*cos(d*x+c)^(1/2)*csc(d*x
+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/
(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/b^2/d/sec(d*x+c)^(1/2)+1/4*(a+b)^(1/2)*(4*A*a*b-3*B*a^2-4*B*b^2)*cos(d*x
+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)+1/2*B*(a+b*cos(d*x+c))^
(1/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/4*(4*A*b-3*B*a)*(a+b*cos(d*x+c))^(
1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1157 vs. $2(539) = 1078$.

Time = 20.96 (sec) , antiderivative size = 1157, normalized size of antiderivative = 2.15

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2
)),x]
```

output
```
(B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) +
 (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-4*a*A*b*Tan[(c + d*x)/2
] - 4*A*b^2*Tan[(c + d*x)/2] + 3*a^2*B*Tan[(c + d*x)/2] + 3*a*b*B*Tan[(c +
 d*x)/2] + 8*A*b^2*Tan[(c + d*x)/2]^3 - 6*a*b*B*Tan[(c + d*x)/2]^3 + 4*a*A
*b*Tan[(c + d*x)/2]^5 - 4*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^2*B*Tan[(c + d*x)
/2]^5 + 3*a*b*B*Tan[(c + d*x)/2]^5 + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8
*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(a + b)] + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(a + b)] - 8*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +...
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3469}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{(4Ab-3aB)\cos^2(c+dx)+2bB\cos(c+dx)+aB}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}{2bd}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{(4Ab-3aB)\cos^2(c+dx)+2bB\cos(c+dx)+aB}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}{2bd}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{(4Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2bB\sin\left(c+dx+\frac{\pi}{2}\right)+aB}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+}}{2bd}\right.$$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{\left(-3Ba^2+4Aba-4b^2B\right)\cos^2(c+dx)-2abB\cos(c+dx)+a(4Ab-3aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(}}{bd\sqrt{\cos(c+dx)}}}{4b}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int \frac{\left(-3Ba^2+4Aba-4b^2B\right)\cos^2(c+dx)-2abB\cos(c+dx)+a(4Ab-3aB}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{\left(-3Ba^2+4Aba-4b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2abB\sin\left(c+dx+\frac{\pi}{2}\right)+a(4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{(-3a^2B+4aAb-4b^2B)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+\int\frac{a(4Ab-3aB)-2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{}}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{(-3a^2B+4aAb-4b^2B)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{a(4Ab-3}{\sin\left(c+dx+\right.}}{2b}}{4b}\right.$$

$\Big\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\int\frac{a(4Ab-3aB)-2abB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(-3a^2B+4aA\right.}{}}{}}{4b}\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(4Ab-3aB)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(-3aB+4Ab+}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{-a(-3aB+4Ab+2bB)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{}}{}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{a(4Ab-3aB)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}}{}}{}}{}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{-\frac{2\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{bd}}{}}{}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(2*b*d) + (-1/2*((2*(a - b)*Sqrt[a + b]*(4*A*b -
 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*(4*A*b
 - 3*a*B + 2*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*Sqrt[a + b]*
(4*a*A*b - 3*a^2*B - 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(b*d))/b + ((4*A*b - 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b
*d*Sqrt[Cos[c + d*x]]))/(4*b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(476) = 952$.

Time = 11.36 (sec) , antiderivative size = 1054, normalized size of antiderivative = 1.96

method	result	size
default	Expression too large to display	1054
parts	Expression too large to display	1092

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/d/b^2*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*cos(d*x+c)
+a)/sec(d*x+c)^(3/2)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*(8+16*sec(d*x+c)+8*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*Ellipt
icPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-6-12*sec(d*x+c)-6*s
ec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*b^2*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*(-8-16*sec(d*x+c)-8*sec(d*x+c)^2)+A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(-csc
(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-4-8*sec(d*x+c)-4*sec(d*x+c)^2)+
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-4-8
*sec(d*x+c)-4*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticE(-csc(d*x+c)+cot(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(3+6*sec(d*x+c)+3*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*Ellip
ticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(3+6*sec(d*x+c)+3*sec(d*
x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*a*b*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**(3/2
)), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\sec(dx+c)^{2}}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/sec(c + d*x)**2,x)
```

3.620 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 433

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2(5a^2Ab-8Ab^3-3a^3B+6ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a-}}}{3a^4\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(a+2b)(4Ab+a(A-3B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{3a^3\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{2(a^2A-4Ab^2+3abB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)\,d}$$

output
```
-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*Ell
ipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^
(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(
a+b)^(1/2)/d/sec(d*x+c)^(1/2)+2/3*(a+2*b)*(4*A*b+a*(A-3*B))*cos(d*x+c)^(1/
2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(
a-b))^(1/2)/a^3/(a+b)^(1/2)/d/sec(d*x+c)^(1/2)+2*b*(A*b-B*a)*sec(d*x+c)^(3
/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^2-4*A*b^2+3*B
*a*b)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3433 vs. 2(433) = 866.

Time = 26.31 (sec) , antiderivative size = 3433, normalized size of antiderivative = 7.93

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(
3/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-5*a^2*A*b + 8*A*b^3 + 3
*a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)) + (2*(-(A*b^3*Sin[c
+ d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) +
(2*A*Tan[c + d*x])/(3*a^2)))/d + (2*((5*A*b)/(3*(a^2 - b^2)*Sqrt[a + b*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(3*a^2*(a^2 - b^2)*Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*B)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) + (2*b^2*B)/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) + (a*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[a +
b*Cos[c + d*x]]) + (7*A*b^2*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a +
b*Cos[c + d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a
+ b*Cos[c + d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Co
s[c + d*x]]) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Co
s[c + d*x]]) + (5*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b
^2)*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x
]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b*B*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])
)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-5*a^2*A*b + 8*A*b^3
+ 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*...
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\downarrow$ 3479

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{Aa^2+3bBa-(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(Ab-a}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{Aa^2+3bBa-(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(Ab-aB}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{Aa^2+3bBa-(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-4Ab^2+2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(Ab}{ad(a^2-b^2)\cos}\right.$$

$\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{-3Ba^3+5Aba^2+6b^2Ba-\left(Aa^2-3bBa+2Ab^2\right)\cos(c+dx)a-8Ab^3}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{a(a^2-b^2)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3Ba^3+5Aba^2+6b^2Ba-\left(Aa^2-3bBa+2Ab^2\right)\cos(c+dx}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a}}{a(a^2-b^2)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-3Ba^3+5Aba^2+6b^2Ba-\left(Aa^2-3bBa+2Ab^2\right)\sin\left(c+dx\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{3a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^3B+5a^2Ab+6ab^2B-8Ab^3\right)\int\frac{\cos(c+dx)\cdot}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^3B+5a^2Ab+6ab^2B-8Ab^3\right)\int\frac{\sin\left(c+c\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^3B+5a^2Ab+6ab^2B-8Ab^3\right)\int\frac{\sin\left(c+c\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\cos(c}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b*(A*b - a*B)*Sin[c + d*x])/(a*(
a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (-1/3*((2*(a -
 b)*Sqrt[a + b]*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(a + 2*b)*(a*A + 4
*A*b - 3*a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a + (2*(a^2*A -
4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d
*x]^(3/2)))/(a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1937 vs. $2(389) = 778$.

Time = 30.93 (sec) , antiderivative size = 1938, normalized size of antiderivative = 4.48

method	result	size
parts	Expression too large to display	1938
default	Expression too large to display	1977

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/3*A/d/(a+b)/(a-b)/a^3*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(
-(a-b)/(a+b))^(1/2))*(5*cos(d*x+c)^3+10*cos(d*x+c)^2+5*cos(d*x+c))+(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*a^2*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(5*cos(d*x
+c)^3+10*cos(d*x+c)^2+5*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3*EllipticE(-csc(d*x+c)+co
t(d*x+c),(-(a-b)/(a+b))^(1/2))*(-8*cos(d*x+c)^3-16*cos(d*x+c)^2-8*cos(d*x+
c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*b^4*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-8
*cos(d*x+c)^3-16*cos(d*x+c)^2-8*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*EllipticF(-csc(d*x
+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+
c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*a^3*b*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(
-5*cos(d*x+c)^3-10*cos(d*x+c)^2-5*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2*EllipticF(-c
sc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(2*cos(d*x+c)^3+4*cos(d*x+c)^2+
2*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*a*b^3*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a...
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}}{\cos\left(dx+c\right)b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2)/(cos(c +
 d*x)*b + a),x)
```

3.621 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 345

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(a^2A-2Ab^2+abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{a^3\sqrt{a+b}d\sqrt{\sec(c+}}$$

$$-\frac{2(2Ab+a(A-B))\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output

```
2*(A*a^2-2*A*b^2+B*a*b)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a+b)^(1/2)/d/sec(d*
x+c)^(1/2)-2*(2*A*b+a*(A-B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a+b)^(1/2)/d/s
ec(d*x+c)^(1/2)+2*b*(A*b-B*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a
+b*cos(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2988 vs. $2(345) = 690$.

Time = 22.96 (sec) , antiderivative size = 2988, normalized size of antiderivative = 8.66

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(
3/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2*A - 2*A*b^2 + a*b*B)
*Sin[c + d*x])/(a^2*(a^2 - b^2)) - (2*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c
 + d*x]))/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/d + (2*(-((a*A)/((a^2 - b
^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) + (2*A*b^2)/(a*(a^2 - b^
2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b*B)/((a^2 - b^2)*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*A*b*Sqrt[Sec[c + d*x]])/((a^2
 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^3*Sqrt[Sec[c + d*x]])/(a^2*(a^2
 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (a*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*
Sqrt[a + b*Cos[c + d*x]]) - (b^2*B*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*Sqrt
[a + b*Cos[c + d*x]]) - (A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 -
b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*
x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b^2*B*Cos[2*(c + d*x)]*
Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a +
 b)*(-2*A*b + a*(A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] - (a^2*A - 2*A*b^2 + a*b*B)*Cos[c + d*x]*(a + b*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a^2*(a^2 - b^2)*d*S...
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{Aa^2+bBa-(Ab-aB)\cos(c+dx)a-2Ab^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+\cdots}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{Aa^2+bBa-(Ab-aB)\cos(c+dx)a-2Ab^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+\cdots}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{Aa^2+bBa-(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2A+abB-2Ab^2)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a(A-B)+2Ab)\int \frac{}{\sqrt{c}}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2A+abB-2Ab^2)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a(A-B)+2A}{a(a^2-b^2)}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2A+abB-2Ab^2)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a(A-B)+2Ab)}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a-b)\sqrt{a+b}(a^2A+abB-2Ab^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{a^2d}}{}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*Sqrt[a + b]*(a^2*A - 2*
A*b^2 + a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*S
qrt[a + b]*(2*A*b + a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))
/(a*(a^2 - b^2)) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(311) = 622$.

Time = 12.94 (sec) , antiderivative size = 1300, normalized size of antiderivative = 3.77

method	result	size
default	Expression too large to display	1300
parts	Expression too large to display	1379

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/d/(a+b)/(a-b)/a^2*sec(d*x+c)^(3/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*((
(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))*a^3*A+(2*(1-cos(d*x+c
))^3*csc(d*x+c)^3-2*csc(d*x+c)+2*cot(d*x+c))*b^3*A+(-(1-cos(d*x+c))^3*csc(
d*x+c)^3+csc(d*x+c)-cot(d*x+c))*b*a^2*A-2*A*a*b^2*(1-cos(d*x+c))^3*csc(d*x
+c)^3+((1-cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*b*a^2*B+(-(1-c
os(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))*b^2*a*B-2*A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+2*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(-cs
c(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-4*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(-csc
(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(-csc(
d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-2*B*(cos(d*x+c)/(1+cos(d*x+...
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x))/(cos(c + d*
x)*b + a),x)
```

3.622 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 324

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{a^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$
$$+\frac{2(A+B)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{a\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*(A*b-B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a+b)^(1/2)/d/sec(d*x+c)^(1/2)+2
*(A+B)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a+b)^(1/2)/d/sec(d*x+c)^(1/2)-2*(A*b-B
*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 15.45 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{b(Ab-aB)\sin(c+dx)}{\sqrt{\sec(c+dx)}}+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a+b)(-Ab+aB)E\left(\text{arcsi}\right.\right.}{}\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(
3/2),x]
```

output
```
(2*((b*(A*b - a*B)*Sin[c + d*x])/Sqrt[Sec[c + d*x]] + (Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(2*(a + b)*(-(A*b) + a*B)*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c +
 d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(a + b)*(A - B)*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[
(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (A*b - a*B)*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c
 + d*x)/2]^2]))/((a^3 - a*b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3440, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3472}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{Ab-aB+(aA-bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{Ab-aB+(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{ 3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)(A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}}{a^2-b^2}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a-b)(A+B)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+(Ab-aB)\int \frac{\sin(c+dx+}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a}}}{a^2-b^2}\right.$$

$$\Big\downarrow \text{ 3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}(A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c}{a+b}}}{}}{a^2-b^2}\right.$$

$$\Big\downarrow \text{ 3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(a-b)\sqrt{a+b}(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*Sqrt[a + b]*(A*b - a*B)
*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(A
+ B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (2*(A*b - a
*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(290) = 580$.

Time = 5.62 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.83

method	result	size
default	Expression too large to display	916
parts	Expression too large to display	986

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/d/a/(a-b)/(a+b)*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(((1-cos(d*x+c))
^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*A*a*b+(-(1-cos(d*x+c))^3*csc(d*x+c)
^3+csc(d*x+c)-cot(d*x+c))*b^2*A+(-(1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)
-cot(d*x+c))*a^2*B+((1-cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))*B
*a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*A*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+2*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b)/((1-cos(d*x+c))...
```

Fricas [F]

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}} \, dx = \int \frac{(B\cos(dx + c) + A)\sqrt{\sec(dx + c)}}{(b\cos(dx + c) + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}} \, dx = \int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{\frac{3}{2}}} \, dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**(3/
2), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}} \, dx = \int \frac{(B\cos(dx + c) + A)\sqrt{\sec(dx + c)}}{(b\cos(dx + c) + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(3/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\cos(dx+c)\,b+a}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)*b + a),x)
```

3.623 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 476

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec}{a-}}}{ab\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(}}{b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*(A*b-B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/(a+b)^(1/2)/d/sec(d*x+c)^(1/2)+
2*(A*b-B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/(a+b)^(1/2)/d/sec(d*x+c)^(1/2)-2
*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+2
*a*(A*b-B*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1050 vs. $2(476) = 952$.

Time = 14.53 (sec) , antiderivative size = 1050, normalized size of antiderivative = 2.21

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sqrt{\sec(c + dx)}}\, dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*
x]]),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b - a*B)*Sin[c + d*x])
/(b*(-a^2 + b^2)) - (2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(b*(-a^2
 + b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*b*Tan[(c + d*x)/2]
+ A*b^2*Tan[(c + d*x)/2] - a^2*B*Tan[(c + d*x)/2] - a*b*B*Tan[(c + d*x)/2]
 - 2*A*b^2*Tan[(c + d*x)/2]^3 + 2*a*b*B*Tan[(c + d*x)/2]^3 - a*A*b*Tan[(c
+ d*x)/2]^5 + A*b^2*Tan[(c + d*x)/2]^5 + a^2*B*Tan[(c + d*x)/2]^5 - a*b*B*
Tan[(c + d*x)/2]^5 + 2*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*B*EllipticPi[-1, ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a^2*B*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-(A*b) +
 a*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
 - b*Tan[(c + d*x)/2]^2)/(a + b)] - b*(a + b)*(A - B)*EllipticF[ArcSin[...
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3440, 3042, 3471, 3042, 3273, 3042, 3274, 3042, 3288, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3471}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{ 3273}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2-b^2}\right)}{b}+\frac{B\int \frac{\sqrt{\sin(c+\cdots}}{\sqrt{a+b\sin(c\cdots}}}{b}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2-b^2}\right)}{b}+\frac{B\int \frac{\sqrt{\sin(\cdots}}{\sqrt{a+b\,\mathrm{si}\cdots}}}{\cdots}\right.$$

$$\Big\downarrow \text{ 3274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\int\frac{}{\sqrt{}}}{a^2-b^2}\right.}{b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(}{a^2-}\right.}{b}\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(}{a^2-}\right.}{b}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2}{}}{}\right.}{b}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(}{}}}{a}\right.}{}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*Sqrt[a + b]*B*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + ((A*b - a*B)*(-(((2*(a - b)*Sqrt
[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b
]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2)) + (2*a*Sin[c +
d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3273
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Simp[d^2/(a^2 - b^2)   Int[Sqrt
[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3471
```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> S
imp[B/b   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + S
imp[(A*b - a*B)/b   Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2)
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(427) = 854$.

Time = 8.88 (sec) , antiderivative size = 1098, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1098
parts	Expression too large to display	1146

input

```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/d/b/(a-b)/(a+b)*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*co
s(d*x+c)+a)/sec(d*x+c)^(1/2)*(B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*EllipticPi(-csc(d*x+c)+cot(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*(-2*cos(d*x+c)-4-2*sec(d*x+c))+B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*
EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(2*cos(d*x+c)+4
+2*sec(d*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b
))^(1/2))*(-cos(d*x+c)-2-sec(d*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticE(-csc(d*x+c)+c
ot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-cos(d*x+c)-2-sec(d*x+c))+B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*E
llipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)+2+sec(d*
x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*a*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)+2+sec(d*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b*EllipticF(-csc(d*x+c)+cot(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)+2+sec(d*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2*EllipticF(-c
sc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)+2+sec(d*x+c))+B*...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\cos{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{3}{2}}\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*sqrt(sec(c + d*
x))), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{B\cos{(dx+c)}+A}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
c))), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{B\cos{(dx+c)}+A}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}}{\cos\left(dx+c\right)\sec\left(dx+c\right)b+\sec\left(dx+c\right)a}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)*sec(c + d*
x)*b + sec(c + d*x)*a),x)
```

3.624 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 560

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{(2aAb-3a^2B+b^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{c}}{\sqrt{a-}}\right.\right.}{ab^2\sqrt{a+b}d\sqrt{\sec(c}}$$

$$-\frac{(2Ab-(3a+b)B)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{b^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(2Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$+\frac{2a(Ab-aB)\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$-\frac{(2aAb-3a^2B+b^2B)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d}$$

output
```
(2*A*a*b-3*B*a^2+B*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/(a+b)^(1/2)/d/sec(
d*x+c)^(1/2)-(2*A*b-(3*a+b)*B)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/(a+b)^(1/2)/d
/sec(d*x+c)^(1/2)-(a+b)^(1/2)*(2*A*b-3*B*a)*cos(d*x+c)^(1/2)*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b^3/d/sec(d*x+c)^(1/2)+2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*co
s(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-(2*A*a*b-3*B*a^2+B*b^2)*(a+b*cos(d*x+c))^
(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1551 vs. $2(560) = 1120$.

Time = 20.59 (sec) , antiderivative size = 1551, normalized size of antiderivative = 2.77

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3
/2)),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-(A*b) + a*B)*Sin[c +
 d*x])/(b^2*(a^2 - b^2)) + (2*(a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x]))
/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (Sqrt[(1 - Tan[(c + d*x)/2]
^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 +
Tan[(c + d*x)/2]^2)]*(2*a^2*A*b*Tan[(c + d*x)/2] + 2*a*A*b^2*Tan[(c + d*x)
/2] - 3*a^3*B*Tan[(c + d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + a*b^2*B*Tan[
(c + d*x)/2] + b^3*B*Tan[(c + d*x)/2] - 4*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a
^2*b*B*Tan[(c + d*x)/2]^3 - 2*b^3*B*Tan[(c + d*x)/2]^3 - 2*a^2*A*b*Tan[(c
+ d*x)/2]^5 + 2*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 -
3*a^2*b*B*Tan[(c + d*x)/2]^5 - a*b^2*B*Tan[(c + d*x)/2]^5 + b^3*B*Tan[(c +
 d*x)/2]^5 - 4*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b
^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] - 4*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +...
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{-\left(\left(-3Ba^2+2Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-\left(\left(-3Ba^2+2Aba+b^2B\right)\cos^2(c+dx)\right)-b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin}{bd\left(a^2-b^2\right)\sqrt{}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\left(3Ba^2-2Aba-b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\text{si}}{bd\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\left(a^2-b^2\right)(2Ab-3aB)\cos^2(c+dx)+2ab(Ab-aB)\cos(c+dx)+a\left(-3Ba^2+2Aba+b^2B\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(-3a^2B+2aAb+b^2B\right)}{bd\sqrt{}}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\left(a^2-b^2\right)(2Ab-3aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-3Ba^2+2Aba+b^2B\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(-3a^2B+2aA\right.}{}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a\left(-3Ba^2+2Aba+b^2B\right)+2ab(Ab-aB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a^2-b^2)(2Ab-3aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(-3a^2B+}{}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)(2Ab-3aB)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{a\left(-3Ba^2+2Aba+b^2B\right)+2ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(-}{}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a\left(-3Ba^2+2Aba+b^2B\right)+2ab(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-b^2\right)(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{2b}}{}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(-3a^2B+2aAb+b^2B\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(2Ab-B(3a+b))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(-3a^2B+2aAb+b^2B\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)(2Ab-B(3a+b))\int \frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{}}{}\right.$$

$\big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(-3a^2B+2aAb+b^2B\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-b^2\right)(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1}{}}}{}}{}}{}\right.$$

$\big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(-3a^2B+2aAb+b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ad}}{}}{}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (((2*(a - b)*S
qrt[a + b]*(2*a*A*b - 3*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d) - (2*(a - b)*Sqrt[a + b]*(2*A*b - (3*a + b)*B)*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/d - (2*Sqrt[a + b]*(a^2 - b^2)*(2*A*b - 3*a*B)*Cot[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/(2*b) - ((2*a*A*b - 3*a^2*B
 + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))
/(b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(507) = 1014$.

Time = 11.39 (sec) , antiderivative size = 1597, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	1597
parts	Expression too large to display	1632

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/d/b^2/(a-b)/(a+b)*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b
*cos(d*x+c)+a)/sec(d*x+c)^(3/2)*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticPi(-csc(d*x+c)+co
t(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(4+8*sec(d*x+c)+4*sec(d*x+c)^2)+A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*b^3*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-4-8*sec
(d*x+c)-4*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*(-6-12*sec(d*x+c)-6*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*E
llipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(6+12*sec(d*x+c)
+6*sec(d*x+c)^2)+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/
(a+b))^(1/2))*(-2-4*sec(d*x+c)-2*sec(d*x+c)^2)+A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^2*EllipticE(-
csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-2-4*sec(d*x+c)-2*sec(d*x+c)^
2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*a^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(3
+6*sec(d*x+c)+3*sec(d*x+c)^2)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b*EllipticE(-csc(d*x+c)+cot...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}}{\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}b+\sec\left(dx+c\right)^{2}a}dx$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)*sec(c + d*
x)**2*b + sec(c + d*x)**2*a),x)
```

3.625 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 607

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{2(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{3a^5(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(16Ab^4-a^4(A-3B)+4ab^3(3A-2B)-9a^3b(A-B)-2a^2b^2(8A+3B))\sqrt{\cos(c+dx)}\csc(c+dx)}{3a^4\sqrt{a+b}\,(a^2-b^2)\,d\sqrt{\sec(c+d}}$$
$$+\frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}$$
$$+\frac{2b(10a^2Ab-6Ab^3-7a^3B+3ab^2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\,(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{2(a^4A-13a^2Ab^2+8Ab^4+8a^3bB-4ab^3B)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^3\,(a^2-b^2)^2\,d}$$

output
```
-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B*a^5+15*B*a^3*b^2-8*B*a*b^4)*cos(
d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(16*A*b
^4-a^4*(A-3*B)+4*a*b^3*(3*A-2*B)-9*a^3*b*(A-B)-2*a^2*b^2*(8*A+3*B))*cos(d*
x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^4/(a+b)^(1/2)/(a^2-b^2)/d/sec(d*x+c)^(1/2)+2/3*b*(A*
b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/
3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2
/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^4-13*A*a^2*b^2+8*A*b^4+8*B*
a^3*b-4*B*a*b^3)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/(a
^2-b^2)^2/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4316 vs. 2(607) = 1214.

Time = 29.60 (sec) , antiderivative size = 4316, normalized size of antiderivative = 7.11

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-8*a^4*A*b + 28*a^2*A*b^
3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sin[c + d*x])/(3*a^4*(a
^2 - b^2)^2) + (2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3*a^2*(
a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-11*a^2*A*b^3*Sin[c + d*x] + 7*A*
b^5*Sin[c + d*x] + 8*a^3*b^2*B*Sin[c + d*x] - 4*a*b^4*B*Sin[c + d*x]))/(3*
a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d +
 (2*((8*a*A*b)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) - (28*A*b^3)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]]) + (16*A*b^5)/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (a^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (5*b^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) - (8*b^4*B)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]) + (a^2*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
d*x]]) + (5*A*b^2*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*
x]]) - (32*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c
 + d*x]]) + (16*A*b^6*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*
Cos[c + d*x]]) - (3*a*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Co
s[c + d*x]]) + (17*b^3*B*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b
*Cos[c + d*x]]) - (8*b^5*B*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a
 + b*Cos[c + d*x]]) + (8*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*...
```

Rubi [A] (verified)

Time = 2.96 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$\Big\downarrow$ 3479

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{4b(Ab-aB)\cos^2(c+dx)-3a(Ab-aB)\cos(c+dx)+3\left(Aa^2+bBa-2Ab^2\right)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b(Ab-aB)\cos^2(c+dx)-3a(Ab-aB)\cos(c+dx)+3\left(Aa^2+bBa-2Ab^2\right)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3a(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(Aa^2+bBa-2Ab^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2l}{3ad\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int \frac{2b\left(-7Ba^3+10Aba^2+3b^2Ba-6Ab^3\right)\cos^2(c+dx)-a\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)+3\left(Aa^4+8bBa^3-1\right.}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{2b\left(-7Ba^3+10Aba^2+3b^2Ba-6Ab^3\right)\cos^2(c+dx)-a\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)+3\left(Aa^4+8bBa^3-13A\right.}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{2b\left(-7Ba^3+10Aba^2+3b^2Ba-6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(Aa^4+8bB\cdots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{a(a^2-b^2)}\cdots}{3a\left(a^2-b^2\right)\cdots}\cdots\right.$$

$$\Big\downarrow \; 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int -\frac{3\left(-3Ba^5+8Aba^4+15b^2Ba^3-28Ab^3a^2-8b^4Ba-\left(Aa^4-6bBa^3+7Ab^2a^2+2b^3Ba-4Ab^4\right)\cos(c+dx)a+16Ab^5\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\cdots}{3a}\cdots}{a(a^2-b^2)}\cdots}{3a\cdots}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3Ba^5+8Aba^4+15b^2Ba^3-28Ab^3a^2-8b^4B\cdots}{\cos^{\frac{3}{2}}\cdots}\cdots}{\cdots}}{a(a^2-b^2)}\cdots}{3a\left(a\cdots\right.}\cdots\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3Ba^5+8Aba^4+15b^2Ba^3-28Ab^3a^2-8b^4B\cdots}{\sin\left(c+d\cdots\right.}\cdots}{\cdots}}{a(a^2-b^2)}\cdots}{3a\cdots}\cdots\right.$$

$$\Big\downarrow \; 3477$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(a-b)\left(-\left(a^4(A-3B)\right)-9a^3b(A-B)-2a^2b^2(8\cdots}{\cdots}}{\cdots}\right.$$

↓ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(a-b)\left(-\left(a^4(A-3B)\right)-9a^3b(A-B)-2a^2b^2(8\cdots}{\cdots}}{\cdots}\right.$$

↓ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-3a^5B+8a^4Ab+15a^3b^2B-28a^2Ab^3-8ab^4B\cdots}{\cdots}}{\cdots}\right.$$

↓ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{\frac{2b\left(-7a^3B+10a^2Ab+3ab^2B-6Ab^3\right)\sin\cdots}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+\cdots}}}{\cdots}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b*(A*b - a*B)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)) + ((2*b*(10*
a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Co
s[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (-(((2*(a - b)*Sqrt[a + b]*(8
*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(16*A
*b^4 - a^4*(A - 3*B) + 4*a*b^3*(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*b^2*(
8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a) + (2*(a^4*A - 1
3*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(a*d*Cos[c + d*x]^(3/2)))/(a*(a^2 - b^2)))/(3*a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3733 vs. $2(555) = 1110$.

Time = 34.25 (sec) , antiderivative size = 3734, normalized size of antiderivative = 6.15

method	result	size
parts	Expression too large to display	3734
default	Expression too large to display	3807

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*A/d/(a-b)^2/(a+b)^2/a^4*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^6*b*EllipticE(-csc(d*x+c)+cot(d*x+c
),(-(a-b)/(a+b))^(1/2))*(-8*cos(d*x+c)^3-16*cos(d*x+c)^2-8*cos(d*x+c))+(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*a^5*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-8*co
s(d*x+c)^4-24*cos(d*x+c)^3-24*cos(d*x+c)^2-8*cos(d*x+c))+(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b^3*E
llipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-8*cos(d*x+c)^4+12*
cos(d*x+c)^3+48*cos(d*x+c)^2+28*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^4*EllipticE(-csc
(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(28*cos(d*x+c)^4+84*cos(d*x+c)^3+
84*cos(d*x+c)^2+28*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^5*EllipticE(-csc(d*x+c)+cot(d
*x+c),(-(a-b)/(a+b))^(1/2))*(28*cos(d*x+c)^4+40*cos(d*x+c)^3-4*cos(d*x+c)^
2-16*cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*a*b^6*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a
+b))^(1/2))*(-16*cos(d*x+c)^4-48*cos(d*x+c)^3-48*cos(d*x+c)^2-16*cos(d*x+c
))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*b^7*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-16
*cos(d*x+c)^4-32*cos(d*x+c)^3-16*cos(d*x+c)^2)+(cos(d*x+c)/(1+cos(d*x+c...
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/
(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(5/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)^{2}}{\cos\left(dx+c\right)^{2}b^{2}+2\cos\left(dx+c\right)ab+a^{2}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x)**2)/(cos(c +
 d*x)**2*b**2 + 2*cos(c + d*x)*a*b + a**2),x)
```

3.626 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 496

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B)\sqrt{\cos(c+dx)}\,\mathrm{csc}}{3a^4(a-b)(a}$$

$$+\frac{2(8Ab^3-3a^3(A-B)+2ab^2(3A-B)-3a^2b(3A+B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{3a^3\sqrt{a+b}\,(a^2-b^2)\,d\sqrt{\sec(c+dx)}}$$

$$+\frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}$$

$$+\frac{2b(8a^2Ab-4Ab^3-5a^3B+ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\,(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4+6*B*a^3*b-2*B*a*b^3)*cos(d*x+c)^(1/2)*cs
c(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(
a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(8*A*b^3-3*a^3*(A-B)+2
*a*b^2*(3*A-B)-3*a^2*b*(3*A+B))*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a+b)^(1/2)/
(a^2-b^2)/d/sec(d*x+c)^(1/2)+2/3*b*(A*b-B*a)*sec(d*x+c)^(1/2)*sin(d*x+c)/a
/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b
^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3891 vs. $2(496) = 992$.

Time = 27.54 (sec) , antiderivative size = 3891, normalized size of antiderivative = 7.84

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^{5/2}}\, dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4*A - 15*a^2*A*b^2 +
 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) - (2
*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos
[c + d*x])^2) - (2*(-8*a^2*A*b^2*Sin[c + d*x] + 4*A*b^4*Sin[c + d*x] + 5*a
^3*b*B*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*C
os[c + d*x]))))/d + (2*(-((a^2*A)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]])) + (5*A*b^2)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) - (8*A*b^4)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (2*a*b*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) + (2*b^3*B)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (3*a*A*b*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[
a + b*Cos[c + d*x]]) + (17*A*b^3*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sq
rt[a + b*Cos[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^
2*Sqrt[a + b*Cos[c + d*x]]) + (a^2*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sq
rt[a + b*Cos[c + d*x]]) - (5*b^2*B*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sq
rt[a + b*Cos[c + d*x]]) + (2*b^4*B*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^
2*Sqrt[a + b*Cos[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/
((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*A*b^3*Cos[2*(c + d*x)]*Sqrt[
Sec[c + d*x]])/(a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +...
```

Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3479$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{3Aa^2+bBa-3(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-}{3ad\left(a^2-b^2\right)\sqrt{\cos(}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\cos(c+dx)a-4Ab^2+2b(Ab-aB)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-}{3ad\left(a^2-b^2\right)\sqrt{\cos(c}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-4Ab^2+2b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(}{3ad\left(a^2-b^2\right)\sqrt{}}\right.$$

$$\Big\downarrow \; 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int \frac{3Aa^4+6bBa^3-15Ab^2a^2-2b^3Ba-\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)a+8Ab^4}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(-5a^3B+8a^2Ab}{ad(a^2-b^2)\sqrt{\cos(}}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3Aa^4+6bBa^3-15Ab^2a^2-2b^3Ba-\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\cos(c+dx)a+8Ab^4}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b(-5a^3B+8a^2Ab+}{ad(a^2-b^2)\sqrt{\cos(c}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3Aa^4+6bBa^3-15Ab^2a^2-2b^3Ba-\left(-3Ba^3+6Aba^2-b^2Ba-2Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+8Ab^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(-5a^3B+8a^2}{ad(a^2-b^2)\sqrt{c}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a-b)\left(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\left(3a^4A+6a^3bB\right.}{a(a^2-b^2)}}{3a\left(a^2\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a-b)\left(-3a^3(A-B)-3a^2b(3A+B)+2ab^2(3A-B)+8Ab^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^4A\right.}{a(a^2-b^2)}}{}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(-3a^3(A-\right.}{}}{}}{}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B-4Ab^3\right)\sin(}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}}{}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b*(A*b - a*B)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (((2*(a -
b)*Sqrt[a + b]*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*
Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(8*A
*b^3 - 3*a^3*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*b*(3*A + B))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*d))/(a*(a^2 - b^2)) + (2*b*(8*a^2*A*b - 4*A*b^
3 - 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2959 vs. $2(450) = 900$.

Time = 15.62 (sec) , antiderivative size = 2960, normalized size of antiderivative = 5.97

method	result	size
default	Expression too large to display	2960
parts	Expression too large to display	3021

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3/d/(a-b)^2/(a+b)^2/a^3*(3*A*a^6*sin(d*x+c)+A*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^6*EllipticE(-csc
(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(8*cos(d*x+c)^3+16*cos(d*x+c)^2+8
*cos(d*x+c))-2*B*a*b^5*cos(d*x+c)^2*sin(d*x+c)+6*A*a^5*b*cos(d*x+c)*sin(d*
x+c)-6*B*a^5*b*cos(d*x+c)*sin(d*x+c)+(3*cos(d*x+c)^2+9*cos(d*x+c)-6)*sin(d
*x+c)*A*a^4*b^2+(-15*cos(d*x+c)^2-5*cos(d*x+c)+3)*sin(d*x+c)*A*a^2*b^4+(6*
cos(d*x+c)^3+27*cos(d*x+c)^2+36*cos(d*x+c)+15)*A*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*b^2*EllipticF
(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(15*cos(d*x+c)^3+28*cos(d*x+
c)^2+11*cos(d*x+c)-2)*A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3*b^3*EllipticF(-csc(d*x+c)+cot(d*x+c),(
-(a-b)/(a+b))^(1/2))+(-2*cos(d*x+c)^3-12*cos(d*x+c)^2-18*cos(d*x+c)-8)*A*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*a^2*b^4*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-3*
cos(d*x+c)^3-12*cos(d*x+c)^2-15*cos(d*x+c)-6)*B*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^5*b*EllipticF(-c
sc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^3-13*cos(d*x+c)^
2-8*cos(d*x+c)-1)*B*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*b^2*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-
b)/(a+b))^(1/2))+(-cos(d*x+c)^3+3*cos(d*x+c)+2)*B*(1/(a+b)*(a+b*cos(d*x...
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/
(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(5/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}\,\sec\left(dx+c\right)}{\cos\left(dx+c\right)^{2}b^{2}+2\cos\left(dx+c\right)ab+a^{2}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x))/(cos(c + d*
x)**2*b**2 + 2*cos(c + d*x)*a*b + a**2),x)
```

3.627 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 469

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(}{3a^3(a-b)(a+b)^3}$$

$$-\frac{2(2Ab^2-3a^2(A+B)+ab(3A+B))\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+}{a-}\right.}{3a^2\sqrt{a+b}\,(a^2-b^2)\,d\sqrt{\sec(c+dx)}}$$

$$+\frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

$$-\frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\,(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*Ellipt
icE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/
2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b
)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(2*A*b^2-3*a^2*(A+B)+a*b*(3*A+B))*cos
(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(
d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a^2/(a+b)^(1/2)/(a^2-b^2)/d/sec(d*x+c)^(1/2)+2/3*b*
(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
-2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/a/(a^
2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3493 vs. $2(469) = 938$.

Time = 26.05 (sec) , antiderivative size = 3493, normalized size of antiderivative = 7.45

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-6*a^2*A*b + 2*A*b^3 +
3*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) + (2*(-(A*b*Sin[c +
 d*x]) + a*B*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-
5*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 2*a^3*B*Sin[c + d*x] + 2*a*b
^2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*((-2
*a*A*b)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A
*b^3)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a
^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b^2*B
)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*S
qrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b^2*Sqr
t[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqr
t[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*B*S
qrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*B*Sqr
t[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*A*b^2*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]
]) + (2*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sq
rt[a + b*Cos[c + d*x]]) + (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^
2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(2*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B...
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$\downarrow$ 3479

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{3Aa^2-bBa-3(Ab-aB)\cos(c+dx)a-2Ab^2}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3Aa^2-bBa-3(Ab-aB)\cos(c+dx)a-2Ab^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3Aa^2-bBa-3(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right.$$

$\downarrow$ 3472

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{-3Ba^3+6Aba^2-b^2Ba+\left(3Aa^2-4bBa+Ab^2\right)\cos(c+dx)a-2Ab^3}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\sin(c+}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+d}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{-3Ba^3+6Aba^2-b^2Ba+\left(3Aa^2-4bBa+Ab^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-2Ab^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\sin}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(-3a^2(A+B)+ab(3A+B)+2Ab^2\right)\int}{a^2-b^2}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(-3a^2(A+B)+ab(3A+B\right.}{a^2-b^2}}{3a\left(a^2-b\right.}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(-3a^2(A+B)+ab(3A-\right.}{}}{a^2-b^2}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\frac{2(a-b)\sqrt{a+b}\left(-3a^3B+6a^2Ab-ab^2B-2Ab^3\right)\cot(c+dx)\sqrt{}}{}}{}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((2*(a -
b)*Sqrt[a + b]*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(2*A*b^2 - 3*a^2*(A +
 B) + a*b*(3*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^
2) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sin[c + d*x])/((a^2 - b^
2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2438 vs. $2(423) = 846$.

Time = 11.18 (sec) , antiderivative size = 2439, normalized size of antiderivative = 5.20

method	result	size
default	Expression too large to display	2439
parts	Expression too large to display	2503

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/a^2*((-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^5
*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-6*cos(d*x+c)^3-1
8*cos(d*x+c)^2-18*cos(d*x+c)-6)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^3*b^2*EllipticE(-csc(d*x+c)+co
t(d*x+c),(-(a-b)/(a+b))^(1/2))+2*A*b^5*cos(d*x+c)^2*sin(d*x+c)-3*B*a^5*cos
(d*x+c)*sin(d*x+c)-2*B*a^2*b^3*cos(d*x+c)^2*sin(d*x+c)+B*a*b^4*cos(d*x+c)^
2*sin(d*x+c)+6*A*a^4*b*cos(d*x+c)*sin(d*x+c)+(-6*cos(d*x+c)^3-10*cos(d*x+c
)^2-2*cos(d*x+c)+2)*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(
a-b)/(a+b))^(1/2))+(2*cos(d*x+c)^3+6*cos(d*x+c)^2+6*cos(d*x+c)+2)*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*a*b^4*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(3*cos(d*x+
c)^3+9*cos(d*x+c)^2+9*cos(d*x+c)+3)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^4*b*EllipticE(-csc(d*x+c)+
cot(d*x+c),(-(a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(5*cos(d*x+c)-7)*A*
a^3*b^2+sin(d*x+c)*cos(d*x+c)*(-6*cos(d*x+c)-2)*A*a^2*b^3+sin(d*x+c)*cos(d
*x+c)*(-cos(d*x+c)+3)*A*a*b^4+sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)+4)*B*a^
4*b+sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)-1)*B*a^3*b^2+(3*cos(d*x+c)^2+6*cos
(d*x+c)+3)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)...
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/
(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{(A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^{5/2}}\,dx$$

input
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(5/
2),x)
```

output
```
int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\cos(dx+c)^2\,b^2+2\cos(dx+c)\,ab+a^2}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)**2*b**2 +
2*cos(c + d*x)*a*b + a**2),x)
```

3.628 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 431

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2(3a^2A+Ab^2-4abB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(a(3A+B)-b(A+3B))\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\text{se}}{a}}}{3a(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2(Ab-aB)\sin(c+dx)}{3(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

$$+\frac{2(3a^2A+Ab^2-4abB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/3*(3*A*a^2+A*b^2-4*B*a*b)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/
2)/d/sec(d*x+c)^(1/2)+2/3*(a*(3*A+B)-b*(A+3*B))*cos(d*x+c)^(1/2)*csc(d*x+c
)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a
-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/
d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2/3*(3*A*a^2+A*b^2-4*B*a*b)*sec(
d*x+c)^(1/2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3155 vs. $2(431) = 862$.

Time = 23.66 (sec) , antiderivative size = 3155, normalized size of antiderivative = 7.32

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}\sqrt{\sec(c + dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*
x]]),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2*A + A*b^2 - 4*a*b
*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)^2) + (2*(-(a*A*b*Sin[c + d*x]) + a^2*B*
Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(2*a^2*A*b*S
in[c + d*x] + 2*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 5*a*b^2*B*Sin[c
+ d*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - (2*((a^2*A)/((a^2
 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*b^2)/(3*(a^2 -
 b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*a*b*B)/(3*(a^2 -
 b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a*A*b*Sqrt[Sec[c +
 d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (A*b^3*Sqrt[Sec[c + d
*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (a^2*B*Sqrt[Sec[c + d
*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (b^2*B*Sqrt[Sec[c + d*x
]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (a*A*b*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (A*b^3*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]
]) - (4*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a
 + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(3*
a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + ...
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3440, 3042, 3478, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3478}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{Ab-aB-3(aA-bB)\cos(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{Ab-aB-3(aA-bB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{Ab-aB-3(aA-bB)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3472}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(3a^2A-4abB+Ab^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-2(\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(3a^2A-4abB+Ab^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{(3a^2A-4abB+Ab^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a(3A+B)-b(A+3B))\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}}{3(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{(3a^2A-4abB+Ab^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a(3A+B)-b(A+3B))\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\cdots}dx}{a^2-b^2}}{3(a^2-b^2)}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{(3a^2A-4abB+Ab^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a(3A+B)-b(A+3B))\cot(c+dx)\cdots}{\cdots}}{a^2-b^2}}{\cdots}\right.$$

$\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2A-4abB+Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\cdots\right)}{a^2d}\cdots}{\cdots}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (((2*(a - b)*
Sqrt[a + b]*(3*a^2*A + A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(a*(3*A + B) - b*(A + 3*B))*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (2*(3*a^2*A + A*b^2 - 4*a*
b*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]))/(3*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3478
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(
m + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*
Sin[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1969 vs. $2(385) = 770$.

Time = 7.43 (sec) , antiderivative size = 1970, normalized size of antiderivative = 4.57

method	result	size
default	Expression too large to display	1970
parts	Expression too large to display	2022

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3/d/(a-b)^2/(a+b)^2/a*(a+b*cos(d*x+c))^(1/2)/((1+cos(d*x+c))*a^2+cos(d*x
+c)*(2*cos(d*x+c)+2)*b*a+cos(d*x+c)^2*(1+cos(d*x+c))*b^2)/sec(d*x+c)^(1/2)
*(2*A*a*b^3*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*(2*cos(d*x+c)-4)*b*a^3*A+sin(
d*x+c)*(-3*cos(d*x+c)+1)*A*a^2*b^2+sin(d*x+c)*(-5*cos(d*x+c)+3)*B*a^2*b^2+
4*B*a*b^3*cos(d*x+c)*sin(d*x+c)+(-cos(d*x+c)^2-2*cos(d*x+c)-1)*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b
^4*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-3*cos(d*x+c)^2
-6*cos(d*x+c)-3)*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a*b^3*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/
(a+b))^(1/2))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*a^3*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+
b))^(1/2))*(-3*cos(d*x+c)^2-9*cos(d*x+c)-9-3*sec(d*x+c))+A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a^2*b^2
*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(-3*cos(d*x+c)^2-7
*cos(d*x+c)-5-sec(d*x+c))+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*b^3*EllipticE(-csc(d*x+c)+cot(d*x+c)
,(-(a-b)/(a+b))^(1/2))*(-cos(d*x+c)^2-3*cos(d*x+c)-3-sec(d*x+c))+B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*a^3*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(4*cos(d*x+c
)+8+4*sec(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos...
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^
3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c)
)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
c))), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}}{\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)b^{2}+2\cos\left(dx+c\right)\sec\left(dx+c\right)ab}$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)**2*sec(c +
 d*x)*b**2 + 2*cos(c + d*x)*sec(c + d*x)*a*b + sec(c + d*x)*a**2),x)
```

3.629 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 602

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(4Ab^3+3a^3B-7ab^2B)\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(\arcsin\Big(\cdots}{3a(a-b)b^2(a+b)^{3/2}d\cdots}$$

$$-\frac{2(3Ab^3+3a^3B+a^2bB-ab^2(A+6B))\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\cdots\right.}{3a(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\cdots}{\cdots}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$+\frac{2a(Ab-aB)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

$$-\frac{2a(4Ab^3+3a^3B-7ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*(4*A*b^3+3*B*a^3-7*B*a*b^2)*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^2/(a+b
)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(3*A*b^3+3*B*a^3+B*a^2*b-a*b^2*(A+6*B))*cos
(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(
d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^2/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2*(a+b)^
(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)+2/3*a*(A
*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-2
/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/(a^2-b^2)
^2/d/(a+b*cos(d*x+c))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1496 vs. $2(602) = 1204$.

Time = 16.65 (sec) , antiderivative size = 1496, normalized size of antiderivative = 2.49

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3
/2)),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(4*A*b^3 + 3*a^3*B - 7*a*
b^2*B)*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)^2) - (2*(-(a^2*A*b*Sin[c + d*x])
+ a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-
(a^3*A*b*Sin[c + d*x]) + 5*a*A*b^3*Sin[c + d*x] + 4*a^4*B*Sin[c + d*x] - 8
*a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
+ (2*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(
c + d*x)/2]^2)]*(-4*a*A*b^3*Tan[(c + d*x)/2] - 4*A*b^4*Tan[(c + d*x)/2] -
3*a^4*B*Tan[(c + d*x)/2] - 3*a^3*b*B*Tan[(c + d*x)/2] + 7*a^2*b^2*B*Tan[(c
 + d*x)/2] + 7*a*b^3*B*Tan[(c + d*x)/2] + 8*A*b^4*Tan[(c + d*x)/2]^3 + 6*a
^3*b*B*Tan[(c + d*x)/2]^3 - 14*a*b^3*B*Tan[(c + d*x)/2]^3 + 4*a*A*b^3*Tan[
(c + d*x)/2]^5 - 4*A*b^4*Tan[(c + d*x)/2]^5 + 3*a^4*B*Tan[(c + d*x)/2]^5 -
 3*a^3*b*B*Tan[(c + d*x)/2]^5 - 7*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 7*a*b^3*B
*Tan[(c + d*x)/2]^5 + 6*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^2*B*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^4*B*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 6*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b...
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3530, 3042, 3288, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{3(a^2-b^2)B\cos^2(c+dx)-3b(Ab-aB)\cos(c+}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}}{3b(a^2-b^2)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3(a^2-b^2)B\cos^2(c+dx)-3b(Ab-aB)\cos(c+dx)+a(Ab-aB)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3b(a^2-b^2)}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{}}{3bd(a^2-b^2)(a+b\cos(c}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3(a^2-b^2)B\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b(a^2-b^2)}+\frac{2a(Ab-aB)\sin(c+}{3bd(a^2-b^2)(a+b}\right.$$

$$\Big\downarrow \text{3530}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{ab(Ab-aB)-3\left(Ab^3+a\left(a^2-2b^2\right)B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{b}+\frac{3B(a^2-b^2)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b(a^2-b^2)}+\frac{2a(Ab-aB)}{3bd(a^2-b^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{ab(Ab-aB)-3\left(Ab^3+a\left(a^2-2b^2\right)B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{3B(a^2-b^2)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b(a^2-b^2)}+\frac{2a(Ab-}{3bd(a}\right.$$

$\Big\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{ab(Ab-aB)-3\left(Ab^3+a\left(a^2-2b^2\right)B\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}-\frac{6B\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(}{}}}{}}{3b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3472

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{a\left(3Ba^3-7b^2Ba+4Ab^3\right)+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}-}{3b}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{a\left(3Ba^3-7b^2Ba+4Ab^3\right)+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}\right.$$

$\Big\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(3a^3B-7ab^2B+4Ab^3\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(3a^3B+a^2bB-ab^2(A+6B)+3Ab^3\right)\int \frac{}{\sqrt{\cos(c+dx)}\sqrt{}}}{a^2-b^2}}{b}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{a\left(3a^3B-7ab^2B+4Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(3a^3B+a^2bB-ab^2(A+6B)+3Ab^3\right)\int\frac{}{\sqrt{s}}}{a^2-b^2}}{}}{b}\right.$$

$\Big\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(3a^3B-7ab^2B+4Ab^3\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^3B+a^2bB-ab^2(A+6B)+3Ab^3\right)}{}}{a^2-b^2}\right.$$

$\Big\downarrow$ 3473

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^3B-7ab^2B+4Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec}{a+}}}{}}{}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + ((-6*Sqrt[
a + b]*(a^2 - b^2)*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2
*d) + (((2*(a - b)*Sqrt[a + b]*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*(3*A*b^3 + 3*a^3*
B + a^2*b*B - a*b^2*(A + 6*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))
/(a^2 - b^2) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sin[c + d*x])/((a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/b)/(3*b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3530
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b
*Sin[e + f*x]], x], x] + Simp[1/b   Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((
a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d,
e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2666 vs. $2(541) = 1082$.

Time = 11.91 (sec) , antiderivative size = 2667, normalized size of antiderivative = 4.43

method	result	size
default	Expression too large to display	2667
parts	Expression too large to display	2722

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3/d/(a-b)^2/(a+b)^2/b^2*(a+b*cos(d*x+c))^(1/2)/((1+cos(d*x+c))*a^2+cos(d
*x+c)*(2*cos(d*x+c)+2)*b*a+cos(d*x+c)^2*(1+cos(d*x+c))*b^2)/sec(d*x+c)^(3/
2)*(B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*a^5*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*(-6-12*sec(d*x+c)-6*sec(d*x+c)^2)-3*B*a^5*tan(d*x+c)-7*B*a*b^4*sin(d*x+
c)-4*A*a^2*b^3*tan(d*x+c)+A*a*b^4*(-5*sin(d*x+c)+3*tan(d*x+c))+B*a^4*b*(-4
*sin(d*x+c)+2*tan(d*x+c))+B*a^3*b^2*(3*sin(d*x+c)+7*tan(d*x+c))+B*a^2*b^3*
(8*sin(d*x+c)-6*tan(d*x+c))+A*a^3*b^2*(sin(d*x+c)+tan(d*x+c))+4*A*b^5*sin(
d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(cos(d*x+c)+2+sec(d*x+c))*B*a^4*b*EllipticPi(-csc(d*x+c)+
cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(3+cos(d*x+c)+3*sec(d*x+c)+s
ec(d*x+c)^2)*B*a^4*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2)
)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*(3+cos(d*x+c)+3*sec(d*x+c)+sec(d*x+c)^2)*A*a*b^4*EllipticE(-cs
c(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(3+cos(d*x+c)+3*sec(d*
x+c)+sec(d*x+c)^2)*B*a^2*b^3*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b
))^(1/2))+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*(5+2*cos(d*x+c)+4*sec(d*x+c)+sec(d*x+c)^2)*B*a*b^4*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}}{\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}b^{2}+2\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}a$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)**2*sec(c +
 d*x)**2*b**2 + 2*cos(c + d*x)*sec(c + d*x)**2*a*b + sec(c + d*x)**2*a**2)
,x)
```

3.630 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 733

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B\right)\sqrt{\cos(c+dx)}\,\mathrm{c}}{3a(a-b)b^3}$$

$$+\frac{\left(3b^3(4A-B)+15a^3B-ab^2(2A+21B)-a^2(6Ab-5bB)\right)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\right.}{3(a-b)b^3(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(2Ab-5aB)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+}}}{b^4d\sqrt{\sec(c+dx)}}$$

$$+\frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}$$

$$+\frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$-\frac{\left(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B\right)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2d}$$

output
```
1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*cos(d*x+c)^(1/2)*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/a/(a-b)/b^3/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+1/3*(3*b^3*(4*A-B)+15*
B*a^3-a*b^2*(2*A+21*B)-a^2*(6*A*b-5*B*b))*cos(d*x+c)^(1/2)*csc(d*x+c)*Elli
pticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(
1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/
b^3/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-(a+b)^(1/2)*(2*A*b-5*B*a)*cos(d*x+c)^(1
/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/b^4/d/sec(d*x+c)^(1/2)+2/3*a*(A*b-B*a)*sin(d*x+c)/b/
(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+2/3*a*(2*A*a^2*b-6*A*b
^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)/
sec(d*x+c)^(1/2)-1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2318 vs. $2(733) = 1466$.

Time = 23.35 (sec) , antiderivative size = 2318, normalized size of antiderivative = 3.16

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}\sec^{\frac{5}{2}}(c + dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5
/2)),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-3*a^2*A*b + 7*A*b^3
+ 6*a^3*B - 10*a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2) + (2*(-(a^3*A*
b*Sin[c + d*x]) + a^4*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)*(a + b*Cos[c +
d*x])^2) + (2*(-4*a^4*A*b*Sin[c + d*x] + 8*a^2*A*b^3*Sin[c + d*x] + 7*a^5*
B*Sin[c + d*x] - 11*a^3*b^2*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(a + b*
Cos[c + d*x]))))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(6*a^
4*A*b*Tan[(c + d*x)/2] + 6*a^3*A*b^2*Tan[(c + d*x)/2] - 14*a^2*A*b^3*Tan[(
c + d*x)/2] - 14*a*A*b^4*Tan[(c + d*x)/2] - 15*a^5*B*Tan[(c + d*x)/2] - 15
*a^4*b*B*Tan[(c + d*x)/2] + 26*a^3*b^2*B*Tan[(c + d*x)/2] + 26*a^2*b^3*B*T
an[(c + d*x)/2] - 3*a*b^4*B*Tan[(c + d*x)/2] - 3*b^5*B*Tan[(c + d*x)/2] -
12*a^3*A*b^2*Tan[(c + d*x)/2]^3 + 28*a*A*b^4*Tan[(c + d*x)/2]^3 + 30*a^4*b
*B*Tan[(c + d*x)/2]^3 - 52*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 6*b^5*B*Tan[(c +
 d*x)/2]^3 - 6*a^4*A*b*Tan[(c + d*x)/2]^5 + 6*a^3*A*b^2*Tan[(c + d*x)/2]^5
 + 14*a^2*A*b^3*Tan[(c + d*x)/2]^5 - 14*a*A*b^4*Tan[(c + d*x)/2]^5 + 15*a^
5*B*Tan[(c + d*x)/2]^5 - 15*a^4*b*B*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*B*Tan[
(c + d*x)/2]^5 + 26*a^2*b^3*B*Tan[(c + d*x)/2]^5 + 3*a*b^4*B*Tan[(c + d*x)
/2]^5 - 3*b^5*B*Tan[(c + d*x)/2]^5 - 12*a^4*A*b*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b^3*...
```

Rubi [A] (verified)

Time = 3.64 (sec) , antiderivative size = 719, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3440, 3042, 3468, 27, 3042, 3526, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3440}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$\downarrow$ 3468

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+2Aba+3b^2B\right)\cos^2(\right.\right.}{2(a+b\cos}}{3b\left(a^2-\right.}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\left(\left(-5Ba^2+2Aba+3b^2B\right)\cos^2(c+dx)\right)-3b(Ab-aB)\cos(c+dx)+3a(Ab-aB)\right)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}+\frac{2a}{3}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5Ba^2-2Aba-3b^2B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-3b(Ab-aB)\sin\left(c+dx+\frac{\pi}{2}\right)+3a(Ab-aB)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3526

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{-\left(\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-\right.\right.}{}}{}}{3b\left(a^2-b^2\right.}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{-\left(\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)\cos^2(c+dx)\right)+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\cos(c+dx)+a\left(-5Ba\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\left(15Ba^4-6Aba^3-26b^2Ba^2+14Ab^3a+3b^4B\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(2Ba^3+Aba^2-6b^2Ba+3Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-5\right.}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3540

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\cos^2(c+dx)+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\cos(c+dx)+a\left(-15Ba^4+6Aba^3+26b^2Ba^2\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}}{b(a^2-b^2)}}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(-15Ba^4+6Aba^3+\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}}{b(a^2-b^2)}}{}\right.$$

$\Big\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+\int \frac{a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)+2ab\left(-5Ba^3+2Aba^2\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}}{b(a^2-b^2)}}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(a^2-b^2\right)^2(2Ab-5aB)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)+2ab\left(-5Ba^3+2\cdots\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+d\cdots\right.}}}{2b}}{b(a^2-\cdots}}{\cdots}\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{a\left(-15Ba^4+6Aba^3+26b^2Ba^2-14Ab^3a-3b^4B\right)+2ab\left(-5Ba^3+2Aba^2+9b^2Ba-6Ab^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a(a-b)\left(15a^3B-a^2(6Ab-5bB)-ab^2(2A+21B)+3b^3(4A-B)\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+a\left(-15a^4B+6a^3Ab+\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a(a-b)\left(15a^3B-a^2(6Ab-5bB)-ab^2(2A+21B)+3b^3(4A-B)\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(-15a^4B\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{a\left(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^2-b^2\right)^2(2Ab-}{}}{}}{}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(}}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{}\right.$$

input
```
Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x
]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(A*b - a*B)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + ((2*a*(2*a
^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (((2*(a - b)*Sqrt[a + b]*(6*a^
3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(a*d) + (2*(a - b)*Sqrt[a + b]*(3*b^3*(4*A - B) + 15*a^
3*B - a*b^2*(2*A + 21*B) - a^2*(6*A*b - 5*b*B))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/d - (6*Sqrt[a + b]*(a^2 - b^2)^2*(2*A*b - 5*a*B)*Cot[c + d*x]*E
llipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/(2*b) - ((6*a^3*A*b - 14*a*A*b^3
 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(b*d*Sqrt[Cos[c + d*x]]))/(b*(a^2 - b^2)))/(3*b*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4166 vs. $2(666) = 1332$.

Time = 17.74 (sec) , antiderivative size = 4167, normalized size of antiderivative = 5.68

method	result	size
default	Expression too large to display	4167
parts	Expression too large to display	4206

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/3/d/(a-b)^2/(a+b)^2/b^3*(a+b*cos(d*x+c))^(1/2)/((1+cos(d*x+c))*a^2+cos(
d*x+c)*(2*cos(d*x+c)+2)*b*a+cos(d*x+c)^2*(1+cos(d*x+c))*b^2)/sec(d*x+c)^(5
/2)*(B*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*b^6*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*
(3+6*sec(d*x+c)+3*sec(d*x+c)^2)-3*B*b^6*sin(d*x+c)+B*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b^4*Ellip
ticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(18+48*sec(d*x+c)+42*sec
(d*x+c)^2+12*sec(d*x+c)^3)+A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^5*EllipticE(-csc(d*x+c)+cot(d*x+c
),(-(a-b)/(a+b))^(1/2))*(14+28*sec(d*x+c)+14*sec(d*x+c)^2)+B*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^5
*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(12+24*sec(d*x+c)+
12*sec(d*x+c)^2)+A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a^4*b^2*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*(12+24*sec(d*x+c)+12*sec(d*x+c)^2)+A*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b^4*E
llipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(-24-48*sec(d*x+
c)-24*sec(d*x+c)^2)+A*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*b^2*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(
a-b)/(a+b))^(1/2))*(-6-18*sec(d*x+c)-18*sec(d*x+c)^2-6*sec(d*x+c)^3)+A*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}\sec^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5
/2)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2
)),x)
```

output
```
int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)b+a}}{\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}b^{2}+2\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}a}$$

input
```
int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)**2*sec(c +
 d*x)**3*b**2 + 2*cos(c + d*x)*sec(c + d*x)**3*a*b + sec(c + d*x)**3*a**2)
,x)
```

3.631 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 266

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{co}}}{\sqrt{a+b}\sqrt{\mathrm{c}}}\right.\right.}{a^2d\sqrt{\sec(c+dx}}$$

$$-\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(\mathrm{c}}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

output
```
2*(a-b)*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*(
a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.12

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\left(\frac{2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}\right.$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+}{a+}\right.\right.}{}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x
])^(3/2),x]
```

output
```
B*((2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a
+ b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2011, 3042, 4710, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 2011$$

$$B\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$B\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4710$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3280$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)$$

$$\Big\downarrow \; 3042$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+\cdots}}\right.$$

$$\Big\downarrow \; 3295$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\mathrm{se}\cdots}{\cdots}}}{\cdots}\right.$$

$$\Big\downarrow \; 3473$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+\cdots}}{\sqrt{a+b}\sqrt{\cos(c\cdots}}\right.\right.}{a^2d}\right.$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/
2),x]
```

output
```
B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a - b)*Sqrt[a + b]*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(a*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Maple [A] (verified)

Time = 28.58 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.44

method	result
default	$\frac{2B\left(\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,a\,\mathrm{EllipticE}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+\left(\cos(dx\right.\right.}{}$
parts	Expression too large to display

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2*B/d/a*((cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*EllipticE(-csc(d*x+c)+cot
(d*x+c),(-(a-b)/(a+b))^(1/2))+(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b*Elli
pticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+(-cos(d*x+c)^2-2*cos(d*
x+c)-1)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*a*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+c
os(d*x+c)*sin(d*x+c)*b+sin(d*x+c)*a)*(a+b*cos(d*x+c))^(1/2)*cos(d*x+c)*sec
(d*x+c)^(3/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*cos(d*x+c)+a)
```

Fricas [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx + c) + Ba)\sec(dx + c)^{\frac{3}{2}}}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
integral(B*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(aB + bB\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx + c) + Ba)\sec(dx + c)^{\frac{3}{2}}}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
^(3/2), x)
```

Giac [F]

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)
^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(B\,a+B\,b\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
^(3/2),x)
```

output
```
int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}\,\sec(dx+c)}{\cos(dx+c)\,b+a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a)*sec(c + d*x))/(cos(c + d*
x)*b + a),x)*b
```

3.632 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 130

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\cdots\right.\right.}{ad\sqrt{\sec(c+d}\cdots}$$

output
```
2*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.80

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a}{a-}\cdots\right.}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\cdots}$$

input
```
Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x
])^(3/2),x]
```

output
```
(2*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2011, 3042, 4710, 3042, 3295}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4710}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3295}$$

$$\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

input
```
Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/
2),x]
```

output
```
(2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Maple [A] (verified)

Time = 13.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{2B\sqrt{\sec(dx+c)}\,\text{EllipticF}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,\cos(dx+c)}{d\sqrt{a+b\cos(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	110
parts	Expression too large to display	984

input
```
int((B*a+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-2*B/d*sec(d*x+c)^(1/2)*EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1
/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(a+b*cos(d*x+c))^(1/2
)*cos(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Fricas [F]

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
integral(B*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(aB + bB\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}}\,dx = B\int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + b\cos(c + dx)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x
)
```

output
```
B*Integral(sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(aB + bB\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx + c) + Ba)\sqrt{\sec(dx + c)}}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)
^(3/2), x)
```

Giac [F]

$$\int \frac{(aB + bB\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{(Bb\cos(dx + c) + Ba)\sqrt{\sec(dx + c)}}{(b\cos(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)
^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(aB + bB\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(B\,a + B\,b\,\cos(c + d\,x))}{(a + b\,\cos(c + d\,x))^{3/2}}\,dx$$

input
```
int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
^(3/2),x)
```

output
```
int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(aB + bB\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c)\,b + a}}{\cos(dx + c)\,b + a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)*b + a),x)*
b
```

3.633 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 137

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{bd\sqrt{\sec(c+dx)}}$$

output
```
-2*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = -\frac{2B\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)-2\,\text{EllipticPi}\left(-1,\arcsin\right.\right.}{d\sqrt{\frac{1}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c
+ d*x]]),x]
```

output
```
(-2*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqr
t[1 + Sec[c + d*x]])/(d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2011, 3042, 4710, 3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3288}$$

$$-\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\right.}{bd\sqrt{\sec(c+dx)}}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]
]),x]
```

output
```
(-2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Maple [A] (verified)

Time = 11.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

method	result
default	$-\frac{2B\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\left(-\text{EllipticF}\left(-\csc(dx+c)+\cot(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)+2\,\text{EllipticPi}\left(-\csc(dx+c)+\cot(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)}{d\sqrt{a+b\cos(dx+c)}\,\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$
parts	Expression too large to display

input
```
int((B*a+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-2*B/d/(a+b*cos(d*x+c))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*(-EllipticF(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))+2*EllipticPi(
-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/sec(d*x+c)^(1/2)/(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)
```

Fricas [F]

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,
algorithm="fricas")
```

output
```
integral(B/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Sympy [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = B\int \frac{1}{\sqrt{a+b\cos{(c+dx)}}\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x
)
```

output
```
B*Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{Bb\cos{(dx+c)} + Ba}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*
x + c))), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{Bb\cos{(dx+c)} + Ba}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*
x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{B\,a + B\,b\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^
(3/2)),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^
(3/2)), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\cos(dx+c)\,b+a}}{\cos(dx+c)\sec(dx+c)\,b+\sec(dx+c)\,a}dx\right)b$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)*sec(c + d*
x)*b + sec(c + d*x)*a),x)*b
```

3.634 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 479

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd\sqrt{\sec(c+dx)}}$$

$$+\frac{a\sqrt{a+b}B\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{aB\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{a+b\cos(c+dx)}}$$

output
```
-(a-b)*(a+b)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+(a+b
)^(1/2)*B*cos(d*x+c)^(1/2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+a*(a+b)^(1/2)*B*c
os(d*x+c)^(1/2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),(a+b)/b,(-(a+b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+B*sin(d*x+c)/d/(a
+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*B*sec(d*x+c)^(1/2)*sin(d*x+c)/b/d/
(a+b*cos(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.65 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.49

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2B\cos(c+dx)\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}}{}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x
]^(3/2)),x]
```

output
```
(2*B*Cos[c + d*x]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(Cos[(c + d*x)/2
]^2*Sec[c + d*x])^(3/2)*((a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 2*a
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a + b*Cos[c + d*x])*Sqrt[Cos[
c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(b*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2011, 3042, 4710, 3042, 3299, 3042, 3288, 3482, 27, 3042, 3472, 25, 27, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3299}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)$$

$\downarrow$ 3288

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{}\right.$$

$\downarrow$ 3482

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{2}\int \frac{2\left(\cos(c+dx)a^2+ba\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{}}{}\right.$$

$\downarrow$ 27

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos(c+dx)a^2+ba}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{a}}{}\right.$$

$\downarrow$ 3042

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a^2+ba}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+}{}\right.$$

$\downarrow$ 3472

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{a\left(a^2-b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\right.$$

$\downarrow$ 25

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int\frac{a(a^2-b^2)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\right.$$

$\downarrow$ 27

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}\right.$$

$\downarrow$ 3042

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b}}}{2b}\right.$$

$\downarrow$ 3280

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{}}{d\sqrt{a+b\cos}}}{2b}\right.$$

$\downarrow$ 3042

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+}{2b}\right.$$

$\downarrow$ 3295

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx}{a-b}}}{}\right.}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}\right.}{}\right.$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2
)),x]
```

output
```
B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (-(a*((2*(a - b)*Sqrt[a + b]*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(a*d))) + (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) + (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a
+ b*Cos[c + d*x]]))/(2*b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3299
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> Simp[(-a)*(d/(2*b))   Int[Sqrt[d*Sin[e + f*x]]/Sqr
t[a + b*Sin[e + f*x]], x], x] + Simp[d/(2*b)   Int[Sqrt[d*Sin[e + f*x]]*((a
 + 2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e
, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Maple [A] (verified)

Time = 14.59 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.78

method	result
default	$-\frac{B\sqrt{a+b\cos(dx+c)}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\,a\,\text{EllipticPi}\left(-\csc(dx+c)+\cot(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\left(-2-4\sec(dx+c)\right.\right.}{}$
parts	Expression too large to display

input
```
int((B*a+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-B/d/b*(a+b*cos(d*x+c))^(1/2)/(cos(d*x+c)^2*b+a*cos(d*x+c)+b*cos(d*x+c)+a)
/sec(d*x+c)^(3/2)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*a*EllipticPi(-csc(d*x+c)+cot(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*(-2-4*sec(d*x+c)-2*sec(d*x+c)^2)+(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*a*EllipticE(-csc(d*
x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(1+2*sec(d*x+c)+sec(d*x+c)^2)+(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*b*EllipticE(-csc(d*x+c)+cot(d*x+c),(-(a-b)/(a+b))^(1/2))*(1+2*sec(d*x+c)
+sec(d*x+c)^2)-sin(d*x+c)*b-a*tan(d*x+c))
```

Fricas [F]

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,
algorithm="fricas")
```

output
```
integral(B/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{3}{2}}\sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c
)^(3/2)), x)
```

Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{(b\cos(dx + c) + a)^{\frac{3}{2}}\sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,
algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c
)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{B\,a + B\,b\,\cos(c + d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a + b\,\cos(c + d\,x))^{3/2}}\,dx$$

input
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^
(3/2)),x)
```

output
```
int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^
(3/2)), x)
```

Reduce [F]

$$\int \frac{aB + bB\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \left(\int \frac{\sqrt{\sec(dx + c)}\,\sqrt{\cos(dx + c)\,b + a}}{\cos(dx + c)\sec(dx + c)^2\,b + \sec(dx + c)^2\,a}dx\right) b$$

input
```
int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

output
```
int((sqrt(sec(c + d*x))*sqrt(cos(c + d*x)*b + a))/(cos(c + d*x)*sec(c + d*
x)**2*b + sec(c + d*x)**2*a),x)*b
```

3.635 $\int (a+b\cos(e+fx))^n (A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int (a+b\cos(e+fx))^n (A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = (c\cos(e+fx))^m (c\sec(e+fx))^m \text{Int}\left((c\cos(e+fx))^{-m}(a+b\cos(e+fx))^n (A+B\cos(e+fx)),x\right)$$

output
```
(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Defer(Int)((a+b*cos(f*x+e))^n*(A+B*cos(f
*x+e))/((c*cos(f*x+e))^m),x)
```

Mathematica [N/A]

Not integrable

Time = 31.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (a+b\cos(e+fx))^n (A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (a+b\cos(e+fx))^n (A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

input
```
Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x
]
```

output
```
Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,
x]
```

Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3440, 3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+B\cos(e+fx))(c\sec(e+fx))^m(a+b\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3440}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int (c\cos(e+fx))^{-m}(a+b\cos(e+fx))^n(A+B\cos(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3486}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int (c\cos(e+fx))^{-m}(a+b\cos(e+fx))^n(A+B\cos(e+fx))dx$$

input
```
Int[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int (a+b\cos(fx+e))^n (A+B\cos(fx+e)) (c\sec(fx+e))^m \, dx$$

input
```
int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^n(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x
)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**n*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^n(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m,
x)
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^n(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m,
x)
```

Mupad [N/A]

Not integrable

Time = 42.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int \left(\frac{c}{\cos(e+f\,x)}\right)^m (A+B\,\cos(e+f\,x))\,(a+b\,\cos(e+f\,x))^n\,dx$$

input
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n,x)
```

output
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^m\left(\left(\int \sec(fx+e)^m(\cos(fx+e)\,b+a)^n\cos(fx+e)\,dx\right)b + \left(\int \sec(fx+e)^m(\cos(fx+e)\,b+a)^n\,dx\right)a\right)$$

input
```
int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
c**m*(int(sec(e + f*x)**m*(cos(e + f*x)*b + a)**n*cos(e + f*x),x)*b + int(
sec(e + f*x)**m*(cos(e + f*x)*b + a)**n,x)*a)
```

3.636 $\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 33, antiderivative size = 644

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx =$$

$$-\frac{c^6(4a^3Ab(15-8m+m^2)+a^4B(15-8m+m^2)+4aAb^3(10-7m+m^2)+6a^2b^2B(10-7m+m^2)}{f(2-m)(4-m)(6}$$

$$-\frac{c^5(a^4A(8-6m+m^2)+6a^2Ab^2(4-5m+m^2)+4a^3bB(4-5m+m^2)+Ab^4(3-4m+m^2)+4ab^3}{f(1-m)(3-m)(5-}$$

$$-\frac{ac^5(4a^2Ab(3-4m+m^2)+a^3B(3-4m+m^2)+2Ab^3(4-2m+m^2)+ab^2B(8-13m+5m^2))\,(c\,\mathrm{se}}{f(1-m)(2-m)(4-m)}$$

$$-\frac{a^2c^5(2abB(1-m)^2+a^2A(2-m)^2+Ab^2(6-m+m^2))\sec(e+fx)(c\sec(e+fx))^{-5+m}\tan(e+fx)}{f(1-m)(2-m)(3-m)}$$

$$-\frac{ac^5(aB(1-m)-Ab(2+m))(c\sec(e+fx))^{-5+m}(b+a\sec(e+fx))^2\tan(e+fx)}{f(1-m)(2-m)}$$

$$-\frac{aAc^5(c\sec(e+fx))^{-5+m}(b+a\sec(e+fx))^3\tan(e+fx)}{f(1-m)}$$

output
```
-c^6*(4*a^3*A*b*(m^2-8*m+15)+a^4*B*(m^2-8*m+15)+4*a*A*b^3*(m^2-7*m+10)+6*a
^2*b^2*B*(m^2-7*m+10)+b^4*B*(m^2-6*m+8))*hypergeom([1/2, 3-1/2*m],[4-1/2*m
],cos(f*x+e)^2)*(c*sec(f*x+e))^(-6+m)*sin(f*x+e)/f/(2-m)/(4-m)/(6-m)/(sin(
f*x+e)^2)^(1/2)-c^5*(a^4*A*(m^2-6*m+8)+6*a^2*A*b^2*(m^2-5*m+4)+4*a^3*b*B*(
m^2-5*m+4)+A*b^4*(m^2-4*m+3)+4*a*b^3*B*(m^2-4*m+3))*hypergeom([1/2, 5/2-1/
2*m],[7/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-5+m)*sin(f*x+e)/f/(1-m)/(3
-m)/(5-m)/(sin(f*x+e)^2)^(1/2)-a*c^5*(4*a^2*A*b*(m^2-4*m+3)+a^3*B*(m^2-4*m
+3)+2*A*b^3*(m^2-2*m+4)+a*b^2*B*(5*m^2-13*m+8))*(c*sec(f*x+e))^(-5+m)*tan(
f*x+e)/f/(1-m)/(2-m)/(4-m)-a^2*c^5*(2*a*b*B*(1-m)^2+a^2*A*(2-m)^2+A*b^2*(m
^2-m+6))*sec(f*x+e)*(c*sec(f*x+e))^(-5+m)*tan(f*x+e)/f/(1-m)/(2-m)/(3-m)-a
*c^5*(a*B*(1-m)-A*b*(2+m))*(c*sec(f*x+e))^(-5+m)*(b+a*sec(f*x+e))^2*tan(f*
x+e)/f/(1-m)/(2-m)-a*A*c^5*(c*sec(f*x+e))^(-5+m)*(b+a*sec(f*x+e))^3*tan(f*
x+e)/f/(1-m)
```

Mathematica [A] (verified)

Time = 4.63 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.49

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

$$= \frac{\cot(e+fx)\left(\frac{b^4B\cos^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),\sec^2(e+fx)\right)}{-5+m}+\frac{b^3(Ab+4aB)\cos^4(e+fx)\,\text{Hypergeometric2F}}{-4+}\right.}{}$$

input
```
Integrate[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x
]
```

output
```
(Cot[e + f*x]*((b^4*B*Cos[e + f*x]^5*Hypergeometric2F1[1/2, (-5 + m)/2, (-
3 + m)/2, Sec[e + f*x]^2])/(-5 + m) + (b^3*(A*b + 4*a*B)*Cos[e + f*x]^4*Hy
pergeometric2F1[1/2, (-4 + m)/2, (-2 + m)/2, Sec[e + f*x]^2])/(-4 + m) + a
*((2*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2,
 (-1 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a*((2*b*(3*A*b + 2*a*B)*Cos[e + f
*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) +
a*(((4*A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/
2, Sec[e + f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2,
 Sec[e + f*x]^2])/m))))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 4.32 (sec) , antiderivative size = 617, normalized size of antiderivative = 0.96, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 3439, 3042, 4514, 25, 3042, 4584, 3042, 4564, 25, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^4\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3439}$$

$$c^5\int (c\sec(e+fx))^{m-5}(b+a\sec(e+fx))^4(B+A\sec(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$c^5\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4\left(B+A\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4514}$$

$$c^5\left(-\frac{\int -(c\sec(e+fx))^{m-5}(b+a\sec(e+fx))^2\left(a(aB(1-m)-Ab(m+2))\sec^2(e+fx)+\left(A(2-m)a^2+b(\right.\right.}{1-m}\right.$$

$$\Big\downarrow \text{25}$$

$$c^5\left(\frac{\int (c\sec(e+fx))^{m-5}(b+a\sec(e+fx))^2\left(a(aB(1-m)-Ab(m+2))\sec^2(e+fx)+\left(A(2-m)a^2+b(Ab+\right.\right.}{1-m}\right.$$

$$\Big\downarrow \text{3042}$$

$$c^5\left(\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(a(aB(1-m)-Ab(m+2))\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(A(2\right.\right.}{1-m}\right.$$

$$\Big\downarrow \text{4584}$$

$$c^5\left(-\frac{\frac{\int (c\sec(e+fx))^{m-5}(b+a\sec(e+fx))\left(-a\left(A(m^2-m+6)b^2+2aB(1-m)^2b+a^2A(2-m)^2\right)\sec^2(e+fx)-\left(B\left(m^2-4m+3\right)a^3+Ab\left(3m^2-12m\right.\right.\right.}{2-m}}{}\right.$$

$\downarrow$ 3042

$$c^5\left(-\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(-a\left(A(m^2-m+6)b^2+2aB(1-m)^2b+a^2A(2-m)^2\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(-B\left(m^2-4m+3\right)a^3\right.\right.}{}\right.$$

$\downarrow$ 4564

$$c^5\left(-\frac{\frac{a^2\tan(e+fx)\sec(e+fx)\left(a^2A(2-m)^2+2abB(1-m)^2+Ab^2\left(m^2-m+6\right)\right)(c\sec(e+fx))^{m-5}}{f(3-m)}-\frac{\int -(c\sec(e+fx))^{m-5}\left((3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5\right.\right.}{}}{}\right.$$

$\downarrow$ 25

$$c^5\left(-\frac{\frac{\int (c\sec(e+fx))^{m-5}\left((3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5-m)ma-b^2B\left(m^2-3m+2\right)\right)b^2-a(3-m)\left(B\left(m^2-4m+3\right)a^3+4Ab\left(m^2-4m+3\right)a^2+b^2B\left(5m^2-\right.\right.\right.}{}}{}\right.$$

$\downarrow$ 3042

$$c^5\left(-\frac{\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left((3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5-m)ma-b^2B\left(m^2-3m+2\right)\right)b^2-a(3-m)\left(B\left(m^2-4m+3\right)a^3+4Ab\left(m^2-4m+3\right)a^2+b^2B\left(\right.\right.\right.}{}}{}\right.$$

$\downarrow$ 4535

$$c^5\left(-\frac{\frac{\int (c\sec(e+fx))^{m-5}\left(b^2(3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5-m)ma-b^2B\left(m^2-3m+2\right)\right)-a(3-m)\left(B\left(m^2-4m+3\right)a^3+4Ab\left(m^2-4m+3\right)a^2+b^2B\left(5m^2-\right.\right.\right.}{}}{}\right.$$

$\downarrow$ 3042

$$c^5\left(-\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left(b^2(3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5-m)ma-b^2B\left(m^2-3m+2\right)\right)-a(3-m)\left(B\left(m^2-4m+3\right)a^3+4Ab\left(m^2-4m+3\right)a^2+b^2B\left(\cdots\right.\right.\right.}{\cdots}\right.$$

$\downarrow$ 4259

$$c^5\left(-\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left(b^2(3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5-m)ma-b^2B\left(m^2-3m+2\right)\right)-a(3-m)\left(B\left(m^2-4m+3\right)a^3+4Ab\left(m^2-4m+3\right)a^2+b^2B\left(\cdots\right.\right.\right.}{\cdots}\right.$$

$\downarrow$ 3042

$$c^5\left(-\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left(b^2(3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5-m)ma-b^2B\left(m^2-3m+2\right)\right)-a(3-m)\left(B\left(m^2-4m+3\right)a^3+4Ab\left(m^2-4m+3\right)a^2+b^2B\left(\cdots\right.\right.\right.}{\cdots}\right.$$

$\downarrow$ 3122

$$c^5\left(-\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-5}\left(b^2(3-m)\left(-B\left(m^2-6m+5\right)a^2+2Ab(5-m)ma-b^2B\left(m^2-3m+2\right)\right)-a(3-m)\left(B\left(m^2-4m+3\right)a^3+4Ab\left(m^2-4m+3\right)a^2+b^2B\left(\cdots\right.\right.\right.}{\cdots}\right.$$

$\downarrow$ 4534

$$c^5\left(-\frac{-\frac{\left(m^2-4m+3\right)\left(a^4B\left(m^2-8m+15\right)+4a^3Ab\left(m^2-8m+15\right)+6a^2b^2B\left(m^2-7m+10\right)+4aAb^3\left(m^2-7m+10\right)+b^4B\left(m^2-6m+8\right)\right)\int (c\sec(e+fx))^{m-5}dx}{4-m}+\cdots}{\cdots}\right.$$

$\downarrow$ 3042

$$c^5\left(-\frac{-\frac{\left(m^2-4m+3\right)\left(a^4B\left(m^2-8m+15\right)+4a^3Ab\left(m^2-8m+15\right)+6a^2b^2B\left(m^2-7m+10\right)+4aAb^3\left(m^2-7m+10\right)+b^4B\left(m^2-6m+8\right)\right)\int\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-}}{4-m}}{}\right.$$

$\downarrow$ 4259

$$c^5\left(-\frac{-\frac{\left(m^2-4m+3\right)\left(a^4B\left(m^2-8m+15\right)+4a^3Ab\left(m^2-8m+15\right)+6a^2b^2B\left(m^2-7m+10\right)+4aAb^3\left(m^2-7m+10\right)+b^4B\left(m^2-6m+8\right)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e}{4-m}}{}\right.$$

$\downarrow$ 3042

$$c^5\left(-\frac{-\frac{\left(m^2-4m+3\right)\left(a^4B\left(m^2-8m+15\right)+4a^3Ab\left(m^2-8m+15\right)+6a^2b^2B\left(m^2-7m+10\right)+4aAb^3\left(m^2-7m+10\right)+b^4B\left(m^2-6m+8\right)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e}{4-m}}{}\right.$$

$\downarrow$ 3122

$$c^5\left(-\frac{\frac{a^2\tan(e+fx)\sec(e+fx)\left(a^2A(2-m)^2+2abB(1-m)^2+Ab^2\left(m^2-m+6\right)\right)(c\sec(e+fx))^{m-5}}{f(3-m)}+\frac{a(3-m)\tan(e+fx)\left(a^3B\left(m^2-4m+3\right)+4a^2Ab\left(m^2-4m+\right.\right.}{}}{}\right.$$

input
```
Int[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
c^5*(-((a*A*(c*Sec[e + f*x])^(-5 + m)*(b + a*Sec[e + f*x])^3*Tan[e + f*x])
/(f*(1 - m))) + (-((a*(a*B*(1 - m) - A*b*(2 + m))*(c*Sec[e + f*x])^(-5 + m
)*(b + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(2 - m))) - ((a^2*(2*a*b*B*(1 -
m)^2 + a^2*A*(2 - m)^2 + A*b^2*(6 - m + m^2))*Sec[e + f*x]*(c*Sec[e + f*x]
)^(-5 + m)*Tan[e + f*x])/(f*(3 - m)) + ((c*(3 - 4*m + m^2)*(4*a^3*A*b*(15
- 8*m + m^2) + a^4*B*(15 - 8*m + m^2) + 4*a*A*b^3*(10 - 7*m + m^2) + 6*a^2
*b^2*B*(10 - 7*m + m^2) + b^4*B*(8 - 6*m + m^2))*Hypergeometric2F1[1/2, (6
 - m)/2, (8 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-6 + m)*Sin[e + f*x]
)/(f*(4 - m)*(6 - m)*Sqrt[Sin[e + f*x]^2]) + ((2 - m)*(a^4*A*(8 - 6*m + m^
2) + 6*a^2*A*b^2*(4 - 5*m + m^2) + 4*a^3*b*B*(4 - 5*m + m^2) + A*b^4*(3 -
4*m + m^2) + 4*a*b^3*B*(3 - 4*m + m^2))*Hypergeometric2F1[1/2, (5 - m)/2,
(7 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*Sin[e + f*x])/(f*(5 -
 m)*Sqrt[Sin[e + f*x]^2]) + (a*(3 - m)*(4*a^2*A*b*(3 - 4*m + m^2) + a^3*B*
(3 - 4*m + m^2) + 2*A*b^3*(4 - 2*m + m^2) + a*b^2*B*(8 - 13*m + 5*m^2))*(c
*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(4 - m)))/(3 - m))/(2 - m))/(1 -
m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4514
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(m + n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*
Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1)
)*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x]
, x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2
- b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

rule 4584
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Simp[1/(m + n + 1)   Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a
*B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
 - b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Maple [F]

$$\int (a+b\cos(fx+e))^4 (A+B\cos(fx+e)) (c\sec(fx+e))^m dx$$

input
```
int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^4(c\sec(fx+e))^m dx$$

input
```
integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="fricas")
```

output
```
integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^
4 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^
2)*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*sec(f*x + e))^m,
x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**4*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^4(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m,
x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^4(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int \left(\frac{c}{\cos(e+f\,x)}\right)^m (A+B\,\cos(e+f\,x))\,(a+b\,\cos(e+f\,x))^4\,dx$$

input
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4,x)
```

output
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4, x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^m\left(\left(\int \sec(fx+e)^m\,dx\right)a^5+5\left(\int \sec(fx+e)^m\cos(fx+e)\,dx\right)a^4b + \left(\int \sec(fx+e)^m\cos(fx+e)^5\,dx\right)b^5+5\left(\int \sec(fx+e)^m\cos(fx+e)^4\,dx\right)a\,b^4 + 10\left(\int \sec(fx+e)^m\cos(fx+e)^3\,dx\right)a^2b^3 + 10\left(\int \sec(fx+e)^m\cos(fx+e)^2\,dx\right)a^3b^2\right)$$

input
```
int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
c**m*(int(sec(e + f*x)**m,x)*a**5 + 5*int(sec(e + f*x)**m*cos(e + f*x),x)*
a**4*b + int(sec(e + f*x)**m*cos(e + f*x)**5,x)*b**5 + 5*int(sec(e + f*x)*
*m*cos(e + f*x)**4,x)*a*b**4 + 10*int(sec(e + f*x)**m*cos(e + f*x)**3,x)*a
**2*b**3 + 10*int(sec(e + f*x)**m*cos(e + f*x)**2,x)*a**3*b**2)
```

3.637 $\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 33, antiderivative size = 455

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = -\frac{c^5(a^3A(8-6m+m^2)+3aAb^2(4-5m+m^2)+3a^2bB(4-5m+m^2)+b^3B(3-4m+m^2))\,\text{Hyperg}}{f(1-m)(3-m)(5-m)\sqrt{\sin^2(e}} - \frac{c^4(Ab^3(2-m)+3ab^2B(2-m)+3a^2Ab(3-m)+a^3B(3-m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-m}{2},\frac{6-m}{2},\text{c}\right.}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}} - \frac{ac^4(3abB(1-m)+a^2A(2-m)-2Ab^2m)\,(c\sec(e+fx))^{-4+m}\tan(e+fx)}{f(1-m)(3-m)} - \frac{a^2c^4(aB(1-m)-Ab(1+m))\sec(e+fx)(c\sec(e+fx))^{-4+m}\tan(e+fx)}{f(1-m)(2-m)} - \frac{aAc^4(c\sec(e+fx))^{-4+m}(b+a\sec(e+fx))^2\tan(e+fx)}{f(1-m)}$$

output
```
-c^5*(a^3*A*(m^2-6*m+8)+3*a*A*b^2*(m^2-5*m+4)+3*a^2*b*B*(m^2-5*m+4)+b^3*B*
(m^2-4*m+3))*hypergeom([1/2, 5/2-1/2*m],[7/2-1/2*m],cos(f*x+e)^2)*(c*sec(f
*x+e))^(-5+m)*sin(f*x+e)/f/(1-m)/(3-m)/(5-m)/(sin(f*x+e)^2)^(1/2)-c^4*(A*b
^3*(2-m)+3*a*b^2*B*(2-m)+3*a^2*A*b*(3-m)+a^3*B*(3-m))*hypergeom([1/2, 2-1/
2*m],[3-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-4+m)*sin(f*x+e)/f/(2-m)/(4-m
)/(sin(f*x+e)^2)^(1/2)-a*c^4*(3*a*b*B*(1-m)+a^2*A*(2-m)-2*A*b^2*m)*(c*sec(
f*x+e))^(-4+m)*tan(f*x+e)/f/(1-m)/(3-m)-a^2*c^4*(a*B*(1-m)-A*b*(1+m))*sec(
f*x+e)*(c*sec(f*x+e))^(-4+m)*tan(f*x+e)/f/(1-m)/(2-m)-a*A*c^4*(c*sec(f*x+e
))^(-4+m)*(b+a*sec(f*x+e))^2*tan(f*x+e)/f/(1-m)
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.57

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$= \frac{\cot(e+fx)\left(\frac{b^3B\cos^4(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-4+m),\frac{1}{2}(-2+m),\sec^2(e+fx)\right)}{-4+m}+\frac{b^2(Ab+3aB)\cos^3(e+fx)\,\text{Hypergeometric2F}\ldots}{-3+\ldots}\right.}{}$$

input
```
Integrate[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x
]
```

output
```
(Cot[e + f*x]*((b^3*B*Cos[e + f*x]^4*Hypergeometric2F1[1/2, (-4 + m)/2, (-
2 + m)/2, Sec[e + f*x]^2])/(-4 + m) + (b^2*(A*b + 3*a*B)*Cos[e + f*x]^3*Hy
pergeometric2F1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a
*((3*b*(A*b + a*B)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2,
Sec[e + f*x]^2])/(-2 + m) + a*(((3*A*b + a*B)*Cos[e + f*x]*Hypergeometric2
F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2])/(-1 + m) + (a*A*Hypergeome
tric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2])/m)))*(c*Sec[e + f*x])^m*Sqrt
[-Tan[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 3439, 3042, 4514, 25, 3042, 4564, 25, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \ 3439$$

$$c^4\int (c\sec(e+fx))^{m-4}(b+a\sec(e+fx))^3(B+A\sec(e+fx))dx$$

$$\Big\downarrow \ 3042$$

$$c^4\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(B+A\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 4514$$

$$c^4\left(-\frac{\int -(c\sec(e+fx))^{m-4}(b+a\sec(e+fx))\left(a(aB(1-m)-Ab(m+1))\sec^2(e+fx)+\left(A(2-m)a^2+b(A\right.\right.}{1-m}\right.$$

$$\Big\downarrow \ 25$$

$$c^4\left(\frac{\int(c\sec(e+fx))^{m-4}(b+a\sec(e+fx))\left(a(aB(1-m)-Ab(m+1))\sec^2(e+fx)+\left(A(2-m)a^2+b(Ab+\right.\right.}{1-m}\right.$$

$$\Big\downarrow \ 3042$$

$$c^4\left(\frac{\int\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(a(aB(1-m)-Ab(m+1))\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(A(2\right.\right.}{1-m}\right.$$

$$\Big\downarrow \ 4564$$

$$c^4\left(\frac{-\frac{\int -(c\sec(e+fx))^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\sec^2(e+fx)+\left(B(3-m)a^3+3Ab(3\right.\right.}{2-m}}{1-m}\right.$$

$\downarrow$ 25

$$c^4\left(\frac{\frac{\int (c\sec(e+fx))^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\sec^2(e+fx)+\left(B(3-m)a^3+3Ab(3-m)\right.\right.}{2-m}}{1-m}\right.$$

$\downarrow$ 3042

$$c^4\left(\frac{\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(B(3-m)a^3+3A\right.\right.}{2-m}}{1-}\right.$$

$\downarrow$ 4535

$$c^4\left(\frac{\frac{\int (c\sec(e+fx))^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\sec^2(e+fx)\right)dx+\frac{(1-m)\left(a^3B(3-m)+3a\right.}{}}{2-m}}{1-m}\right.$$

$\downarrow$ 3042

$$c^4\left(\frac{\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{(1-m)\left(a^3B(\right.}{}}{2-m}}{1-}\right.$$

$\downarrow$ 4259

$$c^4\left(\frac{\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{(1-m)\left(a^3B(\right.}{}}{2-m}}{}\right.$$

$\downarrow$ 3042

$$c^4\left(\frac{\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{(1-m)\left(a^3B(\right.}{}}{2-m}}{}\right.$$

$\downarrow$ 3122

$$c^4\left(\frac{\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}\left((bB(1-m)+aA(4-m))(2-m)b^2+a(2-m)\left(A(2-m)a^2+3bB(1-m)a-2Ab^2m\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx-\frac{(1-m)\sin(e+}{}}{2-m}}{}\right.$$

$\downarrow$ 4534

$$c^4\left(\frac{\frac{\frac{(2-m)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m+4\right)+b^3B\left(m^2-4m+3\right)\right)\int(c\sec(e+fx))^{m-4}dx}{3-m}-\frac{a(2-m)\tan(e+fx)\left(a^2A(2-m)+3abB\right.}{f(3-}}{}}{}\right.$$

$\downarrow$ 3042

$$c^4\left(\frac{\frac{\frac{(2-m)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m+4\right)+b^3B\left(m^2-4m+3\right)\right)\int\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-4}dx}{3-m}-\frac{a(2-m)\tan(e+fx)\left(a^2A(2-m)+\right.}{}}{}}{}\right.$$

$\downarrow$ 4259

$$c^4\left(\frac{\frac{\frac{(2-m)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m+4\right)+b^3B\left(m^2-4m+3\right)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int\left(\frac{\cos(e+fx)}{c}\right)^{4-m}dx}{3-m}-\frac{a(2-}{}}{}}{}\right.$$

$\downarrow$ 3042

$$c^4\left(\frac{\frac{\frac{(2-m)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m+4\right)+b^3B\left(m^2-4m+3\right)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{c}\right)^{4-m}dx}{3-m}-\ldots}{\ldots}}{\ldots}\right.$$

$$\Big\downarrow \; 3122$$

$$c^4\left(\frac{\frac{-\frac{a(2-m)\tan(e+fx)\left(a^2A(2-m)+3abB(1-m)-2Ab^2m\right)(c\sec(e+fx))^{m-4}}{f(3-m)}-\frac{c(2-m)\sin(e+fx)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m\ldots\right.\right.}{f(3-\ldots}}{\ldots}}{\ldots}\right.$$

input
```
Int[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
c^4*(-((a*A*(c*Sec[e + f*x])^(-4 + m)*(b + a*Sec[e + f*x])^2*Tan[e + f*x])
/(f*(1 - m))) + (-((a^2*(a*B*(1 - m) - A*b*(1 + m))*Sec[e + f*x]*(c*Sec[e
+ f*x])^(-4 + m)*Tan[e + f*x])/(f*(2 - m))) + (-(((c*(2 - m)*(a^3*A*(8 - 6*
m + m^2) + 3*a*A*b^2*(4 - 5*m + m^2) + 3*a^2*b*B*(4 - 5*m + m^2) + b^3*B*(
3 - 4*m + m^2))*Hypergeometric2F1[1/2, (5 - m)/2, (7 - m)/2, Cos[e + f*x]^
2]*(c*Sec[e + f*x])^(-5 + m)*Sin[e + f*x])/(f*(3 - m)*(5 - m)*Sqrt[Sin[e +
 f*x]^2])) - ((A*b^3*(2 - m) + 3*a*b^2*B*(2 - m) + 3*a^2*A*b*(3 - m) + a^3
*B*(3 - m))*(1 - m)*Hypergeometric2F1[1/2, (4 - m)/2, (6 - m)/2, Cos[e + f
*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e + f*x])/(f*(4 - m)*Sqrt[Sin[e + f*x
]^2]) - (a*(2 - m)*(3*a*b*B*(1 - m) + a^2*A*(2 - m) - 2*A*b^2*m)*(c*Sec[e
+ f*x])^(-4 + m)*Tan[e + f*x])/(f*(3 - m)))/(2 - m))/(1 - m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4514
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(m + n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*
Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1)
)*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x]
, x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2
- b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

Maple [F]

$$\int (a+b\cos (fx+e))^3 (A+B\cos (fx+e)) (c\sec (fx+e))^m dx$$

input
```
int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^3(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="fricas")
```

output
```
integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^
3 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e
))*(c*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (c\sec(e+fx))^m\,(A+B\cos(e+fx))\,(a+b\cos(e+fx))^3\,dx$$

input
```
integrate((a+b*cos(f*x+e))**3*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

output
```
Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**3,
 x)
```

Maxima [F]

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^3(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m,
x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^3(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int \left(\frac{c}{\cos(e+f\,x)}\right)^m\,(A+B\,\cos(e+f\,x))\,\left(a+b\,\cos(e+f\,x)\right)^3\,dx$$

input
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3,x)
```

output
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^m\left(\left(\int \sec(fx+e)^m\,dx\right)a^4+4\left(\int \sec(fx+e)^m\cos(fx+e)\,dx\right)a^3b + \left(\int \sec(fx+e)^m\cos(fx+e)^4\,dx\right)b^4+4\left(\int \sec(fx+e)^m\cos(fx+e)^3\,dx\right)a\,b^3 + 6\left(\int \sec(fx+e)^m\cos(fx+e)^2\,dx\right)a^2b^2\right)$$

input
```
int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
c**m*(int(sec(e + f*x)**m,x)*a**4 + 4*int(sec(e + f*x)**m*cos(e + f*x),x)*
a**3*b + int(sec(e + f*x)**m*cos(e + f*x)**4,x)*b**4 + 4*int(sec(e + f*x)*
*m*cos(e + f*x)**3,x)*a*b**3 + 6*int(sec(e + f*x)**m*cos(e + f*x)**2,x)*a*
*2*b**2)
```

3.638 $\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 33, antiderivative size = 327

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = -\frac{c^4(b^2B(2-m)+2aAb(3-m)+a^2B(3-m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-m}{2},\frac{6-m}{2},\cos^2(e+fx)\right)(c\sec}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}} - \frac{c^3(Ab^2(1-m)+2abB(1-m)+a^2A(2-m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\cos^2(e+fx)\right)(c\sec}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}} - \frac{ac^3(aB(1-m)-Abm)(c\sec(e+fx))^{-3+m}\tan(e+fx)}{f(1-m)(2-m)} - \frac{aAc^3(c\sec(e+fx))^{-3+m}(b+a\sec(e+fx))\tan(e+fx)}{f(1-m)}$$

output
```
-c^4*(b^2*B*(2-m)+2*a*A*b*(3-m)+a^2*B*(3-m))*hypergeom([1/2, 2-1/2*m],[3-1
/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-4+m)*sin(f*x+e)/f/(2-m)/(4-m)/(sin(f*
x+e)^2)^(1/2)-c^3*(A*b^2*(1-m)+2*a*b*B*(1-m)+a^2*A*(2-m))*hypergeom([1/2,
3/2-1/2*m],[5/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-3+m)*sin(f*x+e)/f/(1
-m)/(3-m)/(sin(f*x+e)^2)^(1/2)-a*c^3*(a*B*(1-m)-A*b*m)*(c*sec(f*x+e))^(-3+
m)*tan(f*x+e)/f/(1-m)/(2-m)-a*A*c^3*(c*sec(f*x+e))^(-3+m)*(b+a*sec(f*x+e))
*tan(f*x+e)/f/(1-m)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.63

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$= \frac{\cot(e+fx)\left(\frac{b^2B\cos^3(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sec^2(e+fx)\right)}{-3+m}+\frac{b(Ab+2aB)\cos^2(e+fx)\operatorname{Hypergeometric2F1}}{-2+m}\right.}{}$$

input
```
Integrate[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x
]
```

output
```
(Cot[e + f*x]*((b^2*B*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2, (-
1 + m)/2, Sec[e + f*x]^2])/(-3 + m) + (b*(A*b + 2*a*B)*Cos[e + f*x]^2*Hype
rgeometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) + a*(((2*A*b
 + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e +
 f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e + f
*x]^2])/m))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 3439, 3042, 4514, 25, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$
$$\Big\downarrow \text{3439}$$

$$c^3\int (c\sec(e+fx))^{m-3}(b+a\sec(e+fx))^2(B+A\sec(e+fx))dx$$

$\downarrow$ 3042

$$c^3\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-3}\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(B+A\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 4514

$$c^3\left(-\frac{\int -(c\sec(e+fx))^{m-3}\left(a(aB(1-m)-Abm)\sec^2(e+fx)+\left(A(2-m)a^2+b(Ab+2aB)(1-m)\right)\sec(e\right.}{1-m}\right.$$

$\downarrow$ 25

$$c^3\left(\frac{\int (c\sec(e+fx))^{m-3}\left(a(aB(1-m)-Abm)\sec^2(e+fx)+\left(A(2-m)a^2+b(Ab+2aB)(1-m)\right)\sec(e+fx\right.}{1-m}\right.$$

$\downarrow$ 3042

$$c^3\left(\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-3}\left(a(aB(1-m)-Abm)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(A(2-m)a^2+b(Ab+2aB)(1-m)\right)\right.}{1-m}\right.$$

$\downarrow$ 4535

$$c^3\left(\frac{\frac{\left(a^2A(2-m)+b(1-m)(2aB+Ab)\right)\int (c\sec(e+fx))^{m-2}dx}{c}+\int (c\sec(e+fx))^{m-3}\left(a(aB(1-m)-Abm)\sec^2(e+fx)+b\right.}{1-m}\right.$$

$\downarrow$ 3042

$$c^3\left(\frac{\frac{\left(a^2A(2-m)+b(1-m)(2aB+Ab)\right)\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-2}dx}{c}+\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-3}\left(a(aB(1-m)-Abm)\csc\left(e\right.\right.}{1-m}\right.$$

$\downarrow$ 4259

$$c^3\left(\frac{\frac{\left(a^2A(2-m)+b(1-m)(2aB+Ab)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int \left(\frac{\cos(e+fx)}{c}\right)^{2-m}dx}{c}+\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-3}\left(a(aB(1-\right.}{1-m}\right.$$

$\downarrow$ 3042

$$c^3\left(\frac{\frac{\left(a^2A(2-m)+b(1-m)(2aB+Ab)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{c}\right)^{2-m}dx}{c}+\int\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-3}\left(a(aB\right.}{1-m}\right.$$

$$\Big\downarrow \; 3122$$

$$c^3\left(\frac{\int\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-3}\left(a(aB(1-m)-Abm)\csc\left(e+fx+\frac{\pi}{2}\right)^2+b(bB(1-m)+aA(3-m))\right)dx-\frac{\sin}{}}{1-m}\right.$$

$$\Big\downarrow \; 4534$$

$$c^3\left(\frac{\frac{(1-m)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)\int(c\sec(e+fx))^{m-3}dx}{2-m}-\frac{\sin(e+fx)\left(a^2A(2-m)+b(1-m)(2aB+Ab)\right)(c\sec(e+fx))^{m-3}\,\mathrm{H}}{f(3-m)\sqrt{\sin^2(e+f}}}{1-m}\right.$$

$$\Big\downarrow \; 3042$$

$$c^3\left(\frac{\frac{(1-m)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)\int\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-3}dx}{2-m}-\frac{\sin(e+fx)\left(a^2A(2-m)+b(1-m)(2aB+Ab)\right)(c\sec(e+fx))^{m}}{f(3-m)\sqrt{\sin^2(}}}{1-m}\right.$$

$$\Big\downarrow \; 4259$$

$$c^3\left(\frac{\frac{(1-m)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int\left(\frac{\cos(e+fx)}{c}\right)^{3-m}dx}{2-m}-\frac{\sin(e+fx)\left(a^2A(2-m)+b(1-m)(2a\right.}{}}{1-m}\right.$$

$$\Big\downarrow \; 3042$$

$$c^3\left(\frac{\frac{(1-m)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{c}\right)^{3-m}dx}{2-m}-\frac{\sin(e+fx)\left(a^2A(2-m)+b(1-\right.}{}}{1-}\right.$$

$$\Big\downarrow \; 3122$$

$$c^3\left(-\frac{c(1-m)\sin(e+fx)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)(c\sec(e+fx))^{m-4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-m}{2},\frac{6-m}{2},\cos^2(e+fx)\right)}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+\right.$$

input
```
Int[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
c^3*(-((a*A*(c*Sec[e + f*x])^(-3 + m)*(b + a*Sec[e + f*x])*Tan[e + f*x])/(
f*(1 - m))) + (-((c*(b^2*B*(2 - m) + 2*a*A*b*(3 - m) + a^2*B*(3 - m))*(1 -
 m)*Hypergeometric2F1[1/2, (4 - m)/2, (6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e
+ f*x])^(-4 + m)*Sin[e + f*x])/(f*(2 - m)*(4 - m)*Sqrt[Sin[e + f*x]^2])) -
 ((b*(A*b + 2*a*B)*(1 - m) + a^2*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)
/2, (5 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*
(3 - m)*Sqrt[Sin[e + f*x]^2]) - (a*(a*B*(1 - m) - A*b*m)*(c*Sec[e + f*x])^
(-3 + m)*Tan[e + f*x])/(f*(2 - m)))/(1 - m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4514
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(m + n)   Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*
Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1)
)*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x]
, x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2
- b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [F]

$$\int (a+b\cos(fx+e))^2 (A+B\cos(fx+e)) (c\sec(fx+e))^m \, dx$$

input
```
int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="fricas")
```

output
```
integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2
+ (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (c\sec(e+fx))^m\,(A+B\cos(e+fx))\,(a+b\cos(e+fx))^2\,dx$$

input
```
integrate((a+b*cos(f*x+e))**2*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

output
```
Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**2,
 x)
```

Maxima [F]

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m,
x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$= \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^2(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorith
m="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$= \int \left(\frac{c}{\cos(e+fx)}\right)^m (A+B\cos(e+fx))\,(a+b\cos(e+fx))^2\,dx$$

input
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2,x)
```

output
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$= c^m\left(\left(\int \sec(fx+e)^m\,dx\right)a^3+3\left(\int \sec(fx+e)^m\cos(fx+e)\,dx\right)a^2b + \left(\int \sec(fx+e)^m\cos(fx+e)^3\,dx\right)b^3+3\left(\int \sec(fx+e)^m\cos(fx+e)^2\,dx\right)a\,b^2\right)$$

input
```
int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
c**m*(int(sec(e + f*x)**m,x)*a**3 + 3*int(sec(e + f*x)**m*cos(e + f*x),x)*
a**2*b + int(sec(e + f*x)**m*cos(e + f*x)**3,x)*b**3 + 3*int(sec(e + f*x)*
*m*cos(e + f*x)**2,x)*a*b**2)
```

3.639 $\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 31, antiderivative size = 217

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = -\frac{c^3(bB(1-m)+aA(2-m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\cos^2(e+fx)\right)(c\sec(e+fx))^{-3+m}\sin}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}} - \frac{(Ab+aB)c^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(e+fx)\right)(c\sec(e+fx))^{-2+m}\sin(e+fx)}{f(2-m)\sqrt{\sin^2(e+fx)}} - \frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)}$$

output
```
-c^3*(b*B*(1-m)+a*A*(2-m))*hypergeom([1/2, 3/2-1/2*m],[5/2-1/2*m],cos(f*x+
e)^2)*(c*sec(f*x+e))^(-3+m)*sin(f*x+e)/f/(1-m)/(3-m)/(sin(f*x+e)^2)^(1/2)-
(A*b+B*a)*c^2*hypergeom([1/2, 1-1/2*m],[2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+
e))^(-2+m)*sin(f*x+e)/f/(2-m)/(sin(f*x+e)^2)^(1/2)-a*A*c^2*(c*sec(f*x+e))^
(-2+m)*tan(f*x+e)/f/(1-m)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\frac{\cot(e+fx)\left(bB(-1+m)m\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+m),\frac{m}{2},\sec^2(e+fx)\right)+(-2+\right.}{}$$

input
```
Integrate[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
(Cot[e + f*x]*(b*B*(-1 + m)*m*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 +
m)/2, m/2, Sec[e + f*x]^2] + (-2 + m)*((A*b + a*B)*m*Cos[e + f*x]*Hypergeo
metric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2] + a*A*(-1 + m)*Hyper
geometric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2]))*(c*Sec[e + f*x])^m*Sqr
t[-Tan[e + f*x]^2])/(f*(-2 + m)*(-1 + m)*m)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 3439, 3042, 4485, 25, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3439}$$

$$c^2\int (c\sec(e+fx))^{m-2}(b+a\sec(e+fx))(B+A\sec(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$c^2 \int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-2} \left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right) \left(B+A\csc\left(e+fx+\frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \; 4485$$

$$c^2\left(-\frac{\int -(c\sec(e+fx))^{m-2}(bB(1-m)+(Ab+aB)\sec(e+fx)(1-m)+aA(2-m))dx}{1-m}-\frac{aA\tan(e+fx)(c}{f(1-}\right.$$

$$\Big\downarrow \; 25$$

$$c^2\left(\frac{\int (c\sec(e+fx))^{m-2}(bB(1-m)+(Ab+aB)\sec(e+fx)(1-m)+aA(2-m))dx}{1-m}-\frac{aA\tan(e+fx)(c\sec(}{f(1-m)}\right.$$

$$\Big\downarrow \; 3042$$

$$c^2\left(\frac{\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-2}\left(bB(1-m)+(Ab+aB)\csc\left(e+fx+\frac{\pi}{2}\right)(1-m)+aA(2-m)\right)dx}{1-m}-\frac{aA\tan(e}{}\right.$$

$$\Big\downarrow \; 4274$$

$$c^2\left(\frac{(aA(2-m)+bB(1-m))\int (c\sec(e+fx))^{m-2}dx+\frac{(1-m)(aB+Ab)\int (c\sec(e+fx))^{m-1}dx}{c}}{1-m}-\frac{aA\tan(e+fx)(c\sec}{f(1-m}\right.$$

$$\Big\downarrow \; 3042$$

$$c^2\left(\frac{(aA(2-m)+bB(1-m))\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-2}dx+\frac{(1-m)(aB+Ab)\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m-1}dx}{c}}{1-m}-\frac{aA\tan(e+}{}\right.$$

$$\Big\downarrow \; 4259$$

$$c^2\left(\frac{\frac{(1-m)(aB+Ab)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int \left(\frac{\cos(e+fx)}{c}\right)^{1-m}dx}{c}+(aA(2-m)+bB(1-m))\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+}{1-m}\right.$$

$$\Big\downarrow \; 3042$$

$$c^2\left(\frac{\frac{(1-m)(aB+Ab)\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(e+fx))^m\int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{c}\right)^{1-m}dx}{c}+(aA(2-m)+bB(1-m))\left(\frac{\cos(e+fx)}{c}\right)^m(c\sec(}{1-m}\right.$$

$$\Big\downarrow \text{3122}$$

$$c^2\left(\frac{-\frac{c\sin(e+fx)(aA(2-m)+bB(1-m))(c\sec(e+fx))^{m-3}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\cos^2(e+fx)\right)}{f(3-m)\sqrt{\sin^2(e+fx)}}-\frac{(1-m)(aB+Ab)\sin(e+fx)(c\mathrm{s}}{}}{1-m}\right.$$

input
```
Int[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
c^2*((-((c*(b*B*(1 - m) + a*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (
5 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(3 -
m)*Sqrt[Sin[e + f*x]^2])) - ((A*b + a*B)*(1 - m)*Hypergeometric2F1[1/2, (2
 - m)/2, (4 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-2 + m)*Sin[e + f*x]
)/(f*(2 - m)*Sqrt[Sin[e + f*x]^2]))/(1 - m) - (a*A*(c*Sec[e + f*x])^(-2 +
m)*Tan[e + f*x])/(f*(1 - m)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
 - a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [F]

$$\int (a+b\cos(fx+e))(A+B\cos(fx+e))(c\sec(fx+e))^m\,dx$$

input
```
int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm=
"fricas")
```

output
```
integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*sec(f*x
+ e))^m, x)
```

Sympy [F]

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (c\sec{(e+fx)})^m\,(A+B\cos{(e+fx)})\,(a+b\cos{(e+fx)})\,\,dx$$

input
```
integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

output
```
Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

Maxima [F]

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (B\cos{(fx+e)}+A)(b\cos{(fx+e)}+a)(c\sec{(fx+e)})^m\,\,dx$$

input
```
integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm=
"giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int \left(\frac{c}{\cos(e+f\,x)}\right)^m (A+B\,\cos(e+f\,x))\,(a+b\,\cos(e+f\,x))\,dx$$

input
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)),x)
```

output
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^m\left(\left(\int \sec(fx+e)^m\,dx\right)a^2 + 2\left(\int \sec(fx+e)^m\cos(fx+e)\,dx\right)ab + \left(\int \sec(fx+e)^m\cos(fx+e)^2\,dx\right)b^2\right)$$

input
```
int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
c**m*(int(sec(e + f*x)**m,x)*a**2 + 2*int(sec(e + f*x)**m*cos(e + f*x),x)*
a*b + int(sec(e + f*x)**m*cos(e + f*x)**2,x)*b**2)
```

3.640 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 299

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = -\frac{(Ab-aB)\,\text{AppellF1}\left(\frac{1}{2},\frac{m}{2},1,\frac{3}{2},\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{m/2}(c\sec(e+f}{(a^2-b^2)\,cf}$$
$$+\frac{a(Ab-aB)\,\text{AppellF1}\left(\frac{1}{2},\frac{1+m}{2},1,\frac{3}{2},\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{1+m}{2}}(c\sec(e+fx))^{1+m}\,\text{si}}{b\,(a^2-b^2)\,cf}$$
$$-\frac{Bc\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(c\sec(e+fx))^{-1+m}\sin(e+fx)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

output
```
-(A*b-B*a)*AppellF1(1/2,1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2/(a^2-b^
2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*m)*(c*sec(f*x+e))^(1+m)*sin(f*x+e)/(a^2
-b^2)/c/f+a*(A*b-B*a)*AppellF1(1/2,1/2+1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f
*x+e)^2/(a^2-b^2))*(cos(f*x+e)^2)^(1/2+1/2*m)*(c*sec(f*x+e))^(1+m)*sin(f*x
+e)/b/(a^2-b^2)/c/f-B*c*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(f*x+e)^
2)*(c*sec(f*x+e))^(-1+m)*sin(f*x+e)/b/f/(1-m)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10630 vs. 2(299) = 598.

Time = 36.70 (sec) , antiderivative size = 10630, normalized size of antiderivative = 35.55

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x
]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3439, 3042, 4526, 3042, 4259, 3042, 3122, 4356, 3042, 3302, 3042, 3668, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3439}$$

$$\int \frac{(A\sec(e+fx)+B)(c\sec(e+fx))^m}{a\sec(e+fx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A\csc\left(e+fx+\frac{\pi}{2}\right)+B\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{a\csc\left(e+fx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{ 4526}$$

$$\frac{(Ab-aB)\int \frac{(c\sec(e+fx))^{m+1}}{b+a\sec(e+fx)}dx}{bc}+\frac{B\int (c\sec(e+fx))^m dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\int \frac{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}}{b+a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{bc}+\frac{B\int \left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}$$

$$\Big\downarrow \text{ 4259}$$

$$\frac{(Ab-aB)\int \frac{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}}{b+a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{bc}+\frac{B\left(\frac{\cos(e+fx)}{c}\right)^m (c\sec(e+fx))^m \int \left(\frac{\cos(e+fx)}{c}\right)^{-m}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\int \frac{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}}{b+a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{bc}+\frac{B\left(\frac{\cos(e+fx)}{c}\right)^m (c\sec(e+fx))^m \int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{c}\right)^{-m}dx}{b}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{(Ab-aB)\int \frac{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}}{b+a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{bc}-\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{ 4356}$$

$$\frac{(Ab-aB)\cos^{m+1}(e+fx)(c\sec(e+fx))^{m+1}\int \frac{\cos^{-m}(e+fx)}{a+b\cos(e+fx)}dx}{bc}-\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\cos^{m+1}(e+fx)(c\sec(e+fx))^{m+1}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{-m}}{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{bc}-\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{ 3302}$$

$$\frac{(Ab-aB)\cos^{m+1}(e+fx)(c\sec(e+fx))^{m+1}\left(a\int\frac{\cos^{-m}(e+fx)}{a^2-b^2\cos^2(e+fx)}dx-b\int\frac{\cos^{1-m}(e+fx)}{a^2-b^2\cos^2(e+fx)}dx\right)}{bc}-\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$\downarrow$ 3042

$$\frac{(Ab-aB)\cos^{m+1}(e+fx)(c\sec(e+fx))^{m+1}\left(a\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{-m}}{a^2-b^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx-b\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-m}}{a^2-b^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx\right)}{bc}-\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$\downarrow$ 3668

$$\frac{(Ab-aB)\cos^{m+1}(e+fx)(c\sec(e+fx))^{m+1}\left(\frac{a\cos^{-m-1}(e+fx)\cos^2(e+fx)^{\frac{m+1}{2}}\int\frac{\left(1-\sin^2(e+fx)\right)^{\frac{1}{2}(-m-1)}}{a^2-b^2+b^2\sin^2(e+fx)}d\sin(e+fx)}{f}-\frac{b\,c}{}\right.}{bc}$$
$$\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$\downarrow$ 333

$$\frac{(Ab-aB)\cos^{m+1}(e+fx)(c\sec(e+fx))^{m+1}\left(\frac{a\sin(e+fx)\cos^{-m-1}(e+fx)\cos^2(e+fx)^{\frac{m+1}{2}}\text{AppellF1}\left(\frac{1}{2},\frac{m+1}{2},1,\frac{3}{2},\sin^2(e+fx)\right.}{f(a^2-b^2)}\right.}{bc}$$
$$\frac{Bc\sin(e+fx)(c\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x]
```

output
```
-((B*c*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(c*Sec
[e + f*x])^(-1 + m)*Sin[e + f*x])/(b*f*(1 - m)*Sqrt[Sin[e + f*x]^2])) + ((
A*b - a*B)*Cos[e + f*x]^(1 + m)*(c*Sec[e + f*x])^(1 + m)*(-((b*AppellF1[1/
2, m/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(Cos[
e + f*x]^2)^(m/2)*Sin[e + f*x])/((a^2 - b^2)*f*Cos[e + f*x]^m)) + (a*Appel
lF1[1/2, (1 + m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 -
b^2))]*Cos[e + f*x]^(-1 - m)*(Cos[e + f*x]^2)^((1 + m)/2)*Sin[e + f*x])/((
a^2 - b^2)*f)))/(b*c)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3439
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[g^(m + n)   Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d +
c*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c
- a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4526
```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[A/a   Int[
(d*Csc[e + f*x])^n, x], x] - Simp[(A*b - a*B)/(a*d)   Int[(d*Csc[e + f*x])^
(n + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x]
&& NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{(A + B\cos(fx + e))\,(c\sec(fx + e))^m}{a + b\cos(fx + e)}dx$$

input
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

output
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \int \frac{(B\cos(fx+e)+A)(c\sec(fx+e))^m}{b\cos(fx+e)+a}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \int \frac{(c\sec(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e)),x)
```

output
```
Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \int \frac{(B\cos(fx+e)+A)(c\sec(fx+e))^m}{b\cos(fx+e)+a}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0,0]%%%} / %%%{1,[0,0,1,0]%%%}+%%%{-1,[0,0,0,1]%%%
} Error:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \int \frac{\left(\frac{c}{\cos(e+f\,x)}\right)^m\,(A+B\,\cos\left(e+f\,x\right))}{a+b\,\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)),x)
```

output
```
int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = c^m\left(\int \sec\left(fx+e\right)^m dx\right)$$

input
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

output
```
c**m*int(sec(e + f*x)**m,x)
```

3.641 $\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx=\frac{2bB\cos(e+fx)\sqrt{a+b\cos(e+fx)}(c\sec(e+fx))^m\sin(e+fx)}{f(5-2m)}+\frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left(\frac{(c\cos(e+fx))^{-m}\left(\frac{1}{2}ac\left(2bB(1-m)+2aA\left(\frac{5}{2}-m\right)\right)+\frac{1}{2}c\left(b^2B(3-2m)+a(2Ab+aB)(5-2m)\right)}{\sqrt{a+b\cos(e+fx)}}\right.}{c(5-2m)}$$

output
```
2*b*B*cos(f*x+e)*(a+b*cos(f*x+e))^(1/2)*(c*sec(f*x+e))^m*sin(f*x+e)/f/(5-2
*m)+2*(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Defer(Int)((1/2*a*c*(2*b*B*(1-m)+2
*a*A*(5/2-m))+1/2*c*(b^2*B*(3-2*m)+a*(2*A*b+B*a)*(5-2*m))*cos(f*x+e)+1/2*b
*c*(A*b*(5-2*m)+2*a*B*(3-m))*cos(f*x+e)^2)/((c*cos(f*x+e))^m)/(a+b*cos(f*x
+e))^(1/2),x)/c/(5-2*m)
```

Mathematica [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])
^m,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3440, 3042, 3469, 27, 3042, 3544}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3440}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int (c\cos(e+fx))^{-m}(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int \left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3469

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{2\int\frac{(c\cos(e+fx))^{-m}\left(bc(Ab(5-2m)+2aB(3-m))\cos^2(e+fx)+c\left(B(3-2m)b^2+a(2Ab+aB)(5-2m)\right)\cos(e+fx)+ac(aA(5-2m)+2b\right.}{2\sqrt{a+b\cos(e+fx)}}}{c(5-2m)}\right.$$

$\downarrow$ 27

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{\int\frac{(c\cos(e+fx))^{-m}\left(bc(Ab(5-2m)+2aB(3-m))\cos^2(e+fx)+c\left(B(3-2m)b^2+a(2Ab+aB)(5-2m)\right)\cos(e+fx)+ac(aA(5-2m)+2bB\right.}{\sqrt{a+b\cos(e+fx)}}}{c(5-2m)}\right.$$

$\downarrow$ 3042

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{\int\frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}\left(bc(Ab(5-2m)+2aB(3-m))\sin\left(e+fx+\frac{\pi}{2}\right)^2+c\left(B(3-2m)b^2+a(2Ab+aB)(5-2m)\right)\sin\left(e+fx+\frac{\pi}{2}\right)+ac(aA(5\right.}{\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}}}{c(5-2m)}\right.$$

$\downarrow$ 3544

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{\int\frac{(c\cos(e+fx))^{-m}\left(bc(Ab(5-2m)+2aB(3-m))\cos^2(e+fx)+c\left(B(3-2m)b^2+a(2Ab+aB)(5-2m)\right)\cos(e+fx)+ac(aA(5-2m)+2bB\right.}{\sqrt{a+b\cos(e+fx)}}}{c(5-2m)}\right.$$

input
```
Int[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3469
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
 f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(
m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m
+ n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGt
Q[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3544
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d*
Sin[e + f*x])^n*(A + B*Sin[e + f*x] + C*Sin[e + f*x]^2), x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int (a+b\cos(fx+e))^{\frac{3}{2}} (A+B\cos(fx+e)) (c\sec(fx+e))^m \,dx$$

input
```
int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m \,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e))^m \,dx$$

input
```
integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algo
rithm="fricas")
```

output
```
integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(
f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))
^m, x)
```

Giac [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int (B\cos(fx+e)+A)(b\cos(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))
^m, x)
```

Mupad [N/A]

Not integrable

Time = 45.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx=\int\left(\frac{c}{\cos(e+f\,x)}\right)^m(A+B\,\cos(e+f\,x))\,\left(a+b\,\cos(e+f\,x)\right)^{3/2}\,dx$$

input
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2),x)
```

output
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\int(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx=c^m\left(2\left(\int\sec(fx+e)^m\sqrt{\cos(fx+e)\,b+a}\,\cos(fx+e)\,dx\right)ab+\left(\int\sec(fx+e)^m\sqrt{\cos(fx+e)\,b+a}\,\cos(fx+e)^2\,dx\right)b^2+\left(\int\sec(fx+e)^m\sqrt{\cos(fx+e)\,b+a}dx\right)a^2\right)$$

input
```
int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
c**m*(2*int(sec(e + f*x)**m*sqrt(cos(e + f*x)*b + a)*cos(e + f*x),x)*a*b +
 int(sec(e + f*x)**m*sqrt(cos(e + f*x)*b + a)*cos(e + f*x)**2,x)*b**2 + in
t(sec(e + f*x)**m*sqrt(cos(e + f*x)*b + a),x)*a**2)
```

3.642 $\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = (c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left((c\cos(e+fx))^{-m}\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx)),x\right)$$

output
```
(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Defer(Int)((a+b*cos(f*x+e))^(1/2)*(A+B*c
os(f*x+e))/((c*cos(f*x+e))^m),x)
```

Mathematica [N/A]

Not integrable

Time = 72.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

input
```
Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m
,x]
```

output
```
Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m
, x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3440, 3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3440}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int(c\cos(e+fx))^{-m}\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3486}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int(c\cos(e+fx))^{-m}\sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))dx$$

input
```
Int[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \sqrt{a+b\cos(fx+e)}\,(A+B\cos(fx+e))\,(c\sec(fx+e))^m\,dx$$

input
```
int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (B\cos(fx+e)+A)\sqrt{b\cos(fx+e)+a}(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algo
rithm="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
 x)
```

Sympy [N/A]

Not integrable

Time = 8.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (c\sec(e+fx))^m\,(A+B\cos(e+fx))\,\sqrt{a+b\cos(e+fx)}\,dx$$

input
```
integrate((a+b*cos(f*x+e))**(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

output
```
Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x))
, x)
```

Maxima [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (B\cos(fx+e)+A)\sqrt{b\cos(fx+e)+a}(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m
, x)
```

Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int (B\cos(fx+e)+A)\sqrt{b\cos(fx+e)+a}(c\sec(fx+e))^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m
, x)
```

Mupad [N/A]

Not integrable

Time = 42.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\sqrt{a+b\,\cos\left(e+f\,x\right)}\,dx$$

input
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2),x)
```

output
```
int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \sqrt{a+b\cos(e+fx)}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$
$$=c^m\left(\left(\int \sec\left(fx+e\right)^m\sqrt{\cos\left(fx+e\right)b+a}\,\cos\left(fx+e\right)dx\right)b+\left(\int \sec\left(fx+e\right)^m\sqrt{\cos\left(fx+e\right)b+a}dx\right)a\right)$$

input
```
int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

output
```
c**m*(int(sec(e + f*x)**m*sqrt(cos(e + f*x)*b + a)*cos(e + f*x),x)*b + int
(sec(e + f*x)**m*sqrt(cos(e + f*x)*b + a),x)*a)
```

3.643 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = (c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left(\frac{(c\cos(e+fx))^{-m}(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}},x\right)$$

output
```
(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Defer(Int)((A+B*cos(f*x+e))/((c*cos(f*x+
e))^m)/(a+b*cos(f*x+e))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 53.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$$

input
```
Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x
]],x]
```

output
```
Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x
]], x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3440, 3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3440}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int \frac{(c\cos(e+fx))^{-m}(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3486}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int \frac{(c\cos(e+fx))^{-m}(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}dx$$

input
```
Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\cos(fx+e))\,(c\sec(fx+e))^{m}}{\sqrt{a+b\cos(fx+e)}}dx$$

input
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

output
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(B\cos{(fx+e)}+A)(c\sec{(fx+e)})^m}{\sqrt{b\cos{(fx+e)}+a}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

Sympy [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(c\sec{(e+fx)})^m\,(A+B\cos{(e+fx)})}{\sqrt{a+b\cos{(e+fx)}}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(1/2),x)
```

output
```
Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x))
, x)
```

Maxima [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(B\cos(fx+e)+A)(c\sec(fx+e))^m}{\sqrt{b\cos(fx+e)+a}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a)
, x)
```

Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = \int \frac{(B\cos(fx+e)+A)(c\sec(fx+e))^m}{\sqrt{b\cos(fx+e)+a}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a)
, x)
```

Mupad [N/A]

Not integrable

Time = 44.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx=\int \frac{\left(\frac{c}{\cos(e+f\,x)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{\sqrt{a+b\,\cos\left(e+f\,x\right)}}\,dx$$

input
```
int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),x
)
```

output
```
int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),
x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = c^m\left(\int \sec\left(fx+e\right)^m\sqrt{\cos\left(fx+e\right)b+a}dx\right)$$

input
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

output
```
c**m*int(sec(e + f*x)**m*sqrt(cos(e + f*x)*b + a),x)
```

3.644 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)(c\sec(e+fx))^m\sin(e+fx)}{a(a^2-b^2)f\sqrt{a+b\cos(e+fx)}} + \frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left(\frac{(c\cos(e+fx))^{-m}\left(\frac{1}{2}c(a^2A+Ab^2(1-2m)-2abB(1-m))-\frac{1}{2}a(Ab-aB)c\cos(e+fx)-\frac{1}{2}b(Ab}{\sqrt{a+b\cos(e+fx)}}}{a(a^2-b^2)c}$$

output
```
2*b*(A*b-B*a)*cos(f*x+e)*(c*sec(f*x+e))^m*sin(f*x+e)/a/(a^2-b^2)/f/(a+b*co
s(f*x+e))^(1/2)+2*(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Defer(Int)((1/2*c*(A*a
^2+A*b^2*(1-2*m)-2*a*b*B*(1-m))-1/2*a*(A*b-B*a)*c*cos(f*x+e)-1/2*b*(A*b-B*
a)*c*(3-2*m)*cos(f*x+e)^2)/((c*cos(f*x+e))^m)/(a+b*cos(f*x+e))^(1/2),x)/a/
(a^2-b^2)/c
```

Mathematica [N/A]

Not integrable

Time = 39.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx = \int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$$

input
```
Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(
3/2),x]
```

output
```
Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(
3/2), x]
```

Rubi [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3440, 3042, 3479, 27, 3042, 3544}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{3440}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int \frac{(c\cos(e+fx))^{-m}(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}\left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{3479}$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{2\int \frac{(c\cos(e+fx))^{-m}\left(-b(Ab-aB)c(3-2m)\cos^2(e+fx)-a(Ab-aB)c\cos(e+fx)+c\left(Aa^2-2bB(1-m)a+Ab^2(1-2m)\right)\right)}{2\sqrt{a+b\cos(e+fx)}}dx}{ac\left(a^2-b^2\right)}+\frac{2b(A\right.$$

$$\Big\downarrow \ 27$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{\int\frac{(c\cos(e+fx))^{-m}\left(-b(Ab-aB)c(3-2m)\cos^2(e+fx)-a(Ab-aB)c\cos(e+fx)+c\left(Aa^2-2bB(1-m)a+Ab^2(1-2m)\right)\right)}{\sqrt{a+b\cos(e+fx)}}dx}{ac\left(a^2-b^2\right)}+\frac{2b(Ab$$

$$\Big\downarrow \ 3042$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{\int\frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}\left(-b(Ab-aB)c(3-2m)\sin\left(e+fx+\frac{\pi}{2}\right)^2-a(Ab-aB)c\sin\left(e+fx+\frac{\pi}{2}\right)+c\left(Aa^2-2bB(1-m)a+Ab^2(1-2m)\right)\right)}{\sqrt{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{ac\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3544$$

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\left(\frac{\int\frac{(c\cos(e+fx))^{-m}\left(-b(Ab-aB)c(3-2m)\cos^2(e+fx)-a(Ab-aB)c\cos(e+fx)+c\left(Aa^2-2bB(1-m)a+Ab^2(1-2m)\right)\right)}{\sqrt{a+b\cos(e+fx)}}dx}{ac\left(a^2-b^2\right)}+\frac{2b(Ab$$

input
```
Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3/2),x
]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3440
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p   Int[(a + b*Sin[e + f*x])^m*((c +
d*Sin[e + f*x])^n/(g*Sin[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && I
ntegerQ[n])
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3544
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d*
Sin[e + f*x])^n*(A + B*Sin[e + f*x] + C*Sin[e + f*x]^2), x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\cos(fx+e))\,(c\sec(fx+e))^{m}}{(a+b\cos(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

output
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx = \int \frac{(B\cos(fx+e)+A)(c\sec(fx+e))^m}{(b\cos(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m/
(b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx = \int \frac{(c\sec(e+fx))^m\,(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(3/2),x)
```

output
```
Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x))**(3
/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{(B\cos(fx+e)+A)(c\sec(fx+e))^m}{(b\cos(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/
2), x)
```

Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{(B\cos(fx+e)+A)(c\sec(fx+e))^m}{(b\cos(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/
2), x)
```

Mupad [N/A]

Not integrable

Time = 48.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx=\int \frac{\left(\frac{c}{\cos(e+f\,x)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{\left(a+b\,\cos\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),x
)
```

output
```
int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),
x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx=c^m\left(\int \frac{\sec\left(fx+e\right)^m\sqrt{\cos\left(fx+e\right)b+a}}{\cos\left(fx+e\right)b+a}dx\right)$$

input
```
int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

output
```
c**m*int((sec(e + f*x)**m*sqrt(cos(e + f*x)*b + a))/(cos(e + f*x)*b + a),x
)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file